NonLinear Quantum Transport
Quantum Impurities
and Circuit Quantum Electrodynamics

ÉCOLE POLYTECHNIQUE
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cnrs
dépasser les frontières

Trieste July 2014

School on nonlinear Quantum Dynamics

Outline of the Presentation

Quantum Impurities
Dissipation and Dynamics

Circuit Quantum Electrodynamics:
Stochastic Schrodinger Approach
(drive, baths)

NonLinear Quantum Transport:
Brownian motion out of equilibrium

Sample two-state systems

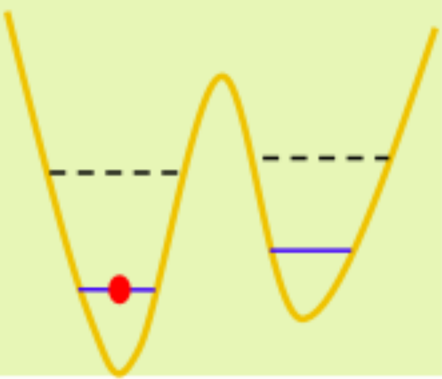
- Intrinsic two-state

Nuclei spin S=1/2

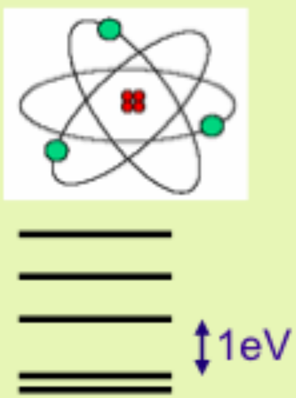
Polarization of photon (electromagnetic cavity)

- Truncated two-state

Particle in double well



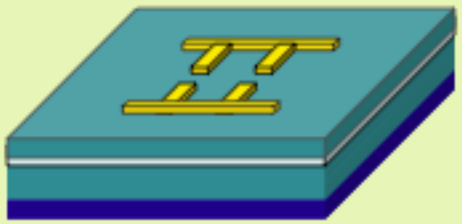
Atom



Condensed matter systems

- rf SQUID

- Quantum dot

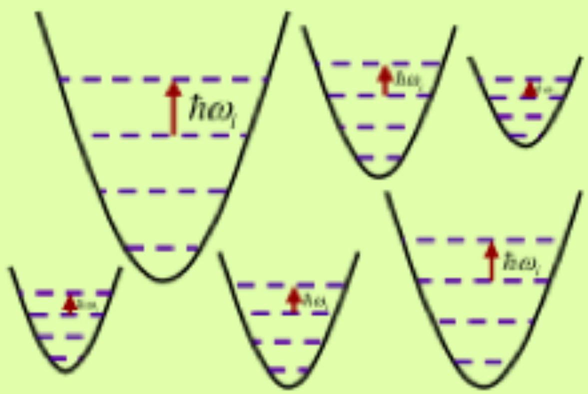


ROLE OF DISSIPATION?

Spin-boson model; analogue of Caldeira-Leggett (CL) problem

- Model the environment by quantum harmonic oscillators

$$H_B = \sum_i \left(\frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2 x_i^2}{2} \right)$$



Bosonic bath

s = 1 ohmic case

$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987)

U. Weiss book, quantum dissipative systems, 1999

$$\frac{1}{2}\left\langle \sum_i \lambda_i x_i(t) \cdot \sum_i \lambda_i x_i(0) \right\rangle_\omega = \hbar J(\omega) \coth(\omega / 2k_B T)$$

Ohmic dissipation

$$J(\omega) = \alpha \pi \hbar \omega^s / 2$$

Dissipation strength

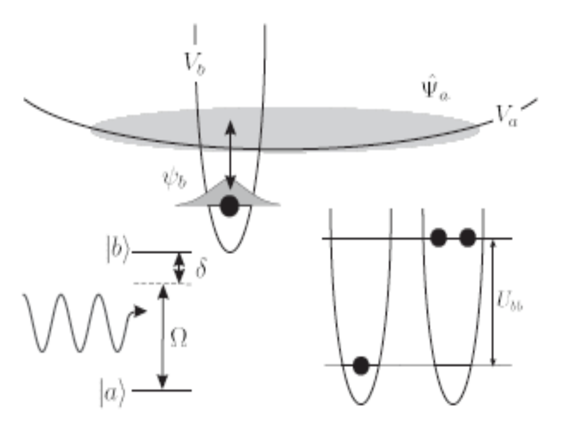
s = 1 ohmic case

s < 1 sub-ohmic situation

s > 1 super-ohmic situation

COLD-ATOMIC Quantum IMPURITIES

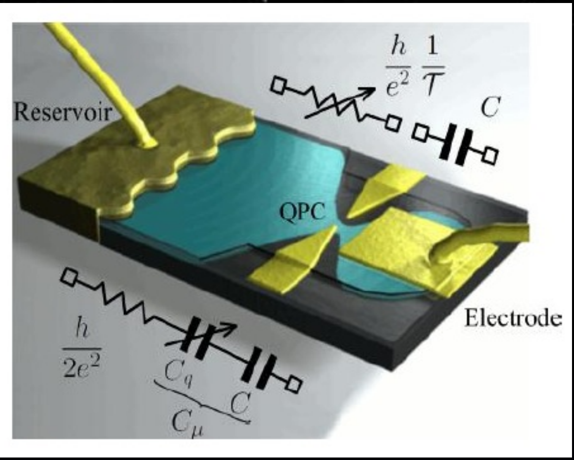
A. Recati et al. PRL **94**, 040404 (2005)
Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)
Single Atom: Ph. Grangier et al. Science **309**, 454 (2005)
A. Fuhrmanek, Y. R. P. Sortais, P. Grangier, A. Browaeys
Phys. Rev. A 82, 023623 (2010).
D. Porras, F. Marquardt, J. von Delft, J. I. Cirac (2007),...
M. Knap et al. Phys. Rev. X 2, 041020 (2012)
M. Knap, D. A. Abanin, E. Demler, PRL 111, 265302 (20
J. Bauer, C. Salomon, E. Demler PRL 111, 215304 (2013)



Talk by E. Demler
Dicke model: lecture J. Keeling

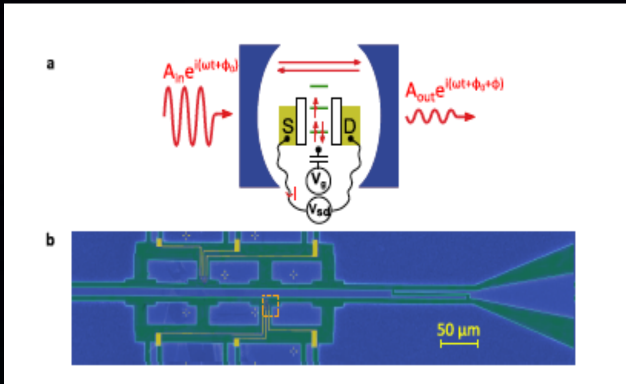
RC circuits

M. Buettiker, H. Thomas, and A. Pretre, Phys. Lett. A 180, 364 - 369,(1993)
J. Gabelli *et al.*, Science **313**, 499 (2006); G. Feve et al. 2007 (LPA ENS)
J. Gabelli et al. Rep. Progress 2012
C. Mora and K. Le Hur, Nature Phys. 6, 697 (2010)
Y. Hamamoto, et al. Phys. Rev. B **81**, (2010) 153305
Y. Etzioni, B. Horovitz, P. Le Doussal, PRL **106**, 166803 (2011)
M. Filippone, KLH, C. Mora; P. Dutt, T. Schmidt, C. Mora, KLH, 2013



Hybrid Photon-Nano Systems, Impurities with Photons

K. Le Hur, Phys. Rev. B 85, 140506(R) (2012)
A. Leclair, F. Lesage, S. Lukyanov and H. Saleur (1997)
M. Goldstein, M. H. Devoret, M. Houzet and L. I. Glazman, 2012
Grenoble: S. Florens, H. Baranger, N. Roch and collaborators
M. Hofheinz et al. arXiv:1102.0131
M. Delbecq et al. PRL 107, 256804 (2011)
M. Schiro & KLH, arXiv 1310.8070, PRB 2014

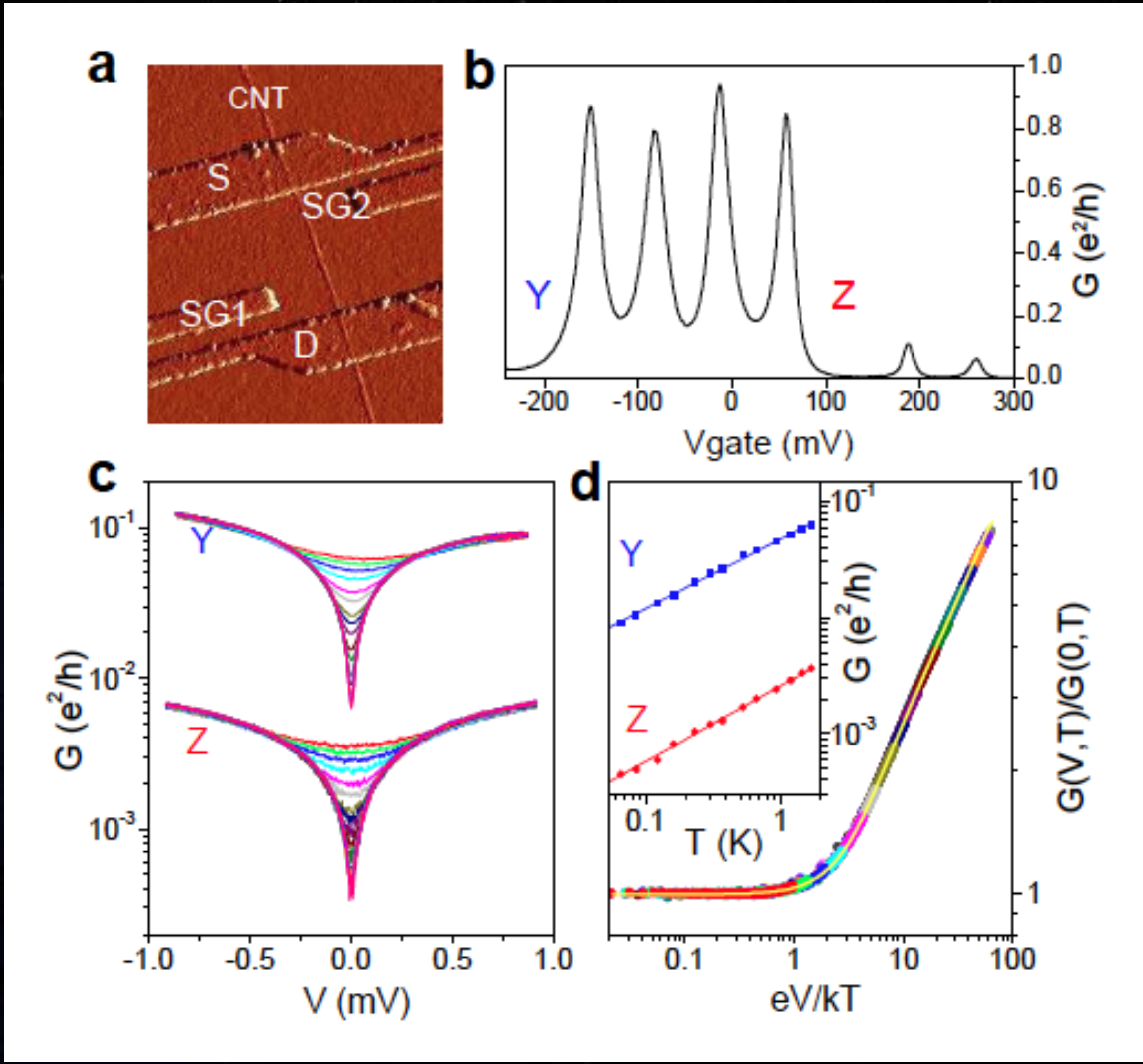


Collaboration with
C.-H. Chung, P. Woelfle,
M. Vojta, G. Finkelstein
PRL 2009, PRB 2013

H. T. Mebrahtu, I. V. Borzenets, H. Zheng, Y. V. Bomze, A. I. Smirnov, S. Florens, H. U. Baranger, G. Finkelstein
Nature Physics, 9 732 (2013)

similar experiments
at LPN Marcoussis
F. Pierre group

Theory: I. Safi &
M. Albert

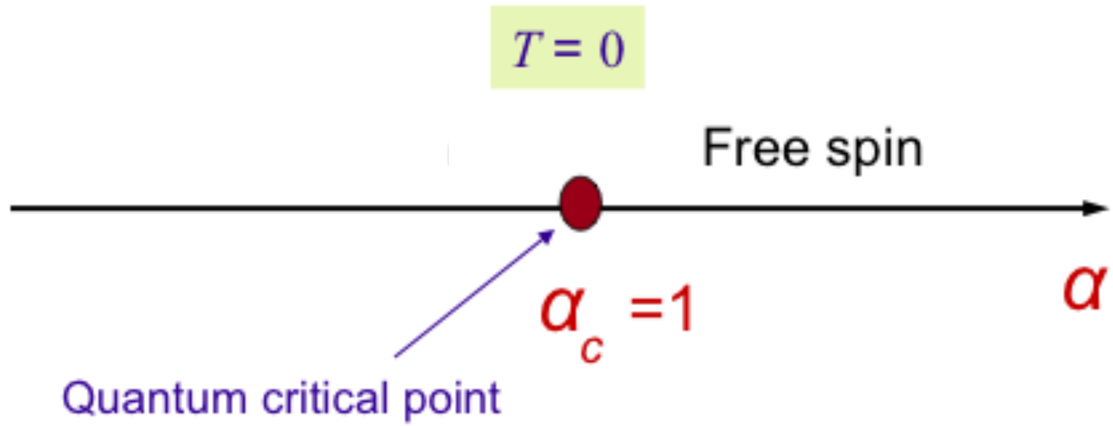


Quantum phase transition in CL model

$$\frac{1}{2}\left\langle \sum_i \lambda_i x_i(t) \cdot \sum_i \lambda_i x_i(0) \right\rangle_\omega = \hbar J(\omega)\coth(\omega / 2k_B T)$$

Ohmic dissipation

$$J(\omega) = \underbrace{\alpha}_{\text{Dissipation strength}} \pi \hbar\omega / 2$$



KLH, chapter in book « Understanding quantum phase transitions », ed. L. Carr, arXiv:0909.4822

