Karyn Le Hur

Centre de Physique Theorique, Ecole Polytechnique and CNRS

4 classes Saclay Lectures Series: 1h30 each

Thanks to Sylvain Ravets, Igor Ferrier-Barbut, Benoit Valiron for invitation

Slides of each lecture accessible at: http:// www.cpht.polytechnique.fr/cpht/lehur/Karyn.LeHur.html

Institut d'Optique Graduate School

Geometry and Topology in the Quantum!

- Class I: Quantum Geometry, Information and Topological Physics from Bloch Sphere (June 9) V
- Class II: Application in Topological Lattice Models and Quantum Matter (June 16)
- Class III: Applications in Transport and Light-Matter Interaction (June 23)
- Class IV: Entangled WaveFunction and Fractional Topology (June 30)

2023

thanks to the team !

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Introduction to Geometry and Topological states from the spin-1/2 particle

Class I: 1h30

- Introduction to Berry curvature and quantum Metric from Bloch sphere
  General geometrical relations for topological state
  Applications in quantum circuits
  Note on classical correspondence

- Implications for Topological Transport, Dynamics and Energetics
  Introduction to Karplus-Luttinger velocity from curved space

- Quantum Dynamo Effect with a Cavity and Many-Body Physics

Berry phase

$$H\psi_n(x) = E_n\psi_n(x),$$

$$\Psi_n(x,t) = \psi_n(x)e^{-iE_nt/\hbar}.$$

$$\boxed{\begin{aligned} H(t)\psi_n(x,t) &= E_n(t)\psi_n(x,t). \\ \Psi_n(x,t) &= \psi_n(x,t)e^{-\frac{i}{\hbar}\int_0^t E_n(t')dt'}e^{i\gamma_n(t)} \end{aligned}}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = H(t)\Psi,$$

$$\frac{\partial\psi_n}{\partial t} = (\nabla_R\psi_n)\cdot\frac{d\mathbf{R}}{dt}$$

Berry phase

$$\boxed{\gamma_n(T) = i\oint\langle\psi_n|\nabla_R\psi_n\rangle\cdot d\mathbf{R}.}$$

D. Griffiths chapter 10.2.2, quantum mechanics

Berry curvature and quantum Metric

We begin with a space, e.g. a two-dimensional space described through the vector **R**=(R_x,R_y)

We introduce a local gauge potential or Berry connection defined as

$\vec{A} = -i \langle \psi | \vec{\nabla} | \psi \rangle$

$|\psi\rangle$: quantum state

This is the analogue of the vector potential in classic mechanics in the sense of (averaged) momentum in quantum physics

$\vec{\nabla} = (\partial_x, \partial_y)$

Berry curvature analogous to the magnetic field

$\mu, \nu = x, y$

$\partial_\mu = \frac{\partial}{\partial R_\mu}$

$$\boxed{F_{\mu\nu} = \frac{\partial}{\partial R_\mu} A_\nu - \frac{\partial}{\partial R_\nu} A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu = -F_{\nu\mu}.}$$

M. V. Berry, Proceedings of the Royal Society A, volume 392, issue 1802 (1984)

Useful relations through eigenstates

$$A_\nu(\mathbf{R}) = -i\langle\psi|\partial_\nu|\psi\rangle.$$

$$\partial_\mu A_\nu = -i\langle\partial_\mu\psi|\partial_\nu\psi\rangle - i\langle\psi|\partial_\mu\partial_\nu\psi\rangle$$

$$F_{\mu\nu} = \frac{\partial}{\partial R_\mu}A_\nu - \frac{\partial}{\partial R_\nu}A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu = -F_{\nu\mu}.$$

Therefore :

$$F_{\mu\nu} = -i\left(\langle\partial_\mu\psi|\partial_\nu\psi\rangle - \langle\partial_\nu\psi|\partial_\mu\psi\rangle\right)$$

Inserting eigenstates

$$H(\mathbf{R})|n\rangle = E_n|n\rangle$$

$$\sum_n |n\rangle\langle n| = 1$$

$$F_{\mu\nu}(\mathbf{R}) = -i \sum_n \left(\langle \partial_\mu \psi | n \rangle \langle n | \partial_\nu \psi \rangle - \langle \partial_\nu \psi | n \rangle \langle n | \partial_\mu \psi \rangle \right)$$

This sum is zero if $|\psi\rangle = |n\rangle$

To relate with general theory of transport and quantum Hall conductivity of crystals in class III :

$|\psi\rangle$: eigenstate
ground state

$$H|\psi\rangle = E_\psi |\psi\rangle$$

$$\partial_\alpha (H|\psi\rangle) = \partial_\alpha H|\psi\rangle + H\partial_\alpha|\psi\rangle$$

$$\langle n|\partial_\alpha (H|\psi\rangle) = \langle n|\partial_\alpha H|\psi\rangle + \langle n|H\partial_\alpha|\psi\rangle$$

$$E_\psi \langle n|\partial_\alpha|\psi\rangle = \langle n|\frac{\partial H}{\partial R_\alpha}|\psi\rangle + E_n \langle n|\partial_\alpha \psi\rangle$$

Therefore :

$$\langle n|\partial_\alpha|\psi\rangle = -\frac{\langle n|\frac{\partial H}{\partial R_\alpha}|\psi\rangle}{(E_n - E_\psi)}$$

$$\langle n|\partial_\alpha\psi\rangle = -\frac{\left\langle n\left|\frac{\partial H}{\partial R_\alpha}\right|\psi\right\rangle}{(E_n - E_\psi)}$$

$$\langle\partial_\alpha\psi|n\rangle = \frac{\left\langle \psi\left|\frac{\partial H}{\partial R_\alpha}\right|n\right\rangle}{(E_n - E_\psi)}$$

$$F_{\mu\nu}(\mathbf{R}) = -i\sum_n \left(\langle\partial_\mu\psi|n\rangle\langle n|\partial_\nu\psi\rangle - \langle\partial_\nu\psi|n\rangle\langle n|\partial_\mu\psi\rangle\right).$$

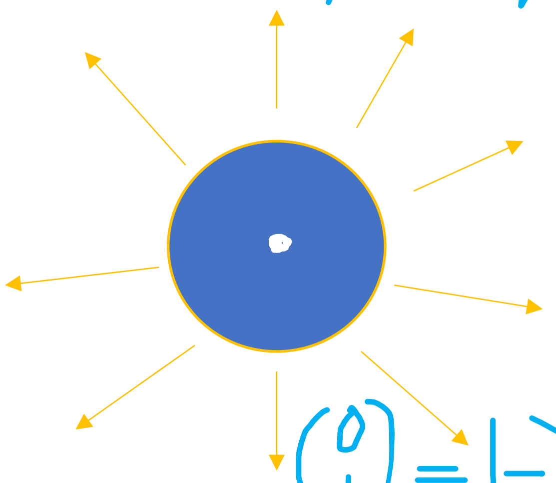
Second order calculation
In conductivity:
See Class III

$$\boxed{F_{\mu\nu} = i\sum_{n\neq\psi}\frac{\left(\left\langle n\left|\frac{\partial H}{\partial R_\mu}\right|\psi\right\rangle\left\langle\psi\left|\frac{\partial H}{\partial R_\nu}\right|n\right\rangle - \mu\leftrightarrow\nu\right)}{(E_n - E_\psi)^2}}$$

