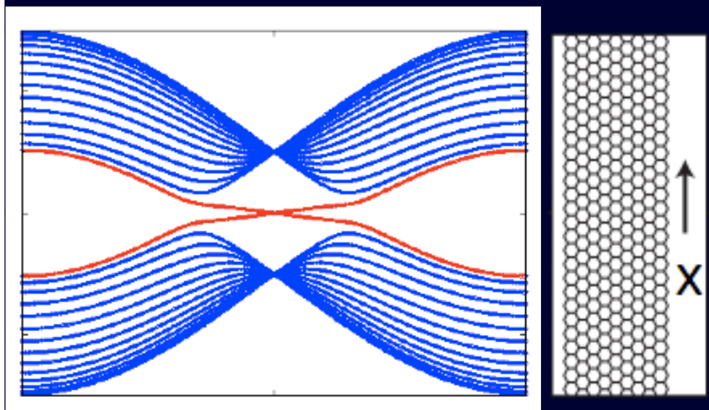
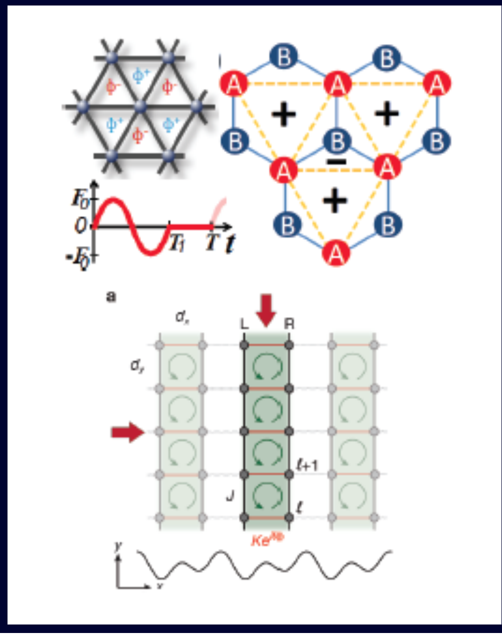
November 2014: Workshop on Dirac Matter & Topological Phases

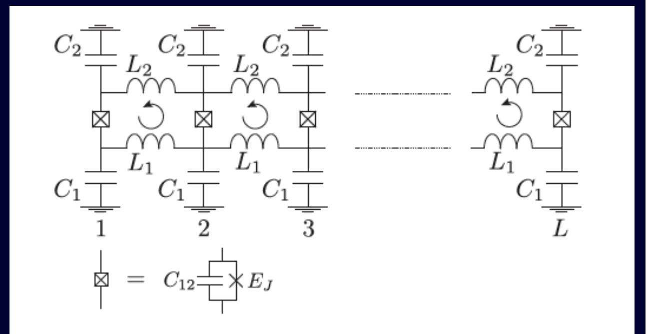
Topological Phases, Mott Physics & Artificial Gauge Fields

Karyn Le Hur

CPHT Ecole Polytechnique, France & CNRS







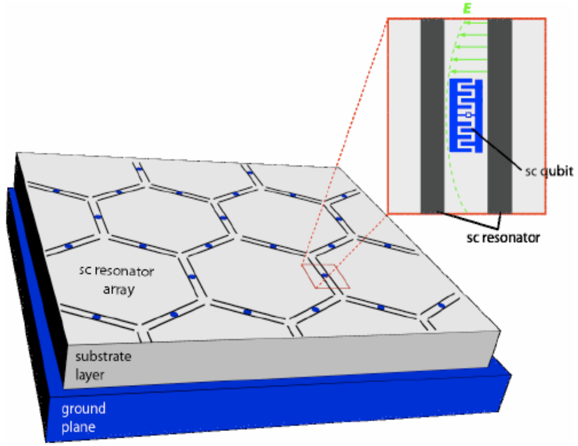
Our Program: Topological Phases, Interaction Effects & Gauge Fields

From Materials, to Ultra-Cold Atoms and Photon Simulators

Our first paper: Majorana fermions in SC Graphene
D. Bergman and KLH, PRB 2009
following P. Ghaemi & F. Wilczek

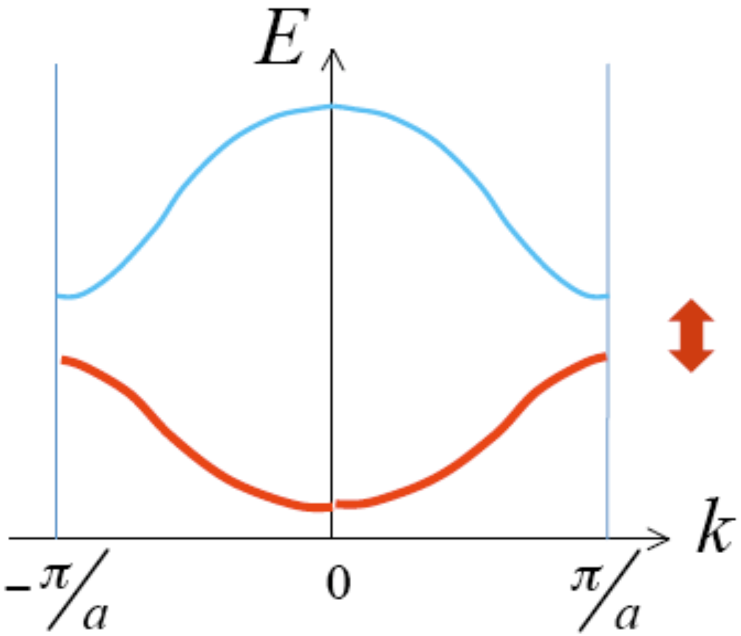
Outline of The Talk:

- **Topological Insulator in graphene lattices**
- **Kane-Mele Model, Interaction Effects**
- **Topological Mott Insulators**
- **Ultra-Cold Atoms**
- **Photons**
- **Ladders**

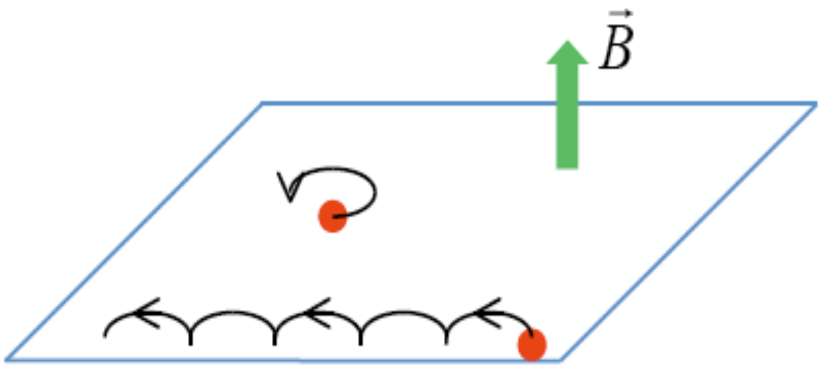


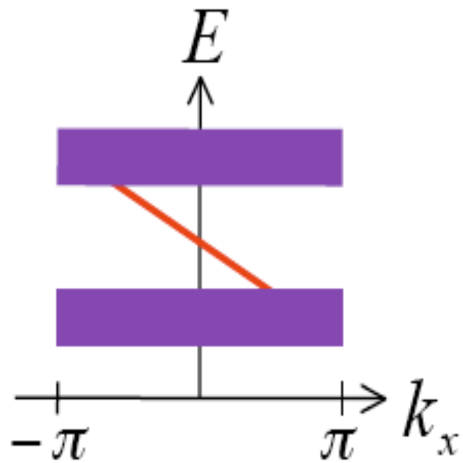
J. Koch & KLH, 2009

Topological Insulators

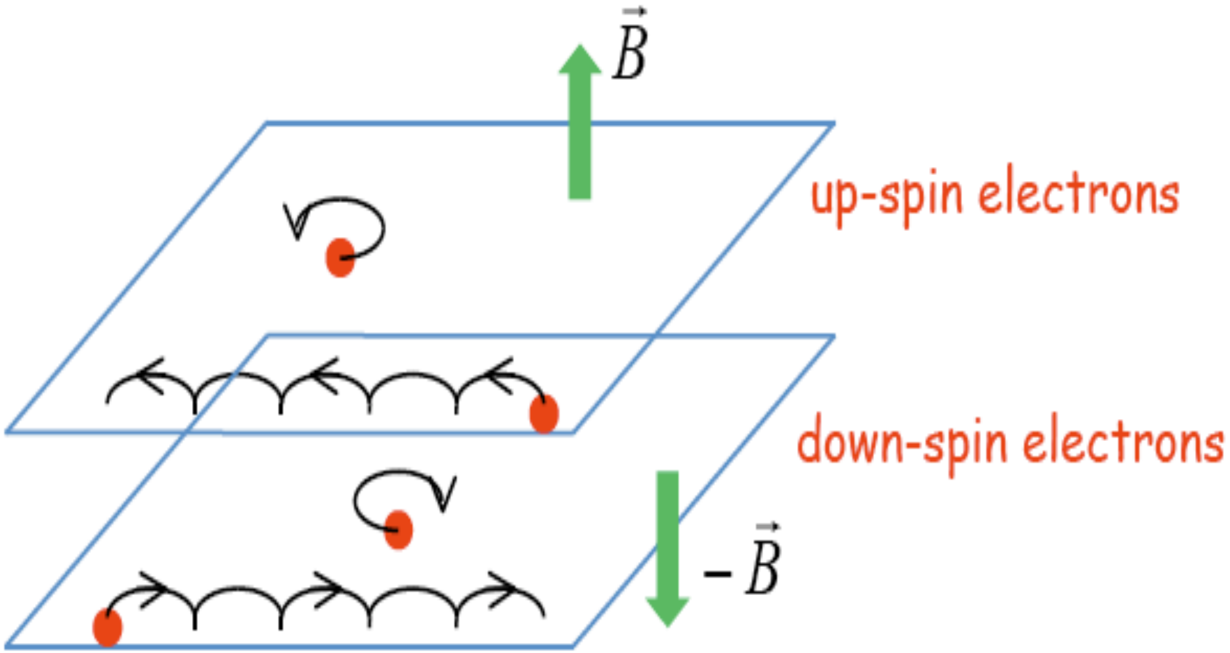


That's all? No





- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda \vec{L} \cdot \vec{\sigma}$
- Gapless helical edge mode (Kramers pair)



