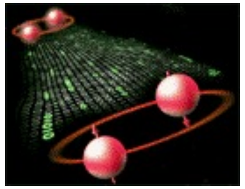
Curved Space Topometry and Fractional Entangled Bloch bands applied to Semimetals



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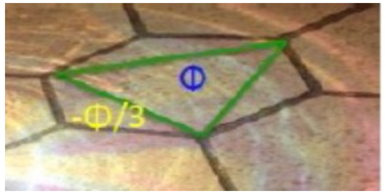
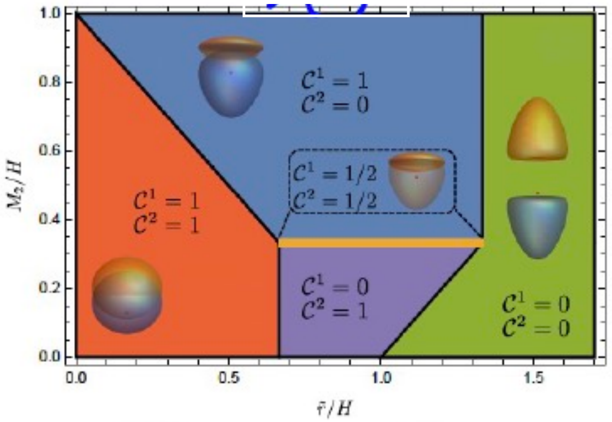
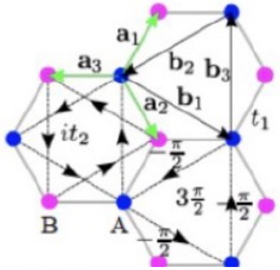
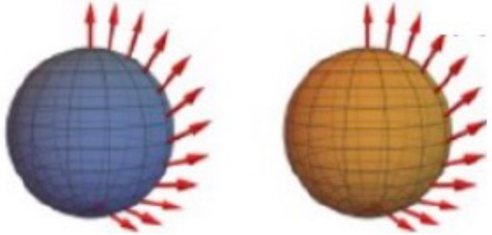


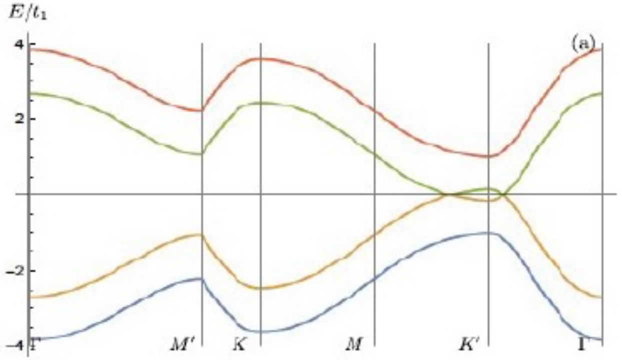


Presentation Aspen 2022

"New Directions in Strong Correlation Physics"







Thanks to ANR and DFG

Periodic Table of Topological Invariants: Application of Topology to Physics
Topological Insulators and Superconductors

See Andrei B. Bernevig book

Quantum Hall system
Quantum Anomalous Hall Effect

Kitaev p-wave superconductor

p+ip superconductor

Symmetry Class	Time reversal symmetry	Particle hole symmetry	Chiral symmetry
A	No	No	No
AIII	No	No	Yes
AI	Yes, $T^2 = 1$	No	No
BDI	Yes, $T^2 = 1$	Yes, $C^2 = 1$	Yes
D	No	Yes, $C^2 = 1$	No
DIII	Yes, $T^2 = -1$	Yes, $C^2 = 1$	Yes
AII	Yes, $T^2 = -1$	No	No
CII	Yes, $T^2 = -1$	Yes, $C^2 = -1$	Yes
C	No	Yes, $C^2 = -1$	No
CI	Yes, $T^2 = 1$	Yes, $C^2 = -1$	Yes

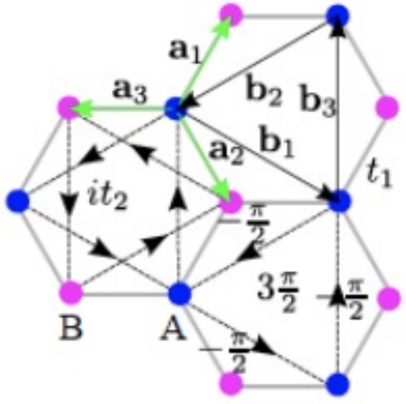
Topological states are characterized by a topological invariant linked to transport
Topology can also be achieved by applying a radial magnetic field on a (Bloch) sphere

Analogy to Gauss' law for electromagnetism
Realized with current technology, spin-1/2 in curved space

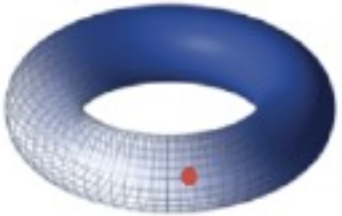
Monopoles

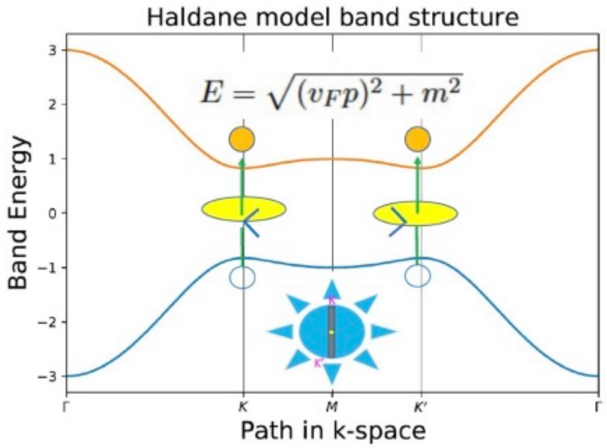
Spin-1/2 analogy

Haldane Model 1988, Nobel prize 2016
Kane-Mele Model 2005



Graphene





$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}(\mathbf{k}) \cdot \hat{\sigma},$$

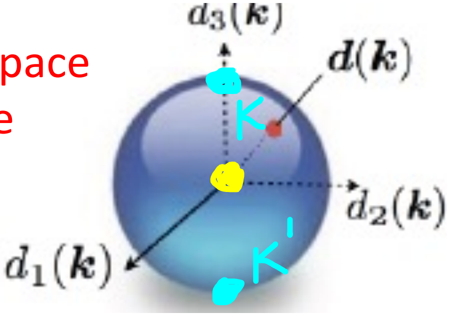
We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\,\mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\,\mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\,\mathbf{b_i} \right).$$

$m \sim 3\sqrt{3}\, t_2$
$\Phi = \frac{\pi}{2}$

The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

SO(3): parameter space
SU(2): Bloch sphere



$$\boxed{C_- = \frac{1}{4\pi} \int_{\mathrm{BZ}} d\mathbf{k}\, \hat{\mathbf{d}} \cdot \left(\partial_1 \hat{\mathbf{d}} \times \partial_2 \hat{\mathbf{d}} \right)}$$

