Low-dimensional correlated electron Systems

Karyn Le Hur, Yale University

Boulder Summer School, 2008

Brief history of electrons

- 1897: Discovery of the electron by Thomson

  Charge measurement of the electron by Millikan

  "oil droplet experiment"

- 1900: Theory of metals by Drude

  $j = \sigma E$ and $\sigma = n e^2 \tau / m$ and mean-free path $l = v\tau$

  classical partition of energy gives $mv^2/2 = 3k_BT/2$

v=0 at T=0 ?

Quantum mechanics…

- Electrons fill successive energy states in pairs of opposite spins, up to the Fermi energy $\sim E_F$

Sommerfeld, Theory of Metals (1928)

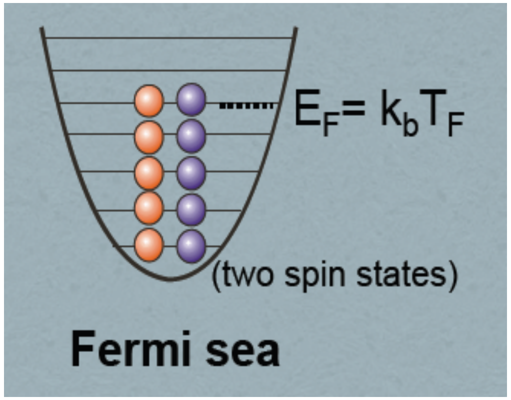
Only electrons within $k_B T$ of E_F

participate in the transport

$v_F{}^2 = 2E_F/m \sim n^{2/3}$

($v_F \sim 3.10^8$ cm/s)

Pauli exclusion



Fermi gas

<u>Second quantization:</u>

$$\{c_k, c_{k'}^\dagger\} = \delta_{kk'}$$

$$k = (\vec{k}, \sigma)$$

$$\{c_k, c_{k'}\} = \{c_k^\dagger, c_{k'}^\dagger\} = 0$$

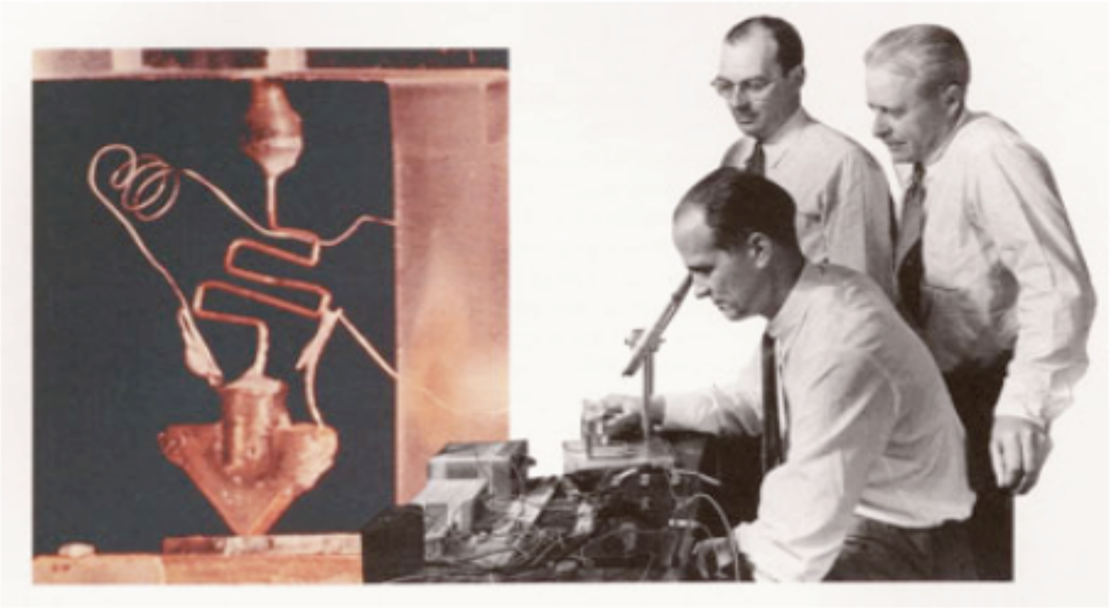
$$N = \int_{-\infty}^{E_F} dE D(E) \rightarrow E_F^{3/2}$$

$$C_v = \frac{\partial U}{\partial T} = \frac{\pi^2}{2} k_B \frac{T}{T_F} + \ldots$$

Specific heat & Pauli susceptibility related through:

$$\frac{T\chi}{C_v} = \frac{3\mu_B^2}{\pi^2 k_B^2}$$

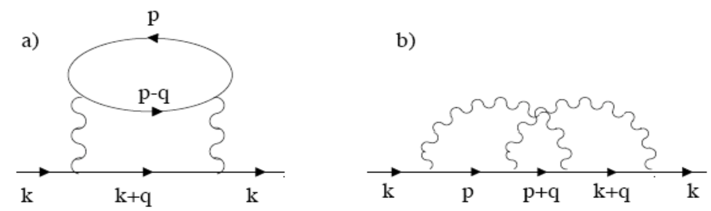
Invention of Transistor, 1947



1947: Invention of the Transistor

Bell Laboratory, William Schokley, John Bardeen, Walter Brattain

! Small angles



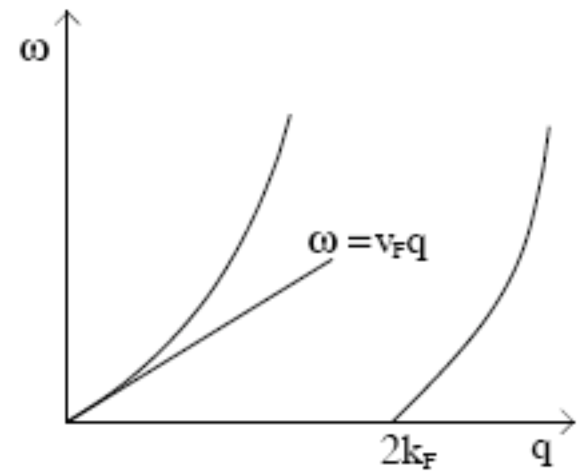
$$\mathrm{Im}\Sigma^R(\varepsilon) = -\frac{2}{(2\pi)^{D+1}}\int_0^{\varepsilon} d\omega \int d^d q\, \mathrm{Im}G^R(\varepsilon-\omega, \mathbf{k}-\mathbf{q})\, \mathrm{Im}V^R(\omega, \mathbf{q})$$