No spin rotation symmetry

Microscopic Description: Simple Standard Model, Kane-Mele

Time reversal invariant of **Haldane model** (1988): **Kane-Mele model**

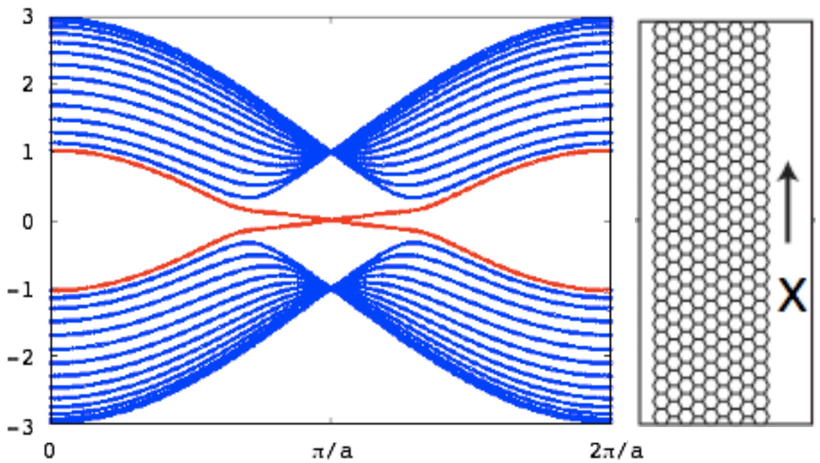
Kane & Mele, PRL 95, 226801 (2005)

see also: **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials) TALKS WEDNESDAY**

Very active field CITE REFERENCES

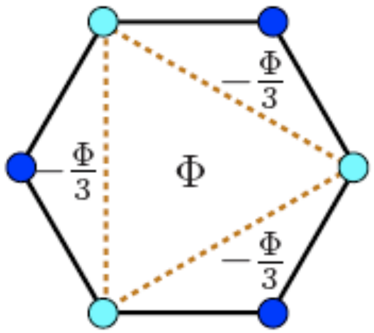
$$\mathcal{H} = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^z_{\sigma\sigma'} c^\dagger_{i\sigma} c_{j\sigma'} \quad \nu_{ij} = \pm 1$$

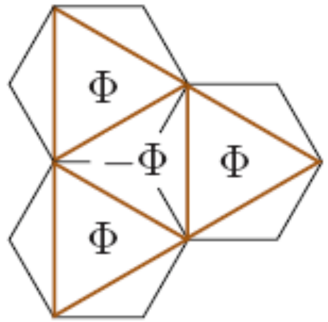
strip geometry:



edge states: Kramers's pair

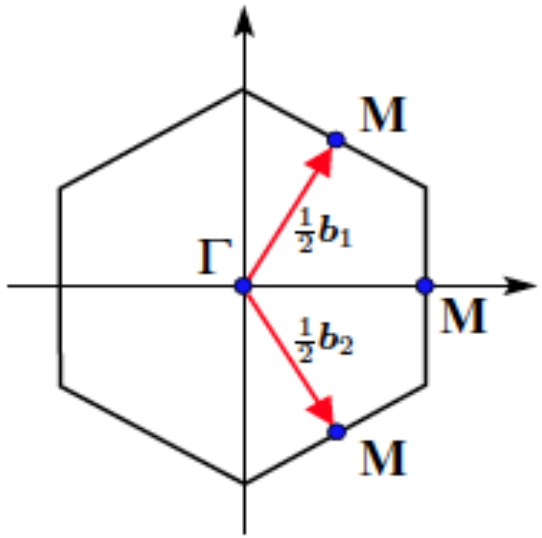
$$\mathcal{H} \propto \Psi^\dagger_k \sigma^z \tau^z \Psi_k$$





GRAPHENE LATTICE with ZERO NET FLUX

Z_2 Invariant: Spin Chern number



Time-reversal invariant
Points of the Brillouin zone

Following L. Fu and C. Kane:

$$\mathcal{H}_{\mathbf{k}} = \sum_{a=1}^{5} d_a(\mathbf{k})\Gamma^a$$

$$\mathcal{P} = \tau^x \otimes I = \Gamma^1$$

$$\mathcal{T} = i(I \otimes \sigma^y)K$$

Time-Reversal & Inversion Symmetry

Z_2 invariant given by (here, ν=0 or 1):

$$(-1)^\nu = \prod_{i=1}^{4} -\mathrm{sign}[d_1(\mathbf{\Gamma}_i)]$$

Single-particle band structure: see also Balents & Moore

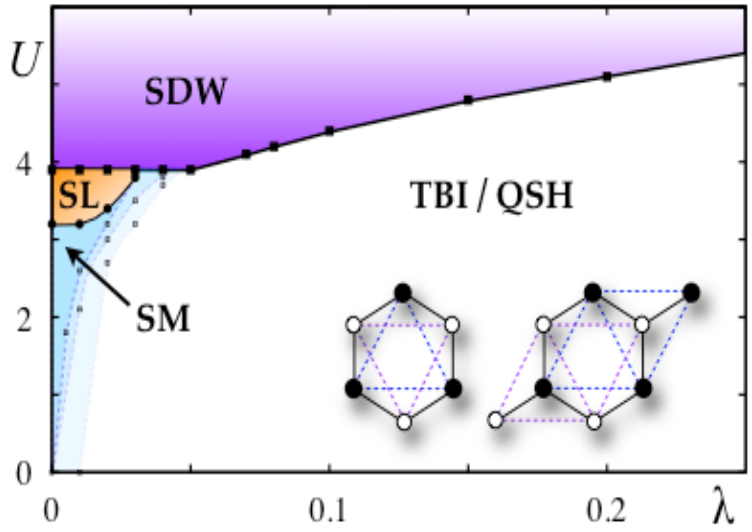
Phase Diagram: "Kane-Mele-Hubbard"

S. Rachel & KLH, PRB 2010

Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

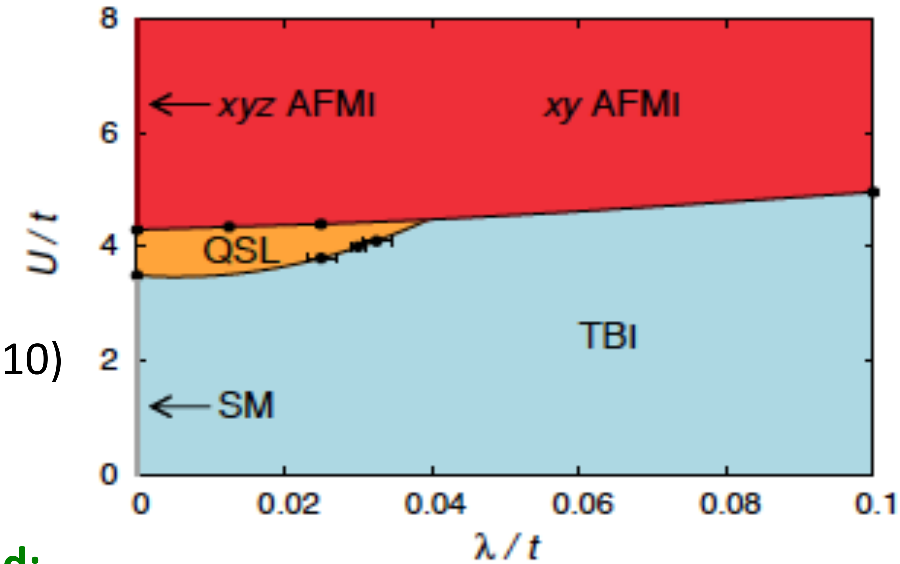
Real-space version
QMC continous-time
Impurity solver



3D XY
S. Rachel & KLH, 2010
Griset & C. Xu, 2011
D.-H. Lee, 2011

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)

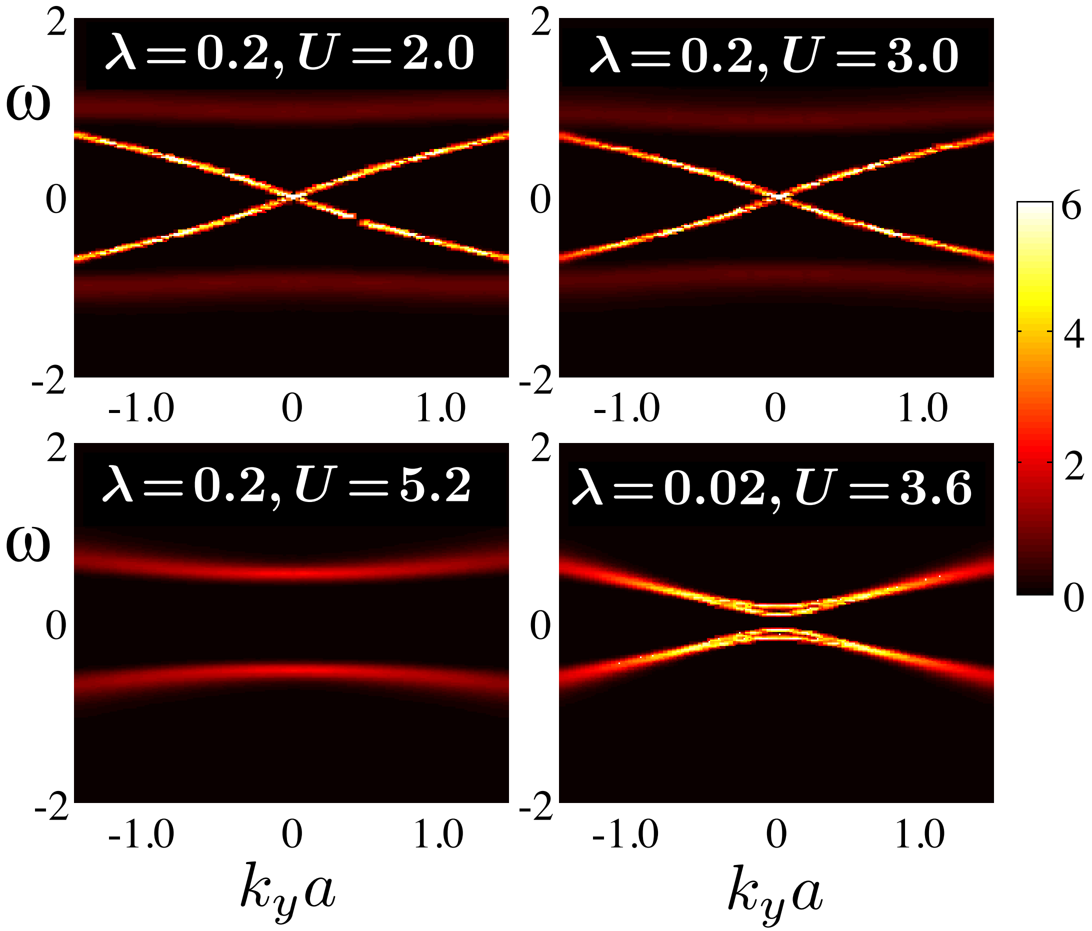


M. Hohenadler et al.
arXiv:1111.3949

Phys. Rev. Lett. **106**,
100403 (2011)

