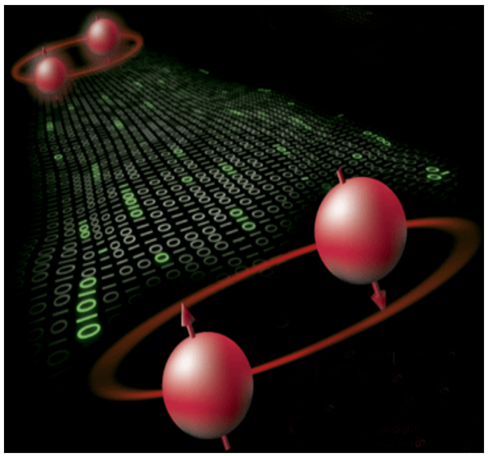
Many-Body Quantum Dynamics & Topology



Karyn Le Hur

CPHT

Ecole Polytechnique

& CNRS, France

Complex Quantum Systems & Quantum Information

Talk in Honor of Adilet Imambekov
Harvard Sept 22nd 2013

Outline of the Presentation

Spins in a Many-Body Environment: Examples Kondo Systems and Spin-Boson Systems

- Entanglement in these quantum systems
  Many-Body Dynamics and Non-Markovian Effects

One important paper for this memorial conference
Peter P. Orth, Adilet Imambekov, Karyn Le Hur Phys. Rev. B **87**, 014305 (2013) **(Review)**
New developments at Ecole Polytechnique, with Loic Henriet & Zoran Ristivojevic

Another approach: P. Dutt, J. Koch, Jong Han and KLH, Annals of Physics **326**, 2963-99 (2011)

Novel Topological Phases of Light and Bosons

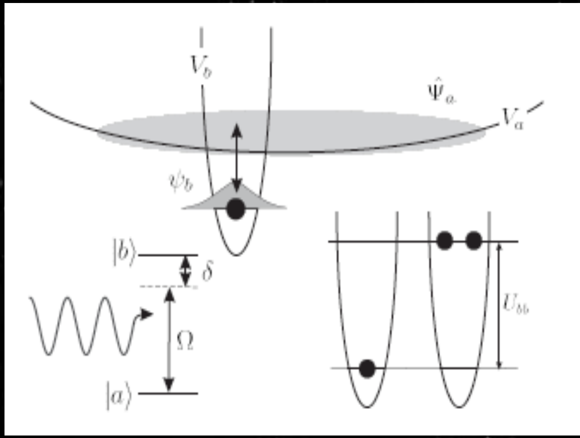
PhD thesis of Alexandru Petrescu

A. Petrescu, A. A. Houck, K. Le Hur Phys. Rev. A 86, 053804 (2012)
A. Petrescu & K. Le Hur, arXiv:1306.5986 (to be published in PRL)

Recent Developments: dynamics

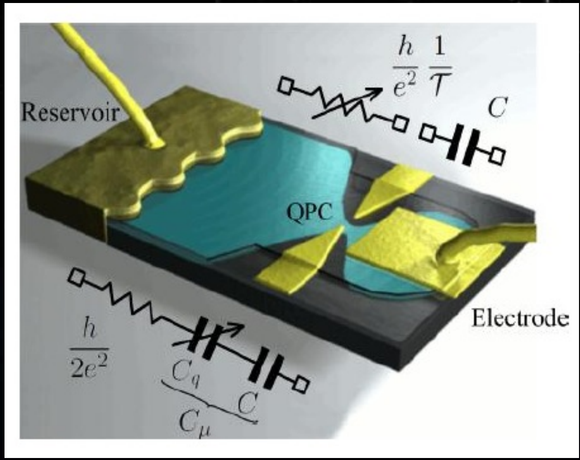
COLD-ATOMIC Quantum IMPURITIES

A. Recati et al. PRL **94**, 040404 (2005)
Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)
Single Atom: Ph. Grangier et al. Science **309**, 454 (2005)
A. Fuhrmanek, Y. R. P. Sortais, P. Grangier, A. Browaeys Phys. Rev. A 82, 023623 (2010).
D. Porras, F. Marquardt, J. von Delft, J. I. Cirac (2007),…
M. Knap et al. Phys. Rev. X 2, 041020 (2012)
M. Knap, D. A. Abanin, E. Demler, arXiv:1306.2947
J. Bauer, C. Salomon, E. Demler arXiv:1308.0603



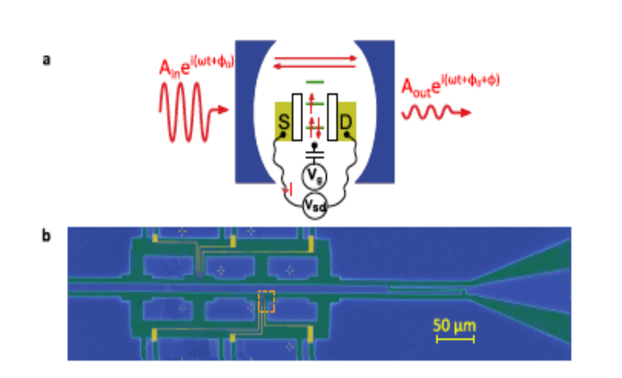
RC circuits & topological insulators

M. Buettiker, H. Thomas, and A. Pretre, Phys. Lett. A 180, 364 - 369,(1993)
J. Gabelli *et al.*, Science **313**, 499 (2006); G. Feve et al. 2007
J. Gabelli et al. Rep. Progress 2012
C. Mora and K. Le Hur, Nature Phys. 6, 697 (2010)
Y. Hamamoto, et al. Phys. Rev. B **81**, (2010) 153305
Y. Etzioni, B. Horovitz, P. Le Doussal, PRL **106**, 166803 (2011)
I. Garate & KLH, PRB 2012

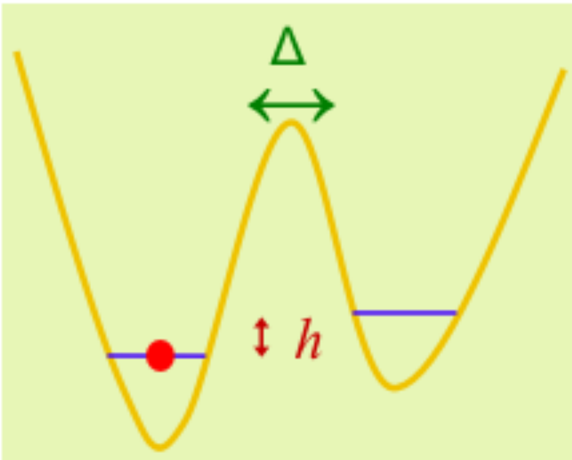


Kondo Effect with Photons

K. Le Hur, Phys. Rev. B 85, 140506(R) (2012)
A. Leclair, F. Lesage, S. Lukyanov and H. Saleur (1997)
M. Goldstein, M. H. Devoret, M. Houzet and L. I. Glazman, 2012
H. Zheng, D. Gauthier, H. U. Baranger, Phys. Rev. A 82, 063816 (2010)
M. Hofheinz et al. arXiv:1102.0131
M. Delbecq et al. PRL 107, 256804 (2011)
M. Schiro & KLH, in preparation

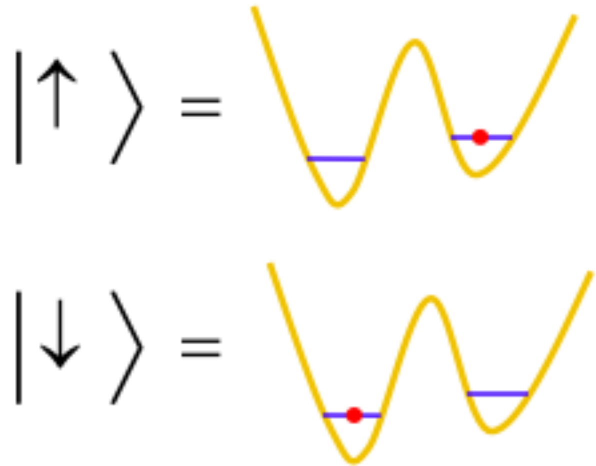


Artificial spin S=1/2



$$H_S = hS_z + \Delta(S_+ + S_-)$$

Spin S=1/2 description



particle localized ↔ spin polarized

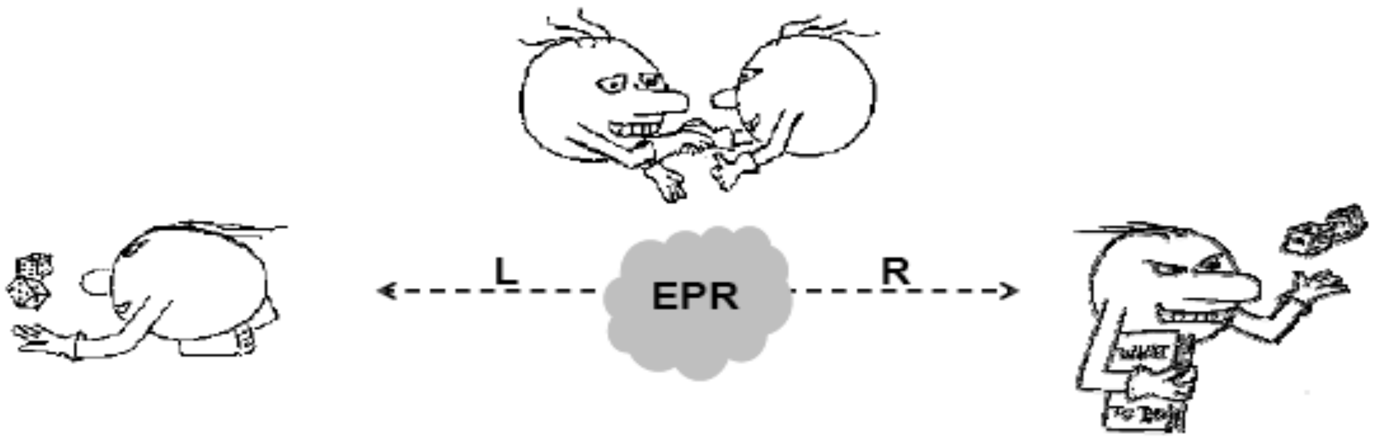
What is the effect of environment?