<u>3D:</u>

$$\mathrm{Im}\Pi^R(\omega,q) = -\nu_3 \frac{\omega}{v_F q}\theta\left(q - |\omega/v_F|\right)$$

$$\mathrm{Im}V^R(\omega,q) = -U^2 \mathrm{Im}\Pi^R(\omega,q)$$

$$\left(\mathrm{Re}\Sigma^R\right)_{\mathrm{non-an}} \propto \varepsilon^3 \ln|\varepsilon|$$



<u>2D:</u>

$$-\mathrm{Im}\Sigma^R(\omega) \sim \frac{U^2}{v_F^2} m \int_0^{\varepsilon} d\omega\,\omega \int_{\sim|\omega|/v_F}^{\sim k_F} \frac{dq}{q} \sim \frac{U^2}{v_F} m \varepsilon^2 \ln \frac{E_F}{|\varepsilon|}.$$

$$\mathrm{Re}\Sigma \propto \varepsilon|\varepsilon|$$

D. Maslov, Les Houches, 2005

Landau Fermi liquid 1957-58

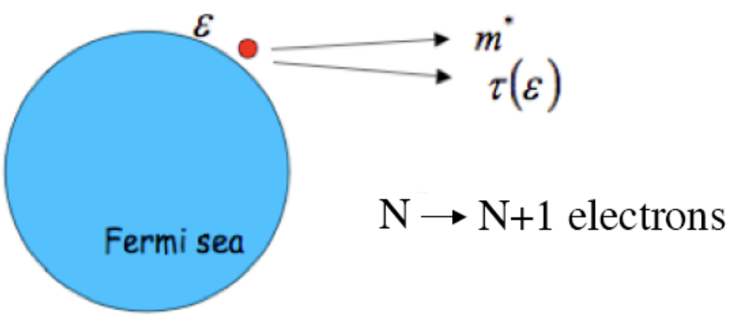
Interacting system of electrons explained by « **quasiparticles** »

« adiabatic evolution of the fermion into an interactive environment »

Same quantum numbers as free electrons

Renormalization of quasiparticle mass increases the specific heat

$C_v^* = C_v\, m^*/m$ (but still linear in T)

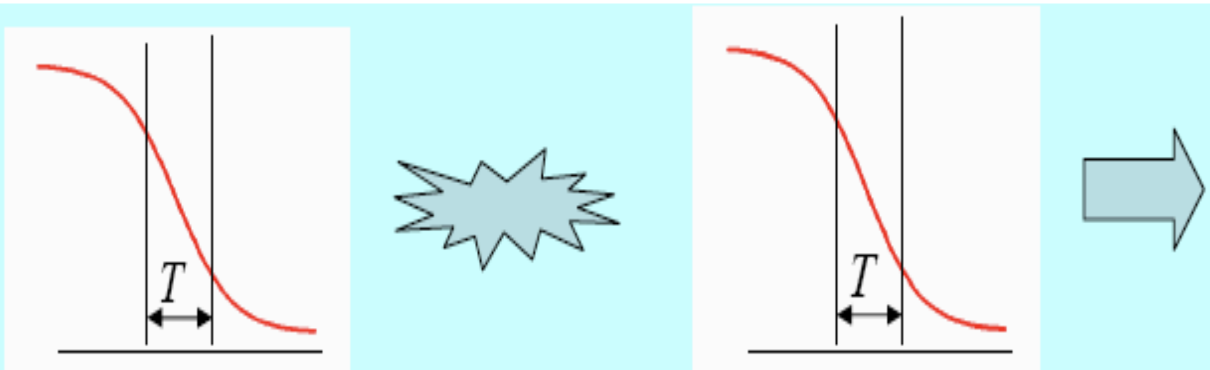


Quasiparticle lifetime: in ε^2 or T^2
Phase space argument

Fermi liquid justified as long as:
$\tau(\varepsilon) \gg \hbar/\varepsilon$

Book by Pines and Nozières (1966)

Phase space argument



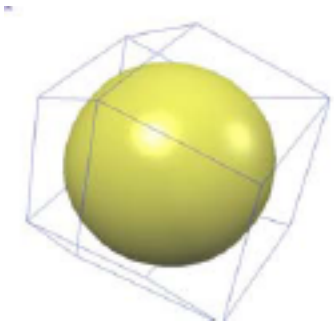
$$\frac{1}{\tau_{ee}} \propto T^2$$

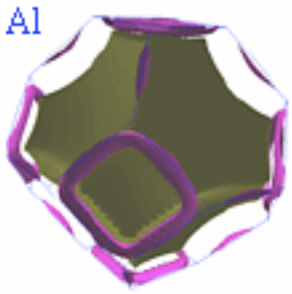
$$\frac{1}{\tau_{ee}} = 2\pi \sum_{\varepsilon',\omega} \int d^D q\, |V(\omega,q)|^2\, \underbrace{n_\varepsilon (1 - n_{\varepsilon-\omega})}_{T}\, \underbrace{n_{\varepsilon'} (1 - n_{\varepsilon'+\omega})}_{T}$$

Particles can only scatter into unoccupied levels (Pauli principle)

Applications

- Good metals
- He_3
- Semiconducting heterostructures
- Heavy Fermions; Kondo physics





