Introduction to Topology

Karyn Le Hur

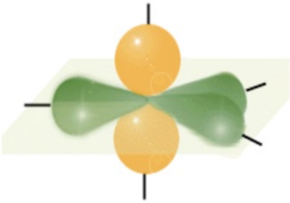
Ecole Polytechnique, CPHT and CNRS

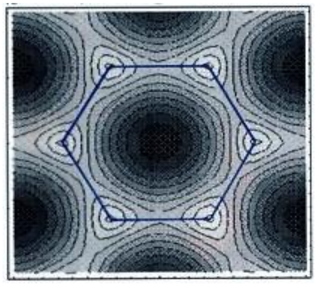
Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

Aile 0, 10.21 CPHT
karyn.le-hur@polytechnique.edu

Silke Biermann & Luca Perfetti

We start then from the graphene
Sessions (days) 4 and 5





Bloch's theorem

Amphi 04

Schrödinger equation in a periodic potential: $V(\vec{r}+\vec{R}) = V(\vec{r})$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right]\psi(\vec{r}) = \varepsilon\,\psi(\vec{r})$$

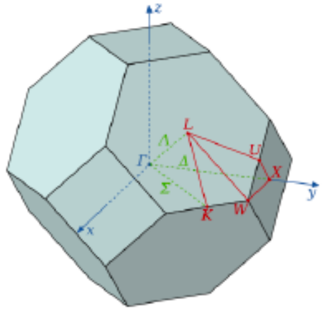
Wave-functions take the following form:

$$\boxed{\psi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}.\vec{r}}\,u_{n\vec{k}}(\vec{r})}$$

n: numbers the different eigenstates for same k.
k ϵ 1. Brillouin zone ("1.BZ")

With u(r) a lattice-periodic function:

$$u(\vec{r}+\vec{R}) = u(\vec{r})$$

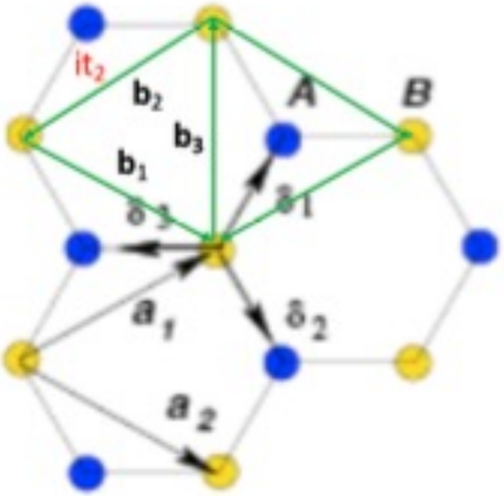


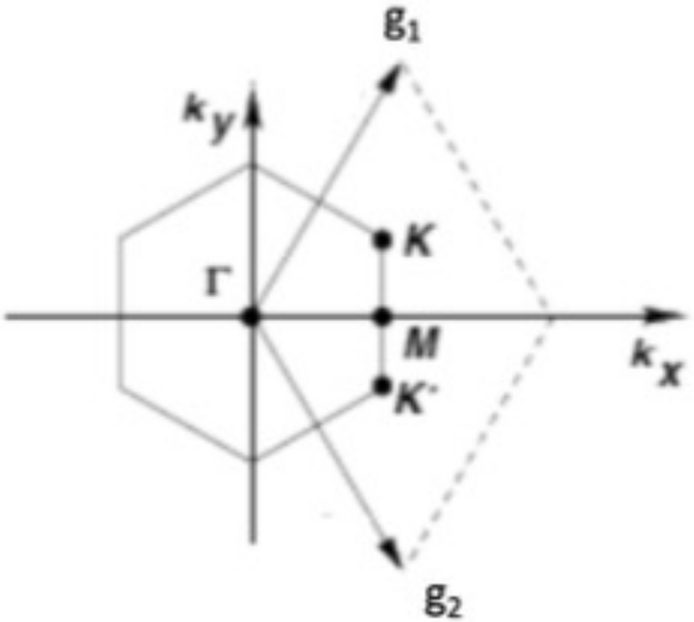
1^{st} BZ = Wigner-Seitz cell of the reciprocal lattice = {k-points that are closer to K=0 than to any other point of the reciprocal lattice}

Special about honeycomb lattice

1 plane of graphene (3D graphite; present research 2 planes and Moire magic angles…)

$$\begin{aligned}\mathbf{b}_1 &= \tfrac{a}{2}(3,-\sqrt{3})\\ \mathbf{b}_2 &= -\tfrac{a}{2}(3,\sqrt{3})\\ \mathbf{b}_3 &= (0,\sqrt{3}a)\end{aligned}$$





$$\boldsymbol{\delta}_1 - \boldsymbol{\delta}_3 = -\mathbf{b}_2$$
$$\boldsymbol{\delta}_2 - \boldsymbol{\delta}_3 = \mathbf{b}_1$$

2 Triangular lattices from Translation Operators in two dimensions
defined through the Bravais lattice vectors $\mathbf{a}_i$ and equivalently $\mathbf{b}_i$: `2 sublattices'

Within these definitions: $\vec{\delta}_1 = \frac{a}{2}(1,\sqrt{3}), \quad \vec{\delta}_2 = \frac{a}{2}(1,-\sqrt{3}), \quad \vec{\delta}_3 = (-a, 0)$

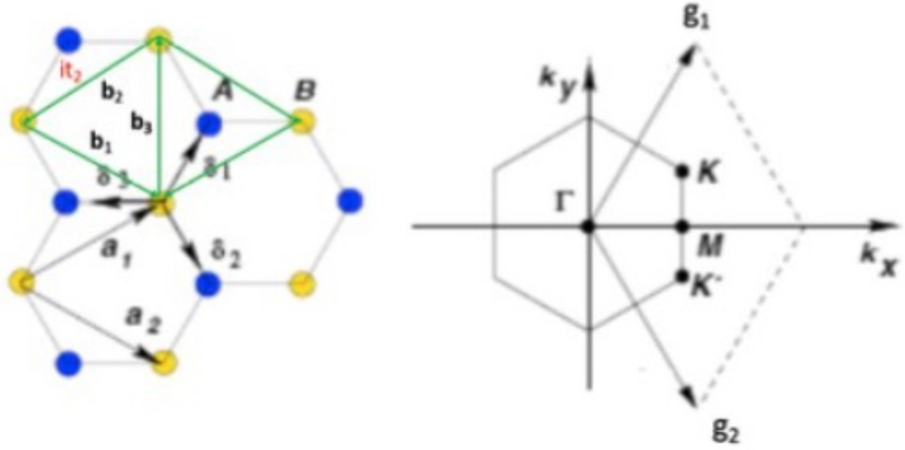
Reminder

$$\psi_{j\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_m} e^{i\mathbf{k}\cdot\mathbf{R}_m} \Phi_j(\mathbf{r} - \mathbf{R}_m)$$

The restricted Bloch wave is

$$\psi_{j\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_m} e^{i\mathbf{k}\cdot\mathbf{R}_m} \Phi_j(\mathbf{r} - \mathbf{R}_m).$$

From Bloch theorem, $\psi(\mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{r}} u(\mathbf{r})$ with $u(\mathbf{r}) = \sum_j c_j \Phi_j(\mathbf{r})$. The functions Φ are centered at a site $\mathbf{R}_m$ — meaning that $\Phi_j(\mathbf{r} - \mathbf{R}_m)$ refers to an electronic wave-function around the site $\mathbf{R}_m$ — and periodic if we apply the translation operator of vector $\mathbf{b}_j$. From symmetries, the honeycomb lattice can be viewed as formed with two triangular lattices made of A and B sites respectively. In this description, N represents the number of A or B sites and a particle has equal probabilities to occupy a site such that $c_j = 1/\sqrt{N}$.



$$\sum_{i=A or B sites} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{R}_i} = N\delta(\mathbf{k} - \mathbf{k}').$$

From the definitions, the Hamiltonian takes the form[1]

$$H = -t \sum_{\mathbf{R}_m} \sum_{\boldsymbol{\delta}_j} |\Phi(\mathbf{R}_m)\rangle\langle\Phi(\mathbf{R}_m + \boldsymbol{\delta}_j)| + h.c.$$

After Fourier transform, the Hamiltonian takes the form $H = \sum_{\mathbf{k}} H(\mathbf{k})$ with

$$H_{\mathbf{k}} = -t \sum_{\boldsymbol{\delta}_j} e^{-i\mathbf{k}\cdot\boldsymbol{\delta}_j} |\psi_{A\mathbf{k}}\rangle\langle\psi_{B\mathbf{k}}| + h.c.$$

$$H(\mathbf{k}) = \begin{pmatrix} 0 & -t\sum_{\delta_j} e^{-i\mathbf{k}\cdot\delta_j} \\ -t\sum_{\delta_j} e^{i\mathbf{k}\cdot\delta_j} & 0 \end{pmatrix}$$

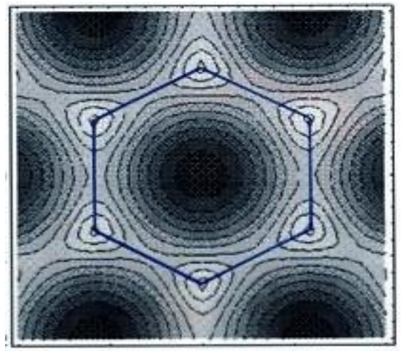
Close to the Dirac points PC

$$E^2 = t^2 \left(\sum_{\delta_j} e^{i\mathbf{k}\cdot\delta_j}\right)\cdot\left(\sum_{\delta_j} e^{-i\mathbf{k}\cdot\delta_j}\right)$$