- Classical phase transition
  tuned by temperature

  liquid-solid
  liquid-gas
  …

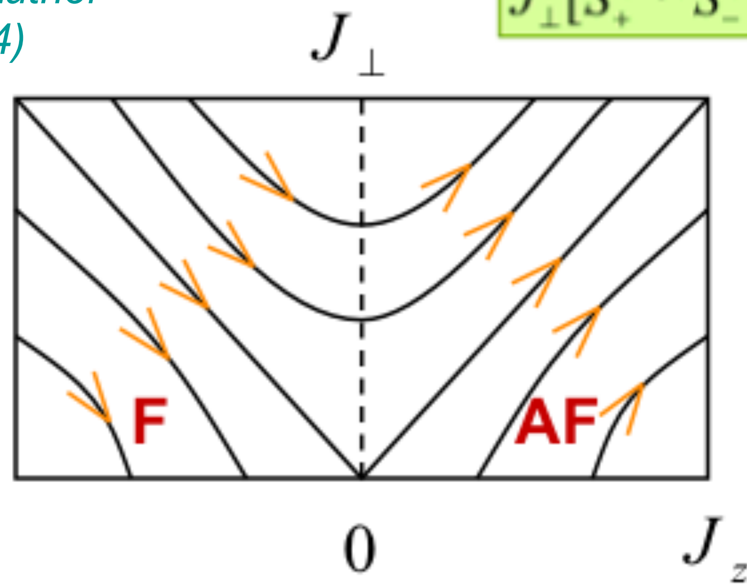
- Quantum phase transition at $T = 0$
  tuned by intrinsic parameters
  - *Metal-insulating*
    in 2DEG
  - *Insulating-superconducting*
    in high-temperature superconducting cuprates
  - *Spin ordered - disordered*
    in quantum spin models

Analogy to another quantum impurity Kondo problem

V.J. Emery & A. Luther
PRB 9, 215 (1974)

$$J_\perp [S_+^{(imp)} S_-^{(e)} + h.c.] + J_z S_z^{(imp)} S_z^{(e)}$$

Perturbative calculations

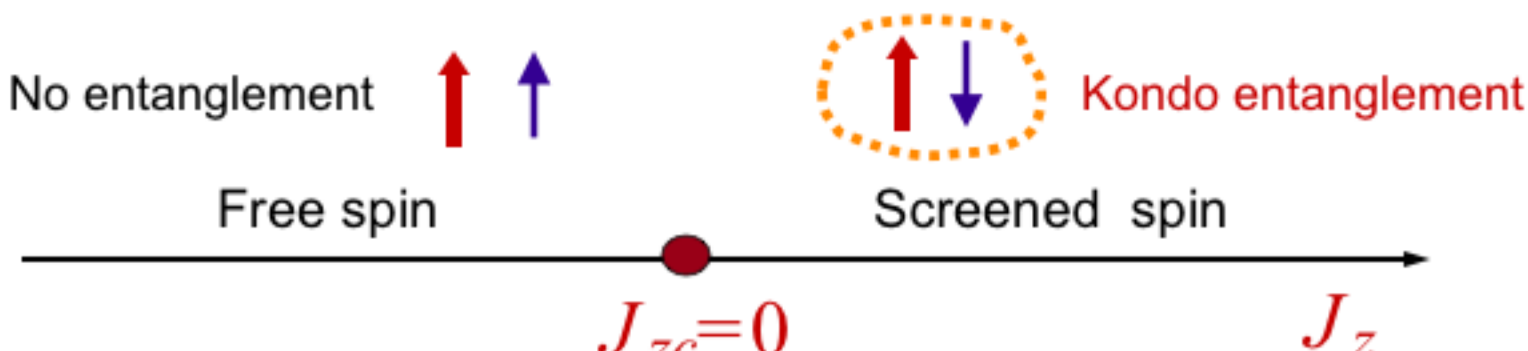


$$\mathbf{S}^e\,(\mathbf{x}=0) = \sum_{\substack{\mathbf{k},\mathbf{k}'\\ \alpha,\beta}} \mathbf{c}_{\mathbf{k}\alpha}^{\dagger} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} \mathbf{c}_{\mathbf{k}'\beta}$$

Bethe Ansatz:
See N. Andrei; A. Tsvelik; P. Wiegmann
CFT: Affleck-Ludwig

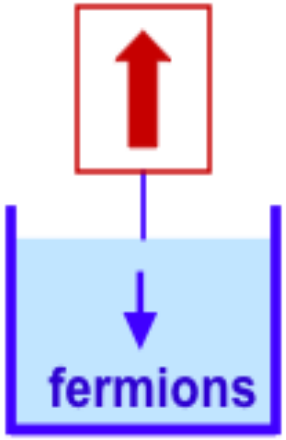
See also K. Wilson, NRG
Ph. Nozières, **Fermi liquid**, 1974
Coqblin-Schrieffer

Small $J_\perp$



$$H_{Kondo} = hS_z^{(imp)} + J_\perp[S_+^{(imp)}S_-^{(e)} + h.c.] + J_z S_z^{(imp)} S_z^{(e)} + H_e$$

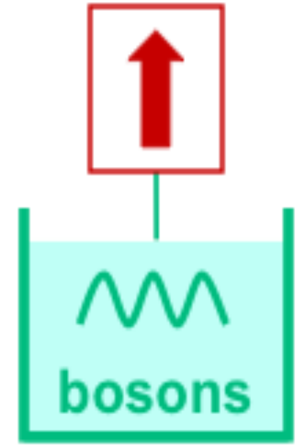
$$H_{CL} = \Delta S_x + hS_z + S_z \sum_i \lambda_i x_i + H_B$$



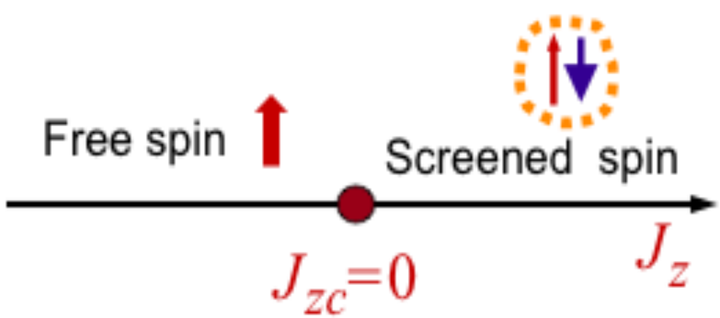
α = ½ corresponds to Toulouse limit

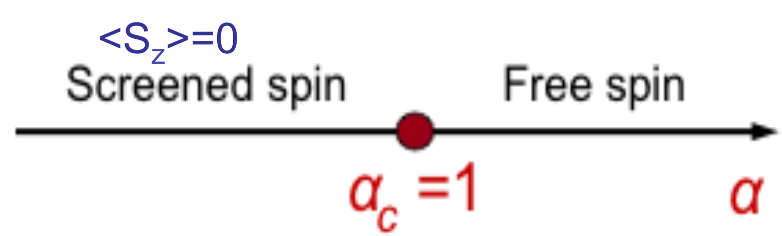
Free Resonant level

"Simple calculations"



$$\boxed{\begin{aligned} J_z &\propto 1-\sqrt{\alpha} \\ J_\perp &\propto \Delta \end{aligned}}$$





S. Chakravarty PRL 49, 681 (1982); A.J. Bray and I.A. Moore PRL 49, 1545 (1982)

Guinea, Hakim, Muramatsu PRB 1986

Kosterlitz-Thouless transition:

2D XY models: Superconductors, ^{4}He, Cold atomic bosons

$$H = -J \sum_{<i;j>} \cos(\varphi_i - \varphi_j)$$

SC order parameter = $|\Psi|\exp(i\varphi)$

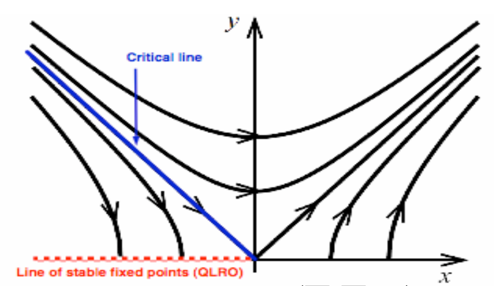
$$S_x + iS_y = \exp(i\varphi)$$

KT transition: High Temperature disordered phase (free vortices)

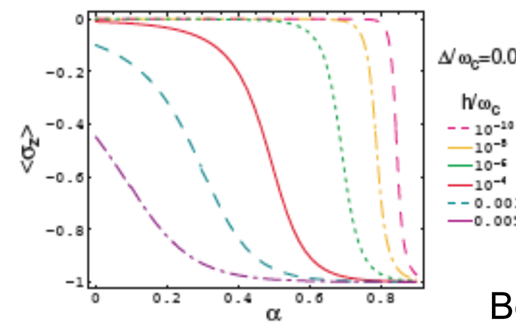
Low-Temperature quasi-long range order

Universal Jump of Superfluid density at T_{KT}

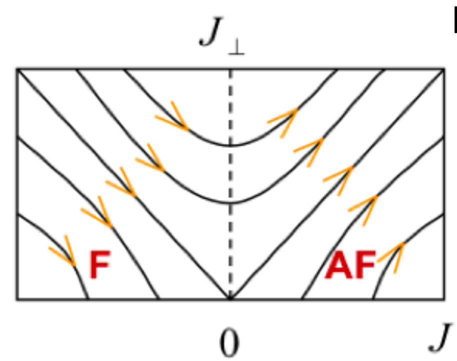
(vortex fugacity)



(T-T$_{KT}$)



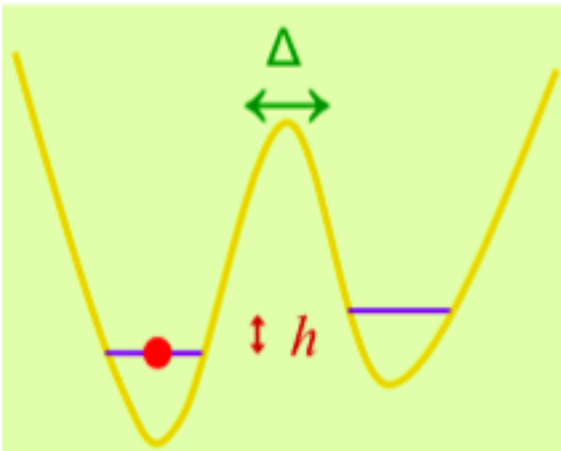
Bethe Ansatz
NRG

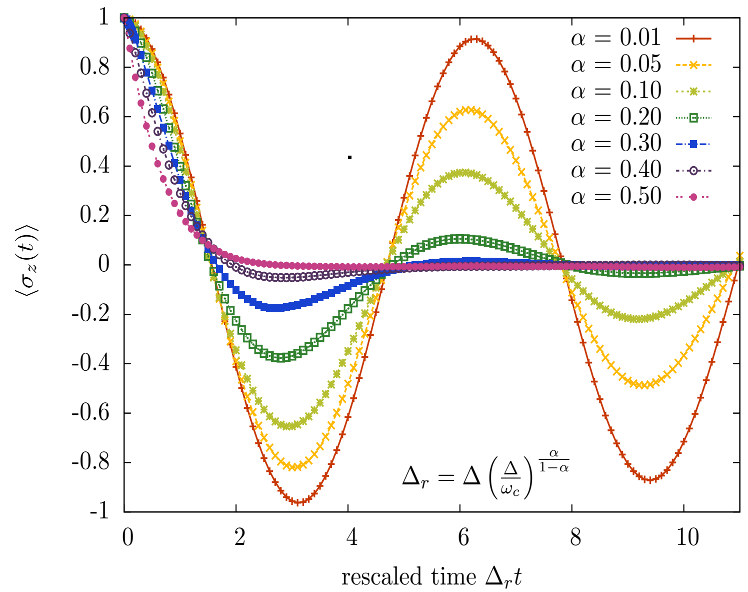


$h\theta(-t)S_z$

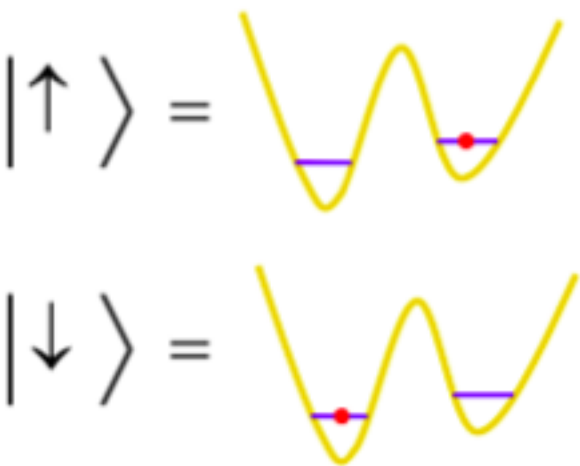
α=1/2:Dynamical crossover

P(0)=1





td-NRG



α=0: P(t) = 1/2(cos(Δ t)+1)
Small α: Master Equations

α=1/2: Loss of oscillations
(true for all h ≠0)
P(t) decreases as exp(-T_Kt)

$$H = -\frac{\Delta}{2}\sigma_x + \frac{h}{2}\sigma_z + \frac{1}{2}\sigma_z \sum_i c_i x_i + H_{osc}$$

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987)

To study the spin dynamics it is convenient to perform a polaronic transformation $U = \exp(-i\sigma_z\Omega/2)$ where $\Omega = \sum_i (c_i/m_i\omega_i^2)p_i$, such that the transformed Hamiltonian $H' = U^{-1}HU$ takes the precise form [1]

$$H' = -\frac{1}{2}\Delta\left(\sigma_+ e^{-i\Omega} + \sigma_- e^{i\Omega}\right) + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2}m_i\omega_i^2 x_i^2\right). \tag{1.11}$$

In the Heisenberg picture, the equations of motion for $\sigma_\pm(t)$ are easily obtained. Integrating and substituting them into the equation of motion for the transverse polarization $\sigma_z(t)$, then one gets the exact formula:

$$\dot{\sigma}_z(t) = -\frac{1}{2}\Delta^2 \int_{-\infty}^{t} \left(e^{-i\Omega(t)}e^{i\Omega(t')}\sigma_z(t') + \sigma_z(t')e^{-i\Omega(t')}e^{i\Omega(t)}\right) dt'. \tag{1.12}$$

On the other hand, to solve this equation, one usually uses approximations [17]. The first approximation generally consists to insert the free bath dynamics when computing the commutator:

$$[\Omega(t), \Omega(t')] = i\sum_j \left(\frac{c_j^2}{m_j\omega_j^3}\right) \sin(\omega_j(t-t')). \tag{1.13}$$

The next step is to average (1.12) with respect to the bath and to decouple the environmental exponentials from the spin. Using that:

$$\langle \Omega(t)\Omega(t') + \Omega(t')\Omega(t)\rangle = \sum_j \frac{c_j^2}{m_j\omega_j^3} \coth\left(\frac{1}{2}\beta\omega_j\right)\cos(\omega_j(t-t')), \tag{1.14}$$

this leads to the evolution equation [17]:

$P(t) = \langle\sigma_z(t)\rangle$

$$\dot{P}(t) + \int^{t} \mathcal{F}(t-t')P(t')dt' = 0, \tag{1.15}$$

where the function $\mathcal{F}$ obeys $\mathcal{F}(t) = \Delta^2 \cos(A_1(t)) \exp -(A_2(t))$, and

$$A_1(t) = \frac{1}{\pi}\int_0^{+\infty} \sin(\omega t)\frac{J(\omega)}{\omega^2}d\omega \tag{1.16}$$

$$A_2(t) = \frac{1}{\pi}\int_0^{+\infty} (1-\cos(\omega t))\coth\left(\frac{\beta\omega}{2}\right)\frac{J(\omega)}{\omega^2}d\omega.$$

Through the Laplace transform one obtains (C denotes a Bromwich contour):

$$P(t) = \frac{1}{2\pi i}\int_C d\lambda e^{\lambda t}\frac{1}{\lambda + \mathcal{F}(\lambda)}. \tag{1.17}$$

At zero temperature and in the scaling limit $\Delta/\omega_c \ll 1$, one finds [1]:

Many-body
Lamb shift: **important**

$$\mathcal{F}(\lambda) = \Delta_e\left(\frac{\Delta_e}{\lambda}\right)^{1-2\alpha}, \tag{1.18}$$

where $\Delta_e = \Delta_r\left(\cos(\pi\alpha)\Gamma(1-2\alpha)\right)^{\frac{1}{2(1-\alpha)}}$; we have introduced the renormalized transverse field $\Delta_r = \Delta(\Delta/\omega_c)^{\alpha/1-\alpha}$ which is proportional to the Kondo energy scale T_K. This expression of $P(t)$ coincides with the formula of $P(t)$ obtained via the Non-Interacting Blip Approximation (NIBA) [1].