Dimensional Reduction and New Physics

3 Lectures :

1D physics for electrons

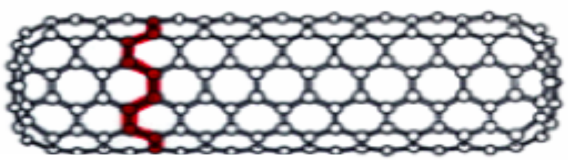
Luttinger paradigm, Mott transition, Spin Chains

Dimensional crossovers, exotic SC, challenges in 2D

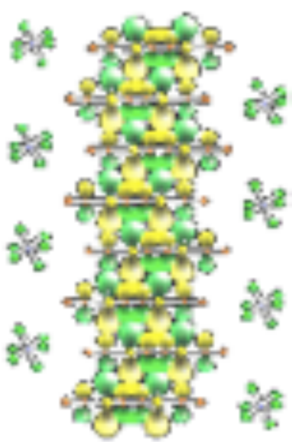
Carbon nanotubes & graphene: From Luttinger to 2D Dirac fermions

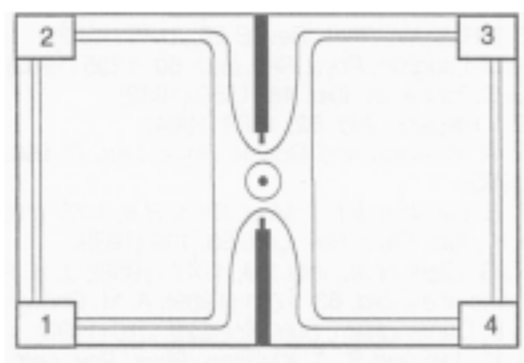
1D prototypes

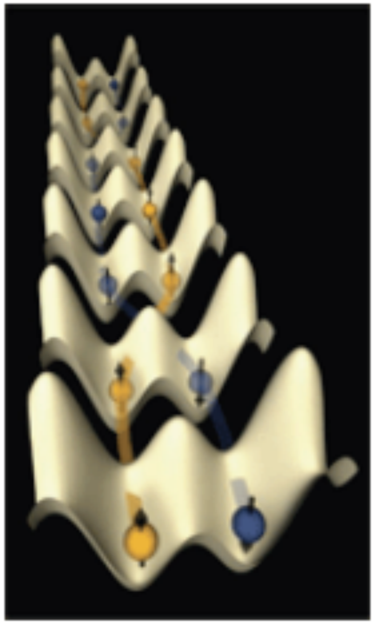
- Why is 1D relevant? A variety of Systems



Carbon Nanotubes
Quantum wires







TMTSF molecules

Edge states

Atoms in optical lattices

Green function approach…

Free electron gas:

$$G_{ret}(\vec{k},t) = -i\theta(t)\exp -i\xi_{\vec{k}}t$$

$$A(\vec{k},\omega) = -\frac{1}{\pi}\Im m G_{ret}(\vec{k},\omega) = \delta(\omega-\xi_{\vec{k}})$$

$$G_{ret}(\vec{k},\omega) = \frac{1}{\omega-\xi_{\vec{k}}+i0^+}$$

electron pole in spectral function

Fermi liquid:

$$G_{ret}(\vec{k},\omega) = \frac{Z_{\vec{k}}}{\omega-\xi^*_{\vec{k}}+i\Gamma_{\vec{k}}}+G_{inc}(\vec{k},\omega)$$

$$G_{ret}(\vec{k},\omega) = \frac{1}{\omega-\xi_{\vec{k}}-\Sigma(\vec{k},\omega)}$$

$$\boxed{\Gamma_{\vec{k}} \propto \Im m\Sigma(\vec{k},\omega)}$$

No Landau quasiparticle in 1D

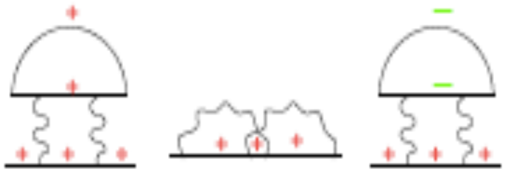
Karyn Le Hur, PRL 2005
Karyn Le Hur, PRB **74**, 165104 (2006)

- <u>Lifetime in 1D</u>?

Take an electron with energy E, wavevector k: $E = \pm v_F k$

energy conservation = momentum conservation

$$\Im m \prod_{\pm}(q,\omega) = \frac{\omega}{2v_F}\delta(\omega \mp v_F q)$$



$$\Im m \Sigma(k \rightarrow 0, E, T) \sim -\left(\frac{Ua}{v_F}\right)^2 \max(E,T)$$

Small angle diagrams cancel each other for spinless electrons!

2 regimes for Green's function

$$\Re e\Sigma(E) \to \alpha^2 \ln(|E|) \quad \text{where } \alpha \sim Ua/v_F$$

$$\mathcal{Z}(k,E) \sim (E - v_F k)^{\alpha^2} \to m/m^* \to \mathcal{Z}(k=0,E)$$

T=0 Mass renormalization effects dominate
power law decay of the electron Green function in time

tT >>1 $\Im m\Sigma$ more important $\tau_F^{-1} \propto \alpha^2 T$
Exponential decay of the electron Green's function

(Spinful electrons: forward diagrams donot cancel); need exact solution: spin-charge separation…

Interference measurements

GaAs 1D ballistic ring
Hansen et al. PRB 2001

L: half-perimeter of the ring
τ: traversal time $\sim L/v_F$

τT>>1: Aharonov-Bohm interference decays as $\exp -\tau/\tau_F$

Exact (Luttinger) theory predicts that τ_F^{-1} remains linear in T at all the orders in U