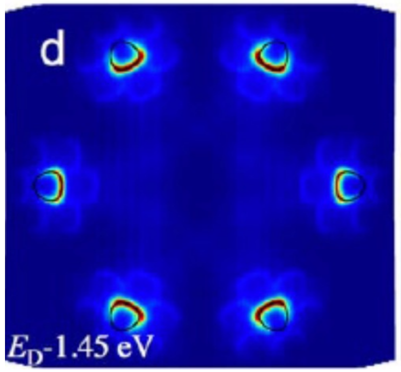
$$k_x = K_x + p_x = \frac{2\pi}{3a} + p_x$$

$$k_y = K_y + p_y = \frac{2\pi}{3\sqrt{3}a} + p_y.$$

Wallace, 1947



$$\boxed{E^2 \approx \frac{9}{4}(ta)^2(p_x^2 + p_y^2)}$$

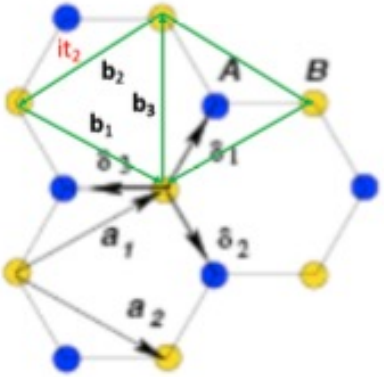


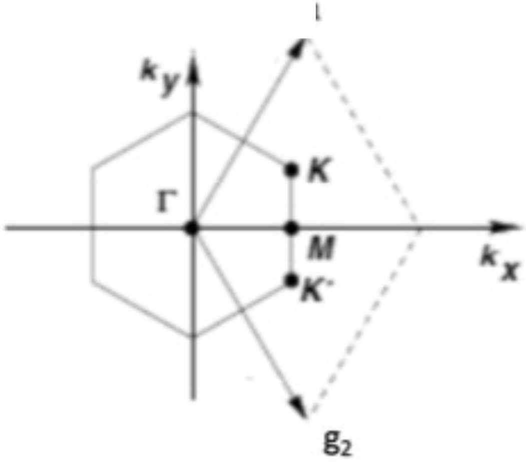
implying

$$E(p) = \pm\hbar v_F|\mathbf{p}|$$

$$v_F = \frac{1}{\hbar}\frac{\partial E}{\partial|\mathbf{p}|} = \frac{3}{2\hbar}ta$$

$\simeq 10^6\, m/s \ll c$





A. Bostwick et al.
Nature Physics **3** 36 (2007)
Photoemission

Similar Dirac points in high-Tc Superconductors

Linear energy dispersion

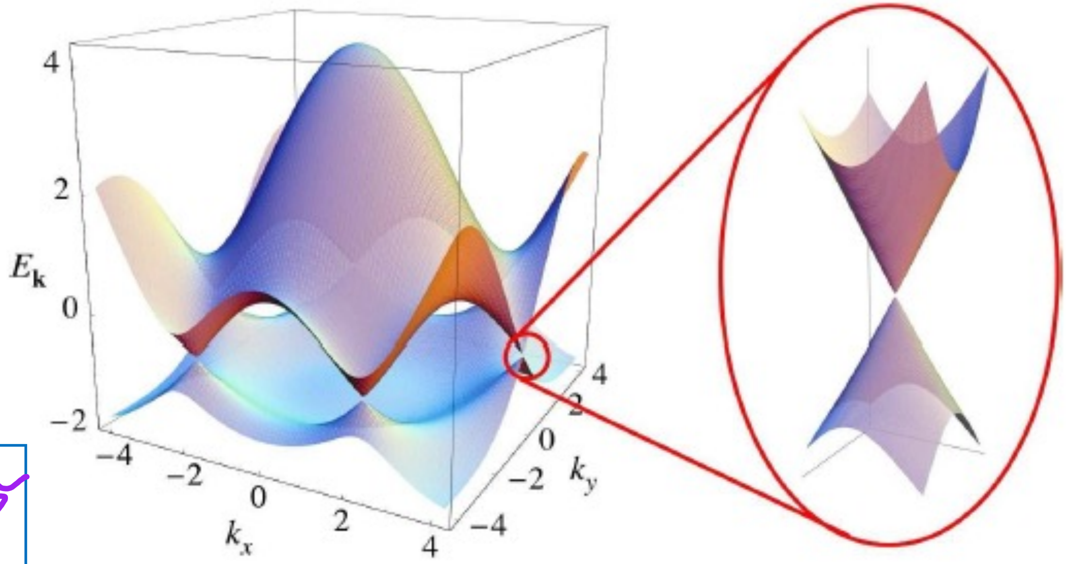
$$H = \hbar v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$H(\mathbf{p})|\psi(\mathbf{p})\rangle = \pm\hbar v_F |\mathbf{p}||\psi(\mathbf{p})\rangle$$

$$\Pi = (p_x + i p_y) = |\vec{p}| e^{i\tilde{\varphi}}$$

$$H = v_F (p_x \sigma_x + \sigma_y p_y)$$

$\vec{\sigma}$: Pauli matrices



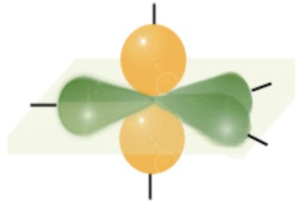
Relation with Dirac Equation in 2D:

$$\boxed{-iv_F\hbar\nabla\cdot\boldsymbol{\sigma}|\Phi(\mathbf{r})\rangle = E|\Phi(\mathbf{r})\rangle.}$$

The isospin **σ** (helicity) acts on each branch (sublattice)

Useful Review:
Rev. Mod. Phys. **81**, 109 (2009).

6 electrons in carbon:
2 in s1, 3 in sp2, 1 in p_z

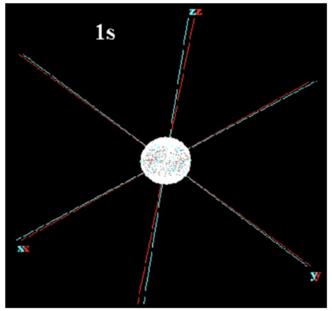


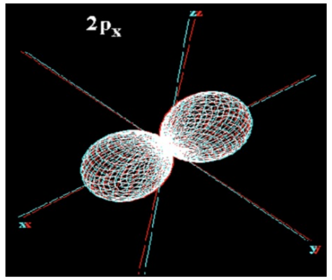
Question:
Where is E_F for graphene?

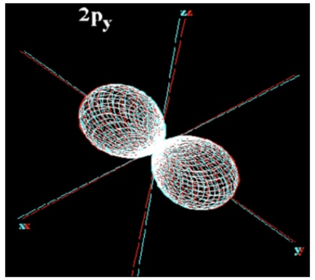
Answer:
E_F=0
Graphene is a semimetal

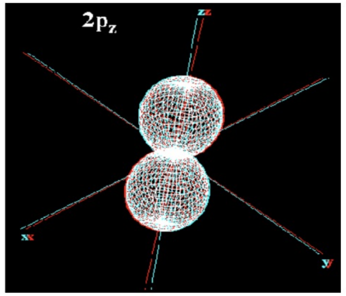
sp2 hybridization in graphene

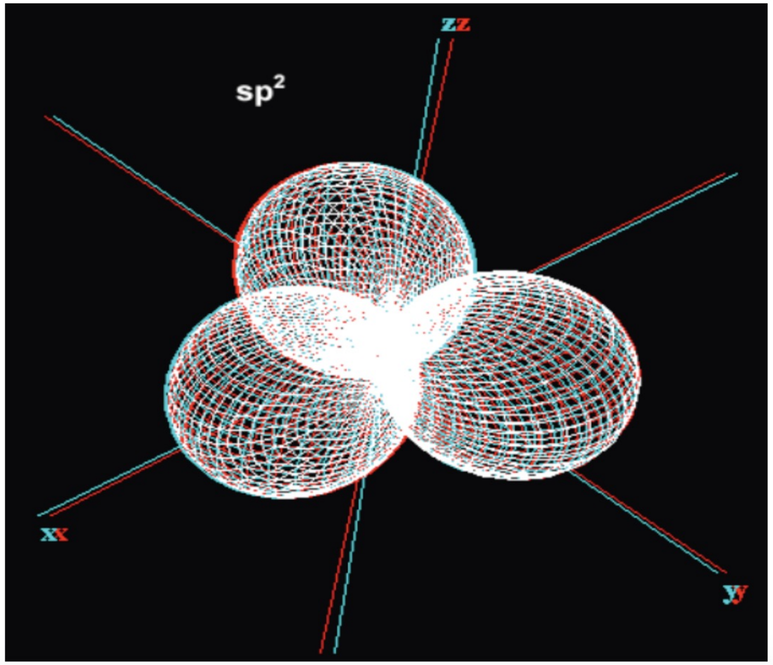
6
electrons
C





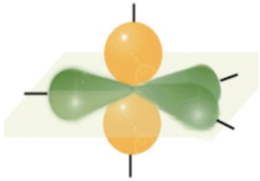






Within graphite, 2s and 2p orbitals undergo a sp2 hybridization

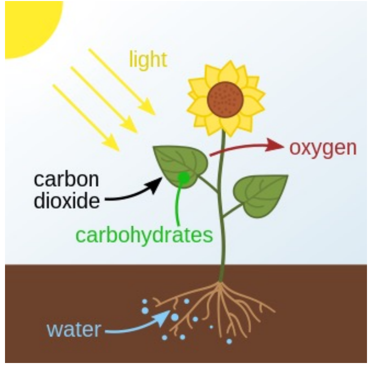
The geometry of the hybridized orbital is trigonal planar:
3 nearest neighbors



