Topological Proximity Effects

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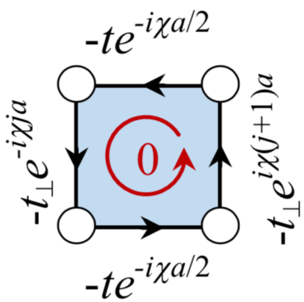
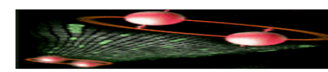
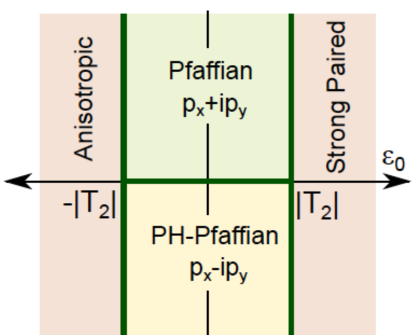
Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

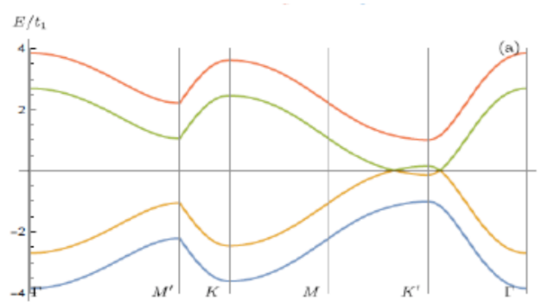


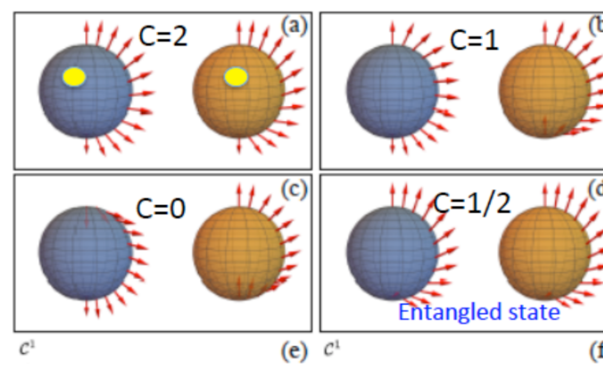
Cambridge October 22, 2020
zoom Meeting



thanks to ANR, DFG







Plan for the presentation

Topological p-wave SC from proximity effect

A coupled-wires model and px+ipy superconductivity

Entanglement induced from an s-wave superconductor

New proximity Effect from the curved space

Poincare-Bloch sphere models from the k-space

Fractional Topology from Entanglement

Relation to new bilayer systems and Z_2 symmetry

Topological proximity effect in graphene from Haldane model:

Peng Cheng, Philipp Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg and Karyn Le Hur, Phys. Rev. B 100, 08110 (R) (2019).

Relation to Cambridge: recent work on p-wave SC in graphene coupled to an high-Tc superconductor

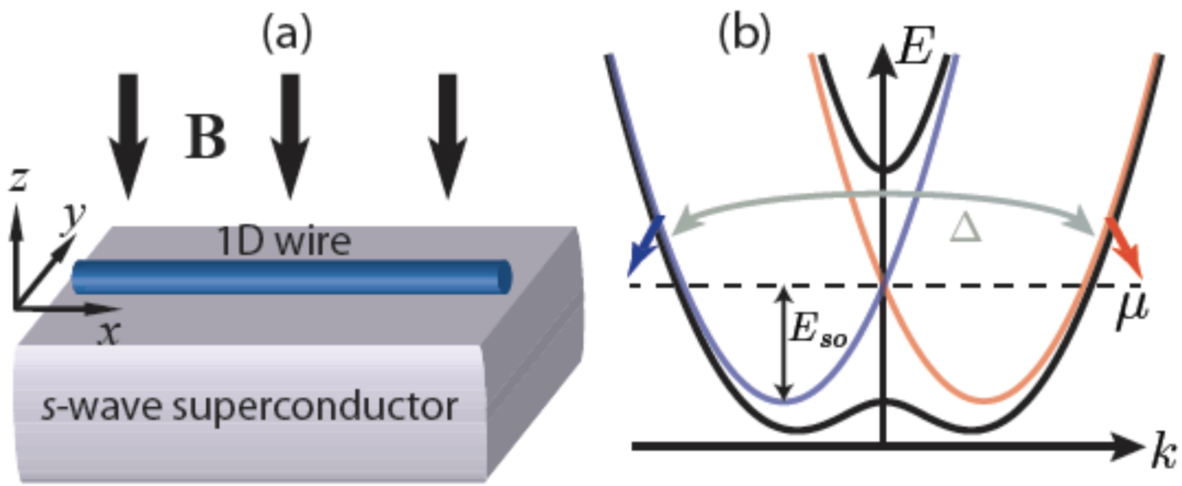
Angelo Di Bernardo et al. Nature Communications 8, 14024 (2017)