Entangling a Spin to its Environment: Novel Phases

Decoherence of the quantum superposition

Non-Markovian Effects and Spin Dynamics

What is Entanglement?
Spooky action at Distance (Einstein)



Simple example: 2 Qbits forming a singlet pair

$$|\Psi_S\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle)$$

Wave function is NOT factorizable into individual wave functions…

2 spins: detection lies on (Bell's) spin correlations (A. Aspect et al)

$$|\Psi_G\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\chi_\uparrow\rangle \pm |\downarrow\rangle|\chi_\downarrow\rangle)$$

Here: A = spin and B = quantum environment

From Thermal to Quantum Limit

$$\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i| = \frac{1}{Z}\sum_i e^{-\beta E_i}|\Psi_i\rangle\langle\Psi_i|$$

$$E = -\mathrm{Tr}(\rho\log\rho) = -\sum_i p_i \log p_i$$

At T=0 all coefficients are zero except for the ground state

$$\rho = |\Psi_G\rangle\langle\Psi_G|$$

$$\rho_A = \mathrm{Tr}_B|\Psi_G\rangle\langle\Psi_G|$$

Measurement on
Spin (A) ONLY

Entanglement & Spin Observables

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1+\langle S_z\rangle & \langle S_x\rangle + i\langle S_y\rangle \\ \langle S_x\rangle - i\langle S_y\rangle & 1-\langle S_z\rangle \end{pmatrix}$$

$$\rho_{\uparrow\uparrow} = |\langle\Psi_G|\uparrow\rangle|^2 = \frac{1}{2}(1+\langle\Psi_G|S_z|\Psi_G\rangle)$$

$$\rho_{\uparrow\downarrow} = \langle\Psi_G|\uparrow\rangle\langle\downarrow|\Psi_G\rangle$$

$$\mathrm{Tr}(\rho_A \log_2 \rho_A) = \sum_{i=+,-} p_i \log_2 p_i$$

$$p_i = \frac{1}{2}\left(1 \pm (\langle S_x\rangle^2 + \langle S_z\rangle^2)^{1/2}\right)$$

Seems easy to characterize and quantify…

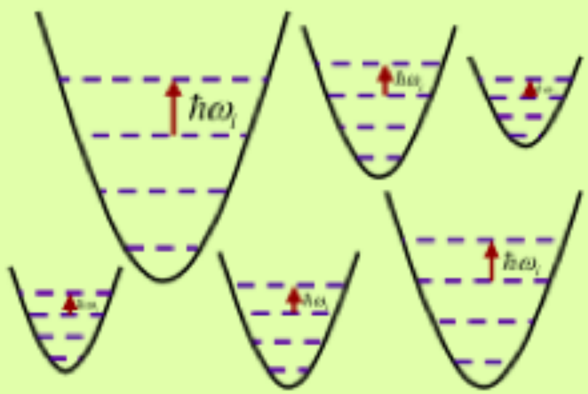
Celebrated Example of Environment

- Model the environment by quantum harmonic oscillators

$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987)
U. Weiss book, quantum dissipative systems, 1999

$$H_B = \sum_i \left(\frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2 x_i^2}{2} \right)$$



Bosonic bath

One important quantity is the frequency dependence of the associated coupling spectrum

$$J(\omega) = \sum_i |\lambda_i|^2 m_i \, \delta(\omega - \omega_i)$$

Exact mapping to Kondo model & Ising Model with long-range forces

F. Guinea, V. Hakim A. and Muramatsu, Phys. Rev. B 32, **4410-4418 (1985)**

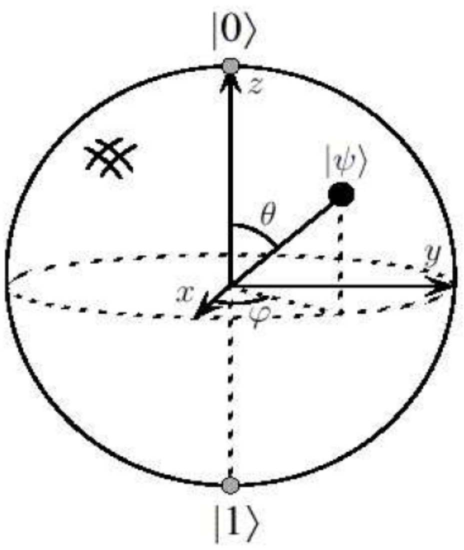
s=1 ohmic case $J(\omega) \propto \alpha\omega^s$

Product State when α = 0

$$|\Psi_G\rangle = |\Psi_{A=spin}\rangle \otimes |\Psi_{B=bath}\rangle$$

$$\langle S_z \rangle = -\frac{h}{\sqrt{\Delta^2 + h^2}} \qquad \langle S_x \rangle = -\frac{\Delta}{\sqrt{\Delta^2 + h^2}}$$

$$\langle S_z \rangle^2 + \langle S_x \rangle^2 = 1$$



Bloch sphere

Spin lies in a pure state: E=0

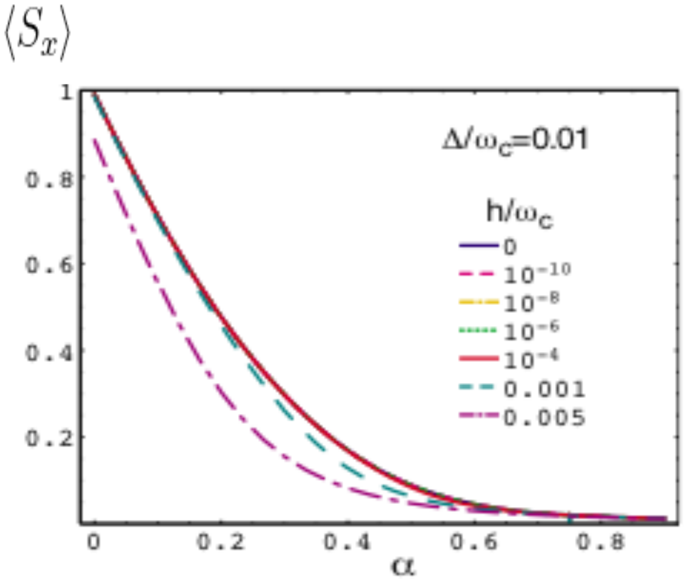
p_+=1 and p_-=0

Introducing the bath produces <u>decoherence & uncertainty</u>

$$S^+ \rightarrow S^+ e^{i\Omega} \qquad \Omega = \sum_\alpha \frac{\lambda_\alpha}{\hbar m_\alpha \omega_\alpha^2} p_\alpha$$

Results for Decoherence & Entanglement at h=0

P. Cedraschi and M. Buettiker *Ann. Phys. N. Y.* ***289****, 1 (2001)*
Karyn Le Hur, arXiv: 0711.2301 (Annals of Physics, 2008)
A. Kopp and K. Le Hur, PRL ***98****, 220401 (2007)*



Bethe Ansatz & NRG

$$|\Psi_G\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\chi_\uparrow\rangle \pm |\downarrow\rangle|\chi_\downarrow\rangle)$$

$$\langle S_x \rangle = 1 - \mathcal{O}(\alpha)$$ (small α) Polaronic effect

For all couplings:

$$\langle \Psi_G | S_x | \Psi_G \rangle = \frac{\partial E_G}{\partial \Delta}$$

Quantum decoherence at $\alpha = 1/2$

Thus, one predicts Maximal entanglement at $E \sim 1$

$1 \geq \alpha > 1/2$ Complete screening of the spin

No Rabi oscillations: $\alpha = 1/2$ pure exponential relaxation

Non-Markovian Dynamics: Feynman-Vernon path integral approach

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987); U. Weiss book, quantum dissipative systems, 1999

We integrate out the **BATH** (quadratic action) and follow the spin real-time dynamics

$$\langle \sigma_f | \rho_S(t) | \sigma'_f \rangle = \int \mathcal{D}\sigma(.) \int \mathcal{D}\sigma'(.) \mathcal{A}(\sigma) \mathcal{A}^*(\sigma') F[\sigma, \sigma']$$

The bath effect is all contained in the **INFLUENCE FUNCTIONAL**:

$$F[\sigma,\sigma'] = \exp\Big(-\frac{1}{\pi}\int_{t_0}^{t} ds \int_{t_0}^{s} ds' \big[-iL_1(s-s')\xi(s)\eta(s') + L_2(s-s')\xi(s)\xi(s')\big]\Big),$$

$$\pi\langle X(t)X(0)\rangle_T = L_2(t) - iL_1(t)$$

$$X = \sum_n \lambda_n (b_n^\dagger + b_n)$$

$$\eta(s) = \frac{1}{2}[\sigma(s) + \sigma'(s)]$$

$$\xi(s) = \frac{1}{2}[\sigma(s) - \sigma'(s)]$$

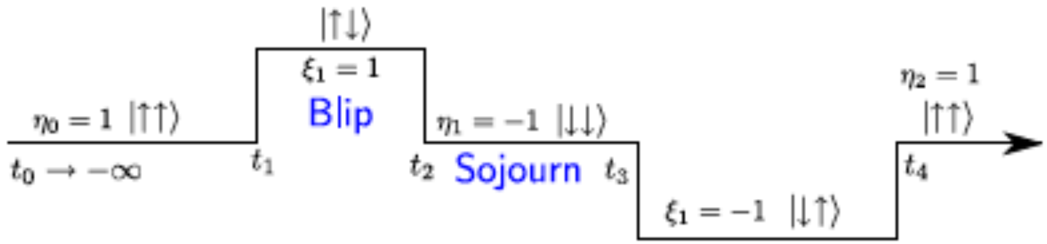
$$L_1(t) = \int_0^\infty d\omega J(\omega) \sin \omega t$$

$$L_2(t) = \int_0^\infty d\omega J(\omega) \cos \omega t \coth \beta\omega/2$$

Parametrization of the Spin Path

$$\xi(t) = \sum_{j=1}^{2n} \Xi_j \theta(t - t_j)$$

$$\eta(t) = \sum_{j=0}^{2n} \Upsilon_j \theta(t - t_j)$$



The variables $\{\Xi_1, \ldots, \Xi_{2n}\} = \{\xi_1, -\xi_1, \ldots, -\xi_n\}$ with $\xi_j = \pm 1$ describe the n off-diagonal or "blip" parts of the path spent in the states $\{|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle\}$ during times $t_{2m-1} < t < t_{2m}$ $(m = 1, \ldots, n)$, where $\xi(t) = \pm 1$ and $\eta(t) = 0$. The variables $\{\Upsilon_0, \ldots, \Upsilon_{2n}\} = \{\eta_0, -\eta_0, \ldots, \eta_n\}$, on the other hand, characterize the $(n+1)$ diagonal or "sojourn" parts of the path during times $t_{2m} < t < t_{2m+1}$ $(m = 0, \ldots, n)$, where $\eta(t) = \pm 1$ and $\xi(t) = 0$.

$$p(t) = 1 + \sum_{n=1}^{\infty} \left(\frac{i\Delta}{2}\right)^{2n} \int_{t_0}^{t} dt_{2n} \ldots \int_{t_0}^{t_2} dt_1 \sum_{\{\Xi_j\}} \sum_{\{\Upsilon_j\}'} \mathcal{F}_n[\{\Xi_j\}, \{\Upsilon_j\}, \{t_j\}]$$

$$\mathcal{Q}_1 = \exp\left[\frac{i}{\pi} \sum_{j>k\geq 0}^{2n} \Xi_j \Upsilon_k Q_1(t_j - t_k)\right]$$

$$\mathcal{Q}_2 = \exp\left[\frac{1}{\pi} \sum_{j>k\geq 1}^{2n} \Xi_j \Xi_k Q_2(t_j - t_k)\right]$$

$$F_n\Big[\{\Xi_j\}, \{\Upsilon_j\}, \{t_j\}\Big] = \mathcal{Q}_1 \mathcal{Q}_2$$

$$\ddot{Q}_{1,2} = L_{1,2}$$

Main Idea to solve the Blip-Blip Interaction: Hubbard Stratonovitch transformation

Peter P. Orth, Adilet Imambekov, Karyn Le Hur Phys. Rev. B **87**, 014305 (2013) **(Review)**
See also G. B. Lesovik, A. V. Lebedev, A. Imambekov JETP Lett. 75, p. 474, (2002);
A. Imambekov, V. Gritsev, E. Demler, Phys. Rev. A 77, 063606 (2008).
J.T. Stockburger, H. Grabert Phys. Rev. Lett. 88, 170407 (2002).
Non-Markovian Approach

$$Q_2 = \exp\Big\{-n\alpha\big[\ln(1+4\omega_c^2 t_{\rm tot}^2) + G\big]\Big\} \int d\mathcal{S} \exp\Big\{i\sum^{2n} \Xi_j h_s(\tau_j)\Big\}$$

The function $h_s(\tau)$ contains information about the environmer

tions and eigenvalues of the bath correlation function $Q_2(t)$. It also depends on

For the Ohmic Bath, this results in $Q_1(t) = 2\pi\alpha \tan^{-1}(\omega_c t) \approx \alpha\pi^2$

$$p(\tau) = 1 + \int d\mathcal{S} \sum_{n=1}^{\infty} \left(\frac{i\Delta t_{\rm tot} e^{-(\alpha/2)\left[\ln(1+4\omega_c^2 t_{\rm tot}^2)+G\right]}}{2} \right)^{2n}$$

$$\times \int_0^{\tau} d\tau_{2n} \cdots \int_0^{\tau_2} d\tau_1 \sum_{\{\xi_j,\eta_j\}} \exp\Big[i\pi\alpha \sum_{k=0}^{n-1} \eta_k \xi_{k+1}\Big] \prod_{j=1}^{2n} \exp\left[i\,\Xi_j h(\tau_j)\right]$$