1D: Hard-core bosons ➔ fermions

Jordan-Wigner, 1928
(See notes)

$$c_j = \exp\left(i\pi \sum_{j'<j} n_{j'}\right) b_j$$

$$[b_i, b_j] = [b_i^\dagger, b_j^\dagger] = [b_i^\dagger, b_j] = 0 \text{ for } i \neq j$$

$$\{b_j^\dagger, b_j\} = 1$$

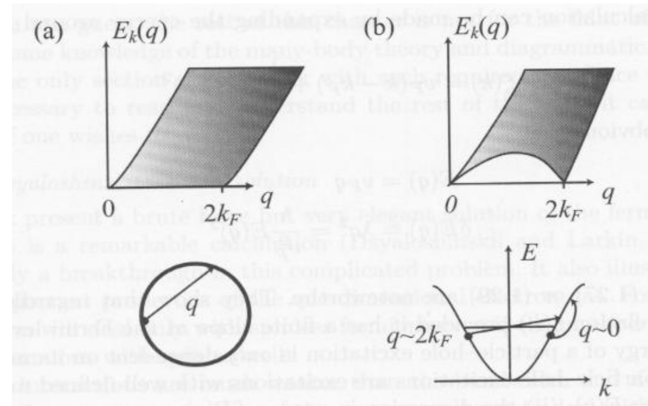
XY model in 1D becomes free fermions (2D more difficult!)

Long-distance properties of 1D Hubbard model: Let us forget the hard-core constraint (?). From electron to (free) boson wave theory: Bosonization

Luttinger Paradigm

Tomonaga 1950 following Bloch 1933

particle/hole pair “excitations”



$$\rho^{e}_{+}(q) = \sum_{k} a^{\dagger}_{k+q}\, a_{k} \quad \textbf{and} \quad \rho^{e}_{-}(q) = \sum_{k} b^{\dagger}_{k+q}\, b_{k}$$

<u>Plasmon waves:</u>

$$\omega(q) = |q| \sqrt{\left(v_F + \frac{g_4(q)}{2\pi}\right)^2 - \left(\frac{g_2(q)}{2\pi}\right)^2}$$

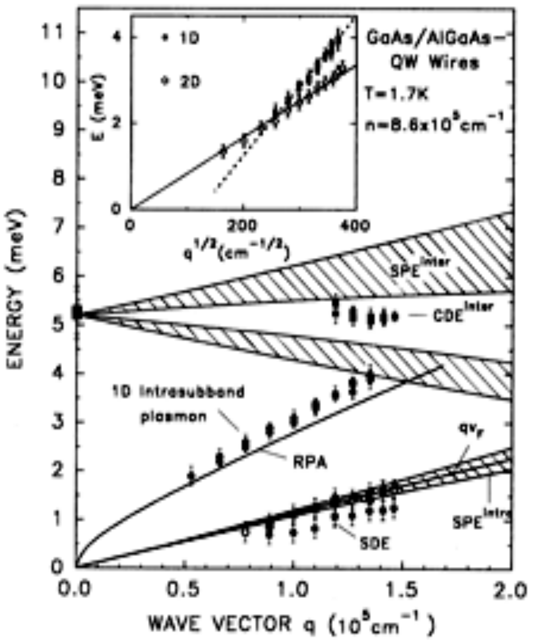
g_4: forward scattering and g_2: backward scattering

Electrons in 1D essentially behave as boson waves

<u>See Notes and references!</u>

Brief Summary

Goni et al. (1991)



Particle-hole pairs → <u>Plasmons</u>
Linear spectrum at small q

Charge plasmons propagate at the « speed » v:
$vg = v_F$ and $g \approx 1\text{-}U/E_F$ (g<1 is the Luttinger parameter)

Spin-Charge separation: Hubbard model
Spin part: Luttinger model with g_s=1 and v_s=v_F

$$H = \frac{v}{2} \int_0^L dx \left[\frac{1}{g} (\partial_x \phi)^2 + g (\partial_x \theta)^2 \right]$$

Wave (string) theory

$$\partial_t^2 \theta = v^2 \partial_x^2 \theta$$

Φ Charge mode
Θ Superfluid phase

$k_B T << (v, v_s)/a$

Absence of order at finite T

Large S, spin wave theory, for ferromagnetic spin chain

J: Heisenberg coupling

$$S_i^z = (S - a_i^\dagger a_i)$$

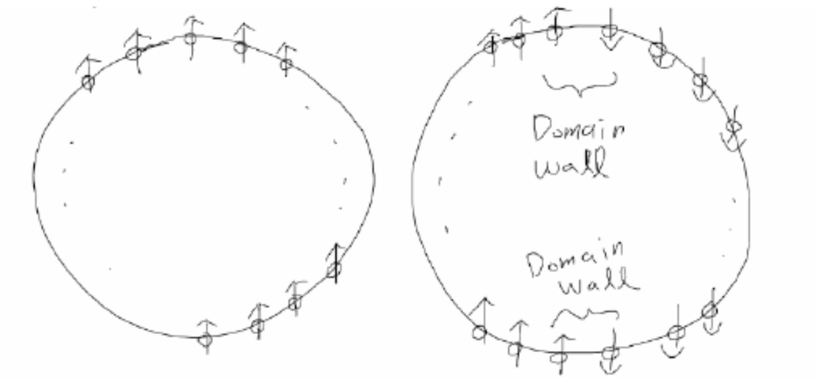
$$S_i^- \approx \sqrt{2S} a_i^\dagger \;\; S^+ \approx \sqrt{2S} a_i$$

$$\langle S_i^z \rangle = S - \frac{1}{N} \sum_{\vec{q}} \frac{1}{e^{\beta E(\vec{q})} - 1}$$

$$\frac{1}{\beta J} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2}$$

DIVERGENT in IR

Absence of order II



$E = E_0$	$E = E_0 + J$
$S = 0$	$S \propto \log N$
$F_0 = E - TS = E_0$	$F_1 = E - TS = E_0 + J - T k_B \log N$

