Karyn Le Hur

Centre de Physique Theorique, Ecole Polytechnique and CNRS

4 classes Saclay Lectures Series: 1h30 each

Thanks to Sylvain Ravets, Igor Ferrier-Barbut, Benoit Valiron for invitation
Thanks for the questions …

Institut d'Optique Graduate School

Geometry and Topology in the Quantum!

- Class I: Quantum Geometry, Information and Topological Physics from Bloch Sphere (June 9)
- Class II: Application in Topological Lattice Models and Quantum Matter (June 16)
- Class III: Applications in Transport and Light-Matter Interaction (June 23) √
- Class IV: Entangled WaveFunction and Fractional Topology (June 30)

2023

thanks to the team!

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Applications in Transport & Light-Matter Interaction

- Geometrical Responses in the Plane and quantum metric
- Derivation of quantum Hall conductance from Kubo formula
- Relations to circularly polarized light and geometry on the sphere

Application to Haldane model, topological insulators, quantum Hall effect

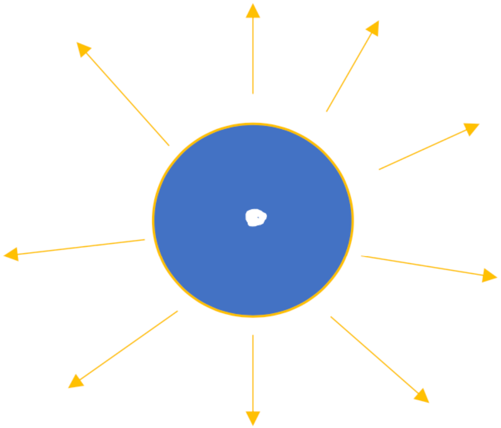
Similar applications with cavity or circuit quantum electrodynamics

Addition: Skin effect as a classical analogue of Dirac monopole

Relation Geometry on a sphere and in plane

Recap of Classes I and II

$$\boxed{H = -\vec{\mathbf{d}} \cdot \vec{\boldsymbol{\sigma}}}$$



$$\vec{\mathbf{d}}(\varphi, \theta) = d(\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta) = (d_x, d_y, d_z)$$

$$|\psi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2} e^{i\frac{\varphi}{2}} \end{pmatrix}, \quad |\psi_-\rangle = \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2} e^{i\frac{\varphi}{2}} \end{pmatrix}$$

$$E_+ = -|\vec{d}|$$

$$E_- = +|\vec{d}|$$

$$A_\varphi = A_\varphi^c = -i\langle\psi|\partial_\varphi|\psi\rangle \qquad F_{\theta\varphi} = \partial_\theta A_\varphi$$

$$|\psi\rangle = |\psi_+\rangle$$

$$A_\varphi^c = -\frac{\cos\theta}{2}$$

$$C = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sin\theta}{2}\, d\theta$$

$$= 1!$$

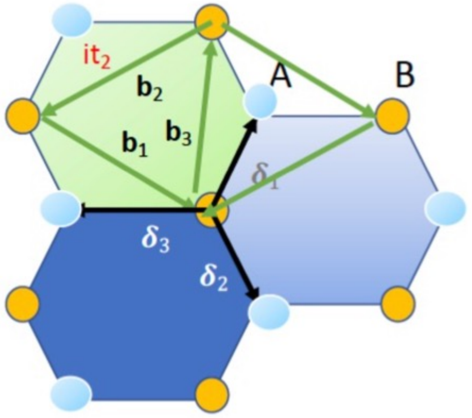
$$F_{\theta\varphi} = (\sin\theta)\frac{1}{2}$$

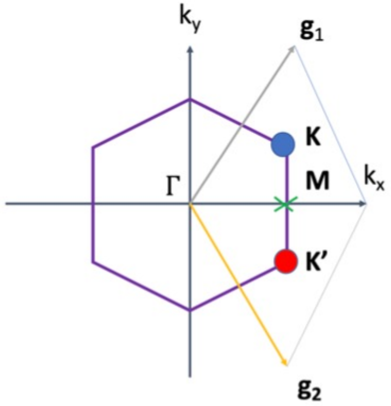
$$\boxed{C = A_\varphi(\pi) - A_\varphi(0)}$$

Honeycomb lattice and Dirac approximation

Wallace, 1947

Useful Review: A. Castro-Neto et al.
Rev. Mod. Phys. **81**, 109 (2009)





$$H = \sum_{\mathbf{p}} \Psi^\dagger(\mathbf{p}) H(\mathbf{p}) \Psi(\mathbf{p}) \text{ with } \Psi(\mathbf{p}) = (c_{A\mathbf{p}}, c_{B\mathbf{p}})$$

$$H(\mathbf{p}) = \hbar v_F (p_x \sigma_x + \zeta p_y \sigma_y),$$

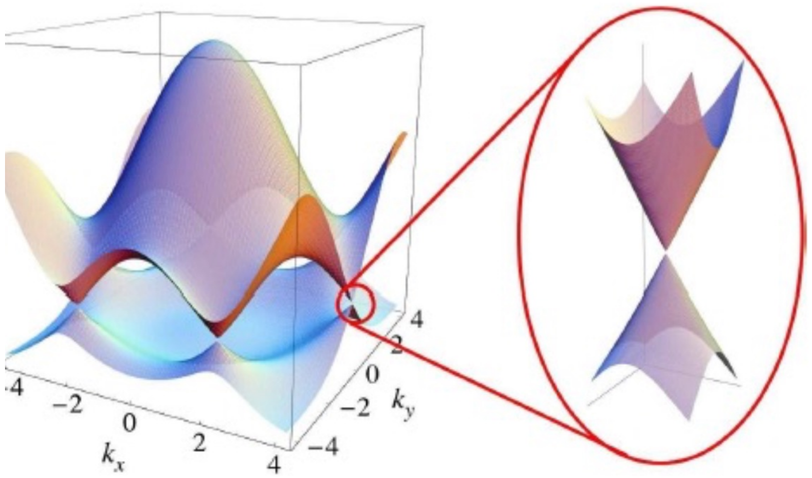
$$\partial_{p_x} H = \frac{\partial H}{\partial p_x} = \hbar v_F \sigma_x \text{ and } \partial_{\zeta p_y} H = \frac{\partial H}{\partial(\zeta p_y)} = \hbar v_F \sigma_y,$$

$$K = \left(\frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a}\right) \qquad K' = \left(\frac{2\pi}{3a}, -\frac{2\pi}{3\sqrt{3}a}\right)$$

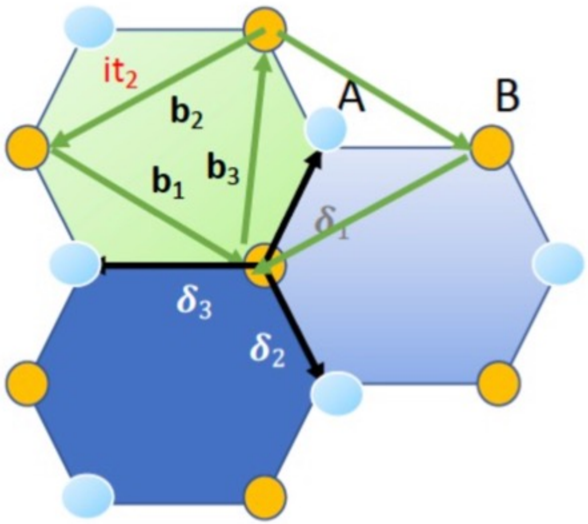
$$i\hbar\gamma^\mu \partial_\mu \psi - mc\psi = 0,$$

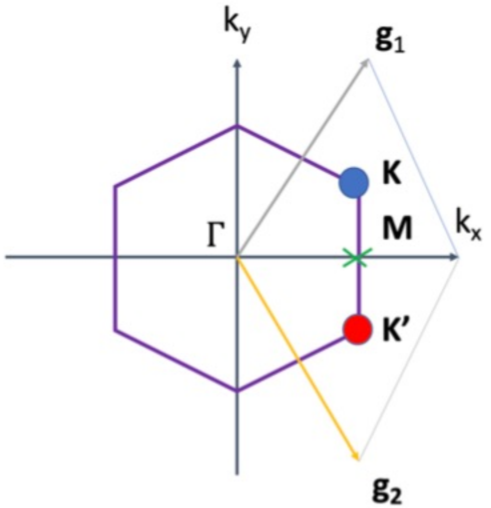
$$E(\mathbf{p}) = \pm \hbar v_F |\mathbf{p}|$$

$$p_x = |\mathbf{p}| \cos\tilde{\varphi} \text{ and } p_y = |\mathbf{p}| \sin\tilde{\varphi}$$