D. J. Thouless, M. Kohmoto, M. P. Nightingale and M. den Nijs, 1982

Spin-1/2 in the presence of a radial field

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |+\rangle_z$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = |-\rangle_z$

$E_+ = -|\vec{d}|$

$E_- = +|\vec{d}|$

Sphere acting on Parameters space of the Magnetic field

$$\boxed{H = -\vec{\mathbf{d}} \cdot \vec{\boldsymbol{\sigma}}}$$

$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$
Pauli Matrices

$$\vec{\mathbf{d}}(\varphi, \theta) = d(\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta) = (d_x, d_y, d_z).$$

Two eigenstates with energy +/- $|\vec{d}|$

$$|\psi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2} e^{i\frac{\varphi}{2}} \end{pmatrix}, \quad |\psi_-\rangle = \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2} e^{i\frac{\varphi}{2}} \end{pmatrix}$$

Do we have a topological state here on the Riemann, Poincare, Bloch sphere?

Relations between coordinates

Spherical coordinates

$$A_\phi = 0$$

$$A^S_\varphi = -i \langle \psi | \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} | \psi \rangle$$

$$= \frac{1}{\sin\theta} A^C_\varphi$$

$$\boxed{A^C_\varphi = -i \langle \psi | \partial_\varphi | \psi \rangle}$$

$$F^S_{\theta\varphi} = \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta A^S_\varphi \right) = \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} A^C_\varphi$$

Singularity in the core of the sphere can be captured through Gauss law or Chern number

$$C = \frac{1}{2\pi} \oiint \vec{F} \cdot \vec{d}^{(2)}\sigma = \frac{1}{2\pi} \iint \frac{\partial}{\partial\theta} A^C_\varphi \, d\theta \, d\varphi$$

For a discussion on this, see e.g. P. Roushan et al. Nature 515, 241-244 (2014)

$$|\psi\rangle = |\psi_+\rangle \qquad C = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sin\theta}{2}\, d\theta$$

$$A_\varphi^c = -\frac{\cos\theta}{2} \qquad = 1\,!$$

Dirac monopole
1931
Pierre Curie
1831

Ehrenfest Theorem for spin-1/2 also gives $\langle \sigma_z \rangle = \cos\theta$

$$\boxed{F_{\theta\varphi} = \frac{\sin\theta}{2}}$$
$$= \partial_\theta A_\varphi^c$$

We obtain interesting additional formulae for 1 spin:

$$C = \int_0^{\pi} \frac{\sin\theta}{2}\, d\theta = -\frac{1}{2}\left[\cos\theta\right]_0^{\pi}$$

$$C = A_\varphi^c(\pi) - A_\varphi^c(0) \quad (1)$$

Related to Berry phases

$$= \frac{1}{2}\left(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle\right) = -\frac{1}{2}\int_0^{\pi} \frac{\partial \langle\sigma_z\rangle}{\partial\theta}\, d\theta \quad (2)$$

We have introduced the green formulae in L. Henriet, A. Slocchi, P. P. Orth, K. Le Hur, Phys. Rev. B 95, 054307 (2017) (2)
J. Hutchinson and K. Le Hur, Communication Physics 4, 144 (2021), Nature (1)

Proof generalizable to multispheres with interactions from geometry, class IV

Application in circuit QED with 1 artificial atom

D. Schroer et al. PRL 2014 (Boulder, K. Lehnert)
P. Roushan et al. Nature (John Martinis, Santa Barbara) 2014
Theory: A. Polkovnikov, V. Gritsev, M. Kolodrubetz

$$H/\hbar = \frac{1}{2}\left[\overbrace{\Delta}^{\cos\theta}\, \boldsymbol{\sigma_z} + \Omega\, \boldsymbol{\sigma_x} \cos\phi + \overbrace{\Omega}^{\sin\theta}\, \boldsymbol{\sigma_y} \sin\phi\right] ,$$

$$\theta(t) = \pi t / t_{\text{ramp}}$$

time $\theta(t)$!

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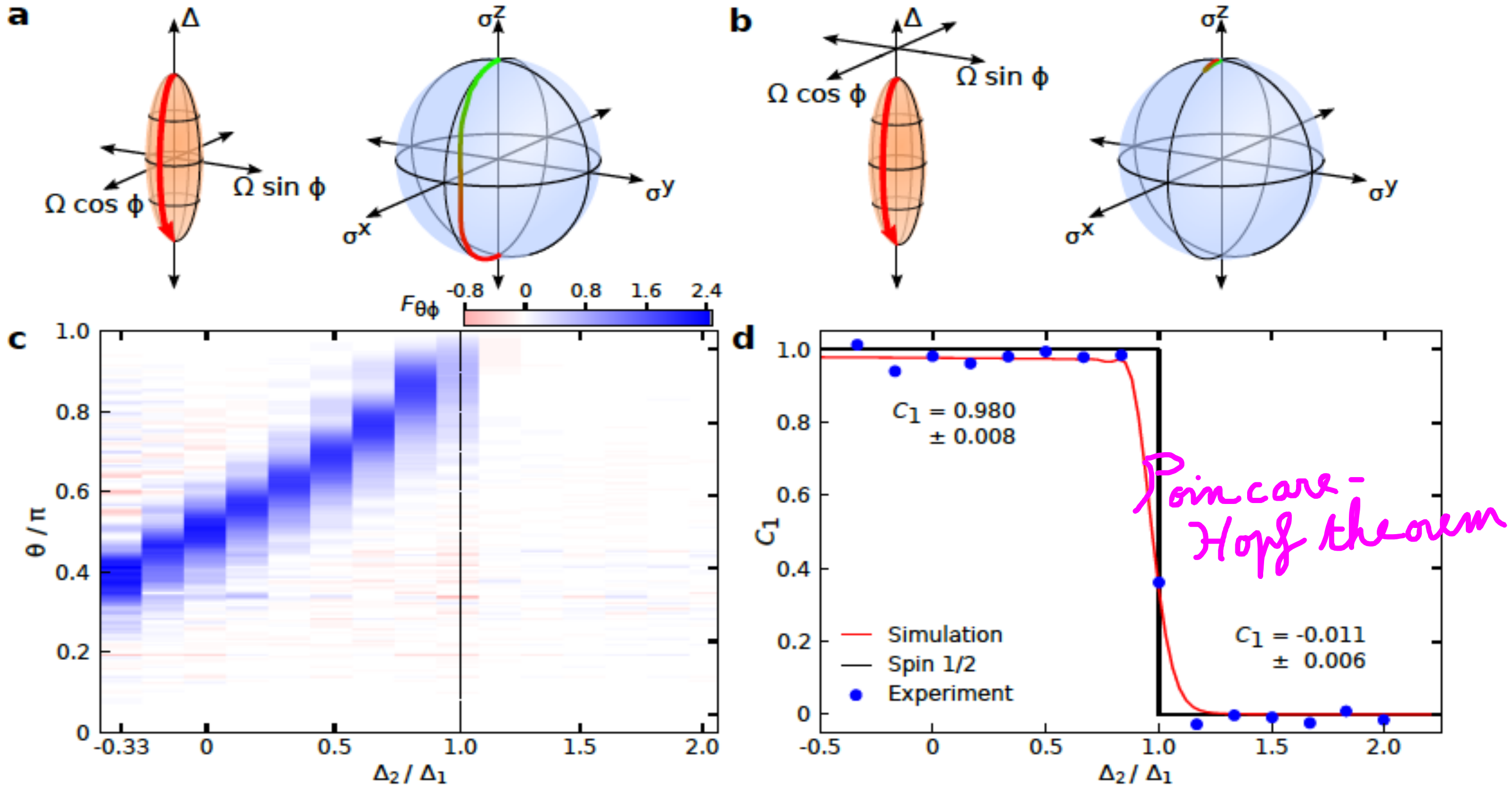
Ecole Polytechnique
Quantum Circuits