$F_1 < F_0$ for any finite T

can continue with more domains: absence of phase transition at finite T…

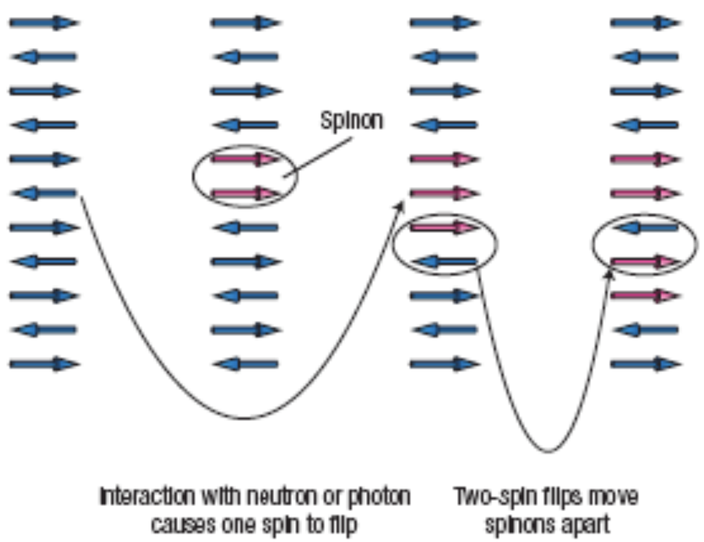
Mermin-Wagner Theorem : No long range order (i.e. no phase transition) in 1D or 2D

New excitations "spinons"

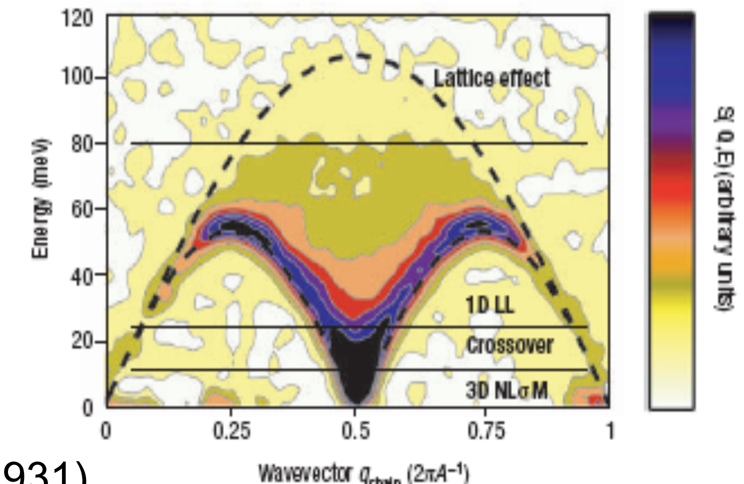
AF Neel order unstable in 1D: breakdown of Holstein-Primakoff spin-wave theory

I. Zaliznyak, Nature Materials **4**, 273 (2005)

Example: $KCuF_3$

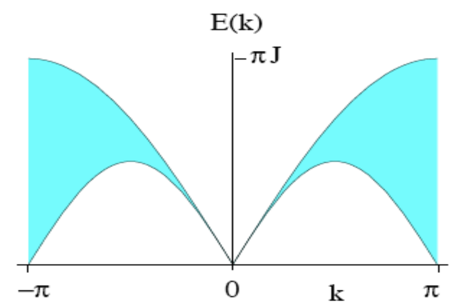


H. Bethe (1931)



Magnon decomposes itself into 2 deconfined spinons: continuum of excitations around π

$$\langle \vec{S}(x)\vec{S}(0)\rangle \approx \frac{\cos(2k_F x)}{x^{1+g}}$$



Renormalization Group

R. Shankar, Rev. Mod. Phys. **66**, 129 (1994)

Gaussian theory unrenormalized

In spin sector, extra backward scattering term which is marginally irrelevant for repulsive interactions

For attractive interactions, it opens a spin gap:
1D BCS state or Luther-Emery liquid

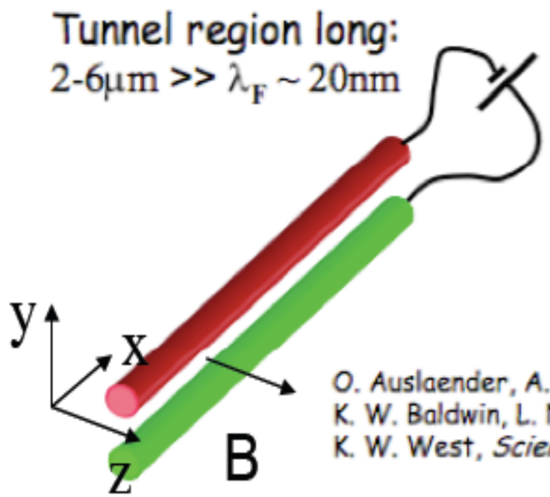
C. Bourbonnais and L. G. Caron, Int. J. Mod Phys B **5** 1033 (1991)
J. Solyom, Adv Phys **28**, 209 (1979)

Next Step:

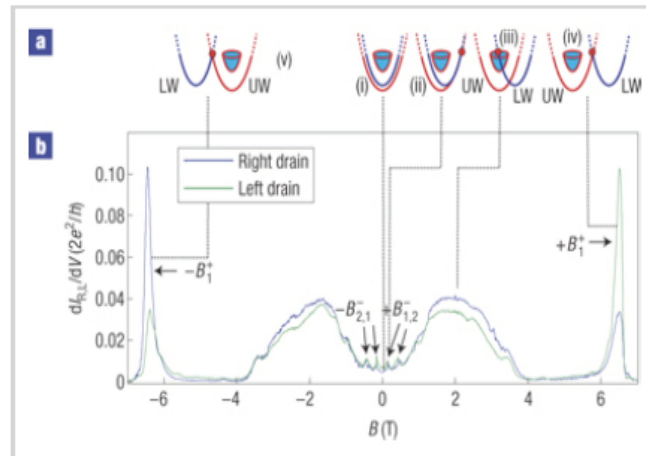
- Electron spectral function
- Spin-charge separation
- Charge fractionalization

<u>New tools to probe low-dimensional Systems:</u>
Momentum-resolved tunneling, cold atomic systems

Momentum-resolved tunneling



O. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, *Science* **295**, 825 (2002).