Absence of spin liquid:
S. Sorella et al. Scientific Reports 2012; S. R. Hassan & D. Senechal PRL 2013

(No) Edge States in A(k,ω)



CDMFT

Real-space version
QMC continous-time
Impurity solver

Some Reviews (not full list):
G. Kotliar et al, RMP 2006
T. Maier et al, RMP 2005
A.-M. Tremblay, B.-S. Kyung,
D. Senechal, 2006

DMFT:
A. Georges, G. Kotliar,
W. Krauth & M. Rozenberg et al.;
Metzner & Vollhardt

Wei Wu, Stephan Rachel, Wu-Ming Liu and KLH, PRB **85**, 205102 (2012)
See also Yamaji & Imada, 2011; Yu, Xie & Li 2011; Zheng, Zhang & C. Wu, 2011

Edge Theory & Mott Transition

C. Xu & J. Moore; C. Wu, A. Bernevig & S.-C. Zhang;…

$$H_0 = v_F \int dx \left(\psi^\dagger_{R\uparrow} i\partial_x \psi_{R\uparrow} - \psi^\dagger_{L\downarrow} i\partial_x \psi_{L\downarrow} \right)$$

$\psi^\dagger_{R\uparrow}\psi_{L\downarrow} + \text{h.c.}$ (**elastic**) Backscattering forbidden

$$H_I = U \int dx \left(\psi^\dagger_{R\uparrow} \psi_{R\uparrow} \psi^\dagger_{L\downarrow} \psi_{L\downarrow} \right)$$

$$H = \int dx \frac{v}{2} \left[\frac{1}{K} \left(\partial_x \phi\right)^2 + K \left(\partial_x \theta\right)^2 \right] - \frac{U m \sin \sqrt{4\pi}\phi}{(\pi a)^2}$$

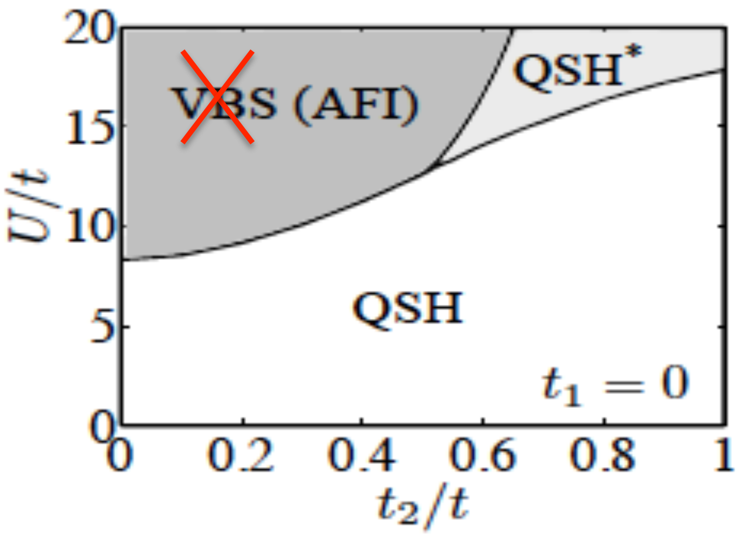
Minimal Model
for TBI/QSH phase

with $m = \langle \psi^\dagger_{R\uparrow} \psi_{L\downarrow} \rangle$

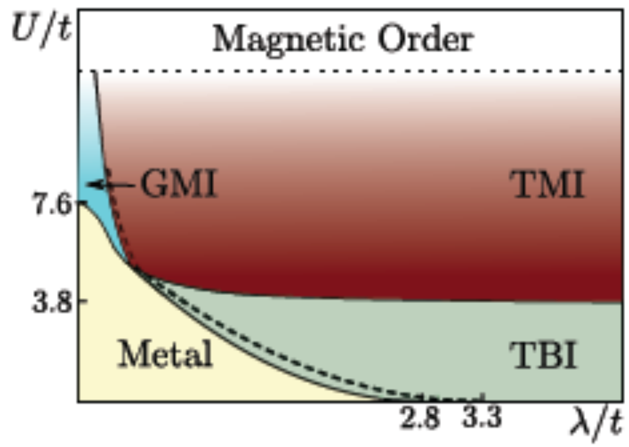
Connection to reality?

- Na_2IrO_3: honeycomb layers (Takagi;Gegenwart;Coldea; Damascelli)

Spin-orbit coupling slightly different: Jackeli and Khaliullin, Phys. Rev. Lett. 102, 017205 (2009); A. Shitade et al., Phys. Rev. Lett. 102, 256403 (2009); Y. Singh et al., Phys. Rev. Lett. 108, 127203 (2012). **(Many other papers to cite here)**



Ruegg & G. Fiete, PRL 2012



D. Pesin & L. Balents, Nature Phys. 2010

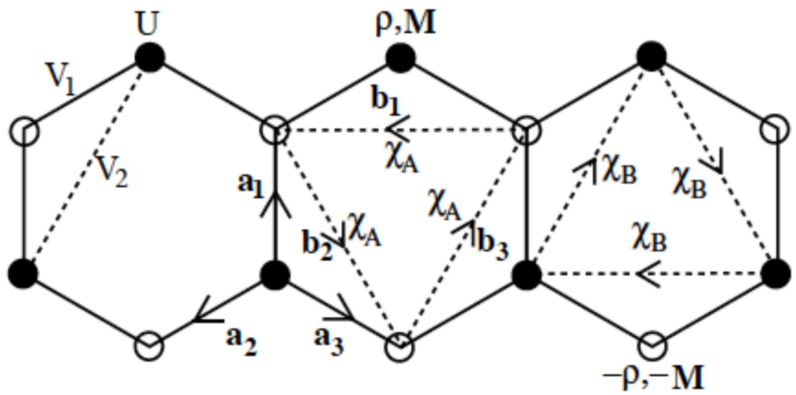
- **Cold atoms (graphene)**

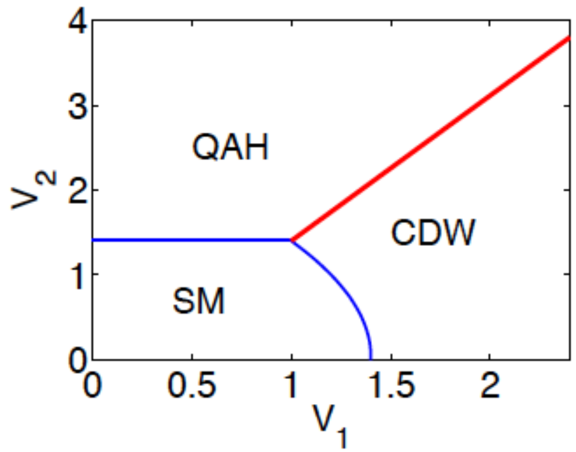
(C. Salomon et al. LKB ENS, L. Tarruell et al. Barcelone Groups ETHZ, Hamburg)

Relevance for 3D pyrochlore Iridates (?) S. Julian (Toronto) Y.B. Kim; D.-H. Lee et al. 2012

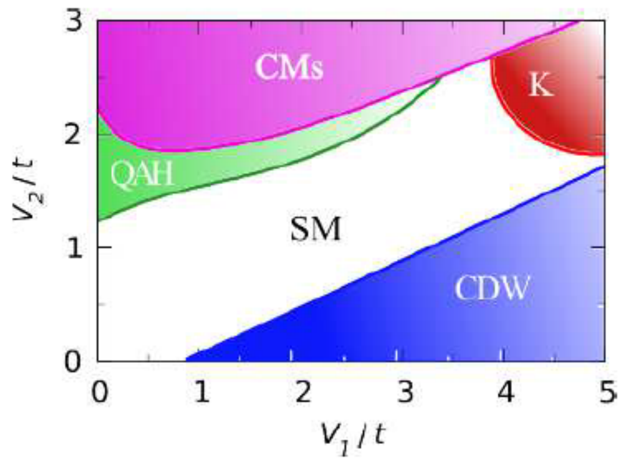
Topological Mott Insulators

S. Raghu[1], Xiao-Liang Qi[1], C. Honerkamp[2], and Shou-Cheng Zhang[1]
[1] Department of Physics, McCullough Building, Stanford University, Stanford, CA 94305-4045 and
[2] Theoretical Physics, Universität Würzburg, D-97074 Würzburg, Germany
(Dated: February 2, 2008)





How to realize a Large V_2 (Mott) coupling ?
Tianhan Liu, Benoît Douçot, KLH: **next slide**



A.G. Grushin et al. 2013
T. Duric et al. 2014 (ED & EPS)

<u>Other 3D topological Mott insulators:</u>

M. W. Young, S.S. Lee, K. Kallin 2008
D. Pesin and L. Balents, 2010
S. Rachel and KLH, 2010
W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents 2013
1D: Spin-1 chain (Haldane, Affleck, …)