Haldane Model 1988





$t_2 e^{i\Phi}$ $\Phi = \frac{\pi}{2}$

$$H_{t_2}(\mathbf{k}) = H^A_{t_2} + H^B_{t_2} = -\sum_{\mathbf{k}}\sum_{\mathbf{b}_j} 2t_2 \sin(\mathbf{k}\cdot\mathbf{b}_j)\sigma_z$$

$$\mathbf{d}(\mathbf{k}) = \left(t\sum_{\boldsymbol{\delta}_i}\cos(\mathbf{k}\cdot\boldsymbol{\delta}_i), t\sum_{\boldsymbol{\delta}_i}\sin(\mathbf{k}\cdot\boldsymbol{\delta}_i), 2t_2\sum_{\mathbf{b}_j}\sin(\mathbf{k}\cdot\mathbf{b}_j)\right)$$

$$\boxed{d_z(\mathbf{K}) = 2t_2\sum_{\mathbf{b}_j}\sin(\mathbf{K}\cdot\mathbf{b_j}) = 3\sqrt{3}t_2 = m.}$$

$$\boxed{\tan\theta = \frac{\hbar v_F|\mathbf{p}|}{m}.}$$

$$-d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta) =$$
$$(\hbar v_F|\mathbf{p}|\cos\tilde{\varphi}, \hbar v_F|\mathbf{p}|\sin(\zeta\tilde{\varphi}), -\zeta m).$$

$\xi = +1$ K

$\xi = -1$ K'

Geometrical properties in the plane

K. Le Hur, Phys. Rev. B 105, 125106 (2022); R. Shankar and H. Mathur, Phys. Rev. Lett. 73, 1565 (1994)

Search for half Skyrmions in Yang-Mills equation
See later for definition of Skyrmion

$$\partial_{p_x} H = \frac{\partial H}{\partial p_x} = \hbar v_F \sigma_x \text{ and } \partial_{\zeta p_y} H = \frac{\partial H}{\partial(\zeta p_y)} = \hbar v_F \sigma_y,$$

class I

$$F_{p_x p_y}(\theta) = i\frac{(\langle\psi_-|\partial_{p_x} H|\psi_+\rangle\langle\psi_+|\partial_{p_y} H|\psi_-\rangle - (p_x \leftrightarrow p_y))}{(E_- - E_+)^2}$$

$\sigma \to 0$

$$\langle\psi_-|\sigma_x|\psi_+\rangle = \sin^2\frac{\theta}{2} e^{i\varphi} - \cos^2\frac{\theta}{2} e^{-i\varphi}$$

$$\langle\psi_+|\sigma_y|\psi_-\rangle = i\cos^2\frac{\theta}{2} e^{i\varphi} + i\sin^2\frac{\theta}{2} e^{-i\varphi}$$

$$\langle\psi_-|\sigma_x|\psi_+\rangle\langle\psi_+|\sigma_y|\psi_-\rangle$$
$$= i\left(\sin^4\frac{\theta}{2} - \cos^4\frac{\theta}{2}\right) + i\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} e^{2i\varphi} - i\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} e^{-2i\varphi}$$

$$\boxed{F_{p_y p_x}(\theta) = -F_{p_x p_y}(\theta) = \frac{(\hbar v_F)^2}{2d^2}\cos\theta.}$$

$\Rightarrow -2A_\varphi(\theta)$

$d = m$

$\theta \to \pi$

$$F_{-p_y p_x}(\theta+\pi) = \frac{(\hbar v_F)^2}{2d^2}\cos(\theta+\pi) = -F_{p_y p_x}(\theta+\pi).$$

$$\boxed{\left(F_{p_y p_x}(0) \pm F_{\pm p_y p_x}(\pi)\right) = C\frac{(\hbar v_F)^2}{m^2}}$$

Verification of formula with approximate form of eigenstates

$$|\psi_+\rangle = \begin{pmatrix} 1 \\ -\frac{1}{2}\frac{\hbar v_F|\mathbf{p}|}{m}e^{i\tilde{\varphi}} \end{pmatrix}$$

$$\tan\theta = \frac{\hbar v_F|\mathbf{p}|}{m} \approx \sin\theta \simeq \theta$$

$$\cos\frac{\theta}{2} \mapsto 1$$

S. Matsuura & S. Ryu, Phys. Rev. B 82, 245113 2010

$$i\partial_{p_y}\langle\psi_+|\partial_{p_x}|\psi_+\rangle = \frac{\hbar^2 v_F^2}{4m^2}$$

$$i\partial_{p_x}\langle\psi_+|\partial_{p_y}|\psi_+\rangle = -\frac{\hbar^2 v_F^2}{4m^2}$$

$$F_{p_y p_x} = \frac{\hbar^2 v_F^2}{2m^2}$$

New geometrical function

Karyn Le Hur, Review ArXiv:2209.15381 Appendix A

$$F_{p_x p_x} = F_{p_y p_y} = 0$$

$$f_{\mu\mu} + f_{\nu\nu} = \sum_{n \neq \psi} \frac{\mathcal{I}_{\mu\mu} + \mathcal{I}_{\nu\nu}}{(E_n - E_\psi)^2}$$

$$\mathcal{I}_{\mu\mu} = \left\langle \psi \left| \frac{\partial H}{\partial R_\mu} \right| n \right\rangle \left\langle n \left| \frac{\partial H}{\partial R_\mu} \right| \psi \right\rangle$$

$$\alpha(\theta) = \left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right)$$

$$\alpha(\theta) = C^2 + 2A'_\varphi(\theta < \theta_c)A'_\varphi(\theta > \theta_c).$$

class I

$$I(\theta) = \mathcal{I}_{p_x p_x} + \mathcal{I}_{p_y p_y} = 2(\hbar v_F)^2\left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right)$$

$$f_{p_x p_x}(0) + f_{p_y p_y}(0) = \frac{2(\hbar v_F)^2 C^2}{2m^2}$$

related to response to circularly polarized light, see later

K. Le Hur, Phys. Rev. B 105, 125106 (2022)

This function is also related to corrections in energy

Quantum metric in the plane

$$g_{ij}dk_i dk_j = 1 - |\langle\psi_+(\mathbf{k} - d\mathbf{k})|\psi_+(\mathbf{k} + d\mathbf{k})\rangle|^2$$

$$\boxed{\begin{aligned} g_{\mu\mu} &= 2\mathrm{Re}(\langle\partial_{k_\mu}\psi_+|\partial_{k_\mu}\psi_+\rangle) \\ &= 2\mathrm{Re}(f_{\mu\mu}) = \frac{1}{2}\frac{\hbar^2 v_F^2}{m^2}C^2. \end{aligned}}$$

Simple calculations on the sphere allow us to reveal geometrical informations on the lattice

$$\alpha(\theta) = C^2 + 2A'_\varphi(\theta < \theta_c)A'_\varphi(\theta > \theta_c).$$

Matsuura and Ryu, 2010 obtain at the pole $\frac{1}{2}\frac{\hbar^2 v_F^2}{m^2}$

Possible applications to Einstein-Field Equation: T. B. Smith, L. Pullasseri, A. Srivastava, Physical Rev. Research 4, 13217 (2022)

1982

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs

Class I
F. Parmentier
Hofstadter model

$$\hat{H}(k_1,k_2) = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x} + \hbar k_1\right)^2 + \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial y} + \hbar k_2 - eBx\right)^2 + U(x,y).$$

$$\psi_{k_1k_2}(x+qa,y)\exp(-2\pi i p y/b - ik_1qa) = \psi_{k_1k_2}(x,y+b)\exp(-ik_2b) = \psi_{k_1k_2}(x,y)$$

$$\boxed{\sigma_H = \frac{ie^2}{A_0\hbar}\sum_{\epsilon_\alpha<E_F}\sum_{\epsilon_\beta>E_F}\frac{(\partial\hat{H}/\partial k_1)_{\alpha\beta}(\partial\hat{H}/\partial k_2)_{\beta\alpha} - (\partial\hat{H}/\partial k_2)_{\alpha\beta}(\partial\hat{H}/\partial k_1)_{\beta\alpha}}{(\epsilon_\alpha-\epsilon_\beta)^2}}$$

$F_{\alpha\beta}$
class I

$$u_{k_1k_2} = \psi_{k_1k_2}\exp(-ik_1x - ik_2y)$$

$$\sigma_H = \frac{ie^2}{2\pi h}\sum\int d^2k\int d^2r\left(\frac{\partial u^*}{\partial k_1}\frac{\partial u}{\partial k_2} - \frac{\partial u^*}{\partial k_2}\frac{\partial u}{\partial k_1}\right)$$

$$= \frac{ie^2}{4\pi h}\sum\oint dk_j\int d^2r\left(u^*\frac{\partial u}{\partial k_j} - \frac{\partial u^*}{\partial k_j}u\right),$$

Quantum Hall Conductivity

How do we show the relation between Kubo formula of transport and sphere?