The last p-orbital forms the π-orbital

Diamond : sp3 hybridization

Useful
For pencils
For tennis Rackets
For bicycles…
For photo-synthesis



Yet be careful with "gas" emission For the planet

Eigenstates and Spin-1/2 quantum mechanics

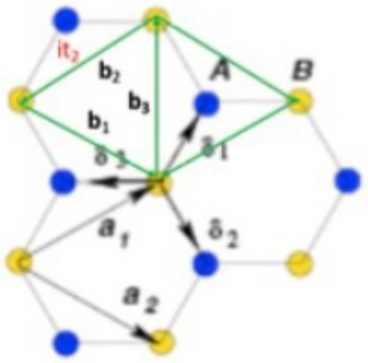
$$H = \hbar v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

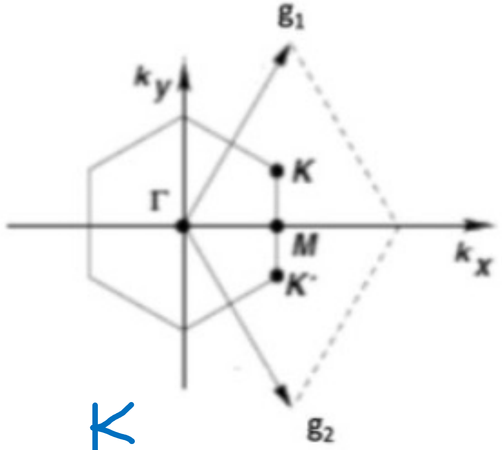
$$H(\mathbf{p})|\psi(\mathbf{p})\rangle = \pm \hbar v_F |\mathbf{p}||\psi(\mathbf{p})\rangle$$

$$p_x + i p_y = |\vec{p}| e^{i\tilde{\varphi}}$$

$$\boxed{\psi_\pm(\mathbf{K}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{i\tilde{\varphi}} \end{pmatrix}.}$$

$$\boxed{\psi_\pm(\mathbf{K}') = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{-i\tilde{\varphi}} \end{pmatrix}.}$$



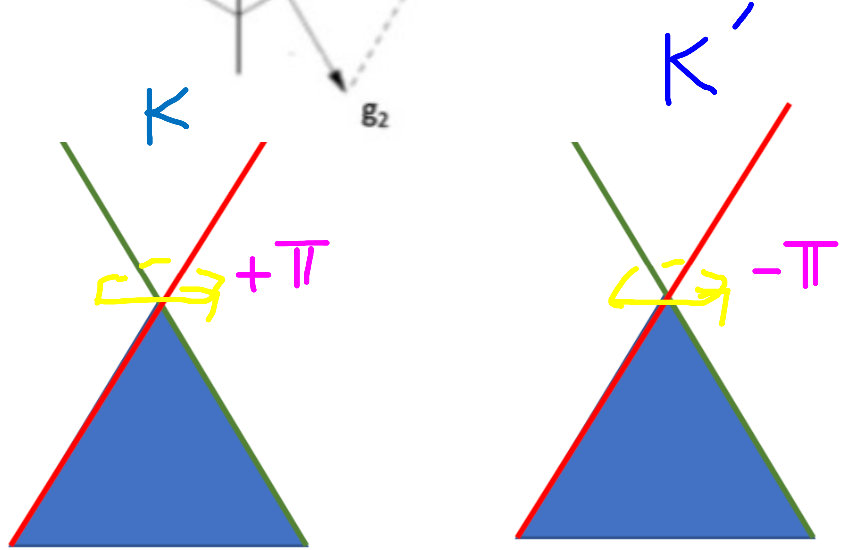


$$\frac{\partial}{\partial \tilde{\varphi}} \psi_+ = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\tilde{\varphi}} \end{pmatrix}$$

$$A_{\tilde{\varphi}} = -i \langle \psi_+(\mathbf{K}) | \frac{\partial}{\partial \tilde{\varphi}} | \psi_+(\mathbf{K}) \rangle = \frac{1}{2},$$

$$\boxed{\gamma(\mathbf{K}) = \int_0^{2\pi} A_{\tilde{\varphi}} d\tilde{\varphi} = \pi = -\gamma(\mathbf{K}').}$$

Berry phase (1984) encircling a Dirac point K



Lower and Upper bands described through an opposite phase

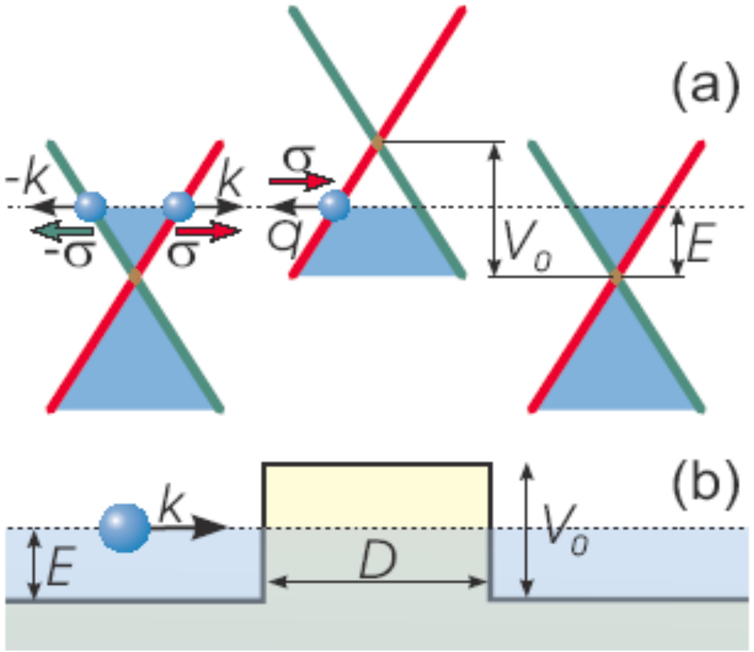
Klein Paradox for graphene:

Perfect transmission

Nobel prize 2010

Katnelson, Novoselov, Geim, Nature Physics 2006

Similar calculation
As in quantum class I



Dirac equation

$$-iv_F\boldsymbol{\sigma}\cdot\nabla\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$\hbar = 1$

The transmission probability
T is directionally-dependent.
For high barriers (V_0 >> E)

$$T(\phi) \simeq \frac{\cos^2\phi}{1-\cos^2(Dq_x)\sin^2\phi}$$

$$q_x = \sqrt{(V_0 - E)^2/(v_F^2) - k_y^2}$$

A slowly varying barrier is more efficient:
See Cheianov & Falko, PRB 2006

If V's are different for different spin orientations (magnetic gates): spin-polarized currents

Topological Phases within the 2*2 Matrix approach

Personnel QUIZ:
Find the associated years for prices?

GaAs

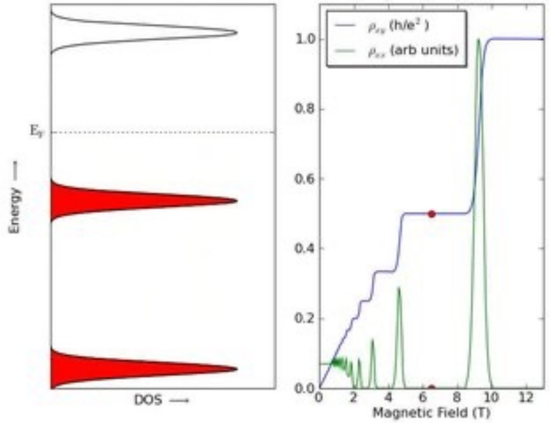
1980 K. Von Klitzing, G. Dorda, M. Pepper
1981 R. Laughlin, D. Tsui, H. Stormer
fractional case

- Quantum Hall Effect, Mosfets
  Uniform magnetic field in z-direction

Analogy to Free particule in a unidimensional vector potential PHY430
Energy levels are related to the 1D harmonic oscillator

$$E = \hbar\omega_c\left(n+\frac{1}{2}\right)$$

graphene ?



F.D.M Haldane
1988

PC

- Topology from the reciprocal space

How to induce a topological phase from the Bloch energy band formalism?

Quantum Hall effect on graphene

McClure, 1954

$\vec{F} = q(\vec{v} \times \vec{B})$

$q>0$

$$\boxed{l_B = \sqrt{\frac{\hbar}{qB}}}$$

$$\omega_c = \frac{v_F}{l_B} = \sqrt{\frac{v_F^2 qB}{\hbar}},$$

$$-i\hbar\nabla \rightarrow -i\hbar\nabla - qA_x$$

$A_x = -By$
$A_y = 0$

$$H = \begin{pmatrix} 0 & -i\hbar v_F\partial_x + \underline{v_F qBy} - \zeta\hbar v_F\partial_y \\ -i\hbar v_F\partial_x + \underline{v_F qBy} + \zeta\hbar v_F\partial_y & 0 \end{pmatrix}$$

$\zeta = \underline{\pm} 1$ $\quad K, K'$

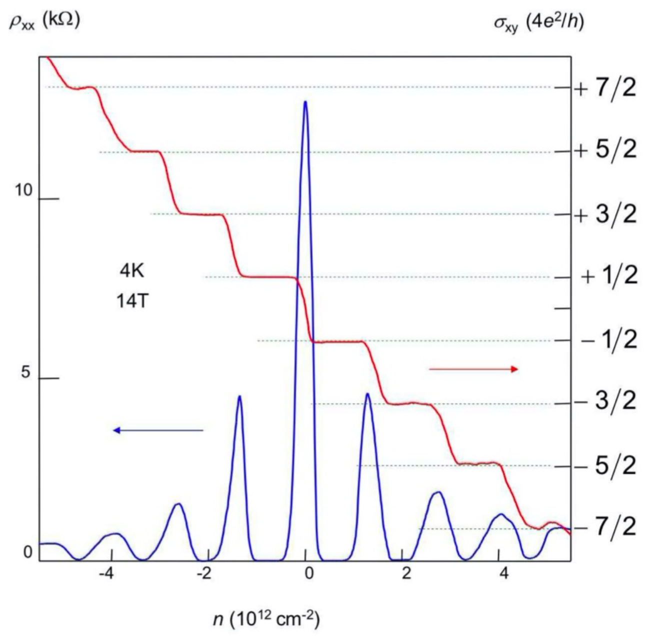
The solutions take the form

$$\Phi(\mathbf{r}) = e^{ikx}\Phi(y),$$

where $\Phi(y)$ associated to the spinor $|\Phi_A(y), \Phi_B(y)\rangle$. Therefore,

$$H = \begin{pmatrix} 0 & \hbar v_F k + v_F qBy - \zeta\hbar v_F\partial_y \\ \hbar v_F k + v_F qBy + \zeta\hbar v_F\partial_y & 0 \end{pmatrix}.$$