Realizing Topological Mott Insulators from RKKY Interaction

Tianhan Liu, Benoît Douçot, KLH arXiv:1409.6237

$$H = -t_f \sum_{\langle i,j \rangle} f_{i\sigma}^\dagger f_{j\sigma} + \mu_f \sum_{j,\sigma=\uparrow,\downarrow} f_{j\sigma}^\dagger f_{j\sigma} + U_f \sum_i f_{i\uparrow}^\dagger f_{i\uparrow} f_{i\downarrow}^\dagger f_{i\downarrow}$$
$$- t_c \sum_{\langle i,j \rangle} c_i^\dagger c_j + \mu_c \sum_j c_j^\dagger c_j + \sum_{j,\sigma=\uparrow,\downarrow} g_{cf} f_{j\sigma}^\dagger f_{j\sigma} c_j^\dagger c_j .$$

<u>**Analogy to KONDO lattices:**</u>

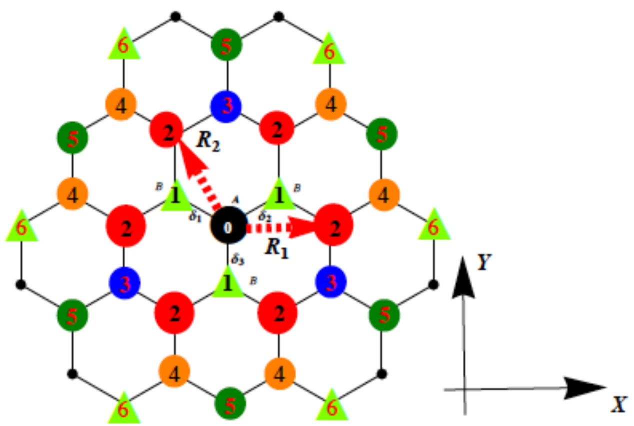
2 types of particles

- **Fast**
- **Slow**

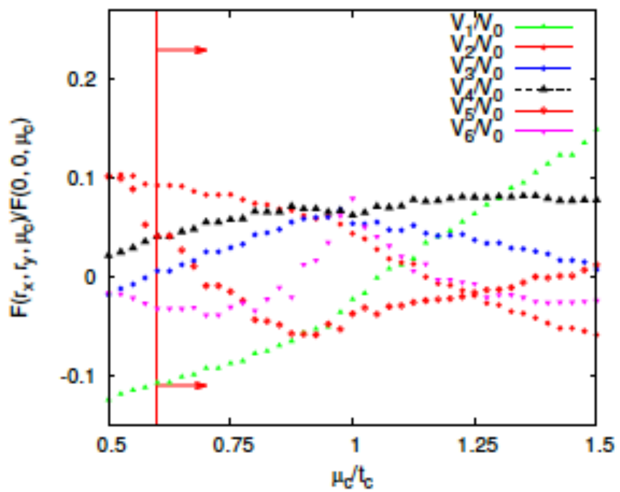
C Fermions are **FAST** particles: **SLOW** particles feel an induced (large V2) interaction

$$H_{int}(\mathbf{r}_{iI} - \mathbf{r}_{jJ}, \mu_c) = \sum_{\mathbf{p},\mathbf{k},i,j,\sigma,\sigma'} \frac{g_{cf}^2 \{ f[\epsilon_c(\mathbf{p})] - f[\epsilon_c(\mathbf{p}-\mathbf{k})] \}}{N^2(\epsilon_c(\mathbf{p}) - \epsilon_c(\mathbf{p}-\mathbf{k}) + i\eta)} e^{i(\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j))} \alpha_{IJ}(\mathbf{k}) f_{iI\sigma}^\dagger f_{iI\sigma} f_{jJ\sigma'}^\dagger f_{jJ\sigma'},$$

$$F(r_x, r_y) = \sum_{\mathbf{p},\mathbf{k}} \frac{t_c \{ f[\epsilon_c(\mathbf{p})] - f[\epsilon_c(\mathbf{p}-\mathbf{k})] \}}{N^2(\epsilon_c(\mathbf{p}) - \epsilon_c(\mathbf{p}-\mathbf{k}) + i\eta)} e^{i(k_x r_x + k_y r_y)} \alpha_{IJ}(\mathbf{k}).$$



SC less dominant Instability here



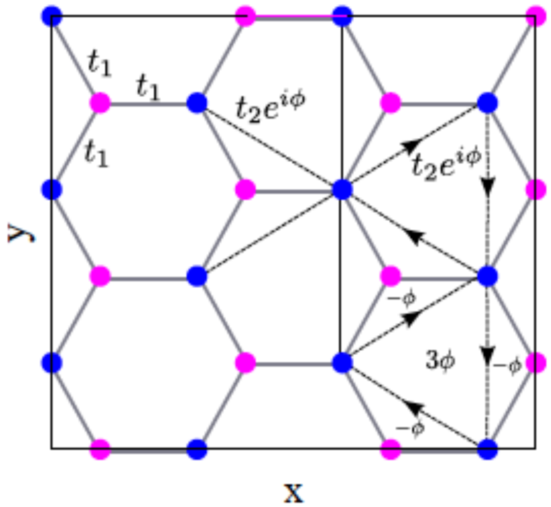
Haldane Model on the Graphene Lattice

Haldane model

$$\mathcal{H}_0 = \sum_i (-1)^i M c_i^\dagger c_i - \sum_{<i,j>} t_1 c_i^\dagger c_j - \sum_{\ll i,j \gg} t_2 e^{i\phi_{ij}} c_i^\dagger c_j$$

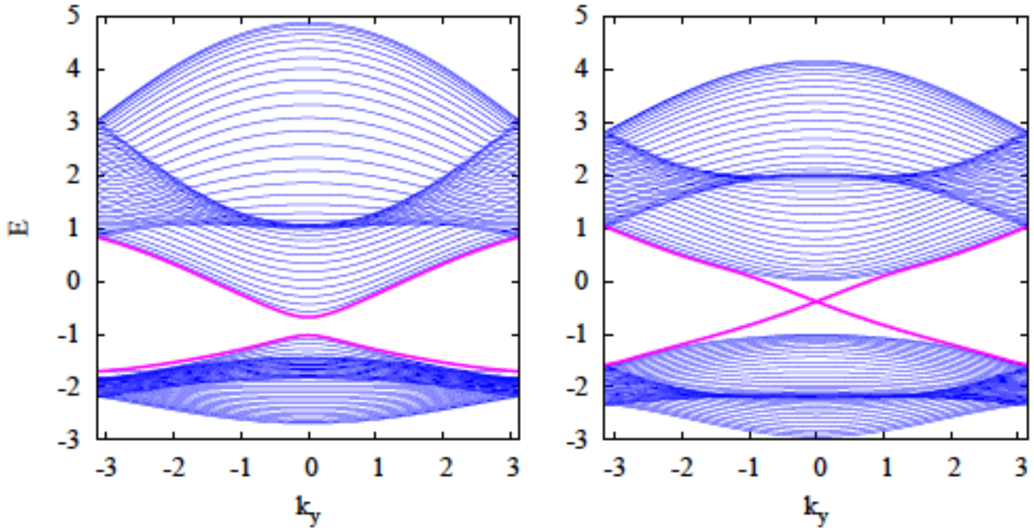
F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

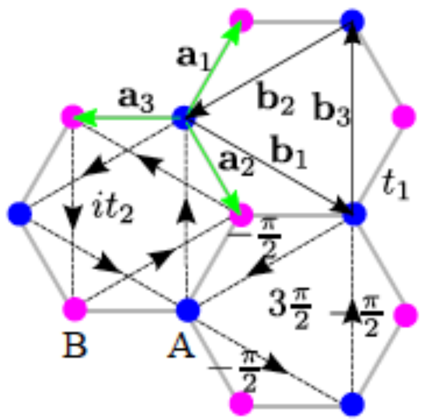
No net flux



Spectrum of the non-interacting model

- t_1 only $\Rightarrow$ Dirac cones
- M or t_2 can open the gap
- Non–trivial topological properties if $M < 3\sqrt{3}t_2 \sin\phi$





$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\left(\mathbf{k}\right) \cdot \hat{\sigma},$$

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\, \mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\, \mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\, \mathbf{b_i} \right).$$

Realized in cold atoms:

Group of T. Esslinger, 2014

arXiv:1406.7874

The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

$$\mathcal{C}_- = \frac{1}{4\pi} \int_{\mathrm{BZ}} d\mathbf{k}\, \hat{\mathbf{d}} \cdot \left(\partial_1 \hat{\mathbf{d}} \times \partial_2 \hat{\mathbf{d}} \right)$$

This is the Chern number of the lower Bloch band, and takes the value $\mathcal{C}_- = 1$. The formula for the upper band is obtained by replacing $\hat{\mathbf{d}}$ by $-\hat{\mathbf{d}}$, and leads to $\mathcal{C}_+ = -1$.