For $\alpha \to 0$, one recovers perfect Rabi oscillations $P(t) = \cos(\Delta t)$ whereas for $\alpha = 1/2$ one gets a pure *relaxation* $P(t) = \exp -(\pi\Delta^2 t/(2\omega_c))$, which is in accordance with the non-interacting resonant level model [1]. For $0 < \alpha < 1/2$, the spin displays coherent oscillations (due to a pair of simple poles) leading to $P_{coh}(t) = a\cos(\zeta t + \phi)\exp(-\gamma t)$ with the quality factor [1]:

$$\frac{\zeta}{\gamma} = \cot\left(\frac{\pi\alpha}{2(1-\alpha)}\right). \tag{1.19}$$

This quality factor has also been found using conformal field theory [18].

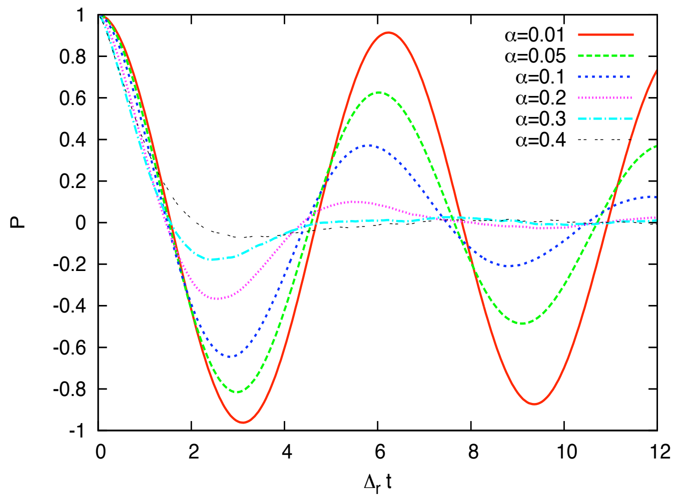
Results: Analytical Approach & tricky NRG numerics

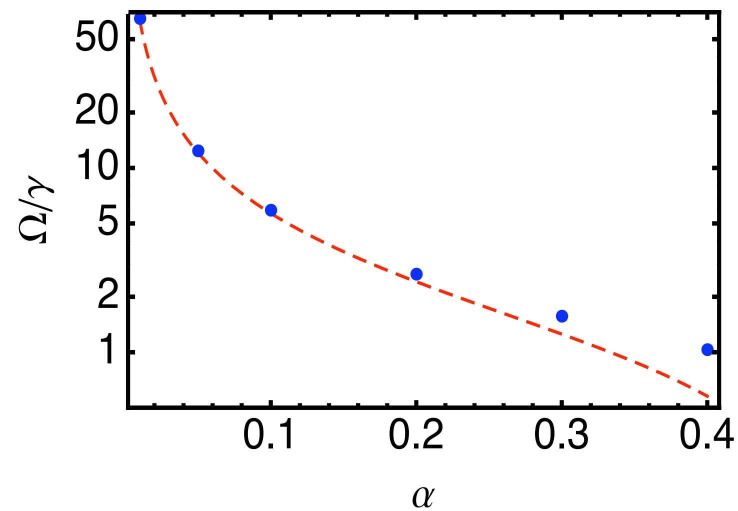
P. Orth, A. Imambekov, K. Le Hur, stochastic Equation, 2010, 2013

D. Roosen, P. Orth, K. Le Hur, W. Hofstetter, time-dependent NRG 2010

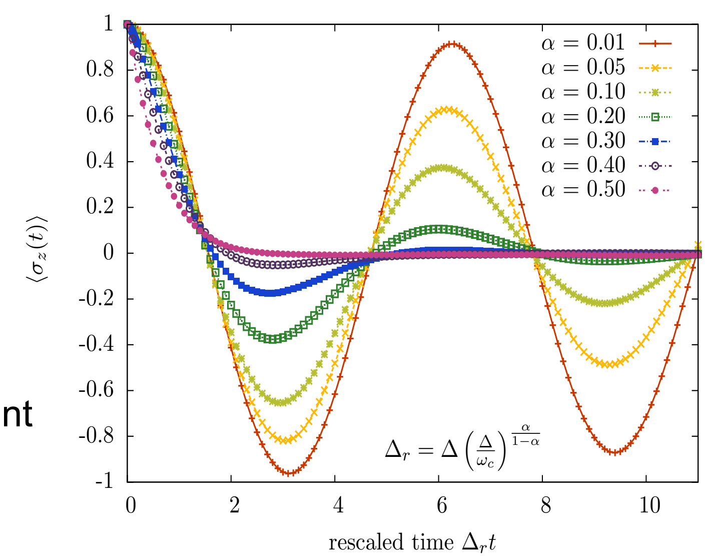
$P=\langle\sigma_z(t)\rangle$

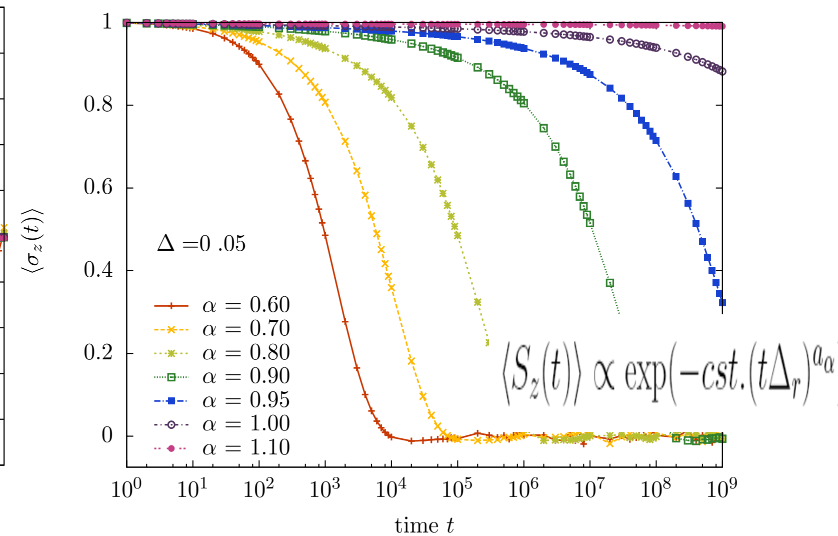
Stochastic Approach (See next)





Time-dependent Wilson NRG

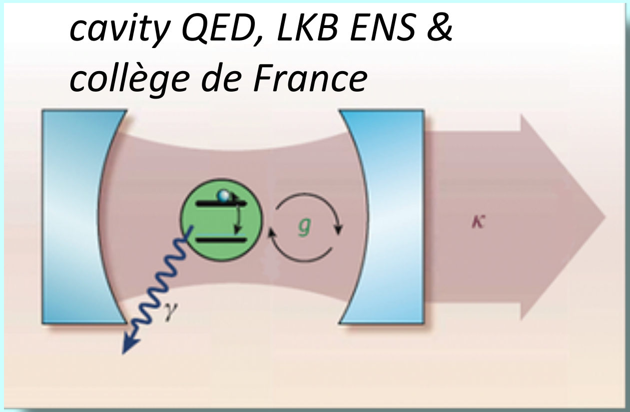




Cavity & Circuit QED: 1 mode of light …

Coupling atoms to the EM field

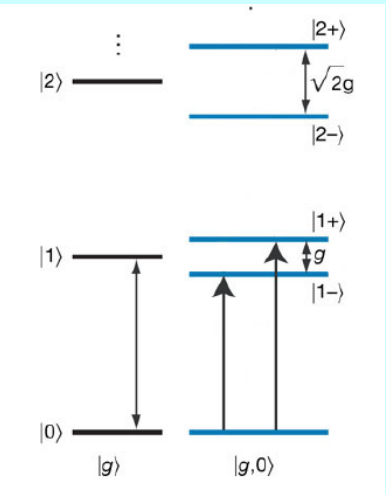
- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity



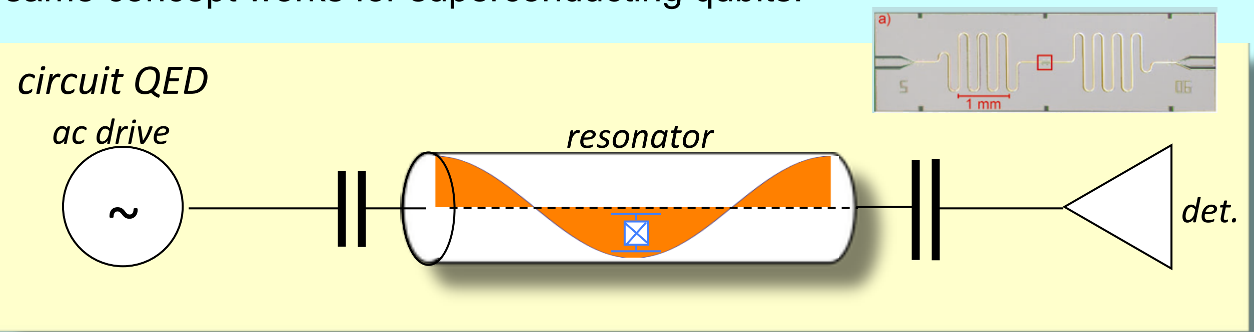
2g = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

Jaynes-Cummings Hamiltonian

$$H = \tfrac{1}{2}\omega_a\sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\text{drive}} + H_{\text{baths}}\right)$$



- same concept works for superconducting qubits!



J. M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. **73**, 565 (2001)
R. J. Schoelkopf, S. M. Girvin, Nature **451**, 664 (2008); D. Vion et al. (SPEC Saclay) 2002; J. Martinis …

Jaynes-Cummings Ladder

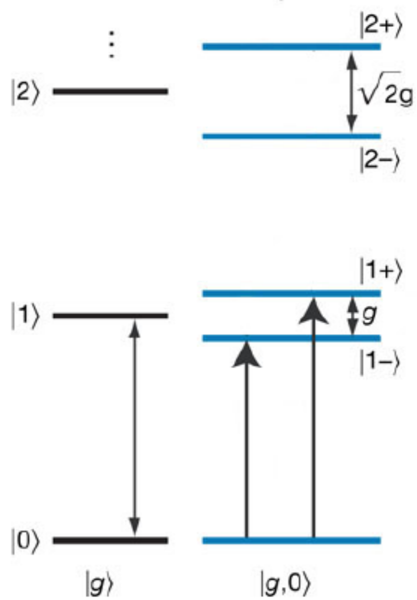
in the base $|n, +_z\rangle$ and $|n+1, -_z\rangle$

$$H = \begin{pmatrix} n\omega_0 + \frac{\Delta}{2} & \frac{g}{2}\sqrt{n+1} \\ \frac{g}{2}\sqrt{n+1} & (n+1)\omega_0 - \frac{\Delta}{2} \end{pmatrix}$$

We have the following eigenvalues and eigenstates ($N > 1$):

$$E_{N+} = N\omega_0 - \frac{\delta}{2} + \frac{1}{2}\sqrt{\delta^2 + Ng^2} \qquad |N+\rangle = \alpha_n|N-1, +_z\rangle + \beta_n|N, -_z\rangle$$

$$E_{N-} = N\omega_0 - \frac{\delta}{2} - \frac{1}{2}\sqrt{\delta^2 + Ng^2} \qquad |N-\rangle = -\beta_n|N-1, +_z\rangle + \alpha_n|N, -_z\rangle$$



Eigenstates are **Polaritons**

$$N = a^\dagger a + \frac{1}{2}(\sigma^z + 1)$$

$$\beta_N = \cos(1/2 \tan^{-1} \tfrac{g\sqrt{N}}{\delta}) \text{ and } \alpha_N = \sin(1/2 \tan^{-1} \tfrac{g\sqrt{N}}{\delta})$$

$\delta = \omega_0 - \Delta$ is the detuning

Photon Blockade: Photons go one by one nonlinearities

Driven & Dissipative **Rabi** Model in circ-QED $\left(\frac{g}{\omega_0} \simeq 10^{-1}\right)$

Coherent drive

$$H = \frac{\Delta}{2}\sigma^z + \omega_0 a^\dagger a + \frac{g}{2}\sigma^x (a + a^\dagger) + c(t)(a + a^\dagger)$$
$$+ \sum_k \omega_k a_k^\dagger a_k + \lambda_k (a_k + a_k^\dagger)\frac{\sigma^x}{2}.$$

The $U(1)$ symmetry of the JC model breaks down to a discrete Z_2 symmetry

Loic Henriet, Zoran Ristivojevic, Peter P. Orth, KLH 2014 (arXiv:1401.4558)

Recent Developments in the strong-coupling limit:

D. Braak: Analytical Solution of Rabi model, Phys. Rev. Lett. **107**,100401 (2011)

F. A. Wolf et al. Phys. Rev. A **87**, 023835 (2013)

A. Moroz, Ann. Phys. (N.Y.) **338**, 319-340 (2013)

M. Tomka et al. arXiv:1307.7876

P. Nataf and C. Ciuti PRL **104**, 023601 (2010)

M. Schiro et al. Phys. Rev. Lett. **109**, 053601 (2012) (Array situation)

Gaussian Bath: Feynman-Vernon path integral approach (1963)

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987); U. Weiss book, quantum dissipative systems, 1999

We integrate out the **BATH** (quadratic action) and follow the spin real-time dynamics

$$\langle \sigma_f | \rho_S(t) | \sigma'_f \rangle = \int \mathcal{D}\sigma(.) \int \mathcal{D}\sigma'(.) \mathcal{A}(\sigma) \mathcal{A}^*(\sigma') F[\sigma, \sigma']$$