K. Novoselov, A. K. Geim
Ph. Kim



<u>Dirac ladder operators</u>

$$\mathcal{O} = \frac{1}{\sqrt{2}}(\hat{r} + \partial_r) = \mathcal{O}_K = \mathcal{O}_{K'}^\dagger$$

$$\mathcal{O}^\dagger = \frac{1}{\sqrt{2}}(\hat{r} - \partial_r) = \mathcal{O}_K^\dagger = \mathcal{O}_{K'}$$

variable r ?

$$\boxed{E = \pm\omega_c^*\sqrt{N}.}$$

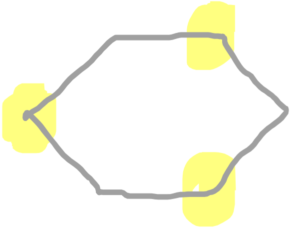
<u>See detailed Note</u>

Simple derivation of Transport

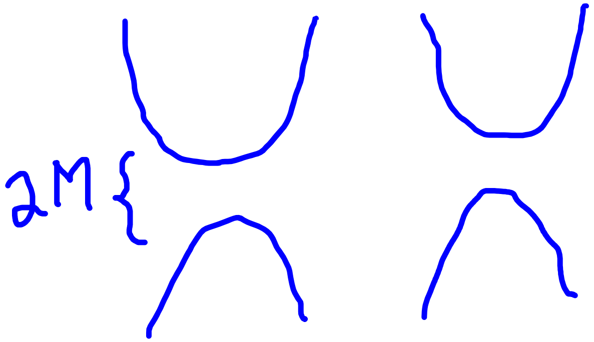
$$\boxed{j_y = J_\perp = \pm\frac{2(2N+1)e^2}{h}E = \sigma_{xy}E.}$$

How to induce a topological phase from Bloch energy bands?

Magnetic field in **k**-space corresponds here to add a staggered potential on the lattice $H = M\sigma^z$



$$H(K) = \hbar v_F \begin{pmatrix} M & p_x - ip_y \\ p_x + ip_y & -M \end{pmatrix}$$



$$E = \pm\sqrt{(\hbar v_F|\mathbf{p}|)^2 + M^2}$$

Semenoff 1984

If we look at energies such that $v_F p \gg M$, energies are unchanged and eigenstates are similar as before such that we can formulate the same arguments as before.

insulator

mass inversion

$$H = \begin{cases} -m\sigma^z & \vec{K} \\ +m\sigma^z & \vec{K}' \end{cases}$$

Route to Topological properties:

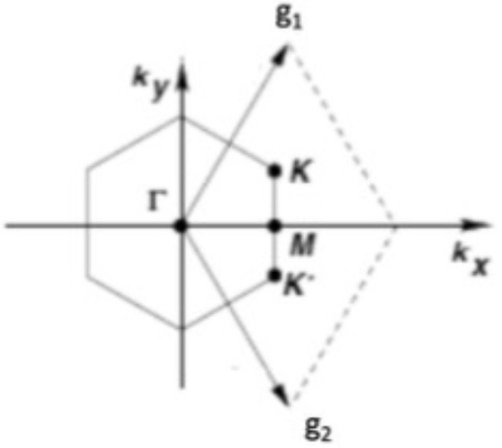
Corresponds to invert upper and lower bands at K': Additive Berry phases at K and K'

Relation to Topological Chern number

See PC

Correspondence $(k_y,k_x) \rightarrow (\theta, \varphi)$

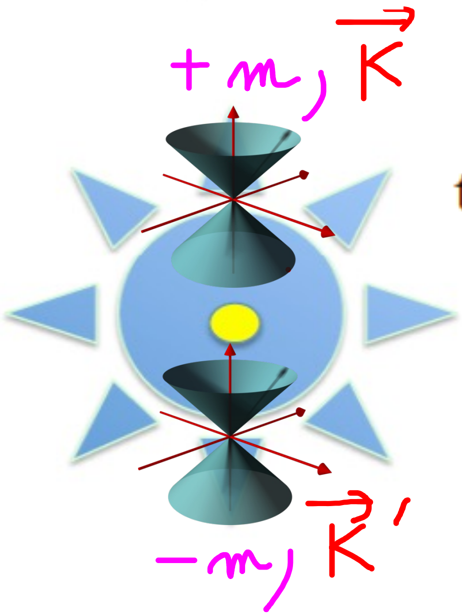
$$\boxed{-d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta) = (v_F|\mathbf{p}|\cos\tilde{\varphi}, v_F|\mathbf{p}|\sin(\zeta\tilde{\varphi}), -\zeta m).}$$



Quantum Class I:

$$\mathcal{H}(\mathbf{k}) = -\mathbf{d}\cdot\boldsymbol{\sigma} = -|\mathbf{d}|\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$|\psi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}, \quad |\psi_-\rangle = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}$$



$$\tan\theta = \frac{v_F|\mathbf{p}|}{m}$$

$$\tilde{\varphi} = \varphi \pm \pi$$

Eigenstates associated to energy +/- $|\mathbf{d}|$

Topology from Electromagnetism on the Sphere & quantum physics

Related to quest of Dirac monopoles and Skyrmions (P. Curie, 1894 ; P. Dirac 1931)
Relation with physics of planets

Another way to apply (radial) magnetism:

$B = B(r)$

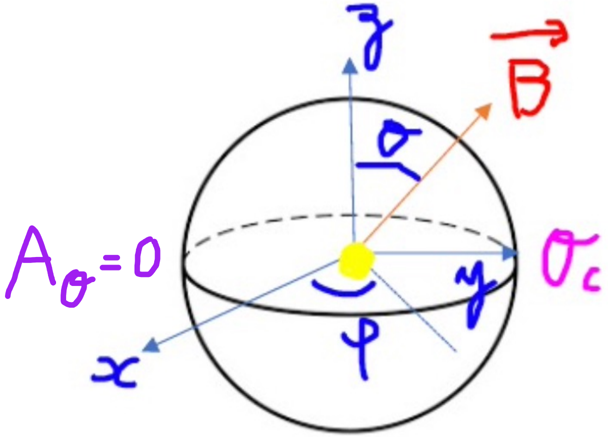
$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} = B\mathbf{e}_r}$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial_\varphi}\right) = B.$$

$$\frac{\partial}{\partial\theta}(A_\varphi \sin\theta) = Br\sin\theta.$$

To solve this equation, we can redefine

$$A'_\varphi = A_\varphi \sin\theta. \qquad \partial A'_\varphi = Br\partial_\theta(\sin\theta).$$



Requires at least
2 joint "domains"

$$\boxed{\begin{aligned} A'_\varphi(\theta < \theta_c) &= -Br(\cos\theta - 1) = 2Br\sin^2\frac{\theta}{2} \\ A'_\varphi(\theta > \theta_c) &= -Br(\cos\theta + 1) = -2Br\cos^2\frac{\theta}{2}. \end{aligned}}$$

T. Wu and C. N. Yang 1975
Book M. Nakahara: Geometry, Topology and Physics

Review: K. Le Hur arXiv:2209.15381

Here, we precisely introduce $\tilde{A}_\varphi(\theta) = -Br\cos\theta$ such that on the two regions we have

$$A'_\varphi(\theta < \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(0)$$
$$A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(\pi),$$

and such that

$$\boxed{A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\pi) - \tilde{A}_\varphi(0).}$$

$$\boxed{F_{\theta\varphi} = \frac{1}{r}\partial_\theta A'_\varphi = B\sin\theta}$$

$$\boxed{\Phi = r^2\left(\int_0^{2\pi} d\varphi\right)\left(\int_0^{\pi} F_{\theta\varphi} d\theta\right) = B(r)(4\pi r^2).}$$

$$B = \frac{q_m}{2r^2} = B(r)$$

$$q_m = \frac{\Phi}{2\pi}$$

Definition similar to
Plane or lattice: useful

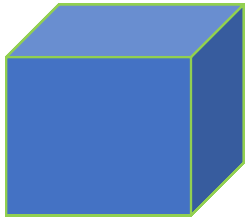
$$\Phi = 2\pi r^2\left(\int_0^{\theta_c^-} F_{\theta\varphi} d\theta + \int_{\theta_c^+}^{\pi} F_{\theta\varphi} d\theta\right).$$

Simple Calculations!

$$\boxed{\Phi = 2\pi r\left(A'_\varphi(\theta_c^-) - A'_\varphi(\theta_c^+)\right) = 2\pi r(\tilde{A}_\varphi(\pi) - \tilde{A}_\varphi(0)).}$$

q_m measures the topological charge

Mathematics of Shapes and Table

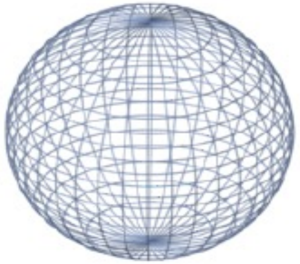


Euler characteristic is defined as

$$\chi = V - E + F$$

where V is the number of vertices (corners), E edges and F faces.