Chiral Bosonic Phases on the Haldane Honeycomb Lattice

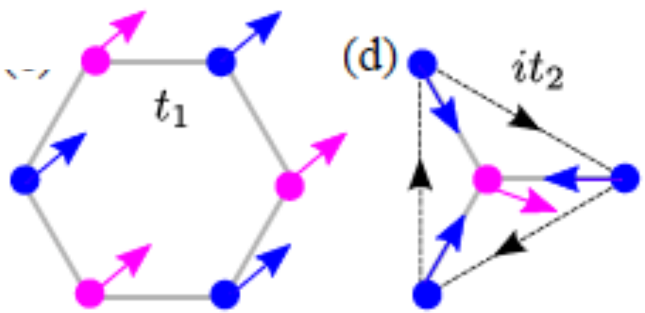
I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411

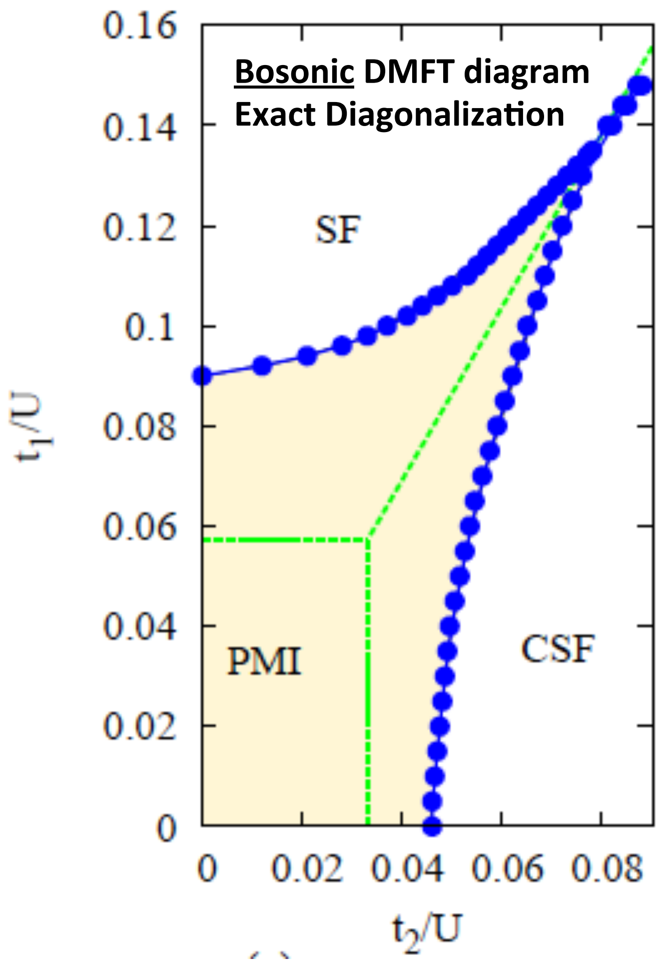
$$\mathcal{H} = \mathcal{H}_{\mathrm{H}} + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu \sum_i \hat{n}_i,$$

Phase-angle variables $b_i^\dagger = \sqrt{n}e^{i\theta_i}$

chiral SF:
nonuniform phase,
plaquette currents

SF:
uniform phase,
"Meissner current"

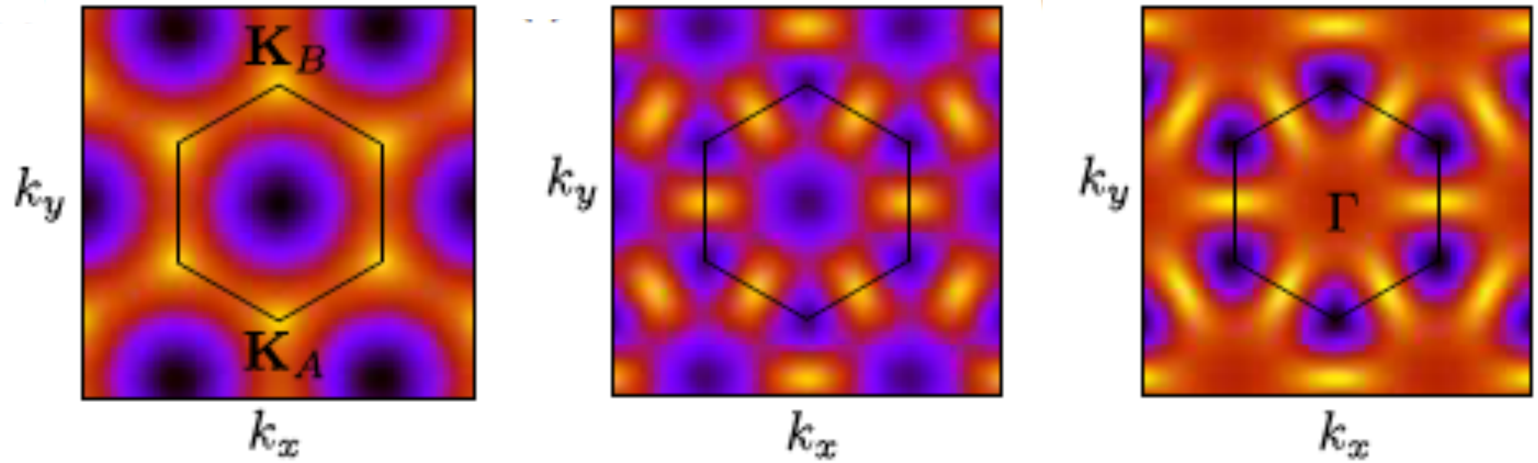




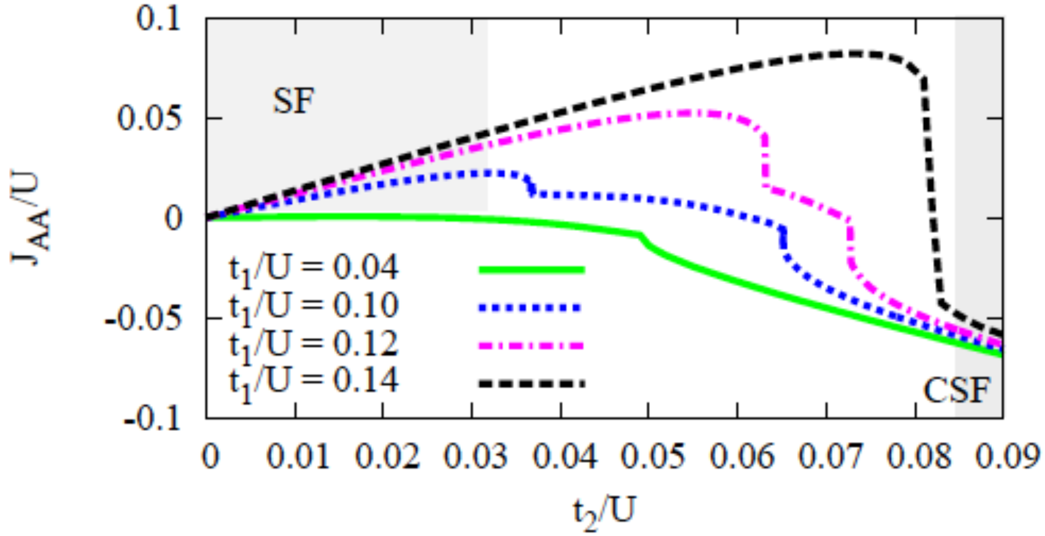
NOT YET A TOPOLOGICAL MOTT PHASE…
See after

Condensation of Bosons

SF CSF



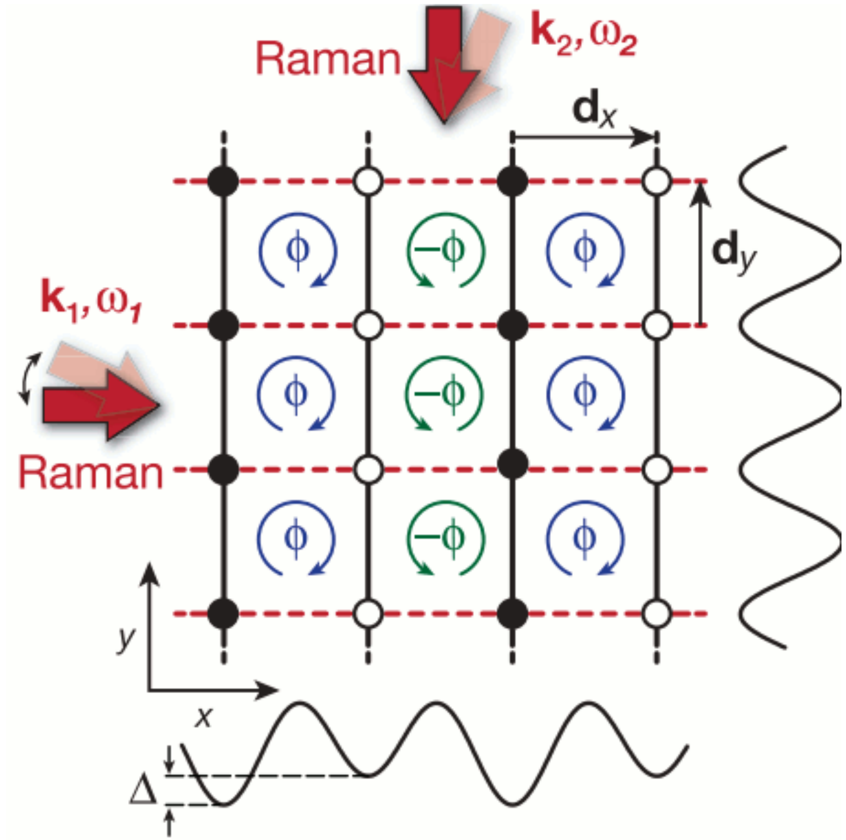
$$J_{AA}^{SF} = -2\, n\, t_2\, \mathrm{Im}\exp(-i\pi/2) = 2nt_2$$



$$J_{AA}^{CSF} = -2\ \mathrm{Im}\left(t_2 e^{i\phi}\left\langle \hat{b}^{\dagger}_{Ai}\hat{b}_{Aj}\right\rangle\right)$$
$$= -2t_2 n \sin\left[\phi - \mathbf{K}_A\cdot(\mathbf{r}_i - \mathbf{r}_j)\right] = -nt_2$$

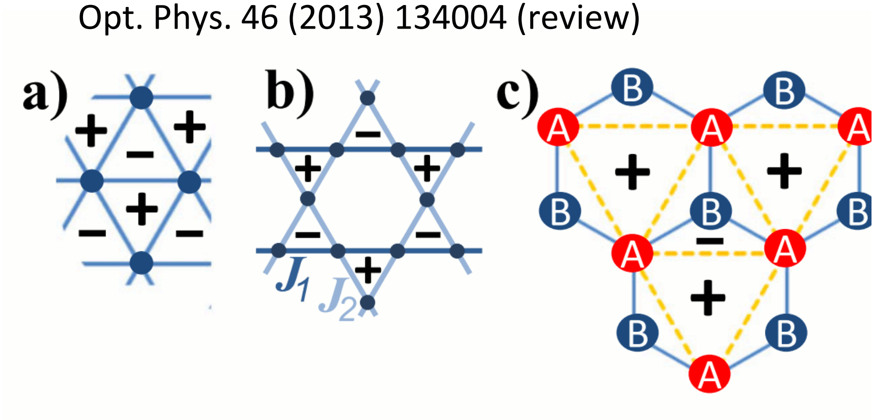
Cold Atoms & Gauge Fields

Goal: strongly correlated atoms in strong artificial magnetic fields.
Maximal flux per plaquette of order π