The bath effect is all contained in the **INFLUENCE** FUNCTIONAL (connection to Ising models, Anderson-Yuval-Hamann and Dyson):

$$F[\sigma, \sigma'] = \exp\left(-\frac{1}{\pi} \int_{t_0}^{t} ds \int_{t_0}^{s} ds' \left[-iL_1(s-s')\xi(s)\eta(s') + L_2(s-s')\xi(s)\xi(s') \right] \right),$$

$$\boxed{\begin{aligned} \pi \langle X(t) X(0) \rangle_T &= L_2(t) - iL_1(t) \\ X &= \textstyle\sum_n \lambda_n (b_n^\dagger + b_n) \text{ + photon part} \end{aligned}}$$

$$\boxed{\begin{aligned} \eta(s) &= \frac{1}{2}[\sigma(s) + \sigma'(s)] \\ \xi(s) &= \frac{1}{2}[\sigma(s) - \sigma'(s)] \end{aligned}}$$

$$L_1(t) = \int_0^\infty d\omega J(\omega) \sin \omega t$$

$$L_2(t) = \int_0^\infty d\omega J(\omega) \cos \omega t \coth \beta\omega/2$$

Stochastic Method: Fast View

J(ω) spectral function of the environment (light & dissipative bath)

$$J(\omega) = \pi g^2 \delta(\omega - \omega_0) + 2\pi\alpha\omega e^{-\frac{\omega}{\omega_c}}$$

Trick: Decouple Interactions through Hubbard-Stratonovitch Transformation
(analogy to disorder averaging)

$$2\langle\sigma^x(t)\rangle - 1 = 2\langle\sigma^x(t)\rangle - 1 = \overline{\langle\Phi_f|Te^{-i\int_0^t dsW(s)}|\Phi_i\rangle}$$

$$W(t) = \frac{\Delta}{2}\begin{pmatrix} 0 & e^{-h_\xi+h_\eta} & -e^{h_\xi+h_\eta} & 0 \\ e^{h_\xi-h_\eta} & 0 & 0 & -e^{h_\xi+h_\eta} \\ -e^{-h_\xi-h_\eta} & 0 & 0 & e^{-h_\xi+h_\eta} \\ 0 & -e^{-h_\xi-h_\eta} & e^{h_\xi-h_\eta} & 0 \end{pmatrix}$$

$$\overline{h_\xi(t)h_\xi(s)} \propto Q_2(t-s)$$
$$\overline{h_\xi(t)h_\eta(s)} \propto iQ_1(t-s)$$

(vector = 4 states of spin reduced density matrix)

Dynamics of the Spin can be obtained via a Schrodinger Equation

$$i\partial_t|\Phi(t)\rangle = W(t)|\Phi(t)\rangle$$

See also G. B. Lesovik, A. V. Lebedev, A. Imambekov JETP Lett. 75, p. 474, (2002).
A. Imambekov, V. Gritsev, E. Demler, Phys. Rev. A 77, 063606 (2008).
J.T. Stockburger, H. Grabert Phys. Rev. Lett. 88, 170407 (2002).
Non-Markovian Approach

Note: This is a numerically exact Approach, Drive & Non-Markovian Effects captured
Little Price to Pay: Numerical Convergence
Different from J. Dalibard, Y. Castin, K. Molmer, Phys. Rev. Lett. **68**, 580 (1992)

Applications: **Landau-Zener problem** with dissipation
Peter Orth, Adilet Imambekov, Karyn Le Hur PRB 2010 and 2013

Dissipative Driven Rabi models: Loic Henriet, Zoran Ristivojevic, Peter P. Orth, KLH
Necessity to introduce 2 stochastic fields 2014 (arXiv:1401.4558 to appear)

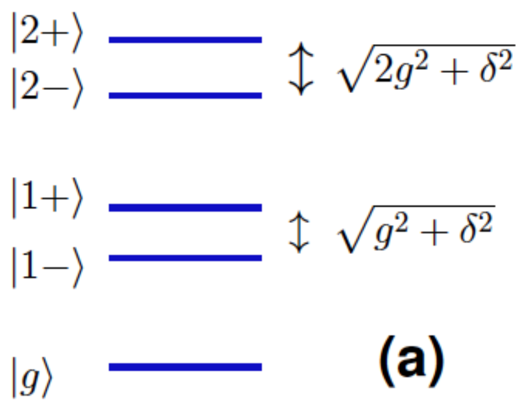
$$\rho_S(t_0) = |+_z\rangle\langle +_z|$$

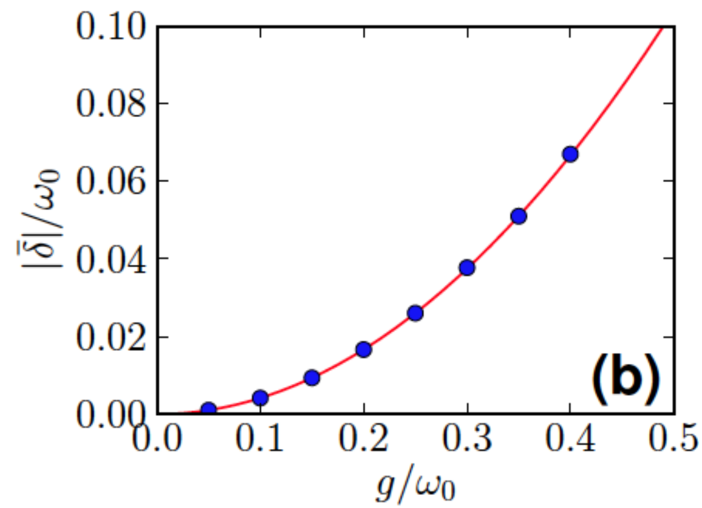
<u>JC case</u>: Rabi oscillations between 1- & 1+ states

$$\langle \sigma^z(t)\rangle = 1 - 2\sin^2\!\left(\sqrt{g^2+\delta^2}\,\tfrac{t}{2}\right)\left[1 - \cos^2\!\left(\tan^{-1}\tfrac{g}{\delta}\right)\right]$$

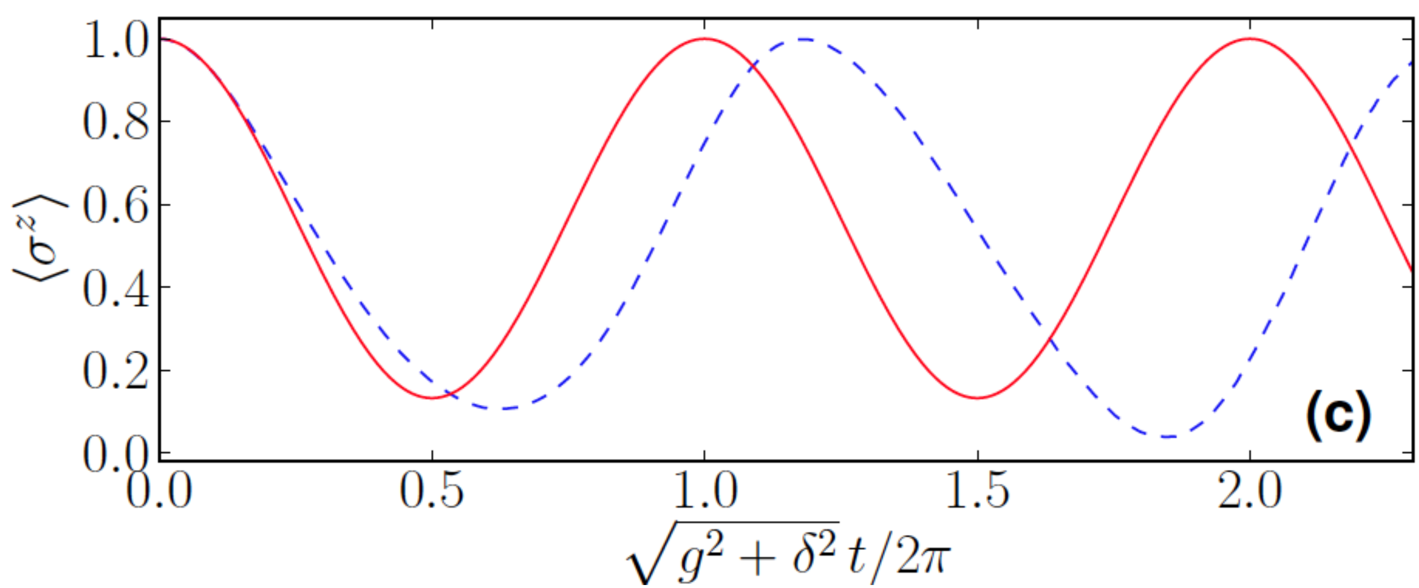
Bloch-Siegert shift

C. Cohen-Tanoudji 1968, Cargèse lectures
S. Haroche 1971



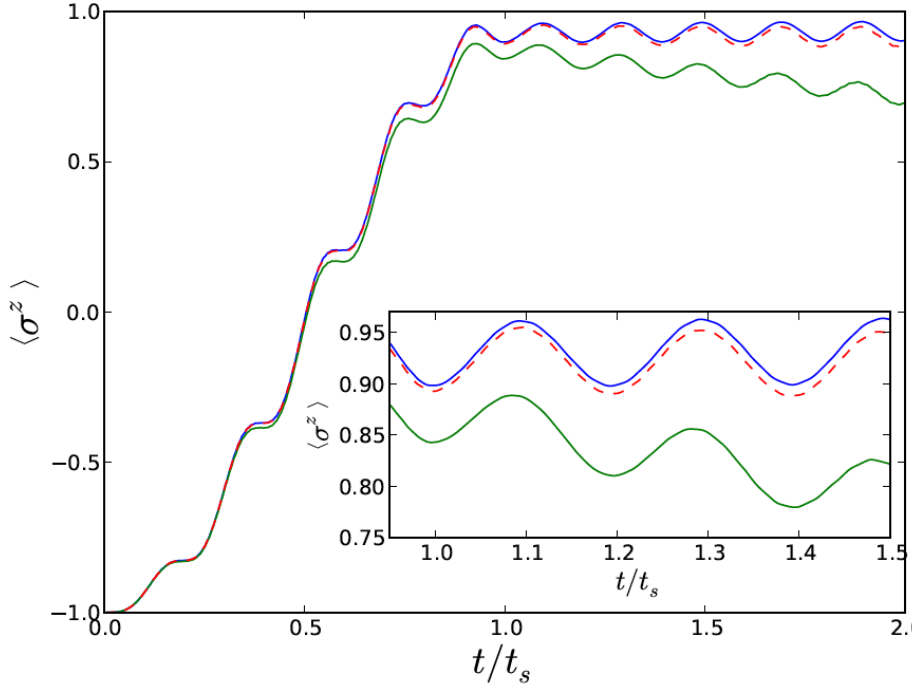


Rabi model in the strong coupling:
New features, not explained with polaritons



L. Henriet
Z. Ristivojevic
P. P. Orth, KLH, 2014
(arXiv:1401.4558)

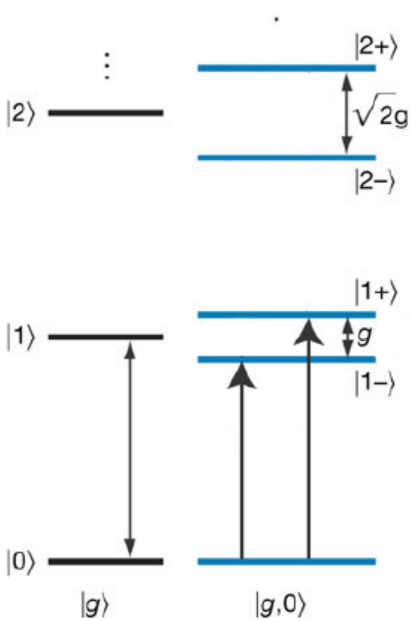
Example of strong AC driving Effects

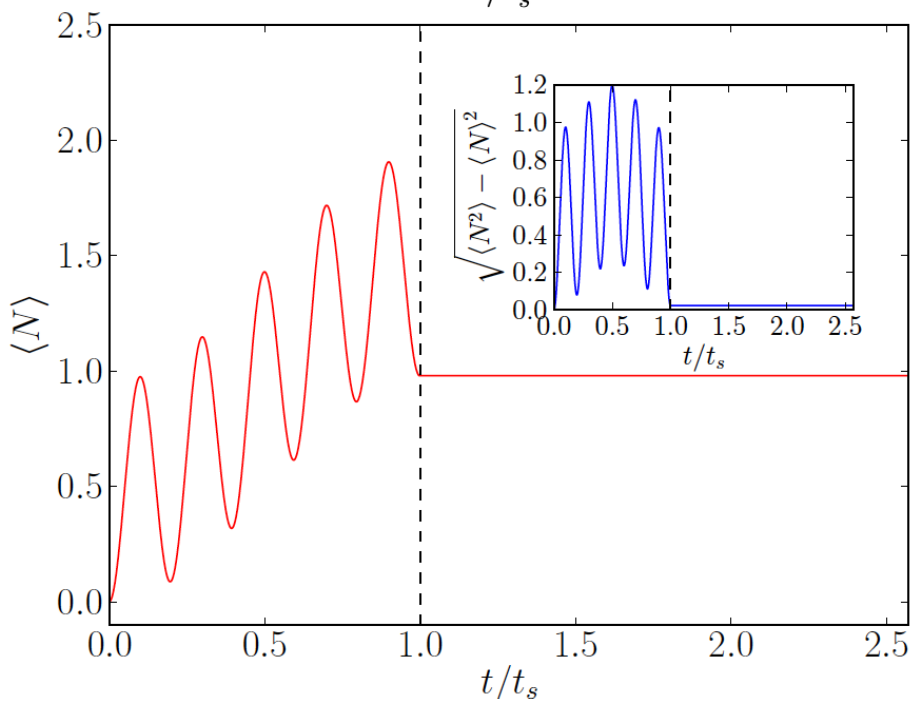


in agreement with Bethe Ansatz

$$N = a^\dagger a + \tfrac{1}{2}(\sigma^z + 1)$$

Small g limit:
Jaynes-Cummings

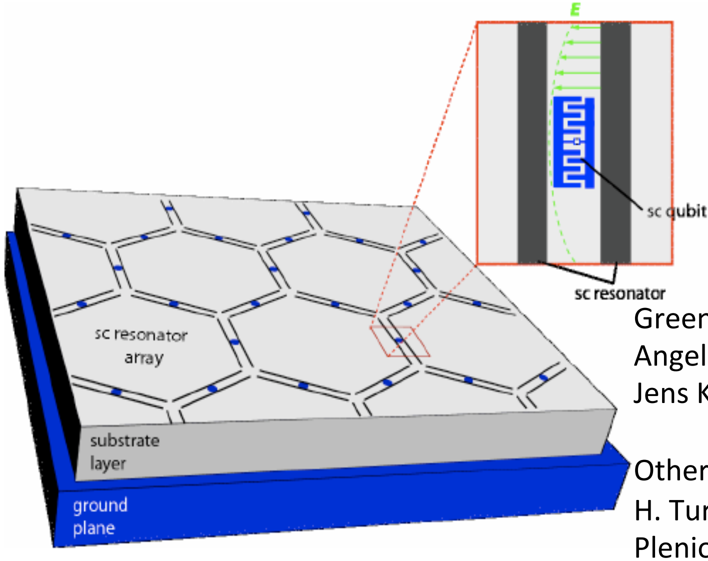




More than 1 cavity extension:

More stochastic Fields or Bigger Matrix

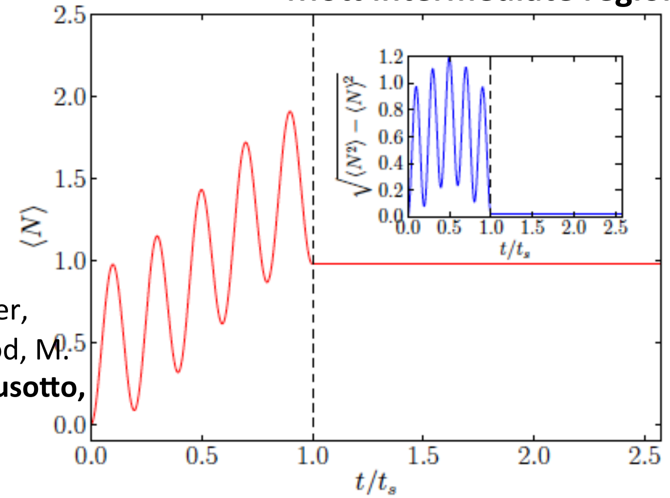
Array Systems



Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)

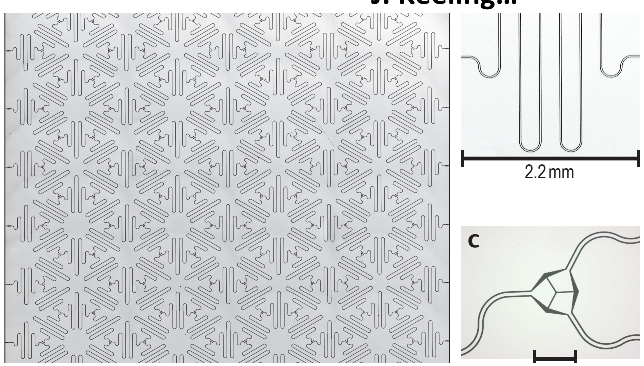
Other groups: R. Fazio, S. Schmidt & G. Blatter, H. Tureci, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik, C. Ciuti, **I. Carusotto, J. Keeling…**

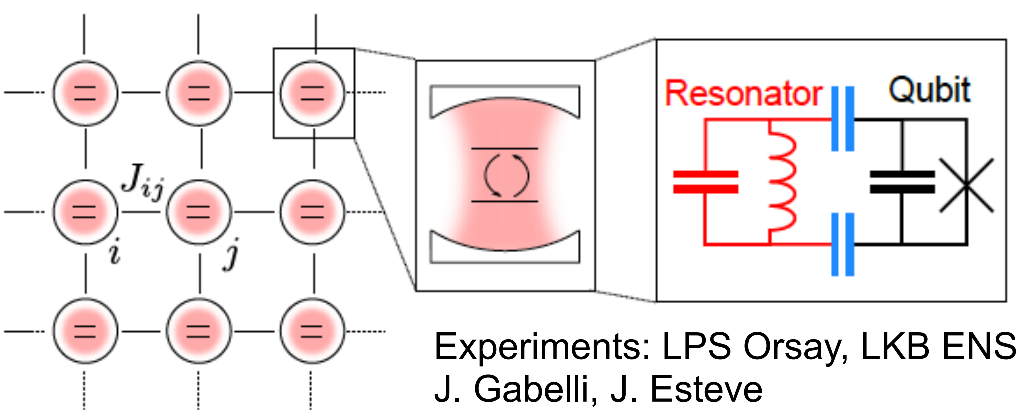
Mott intermediate region



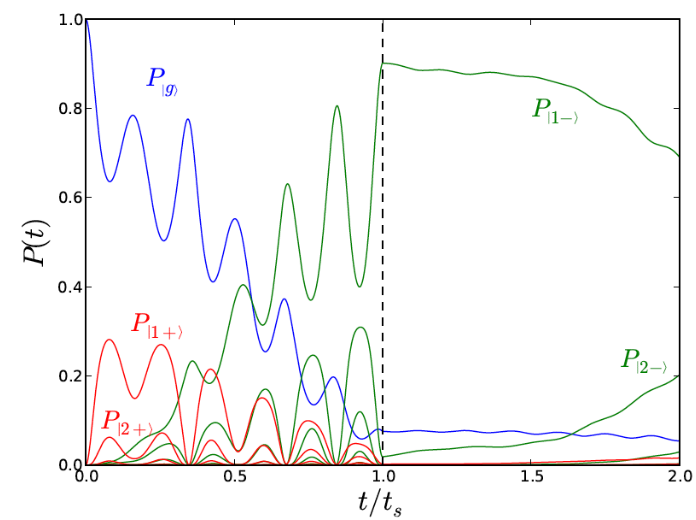
A. Houck's Lab

Princeton





Experiments: LPS Orsay, LKB ENS
J. Gabelli, J. Esteve



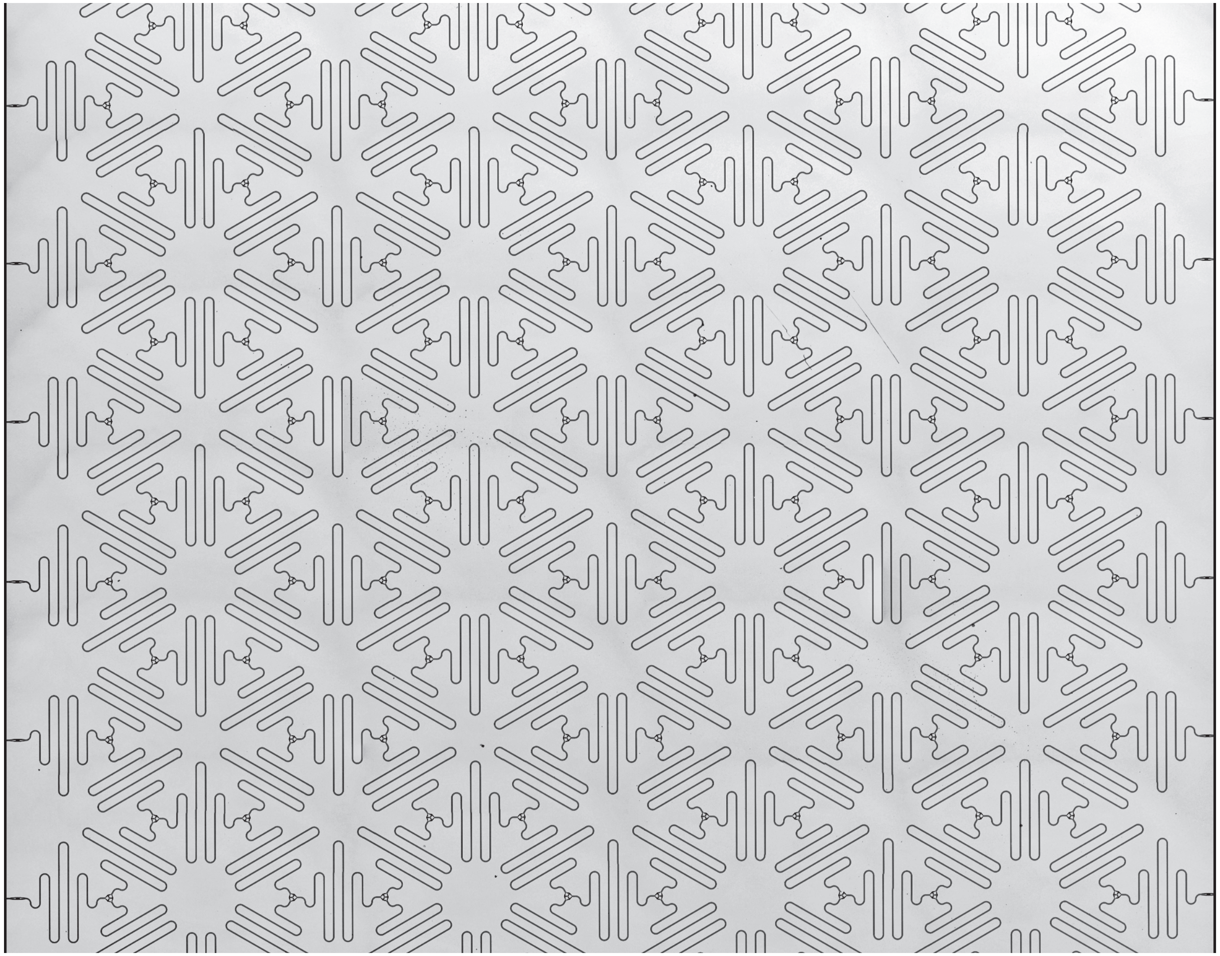
L. Henriet, Z. Ristivojevic, P. Orth, KLH
2014: application to driven Rabi model

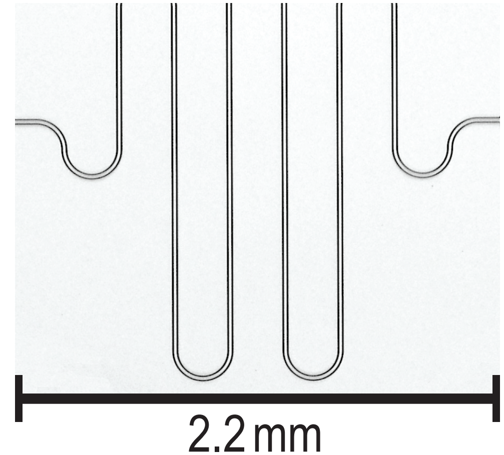
A. Houck lab at princeton

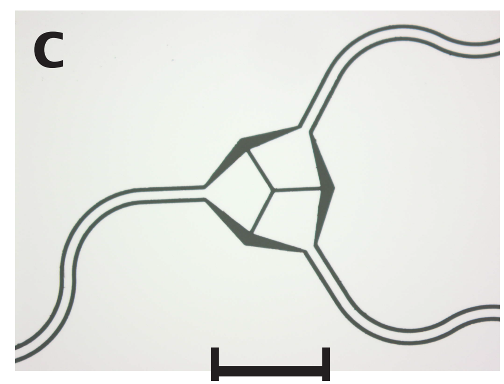
Other Realizations of Dirac Photons in Artificial Graphene:

M. Bellec, U. Kuhl, G. Montambaux, F. Mortessagne PRL 110, 033902 (2013)

T. Jacqmin et al Phys. Rev. Lett **112**, 1116402 (2014) (LPN Marcoussis)





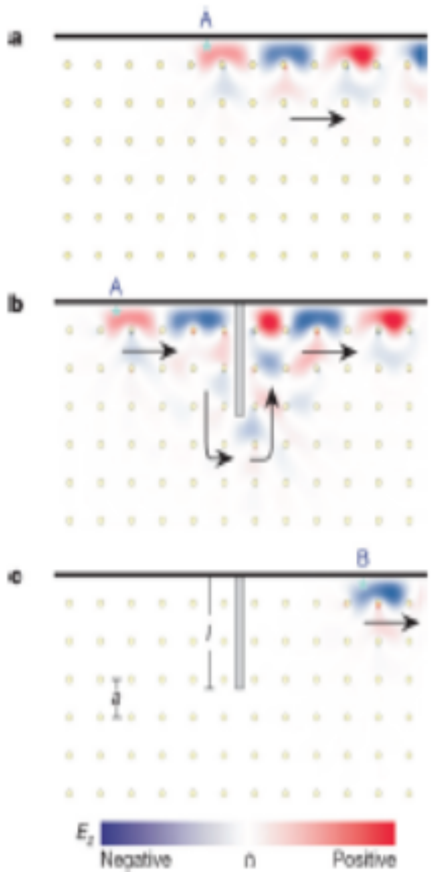


D. L. Underwood, W. E. Shanks, J. Koch and A. A. Houck, Phys. Rev. A 86, 023837 (2012).

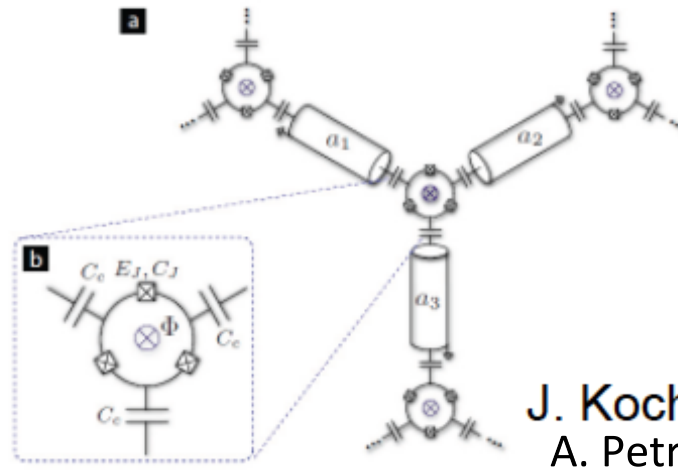
Artificial Gauge Fields with Light

See also M. C. Rechstmann et al Nature 2013
D. Schuster and J. Simon lab Chicago

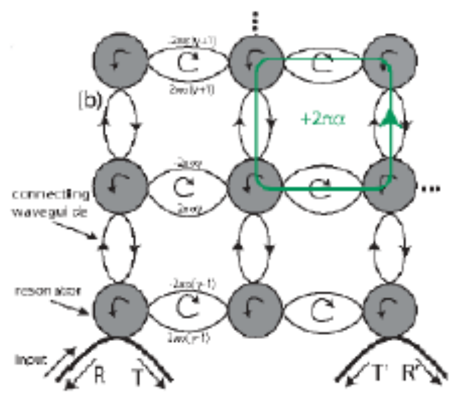
Review:
I. Carusotto
& C. Ciuti
RMP 2012



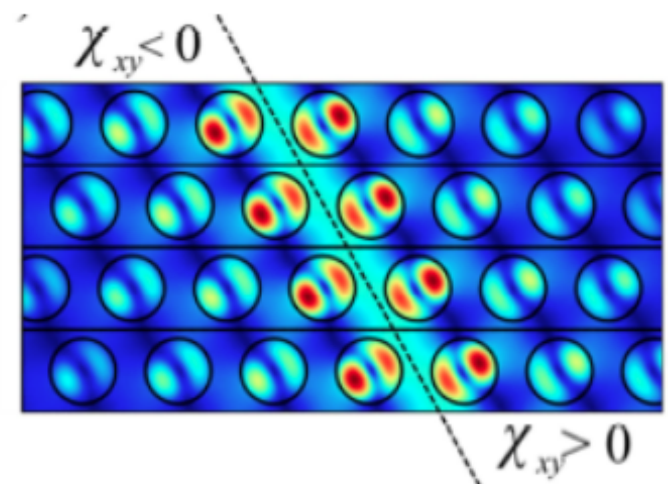
Haldane-Raghu, PRL 2008
Z. Wang et al. Nature 2009