$C = \pm 1$
$\mathbb{Z}$

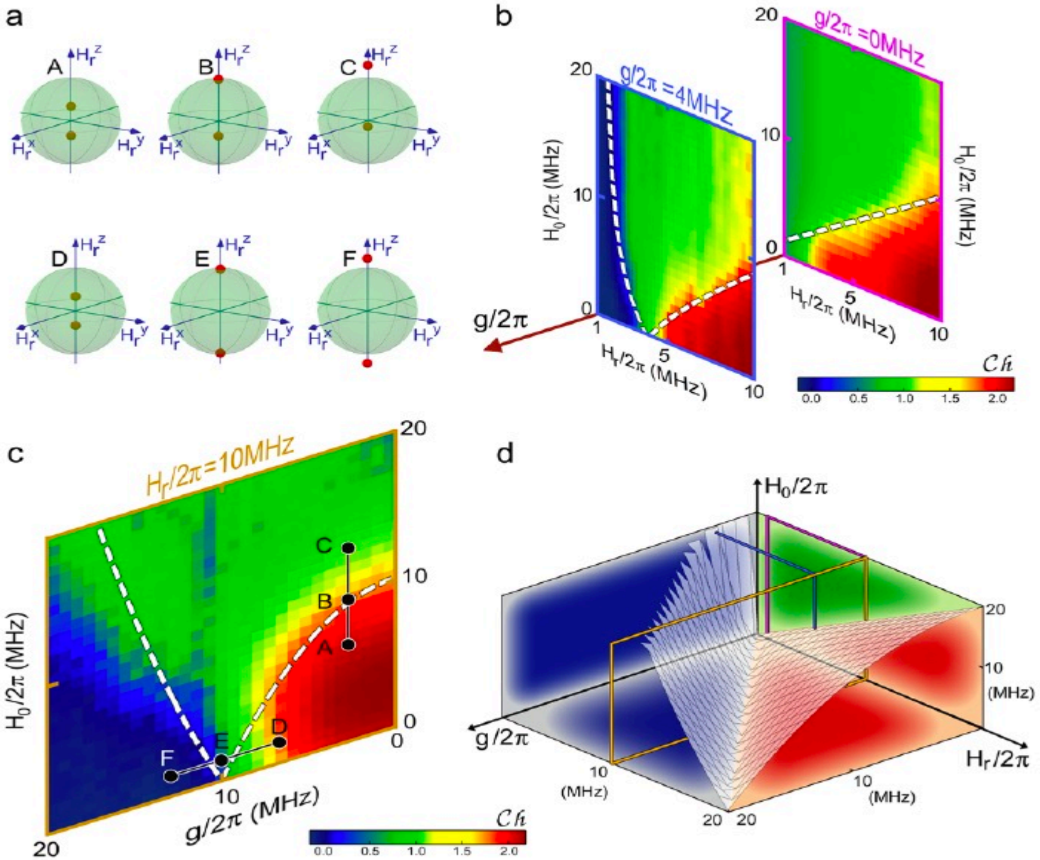
Analog of Bardeen-Cooper-Schrieffer pair from the reciprocal space
"2 giant Bloch spheres in k-space"

The Hamiltonian of this system is given by

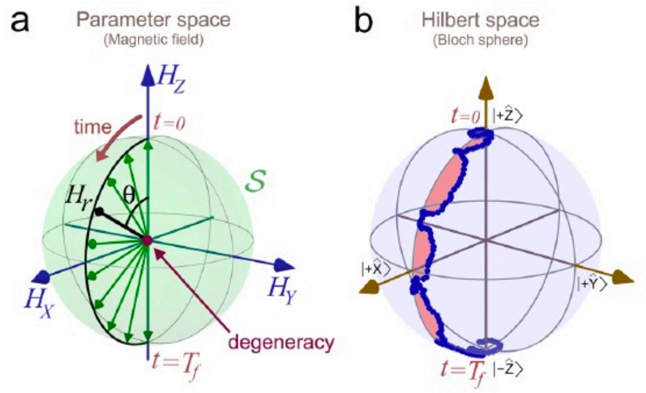
$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)], \tag{5}$$

where 1 and 2 refer to qubit 1 (Q1) and qubit 2 (Q2)

<u>Santa-Barbara "google":</u>
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)



Realization in superconducting circuits
In cavity



Berry, 1984

$$C = \frac{1}{2\pi}\oint \vec{F}\cdot d^2\vec{s}$$

$$\vec{F} = \vec{\nabla}\times\vec{A}$$

$$\vec{A} = i\langle\psi|\vec{\nabla}|\psi\rangle$$

General Topometry: Smooth Fields

Joel Hutchinson and Karyn Le Hur, arXiv:2002.11823
Communications Physics, **4** 144 (2021)

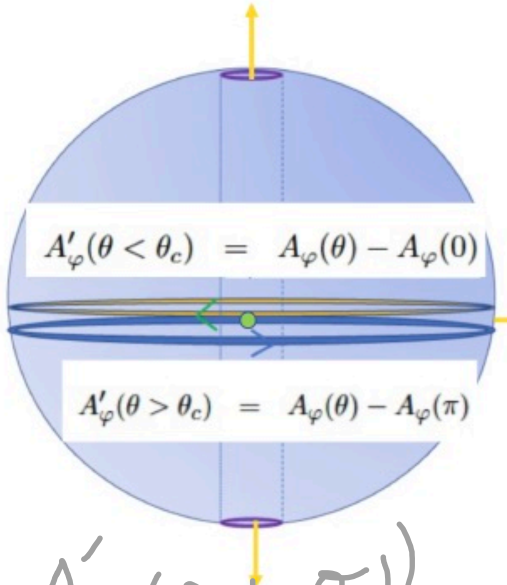
$$\boxed{C = \frac{1}{2\pi}\int\!\!\int_{S^{2\prime}} \nabla \times \mathbf{A}' \cdot d^2\mathbf{s},}$$

(1) $$\frac{1}{2\pi}\int\!\!\int_{north'} \nabla \times \mathbf{A} \cdot d^2\mathbf{s} = \frac{1}{2\pi}\int_0^{2\pi} \left(A_{N\varphi}(\theta,\varphi) - A_\varphi(0)\right) d\varphi.$$

(2) $$\frac{1}{2\pi}\int\!\!\int_{south'} \nabla \times \mathbf{A} \cdot d^2\mathbf{s} = -\frac{1}{2\pi}\int_0^{2\pi} \left(A_{S\varphi}(\theta,\varphi) - A_\varphi(\pi)\right) d\varphi.$$

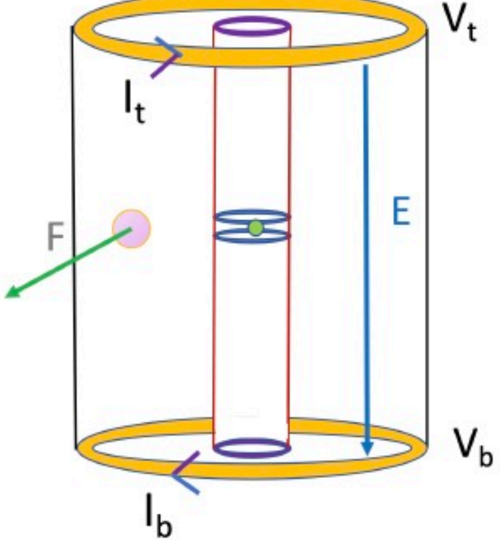
$$\begin{cases} \lim\limits_{\theta \to 0} A_{N\varphi} = \lim\limits_{\theta \to 0} A_{S\varphi} = A_\varphi(0) \\ \lim\limits_{\theta \to \pi} A_{S\varphi} = \lim\limits_{\theta \to \pi} A_{N\varphi} = A_\varphi(\pi) \end{cases}$$

$\theta_c \mapsto 0$



θ_c

$C = 1$

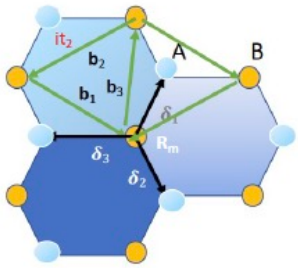


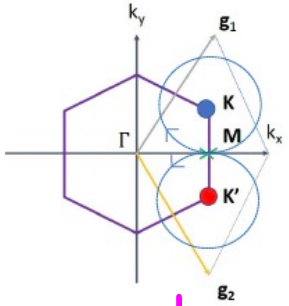
$$C = A_\varphi(0) - A_\varphi(\pi)$$
$$= \frac{1}{2\pi}\int d\varphi \left(A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c)\right)$$

$$C = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi} F_{\varphi\theta}\, d\varphi\, d\theta,$$

Local Interpretation of C^2

Karyn Le Hur, arXiv:2106.15665





M point lattice

$$C = \mathcal{A}_\varphi(0) - \mathcal{A}_\varphi(\pi) = \mathcal{A}'_\varphi(\theta > \theta_c) - \mathcal{A}'_\varphi(\theta < \theta_c).$$

$$C^2 = \mathcal{A}'^2_\varphi(\theta > \theta_c) + \mathcal{A}'^2_\varphi(\theta < \theta_c) - 2\mathcal{A}'_\varphi(\theta > \theta_c)\mathcal{A}'_\varphi(\theta < \theta_c).$$

$$\mathcal{I}\left(\frac{\pi}{2}\right) = \mathcal{I}(M) = \frac{C^2}{2}(2v_F^2)$$

conductivity:

$$F_{p_x p_y}(\theta) = i\frac{(\langle\psi_-|\partial_{p_x}\mathcal{H}|\psi_+\rangle\langle\psi_+|\partial_{p_y}\mathcal{H}|\psi_-\rangle - (p_x \leftrightarrow p_y))}{(E_- - E_+)^2},$$

$$\frac{m^2}{v_F^2}\left(F_{p_x p_y}(0) \pm F_{p_x \pm p_y}(\pi)\right) = \mathcal{A}_\varphi(0) - \mathcal{A}_\varphi(\pi) = C.$$

$$\mathcal{P}(\tilde{\omega}, t) = \frac{4A_0^2}{(\hbar\tilde{\omega})^2} C^2 \sin^2\left(\frac{1}{2}\tilde{\omega} t\right)$$

$$\mathcal{I}(\theta) = \left\langle\psi_+\left|\frac{\partial\mathcal{H}}{\partial p_x}\right|\psi_-\right\rangle\left\langle\psi_-\left|\frac{\partial\mathcal{H}}{\partial p_x}\right|\psi_+\right\rangle + \left\langle\psi_+\left|\frac{\partial\mathcal{H}}{\partial p_y}\right|\psi_-\right\rangle\left\langle\psi_-\left|\frac{\partial\mathcal{H}}{\partial p_y}\right|\psi_+\right\rangle = 2v_F^2\left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right)$$