In fact, the Q_1 function can also be treated in a general way (L. Henriet, Z. Ristivojevic, KLH)

Stochastic Schrodinger Equation

In a 4 by 4 Matrix Form, we obtain: A spin in Random Magnetic Fields

$$p(\tau) = \int d\mathcal{S} \langle \Phi_f | T e^{-i \int_0^\tau ds V(s)} | \Phi_i \rangle$$

$$i \frac{\partial}{\partial \tau} |\Phi(\tau)\rangle = V(\tau) |\Phi(\tau)\rangle$$

Note: This is a numerically exact Approach, Non-Markovian Effects captured
Little Price to Pay: Numerical Convergence, similar to QMC in spirit
Different from J. Dalibard, Y. Castin, K. Molmer, Phys. Rev. Lett. **68**, 580 (1992)

Applications: Landau-Zener problem for Ohmic spin-boson model (Peter, Adilet, Karyn 2010)
Dissipative Rabi models: Loic Henriet, Zoran Ristivojevic, Peter P. Orth, KLH in progress

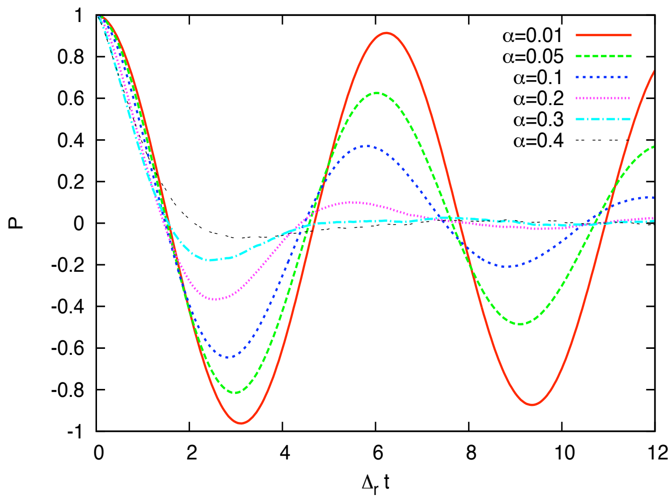
Results: Analytical Approach & tricky NRG numerics

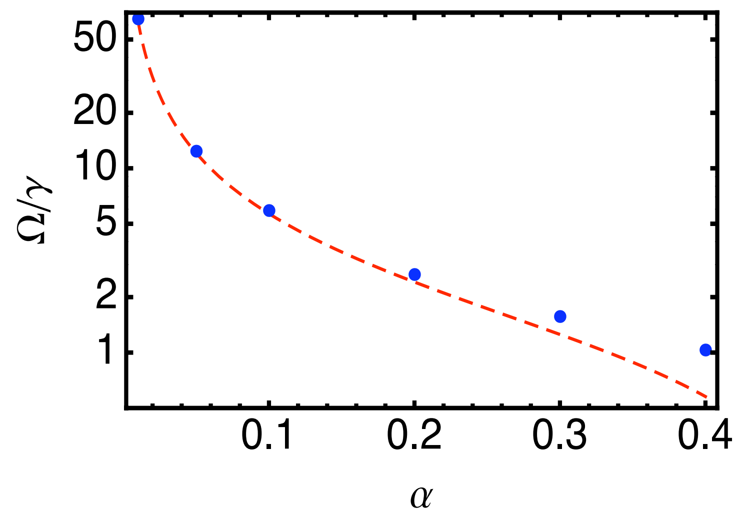
P. Orth, A. Imambekov, K. Le Hur, stochastic Equation

D. Roosen, K. Le Hur, W. Hofstetter, time-dependent NRG

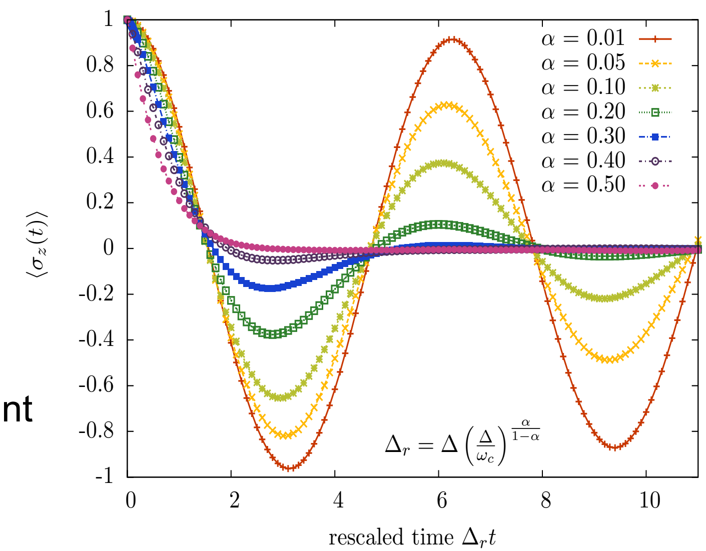
$P=\langle\sigma_z(t)\rangle$

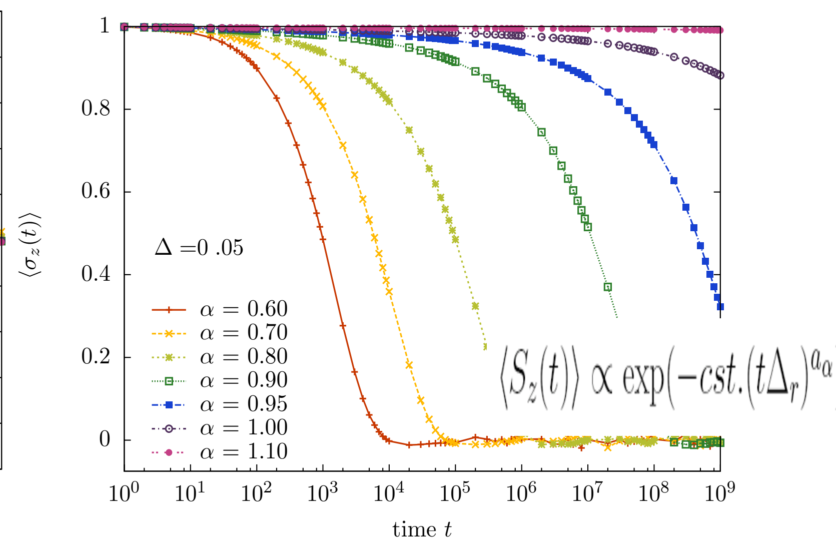
Random Variable Approach





Time-dependent Wilson NRG





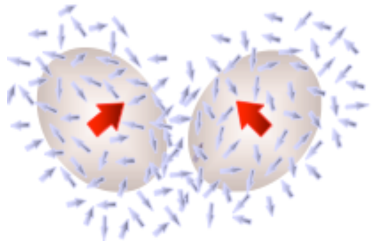
Other Developments

More than one spin and dissipative quantum phase transitions

RKKY interaction between spins and pseudo-spins

P. P. Orth, D. Roosen, W. Hofstetter and KLH 2010

M.R. Delbecq, L.E. Bruhat, J.J. Viennot, S. Datta, A. Cottet and T. Kontos, Nature Communications **4**, 1400 (2013)



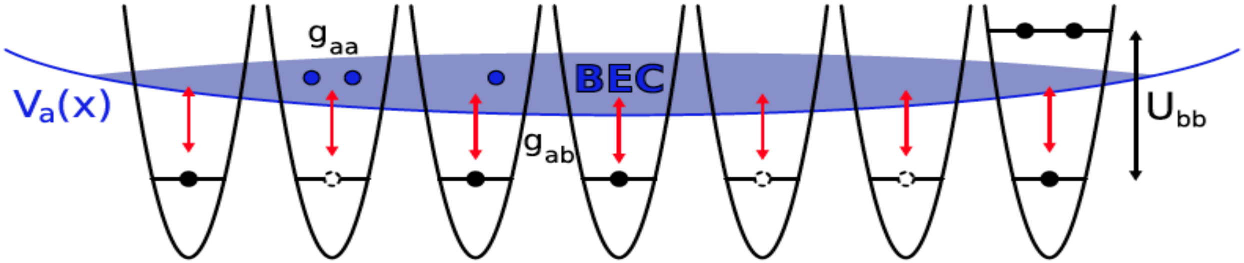
Realization of quantum Ising chain in a transverse field

P. G. de Gennes 1963

Dissipation can be controlled and tuned to ZERO in principle

Potential application to Majorana fermions

Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)



Topological Phases with <u>Bosons</u> & Spin degrees of freedom

Topological "quantum numbers"

1) Tight-Binding models with **Bands** with non-zero Chern number

$$\nu^{(n)} = \frac{1}{2\pi} \int_{\mathrm{BZ}} d^2\mathbf{k} \left(\partial_{\mathbf{k}} \times \mathscr{R}^{(n)}(\mathbf{k}) \right),$$

where the vector field $\mathscr{R}^{(n)}(\mathbf{k})$ is the Berry gauge potential associated to the n^{th} Bloch band,

$$\mathscr{R}^{(n)}(\mathbf{k}) = -i \langle n\mathbf{k} | \partial_{\mathbf{k}} | n\mathbf{k} \rangle.$$

Also true for Bosons on lattices

2) Superconductors or bosonic Superfluids: Meissner Currents

$$\oint \mathbf{A} \cdot d\mathbf{r} = -\frac{\hbar}{2e} \oint \mathrm{grad} S \cdot d\mathbf{r} = n \frac{h}{2e}$$

Take Home Message

Discuss two bosonic models where chiral edge modes appear with topological Quantum numbers:

1) Tight-Binding Models of bosons with Artificial Gauge Fields:

Bloch Bands with Chiral Edge Modes

Applications to Photon Systems and Cold Atoms

A. Petrescu, A. A. Houck, KLH PRA 2012

2) Can we have exotic Mott phases of Bosons with chiral currents?