J. Koch et al, PRA 2010
A. Petrescu et al 2012



M. Hafezi, E. Demler, M. Lukin, J. Taylor 2011
(more recent works as well)



A. MacDonald et al. 2012

Cold Atoms:

Jaksch & Zoller 2003

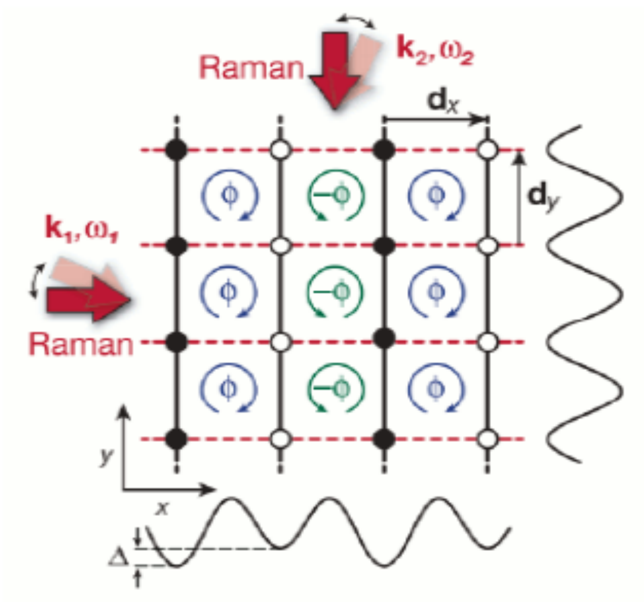
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
I. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- Ways to implement magnetic fields & gauge fields

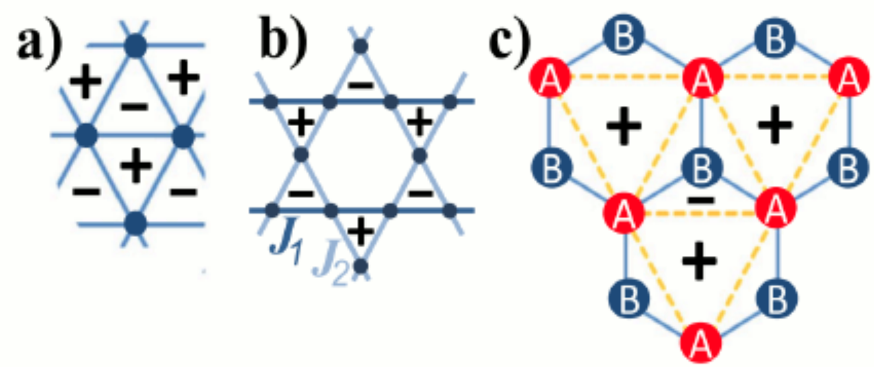
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



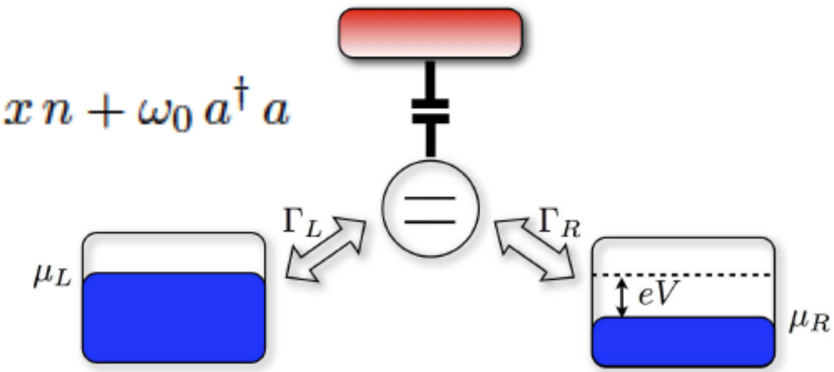
Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)



Floquet Topological Insulators:
Recent review J. Cayssol, B. Dora, F. Simon, R. Moessner, arXiv:1211.5623

Nonlinearities in Hybrid Systems: Brownian motion out of equilibrium

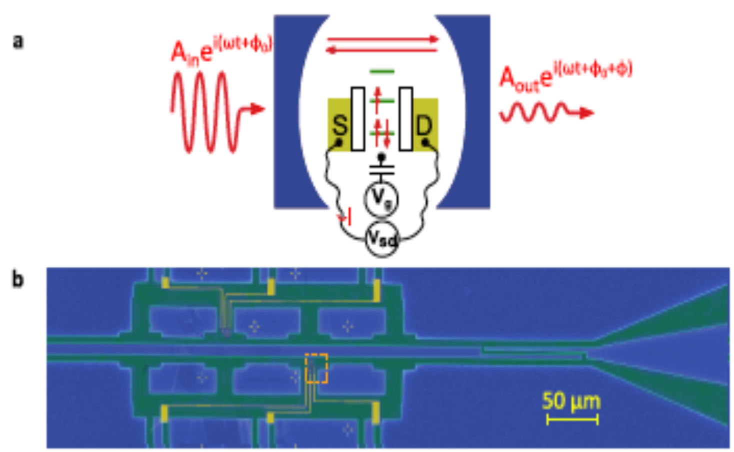
$$\lambda\, x\, n + \omega_0\, a^\dagger\, a$$



Marco Schiro & KLH 2014

FIG. 1: Schematic figure of the hybrid quantum impurity system consisting of a quantum dot hybridized to biased metallic leads and capacitively coupled to an electromagnetic resonator.

Anderson-Holstein model



M. Delbecq et al 2011

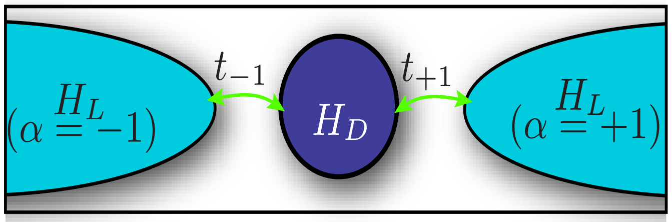
Experiment at ENS Paris

Group of T. Kontos

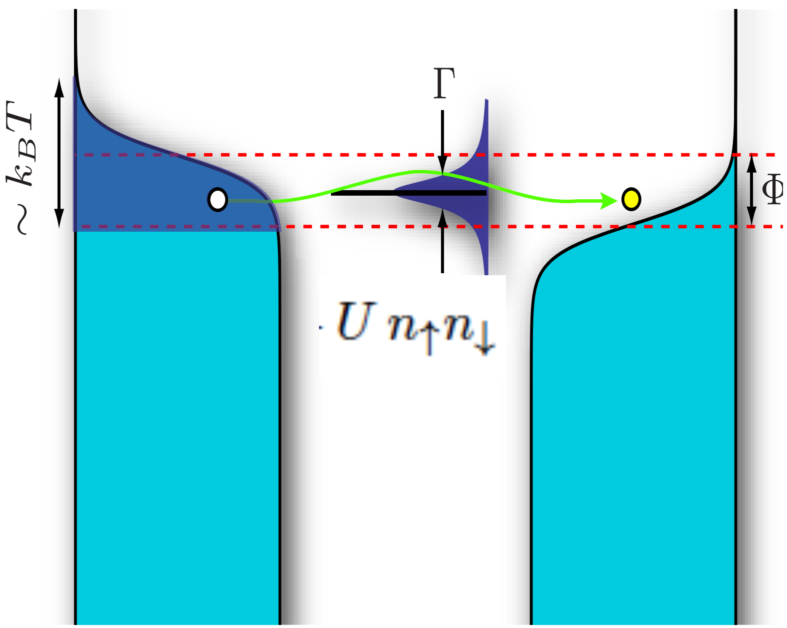
also ETH Zuerich, Princeton LPN Marcoussis

Example of Nonlinear Quantum Transport

P. Dutt, J. Koch, J. E. Han, KLH Annals of Physics 326, 2963-99 (2011)
Generalization to gradients of temperature: P. Dutt & KLH, 2013



Anderson model



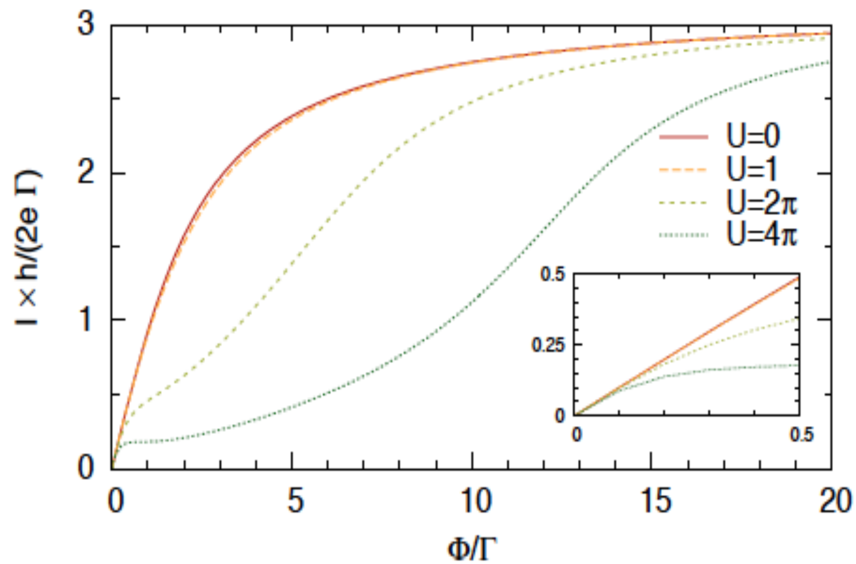


Fig. 6. The current-voltage curves for the Anderson model for $U/\Gamma = 0.0, 1.0, 2\pi$ and 4π, where $\Gamma = 1$. The inset shows the behavior of the curves for low bias. In the limit $\Phi \to 0$ the slope of the curves tend to 1, which corresponds to the value of the conductance quantum.

See also Diagrammatic MC in Keldysh scheme:
P. Werner, T. Oka and A. J. Millis, 2009-2010
M. Schiro & M. Fabrizio, 2008
T. Schmidt, Muehlbacher, Urban and Komnik 2011

Effective Boltzmann-Gibbs description of steady states (other works)
Hershfield 1993; Andrei, Doyon, Schiller, Anders, D. Bernard; C. Aron and G. Kotliar…

Feedback on the circuit Quantum Electrodynamics?

$$\mathcal{H} = \sum_{kl} \omega_k \, b^\dagger_{kl} \, b_{kl} + \left(a + a^\dagger\right) \sum_{kl} g_k \, \left(b^\dagger_{kl} + b_{kl}\right) + \mathcal{H}_{sys}$$

Anderson-Holstein model

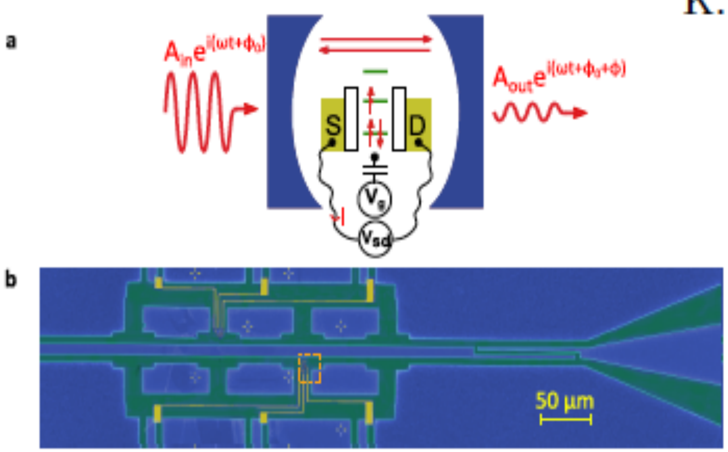
$$\lambda \, x \, n + \omega_0 \, a^\dagger \, a$$

$$t(\omega) \equiv \frac{\langle V_R^{out}(\omega) \rangle}{\langle V_L^{in}(\omega) \rangle} = i \, J(\omega) \, \chi^R_{xx}\,(\omega)$$

Input-Ouput Theory:

A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).

K. Le Hur, Phys. Rev. B **85**, 140506 (2012).



$$\chi^R_{xx}(t) = -i\theta(t) \langle \, [x(t), x(0)] \rangle_{H_{sys}}$$

$$\tan \varphi(\omega) \equiv \frac{\operatorname{Im} t(\omega)}{\operatorname{Re} t(\omega)} = \frac{\operatorname{Re} \chi^R_{xx}(\omega)}{\operatorname{Im} \chi^R_{xx}(\omega)} \, .$$

Transport of electrons & Anderson-Holstein model ; see A. Mitra, Aleiner and A. Millis

The retarded photon Green's function can be written in Fourier space in terms of the photon self-energy $\Pi^R(\omega)$ as

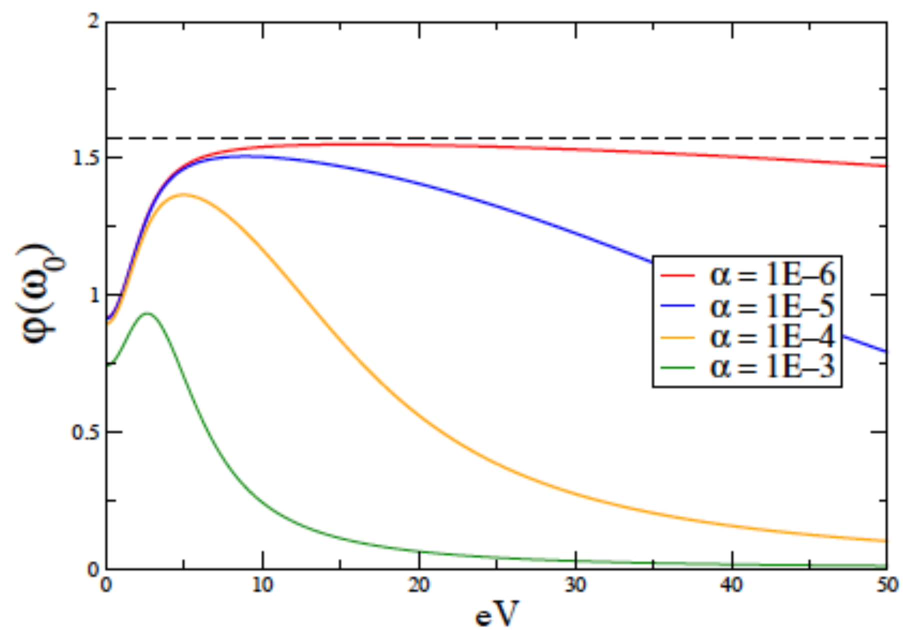
$$\chi^R_{xx}(\omega) = \frac{\omega_0}{\omega^2 - \omega_0^2 - \omega_0 \Pi^R(\omega)} \qquad (15)$$

where $\Pi^R(\omega)$ includes both the effects of frequency renormalization and the damping due to the environment.