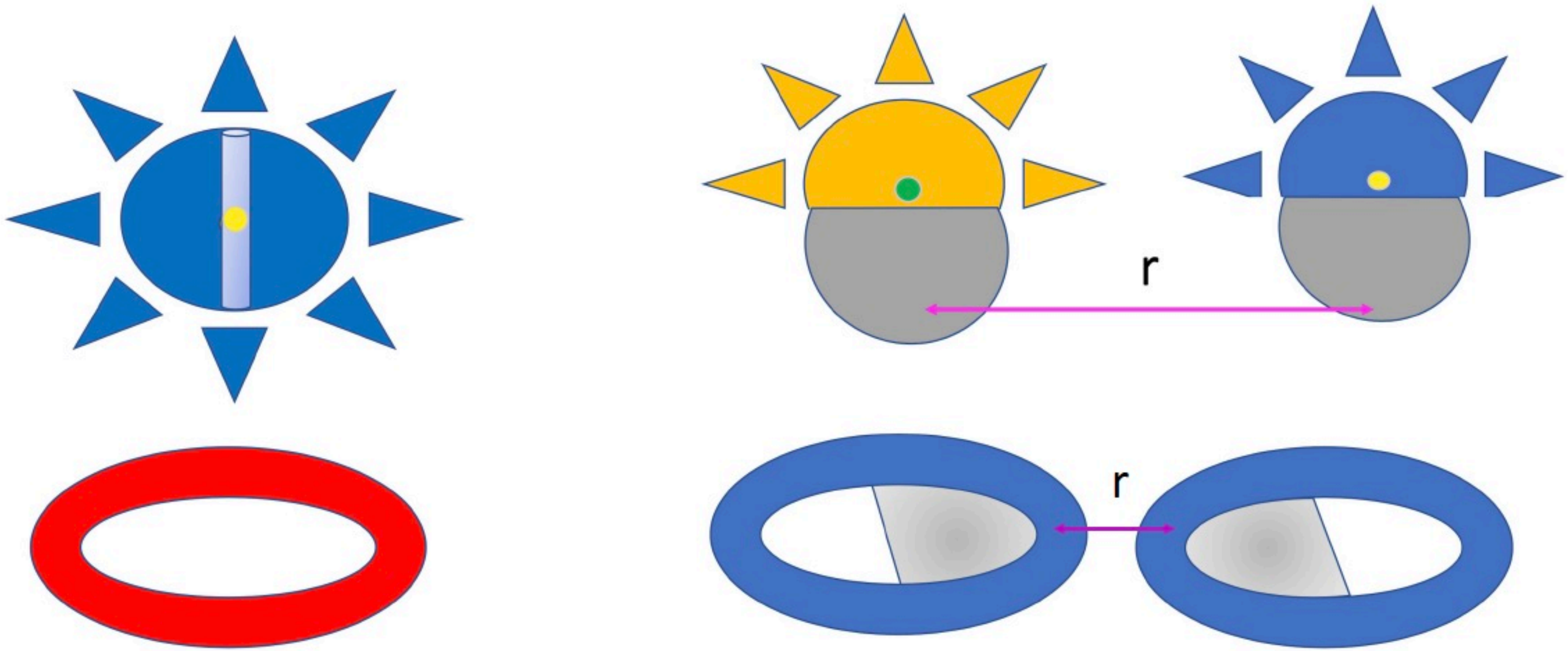
$$\sigma_x = \frac{1}{v_F}\frac{\partial\mathcal{H}}{\partial p_x}$$

$$\sigma_y = \frac{1}{v_F}\frac{\partial\mathcal{H}}{\partial p_y}$$

$$\frac{\mathcal{I}(\theta)}{2v_F^2} = \left(2\mathcal{A}'_\varphi(\theta < \theta_c)\mathcal{A}'_\varphi(\theta > \theta_c) + C^2\right)$$

$$\frac{\mathcal{I}(0) + \mathcal{I}(\pi)}{4v_F^2} = C^2$$

time



To realize fractional topological numbers, we introduce the specific system of interest whose ground state evolves from a *product state* at $\theta = 0$ to an *entangled state* at $\theta = \pi$:

$$\boxed{|\psi(0)\rangle = |\uparrow\rangle_1 |\uparrow\rangle_2 \to |\psi(\pi)\rangle = \frac{e^{i\varphi}}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2).}$$

Relation entanglement and Topology

Key point: Smooth Fields are uniquely defined on the north' and south' regions

Hilbert space representation of 2-spheres' model, $|\Phi_+\rangle_1$ & $|\Phi_-\rangle_i$

$$|\psi\rangle = \sum_{kl} c_{kl}(\theta)|\Phi_k(\varphi)\rangle_1|\Phi_l(\varphi)\rangle_2,$$

Poles: all azimuthal angles are equivalent $c_{kl}=c_{kl}(\theta)$

$$\begin{cases} \theta \to 0 & |\psi\rangle = |\Phi_+\rangle_1|\Phi_+\rangle_2 \\ \theta \to \pi & |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_+\rangle_1|\Phi_-\rangle_2+|\Phi_-\rangle_1|\Phi_+\rangle_2\right) \\ r=0 & |\psi\rangle = |\Phi_-\rangle_1|\Phi_-\rangle_2 \end{cases}$$

$$i\langle\psi(\pi)|\partial_j|\psi(\pi)\rangle = \frac{1}{2}\left(A_{j\varphi}(0)+A_{j\varphi}(\pi, r=0)\right) = A_{j\varphi}(\pi)$$

$$r=0 \quad A_{j\varphi}(0)-A_{j\varphi}(\pi, r=0) = q = C = 1$$

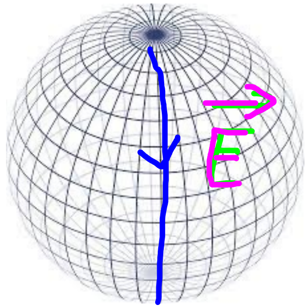
Therefore: $C_j = A_{j\varphi}(0)-A_{j\varphi}(\pi) = A_{j\varphi}(0) - \frac{1}{2}A_{j\varphi}(0) + \frac{q}{2} - \frac{1}{2}A_{j\varphi}(0)$

$= \frac{q}{2}$

Transport on the sphere

Joel Hutchinson and Karyn Le Hur, Communications Physics 4, 144 2021 Nature Journal, open access

$$\mathbf{E} = E\mathbf{e}_{x_\parallel} = -\nabla V$$



$T = \frac{\pi\hbar}{qE}$

$$H = \frac{(\hbar k_\parallel)^2}{2m} + \frac{(\hbar k_\perp)^2}{2m} + qV - \mathbf{d}\cdot\boldsymbol{\sigma}$$

Newton equation $ma_\parallel = \hbar \dot{k}_\parallel = qE$

$$\boxed{\theta(t) = k_\parallel(t) = \frac{q}{\hbar}Et.}$$

Parseval-Plancherel Theorem

$$\boxed{J_\perp = \frac{q}{T}\int_0^T dt \frac{d\langle x_\perp\rangle}{dt} = \frac{q}{T}\left(\langle x_\perp\rangle(T) - \langle x_\perp\rangle(0)\right) = \oint d\varphi \left(J_\varphi(\varphi, T) - J_\varphi(\varphi, 0)\right),}$$

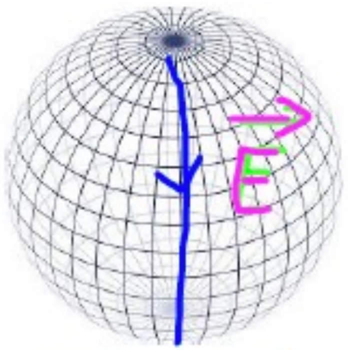
$$J_\varphi(\varphi,\theta) = \frac{iq}{4\pi T}\left(\psi^* \frac{\partial}{\partial\varphi}\psi - \frac{\partial\psi^*}{\partial\varphi}\psi\right) = \frac{iq}{2\pi T}\psi^*\frac{\partial}{\partial\varphi}\psi$$