B plays the role of momentum
Bias voltage V embodies energy (frequency)

Transverse field

Symmetric gauge:

$\vec{A} = (-By, Bx, 0)/2$

Landau gauge:

$A_y = xB$

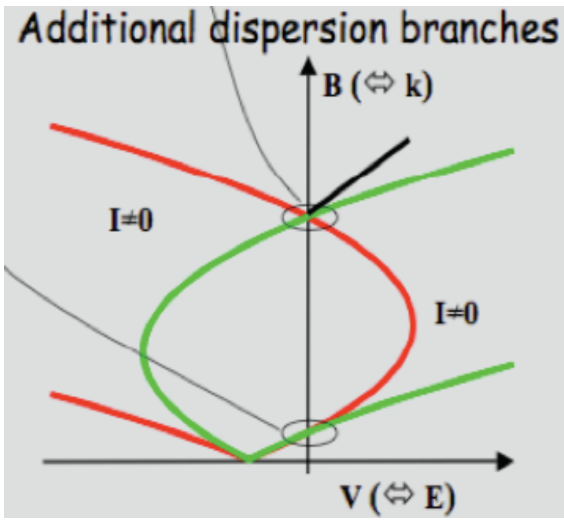
Phase accumulated during tunneling xBd
Boost in momentum $\delta k_x \propto eBd = q_B$

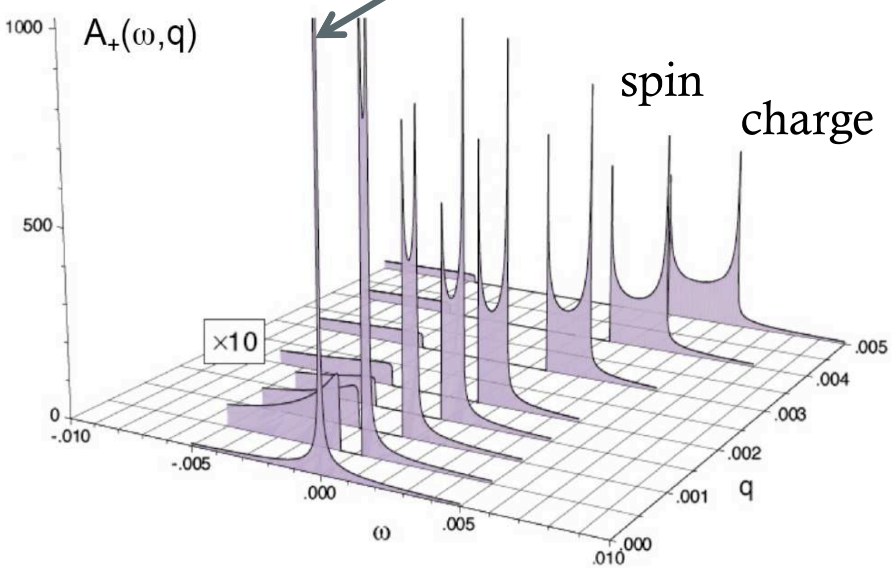
Spin-Charge separation

Conductance dI/dV follows electron spectral function A(k,E=V)
Analogous to a photemission experiment

Absence of quasiparticle peak!

T=0: Branch cut due to mass effects
Finite T: Lorentzian





See Anderson & Ren, Giamarchi's book, K. Le Hur PRB 74, 165104 (2006) for finite T

Charge Fractionalization I

Electron: not a good eigenstate (quasiparticle) of Luttinger theory

$$[H, \Psi^\dagger] \neq E\Psi^\dagger$$

Exact (chiral) eigenstates:

$$L_\pm(x,t) = \exp -i\sqrt{\pi}N_\pm\theta_\pm(x,t)$$

$$\theta_\pm = \theta \mp \phi/g$$

Pham, Lederer, Gabay, 1999

Charge Fractionalization II

Suppose that we inject N electrons in a Luttinger liquid
We denote N_+^e and N_-^e the injected electrons at the 2 Fermi points: $J= (N_+^e - N_-^e)$
This produces right and left excitations with charge N_+ and N_-

One gets simple conservation laws:

$$\boxed{\begin{aligned} (N+ + N-) &= N \\ v(N+ - N-) &= vgJ \end{aligned}}$$

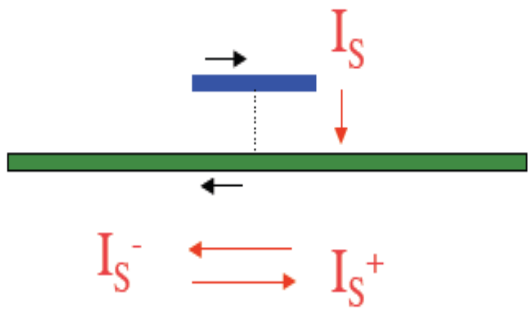
Current in Luttinger theory

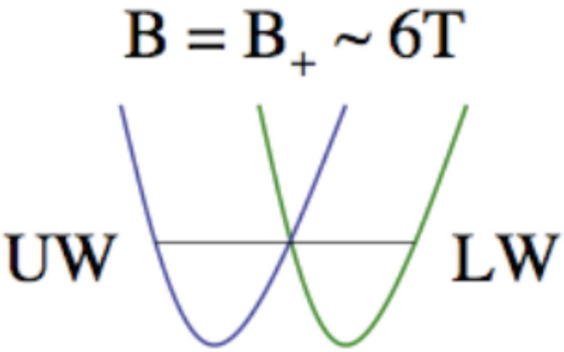
In particular, 1 electron at $-k_F$ gives: $N_- = (1+g)/2 = f$ and $N_+ = (1-g)/2 = (1-f)$