Relation to quantum Hall effect

Useful material
S. Q. Shen, Topological Insulators: Dirac equation
In condensed matters, book Appendix A

Class I: General understanding of quantum Hall conductivity

Karplus-Luttinger velocity 1954

$$\boxed{\mathbf{v} = \frac{e}{\hbar}\mathbf{E}\times\mathbf{F}.}$$

$$\mathbf{j} = \iint \frac{dk_x dk_y}{(2\pi)^2}\mathbf{j}(\mathbf{k}).$$

$$|\mathbf{j}| = \frac{e^2}{h}\iint |(d\mathbf{k}\times\mathbf{F})\cdot\mathbf{E}| = \frac{e^2}{h}C|\mathbf{E}|,$$

$$\boxed{\sigma_{xy} = \frac{e^2}{h}C.}$$

1957

We begin with Kubo formula for electrical conductivity
See book of G. D. Mahan Many Particles Physics chapter 3.8

$$E_\alpha(\vec{r},t) = [-]_\alpha e^{i\vec{q}\cdot\vec{r} - i\omega t}$$

$$J_\alpha(\vec{r},t) = \sum_\beta \sigma_{\alpha\beta}(\vec{q},\omega) E_\beta(\vec{r},t)$$

$$H_{tot} = H - \frac{1}{c}\int d\vec{r}\, j_\alpha(\vec{r}) A_\alpha$$

$$\frac{1}{c}A_\alpha(\vec{r},t) = \frac{-1}{\omega} E_\alpha(\vec{r},t)$$

Interaction Representation $H_{tot} = H + H'$ and $|\psi\rangle$ is the wavefunction when $H' = 0$

$$J_\alpha(\mathbf{r},t) = \sigma_{\alpha\beta} E_\beta$$

$$J_\alpha^{(2)}(\mathbf{r},t) = \langle\psi|S^\dagger(t,-\infty)J_\alpha(\mathbf{r},t)S(t,-\infty)|\psi\rangle$$

$$S(t,-\infty)|\psi\rangle = Te^{-\frac{i}{\hbar}\int_{-\infty}^{t} dt' H'(t')}|\psi\rangle$$

$$J_\alpha^{(2)}(\mathbf{r},t) = -\frac{i}{\hbar}\int_{-\infty}^{t} dt' \langle\psi|[J_\alpha(\mathbf{r},t), H'(t')]|\psi\rangle$$

$$[J_\alpha(\mathbf{r},t), H'(t')] = \frac{i}{\omega} E_\beta(\mathbf{r},t) e^{-i\mathbf{q}.\mathbf{r}} e^{i\omega(t-t')}[J_\alpha(\mathbf{r},t), J_\beta(\mathbf{q},t')]$$

$$J_\alpha^{(2)} = \frac{1}{\hbar}\frac{E_\beta(\mathbf{r},t)}{\omega} e^{-i\mathbf{q}\cdot\mathbf{r}}\int_{-\infty}^{t} dt' e^{i\omega(t-t')}\langle\psi|[J_\alpha(\mathbf{r},t), J_\beta(\mathbf{q},t')]|\psi\rangle$$

$$\sigma_{xy} = \mathrm{lim}_{\omega\to 0}\mathrm{lim}_{\mathbf{q}\to\mathbf{0}}\sigma_{xy}(\mathbf{q},\omega)$$

$$\sigma_{\alpha\beta}(\mathbf{q},\omega) = \frac{i}{\omega}\pi_{\alpha\beta}(\mathbf{q},\omega)$$

$$\boxed{\pi_{\alpha\beta}(\mathbf{q},\omega) = -\frac{i}{\hbar V}\int_{-\infty}^{+\infty} dt e^{i\omega(t-t')}\theta(t-t')\langle\psi|[J_\alpha(\mathbf{q},t), J_\beta(\mathbf{q},t')]|\psi\rangle}$$

2 terms

Fourier transform is defined as $\int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} J_\alpha(\mathbf{r},t) = J_\alpha(\mathbf{q},t)$.
$|\psi\rangle$ is an eigenstate associated to lowest band denoted $+$

(1)

$$\pi_{\alpha\beta}(\mathbf{q},\omega) = -\frac{i}{\hbar V}\int_{-\infty}^{+\infty} dt' e^{i\omega(t-t')}\theta(t-t')\sum_{\mathbf{k},n'}\langle\mathbf{k},+|J_\alpha(\mathbf{q},t)|\mathbf{k},n'\rangle\langle\mathbf{k},n'|J_\beta(\mathbf{q},t')|\mathbf{k},+\rangle.$$

We insert the normalization condition for eigenstates and for the band $+$ anticipate that the result will be zero if $\mathbf{k}' \neq \mathbf{k}$. In two dimensions, the volume is an area.

$q \to 0$

$$\boxed{J_\alpha(t) = e^{\frac{i}{\hbar}tH} J_\alpha e^{-\frac{i}{\hbar}tH}}$$

$$(1) = -\frac{i}{\hbar V}\int_{-\infty}^{+\infty} dt' e^{i\omega(t-t')}\theta(t-t')\sum_{\mathbf{k},n'}\langle \mathbf{k},+|J_\alpha(t)|\mathbf{k},n'\rangle\langle \mathbf{k},n'|J_\beta(t')|\mathbf{k},+\rangle.$$

$$\boxed{J_\alpha(t) = e^{\frac{i}{\hbar}tH}J_\alpha e^{-\frac{i}{\hbar}tH}}$$

$$\langle \mathbf{k},+|e^{-\frac{i}{\hbar}tH}J_\alpha e^{\frac{i}{\hbar}tH}|\mathbf{k},n'\rangle = e^{-\frac{i}{\hbar}(E_k^n - E_k^+)t}\langle \mathbf{k},+|J_\alpha|\mathbf{k},n'\rangle.$$

$$\langle \mathbf{k},n'|e^{\frac{i}{\hbar}t'H}J_\beta e^{-\frac{i}{\hbar}t'H}|\mathbf{k},+\rangle = e^{\frac{i}{\hbar}(E_k^n - E_k^+)t'}\langle \mathbf{k},n'|J_\beta|\mathbf{k},+\rangle.$$

$$(1) = -\frac{i}{\hbar V}\sum_{\mathbf{k},\mathbf{n}'}\int_{-\infty}^{t} dt' e^{i\left(\omega - \frac{E_k^n}{\hbar} + \frac{E_k^+}{\hbar}\right)(t-t')}\langle \mathbf{k},+|J_\alpha|\mathbf{k},n'\rangle\langle \mathbf{k},n'|J_\beta|\mathbf{k},+\rangle.$$

$$\boxed{(1) = -\frac{1}{\hbar V}\sum_{\mathbf{k},\mathbf{n}'}\frac{\langle \mathbf{k},+|J_\alpha|\mathbf{k},n'\rangle\langle \mathbf{k},n'|J_\beta|\mathbf{k},+\rangle}{\omega - \frac{E_k^n}{\hbar} + \frac{E_k^+}{\hbar} + i\epsilon}.}$$

$$\lim_{\omega\to 0}\mathrm{Im}\frac{i(1)}{\omega} = \mathrm{Im}\left(\frac{d}{d\omega}i(1)\right)_{\omega=0}$$

For $\omega = 0$, $(1) = 0$ in the sense that $\frac{\partial \mathbf{A}}{\partial t} = 0$.

$$\boxed{\mathrm{Im}i\left(\frac{d}{d\omega}(1)\right)_{\omega=0} = \mathrm{Im}\frac{i\hbar}{V}\sum_{\mathbf{k},\mathbf{n}'}\frac{\langle \mathbf{k},+|J_\alpha|\mathbf{k},n'\rangle\langle \mathbf{k},n'|J_\beta|\mathbf{k},+\rangle}{(-E_k^n + E_k^+ + i\epsilon)(-E_k^n + E_k^+ + i\epsilon)}.}$$

$$J_\alpha = \frac{e}{\hbar}\frac{\partial H}{\partial k_\alpha}.$$

This relation is certainly valid in a general sense with $E = \sum_\alpha \frac{\hbar^2 k_\alpha^2}{2m}$. For the Dirac Hamiltonian, we also have $E(|\mathbf{k}|) \sim \hbar|\mathbf{k}|v_F$. In this way, this step can be applied for square lattice or honeycomb lattice.