Chiral Mott insulator Arya Dhar et al. PRA A 85, 041602 (2012)

Bosonic Mott insulator with Meissner Currents

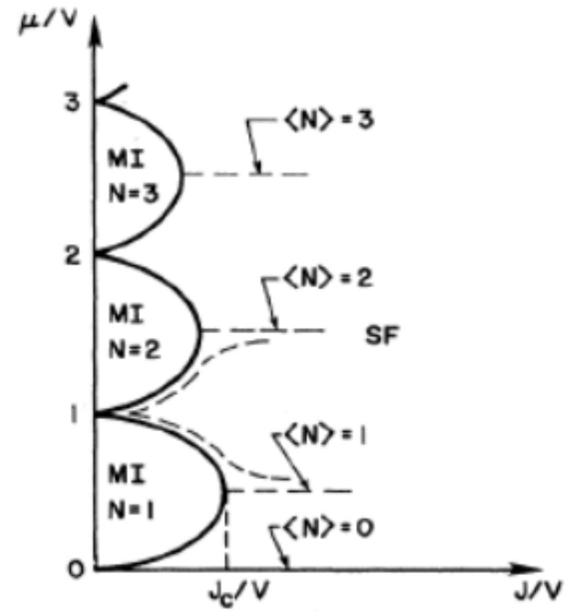
Alex Petrescu and KLH, arXiv:1306.5986 **(to appear in PRL)**

I will start with the point 2)

Mott Physics in Boson Systems: Lattice Effects

Bose-Hubbard model of a single lattice boson:

$$H = -t \sum_{<ij>} b_i^\dagger b_j + \sum_i \frac{U}{2} n_i (n_i - 1) - \mu n_i$$



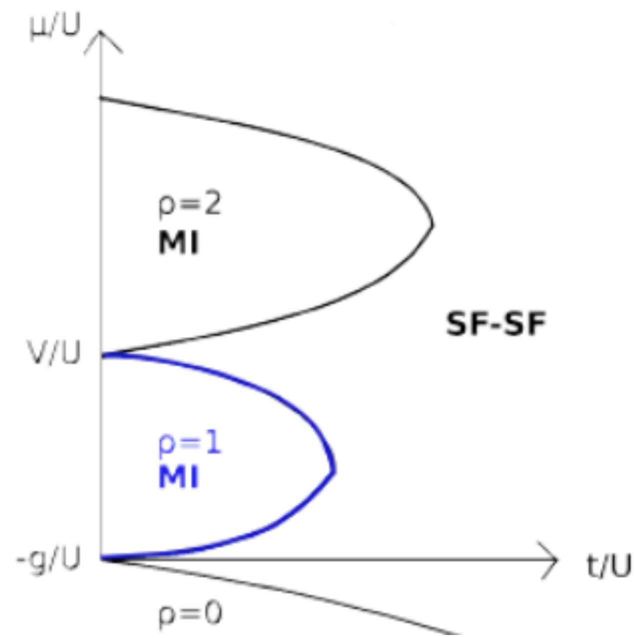
Two-species Bose-Hubbard model:

$$H = -t \sum_{\alpha=1,2} \sum_{<ij>} b_{\alpha i}^\dagger b_{\alpha j} + \sum_{\alpha i} \frac{U}{2} n_{\alpha i} (n_{\alpha i} - 1) - \mu n_{\alpha i}$$

$$+ \sum_i V_\perp n_{1i} n_{2i} - g \sum_i b_{1i}^\dagger b_{2i} + H.c.$$

Mott at ρ=1

Interchain coherence:
Meissner effect



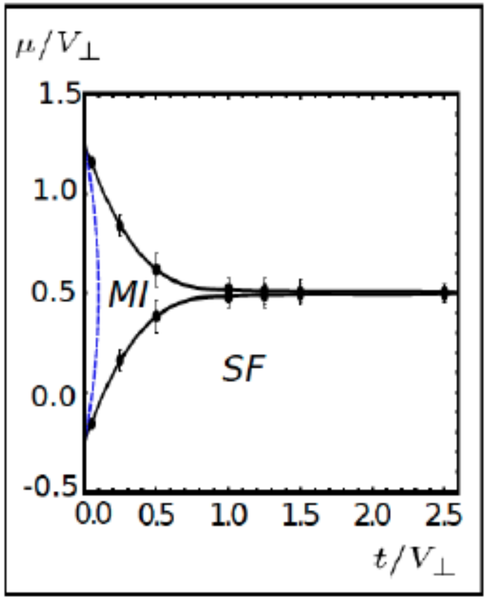
Multicomponent systems: active field in cold atoms

e.g. E. Altman, W. Hofstetter, E. Demler, M. Lukin 2003

Route for Chiral Mott Insulator: Spin Meissner Effect (2)

Mott insulating phase of total density:

$$\rho = b_1^\dagger b_1 + b_2^\dagger b_2$$

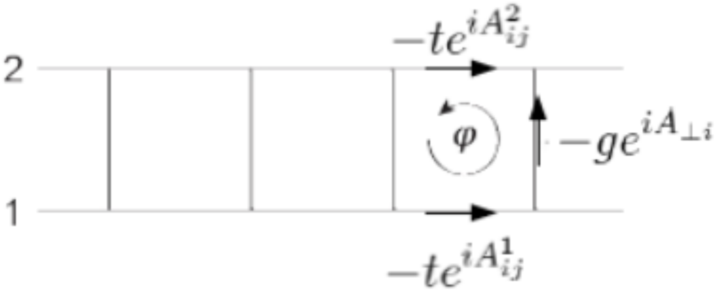


Relative density exhibits fluctuations.

$$\sigma^z = b_1^\dagger b_1 - b_2^\dagger b_2$$

(At ρ=1, spin ½ exchange Hamiltonian)

Example: Ladder System



A. Petrescu and KLH, arXiv:1306.5986

Superfluid Phase, Check

$$j_{\|} = it(-e^{iaA^1_{ij}} b^\dagger_{1i} b_{1j} + e^{iaA^2_{ij}} b^\dagger_{2i} b_{2j}) + \text{H.c.},$$
$$j_{\perp} = -2igb^\dagger_{1i} b_{2i} e^{ia' A_{\perp i}} + \text{H.c.}$$

Outside the Mott lobe, the phase-angle representation is justified $b^\dagger_{1,2i} = \sqrt{n} e^{i\theta_{1,2i}}$ (in this reasoning, $n = \rho/2$ represents the mean (superfluid) density in each species).

The conversion takes the form of a Josephson coupling

$$-g\cos(a' A_{\perp i} + \theta_{1i} - \theta_{2i}).$$

For strong g, the superfluid phases will be pinned by this term such that $a' A_{\perp i} + \theta_{1i} - \theta_{2i} = 0$. Then $j_\perp$ vanishes and furthermore in the small field limit we may expand to obtain the Meissner form of the intraspecies current

$$\langle j_{\|} \rangle = -2tn\, \text{phase}_{ij}.$$

1D: Orignac-Giamarchi, 2001

Mott Regime: Pseudo-spin H

$$H_\sigma = -\sum_{\langle ij \rangle} \left(2J_{xx} (\sigma_i^+ \sigma_j^- e^{iaA_{ij}^\sigma} + \text{H.c.}) - J_z \sigma_z^i \sigma_z^j \right)$$

$$-g \sum_i (\sigma_i^x \cos(a' A_{\perp i}) - \sigma_i^y \sin(a' A_{\perp i})),$$

$$\text{with } J_{xx} = \frac{t^2}{V_\perp} \text{ and } J_z = t^2 \left(-\frac{2}{U} + \frac{1}{V_\perp} \right)$$

$$j_\parallel = 2J_{xx} \big[\cos(A_{ij}^\sigma)(\sigma_i^y \sigma_j^x - \sigma_i^x \sigma_j^y) + \sin(A_{ij}^\sigma)(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \big],$$

$$j_\perp = -2g \left[\cos(a' A_{\perp i}) \sigma_i^y + \sin(a' A_{\perp i}) \sigma_i^x \right].$$

$$\langle j_\parallel \rangle = -2J_{xx} \, \text{phase}_{ij}$$

Meissner currents survive

Solution

- 1D: The Hamiltonian can be bosonized
- Generalization in higher dimensions

Pseudo-spin coherent states

$$|\psi\rangle = \prod_i (\cos\phi_{\sigma i}\,|\uparrow\rangle_i + e^{i\theta_{\sigma i}}\sin\phi_{\sigma i}\,|\downarrow\rangle_i)$$

$$H_\sigma[\theta_\sigma, \phi_\sigma] = \frac{1}{2}\int \frac{d^d x}{a^{d-2}} J_{xx}\left(\nabla\theta_\sigma - A^\sigma\right)^2 - \int \frac{d^d x}{a^d} g\cos\left(\theta_\sigma + a' A_\perp\right)$$

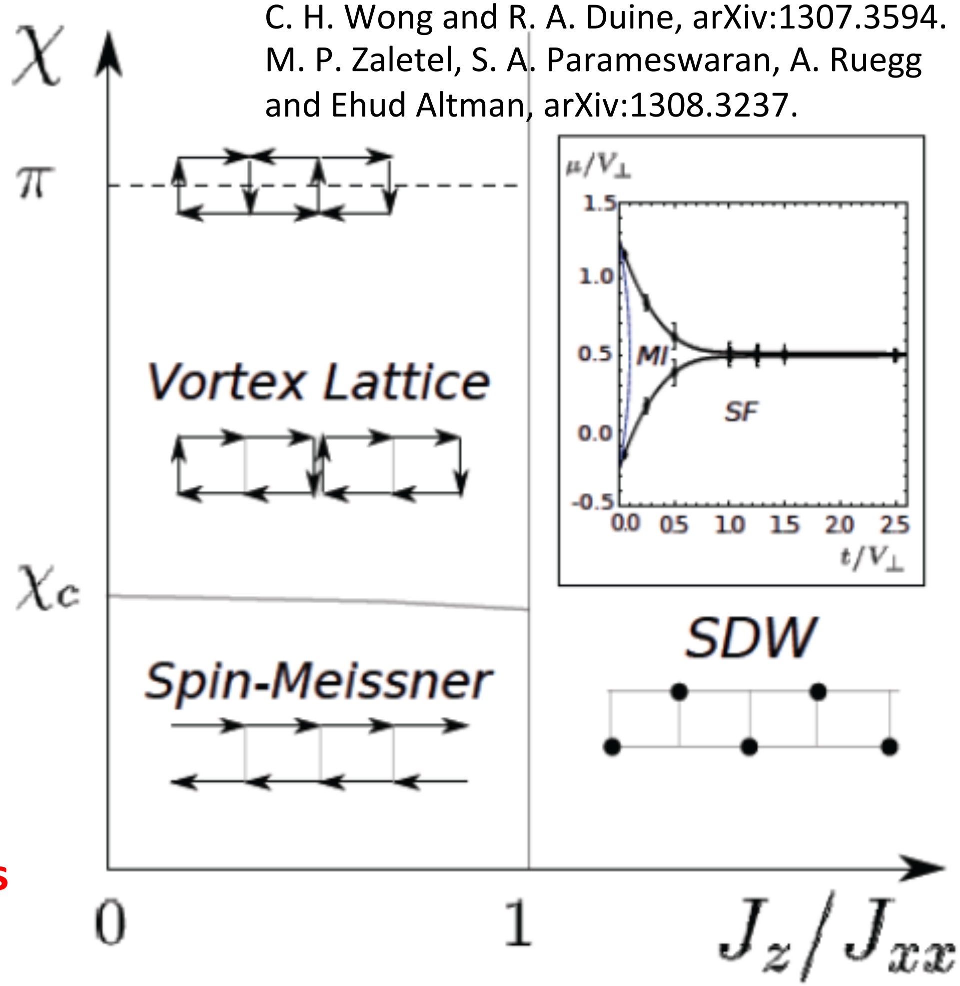
Coupled Ladder Models give the same conclusion (See Supplementary Material)

Chiral Mott Phases

Chiral Mott insulator Arya Dhar et al. PRA A 85, 041602 (2012)

C. H. Wong and R. A. Duine, arXiv:1307.3594.
M. P. Zaletel, S. A. Parameswaran, A. Ruegg and Ehud Altman, arXiv:1308.3237.