$$\Pi^R(t,t') = \Lambda^R(t,t') \equiv \lambda^2 \chi_{el}(t-t') \qquad (17)$$

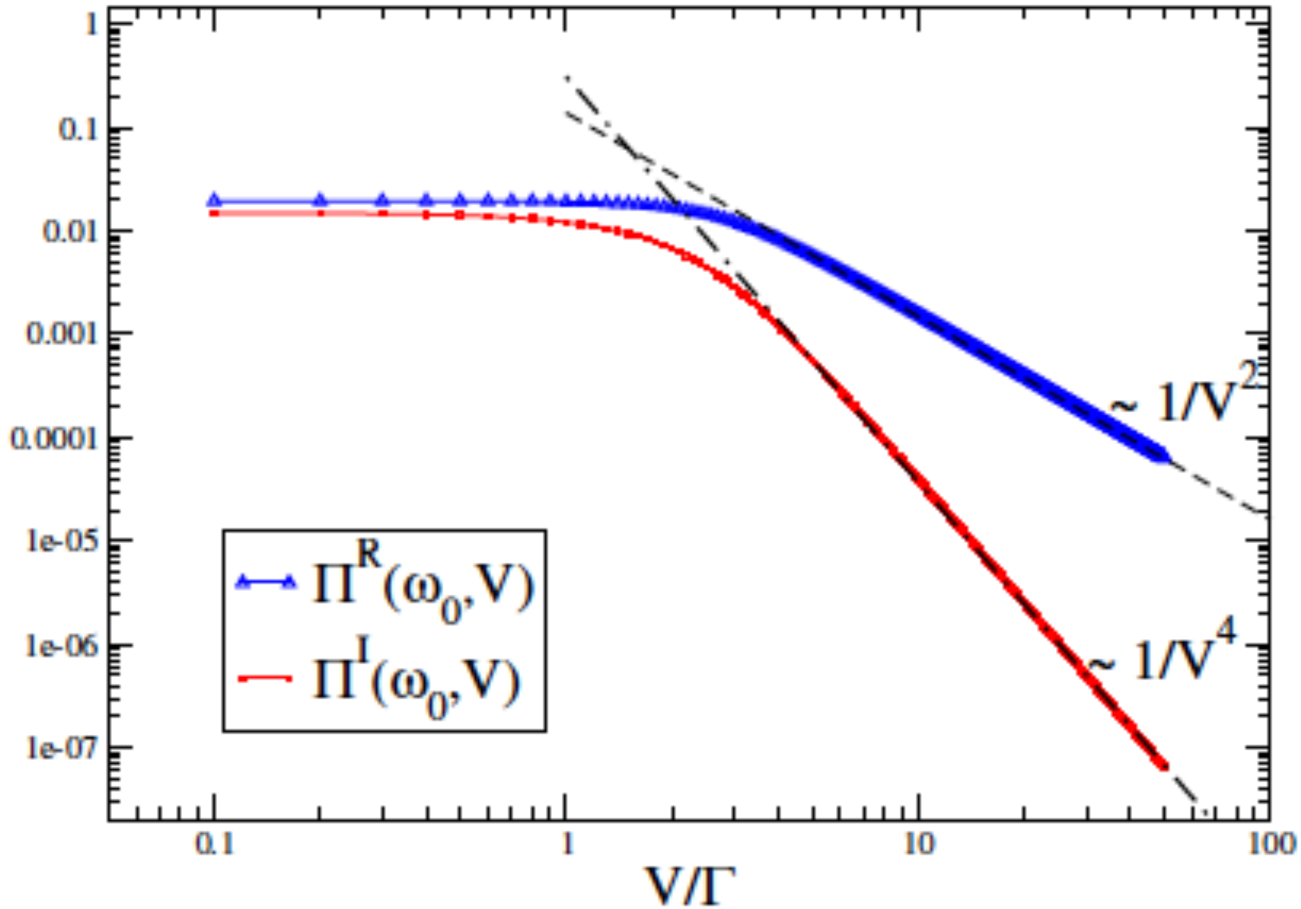
with $\chi_{el}(t-t') = -i\theta(t-t')\langle[n(t),n(t')]\rangle_{el}$ the electronic charge susceptibility. For an Anderson Impurity Model which exhibits a Fermi-Liquid type of ground state this must satisfy the Korringa-Shiba relation[57] which implies

$$\mathrm{Im}\chi_{el}(\omega) = \pi\omega \left[(\mathrm{Re}\chi_{el\uparrow}(0))^2 + (\mathrm{Re}\chi_{el\downarrow}(0))^2 \right] . \qquad (18)$$



At low-frequency: importance in RC circuits
Anderson model, M. Fillipone, KLH, C. Mora
Phys. Rev. Lett. **107**, 176601 (2011)

Marco Schiro & KLH,
Phys. Rev. B **89**, 195127 (2014)



Damping rate of photons at large bias in $1/V^4$

Effective Langevin Description

Integration of electron degrees of freedom (resonant level model; Kondo limit)

$$\boxed{V_{eff}(x) = \frac{\lambda\, x}{2} + \frac{\omega_*^2\, x^2}{2} + \eta\, x^3 + g\, x^4}$$

No interaction On dot

$$\omega_*^2 = \omega_0^2 - \frac{\lambda^2}{\pi} \sum_\alpha \frac{\Gamma_\alpha}{(\varepsilon_0 - \mu_\alpha)^2 + \Gamma^2}$$

$$\eta = \frac{2\lambda^3}{\pi} \sum_\alpha \frac{\Gamma_\alpha\,(\varepsilon_0 - \mu_\alpha)}{(\varepsilon_0 - \mu_\alpha)^2 + \Gamma^2}$$

and finally the anharmonicity

$$g = \frac{2\lambda^4\,\Gamma_\alpha}{\pi} \sum_\alpha \frac{\Gamma^2 - (\varepsilon_0 - \mu_\alpha)^2}{\left[\Gamma^2 + (\varepsilon_0 - \mu_\alpha)^2\right]^3}\,.$$

Kondo Limit: NCA

A. Rosch, J. Kroha and P. Woelfle

T_K being the Kondo temperature

$$\Gamma_\star \sim \frac{V}{\log^2(V/T_K)}\left[1 + \frac{2}{\log(V/T_K)} + \ldots\right]$$

Large bias voltage limit: Brownian motion out of equilibrium

$$\ddot{x}_c = -\omega_0\, x_c - F(x_c) - \gamma(x_c)\dot{x}_c + \xi(t)$$

$$\langle \xi(t)\,\xi(t')\rangle = D(x_c)\delta(t-t')$$

$$T_{eff}(V) = \frac{D(V)}{4\gamma(V)}$$

bias voltage. We see the small and large bias behaviours (compared to the electronic lifetime Γ) are characterized by two different power laws, $T_{eff} \sim V$ at small bias when T_{eff} is almost set by the noise $D(V) \sim V$ while $T_{eff} \sim V^4$ at large voltage when the noise as we have seen saturates while the dissipation decays fast $\gamma(V) \sim 1/V^4$.

A. Kamenev, next week, Keldysh approach

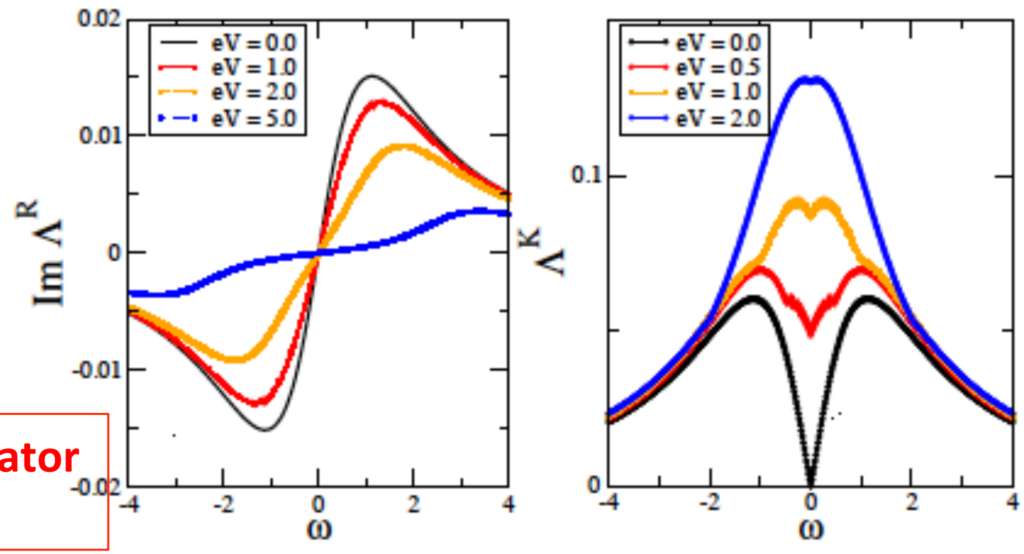
Marco Schiro & KLH,
Phys. Rev. B **89**, 195127 (2014)

$$S_{frict} = \int dt\, dt'\, x_c(t)\, \mathrm{Im}\, \Lambda^R_{x_c}(t-t')\, x_q(t')$$

$$S_{noise} = \int dt\, dt'\, x_q(t)\, \Lambda^K_{x_c}(t-t')\, x_q(t')$$

$$\mathrm{Im}\,\Lambda^R(\omega) = -i\gamma(x_c)\,\omega$$

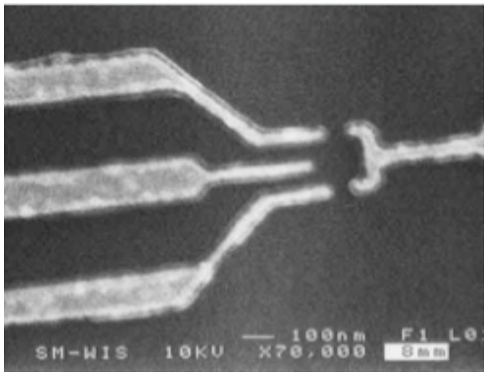
$$\Lambda^K(\omega \to 0; x_c) \equiv iD(x_c)$$



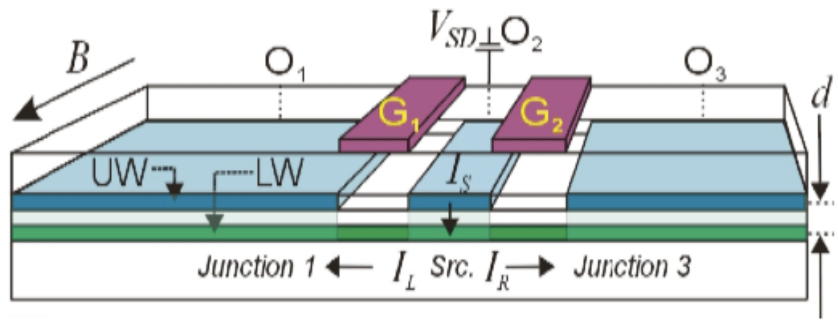
Driven case: Mapping to the Duffing oscillator

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma\cos(\omega t)$$

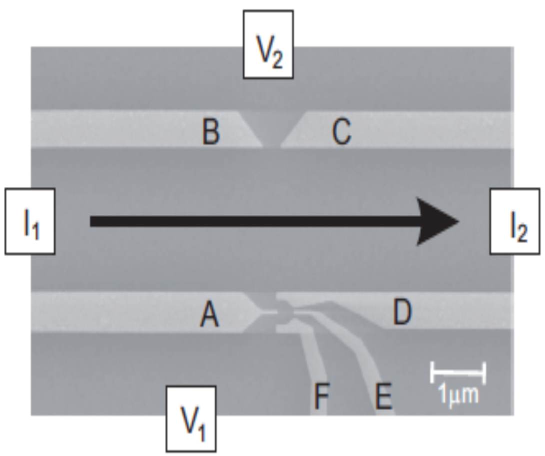
Transport in Nano-Matter: QPCs, dots and quantum wires



Quantum impurity models:
Transport: **current** and **noise** commonly accessible



Low-D Luttinger liquids Luttinger liquid introduction: T. Giamarchi's book; Haldane (1981)
Talk by A. Mirlin, nonlinearities



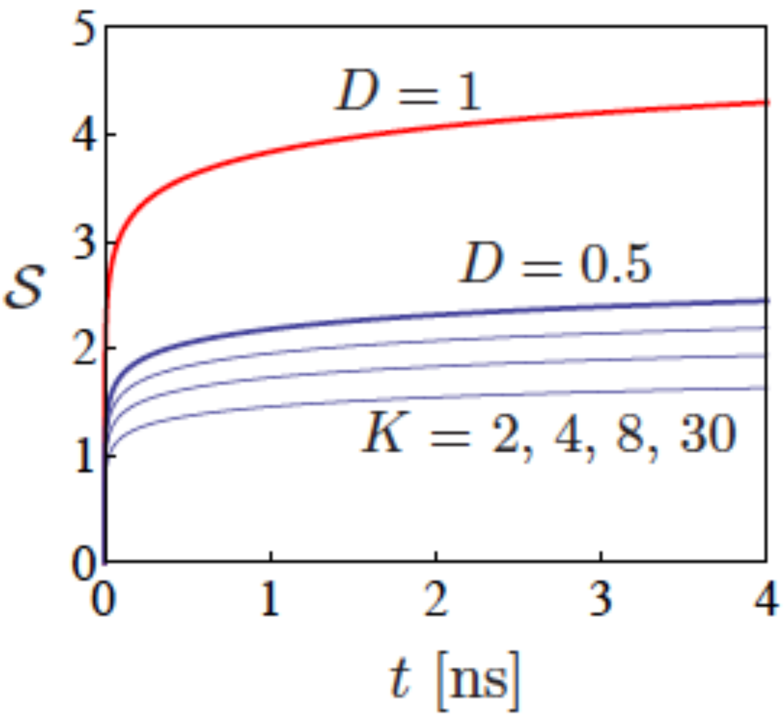
Thermopower:
L. Molenkamp et al (2005)

Quantum wires in cQED cavities
Inducing new long-range physics; probing Majorana fermions
Loic Herviou, Christophe Mora & KLH

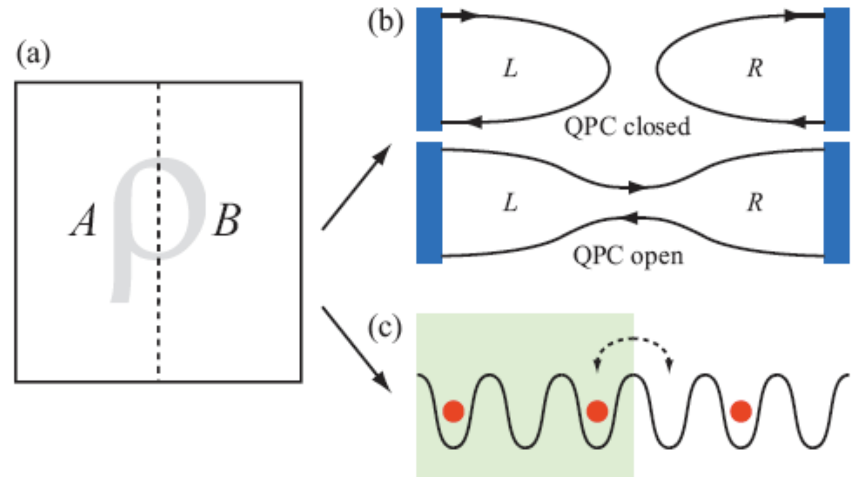
Noise, FCS & Entanglement Entropy

Current and (thermodynamic) entropy production: P. Mehta & N. Andrei

Klich-Levitov, Gaussian case D=1: 2009
H. F. Song, S. Rachel, C. Flindt, N. Laflorencie
I. Klich & KLH, 2012 (general case)