Therefore, for the two-bands model we verify

$$\boxed{\sigma_{xy} = \frac{e^2}{\hbar V}\sum_{\mathbf{k}} F_{k_x k_y} = \frac{e^2}{\hbar}\iint \frac{dk_x dk_y}{(2\pi)^2} F_{k_x k_y} = \frac{e^2}{h}\frac{1}{2\pi}\iint dk_x dk_y F_{k_x k_y} = \frac{e^2}{h}C}$$

Here, we verify the relation with the sphere geometry for the honeycomb lattice within Dirac approximation

$$H(\mathbf{k}) = -\mathbf{d}\cdot\boldsymbol{\sigma}$$

$$J_\alpha = \frac{e}{\hbar}\partial_{k_\alpha} H(\mathbf{k}) = -\frac{e}{\hbar}\sum_\alpha \frac{\partial d_\alpha}{\partial k_\alpha}\sigma_\alpha$$

$$\sigma_{xy} = \frac{e^2}{\hbar V} \sum_{\mathbf{k}} \frac{\partial_{k_x} d_x \partial_{k_y} d_y}{4 d(\mathbf{k})^2} \mathrm{Im}(\langle \psi_- | \sigma_x | \psi_+ \rangle \langle \psi_+ | \sigma_y | \psi_- \rangle)$$

$$\boxed{\mathrm{Im}(\langle \mathbf{k}, n | \sigma_x | \mathbf{k}, m \rangle \langle \mathbf{k}, m | \sigma_y | \mathbf{k}, n \rangle) = 2 \cos\theta = 2 \frac{d_z(\mathbf{k})}{d(\mathbf{k})}}$$

$$\sigma_{xy} = \frac{e^2}{a^2 \, 2 N \hbar} \sum_{\mathbf{k}} \frac{(\partial_{k_x} d_x)(\partial_{k_y} d_y) d_z}{d(\mathbf{k})^3}$$

$$\vec{n} = \frac{\vec{d}}{|\vec{d}|}$$

Equivalently, we have

$$\sigma_{xy} = \frac{e^2}{h} \iint \frac{d^2\mathbf{k}}{4\pi} \frac{(\partial_{k_x} d_x)(\partial_{k_y} d_y) d_z}{d(\mathbf{k})^3}.$$

Winding Number

$$\boxed{W = \iint \frac{d^2\mathbf{k}}{4\pi} (\partial_{k_x} \mathbf{n}) \times (\partial_{k_y} \mathbf{n}) \cdot \mathbf{n} = 1}$$

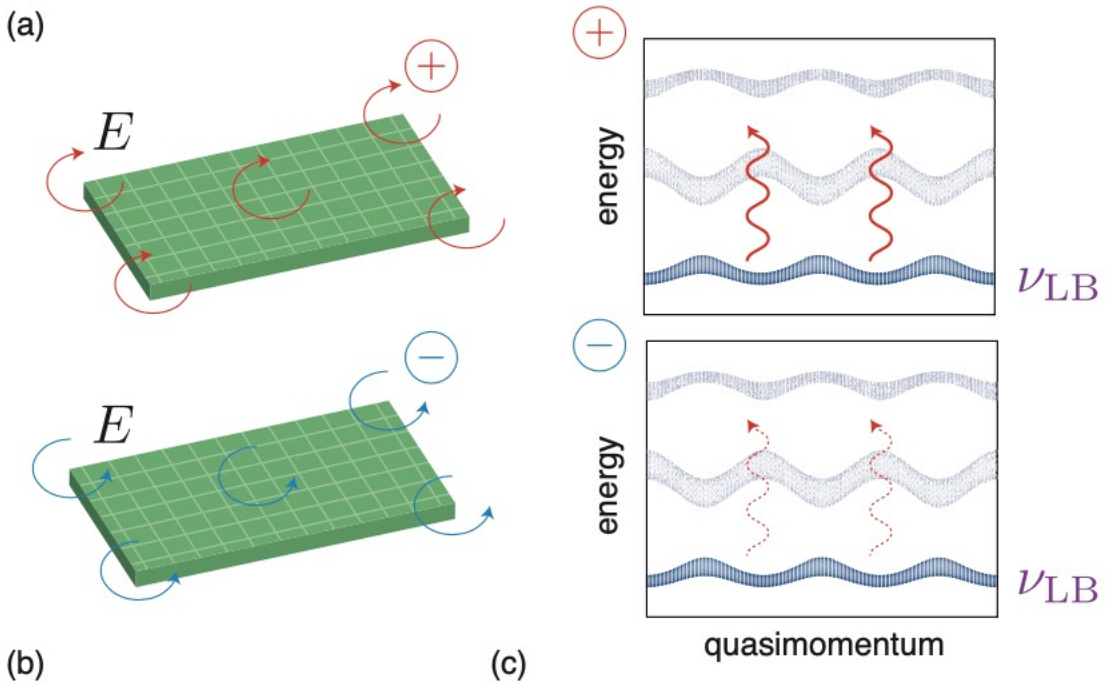
Skyrmion

Probing topology by "heating": Quantized circular dichroism in ultracold atoms

2017

D. T. Tran,[1] A. Dauphin,[2] A. G. Grushin,[3,4] P. Zoller,[5,6,7] and N. Goldman*[1]

L. Asteria, D.-T. Tran, T. Ozawa, M. Tarnowski, B.-S. Rem, N. Flaschner, K. Sengstock, N. Goldman and C. Weitenberg, Nature Physics 15, pages 449-454 (2019).



$$\boxed{\hat{H}_{\pm}(t)=\hat{H}_0+2E\,\left[\cos(\omega t)\hat{x}\pm\sin(\omega t)\hat{y}\right],}$$

$$\hat{R}_{\pm}=\exp\left\{i\frac{2E}{\hbar\omega}\left[\sin(\omega t)\hat{x}\mp\cos(\omega t)\hat{y}\right]\right\}$$

$$\Gamma_{\pm}(\omega)=\sum_{\boldsymbol{k}}\Gamma_{\pm}(\boldsymbol{k};\omega),$$

$$\Gamma_{\pm}(\boldsymbol{k};\omega)=\frac{2\pi}{\hbar}\sum_{n>0}|\mathcal{V}^{\pm}_{n0}(\boldsymbol{k})|^2\,\delta^{(t)}(\varepsilon_n(\boldsymbol{k})-\varepsilon_0(\boldsymbol{k})-\hbar\omega),$$

$$|\mathcal{V}^{\pm}_{n0}(\boldsymbol{k})|^2=(E/\hbar\omega)^2\left|\left\langle n(\boldsymbol{k})\left|\frac{1}{i}\frac{\partial\hat{H}_0}{\partial k_x}\mp\frac{\partial\hat{H}_0}{\partial k_y}\right|0(\boldsymbol{k})\right\rangle\right|^2.$$

$$\boxed{\Delta\Gamma^{\rm int}/A_{\rm syst}=\eta_0E^2,\qquad \eta_0=(1/\hbar^2)\,\nu,}$$

$$\boxed{\Delta\Gamma^{\rm int}=4\pi(E/\hbar)^2\,{\rm Im}\sum_{n>0}\sum_{\boldsymbol{k}}\frac{\langle 0|\partial_{k_x}\hat{H}_0|n\rangle\langle n|\partial_{k_y}\hat{H}_0|0\rangle}{(\varepsilon_0-\varepsilon_n)^2}.}$$