Take a cube. What is the Euler characteristic? Is this non-zero? $\chi = 2$

Sphere		2

- In the presence of a Dirac monopole or Topological charge, $\chi = 2 - 2g = 0$
- Agrees with Gauss-Bonnet theorem

on the surface and Poincare-Hopf Theorem; Similar to a cup or donut
- Equivalent to 2 circles from Stokes' theorem, can be measured at the poles

Physics: g related to quantum Hall conductivity, D. Thouless 1982 quantum Hall effect (Nobel Prize 2016)

Quantum Correspondence & Topological Number:

Michael V. Berry, proceedings of the Royal Society A **392**, Issue 1802 (1984)

$\tilde{A}$	Vector potential	→	Berry connection, equivalent to momentum $\vec{A} = i\langle\psi\|\vec{\nabla}\|\psi\rangle$
$\vec{B}$	Magnetic Field	→	Berry curvature $\vec{F} = \vec{\nabla} \times \vec{A}$

PC

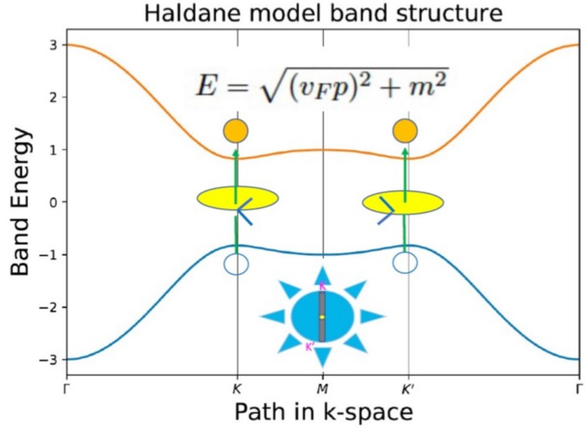
Haldane Model

$$H = -\mathbf{d}\cdot\boldsymbol{\sigma}$$

$$\mathbf{d}(\varphi, \theta) = d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta).$$

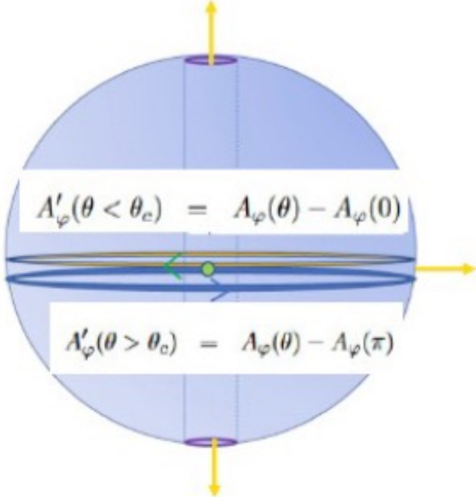
$B = \frac{1}{2}$ $\quad F_{\varphi\theta}(\theta) = \frac{\sin\theta}{2}$

$r = 1$ $\quad A_\varphi(\theta) = \frac{\cos\theta}{2}$

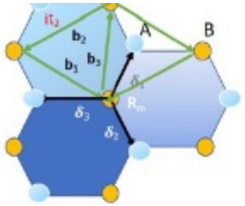


$$\boxed{C = \frac{1}{2\pi}\int\int_{S^2} F_{\varphi\theta}\, d\varphi\, d\theta,}$$

$C = 1\ (\mathbb{Z})$



General proof from geometry (also for multiple spins) :



$$C = A_\varphi(0) - A_\varphi(\pi) = A'_\varphi(\theta > \theta_c) - A'_\varphi(\theta < \theta_c)$$

Joel Hutchinson & Karyn Le Hur, Communications Physics 4, 144 (2021)

And this sphere can be real, adjustable

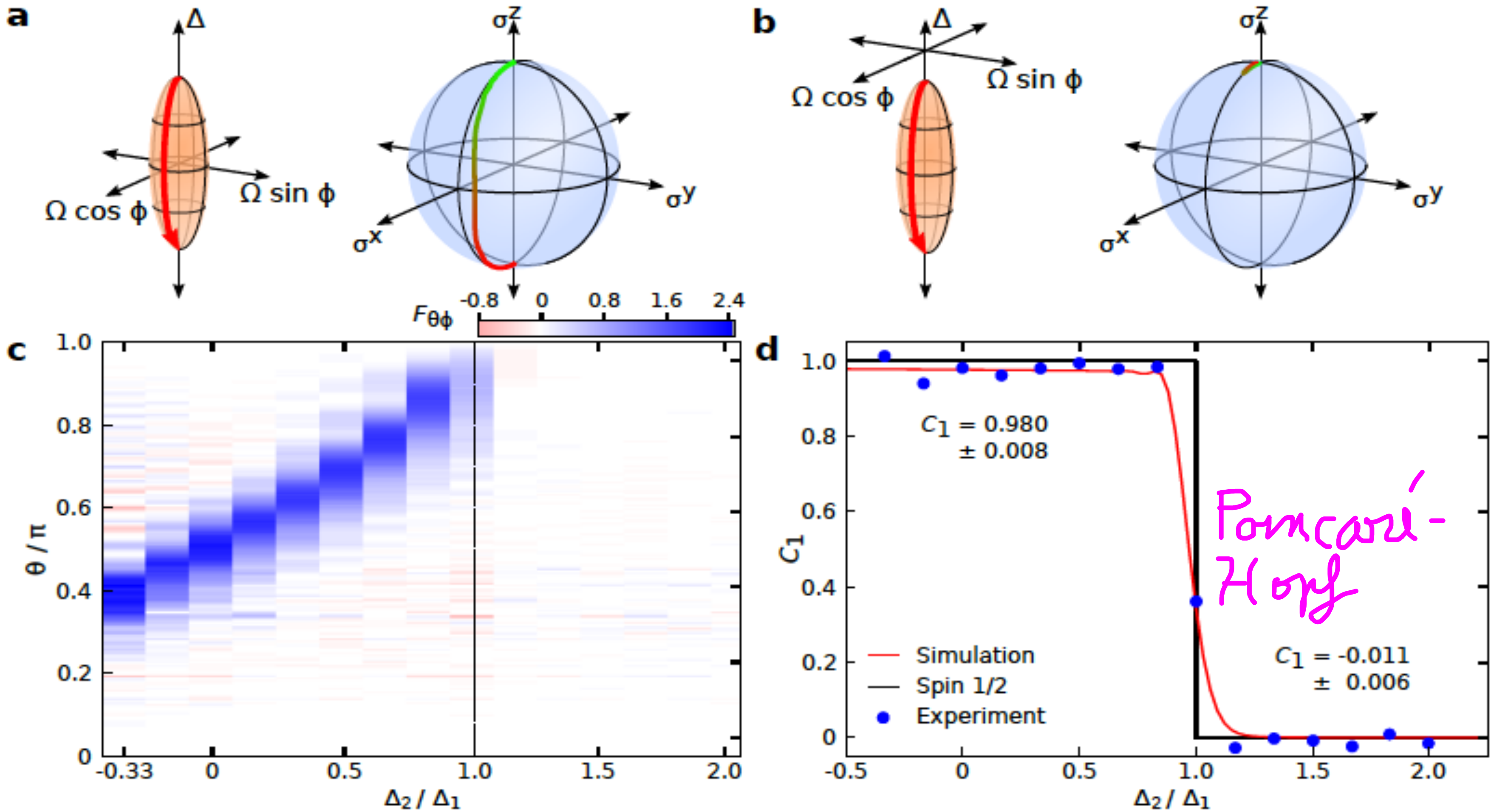
D. Schroer et al. PRL 2014 (Boulder, K. Lehnert)
P. Roushan et al. Nature (John Martinis, Santa Barbara) 2014

Superconducting circuits
And cavities

$$\Delta = \Delta_1 \cos\theta + \Delta_2\ ,\ \ \Omega = \Omega_1 \sin\theta$$

$$\theta(t) = \pi t / t_{\text{ramp}}$$

$$\boldsymbol{H}/\hbar = \frac{1}{2}\left[\Delta\, \boldsymbol{\sigma_z} + \Omega\, \boldsymbol{\sigma_x} \cos\phi + \Omega\, \boldsymbol{\sigma_y} \sin\phi\right]\ ,$$



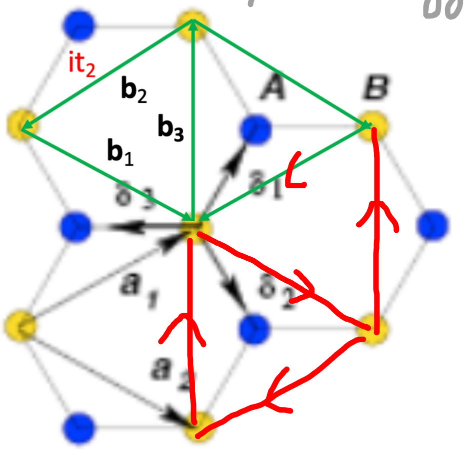
"zero net flux" quantum anomalous Hall effect