Laser-assisted tunneling in optical superlattice. PRL 107, 255301 (2011) (Immanuel Bloch's lab at Muenich)

Non-Abelian Models with interactions
P. P. Orth, D. Cocks et al J. Phys. B: At. Mol. Opt. Phys. 46 (2013) 134004 (review)



$$\hat{H}(t) = -\sum_{\langle ij \rangle} J_{ij} \hat{a}_i^\dagger \hat{a}_j + \sum_i v_i(t) \hat{n}_i + \hat{H}_{\text{on-site}}.$$

Spatially periodic driving induces Peierls phases PRL 109, 145301 (2012) **FLOQUET**
K. Sengstock's lab at Hamburg

A lot of efforts at NIST, Paris, MIT,…
I. Spielman, W. Philipps (NIST)
J. Dalibard, F. Gerbier (Paris)…

Laser-assisted tunneling

Two distinct internal atomic states localized on columns of a square lattice

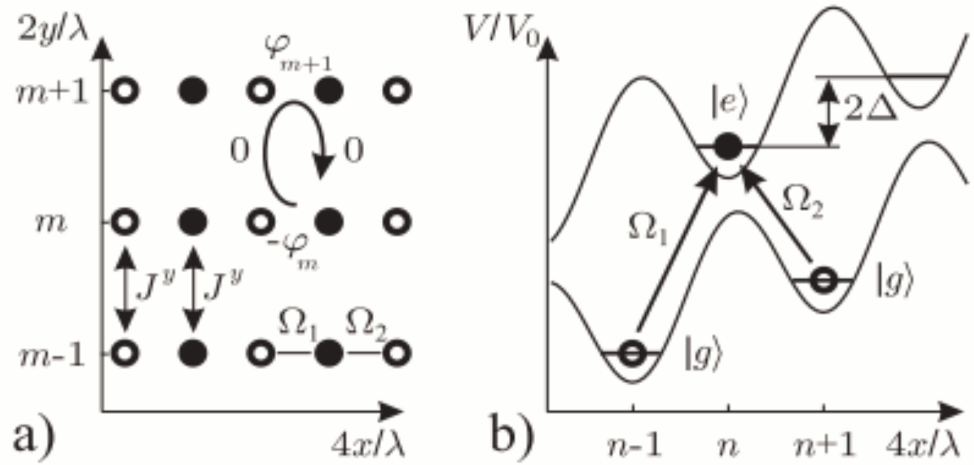


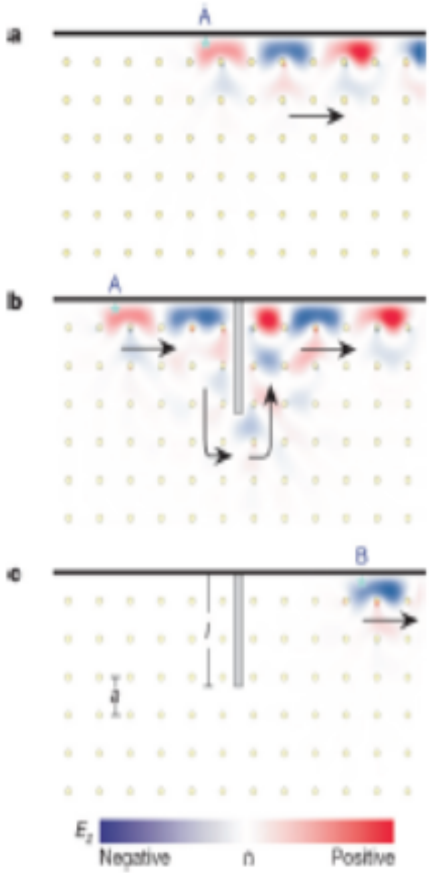
Figure 2. Optical lattice set-up. Open (closed) circles denote atoms in state $|g\rangle$ ($|e\rangle$). (a) Hopping in the y-direction is due to kinetic energy and described by the hopping matrix element J^y being the same for particles in states $|e\rangle$ and $|g\rangle$. Along the x-direction hopping amplitudes are due to the additional lasers. (b) Trapping potential in the x-direction. Adjacent sites are set off by an energy Δ because of the acceleration or a static inhomogeneous electric field. The laser Ω_1 is resonant for transitions $|g\rangle \leftrightarrow |e\rangle$ while Ω_2 is resonant for transitions $|e\rangle \leftrightarrow |g\rangle$ due to the offset of the lattice sites. Because of the spatial dependence of $\Omega_{1,2}$

Jaksch and Zoller, New Journal of Physics **5**, 56.1-11 (2003)

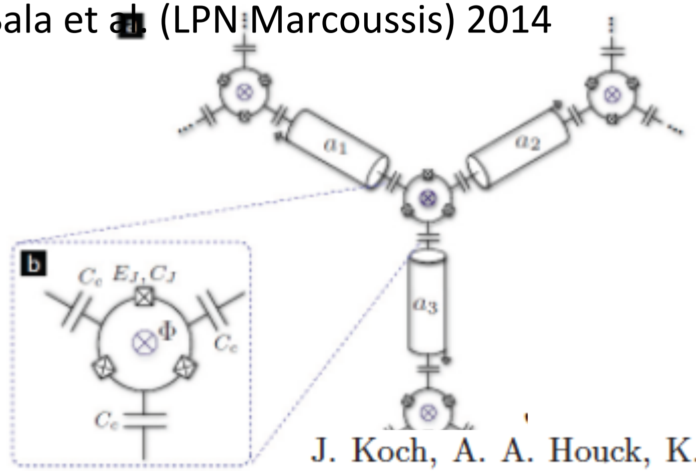
Artificial Gauge Fields with Light

See also M. C. Rechstmann et al Nature 2013 (FLOQUET photonic TIs)
D. Schuster and J. Simon lab Chicago
V. G. Sala et al. (LPN Marcoussis) 2014

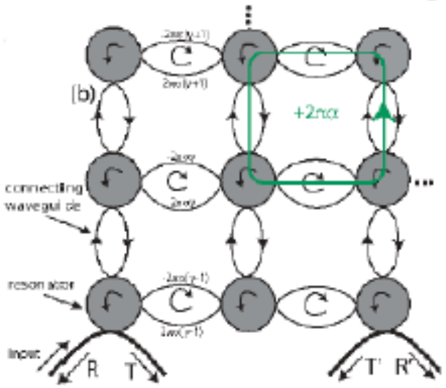
Review:
I. Carusotto
& C. Ciuti
RMP 2012



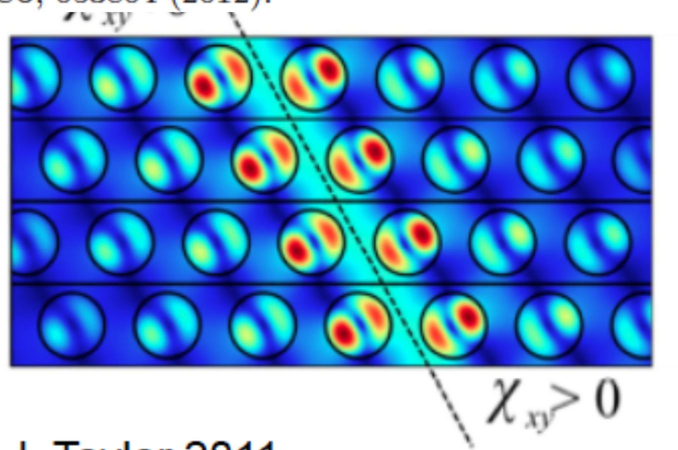
Haldane-Raghu, PRL 2008
Z. Wang et al. Nature 2009



J. Koch, A. A. Houck, K. Le Hur, and S. M. Girvin, Phys. Rev. A **82**, 043811 (2010).
A. Petrescu, A. A. Houck, and K. Le Hur, Phys. Rev. A **86**, 053804 (2012).

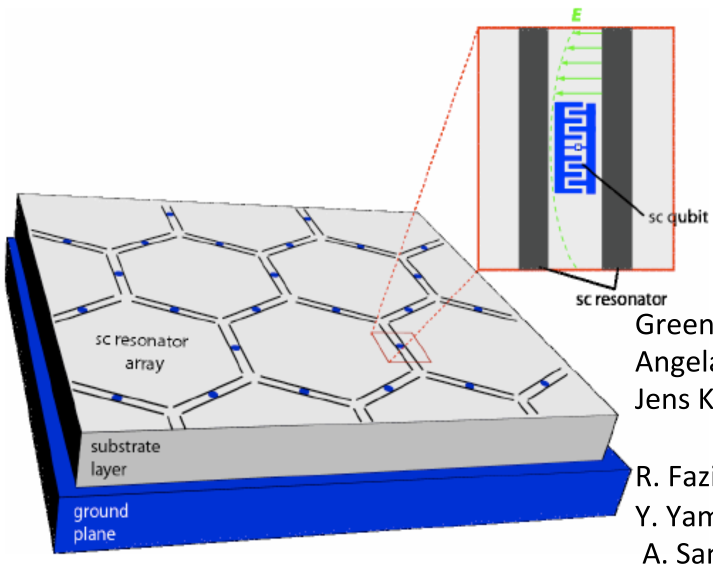


M. Hafezi, E. Demler, M. Lukin, J. Taylor 2011



A. MacDonald et al. 2012

Array cQED Systems



Greentree et al., Nat. Phys. 2, 856 (2006)
Angelakis et al., PRA 76, 031805 (2007)
Jens Koch and KLH, PRA 80, 023811 (2009)

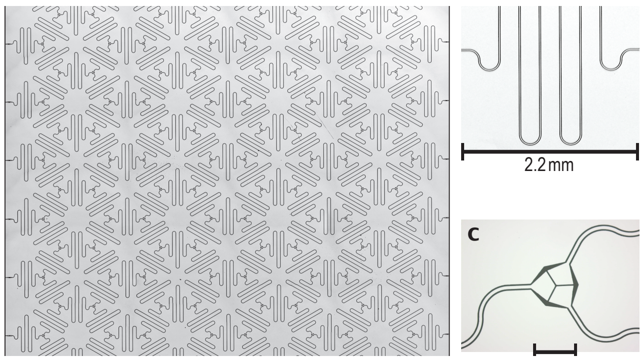
R. Fazio, S. Schmidt & G. Blatter, H. Tureci, S. Bose,
Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons,
A. Sandvik, C. Ciuti, I. Carusotto,J. Keeling, J. Larson,…