D = transparency of the barrier



D=1: Results from CFT (**Calabrese-Cardy**): entropy grows logarithmically with time

D=0.5: Higher cumulants matter, but the entropy maintains its logarithmic growth
noise: Lower bound on the full entanglement entropy

New results for entanglement spectrum:
A. Petrescu et al. arXiv:1405.7816

Yale
2010

<u>Picture</u>
J. F. Dars
A. Papillaut
CNRS
Book Le Plus Grand des Hasards

<u>Ecole Polytechnique, CPHT (since 2012):</u>
Loic Henriet PhD student
Loic Herviou Master and PhD student, co-direction with C. Mora, ENS
Tianhan Liu, PhD, co-direction with B.Douçot LPTHE (topological insulators)
Alexandru Petrescu, Yale and CPHT X (work also on artificial gauge fields)
Zoran Ristivojevic, post-doctoral associate (CNRS Toulouse)

Other Collaborators related to the Talk: P. Orth (KIT Karlsruhe) & A. Imambekov;
M. Schiro (Columbia, IPHT), W. Hofstetter (Frankfurt), M. Filippone (Berlin), C. Mora (LPA ENS),
P. Dutt, T. Schmidt (Basel), J. Koch, J. Han, C.-H. Chung, M. Vojta, P. Woelfle …

Summary of the Presentation

Quantum Impurities (done)

Examples of Non-Trivial Dynamics (done)

Circuit Quantum Electrodynamics (done)

NonLinear Quantum Transport (done)

Supplementary Slides on Stochastic Method

Functional approach

Method

We extend here a non-perturbative stochastic method [9] to evaluate the exact dynamics of the spin.

- Functional integration of bosonic degrees of freedom$\rightarrow$ evaluation of the spin-reduced density matrix $\rho_S(t) = tr_B \left[U(t,t_0)\rho_{tot}(t_0)U^\dagger(t,t_0)\right]$.
- The influence of the environment is contained in the Feynman-Vernon [10] influence functional $F[\sigma,\sigma']$:

$$\langle \sigma_f|\rho_S(t)|\sigma'_f\rangle = \int D\sigma D\sigma' A[\sigma]A[\sigma']^* F[\sigma,\sigma'] \,.$$

Double spin path similar to Keldysh contour

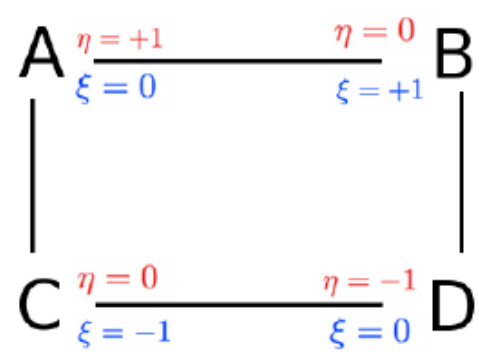
9. G. B. Lesovik et al., JETP Lett, **75**, 474 (2002), J. T. Stockburger et al. Phys. Rev. Lett. **88**, 170407 (2002), P. P. Orth, et al., Phys. Rev. B **87**, 014305 (2013).
10. R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.), **24**, 118, (1963).

FV influence functional-1/3

$$F[\sigma,\sigma'] = \exp\left\{-\frac{1}{\pi}\int_0^t ds \int_0^s ds'[-iL_1(s-s')\xi(s)\eta(s') + L_2(s-s')\xi(s)\xi(s')]\right\}$$

With η and ξ the symmetric and anti-symetric spin paths :

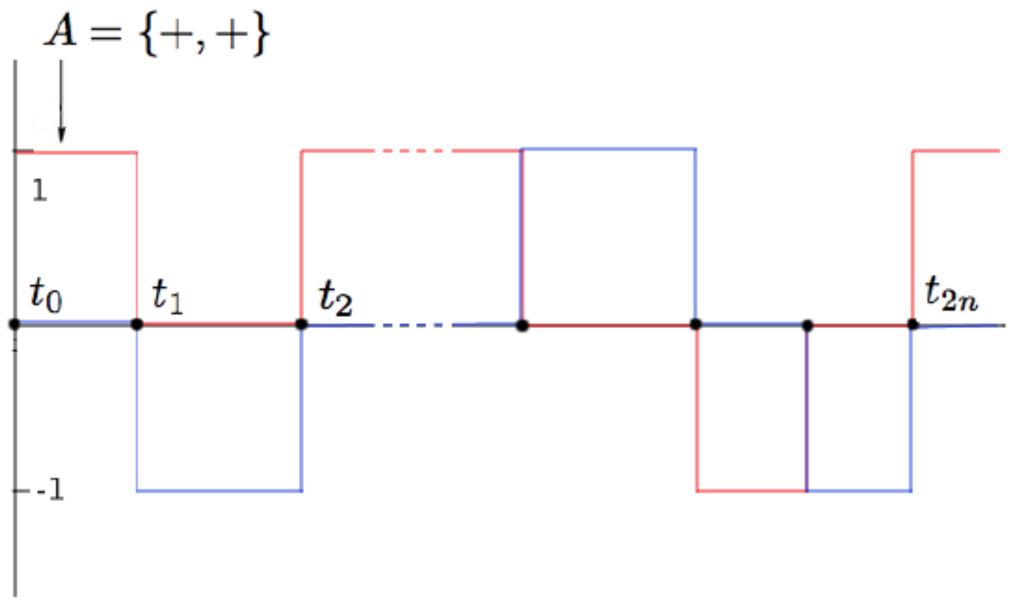
$$\eta(s) = \tfrac{1}{2}\left[\sigma(s)+\sigma'(s)\right]$$
$$\xi(s) = \tfrac{1}{2}\left[\sigma(s)-\sigma'(s)\right]$$



$$L_1(t) = \int_0^\infty d\omega J(\omega)\sin\omega t$$

$$L_2(t) = \int_0^\infty d\omega J(\omega)\cos\omega t \coth\beta\omega/2$$

FV influence functional-2/3



$\xi(t) = \sum_{j=1}^{2n} \Xi_j \theta(t - t_j)$ and $\eta(t) = \sum_{j=0}^{2n} \Upsilon_j \theta(t - t_j)$ [11]

$$\mathcal{Q}_1 = e^{\frac{i}{\pi} \sum_{k=0}^{2n-1} \sum_{j=k+1}^{2n} \Xi_j \Upsilon_k Q_1(t_j - t_k)}$$

$$F_n[\{\Xi_j\}, \{\Upsilon_j\}, \{t_j\}] = \mathcal{Q}_1 \mathcal{Q}_2$$

$$\mathcal{Q}_2 = e^{\frac{1}{\pi} \sum_{k=1}^{2n-1} \sum_{j=k+1}^{2n} \Xi_j \Xi_k Q_2(t_j - t_k)}$$

11. A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg and W. Zwerger, Rev. Mod. Phys, **59**, 1 (1987).

FV influence functional-3/3

Finally, the probability to find back the system in state $|+\rangle$ at time t is given by the development :

$$p(t) = \langle +|\rho_S(t)|+\rangle = \sum_{n=0}^{\infty} \left(\frac{i\Delta}{2}\right)^{2n} \int_{t_0}^{t} dt_{2n} \ldots \int_{t_0}^{t_2} dt_1 \sum_{\{\Xi_j,\Upsilon_j\}} F_n[\{\Xi_j\},\{\Upsilon_j\},\{t_j\}]$$

$$F_n[\{\Xi_j\},\{\Upsilon_j\},\{t_j\}] = e^{\frac{i}{\pi}\sum_{j>k=0}^{2n}\Xi_j\Upsilon_k Q_1(t_j-t_k)}\, e^{\frac{1}{\pi}\sum_{j>k=1}^{2n}\Xi_j\Xi_k Q_2(t_j-t_k)}$$

Stochastic decoupling :

$$F_n[\{\Xi_j\},\{\Upsilon_j\},\{t_j\}] = \overline{\prod_{j=1}^{2n} \exp\left[h_\xi(t_j)\Xi_j + h_\eta(t_j)\Upsilon_j\right]}$$

with h_ξ and h_η two complex gaussian random fields which verify :

$$\overline{h_\xi(t)h_\xi(s)} \propto Q_2(t-s)$$
$$\overline{h_\xi(t)h_\eta(s)} \propto iQ_1(t-s)$$

$$p(t) = \overline{\sum_n \left(\frac{i\Delta}{2}\right)^{2n} \int_{t_0}^{t} dt_{2n} .. \int_{t_0}^{t_2} dt_1 \sum_{\{\Xi_j, \Upsilon_j\}} \prod_{j=1}^{2n} \exp\left[h_\xi(t_j)\Xi_j + h_\eta(t_j)\Upsilon_j\right]}$$

$$p(t) = \overline{\langle \Phi_f | e^{-i\int_0^t ds W(s)} | \Phi_i \rangle}\,; \quad W = V_0 \begin{pmatrix} 0 & e^{-h_\xi + h_\eta} & e^{h_\xi + h_\eta} & 0 \\ e^{h_\xi - h_\eta} & 0 & 0 & e^{h_\xi + h_\eta} \\ e^{-h_\xi - h_\eta} & 0 & 0 & e^{-h_\xi + h_\eta} \\ 0 & e^{-h_\xi - h_\eta} & e^{h_\xi - h_\eta} & 0 \end{pmatrix}$$

W : effective spin Hamiltonian in the space of states $\{A, B, C, D\}$.
$|\Phi_i\rangle = (e^{h_\eta(t_0)}, 0, 0, 0)^T$ and $\langle \Phi_f| = (e^{-h_\eta(t_{2n})}, 0, 0, 0)$ (these choices account for the asymmetry between blips and sojourns).

$p(t)$ is then given by the stochastic average $\overline{\langle \Phi_f | \Phi(t) \rangle}$ where $|\Phi(t)\rangle$ is the solution of the stochastic Schödinger equation :

$$i\partial_t |\Psi\rangle = W|\Psi\rangle,$$

with initial condition $|\Phi_i\rangle$.

Systems of interacting photons: Theory surveys

▶ M. Hartmann et al., Laser & Photonics Review 2, 527 (2008)
A. Tomadin & R. Fazio, J. Opt. Soc. Am B 27, A130 (2010)
J. Larson ; I. Carusotto and C. Ciuti, RMP 2012

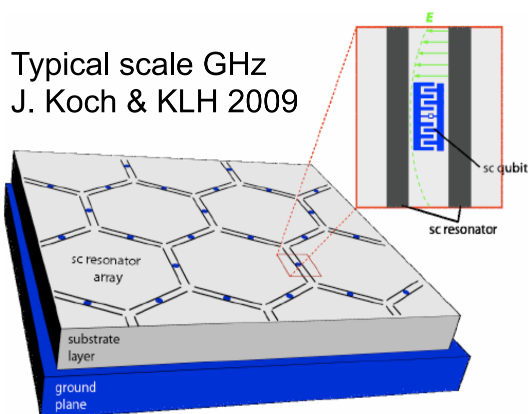
Interacting photons:

M. Lukin, E. Demler et al:
Fermionizing light

realizations: superfluidity of polaritons ***Stanford at Grenoble-EPFL, LKB ENS, LPN Marcoussis, PIttsburg***

* photonic band gap cavities
* arrays of silicon micro-cavities
* fibre based cavities
* cQED Array current realization (A. Houck; H. Tureci; J. Koch 2012 & S. Schmidt, J. Koch 2012)

Typical scale GHz
J. Koch & KLH 2009

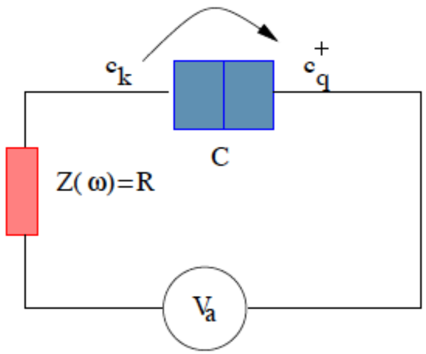


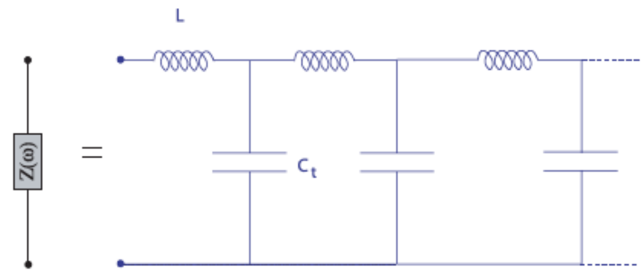
some pros and cons

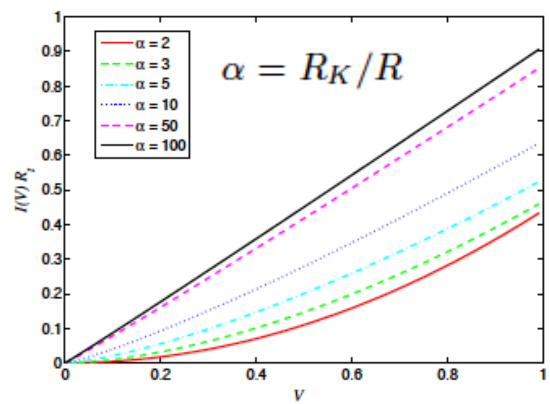
\+ tunability
\+ access to single lattice site
\- must be treated as open system
\+ interesting: transitions between different steady states

Nonlinear Quantum Transport: example

A BASIC EXAMPLE: THE NOISY BARRIER







Analogy with a Luttinger liquid:
I. Safi and H. Saleur,
Phys. Rev. Lett. 93, 126602 (2004)

See Ingold-Nazarov
Single Charge Tunneling Coulomb
Blockade phenomena in Nanostructures
Eds H. Grabert-M. Devoret 1992

FIG. 4: Current-voltage characteristics for the noisy tunnel barrier. For $R \ll R_K$ one observe a clear deviation from Ohm's formula as a result of prominent quantum fluctuations in the environment induced when one tries to add in an electron (hole) in the left electrode. When R increases, charging effects at the tunnel junction will lead to Coulomb blockade.

$$\Gamma(V) = \frac{\exp(-2\gamma_e/\alpha)}{\Gamma(2+2/\alpha)} \frac{V}{R_t} \left[\frac{\pi}{\alpha} \frac{e|V|}{E_c}\right]^{2/\alpha}.$$