Relation with light-matter interaction

Arago & Fresnel

$-$

$\varphi(t) = \pm\,\omega t$

$+$

$$\mathbf{E}_\pm = E_0 e^{-i\omega t} e^{ikz}(\mathbf{e}_x \mp i\mathbf{e}_y)$$

$z = 0$

Jones Formalism

$\mathrm{Re}\, E^x_\pm = E_0 \cos\omega t$

$\mathrm{Re}\, E^y_\pm = \mp E_0 \sin\omega t$

Electric dipole at z=0 interacting with circularly polarized light

$$\delta H_\pm = E_0 e^{\pm i\omega t}|a\rangle\langle b| + h.c. = E_0 e^{\pm i\omega t}\sigma^+ + h.c.$$

Resonance.

$$|b\rangle = e^{\mp i\frac{\omega t}{2}}|b'\rangle$$

$$|a\rangle = e^{\pm i\frac{\omega t}{2}}|a'\rangle$$

$$E_b - E_a = \pm\hbar\omega$$

$\theta = 0$ $+$ $\theta = \pi$ $-$

$$|\psi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}$$

$$|\psi_-\rangle = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}$$

$|a\rangle = \left(\cos\frac{\theta}{2}|\psi_+\rangle - \sin\frac{\theta}{2}|\psi_-\rangle\right)e^{i\varphi/2}$

$|b\rangle = \left(\sin\frac{\theta}{2}|\psi_+\rangle + \cos\frac{\theta}{2}|\psi_-\rangle\right)e^{-i\varphi/2}$

$$\delta H_{\pm} = E_0 e^{\pm i\omega t} \sin\frac{\theta}{2}\cos\frac{\theta}{2} e^{i\varphi}\left(|\psi_+\rangle\langle\psi_+| - |\psi_-\rangle\langle\psi_-|\right) + h.c.$$

$$+ E_0 e^{\pm i\omega t}\cos^2\frac{\theta}{2}|\psi_+\rangle\langle\psi_-| e^{i\varphi} + h.c.$$

$$- E_0 e^{\pm i\omega t}\sin^2\frac{\theta}{2}|\psi_-\rangle\langle\psi_+| e^{i\varphi} + h.c.$$

What do we learn from this representation? Relation with geometry?

- 1st term is going to zero close to the poles and acts as a small renormalization of chemical potential in a time-dependent way; becomes negligible close to the poles

- Class I

$$\boxed{\begin{aligned} A'_\varphi(\theta < \theta_c) &= A_\varphi(\theta) - A_\varphi(0) = \sin^2\frac{\theta}{2} \\ A'_\varphi(\theta > \theta_c) &= A_\varphi(\theta) - A_\varphi(\pi) = -\cos^2\frac{\theta}{2} \end{aligned}}$$

Light-induced inter-bands transition rates

Time-independent terms

$$\Gamma_{\pm}(\omega) = \frac{2\pi}{\hbar} |\langle \psi_+ | \delta H_{\pm} | \psi_- \rangle|^2 \delta(E_b - E_a \mp \hbar\omega)$$

$$\Delta \Gamma = \frac{1}{2\pi} \left(\frac{\Gamma_+(0) + \Gamma_-(\pi)}{2} \right)$$

$$= \frac{1}{\hbar^2} E_0^2 C^2$$

$$\Gamma_{\pm}(\omega) = \frac{2\pi}{\hbar} E_0^2 \alpha(\theta) \delta(E_b - E_a \mp \hbar\omega)$$

$$\alpha(\theta) = \left(\cos^4 \frac{\theta}{2} + \sin^4 \frac{\theta}{2} \right)$$

$$\alpha(\theta) = C^2 + 2A'_{\varphi}(\theta < \theta_c) A'_{\varphi}(\theta > \theta_c)$$

Ph. W. Klein, A. G. Grushin, K. Le Hur, Phys. Rev. B 103, 035114 (2021)
K. Le Hur, Phys. Rev. B 105, 125106 (2022)

Similar transition rates with a local microscope in circuit quantum electrodynamics
J. Legendre and K. Le Hur, to appear (related to Class III, in 1D)

Is this real ?

Close to the poles: Protected Photo-Electric Effect

K. Le Hur, Phys. Rev. B 105, 125106 (2022)

$\theta_c \mapsto 0$

$$\boxed{\begin{aligned} -A'_\varphi(\theta > \theta_c) &= \\ -A'_\varphi(\theta = 0^+) &= C \end{aligned}}$$

$$\delta H_+ = E_0 e^{i\omega t} e^{-i\varphi} C\, |\psi_-\rangle\langle\psi_+| + h.c.$$

$$\tilde{\omega} = \omega - \frac{2m}{\hbar}$$

$$|\psi_+(t)\rangle = e^{\frac{imt}{\hbar}} |\psi_+(0)\rangle - \frac{e^{\frac{i}{\hbar}mt}}{\hbar\tilde{\omega}} E_0 e^{i\varphi} C \left(e^{i\tilde{\omega}t} - 1\right) |\psi_-(0)\rangle$$

$$N_+(t) = |\langle\psi_+(t)|\psi_+(t)\rangle|^2 = N_+(0) - \mathcal{P}(t)$$

$$\boxed{\mathcal{P}(\tilde{\omega}, t) = \frac{4E_0^2}{(\hbar\tilde{\omega})^2}(A'_\varphi(\theta > \theta_c))^2 \sin^2\left(\frac{1}{2}\tilde{\omega}t\right)}$$

$$\boxed{\frac{dN_+}{dt^2} = -\Delta\Gamma}$$

Similar form as nuclear magnetic resonance, may have applications for (protected) optical "imagery"

Relation to Topological Lattice Models

Simple view of photo-induced currents

Honeycomb lattice and Haldane model

$$\hat{P}_{a,b} = \frac{1}{2}(1 \pm \sigma_z)$$

$$\hat{n}_{\lambda} = \psi^{+} \hat{P}_{\lambda} \psi$$

$$\nabla \cdot \mathbf{J} + \frac{\partial \hat{n}}{\partial t} = 0.$$

$e = 1$

$$\boxed{\hat{J}(t) = \frac{d}{dt}(\hat{n}_a(\mathbf{k},t) - \hat{n}_b(\mathbf{k},t))}$$

2*2 matrix

$$\frac{d\hat{n}_a}{dt} = -\frac{d\hat{n}_b}{dt}$$

$$\hat{J}(t) = \frac{1}{2}\frac{d}{dt}\sigma_z(t)$$

$$\hat{J}(t) = v_F\left((p_x + A_x(t))\sigma_y - (\zeta p_y + A_y(t))\sigma_x\right)$$

$$\frac{\partial \hat{J}_i}{\partial x_i} \sim \frac{\hat{J}_i}{a}$$

When evaluating photo-induced currents similar to take px=py=0 or sample on all momenta (average)

$$\boxed{\frac{d}{dt}\sigma_z(t) = \frac{i}{\hbar}[H, \sigma_z(t)]}$$

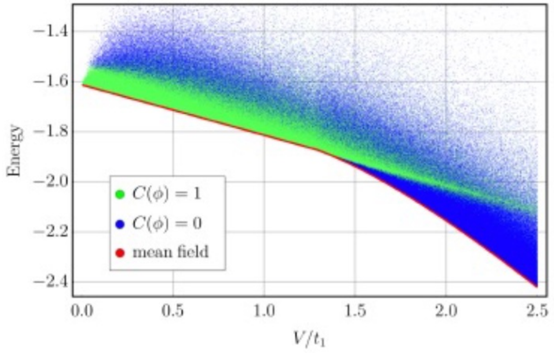
$$\boxed{\hat{J}_{\pm,\zeta}(t) = \frac{1}{2\hbar}A_0 e^{-i\omega t}\left(\frac{\partial H}{\partial(\zeta p_y)} \pm i\frac{\partial H}{\partial p_x}\right) + h.c.}$$

$$\tilde{\Gamma}_{\pm} = \frac{2\pi}{\hbar}\frac{A_0^2}{2\hbar^2}\left|\left\langle \psi_- \middle| \left(\pm i\frac{\partial H}{\partial p_x} + \frac{\partial H}{\partial p_y}\right) \middle| \psi_+ \right\rangle\right|^2$$
$$\times\, \delta(E_-(0) - E_+(0) - \hbar\omega).$$

$$\boxed{\begin{aligned}\frac{\tilde{\Gamma}_+(K,\omega) - \tilde{\Gamma}_-(K',\omega)}{2} &= -\frac{2\pi}{\hbar}\frac{A_0^2}{(\hbar v_F)^2} \\ \times\, m^2 \left(F_{p_y p_x}(0) - F_{-p_y p_x}(\pi)\right) &\delta(E_-(0) - E_+(0) - \hbar\omega).\end{aligned}}$$

$$\left|\frac{\tilde{\Gamma}_+(K) - \tilde{\Gamma}_-(K')}{2}\right| = \frac{2\pi}{\hbar} A_0^2 |C|.$$

When measuring the variation of population in time, since $dN/dt^2 < 0$ then $|C|$ should occur in the response.