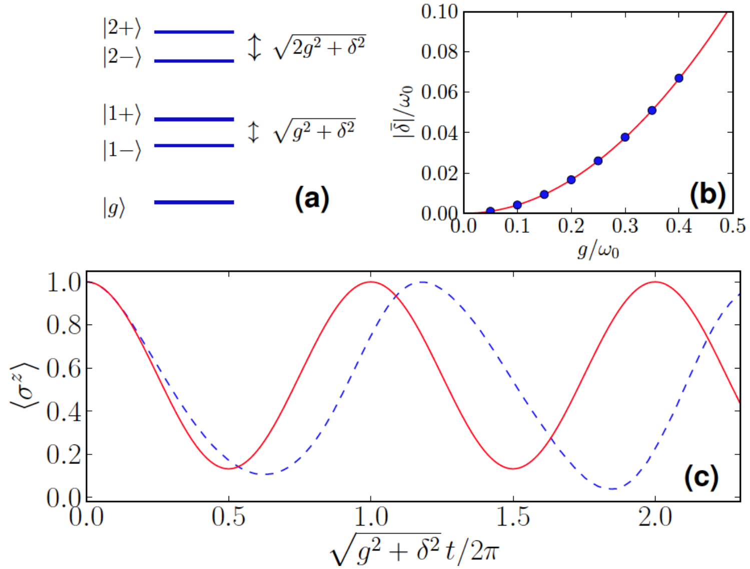
A. Houck, H. E. Tureci
J. Koch Nature Physics
8 292 (2012)**, review**

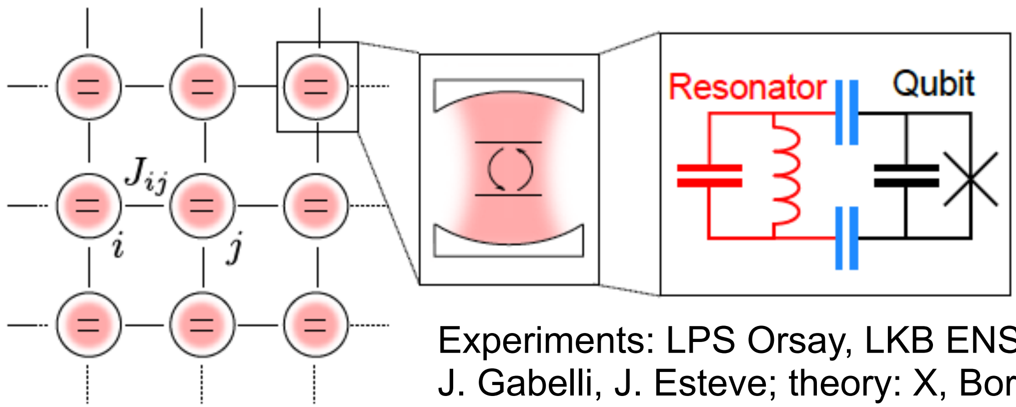
OTHER PHOTON GRAPHENE
AFTERNOON TALKS
M. Bellec (Nice)

A. Houck's
Lab

Princeton





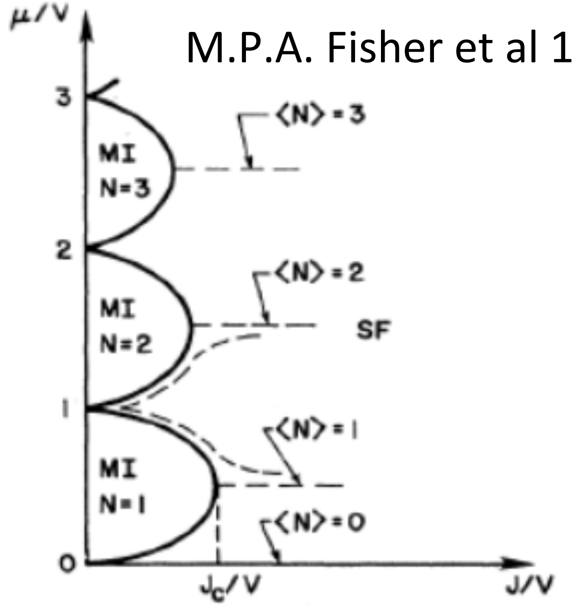


Experiments: LPS Orsay, LKB ENS
J. Gabelli, J. Esteve; theory: X, Bordeaux

L. Henriet, Z. Ristivojevic, P. Orth, KLH
driven and dissipative Rabi model
PRA **90**, 023820 (2014)
Stochastic approaches

Going Back to Low Dimensions: Quantum ladders & Josephson Effect

M.P.A. Fisher et al 1989



Bose-Hubbard model of a single lattice boson:

$$H = -t \sum_{<ij>} b_i^\dagger b_j + \sum_i \frac{U}{2} n_i (n_i - 1) - \mu n_i$$

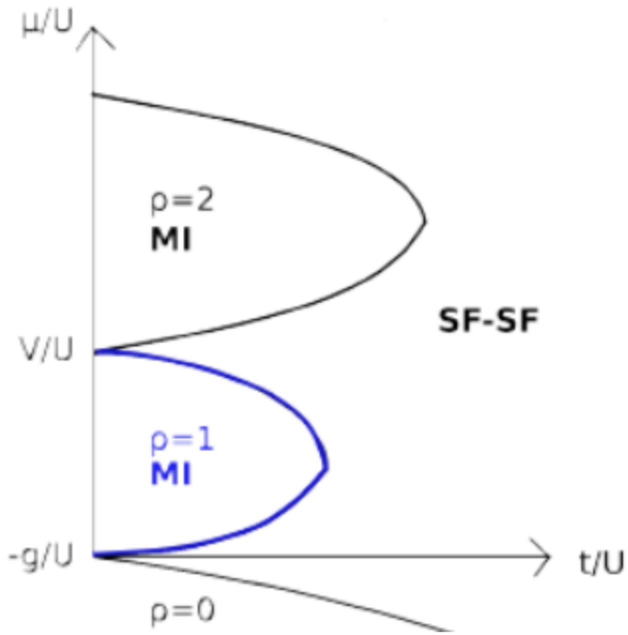
Two-species Bose-Hubbard model:

$$H = -t \sum_{\alpha=1,2} \sum_{<ij>} b_{\alpha i}^\dagger b_{\alpha j} + \sum_{\alpha i} \frac{U}{2} n_{\alpha i}(n_{\alpha i} - 1) - \mu n_{\alpha i}$$

$$+ \sum_i V_\perp n_{1i} n_{2i} - g \sum_i b_{1i}^\dagger b_{2i} + H.c.$$

Mott at ρ=1

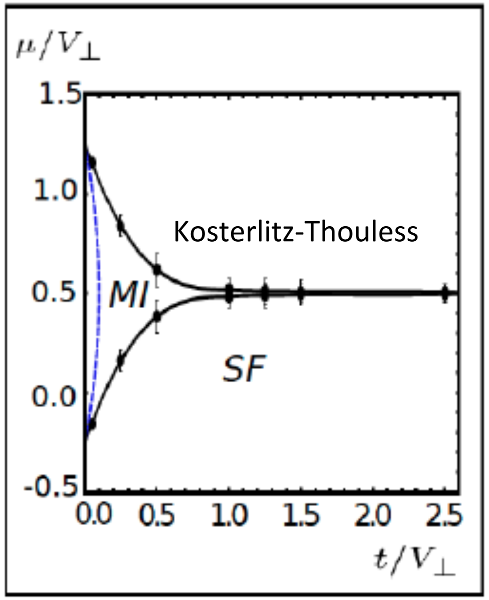
Interchain coherence:
Meissner effect



e.g. E. Altman, W. Hofstetter, E. Demler, M. Lukin 2003

Multicomponent systems: active field in cold atoms

Route for Chiral Mott Insulator: Spin Meissner Effect



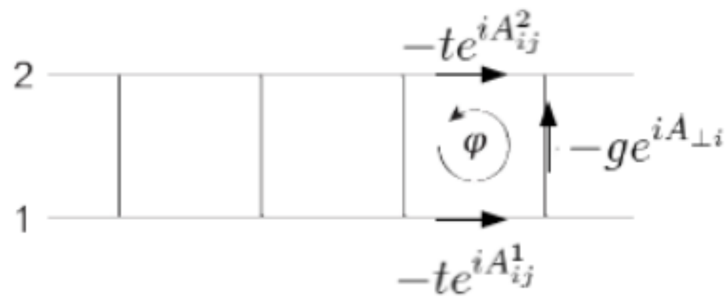
Mott insulating phase of total density:

$$\rho = b_1^\dagger b_1 + b_2^\dagger b_2$$

Relative density exhibits fluctuations.

$$\sigma^z = b_1^\dagger b_1 - b_2^\dagger b_2$$

(At ρ=1, spin ½ exchange Hamiltonian)



A. Petrescu and KLH, arXiv:1306.5986 **(PRL)**

Long Paper, arXiv:1410.