$q = e$

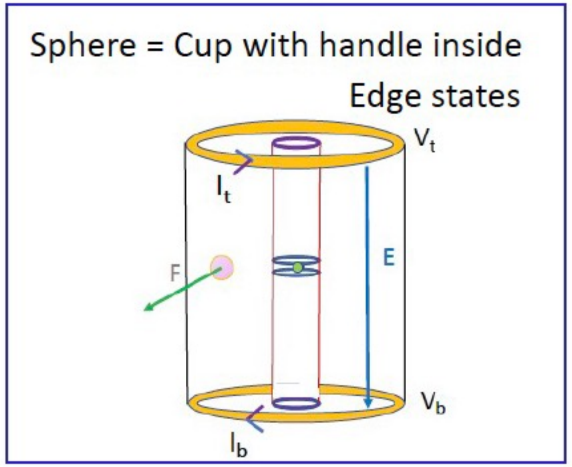
$$\boxed{J_\perp(\theta) = \frac{e}{2\pi T}\oint d\varphi A'_{N\varphi}(\varphi,\theta) = \frac{e}{T}A'_\varphi(\theta < \theta_c),}$$

$$\boxed{J_\perp = \frac{e^2}{h} 2 E}$$

Quantum Hall conductivity



$$T = \frac{\pi\hbar}{qE}$$



$$G = \frac{e^2}{\hbar} = \frac{dI}{dV}$$

$$\Delta P = C = \int_0^T dt \frac{j(t)}{e}, \qquad j(t) = J_\perp$$

We define the vector associated to the Chern number

$$\mathbf{C} = \frac{1}{2\pi}\int d\mathbf{k} \times \mathbf{F},$$

with $\mathbf{F} = \nabla \times \mathbf{A}$ such that $\mathbf{C}$ has the direction of the induced perpendicular current.

$$\hbar\dot{\mathbf{k}} = -e\mathbf{E}, \Rightarrow \mathbf{C} = -\frac{1}{2\pi}\int dt \frac{e}{\hbar}\mathbf{E} \times \mathbf{F} \Rightarrow \mathbf{j}(\mathbf{k}) = -\frac{e^2}{h}\mathbf{E} \times \mathbf{F}.$$

$$\boxed{\mathbf{v} = -\frac{e}{h}\mathbf{E} \times \mathbf{F}.}$$

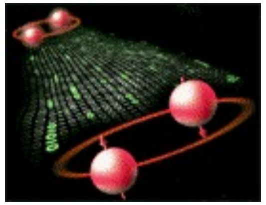
Karplus-Luttinger velocity (1954)

$$|\mathbf{j}| = \frac{e^2}{h}\int (d\mathbf{k} \times \mathbf{F}) \cdot \mathbf{E} = \frac{e^2}{h}C|\mathbf{E}|, \qquad \boxed{\sigma_{xy} = \frac{e^2}{h}C}.$$

Formula generalisable to multispheres or planes
Current response in a plane

C=1/2: measure = projection on one of the 2 Kets of ψ with probability 1/2

Fractional Topological Numbers



Joel Hutchinson and Karyn Le Hur, arXiv:2002.11823
Communications Physics, **4** 144 (2021)

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2$$

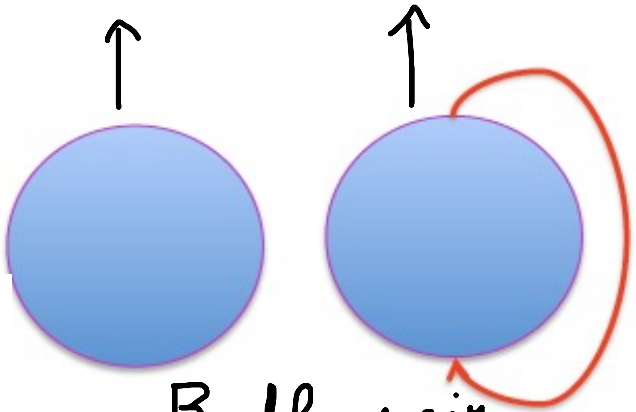
$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i),$$

Geometry :

$$\mathcal{C}^i = -(\mathcal{A}_\phi^i(\pi) - \mathcal{A}_\phi^i(0)).$$

$$\boxed{C^j = \frac{1}{2}}$$

$$\mathcal{C}^j = \frac{1}{2}\Big(\langle \sigma_z^j(\theta = 0)\rangle - \langle \sigma_z^j(\theta = \pi)\rangle\Big).$$



Bell pair

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$

Einstein-Podolsky-Rosen pair
or resonating valence bond

Analogy to cuprates with hot and cold spots in the reciprocal space

Model

Kane-Mele model

$\mathcal{H}^-$ with $\tilde{r}=0$

$\vec{H}_1 = -\vec{H}_2$

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2 .$$

$\mathcal{H}_+$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

$f(\theta) = \text{cst}$

$\theta = 0$ $|\uparrow\uparrow\rangle$

$$\begin{aligned} E_{\uparrow\uparrow} &= -2H - M_1 - M_2 + \tilde{r} f(0) \\ E_{\uparrow\downarrow} &= -M_1 + M_2 - \tilde{r} f(0) \\ E_{\downarrow\uparrow} &= M_1 - M_2 - \tilde{r} f(0) \\ E_{\downarrow\downarrow} &= 2H + M_1 + M_2 + \tilde{r} f(0), \end{aligned}$$

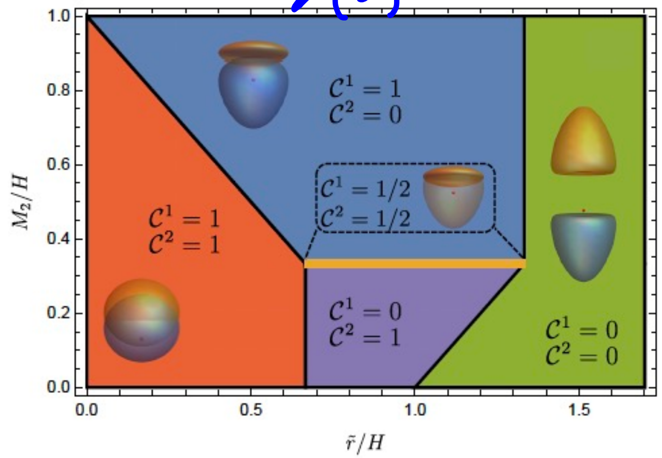
$\theta = \pi$ $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

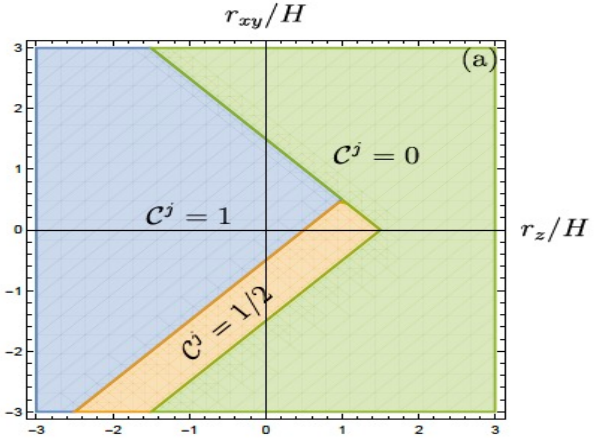
$$\begin{aligned} E_{\uparrow\uparrow} &= 2H - M_1 - M_2 + \tilde{r} f(\pi) \\ E_{\uparrow\downarrow} &= -M_1 + M_2 - \tilde{r} f(\pi) \\ E_{\downarrow\uparrow} &= M_1 - M_2 - \tilde{r} f(\pi) \\ E_{\downarrow\downarrow} &= -2H + M_1 + M_2 + \tilde{r} f(\pi). \end{aligned}$$