Rydberg atoms
Institut d'Optique
College de France, ENS

$\Delta_2 \sigma_z$



L. Henriet, A. Slocchi, P. P. Orth, K. Le Hur, Phys. Rev. B 95, 054307 (2017)

Example of information: metric and curvature

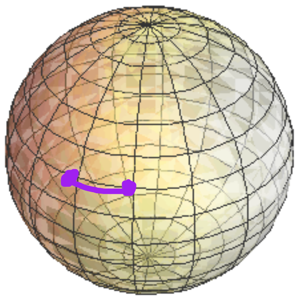
Application in Einstein-Field Equation

T.B. Smith, L. Pullasseri, A. Srivastava, Phys. Rev. Research (2022)

Review: A. Carollo, D. Valenti, B. Spagnolo Physics Reports (2020)

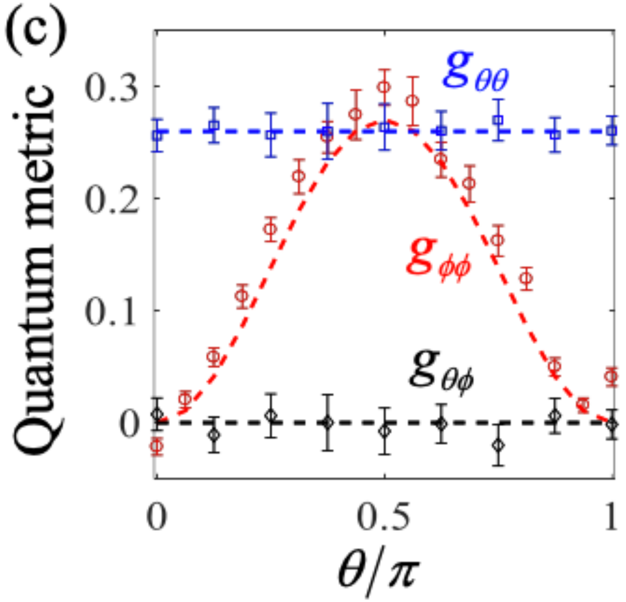
Karyn Le Hur, Review arXiv: 2209.15381
Appendix A

Quantum distance



(plotted for u from 0 to 2π and v from 0 to π)

Circuit QED



$\frac{1}{4}$

X. Tan et al. Phys. Rev. Lett. 122, 210401 (2019)

$$|\langle\psi_+(\theta,\varphi)|\psi_+(\theta,\varphi+d\varphi)\rangle|^2 = I(\theta) + 2\cos(d\varphi)\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$$

New function $I(\theta) = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}$

$$|\langle\psi_+(\theta,\varphi)|\psi_+(\theta,\varphi+d\varphi)\rangle|^2 = 1 - g_{\varphi\varphi}\, d\varphi^2$$

$d\varphi \mapsto 0$

$$\boxed{g_{\varphi\varphi} = \frac{\sin^2\theta}{4} = F_{\theta\varphi}^2}$$

Details of calculations

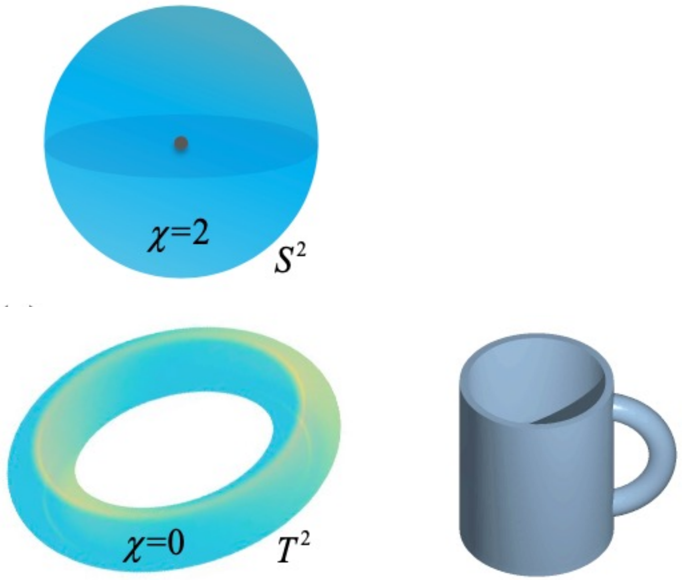
$$|\psi_+(\theta, \varphi + d\varphi)\rangle = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\frac{\varphi + d\varphi}{2}} \\ \sin\frac{\theta}{2} e^{i\frac{\varphi + d\varphi}{2}} \end{pmatrix}$$

$$\langle \psi_+(\theta, \varphi) | \psi_+(\theta, \varphi + d\varphi)\rangle = \cos^2\frac{\theta}{2} e^{-i\frac{d\varphi}{2}} + \sin^2\frac{\theta}{2} e^{i\frac{d\varphi}{2}}$$

$$|\langle \psi_+(\theta, \varphi) | \psi_+(\theta, \varphi + d\varphi)\rangle|^2 = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\cos d\varphi$$