Example: Ladder System

Mott Regime: Pseudo-spin H

$$H_\sigma = -\sum_{\langle ij\rangle}\left(2J_{xx}(\sigma_i^+\sigma_j^- e^{iaA_{ij}^\sigma} + \text{H.c.}) - J_z\sigma_z^i\sigma_z^j\right)$$

$$-g\sum_i(\sigma_i^x\cos(a'A_{\perp i}) - \sigma_i^y\sin(a'A_{\perp i})),$$

$$\text{with } J_{xx} = \frac{t^2}{V_\perp} \text{ and } J_z = t^2\left(-\frac{2}{U} + \frac{1}{V_\perp}\right)$$

$$j_\| = 2J_{xx}\big[\cos(A_{ij}^\sigma)(\sigma_i^y\sigma_j^x - \sigma_i^x\sigma_j^y)$$

$$+\sin(A_{ij}^\sigma)(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)\big],$$

$$j_\perp = -2g\left[\cos(a'A_{\perp i})\sigma_i^y + \sin(a'A_{\perp i})\sigma_i^x\right].$$

$$\langle j_\|\rangle = -2J_{xx}\ \text{phase}_{ij}$$

Meissner currents survive

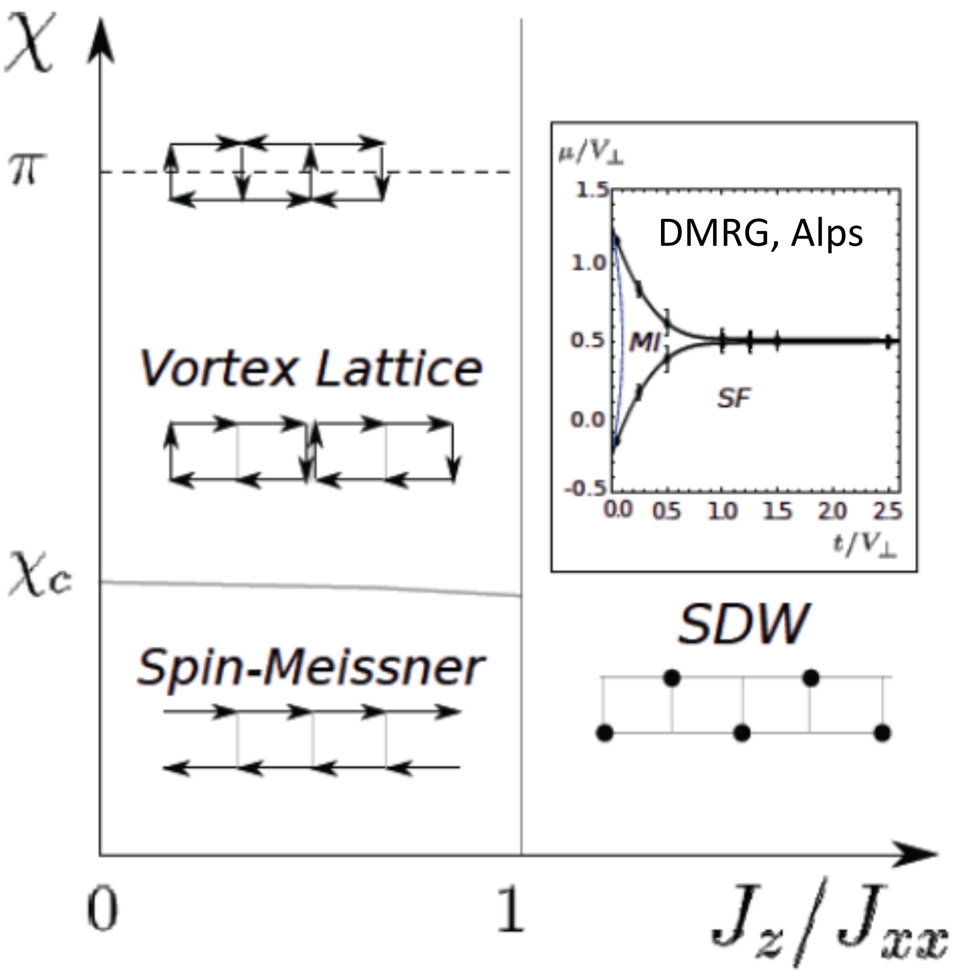
at other filling
A. Tokuno & A. Georges, 2014

Spin Meissner Effect

Similar Hamiltonian
different contexts
I. Garate and I. Affleck

Chiral Mott insulator Arya Dhar et al. PRA A 85, 041602 (2012)

Analogy to DDW phase in high-Tc systems



Possible transitions to Laughlin states

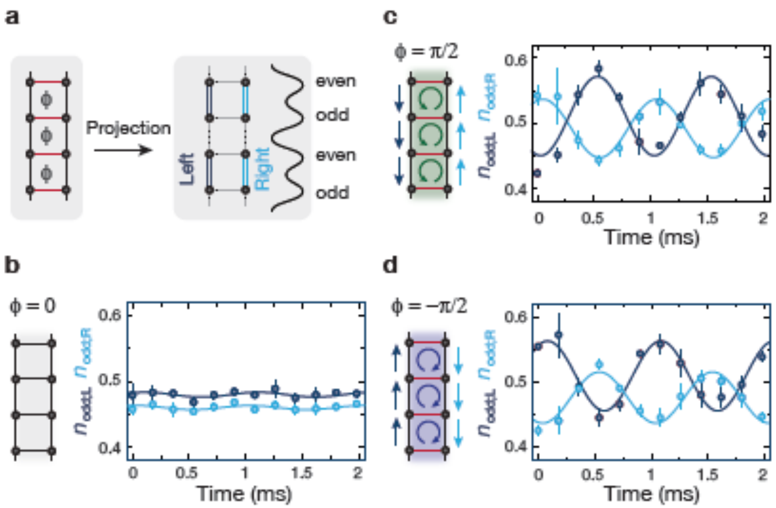
Teo – Kane, PRB 2014

PRL 2013

Bosonic Mott insulator with Meissner Currents Alex Petrescu and KLH, arXiv:1306.5986

Observation of the Meissner effect with ultracold atoms in bosonic ladders

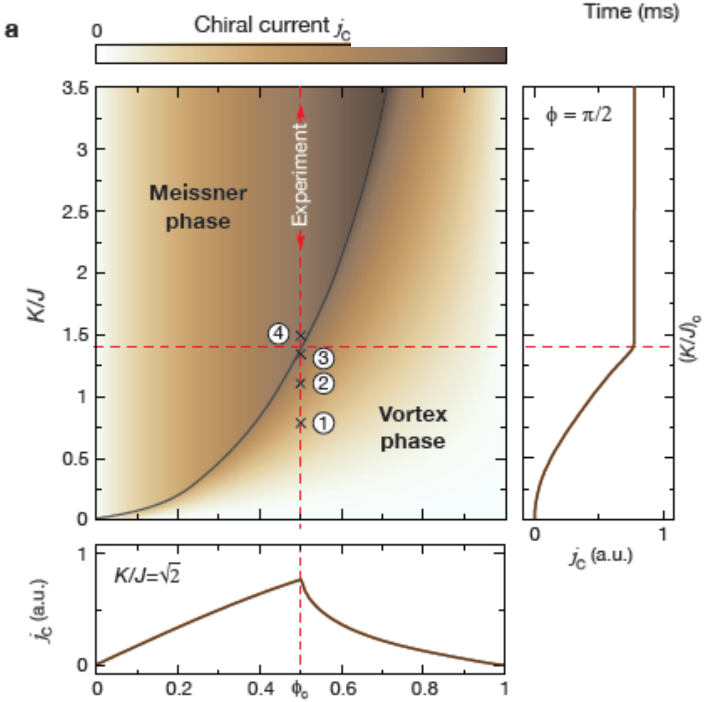
M. Atala[1,2], M. Aidelsburger[1,2], M. Lohse[1,2], J. T. Barreiro[1,2], B. Paredes[3] & I. Bloch[1,2]

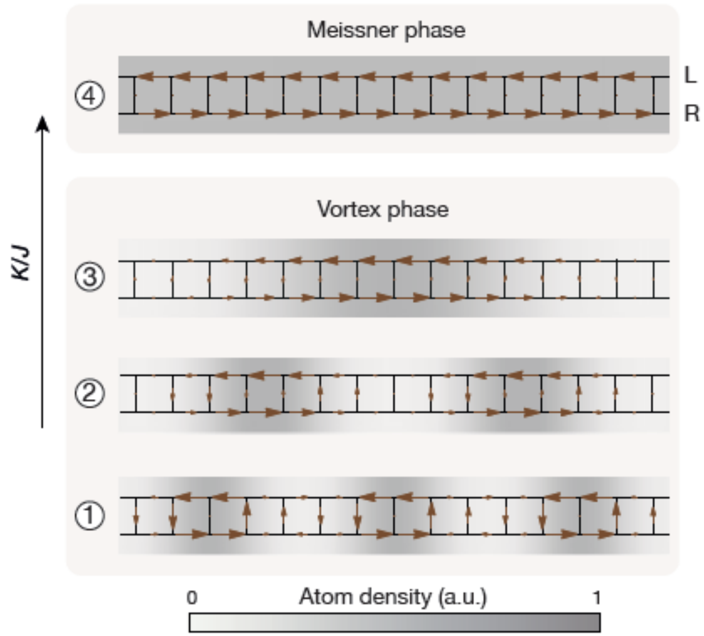


Nature Physics 2014

Theory by E. Orignac & T. Giamarchi 2001
No Mott physics here

See also M. Piraud et al arXiv:1409.7016



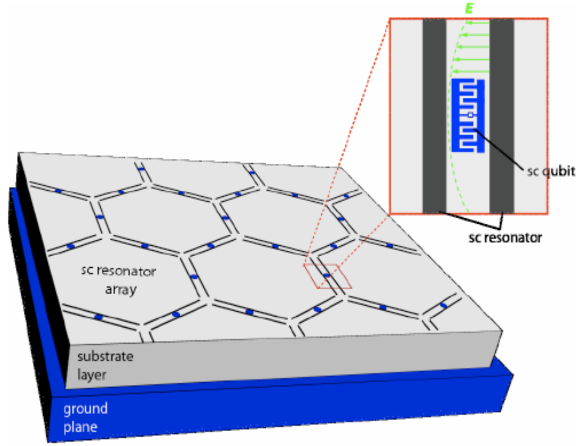


Our Program: Topological Phases, Interaction Effects & Gauge Fields

From Materials, to Ultra-Cold Atoms and Photon Simulators

Main Lines of The Talk:

- Topological Insulator in graphene lattices
- Kane-Mele Model, Interaction Effects
- Topological Mott Insulators
- Ultra-Cold Atoms
- Photons
- Ladders



Some Collaborators (Recent Works, topological phases):

Alexandru Petrescu (PhD 2015: Yale & Ecole Polytechnique)
Tianhan Liu (PhD 2015: LPTHE Jussieu and Ecole Polytechnique)
Benoît Douçot (LPTHE Jussieu)
Ivana Vidanovic and Walter Hofstetter (Frankfurt)