$M_1 = M_2$

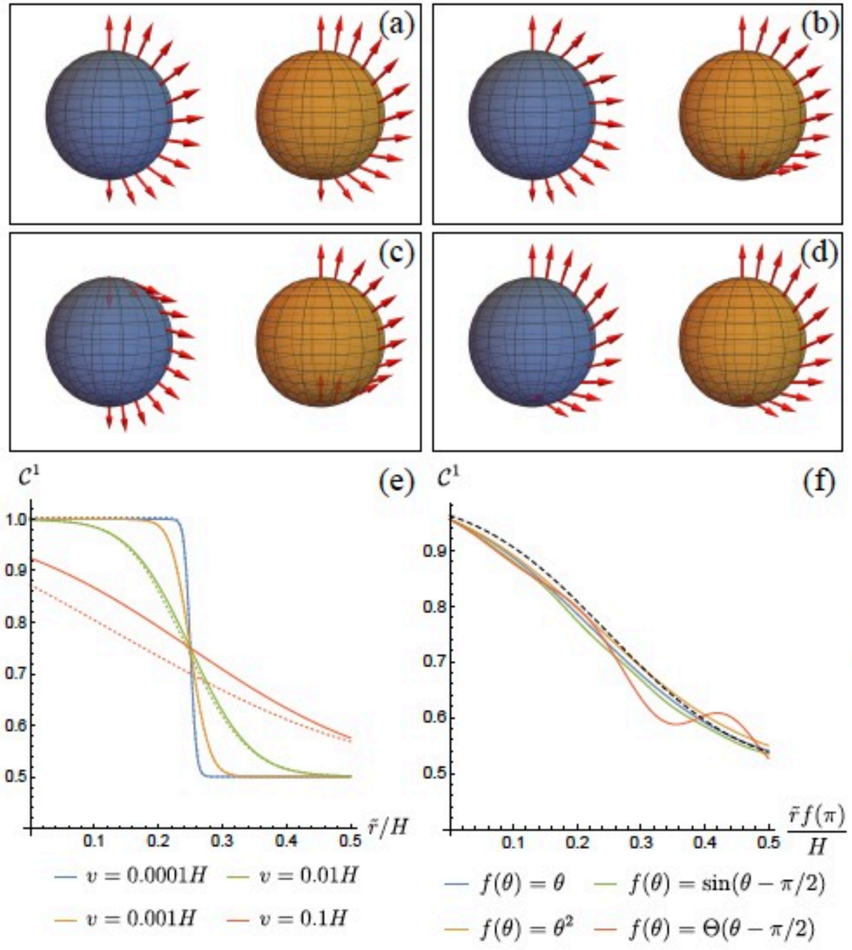
$\mathbb{Z}_2$ symmetry

$$H - M < \tilde{r} < H + M,$$





Time-dependent protocol



dynamics robust to various forms of interactions

$$\mathcal{C}^j \approx \frac{3}{4} + \frac{\pi}{4}\mathrm{Re}\left(e^{i3\pi/4}e^{-\gamma\pi/4}\frac{\mathrm{sgn}(\Delta)\sqrt{\gamma}}{\Gamma(1/2+i\gamma/4)\Gamma(1-i\gamma/4)}\right),$$

$$\boxed{\mathcal{C}^j \to \frac{3}{4} - \frac{1}{4}\mathrm{sgn}(\Delta),}$$

$$\mathcal{H}^+_{\mathrm{eff}} \to \sqrt{2}Hvt\sigma_z + [\tilde{r}f(\pi) - H + M]\sigma_x.$$

$$\lambda \equiv \sqrt{2}Hv, \quad \Delta = \tilde{r}f(\pi) - H + M,$$

Effective Landau-Zener model

$$t \to t - \pi/v.$$

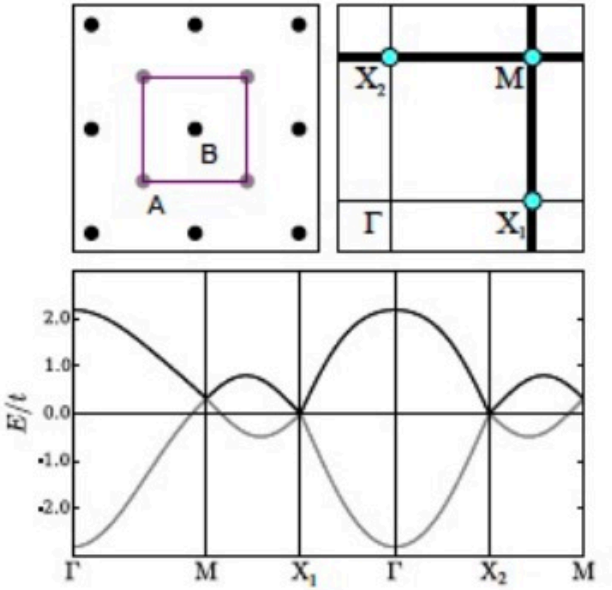
$$\gamma = \lambda^2/\Delta$$

Table & topological semimetals

Dirac Semimetals in Two Dimensions

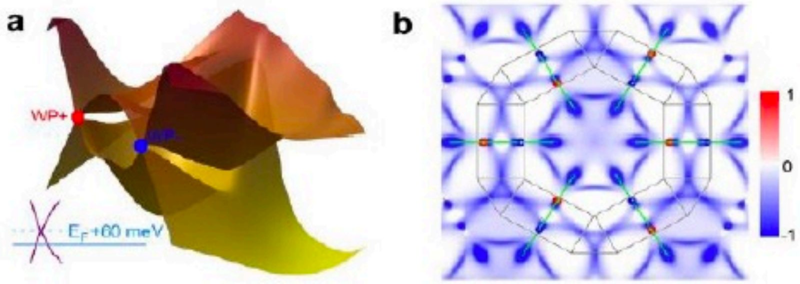
S. M. Young & C. L. Kane, PRL 2005

$$H = 2t\tau_x \cos\frac{k_x}{2}\cos\frac{k_y}{2} + t_2(\cos k_x + \cos k_y) + t^{\mathrm{SO}}\tau_z[\sigma_y \sin k_x - \sigma_x \sin k_y],$$

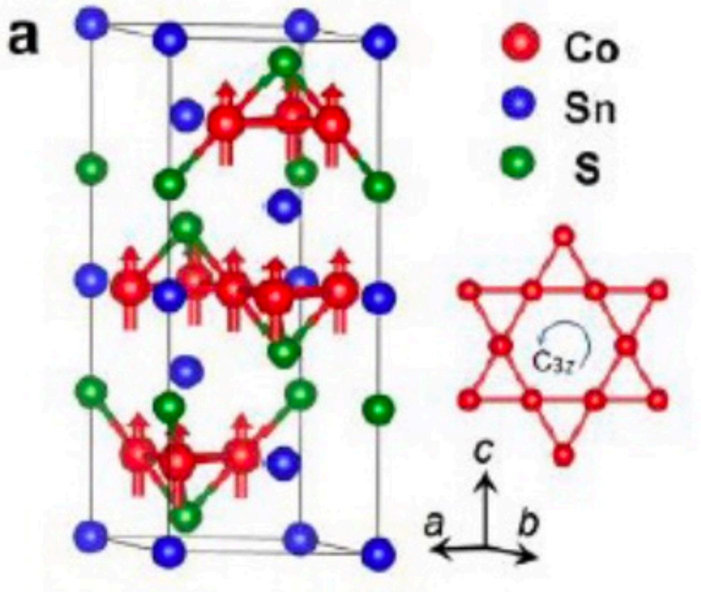


Magnetic Weyl semimetals in 3D: recent

Z. Guguchia et al., group of Zahid Hasan, Princeton
E. Liu et al., group of Claudia Felser Dresden
Figures from Enke Liu, Berry curvature k_x-k_y plane



ferro

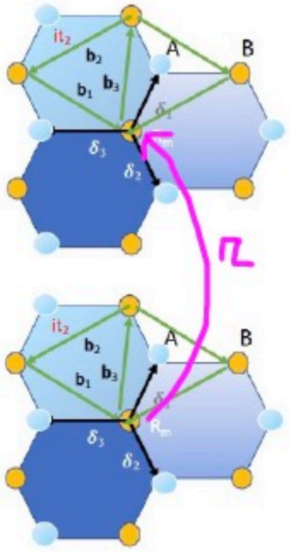


Anomalous Hall conductivity

2D model

Julian Legendre & Karyn Le Hur
Phys. Rev. Research, 2020

Bilayer system with $M_1 = M_2$

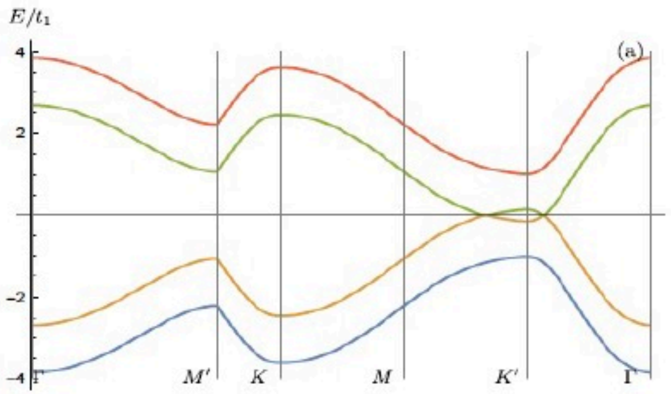


$$\mathcal{H} = (\psi^\dagger_{\boldsymbol{k}1}, \psi^\dagger_{\boldsymbol{k}2}) \mathcal{H}(\boldsymbol{k}) \begin{pmatrix} \psi_{\boldsymbol{k}1} \\ \psi_{\boldsymbol{k}2} \end{pmatrix},$$

where $\psi^\dagger_{\boldsymbol{k}i} \equiv (c^\dagger_{\boldsymbol{k}Ai}, c^\dagger_{\boldsymbol{k}Bi})$ and