$$= I(\theta) + \frac{\sin^2\theta}{2}\cos d\varphi$$

Euler Characteristic Number



$$\chi = 2 - 2g = 0$$

This approach also allows to relate Hawking temperature with χ

Y.-P. Zhang, S.-W. Wei, Y. Xiao-Liu, Physics Letters B 2020

X. Tan et al. Phys. Rev. Lett. 122, 210401 (2019)

$$\mathcal{F}^{\pm}_{\theta\phi} = \pm\tfrac{1}{2}\sin\theta$$

$$C_{\pm} = \frac{1}{2\pi}\int_{S^2} \mathcal{F}^{\pm}_{\theta\phi}\, d\theta d\phi = \pm 1$$

$$\chi = \frac{1}{4\pi}\int_{\mathcal{M}} R\sqrt{\det g}\; d\mu d\nu,$$

Sphere

Ricci Scalar curvature $R = 8$

$$\sqrt{\det g} = \frac{\sin\theta}{4}$$

$$\boxed{\chi = 2|C_{\pm}|}$$

See also Y. Q. Ma et al. Europhysics Letters 103, 2013 10008

2d $\quad R_{\varphi\varphi} = \frac{1}{2} R\, g_{\varphi\varphi}$

$$d\vec{\ell}\cdot d\vec{\varphi} = r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$

$r = 1$

class MathPhys 105, 2021

Ecole Polytechnique

Another way to apply the magnetism

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} = B\mathbf{e}_r}$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial_\varphi}\right) = B.$$

$A_\theta = 0$

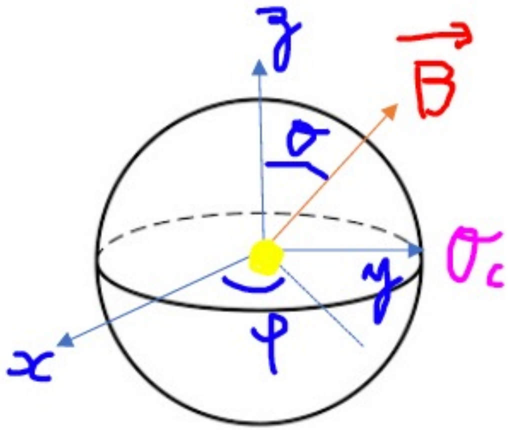
$A'_\varphi = A_\varphi \sin\theta$

$\frac{\partial A'_\varphi}{\partial\theta} = B r \sin\theta$

Nakahara Book 2003

Wu & Yang, 1975

We need at least 2 regions to have $A'_\varphi = 0$ $\theta = 0, \pi$



$$A'_\varphi(\theta < \theta_c) = -Br(\cos\theta - 1) = 2Br\sin^2\frac{\theta}{2}$$

$$A'_\varphi(\theta > \theta_c) = -Br(\cos\theta + 1) = -2Br\cos^2\frac{\theta}{2}.$$

$$A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c) = 2Br.$$

The symbols $\theta < \theta_c$ and $\theta > \theta_c$ in A' can be equivalently understood as $\theta = \theta_c^-$ and $\theta = \theta_c^+$.

New Insight

Karyn Le Hur, Review arXiv: 2209.15381
Sec. II and III

To describe topological properties of the surface from the poles we find it useful to introduce the field

$$\boxed{\tilde{A}_\varphi(\theta) = -Br\cos\theta} \quad (7)$$

which has the property to be smooth on the whole surface. This leads to

$$A'_\varphi(\theta < \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(0) \quad (8)$$
$$A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(\pi),$$

and such that

$$A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\pi) - \tilde{A}_\varphi(0). \quad (9)$$

Correspondence with quantum physics

$$\tilde{A}_\varphi = A_\varphi^c = -\frac{\cos\theta}{2}$$

$$B = \frac{1}{2}$$

$$r = 1$$

$$A'_\varphi(0) = A'_\varphi(\pi) = 0$$

$$\boxed{\begin{aligned} A'_\varphi(\theta < \theta_c) &= A_\varphi(\theta) - A_\varphi(0) = \sin^2\frac{\theta}{2} \\ A'_\varphi(\theta > \theta_c) &= A_\varphi(\theta) - A_\varphi(\pi) = -\cos^2\frac{\theta}{2}. \end{aligned}}$$

$A_\varphi = A_\varphi^c = -\frac{\cos\theta}{2}$

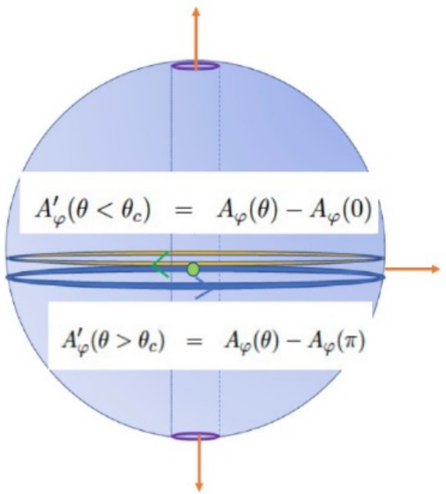
" Handle "

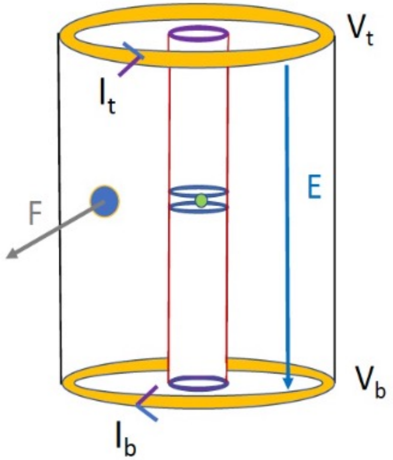
$$C = A'_\varphi(\theta_c^-) - A'_\varphi(\theta_c^+) = 1$$
$$= A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c).$$
$$= (A_\varphi(\pi) - A_\varphi(0)).$$

These relations can be generalized from geometry

J. Hutchinson and K. Le Hur, Communication Physics 4, 144 (2021), Nature

class IV





K. Le Hur, Phys. Rev. B 105, 125106 (2022)

Measurable with light
See Class III

$\alpha(0) = \alpha(\pi) = C^2$

$$\boxed{\begin{aligned} \alpha(\theta) &= \left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right). \\ \alpha(\theta) &= C^2 + 2A'_\varphi(\theta < \theta_c)A'_\varphi(\theta > \theta_c). \end{aligned}}$$

$\alpha(\theta) = I(\theta)$

Topological Transport

Link with the Apple

$(R_x, R_y) = (\theta, \varphi)$
$= (k_{\parallel}, k_{\perp})$

$$H_{\parallel} = \frac{(\hbar k_{\parallel})^2}{2m} + qV - \mathbf{d} \cdot \boldsymbol{\sigma}.$$