Analogy to DDW phase in high-Tc systems



DMRG results
Collaboration with
G. Roux (Orsay)

Topology through Flux Quantization

Realizable in cold atoms And Josephson junctions

Bosonic Mott insulator with Meissner Currents Alex Petrescu and KLH, arXiv:1306.5986

Cold Atoms:

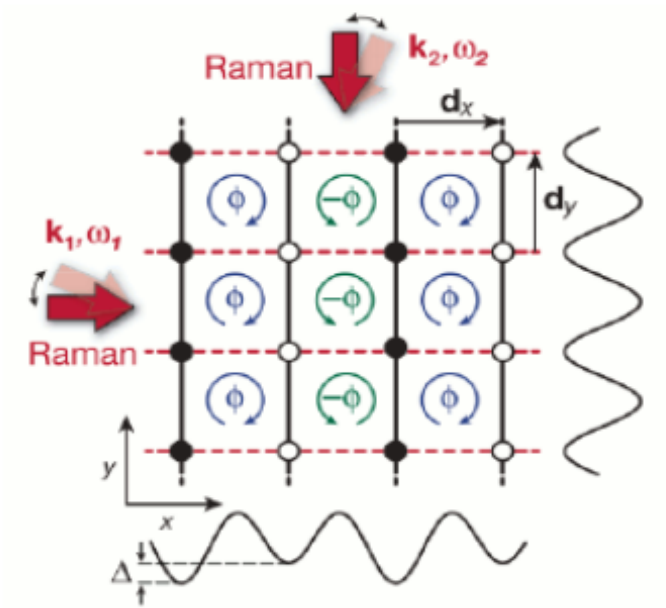
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- Ways to implement magnetic fields & gauge fields

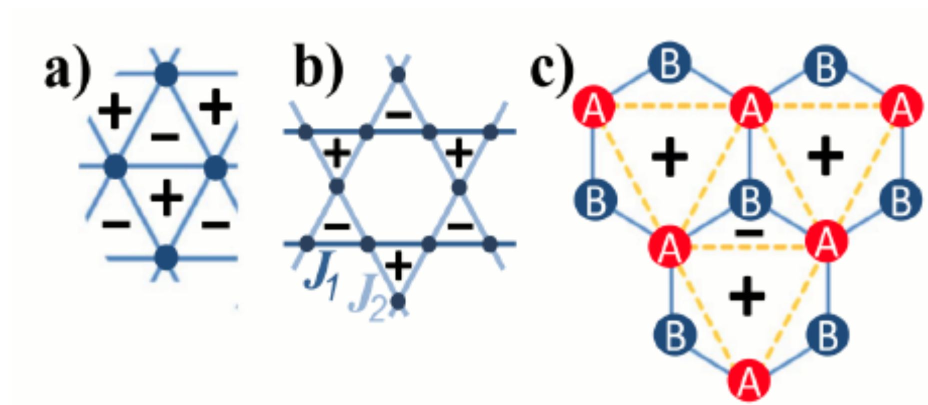
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)

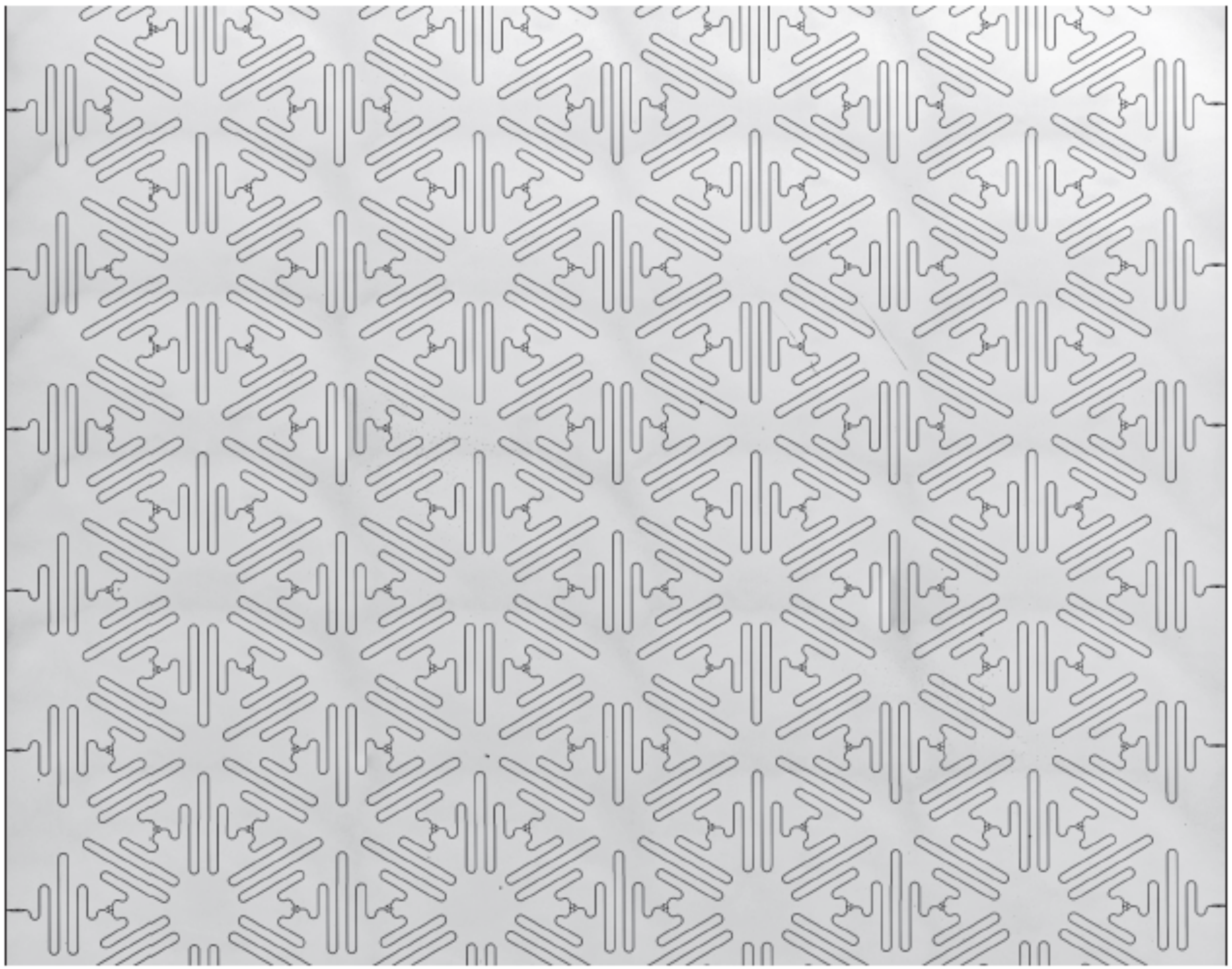


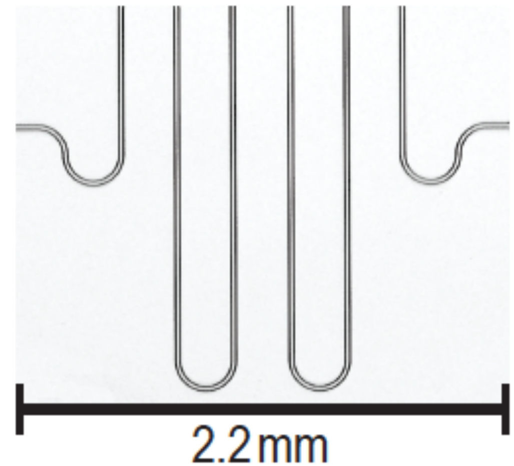
Shaken Optical Lattices

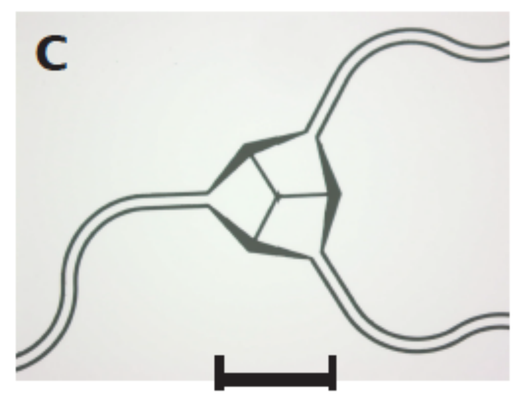
M. Aidelsburger et al., arXiv:1308.0321.
Hirokazu Miyake et al. (MIT), arXiv:1308.1431.

Artificial Graphene with Light

M. C. Rechtsman et al. Nature (2013)
See also M. Bellec, U. Kuhl, Gilles Montambaux, F. Montessagne PRL 2013

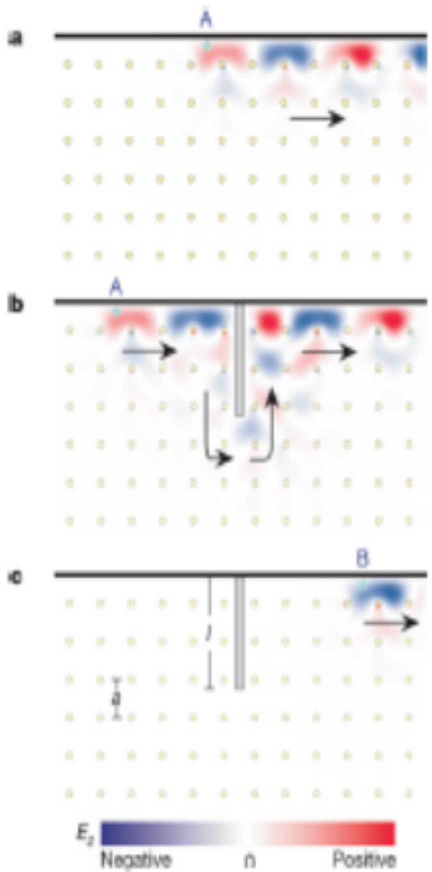




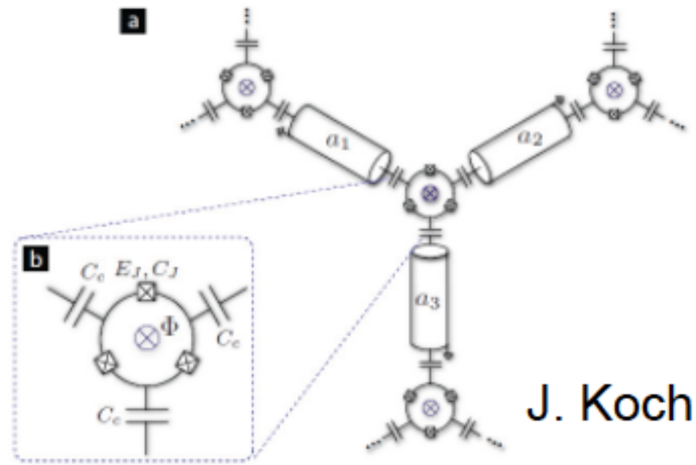


D. L. Underwood, W. Shanks, J. Koch, A. Houck, PRA 2012

Artificial Gauge Fields with Light

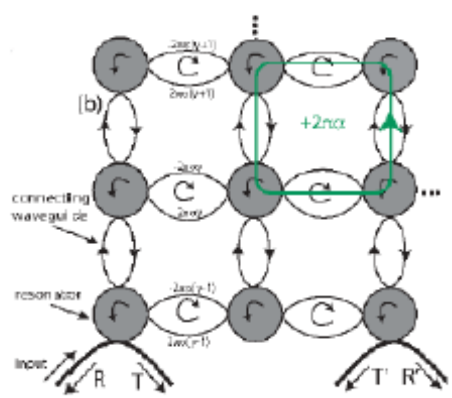


Haldane-Raghu, PRL 2008
Z. Wang et al. Nature 2009

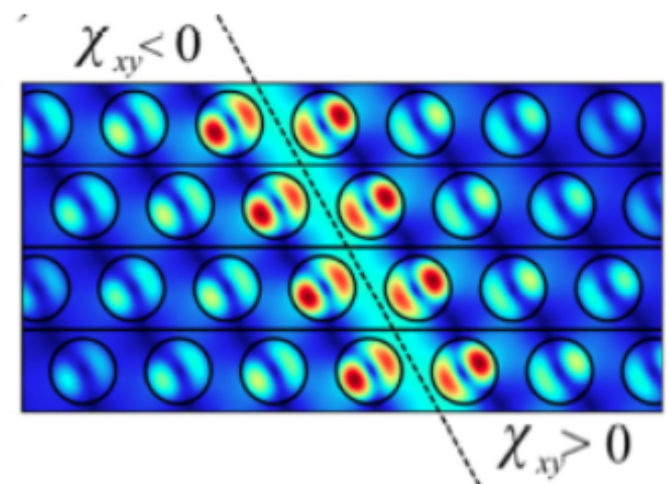


J. Koch et al, PRA 2010

Review:
I. Carusotto
& C. Ciuti
RMP 2012



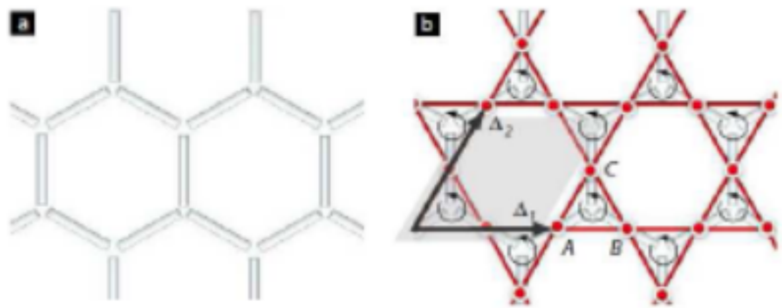
M. Hafezi, E. Demler, M. Lukin, J. Taylor 2011