Karyn Le Hur, Bertrand I. Halperin, Amir Yacoby, Annals of Physics (2008) in press

Gedanken Experiment

We ignore measuring leads…





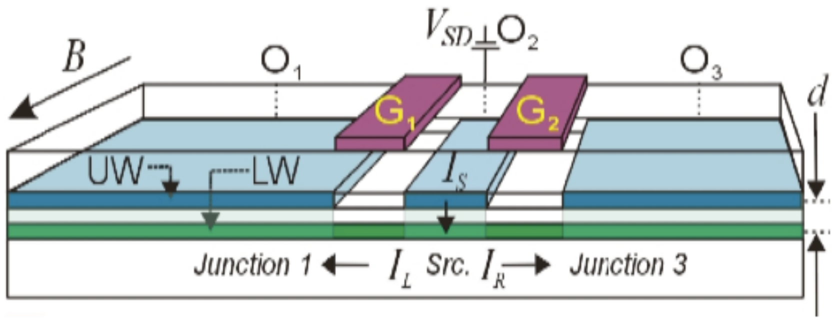
Universal quantity:
$A = (I_S^- - I_S^+)/I_S = (2f\text{-}1) = g$

Shot-noise in the current
$S = 2e^*I$ where $e^*=fe$ or $(1\text{-}f)e$

Observation of electron charge by Millikan 1910
Observation of "fractional charge" at the edges of quantum Hall systems
Saclay (Saminadayar/Glattli) & Weizmann (Heiblum et al.) in 1997

How to measure charge “f”?

H. Steinberg, G. Barak, A. Yacoby, L. Pfeiffer, K.W. West, B. I. Halperin, K. Le Hur



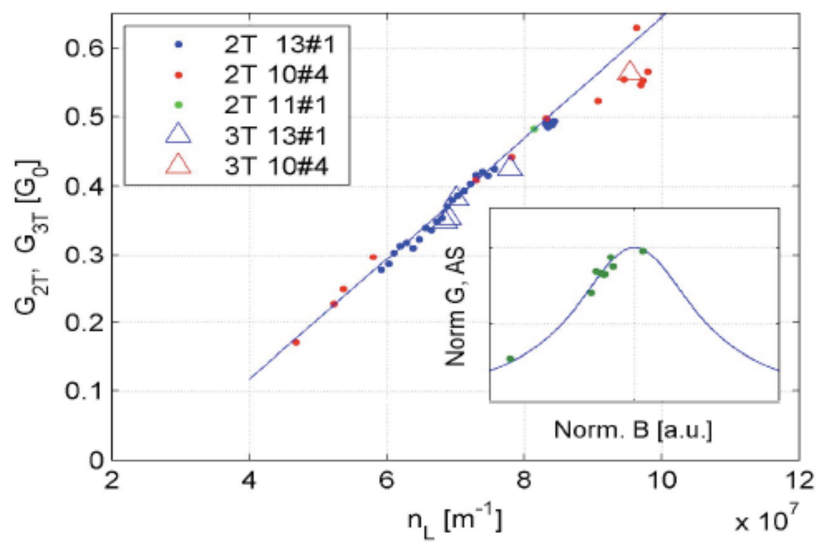
Nature Physics, **4**, 116 (2008)

Measured asymmetry:

$A_S = (I_L\text{-}I_R)/I_S$

One can get rid of couplings with “probes”: 2-terminal conductance

$$\frac{A_S(2e^2/h)}{G_2} = \frac{1}{g}\left(\frac{I_S^- - I_S^+}{I_S}\right) = 1$$



Mott Transition

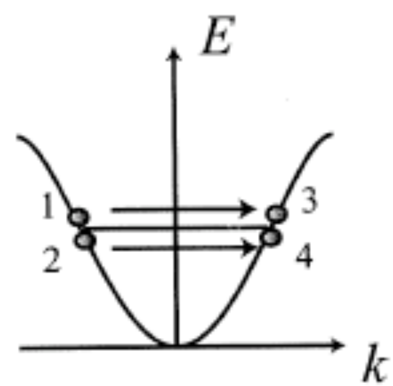
3D: U<<t one expects Fermi liquid
U>>t electrons localize (one per atomic orbital): charge gap ~ U

Theory of the Mott transition? Brinkmann-Rice (1970)
Exact solution in infinite dimensions