Newton :

$$\boxed{\theta(t) = k_{\parallel}(t) = \frac{q}{\hbar} E t.}$$

$$J_{\perp} = \frac{q}{T} \int_0^T \frac{d\langle x_{\perp} \rangle}{dt} dt = \frac{q}{T} \left(\langle x_{\perp} \rangle (T) - \langle x_{\perp} \rangle (0) \right)$$

$$= \oint \left(J_{\varphi}(\varphi, T) - J_{\varphi}(\varphi, 0) \right) d\varphi,$$

Parseval
Plancherel

$$|J_{\perp}| = \frac{e}{T} A'_{\varphi}(\theta < \theta_c)$$

$$J_{\varphi}(\varphi, \theta) = \frac{iq}{4\pi T} \left(\psi^* \frac{\partial}{\partial \varphi} \psi - \frac{\partial \psi^*}{\partial \varphi} \psi \right) = \frac{iq}{2\pi T} \psi^* \frac{\partial}{\partial \varphi} \psi,$$

$$|J_{\perp}(T)| = \frac{e}{T} C.$$

$\theta_c \to \pi$

$A'_{\varphi}(\theta_c^{+}) = 0$

$$|J_{\perp}| T = \frac{e\hbar (k_{\perp} T)}{m} = eC = \Delta P$$

General relations applicable to many-body physics

$$\Delta P = eC = \int_0^T dt j(t).$$

$$\begin{cases} \theta(t) = k_{\parallel}(t) \\ \varphi = k_{\perp} \end{cases}$$

$$C = \frac{1}{2\pi} \iint dk_{\parallel}\, dk_{\perp}\, F_{k_{\parallel} k_{\perp}}$$

$$j = J_{\perp} = -\frac{e}{2} \frac{\partial \langle \sigma_z \rangle}{\partial t}$$

$$\hbar \dot{\mathbf{k}} = e\mathbf{E} = \vec{F}$$

$$\mathbf{C} = \int dt \frac{e}{\hbar} \mathbf{E} \times \mathbf{F}.$$

Also applicable for quantum Hall conductivity

On lattice (D. Thouless; Kohmoto, Niu,…)

Class III

Xiao, Chang, Niu, Rev. Mod. Phys. 2010

$$\mathbf{j} = \iint \frac{dk_x dk_y}{(2\pi)^2} \mathbf{j}(\mathbf{k}).$$

$$\mathbf{j}(\mathbf{k}) = \frac{e^2}{\hbar} \mathbf{E} \times \mathbf{F}.$$

$$|\mathbf{j}| = \frac{e^2}{h} \iint |(d\mathbf{k} \times \mathbf{F}) \cdot \mathbf{E}| = \frac{e^2}{h} C |\mathbf{E}|,$$

$$\boxed{\mathbf{v} = \frac{e}{\hbar} \mathbf{E} \times \mathbf{F}.}$$

$$\boxed{\sigma_{xy} = \frac{e^2}{h} C.}$$

Karplus-Luttinger velocity 1954

Anomalous Hall effect in materials

Nozieres and Lewiner, 1973

Nagaosa et al. Rev. Mod. Phys. 82, 1539 (2010)

Mapping onto Laughlin cylinder

$G = \sigma_{xy}$

Conductance quantum e^2/h
Landauer, Büttiker, Imry

$r = 1$

$$\boxed{\begin{aligned} z &= \cos\theta \\ \varphi &= \varphi \\ A_\varphi &= -\frac{z}{2} \end{aligned}}$$

$$F(\varphi, z) = F(\theta = \tfrac{\pi}{2}, \varphi) = \frac{1}{2}$$

$$C = \frac{1}{2\pi} \iint d\varphi\, dz \cdot F(\varphi, z)$$

$$z \in \left[-\frac{H}{2}, \frac{H}{2}\right]$$

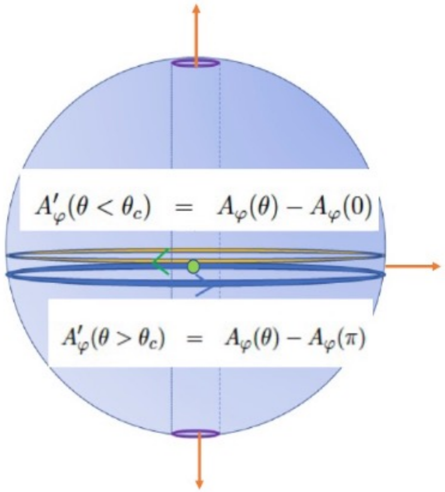
$$A_\varphi(\pi) - A_\varphi(0) = 1$$

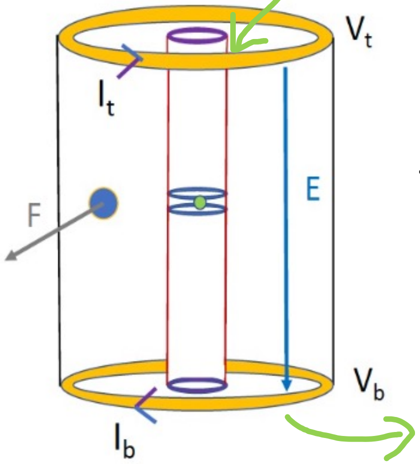
$$A_\varphi(0) = -\frac{1}{2}$$

$$G = \frac{e^2}{h}$$

$$H = 2$$

$$A_\varphi(\pi) = \frac{1}{2}$$





- $-H \cdot E = V_t - V_b$
- $T = \dfrac{h\pi}{eE}$
- $J_\perp = (eC)\dfrac{1}{T} = \dfrac{e^2}{h}(V_t - V_b)$

Topological kinetic Energetics: Gain!

Karyn Le Hur, Review arXiv:2209.15381

We can relate the transverse current to momentum

$$J_\perp = e\langle v_\perp\rangle = \frac{e\hbar k_\perp}{m} = \frac{\hbar \varphi}{m} e$$

$$E_{kin} = \frac{(\hbar k_\perp)^2}{2m}$$

$$\boxed{E_{kin} = \frac{1}{2m}(\hbar k_\perp)^2 = \frac{m}{2T^2}C^2.}$$

We can also define an averaged kinetic energy

$$\bar{E}_{kin} = \frac{1}{\pi}\int_0^\pi \frac{m}{2}\frac{(eE)^2}{\hbar^2}\frac{\sin^4\frac{\theta}{2}}{\theta^2}d\theta \approx \frac{\pi m}{T^2}\frac{C^2}{8},$$

which is slightly reduced but comparable to E_{kin}.