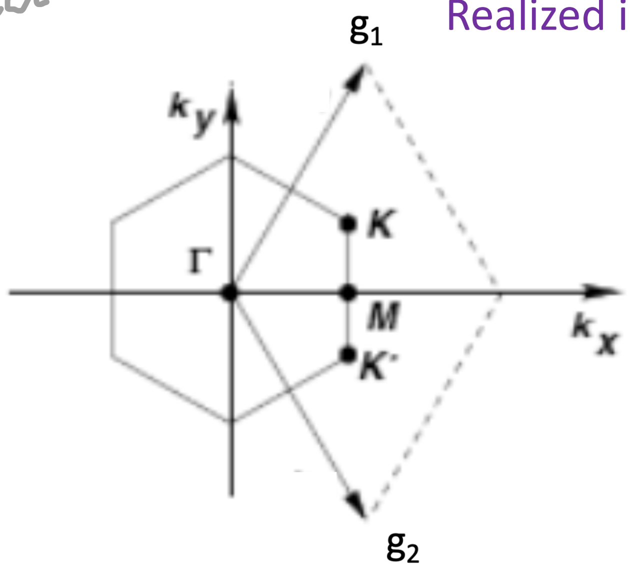
Haldane Model

1988

PC

Realized in quantum materials, graphene, ultra-cold atoms, light systems



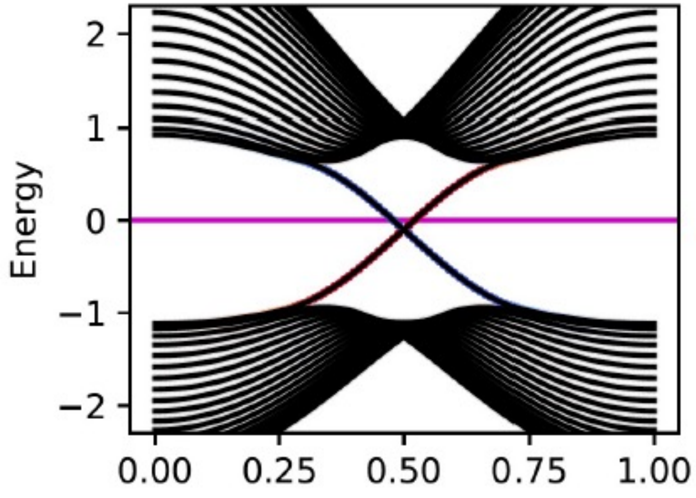


$$\mathcal{H}(\mathbf{k}) = -\mathbf{d}\cdot\boldsymbol{\sigma}$$

$$\mathbf{d} = \Big(t\sum_{\boldsymbol{\delta}_j}\cos(\mathbf{k}\cdot\boldsymbol{\delta}_i),\, t\sum_{\boldsymbol{\delta}_j}\sin(\mathbf{k}\cdot\boldsymbol{\delta}_i),\, +t_2 \sum_{\mathbf{b}_j}\sin(\mathbf{k}\cdot\mathbf{b}_j)\Big).$$

$$+d_z(\mathbf{K}) = 2t_2\sum_{\mathbf{b}_j}\sin(\mathbf{K}\cdot\mathbf{b_j}) = 3\sqrt{3}t_2 = m$$

$$+d_z(\mathbf{K}') = 2t_2\sum_{\mathbf{b}_j}\sin(\mathbf{K}'\cdot\mathbf{b_j}) = -3\sqrt{3}t_2 = -m.$$

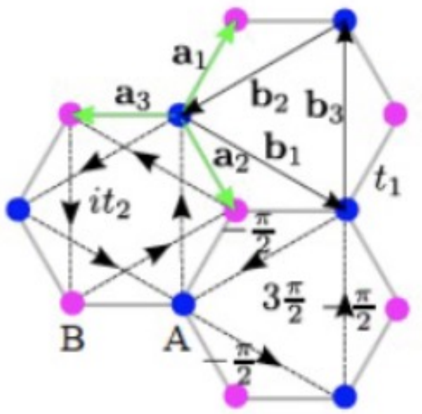


The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

http://www.physics.rutgers.edu/pythtb/

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

The phase $\emptyset = \frac{\pi}{2}$ *can be realized* with lasers, light-matter coupling:
Control at the atomic scale!

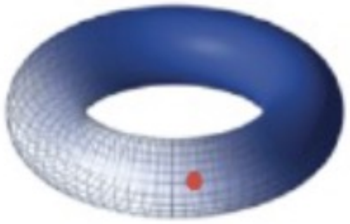


Spin-1/2 analogy

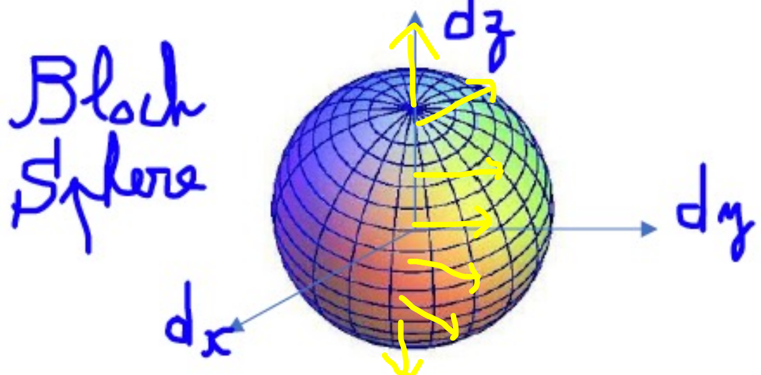
$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}(\mathbf{k}) \cdot \hat{\sigma},$$

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\,\mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\,\mathbf{a_i}, \pm 2t_2 \sum_i \sin \mathbf{k}\,\mathbf{b_i} \right).$$



The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.



$m = \pm 3\sqrt{2}\, t_2$ $\quad + K, \; - K'$

radial magnetic field

$C = 0, \pm 1$

$\mathbb{Z}$ number

Light-induced anomalous Hall effect in graphene

J.W. McIver[1*], B. Schulte[1*], F.-U. Stein[1*], T. Matsuyama[1], G. Jotzu[1], G. Meier[1] and A. Cavalleri[1,2]

Nature 2020

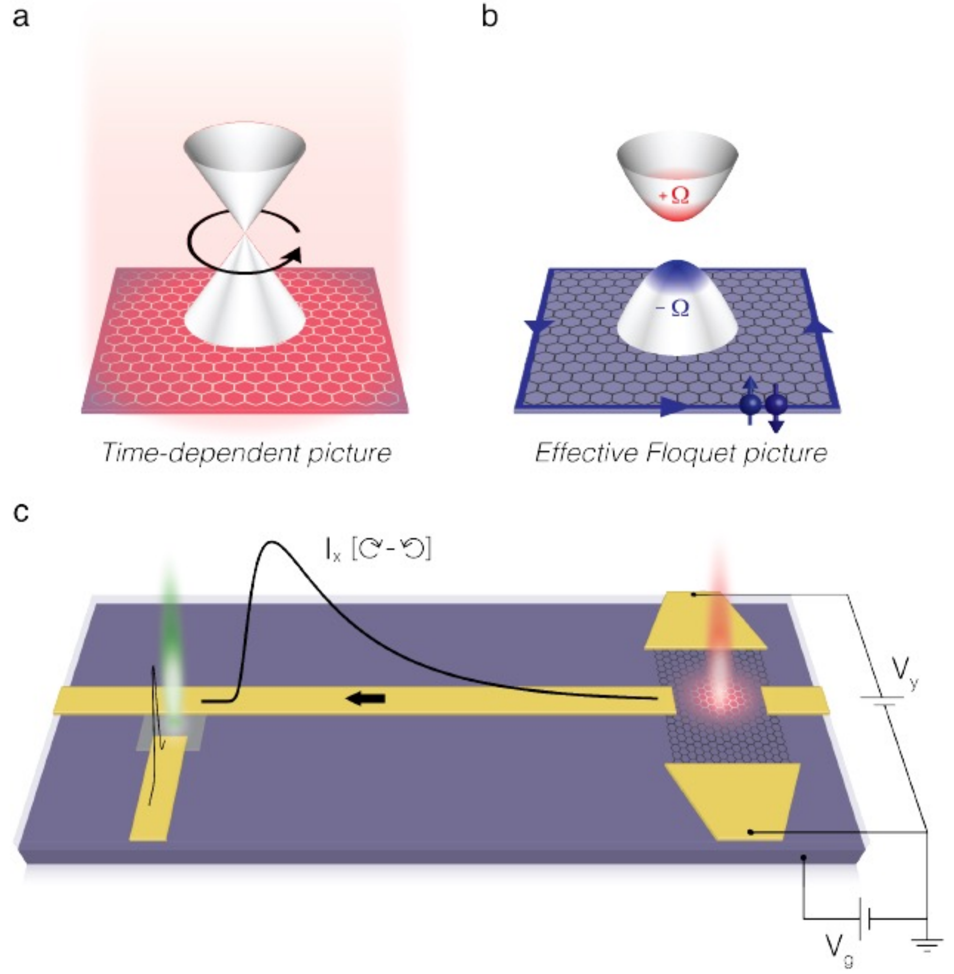


Fig. 1 | Light-induced topological Floquet bands in graphene and device architecture used to detect ultrafast anomalous Hall currents. a, A coherent interaction between graphene and circularly polarized light is predicted to open a topological band gap in the effective Floquet band

Light-Matter Coupling

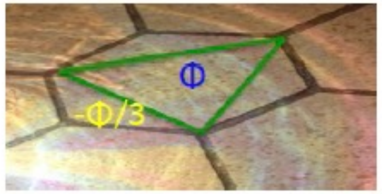
Stages 3A
Joshua Benabou 2021
Han Yu Sit 2020

D. Tran, A. Dauphin, A. G. Grushin, P. Zoller, N. Goldman Sciences Advances 2017

Philipp Klein, Adolfo Grushin, Karyn Le Hur, Phys. Rev. B 2021

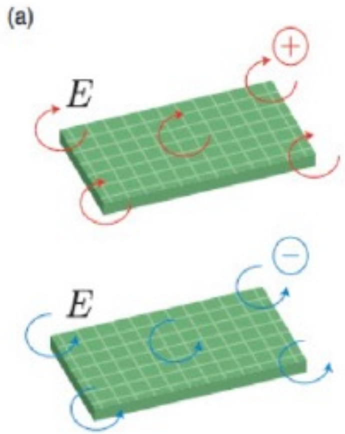
Karyn Le Hur, arXiv: arXiv:2106.15665 C^2