Rigorous solution also in 1D: Lieb-Wu (1968) versus bosonization

-Half-filled band: Mott transition for all U>0, umklapps

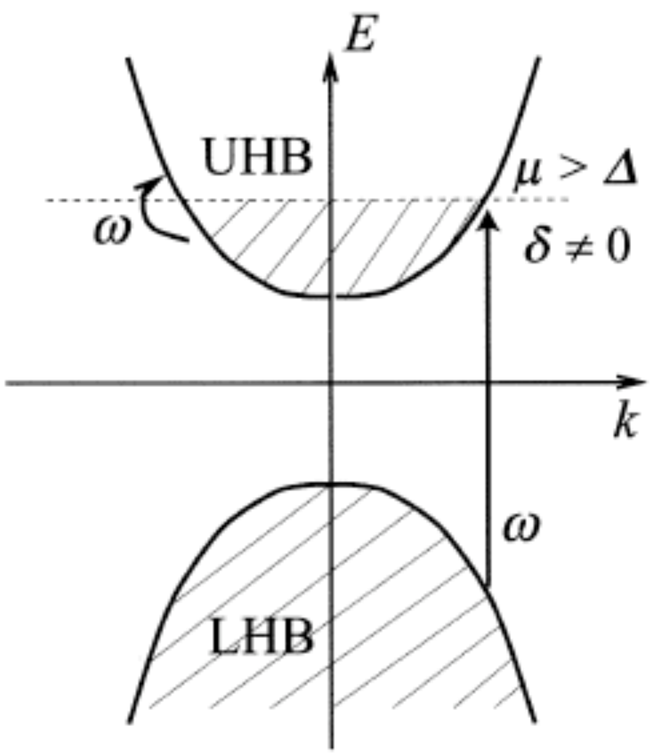
-Mott gap ~ exp-U/E_F at small U



Giamarchi's book

Commensurate-Incommensurate transition

Analogy to a doped band insulator with “holons”: Rigorous proof



g ~ 1/2 for all U and v~δ

Umklapp scattering can be refermionized for g=1/2

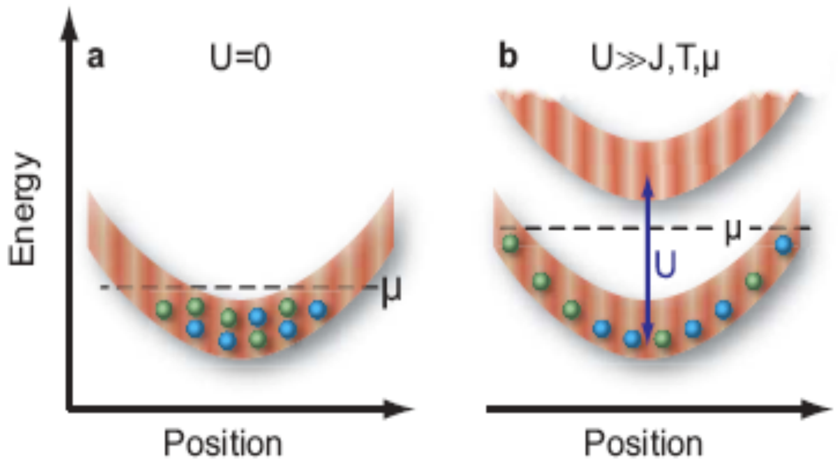
Mott physics irrelevant when doping δ > Mott gap

Review: H.J. Schulz, 1995

Emery, Luther, Peschel, 1976

3D Cold Atomic Fermions

T. Esslinger et al. (ETH Zuerich, 2008)

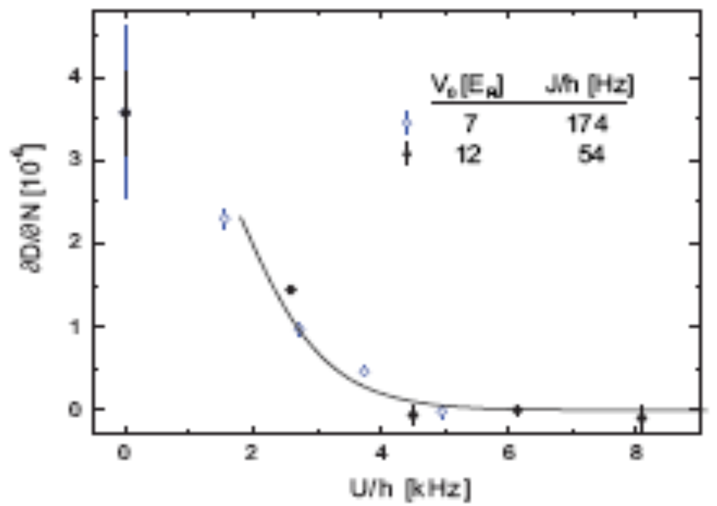


^{40}K

J = hopping

N=5×10^4 atoms, n=1.4

Temperature not too small: $0.2T_F$



Mott transition for bosons: M. Greiner et al.

incompressible

Confinement

<u>Possible 1D phases in higher dimensions:</u>

1)Start with a strong Mott gap along the chains
Suppress the transverse hopping
Case of quasi-1D organic material: TMTTF

K. Le Hur, 2002

2) Long-range interactions: *sliding Luttinger phase*
A. Vishwanath & D. Carpentier, 2000
Emery, Fradkin, Kivelson, cond-mat/0001077

3) Edge states of Quantum Hall systems: Lorentz confinement
Gapless edge U(1) mode

Spin-incoherent regime

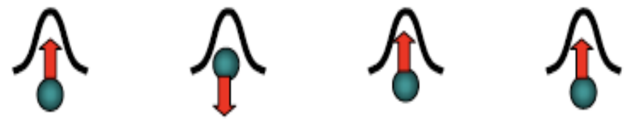
Greg Fiete, Karyn Le Hur, Leon Balents, PRB 2005 & 2006

$r_s \gg 1$: low density n

$$\boxed{r_s = \frac{E_C}{E_K} \sim \frac{m^* e^2}{n\,\hbar^2 \epsilon_d}}$$

$$\boxed{J \ll T \ll \varepsilon_F}$$

Quasi-Wigner crystal regime



- "phonons" $\omega_{ZB} \sim \varepsilon_F\, r_s^{1/2}$
- spin exchange $J \sim \varepsilon_F e^{-\alpha\sqrt{r_s}} \ll \varepsilon_F$

No coherent single-particle propagation
Transport: analogy to spinless electrons (no spin diffusion)

See also K. Matveev (2004)

Conclusion of Lecture 1

1 dimension is far from Fermi liquid: Luttinger paradigm
Rigorous calculations can be performed: Bosonization, Bethe Ansatz,…

-Spin-Charge separation
-Charge fractionalization
-Commensurate-Incommensurate transition: Mott physics
-Interesting electron spectral function

Can be tested in various systems: quantum wires, spin chains, cold atoms

Next step: From 1D to 2D… Lecture 2
Superconductivity close to the Mott state