Fourier Series : See Review

Topological response can be reproduced from an electric field perpendicular to E (semiclassical approach)

$$\mathcal{J}_\perp(\theta) = \alpha\theta \quad \text{for} \quad \theta \in [0;\pi]$$
$$\mathcal{J}_\perp(\theta) = -\alpha\theta + \alpha 2\pi \quad \text{for} \quad \theta \in [\pi;2\pi].$$

$$\mathcal{J}_\perp(\theta) = f_0 + \sum_{n=1}^{+\infty} a_n(-1)^n\cos(n\theta) = \frac{e}{T}A'_\varphi(\theta<\theta_c).$$

Quantum Dynamo Effect, Many-Body physics

$$J(\omega) = \pi \sum_k \lambda_k^2 \delta(\omega - \omega_k);$$

$$= 2\pi\alpha\omega e^{-\omega/\omega_c}$$

$$\boxed{\mathcal{H}_{diss} = \sigma^z \sum_k \frac{\lambda_k}{2}(b_k + b_k^\dagger) + \sum_k \omega_k \left(b_k^\dagger b_k + \frac{1}{2} \right)}$$

Caldeira-Leggett

$$|g\rangle = \frac{1}{\sqrt{p^2+q^2}} \left[p e^{-i\phi} |\uparrow_z\rangle \otimes |\chi_\uparrow\rangle + q |\downarrow_z\rangle \otimes |\chi_\downarrow\rangle \right].$$

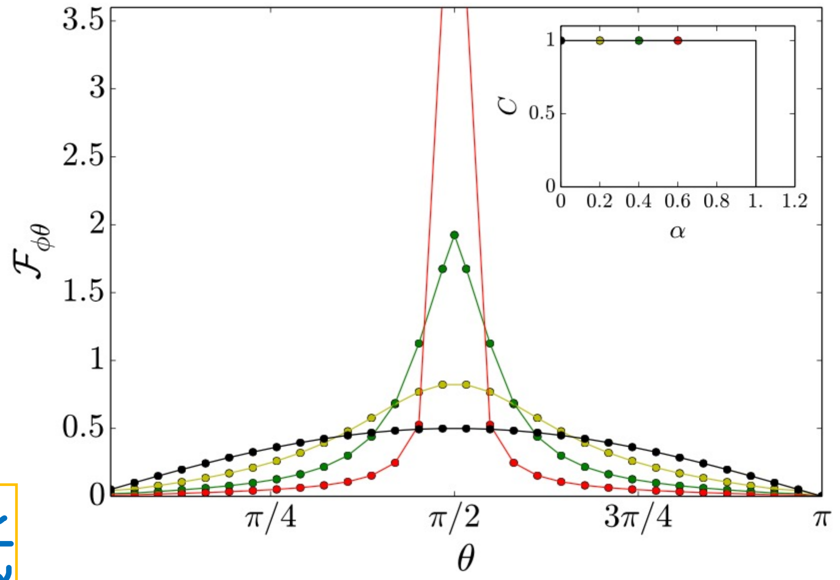
Role of a Quantum universe

$$\mathcal{A}_\phi = \langle g | i\partial_\phi | g \rangle = \frac{p^2}{p^2+q^2}.$$

$$\mathcal{F}_{\phi\theta} = -\partial_\theta \langle \sigma^z \rangle / 2.$$

$$\langle \sigma^z \rangle = (p^2 - q^2)/(p^2 + q^2)$$

$$\boxed{\mathcal{F}_{\phi\theta = \frac{\pi}{2}} = F(\alpha) \left(\frac{\omega_c}{d} \right)^{\frac{\alpha}{1-\alpha}}}$$



Quantum Phase Transition

$$\alpha_c = 1$$

Kondo model (1964), Ising model
Spin localized in 1 state: $|\chi_\uparrow\rangle$ & $|\chi_\downarrow\rangle$
become orthogonal

L. Henriet, A. Slocchi, P. P. Orth, K. Le Hur, Phys. Rev. B 95, 054307 (2017)

Can we produce a quantum dynamo through drive?

$$\mathcal{H}_{\text{single-mode}} = \frac{H}{2}\cos(vt)\sigma^z + \frac{H}{2}\sin(vt)\sigma^x + \frac{\lambda}{2}\sigma^z(b+b^\dagger) + vb^\dagger b\,.$$

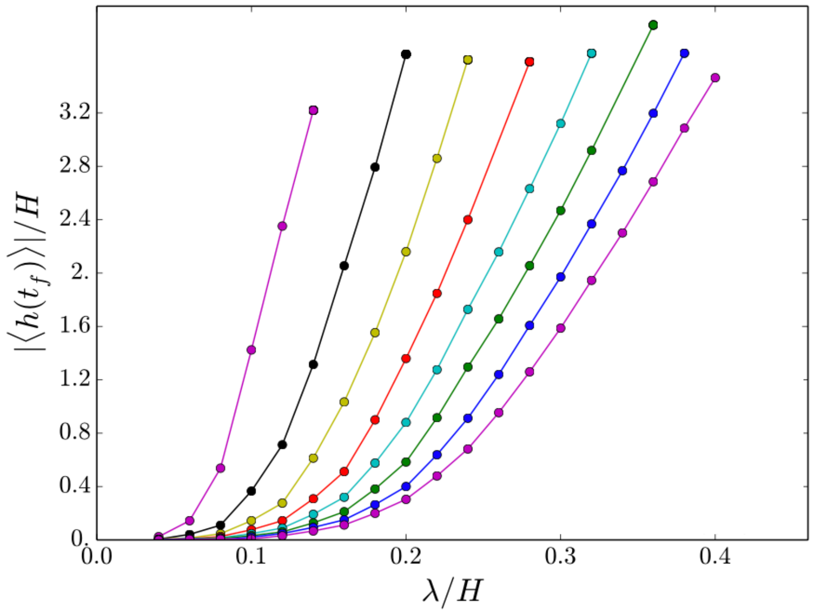
Equilibrium:
shifted modes
$b \mapsto b + \frac{\lambda}{2v}\sigma_z$

$$\partial_t \boldsymbol{\sigma} = \boldsymbol{H} \times \boldsymbol{\sigma},$$

$$\frac{1}{v^2}\partial_t^2 h_{\text{ind}} + h_{\text{ind}} = -\frac{\lambda^2}{v}\langle\sigma_z(t)\rangle,$$

$h_{\text{ind}} = \lambda\langle b + b^+\rangle$

yes, we can!



Mathematical details: New Insight on quantum thermo!