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} (\boldsymbol{d} + M_1 \hat{z}) \cdot \boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\boldsymbol{d} + M_2 \hat{z}) \cdot \boldsymbol{\sigma} \end{pmatrix},$$

$d = 3\sqrt{3}\, t_2 \sin \Phi$



$d - M < r < d + M$

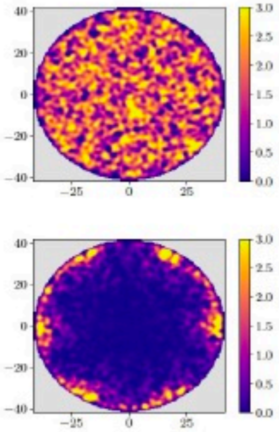
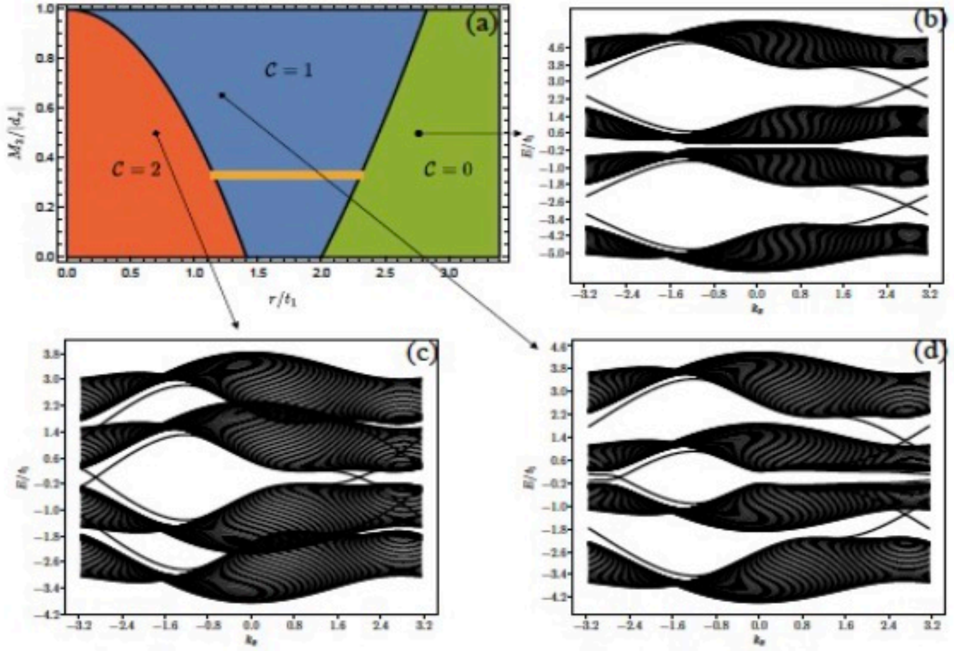
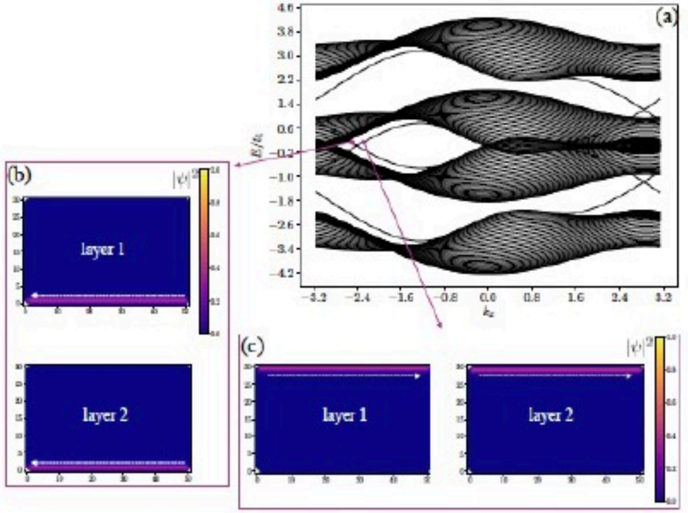


Figure 9: Top: Local density of states for a disk geometry with 30-site radius with $M_1 = M_2 = \sqrt{3}/3t_1$ and $r = 1.4t_1$ showing the edge mode and additional bulk states coming from the nodal ring semimetal in the reciprocal space. Bottom: Local density of states shifted very slightly from the line of symmetry: $M_2 = M_1 + 0.2$, $M_1 = \sqrt{3}/3t_1$ and $r = 1.4t_1$ in the blue region of the phase diagram, showing the single chiral edge mode.





Circular dichroism of light

Jones formalism: average 1 and 0 light responses

Topological semimetal in two dimensions

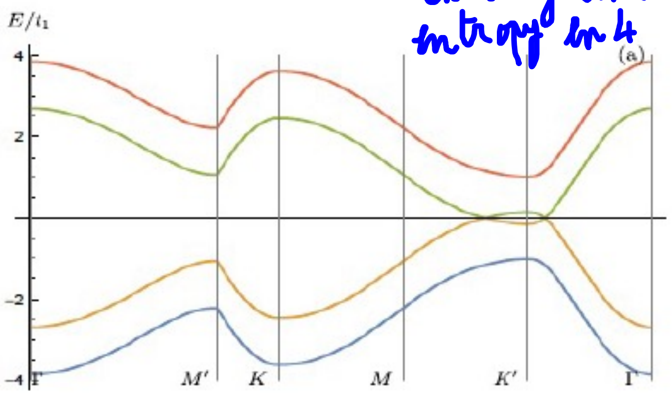
Summary of Geometry

$$(J_{xy})^j = C^j \frac{e^2}{h}$$

$$C^j = \frac{1}{2}$$

This formula is correct and is applicable in a sphere (plane j) from the poles (Dirac points)

entanglement entropy in 4



$$\psi_1 = \frac{1}{\sqrt{2}}(0,-1,0,1), \quad \psi_2 = \frac{1}{\sqrt{2}}(0,1,0,1),$$

$$\psi_3 = \frac{1}{\sqrt{2}}(-1,0,1,0), \quad \psi_4 = \frac{1}{\sqrt{2}}(1,0,1,0).$$

do not modify

$n_B - n_A$

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

Fractional
Topological Bloch band
resolved locally in "planes"

$$A_{j\varphi}(\mathbf{K}') = \frac{1}{2}A_{j\varphi}(\mathbf{K}) + \frac{1}{2}A^{r=0}_{j\varphi}(\mathbf{K}'),$$

$$c^\dagger_{B1}c^\dagger_{B2}|0\rangle = |\uparrow\uparrow\rangle,\ c^\dagger_{A1}c^\dagger_{A2}|0\rangle = |\downarrow\downarrow\rangle,\ c^\dagger_{B1}c^\dagger_{A2}|0\rangle = |\uparrow\downarrow\rangle,\ c^\dagger_{A1}c^\dagger_{B2}|0\rangle = |\downarrow\uparrow\rangle.$$

$$\tilde{C}^j = \frac{1}{2}\langle \underbrace{n^j_{KB} - n^j_{KA}}_{\sigma_z} - n^j_{K'B} + n^j_{K'A}\rangle = \frac{1}{2}$$