A. MacDonald et al. 2012

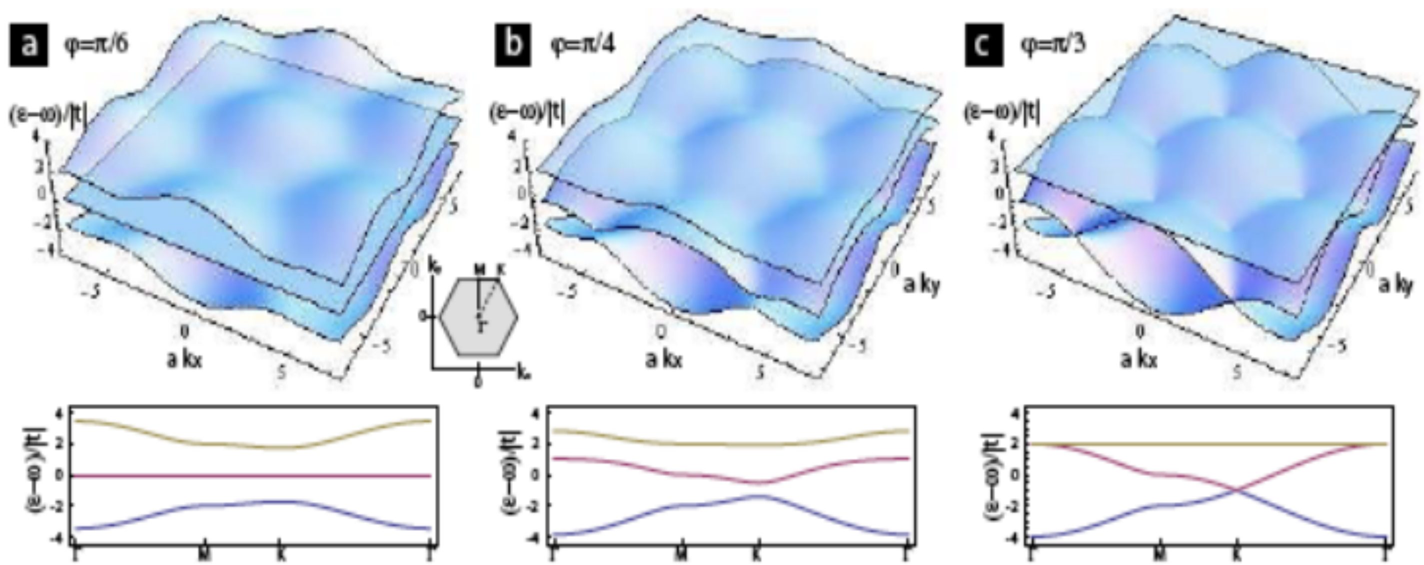
Breaking T-reversal symmetry: Josephson ring circulators

Dice lattice:
J. Vidal, R. Mosseri
and B. Doucot, 1998



J. Koch, A. Houck, KLH
and S. M. Girvin
PRA **82**, 043811 (2010)

ViewPoint:
A. Greentree & A. Martin,
Physics 3, **85** (2010)



Kagome lattice: why interesting...

Flat band (search for ferromagnetism)
A. Mielke; H. Tasaki; E. Lieb

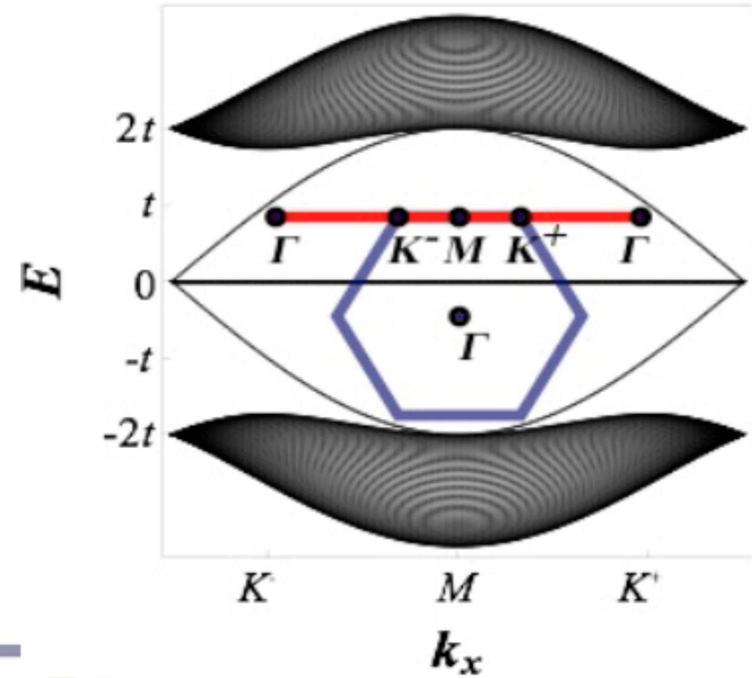
Exotic Topological Phases:
H. M. Guo & M. Franz, PRB 2009
E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011
N. Regnault and A. Bernevig, PRX 2012,...

Spin liquid search, classical degeneracies
Experimentally relevant: 2D Materials (Orsay; Princeton;...)
Cold atoms: Berkeley; see D. Stamper-Kurn group, 2011
L. Balents, Nature 464, 199 (2010)
S. Yang, D. Huse and S. White, Science (2011)
S. Dupenbrock, I. P. McCulloch, U. Schwollwoeck, PRL 2012
D. Poilblanc and N. Schuch (2013)
Work by Laura Messio, Claire Lhuillier, Bernard Bernu, G. Misguich...

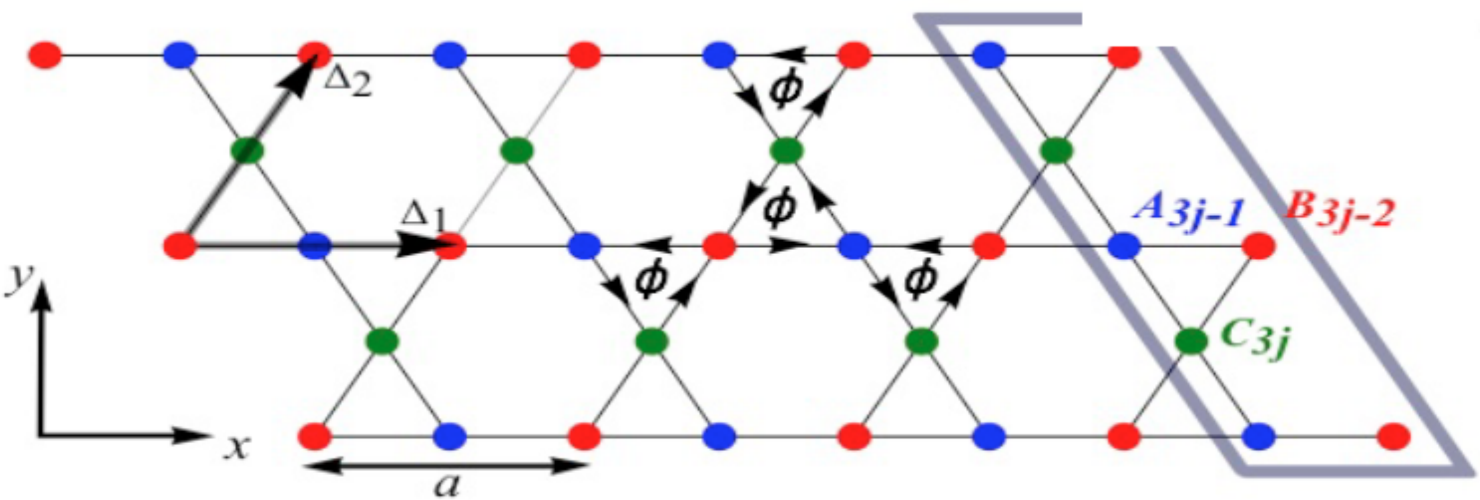
Topological bosons 1: band argument

A. Petrescu, A. Houck, KLH PRA 2012

- Proximity Effect of a trivial band?
- How to probe edge modes?
- Effect of Disorder?
- Effect of Interactions



$\Phi = \pi/6$



Variant of definitions

So far, we have dealt with a single Bloch band. In a multiple band system, we can define the following quantity as a sum over states below some energy:

$$\nu(E) = \frac{1}{2\pi} \sum_n \int_{\mathrm{BZ}} d^2\mathbf{k}\,\theta(E - E_n(\mathbf{k}))\mathscr{F}_{\mathbf{k}}^{(n)},$$

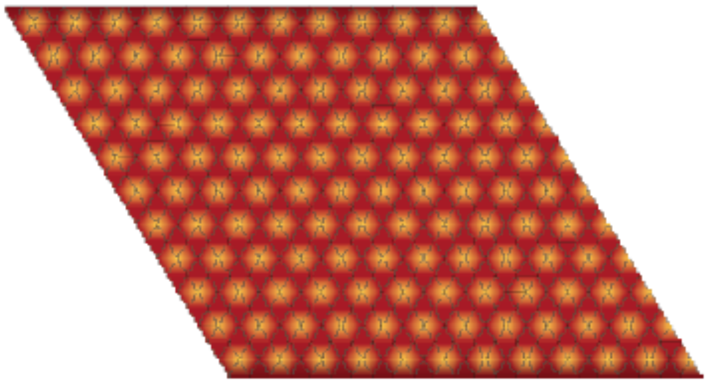
$$\mathscr{F}_{\mathbf{k}} \equiv \partial_{\mathbf{k}} \times \mathscr{R}(\mathbf{k})$$

$$\nu(E) = \frac{1}{2\pi i} \mathrm{Tr}\left\{P_{\mathbf{k}}\left[\partial_{k_x} P_{\mathbf{k}}, \partial_{k_y} P_{\mathbf{k}}\right]\right\}$$

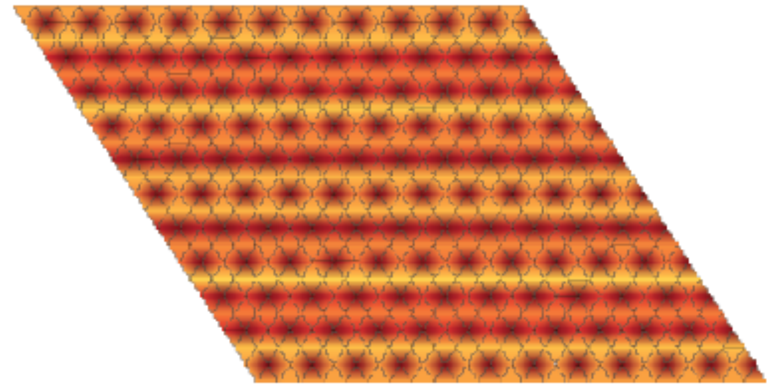
J. Bellissard; E. Prodan

$$\nu(E) = - \lim_{N\to\infty} \frac{2\pi i}{N} \sum_{\mathbf{m}} \langle \mathbf{r}_{\mathbf{m}} | P(E)\left[-i[x, P(E)], -i[y, P(E)]\right] | \mathbf{r}_{\mathbf{m}} \rangle,$$