Long Work: Ephraim Bernhardt, Cyril Elouard, Karyn Le Hur Phys. Rev. A 107, 022219 (2023)

$$\mathcal{H} = \mathcal{H}_S(t) + SR + \mathcal{H}_R \qquad (1)$$

where

$$\mathcal{H}_R = \sum_k \omega_k b_k^\dagger b_k,$$

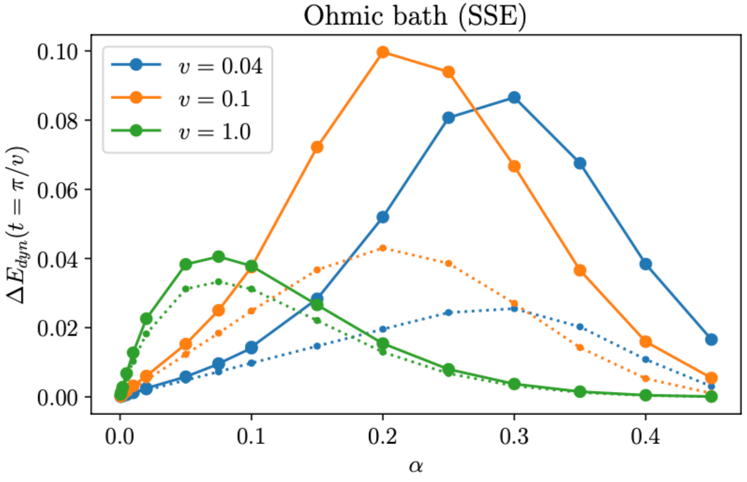
$$R = \sum_k g_k (b_k + b_k^\dagger).$$

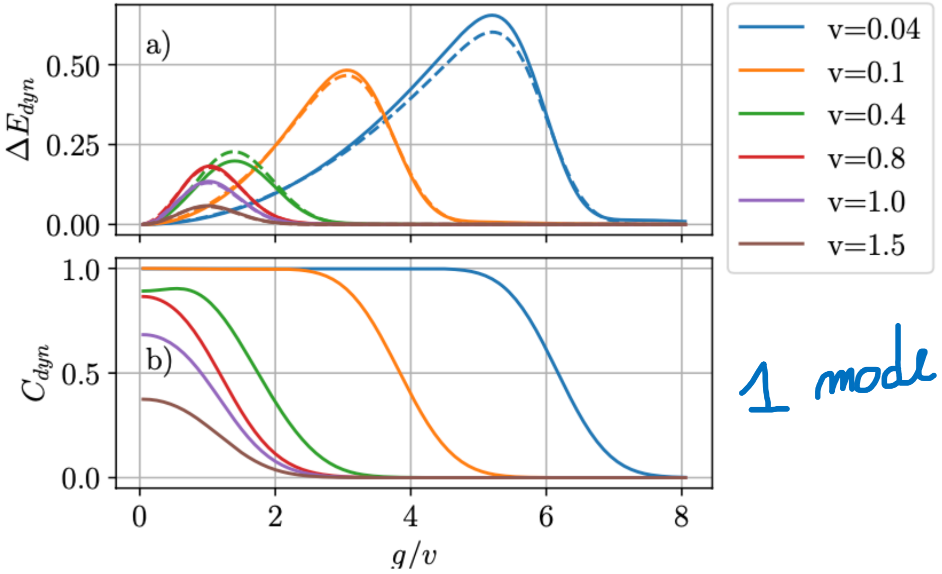
$$W_{\rm dr}(t) = \int_0^t dt' \left\langle \frac{\partial \mathcal{H}(t')}{\partial t} \right\rangle.$$

$$\boxed{\eta = \frac{\Delta E_{\rm dyn}(t)}{W_{\rm dr}(t)}.}$$

$$\boxed{\begin{aligned} W_{\rm dr}(t) &= \Delta E_S + \Delta E_{\rm dyn} + \Delta E_{\rm fluct}. \\ E_{\rm dyn}(t) &= \sum_k \omega_k \left| \langle b_k(t) \rangle + \frac{g_k}{\omega_k} \langle S(t) \rangle \right|^2. \end{aligned}}$$

Quantum Wheel

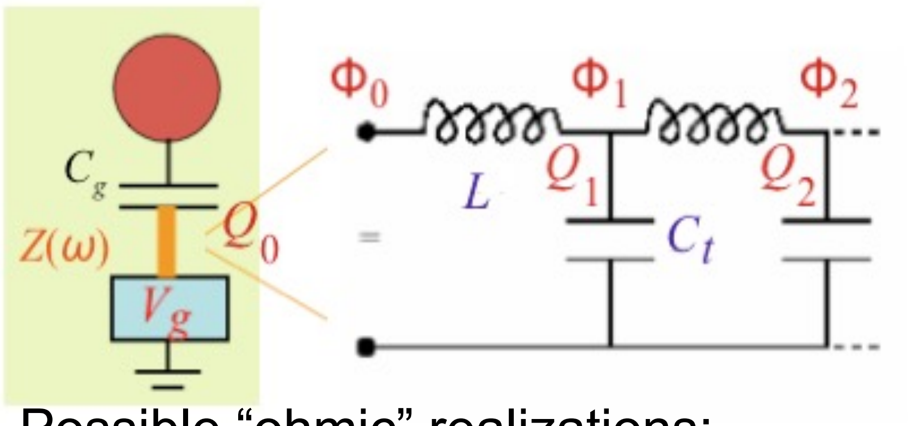




1 mode

Stochastic Schrodinger Equation Method: initiated with P. P. Orth and A. Imambekov; PhD thesis of L. Henriet & E. Bernhardt

Implementations/Realizations…



Transmission line

$J(\omega) \propto R\omega$

Possible "ohmic" realizations:

R. Schoelkopf et al, (2002)

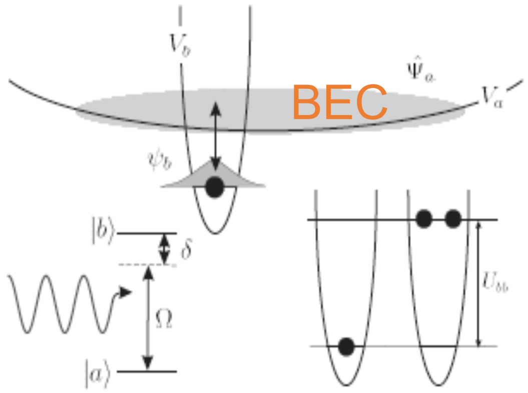
Makhlin et al. Rev. Mod. Physics 73, 357 (2001)

K. Le Hur PRL **92**, 196804 (2004)

M.-R. Li, K. Le Hur, W. Hofstetter, PRL **95**, 086406 (2005)