2 planes' model realized with a Coulomb interaction

Analogy with an Ising Interaction from the reciprocal space

$$\hat{n}_1^i = \frac{1}{2}(1 \pm \sigma_{1z})$$

$$\hat{n}_2^i = \frac{1}{2}(1 \pm \sigma_{2z}),$$

$i = a, b$ sublattices

Projection on occupied bands

$\zeta = +$ $\quad H_{Int}(K) = \lambda \hat{n}_1^a \hat{n}_2^a$

$\zeta = -$ $\quad H_{Int}(K') = \lambda \hat{n}_1^b \hat{n}_2^b$

$$H_{Int}^1 = \frac{\lambda}{4}\sigma_{1z}\sigma_{2z} + \frac{\lambda}{4} + \underbrace{\frac{\lambda}{4}\zeta(\sigma_{1z} + \sigma_{2z})}_{0}.$$

$$H_{int}^2 = \lambda' \left(\hat{n}_1^a(K)\hat{n}_2^b(K') + \hat{n}_1^b(K')\hat{n}_2^a(K) \right)$$

$$E_{aa}(K) + E_{bb}(K') = (-4d + 2\lambda + 2\lambda')$$

$d - M < \lambda + \lambda' < d + M$

equivalent to $\lambda_{eff} = \lambda' + \lambda$ in $\sigma_{1z}\sigma_{2z}$

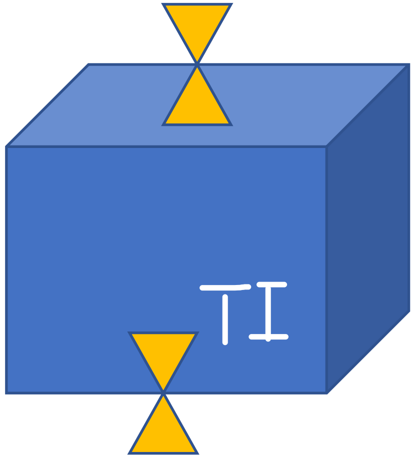
C=1/2

Relation to Magnetoelectric effect in 3D TIs

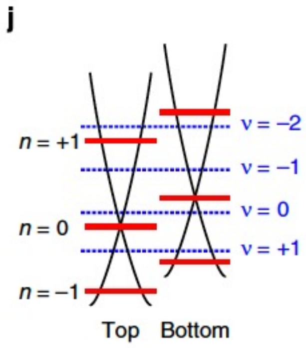
Experiment: R. Toshimi et al. Nature Communications 2015, $(Bi_{1-x}Sb_x)2Te_3$

Axion electrodynamics in 3D topological materials, recent review, A. Sekine and K. Nomura, arXiv:2011.13601

X. Qi, T. Hughes, S.-C. Zhang Phys. Rev. B **78**, 195424 (2008)



$$\sigma_{xy} = \mathrm{sgn}(m_z)\frac{e^2}{2h}$$

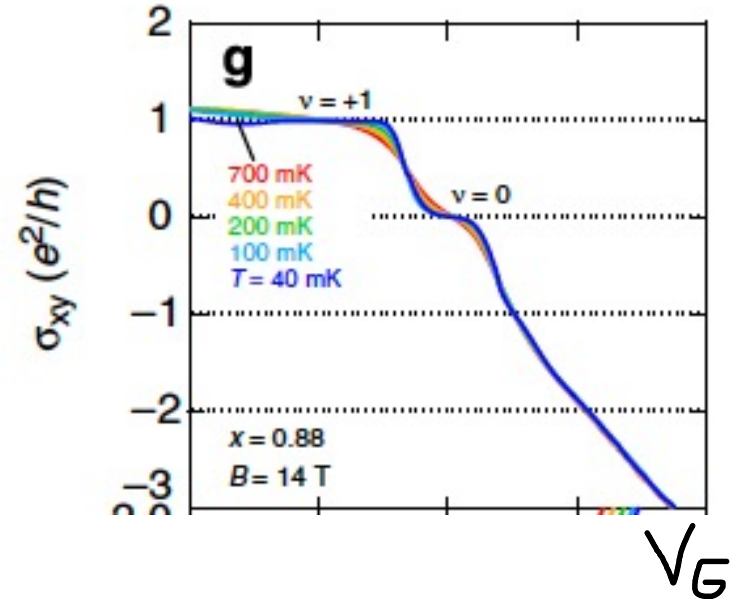


electron density

$$n_e = \frac{\sigma_{xy} B}{e}$$

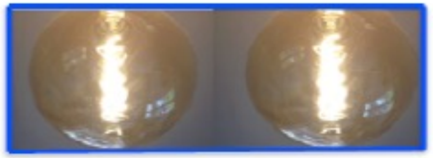
$$\rho_{Top} = \frac{1}{2}\frac{Be^2}{h}$$

$$\rho_{Bott} = -\frac{1}{2}\frac{Be^2}{h}$$

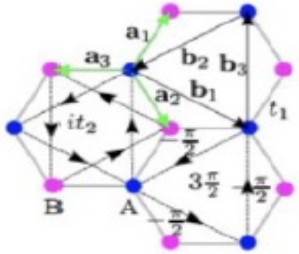


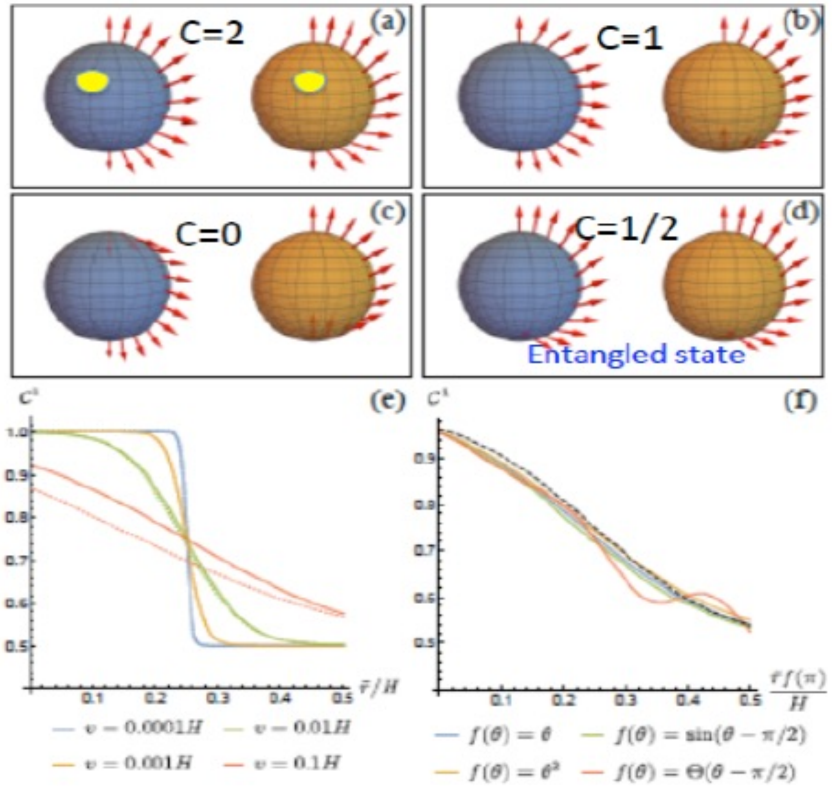
half hemisphere = 1 Dirac cone

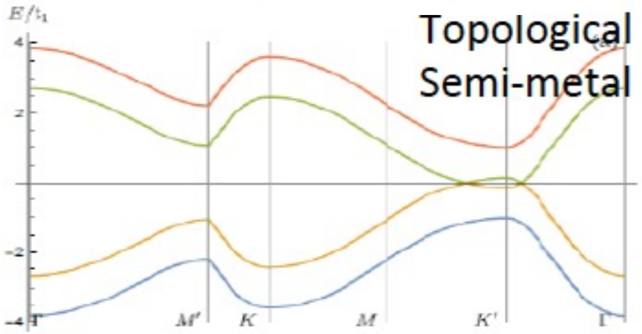
$$C = \frac{q}{2}$$



Spheres realizable in cQED mesoscopic and atomic systems







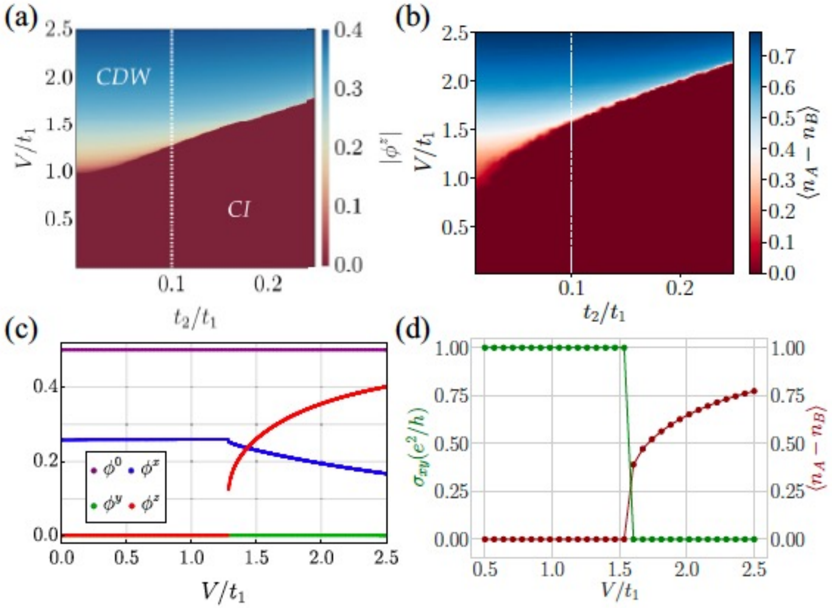
spheres	lattices
Spin 1/2	- 2 sublattices - Nambu basis
Radial Field on S^2	- Haldane model - P-wave SC - Resonating Bonds