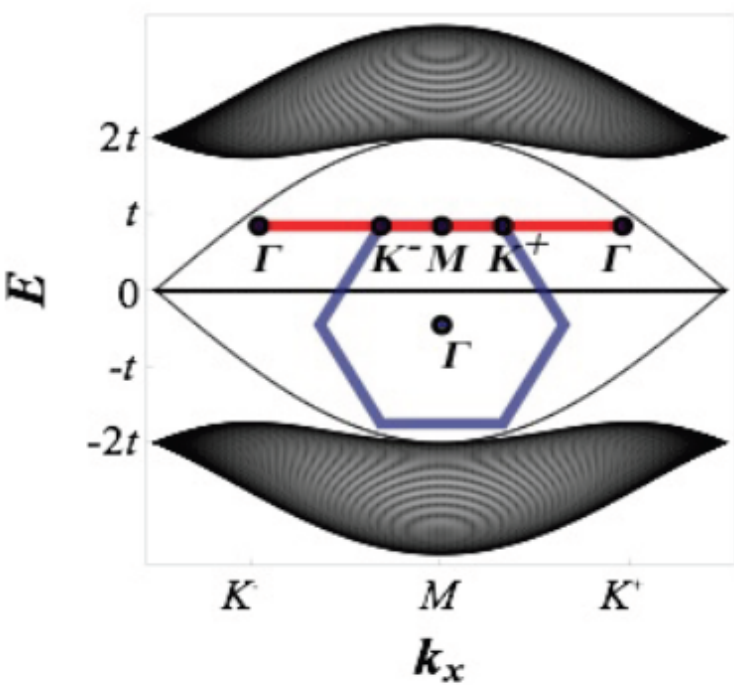
$$\mathbf{r}_{\mathbf{m}} = m_1 \mathbf{\Delta}_1 + m_2 \mathbf{\Delta}_2$$

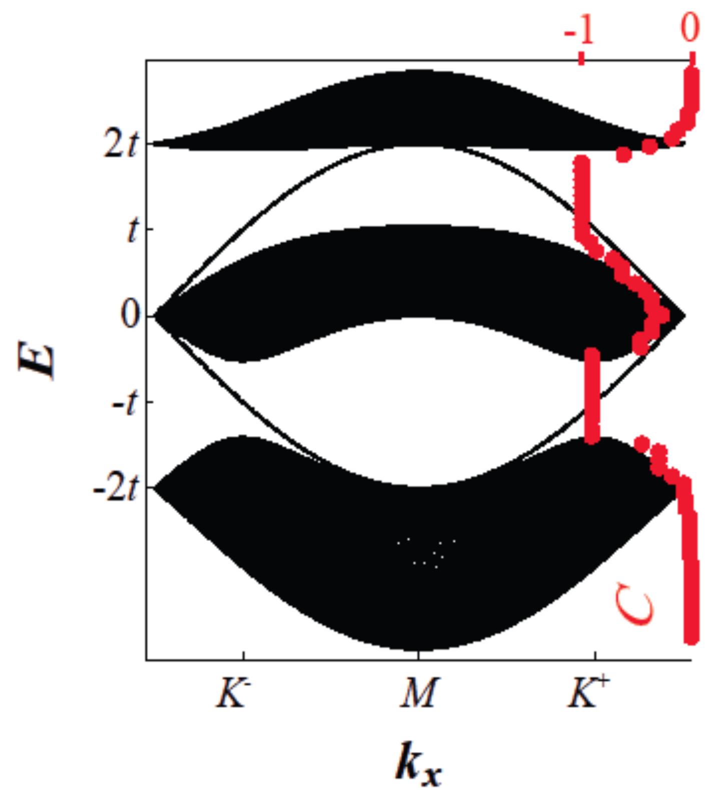


Φ = π/6



Φ = π/4



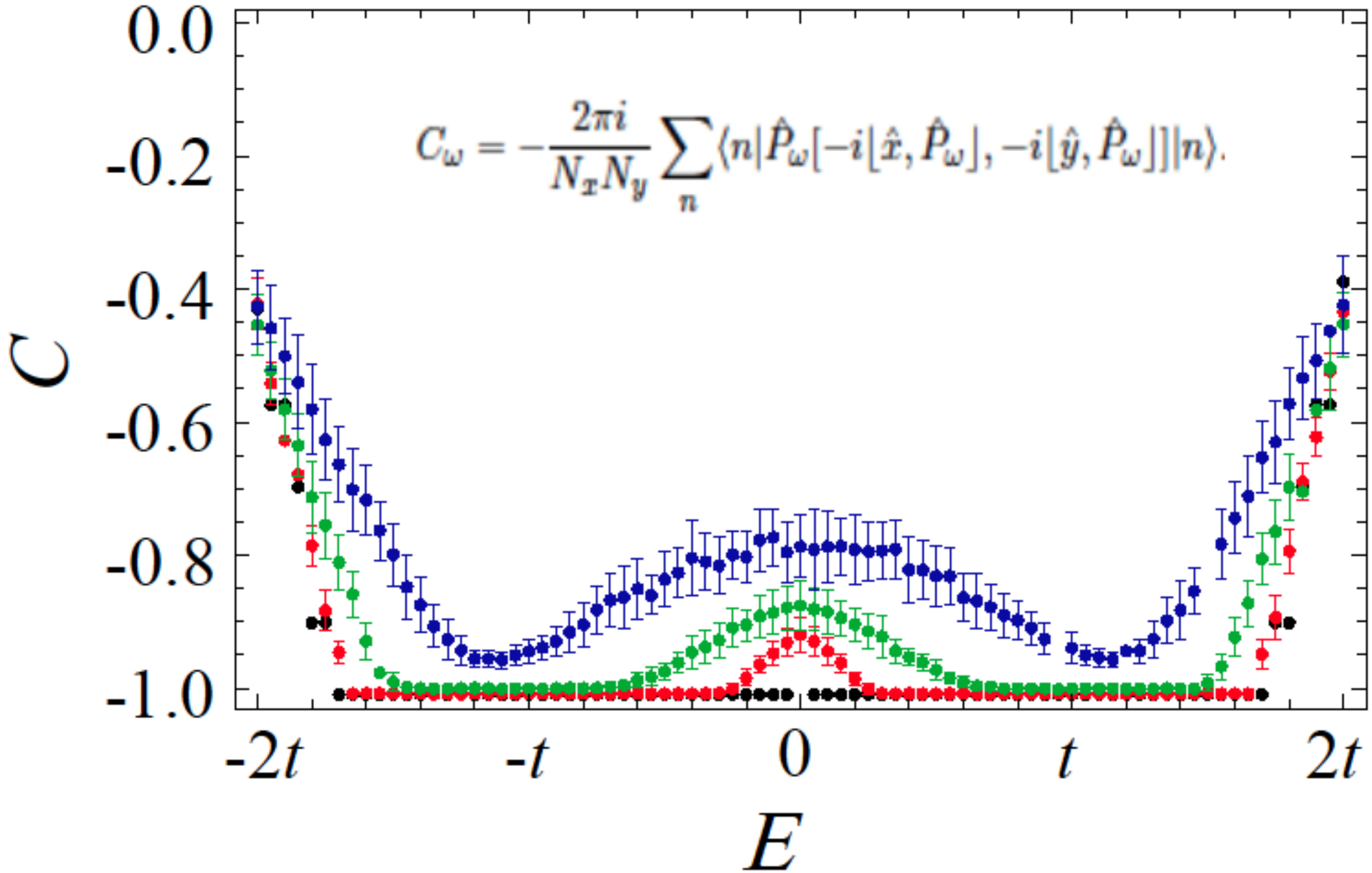


Karplus-Luttinger, 1954

D. Haldane, 2004

See also
D. Bergman
& G. Refael, 2010

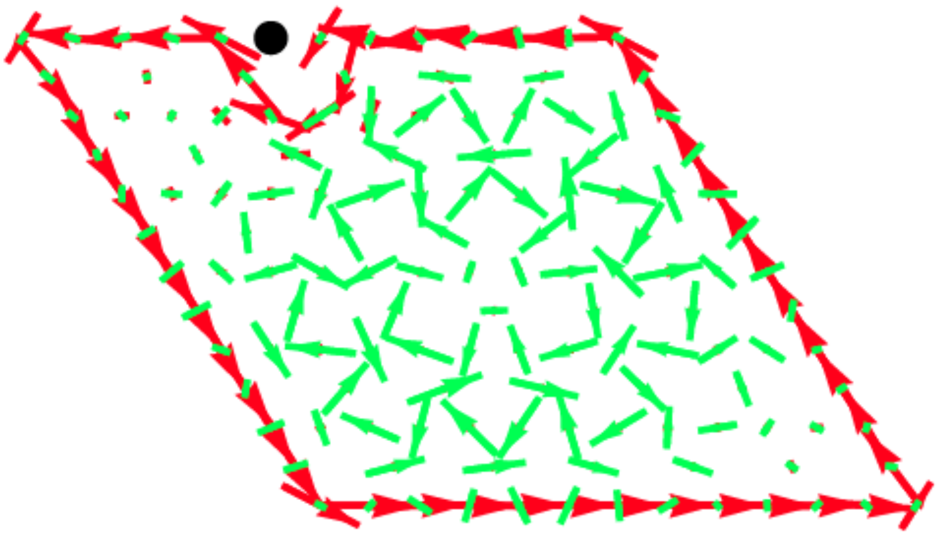
Disordered case at Φ=π/6



Real Space computation of Chern number following J. Bellissard; E. Prodan (non-commutative geometry)

Quantum versus Anomalous Hall

$$j_{mn} = -ic_m^\dagger(t_{mn} + t_{nm}^*)c_n + ic_n^\dagger(t_{mn}^* + t_{nm})c_m$$



Red: situation at Φ = π/4 (gap)
Green: situation at Φ = π/4 (bulk states)

situation at Φ = π/6
disordered case

Chern number **non-quantized** for AHE and measurable…

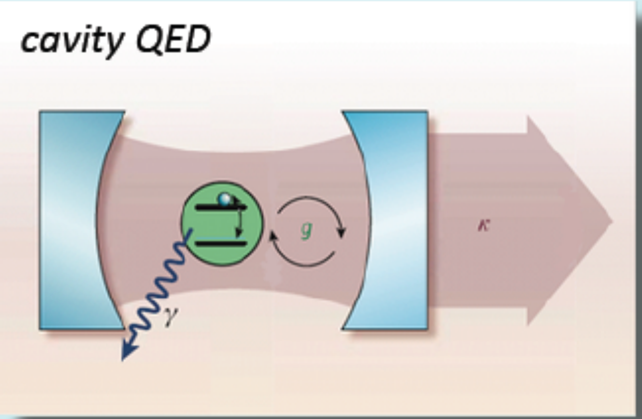
Synthetic B-field: Loops in k space and interference experiment

See also related idea by D. Price and N. Cooper, PRA 2012

Cavity & Circuit QED: 1 cavity a lot of activity...