Circular
Dichroism
Jones Polarizations



$$\mathbf{A} = A_0 e^{-i\omega t}(\mathbf{e}_x \mp i\mathbf{e}_y)$$

$$\delta\mathcal{H}_\pm = A_0 e^{\pm i\omega t}|a\rangle\langle b| + h.c.$$

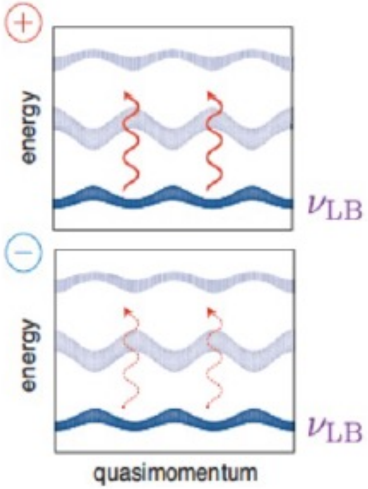


Realization in Hamburg in atoms (C. Weitenberg & K. Sengstock group):
Luca Asteria et al. Nature Physics 2019

Photo-induced currents

Fermi golden's rule

$$\boxed{\tilde{\Gamma}_\pm = \frac{2\pi}{\hbar}\frac{A_0^2}{2\hbar^2}\left|\left\langle\psi_-\left|\left(\frac{\partial H}{\partial p_x} + i\frac{\partial H}{\partial(\zeta p_y)}\right)\right|\psi_+\right\rangle\right|^2 \delta(E_-(0) - E_+(0) - \hbar\omega).}$$



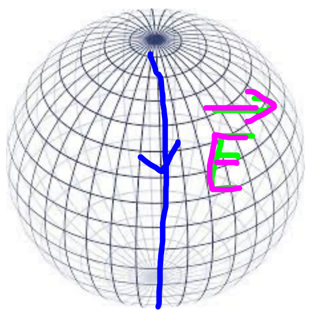
$$|\Delta\tilde{\Gamma}| = \left|\frac{\Gamma_+(K) - \Gamma_-(K')}{2}\right| = \frac{2\pi}{\hbar}\frac{A_0^2}{4m^2}C$$

Relation to quantum Hall conductivity: D. Thouless (Nobel prize in physics 2016), M. Kohmoto, M. P. Nightingale, M. Den Nijs

Transport on the sphere

Joel Hutchinson, Karyn Le Hur, Communications Physics 4, 144 2021 Nature Journal, ArXiv: Feb 28 2020

$$\mathbf{E} = E\mathbf{e}_{x_\parallel} = -\nabla V$$

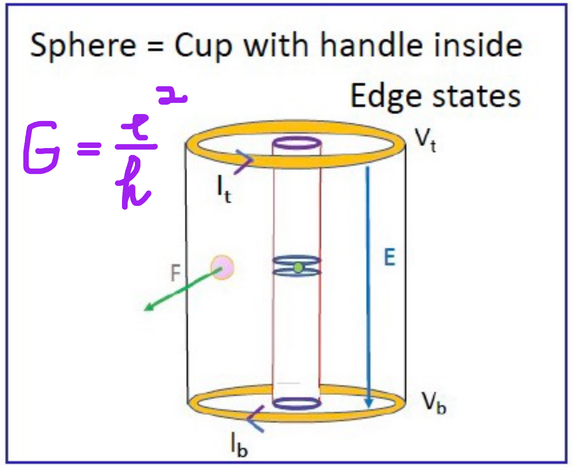


$$T = \frac{\pi\hbar}{qE}$$

$$H = \frac{(\hbar k_\parallel)^2}{2m} + \frac{(\hbar k_\perp)^2}{2m} + qV - \mathbf{d}\cdot\boldsymbol{\sigma}$$

Newton equation $ma_\parallel = \hbar\dot{k}_\parallel = qE$

$$\boxed{\theta(t) = k_\parallel(t) = \frac{q}{\hbar}Et.}$$



Analogy Laughlin cylinder
" Nobel Prize"

Parseval-Plancherel Theorem (quantum mechanics)

$$\boxed{J_\perp = \frac{q}{T}\int_0^T dt \frac{d\langle x_\perp\rangle}{dt} = \frac{q}{T}\left(\langle x_\perp\rangle(T) - \langle x_\perp\rangle(0)\right) = \oint d\varphi\left(J_\varphi(\varphi, T) - J_\varphi(\varphi, 0)\right),}$$

$$J_\varphi(\varphi,\theta) = \frac{iq}{4\pi T}\left(\psi^*\frac{\partial}{\partial\varphi}\psi - \frac{\partial\psi^*}{\partial\varphi}\psi\right) = \frac{iq}{2\pi T}\psi^*\frac{\partial}{\partial\varphi}\psi$$

$q = e$

$$\boxed{J_\perp(\theta) = \frac{e}{2\pi T}\oint d\varphi A'_{N\varphi}(\varphi,\theta) = \frac{e}{T}A'_\varphi(\theta < \theta_c),}$$

$-e$ goes "up"

$$\boxed{J_\perp = \frac{e^2}{h} 2 E(}$$

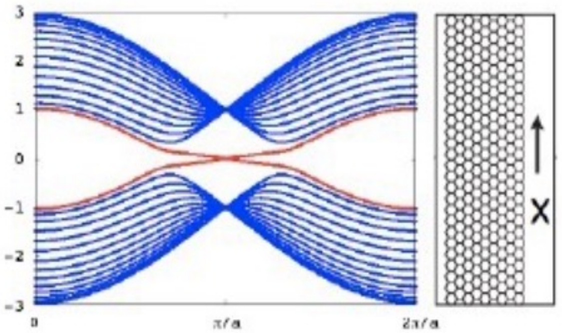
Agrees with transport in the plane: Bloch bands (PC)

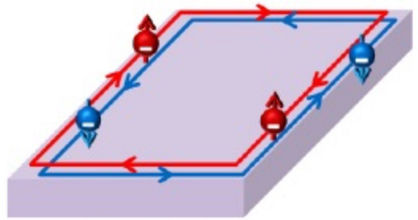
Topological States and Materials

The quantum Hall conductivity can be visualized as a Karplus-Luttinger velocity (1954)
Applications in crystals: Ph. Nozieres and C. Lewiner, 1973; Review M. Nagaosa et al. 2010

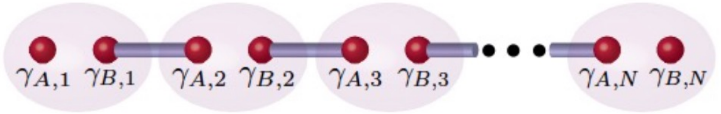
$$\mathbf{v} = \frac{e}{\hbar}\mathbf{E} \times \mathbf{F}.$$

Topological Insulators (Ti): Generalization with Spin-Orbit Coupling
Theory Work of C. L. Kane & E. Mele and L. Fu starting in 2005 and also B. A. Bernevig, X. Qi, T. Hughes & S.-C. Zhang
Realization in "Mercury" Materials in 2D (QSHE) and Bismuth Materials in 3D, … Requires "Spin-Orbit Coupling"
Characterization Through a Z_2 number
Measures the number of "up" – "down" particles at the edges





Topological Superconductors and Majorana Fermions (1937)
Electrons are bound in Cooper Pairs (Bosons, Superfluidity)
Majorana Fermions are their own antiparticles, may be used
For Topological quantum computing

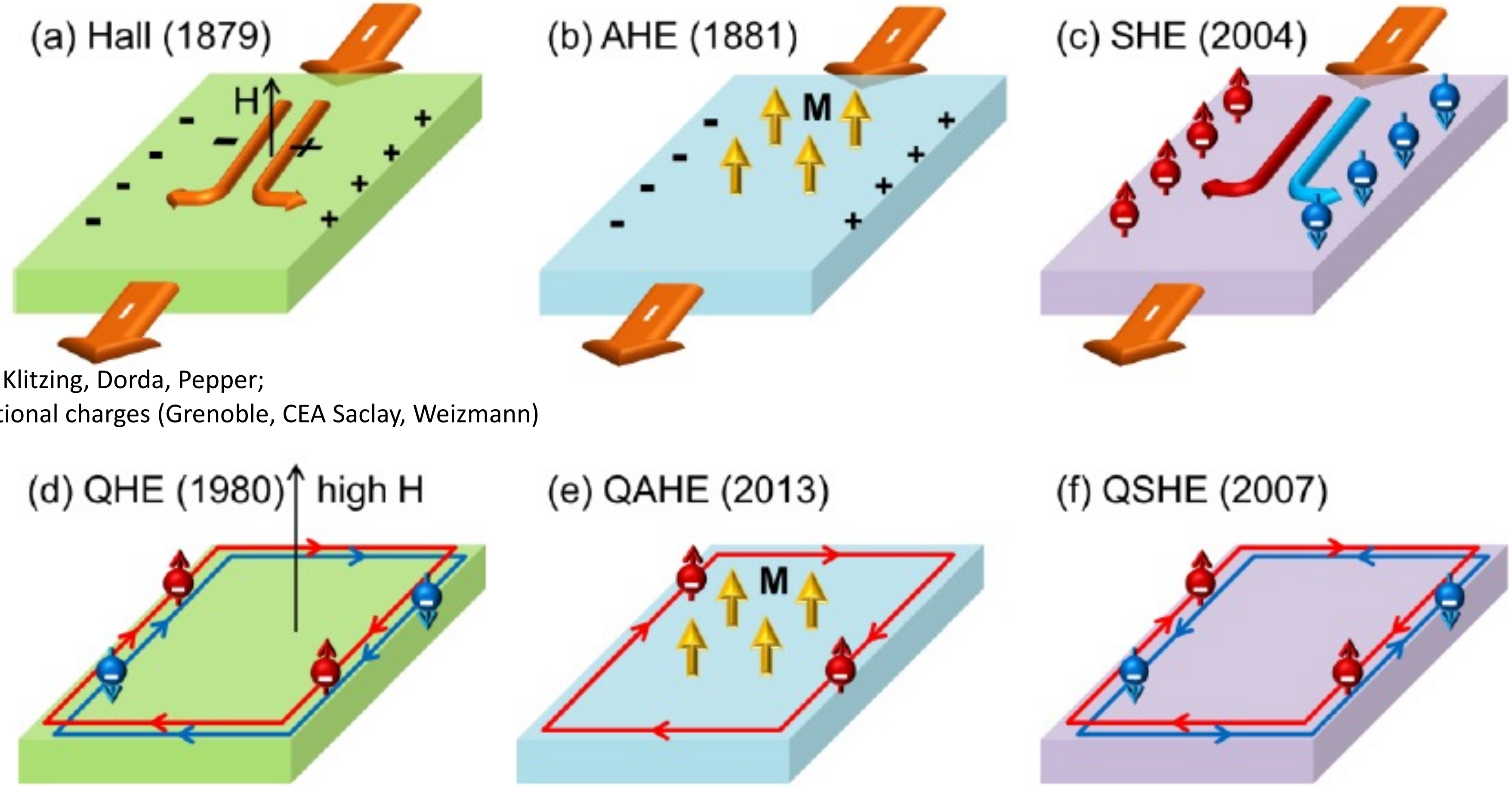


Review: J. Alicea, arXiv:1202.1293

Present research also includes Topological Semimetals

A Lot of questions here
Application for NanoElectronics, Spintronics, and Energy Photovoltaic Effect…