Including interactions

W. Klein, A. G. Grushin, K. Le Hur, Phys. Rev. B 103, 035114 (2021)

Stochastic Variational approach

(see additional slides Class II)

$$\Gamma^{\pm}_{l\to u}\left(\omega_{\boldsymbol{k}},\boldsymbol{k}\right)=\frac{2\pi}{\hbar}\left(\frac{E}{\hbar\omega}\right)^2\left|\mathcal{A}^{\pm}_{l\to u}\right|^2\delta\left(\epsilon^{\boldsymbol{k}}_u-\epsilon^{\boldsymbol{k}}_l-\hbar\omega\right)$$

$$\boxed{\mathcal{A}^{\pm}_{l\to u}=\langle u_{\boldsymbol{k}}|\frac{1}{i}\frac{\partial\mathcal{H}_0}{\partial k_x}\mp\frac{\partial\mathcal{H}_0}{\partial k_y}|l_{\boldsymbol{k}}\rangle}$$

$$\Gamma^{\pm}_{l\to u}\left(\omega_{\boldsymbol{k}}\right)=\sum\nolimits_{\boldsymbol{k}\in\mathrm{BZ}}\Gamma^{\pm}_{l\to u}\left(\omega_{\boldsymbol{k}},\boldsymbol{k}\right)$$

$$\boxed{\frac{1}{2}\int_0^{\infty}\mathrm{d}\omega\sum_{\boldsymbol{k}=\boldsymbol{K},\boldsymbol{K}'}\left(\Gamma^{+}_{l\to u}\left(\omega_{\boldsymbol{k}},\boldsymbol{k}\right)-\Gamma^{-}_{l\to u}\left(\omega_{\boldsymbol{k}},\boldsymbol{k}\right)\right)=\rho C}$$

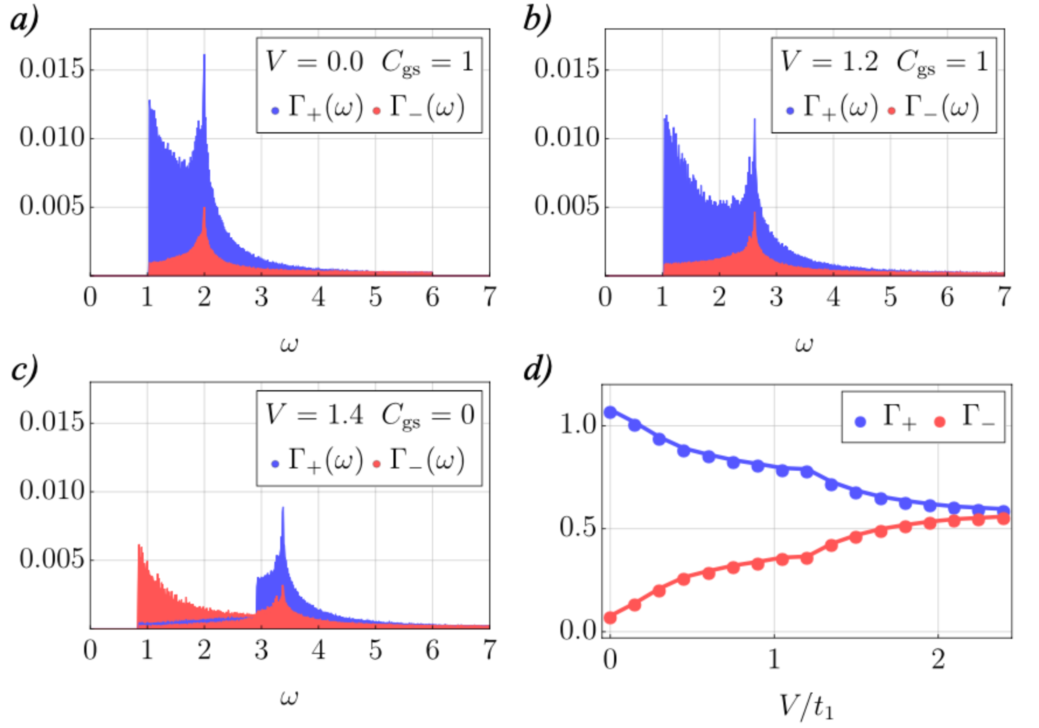


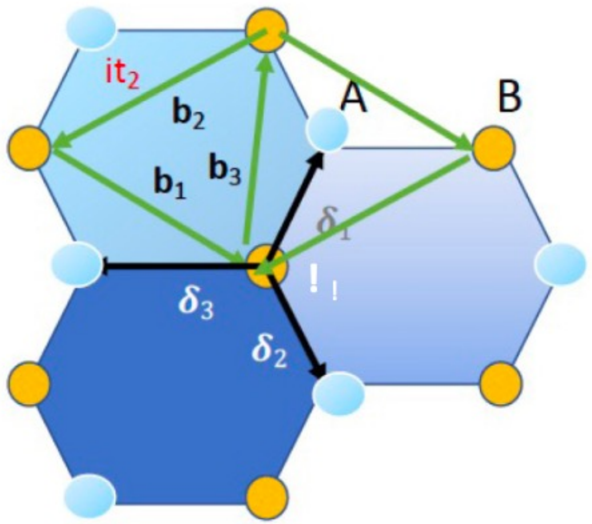
Figure 7. (color online) (a-c) Ground state depletion rate $\Gamma_\pm\equiv\sum_{\boldsymbol{k}\in\mathrm{BZ}}\Gamma^{\pm}_{l\to u}(\omega_{\boldsymbol{k}},\boldsymbol{k})$ as a function of frequency for $t_2=0.1$ and different fixed values of the interaction strength V. (d) Stochastic frequency-integrated depletion rate $\Gamma_\pm\equiv\frac{1}{2}\rho^{-1}\int\mathrm{d}\omega\sum_{\boldsymbol{k}=\boldsymbol{K},\boldsymbol{K}'}\Gamma^{\pm}_{l\to u}(\omega_{\boldsymbol{k}},\boldsymbol{k})$ as a function of V.

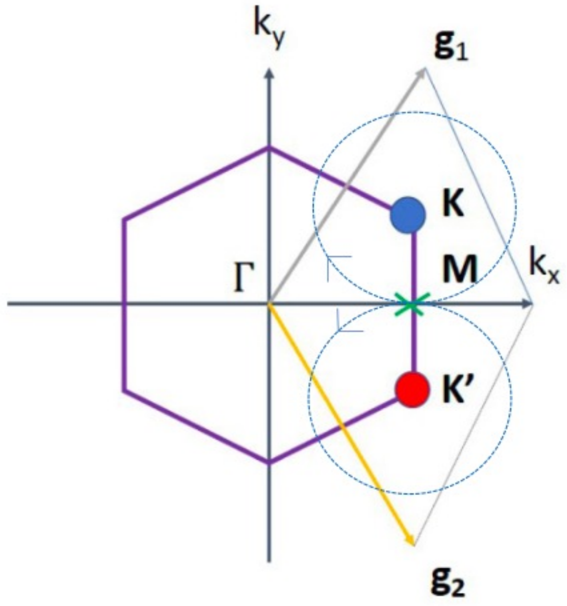
Results with coupling to a microwave port on the lattice