Joel Hutchinson and Karyn Le Hur arXiv:2002.11823 and Communications Physics 4, 144 (2021)
https://www.nature.com/articles/s42005-021-00641-0

Peng Cheng, Philipp W. Klein, Kirill Plekhanov, Klaus Sengstock, Monika Aidelsburger, Christoph Weitenberg, Karyn Le Hur, Phys. Rev. B (2019) , application in bilayer systems

New efforts on interactions and Mott phases

Haldane Hubbard model, fermions

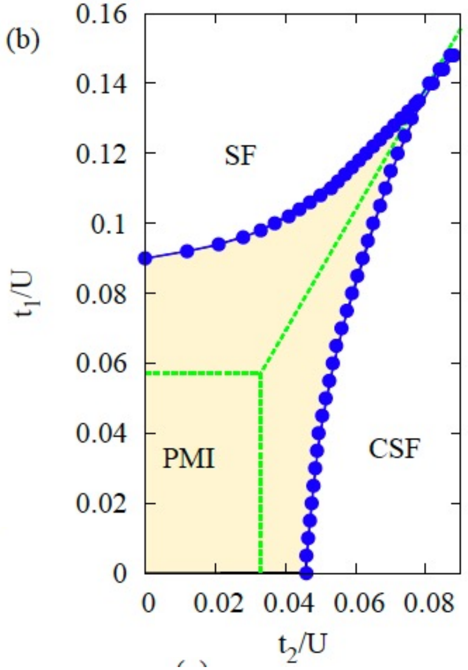


Ph. W. Klein, A. Grushin (DMRG), K. Le Hur Phys. Rev. B 2021
Stochastic approach, path integral variational method

Haldane Hubbard model, bosons

I. Vasic, A. Petrescu,
K. Le Hur, W. Hofstetter, 2015

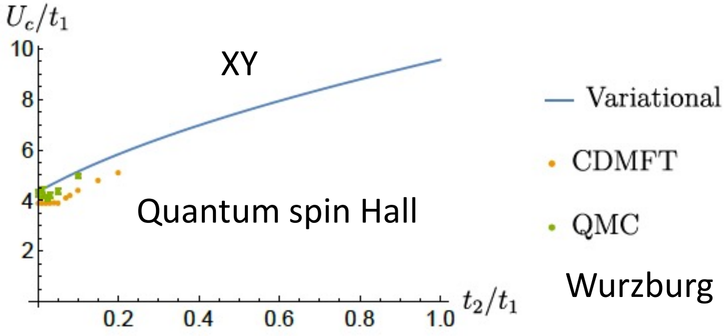
PMI state: Mott state
with topological particle-hole
excitations



DMFT
Cluster perturbation theory
Quantum field theory

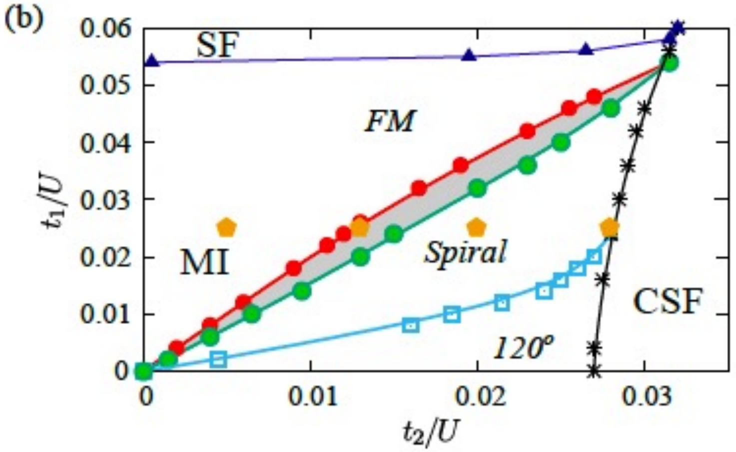
CSF: analog
of FFLO state for
bosons

Kane-Mele-Hubbard Bosons: K. Plekhanov, I. Vasic, A. Petrescu, R. Nirwan, G. Roux, W. Hofstetter, K. Le Hur, PRL 2017; new chiral spin state (peak in $\chi = \langle S_{r_i} \cdot (S_{r_i+u_1} \times S_{r_i+u_2}) \rangle$).



Wurzburg

Kane-Mele-Hubbard Model, fermions



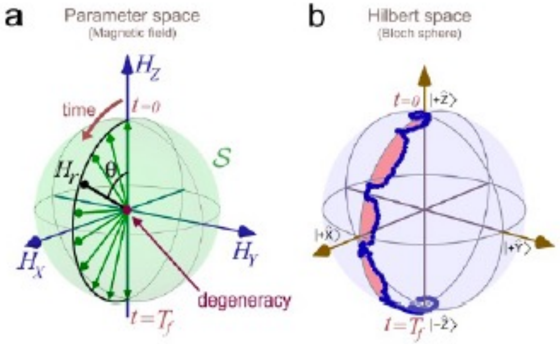
Analytical solution from stochastic approach for Mott transition in 2D, controllable fluctuations
J. Hutchinson, Ph. W. Klein, K. Le Hur 2021; agree with CDMFT W. Wu, S. Rachel, W.-M. Liu, K. Le Hur, 2012

Conclusion of Presentation

From graphene and analogy to spin-1/2 towards topological states

Another way to apply magnetism and smooth fields*

- Topological properties from the poles (K and K' points)
- Insight on Transport and Edge States correspondence
- Light-Matter Response I(θ) & C[2] Karyn Le Hur, arXiv:2106.15665

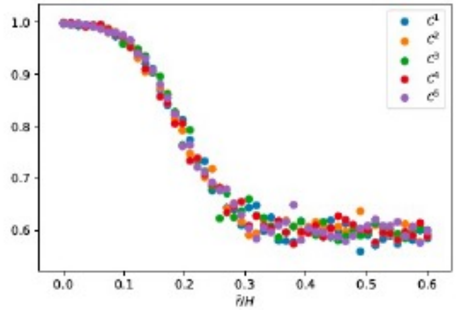


- Interaction Effects from the reciprocal space in a stochastic view & Light Philipp Klein, Adolfo Grushin, Karyn Le Hur, Phys. Rev. B **103**, 035114 (2021)
  Joel Hutchinson, Philipp Klein, Karyn Le Hur, Phys. Rev. B **104**, 075120 (2021)

General "entangled" GHZ states:
Relation with quantum spin liquids?
4 spins: relation to Kitaev Z_2 spin model

- Two spheres and "fractional" topological numbers
- Topological semi-metals:
  Nodal Ring Semi-Metal with Maximum entanglement entropy ln4
  Quantized π *Berry phase* $\rightarrow C = \frac{1}{2}$ *in each plane*, $\sigma_{xy} = Ce^2/h$

*Joel Hutchinson & Karyn Le Hur, Communications Physics 4, 144 (2021)



Even-Odd Effect

Cirq Algorithm,

Figure 12: Partial Chern numbers as a function of the coupling $\tilde{r}$ measured in a five-spins quantum circuit simulation with nearest-neighbour Ising interactions and periodic boundary conditions. To time-evolve the spins (qubits), we use a Trotter decomposition with 800 time steps and sweep velocity $v = 0.03H$. The bias field for all qubits is fixed to $M = 0.6H$. For $\tilde{r} \geq H - M \sim 0.4$ we verify the presence of the topological phase with $C^j = 3/5 = 0.6$ in agreement with Eq. (114).

Thanks to Students, Post-docs & Collaborators
Thanks for your Attention

Application, "Quantum dynamo effect": L. Henriet, A. Slocchi, P. P. Orth, K. Le Hur, 2017