P. Cedraschi and M. Büttiker

Annals of Physics **289**, *1-23 (2001)*

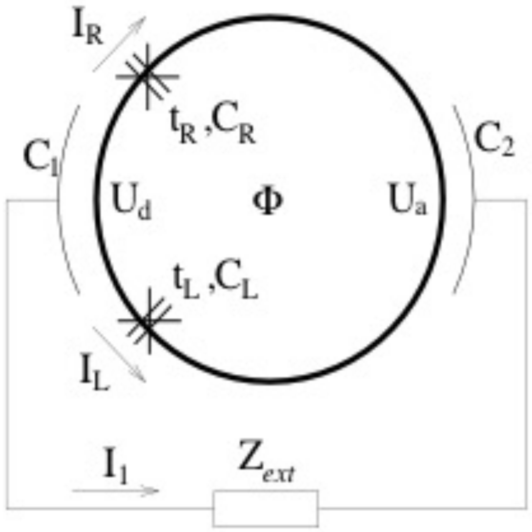


Persistent current

$$I(\alpha) \propto \langle S_x \rangle$$

H. Bouchiat, B. Reulet,…

J. Harris et al.



Cold Atomic Analogue

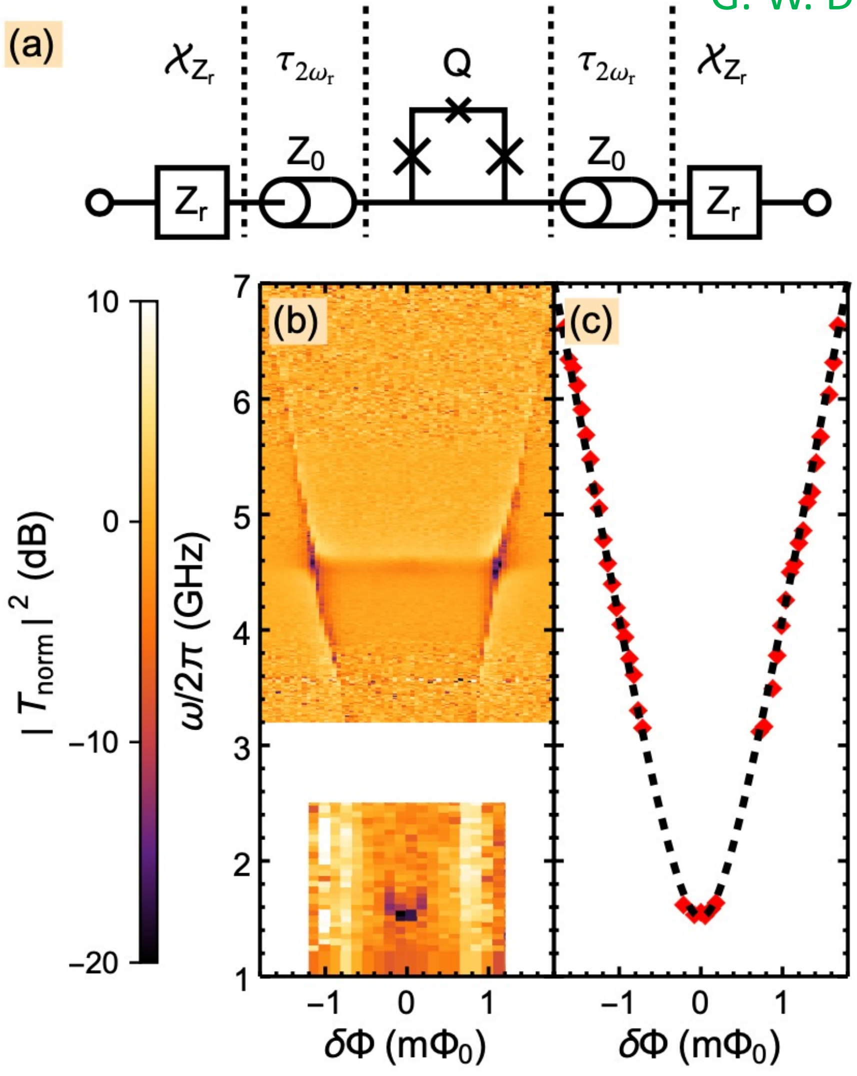
P. Zoller et al. PRL **94**, 040404 (2005)

Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)

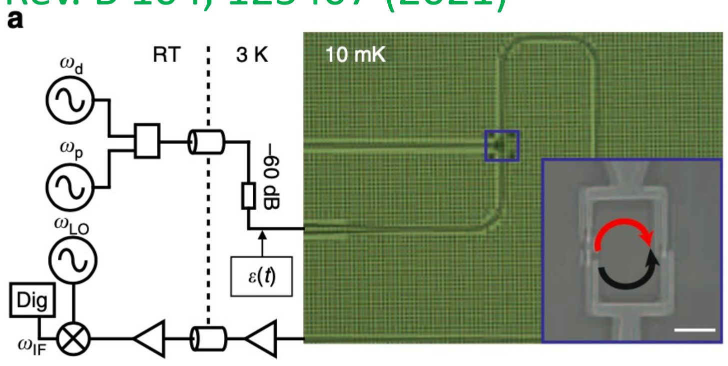
Recent News in circuit Quantum ElectroDynamics

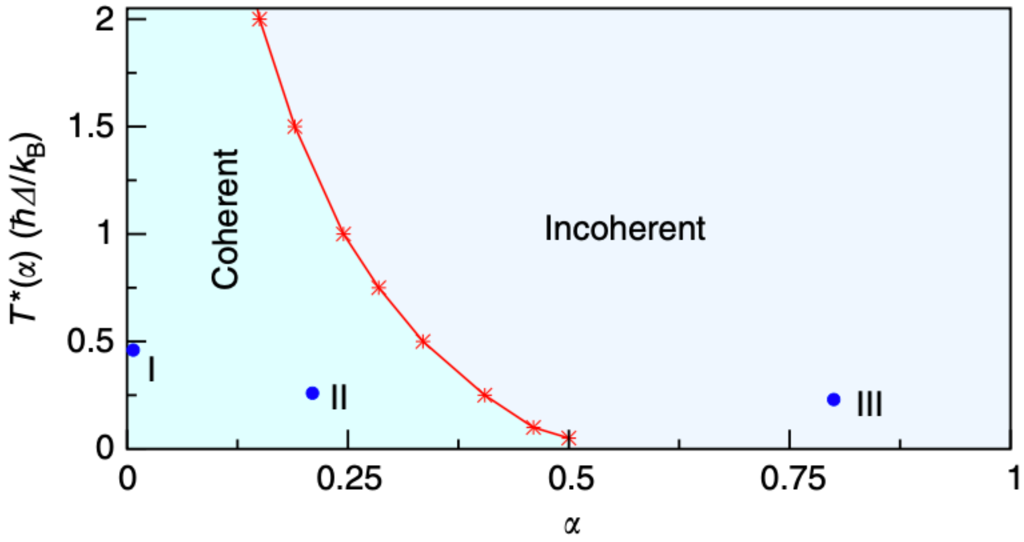
Karyn Le Hur, Phys. Rev. B 85, 140506 (2012)

G.-W. Deng, L. Henriet, Da Wei et al. Phys. Rev. B 104, 125407 (2021)



M. Haeberlein et al. arXiv:1506.0911





L. Magazzu et al. Nature Communications 9, 2298 (2018)

See also S. Leger et al. Nature Communications 10, 5259 (2019)