Topological states of matter: TRANSPORT AT THE EDGES



Von Klitzing, Dorda, Pepper;
fractional charges (Grenoble, CEA Saclay, Weizmann)

C. Z. Chang and M. Li, Topical Review, arXiv:1510.01754
From material science, to cold atoms and photons

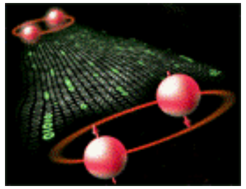
REALIZED AT WURZBURG IN HGTE (Molenkamp)
3D MERCURY ANALOGUES, PRINCETON (Hasan)

Application spintronics: D. Pesin & A. H. Macdonald, Nature Materials 2012

Quantum Theory and Entangled Wave Function

Einstein-Poldosky-Rosen (EPR) Pair and Bell Tests of Quantum Mechanics (1964)

Our Logo at CPHT



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle \pm |\downarrow\rangle \otimes |\uparrow\rangle\right)$$

- Observation with Atoms : Serge Haroche (Nobel prize,) and Schrodinger Cats
  Applications in Rydberg Atoms and Quantum Circuits, cavities

- Applications with Photons (A. Aspect; Ph. Grangier; J. Dalibard)
  Nobel Prize 2022 (A. Aspect; J. Clauser; A. Zeilinger)!
  Quantum Security Protocols: BB84 Codes (Quantum Class I)

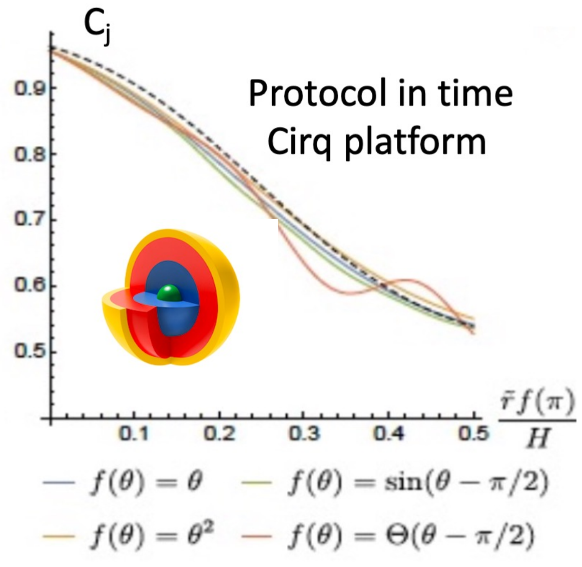
- New Applications in Quantum Information and Algorithms,
  New Interfaces: Everybody is welcome in Science

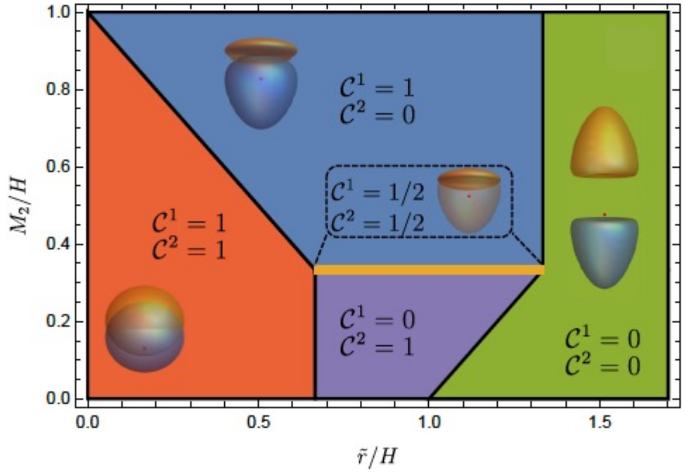
2 spheres

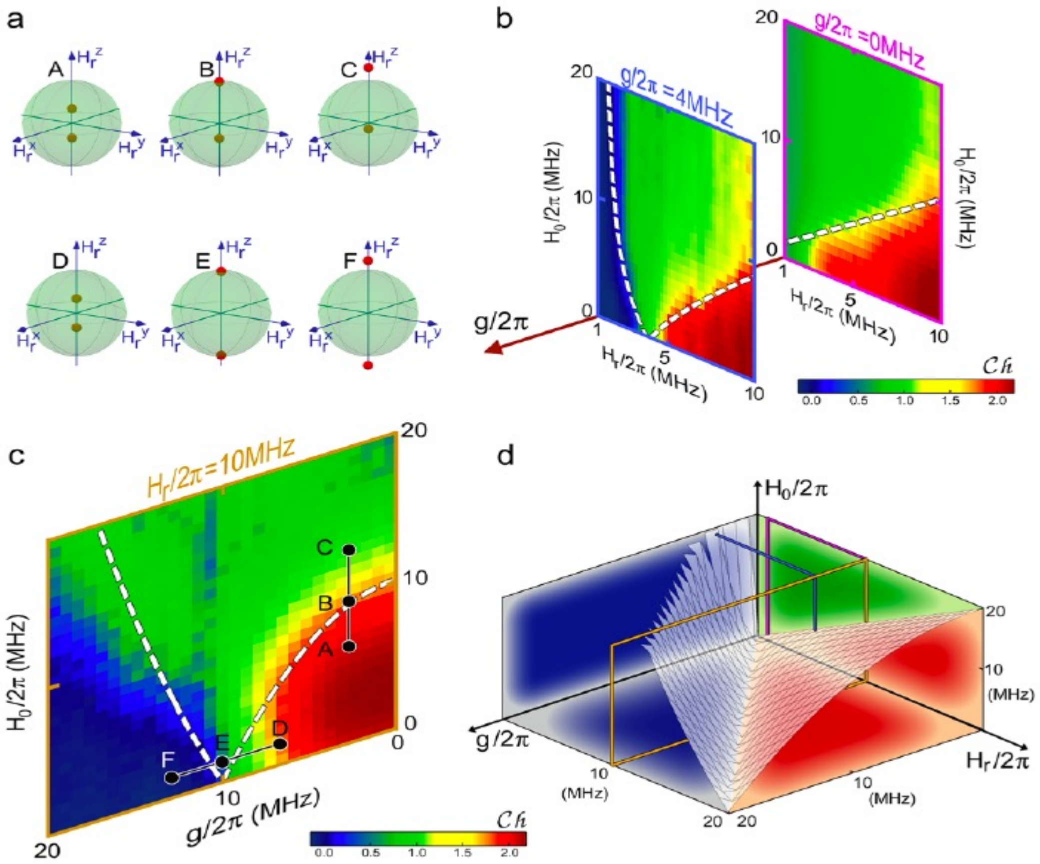
Joel Hutchinson and Karyn Le Hur, Communications Physics 4, 144 2021

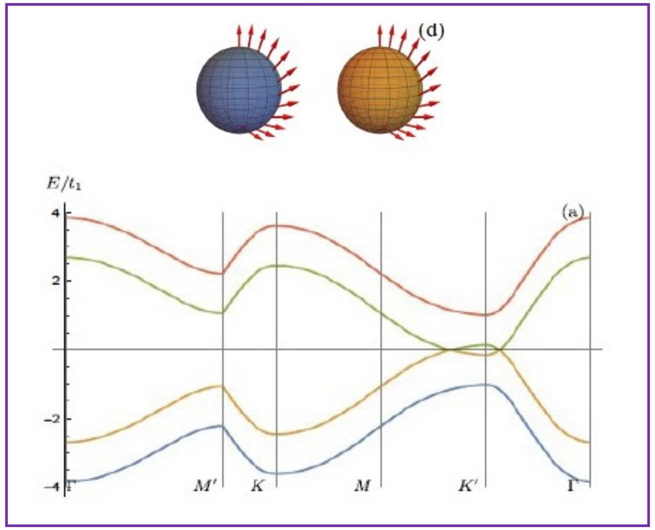
$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

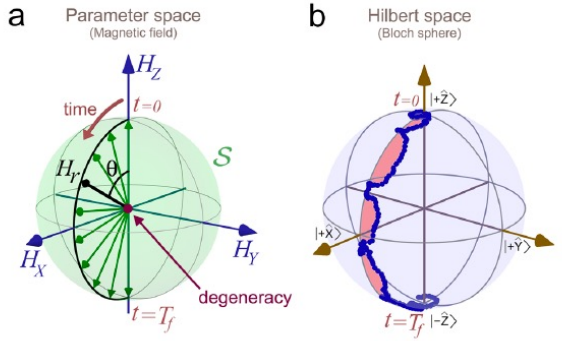








Santa-Barbara:
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)



Review: K. Le Hur arXiv:2209.15381