J. Legendre & K. Le Hur, in preparation

Special response on the lattice at M point

K. Le Hur, Phys. Rev. B 105, 125106 (2022)





$$\mathbf{u}_1 = -\mathbf{b}_2 = \tfrac{a}{2}(3, \sqrt{3})$$

$$\mathbf{u}_2 = \mathbf{b}_1 = \tfrac{a}{2}(3, -\sqrt{3})$$

$$k_x^M = \frac{2\pi}{3a}, k_y^M = 0.$$

$$\mathcal{H}(M) = w\sigma^+ + \text{H.c.}, \qquad d_1 = \tfrac{1}{2}(w + w^*)$$

$$w = t\left(1 + \sum_{i=1}^{2} e^{-i\mathbf{k}\cdot\mathbf{u}_i}\right),$$

Hamiltonian should respect symmetry $k_y \leftrightarrow -k_y$

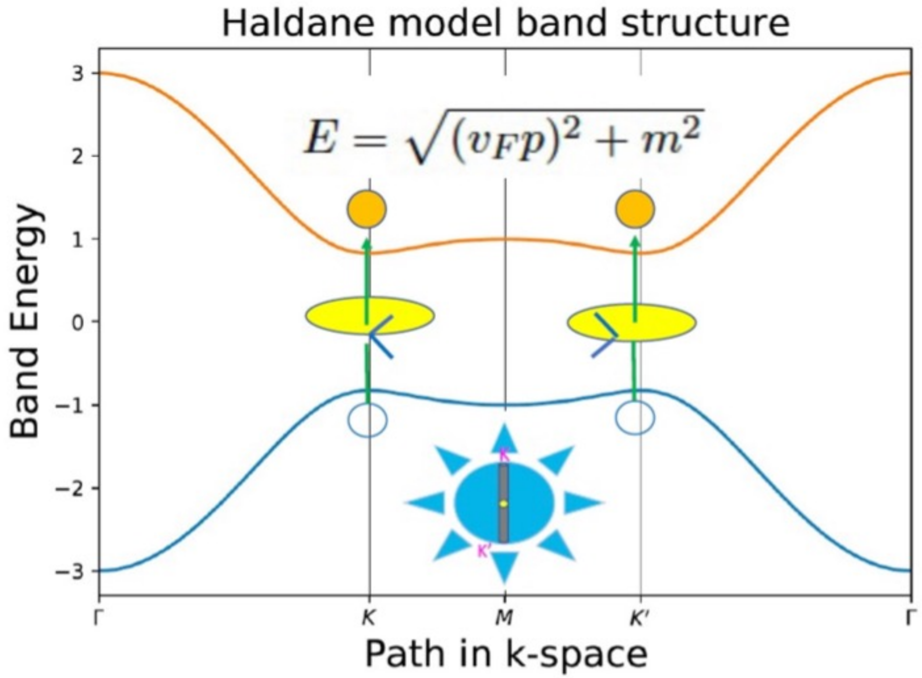
Fu & Kane, 2007

$$\left\langle \psi_+ \left| \frac{\partial H}{\partial k_x} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial H}{\partial k_x} \right| \psi_+ \right\rangle = \mathcal{I}(M),$$

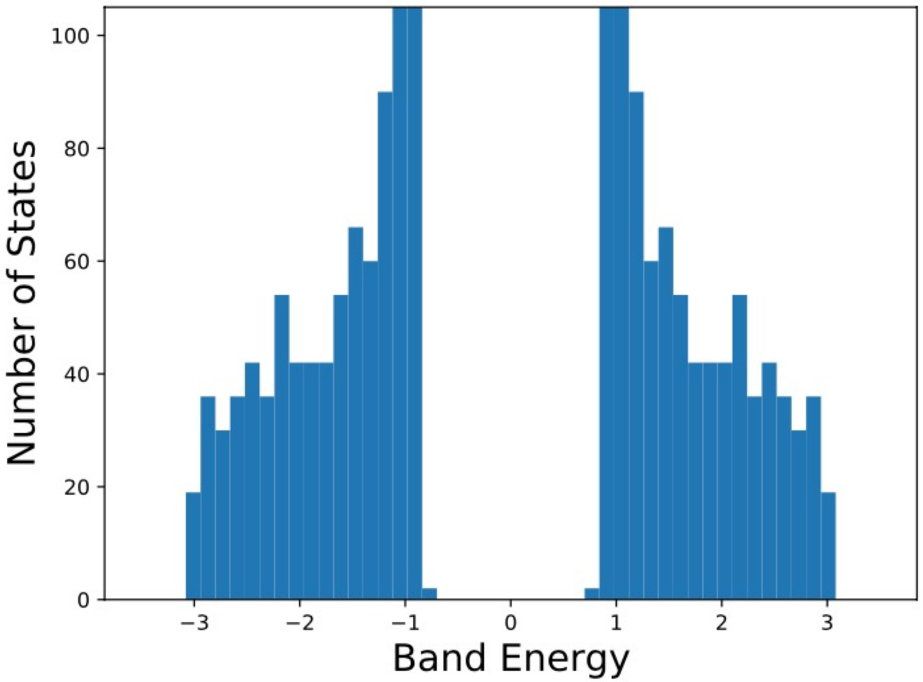
$$\mathcal{I}(M) = \frac{\mathcal{I}(0)}{2} = \frac{\mathcal{I}(\pi)}{2}$$

$$\mathcal{I}(\theta) = \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_+ \right\rangle + \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_+ \right\rangle = 2v_F^2\left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right).$$

$\hbar = 1$ $\quad = \alpha(\theta)\, v_F^2$



Haldane model band structure
$E = \sqrt{(v_F p)^2 + m^2}$
Band Energy
Path in k-space
Γ
K
M
K′
Γ



Number of States
Band Energy

Quantum Hall effect and Light: simple model

K. Le Hur, review arXiv:2209.15381

Class II

$$H = \hbar\omega_c^* \begin{pmatrix} 0 & \mathcal{O}^\dagger \\ \mathcal{O} & 0 \end{pmatrix} \qquad |\Phi_A(y), \Phi_B(y)\rangle$$

$$E = \pm\hbar\omega_c^* \sqrt{N}$$

$$\boxed{\begin{aligned} \left(\hbar\omega_c^*\mathcal{O} + \hbar\omega_c^*(A_0 l_B)e^{\mp i\omega t}\right)\Phi_A &= i\hbar\frac{d}{dt}\Phi_B \\ \left(\hbar\omega_c^*\mathcal{O}^\dagger + \hbar\omega_c^*(A_0 l_B)e^{\pm i\omega t}\right)\Phi_B &= i\hbar\frac{d}{dt}\Phi_A. \end{aligned}}$$

$$\mathcal{O} \to \mathcal{O} + (A_0 l_B)e^{\mp i\omega t},$$

$$\mathcal{O}^\dagger \mathcal{O} \mapsto N + (A_0 l_B)^2$$

1st-order calculation of eigenstates

$$\Phi_\pm(t) = \Phi e^{\mp i\sqrt{N}\hbar\omega_c^* t}$$

$$\left(\hbar\omega_c^*\mathcal{O} + \hbar\omega_c^*(A_0 l_B)e^{\mp i\omega t}\right)\Phi_A = 0$$
$$\left(\hbar\omega_c^*\mathcal{O}^\dagger + \hbar\omega_c^*(A_0 l_B)e^{\pm i\omega t}\right)\Phi_B = 0.$$

For the filled plateau at N=0, the model with 2 rungs is easily solvable

A natural ansatz here is $\Phi_B = 0$ and

$$\boxed{f_A = (A_0 l_B)}$$

$$(A_0 l_B)^2 (\omega_c^* t)^2$$

$$\tilde{\Phi}_A(0) = \Phi_A(0) - f_A \Phi_A(1) e^{-i\omega_c^* t}.$$

Probability to reach upper band at short time

Degeneracy Φ/Φ_0

$\nu = 1$

Fractional quantum Hall effect ν

C. Repellin and N. Goldman, Phys. Rev. Lett. 122, 166801 (2019)

Topological phase from circularly polarized light

K. Le Hur, arXiv:2209.15381

$$U_\pm(t) = e^{\mp i \frac{\omega t}{2}\sigma_z}$$

$$\delta H_\pm = E_0 e^{\pm i\omega t}|a\rangle\langle b| + h.c. = E_0 e^{\pm i\omega t}\sigma^+ + h.c.$$