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity



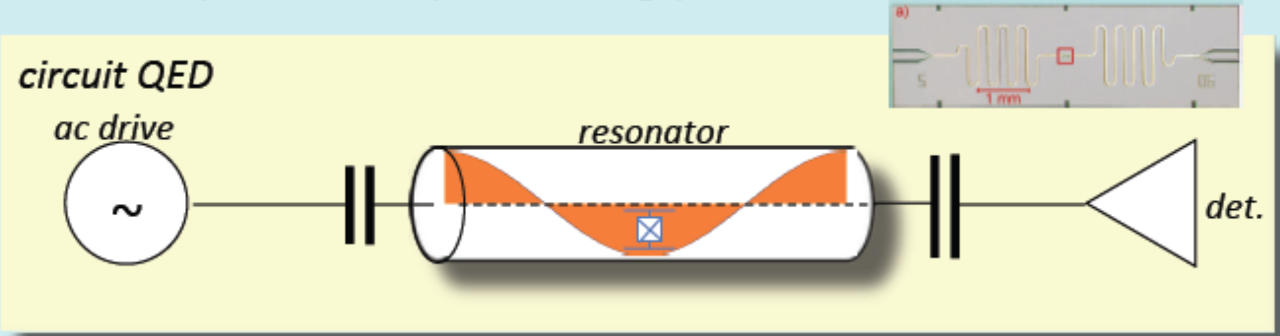
$2g$ = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\omega_a\sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\text{drive}} + H_{\text{baths}}\right)$$

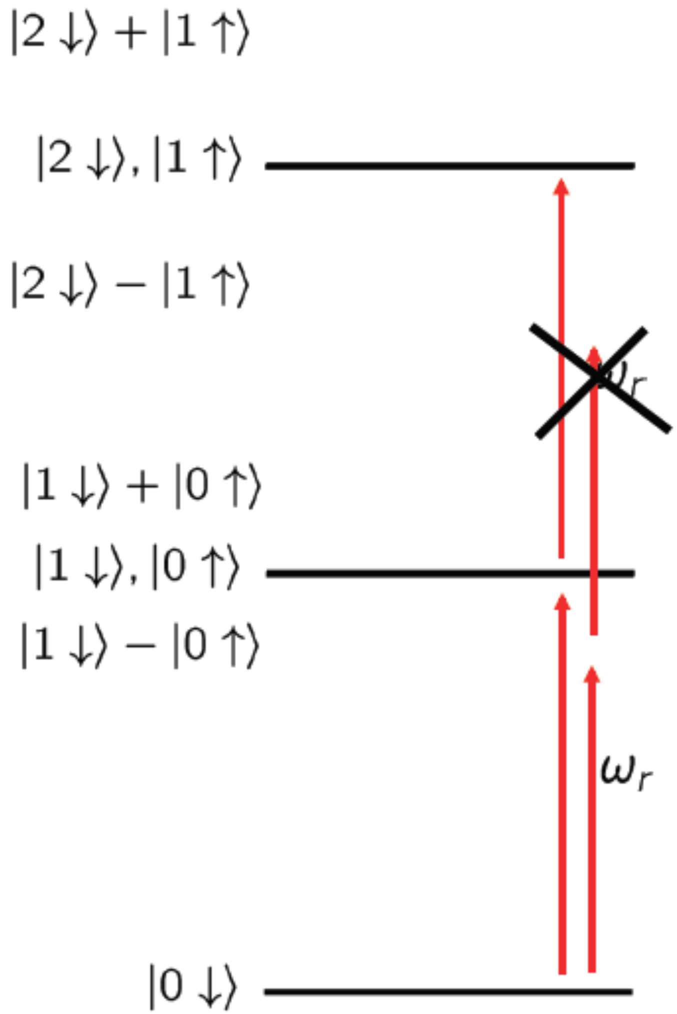
also Dicke model realized experimentally in cold atoms ETH Zuerich

- same concept works for superconducting qubits!



JM Raimond, M. Brune, S. Haroche , Rev. Mod. Phys. **73**, 565 (2001); R. J. Schoelkopf, S.M. Girvin, Nature **451**, 664 (2008)

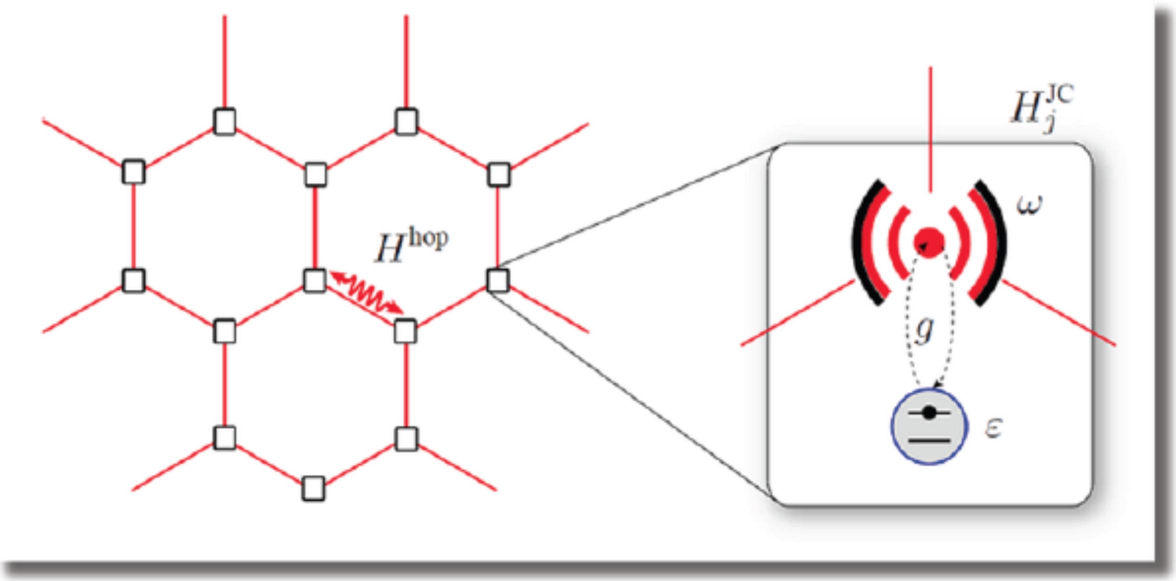
Photon blockade



$$H = \frac{1}{2}\omega_a \sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right)$$

- single atom inside cavity can make spectrum anharmonic!
- hybridized atom/photon object is a *polariton*
- photons have to go one by one!

The Jaynes-Cummings "Lattice" Model



Jaynes-Cummings model: 1963
(famous model in quantum optics)

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)
Loic Henriet, Zoran Ristivojevic, KLH

Other groups: R. Fazio, G. Blatter, H. Tureci & M. Schiro, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik, ...

Jaynes-Cummings lattice model $H = \sum_j H_j^{\mathrm{JC}} + H^{\mathrm{hop}} - \mu N$ ← "chemical potential"

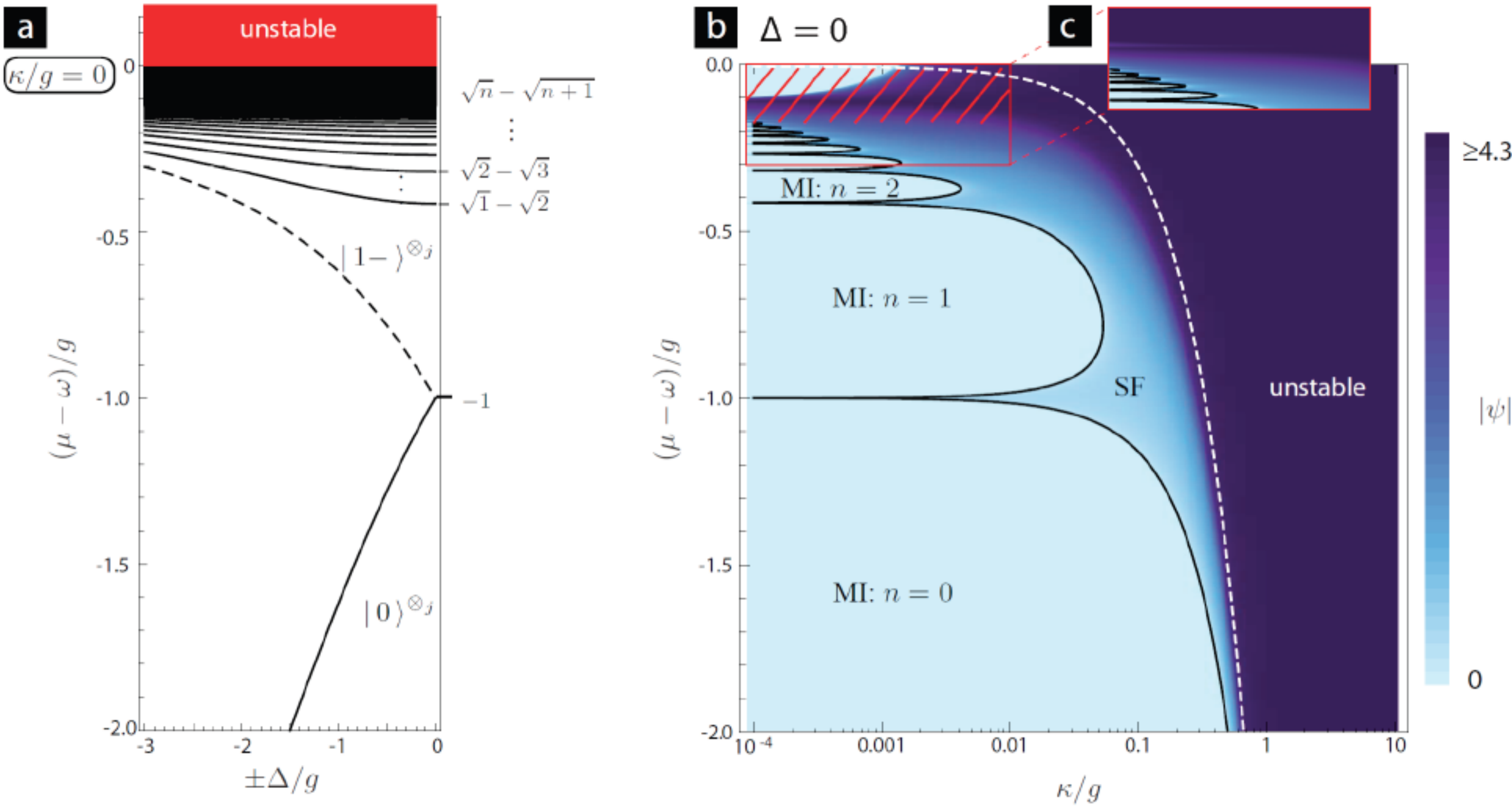
- *Jaynes-Cummings:* $H_j^{\mathrm{JC}} = \omega a_j^\dagger a_j + \varepsilon \sigma_j^+ \sigma_j^- + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j)$
- *nearest-neighbor photon hopping:* $H^{\mathrm{hop}} = -\kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i)$
- *polariton number:* $N = \sum_j (a_j^\dagger a_j + \sigma_j^+ \sigma_j^-)$

Difficulty with photons: Simulate the chemical potential

MFT results for the JC lattice

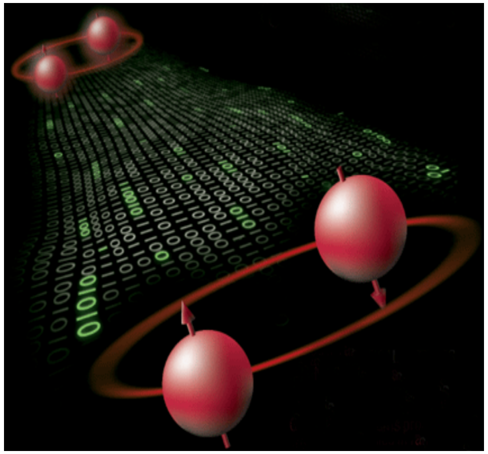
Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)

Needs to engineer the Mott state via driving (pumping)



Jens Koch and KLH, PRA **80** 023811 (2009)

We have closed the Loop…



Many-Body Dynamics: Simple Mean-Field Prototype, Impurity in A Many-Body Environment

Stochastic Approach to Describe Non-Markovian Effects

Bath Useful to Induce (long-range) **Interactions**
Cold Atoms: Controllable Decoherence

Topology: **Bosonic Mott Phase with Meissner Currents**

Topological Phases of Light in cQED Complex geometries
Driven Effects lead to Non-Equilibrium description

Thanks to Adilet
Thanks to collaborators & Thanks to Organisers