To induce a topological phase, i.e. a diagonal term of the form

$\xi d_z \sigma_z$ with $\xi = \pm$ K, K' this requires the superposition of 2 polarizations

$$\delta\tilde{H}_\pm = U_\pm \delta H_\pm U_\pm^{-1} \pm \frac{\hbar\omega}{2}\sigma_z$$

$$U_\pm \delta H_\pm U_\pm^{-1} = E_0 \sigma_x$$

$$U_\pm(-d_x\sigma_x - \check{d}_y\sigma_y)U_\pm^{-1} = -d\sin\theta\sigma_x$$

$$E_0 e^{-ikz}e^{-i\omega t}(\mathbf{e}_x + i\mathbf{e}_y) + E_0 e^{ikz}e^{-i\omega t}(\mathbf{e}_x - i\mathbf{e}_y)$$

$$\mathbf{A} = -A_0(\sin(\omega t)\mathbf{e}_x + \sin(\omega t - \phi)\mathbf{e}_y)$$

$$H_{eff} = \begin{pmatrix} \frac{\hbar\omega}{2}\cos\theta & E_0 - d\sin\theta \\ E_0 - d\sin\theta & -\frac{\hbar\omega}{2}\cos\theta \end{pmatrix}$$

$$\mathbf{d}_{eff} = \left((\tilde{d}+d)\sin\theta, 0, -\frac{\hbar\omega}{2}\cos\theta\right)$$

J. Cayssol, B. Dora, R. Moessner
Phys. Status Solidi RRL 7, 101-108 (2013)

High-Frequency Magnus expansion,
N. Goldman and J. Dalibard, Phys. Rev. X 4, 031027 (2014)

Realization in graphene: J. W. McIver et al.
Nature Physics 16, 38-41 (2020)

Skin Effect on a ball as a classical effect

$$\boxed{\mathbf{E}(t) = E_0 e^{-i\omega t} e^{ikz} \mathbf{e}_\varphi}$$

AC perturbation in the rotating frame

We suppose a metal on the surface : the number of charges is larger in the equatorial plane where the response can be studied starting equivalently in spherical or cylindrical coordinates (r,φ, z)

Newton equation on a charge e:

$$\dot{p}_\varphi(t) = eE_0 e^{-i\omega t} e^{ikz}.$$

$$B_r(t) = +\frac{ik}{e} p_\varphi(t).$$

$z = 0$

$$\boxed{\begin{aligned} \mathrm{Re}p_\varphi(t) &= \frac{eE_0}{\omega}\sin(\omega t) \\ \mathrm{Im}p_\varphi(t) &= -\frac{eE_0}{\omega} 2\sin^2\frac{\omega t}{2}, \end{aligned}}$$

$$\mathrm{Re}B_r(t) = -\frac{k}{e}\mathrm{Im}p_\varphi(t) = \frac{2E_0 k}{\omega}\sin^2\frac{\omega t}{2}$$

$$\boxed{\bar{B}_r = \frac{1}{T}\int_0^T \mathrm{Re}B_r(t)dt = \frac{E_0 k}{\omega} = \frac{E_0}{c}.}$$

At time t=0, we fix B_r=0 to study only the response to the circularly polarized field.

$T = 2\pi/\omega$ Floquet period

Maxwell-Faraday equation:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{1}{2}\mu_0^{-1}\bar{B}_r\bar{B}_r = \frac{1}{2}\epsilon_0|E_0|^2$$

e^{ikz}

Ampere's law

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 \bar{\mathbf{J}}$$

$$\mu_0 \bar{\mathbf{J}} = \frac{ik}{c} E_0 e^{ikz} \mathbf{e}_{\varphi}.$$

gives current closer to the poles

The Maxwell-Faraday equation also induces a term B_z in agreement with the loops of current

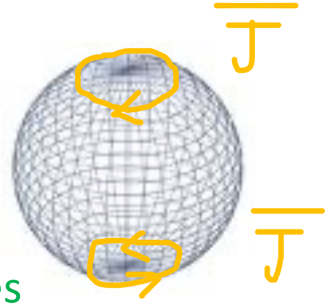
$z = \cos\theta$

From the equatorial plane, then this gives rise to

$$\mathrm{Re}B_z = -\frac{E_0}{\omega}\sin(\omega t) - \frac{2kE_0}{\omega}\sin^2\frac{\omega t}{2}\cos\theta,$$

$$\boxed{\bar{B}_z = -\frac{E_0}{c}\cos\theta}$$

$$\bar{\mathbf{B}} = \mathrm{Re}\mathbf{B} = \frac{E_0}{c}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$



$$\boxed{q_m = \frac{2E_0}{c}}$$

Karyn Le Hur, Review ArXiv:2209.15381, Section II, preliminaries

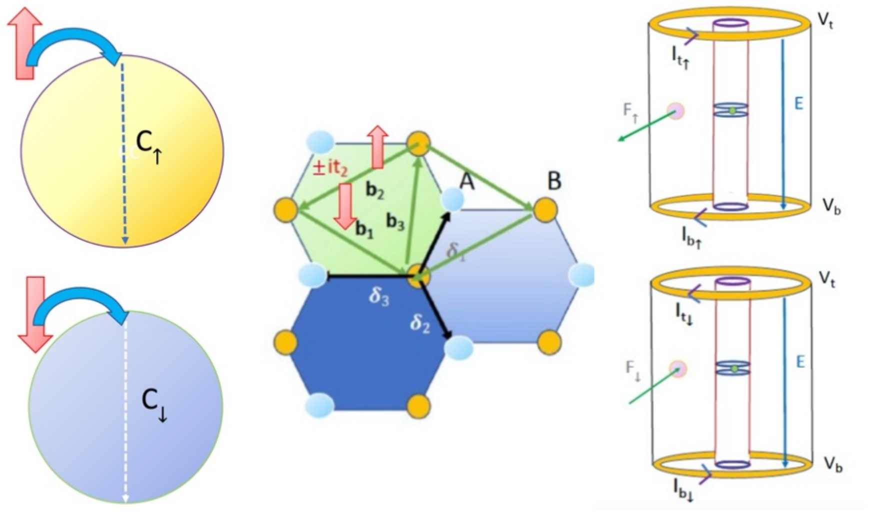
Review on response to AC perturbations: Michele Filippone, Arthur Marguerite, Karyn Le Hur, Gwendal Feve, Christophe Mora
Mesoscopic quantum RC circuit, Entropy 22 (8), 847 (2020)

Light response in topological insulators

$$|u_\uparrow(\mathbf{K})\rangle = \frac{1}{\sqrt{(E+m)^2 + v_F^2|\mathbf{p}|^2}} \begin{pmatrix} v_F|\mathbf{p}| \\ (E+m)e^{i\tilde{\varphi}} \end{pmatrix}$$

$$\mathbf{k} = \mathbf{K} + \mathbf{p}.$$

$$|u_\downarrow(\mathbf{K})\rangle = \frac{1}{\sqrt{(E-m)^2 + v_F^2|\mathbf{p}|^2}} \begin{pmatrix} v_F|\mathbf{p}| \\ (E-m)e^{i\tilde{\varphi}} \end{pmatrix}$$



$$C_s = C_1 - C_2$$

Kane-Mele Pfaffian 2005

$$Pf_{\uparrow\downarrow} = \langle u_\uparrow(\mathbf{p})|u_\uparrow(-\mathbf{p})\rangle^*$$

D. N. Sheng, Z. Y. Weng, L. Sheng, F. D. M. Haldane
Phys. Rev. Lett. 97, 036808 (2006)

$$P(\mathbf{k}) = \frac{v_F|\mathbf{p}|}{m} \approx \sin\theta$$

$$C_s = \langle s_z(0)\rangle - \langle s_z(\pi)\rangle = -\int_0^{\frac{\pi}{v}} \frac{\partial\langle s_z\rangle}{\partial t}dt$$

Response to circularly polarized light measures the "zeroes"
Of Pfaffian at the poles (Dirac points) and Z_2 topological spin number

$$J_\perp^1 - J_\perp^2 = \frac{2q^2}{h}C_s E.$$

$$\alpha(\theta) = C^2 + 2A'_\varphi(\theta < \theta_c)A'_\varphi(\theta > \theta_c).$$

K. Le Hur, Phys. Rev. B 105, 125106 (2022)