1 Wire

Realizations: OK
Delft, L. Kouwenhoven's group
Copenhagen: C. Marcus's group

Yu. Oreg, G. Refael, F. Von Oppen
R. Lutchyn, J. Sau, S. Das Sarma
Review: J. Alicea, arXiv:1202.1293

Progress in cold atoms
M. Aidelsburger review



$$H = H_{\rm wire} + H_\Delta$$

Rashba spin-orbit coupling

$$H_{\rm wire} = \int dx \psi^\dagger \left(-\frac{\partial_x^2}{2m} - \mu - i\alpha\sigma^y \partial_x + h\sigma^z \right) \psi$$

$$H_\Delta = \int dx \Delta(\psi_\uparrow \psi_\downarrow + H.c.).$$

Zeeman effects
can also be in
x direction (see later)

Kitaev p-wave Superconductor

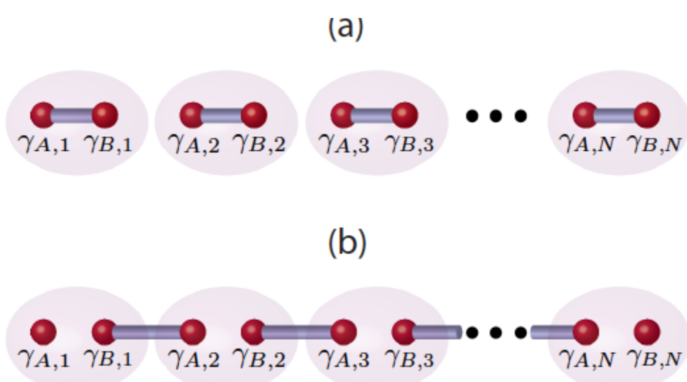
$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2}\sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.),$$

$$\mathcal{H}_k = \mathrm{h}(k)\cdot\boldsymbol{\sigma}$$

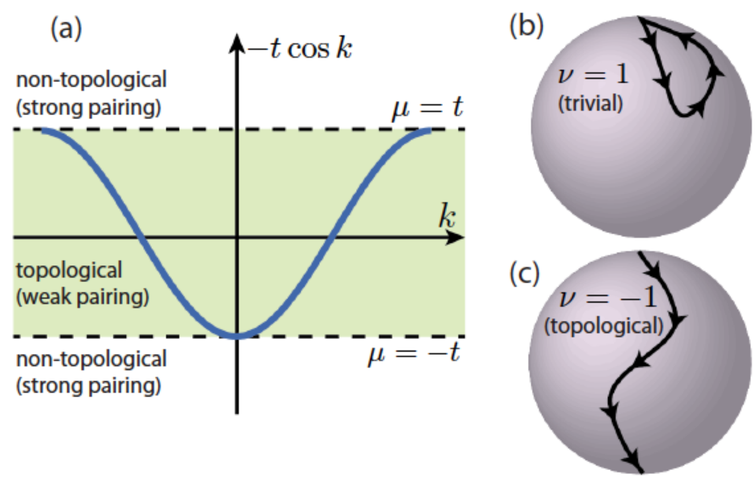
$$H = \frac{1}{2}\sum_{k\in BZ} C_k^\dagger \mathcal{H}_k C_k, \quad \mathcal{H}_k = \begin{pmatrix} \epsilon_k & \tilde{\Delta}_k^* \\ \tilde{\Delta}_k & -\epsilon_k \end{pmatrix},$$

with $\epsilon_k = -t\cos k - \mu$ the kinetic energy and $\tilde{\Delta}_k = -i\Delta e^{i\phi}\sin k$ the Fourier-transformed pairing potential. The

$$c_x = \frac{e^{-i\phi/2}}{2}(\gamma_{B,x} + i\gamma_{A,x}).$$



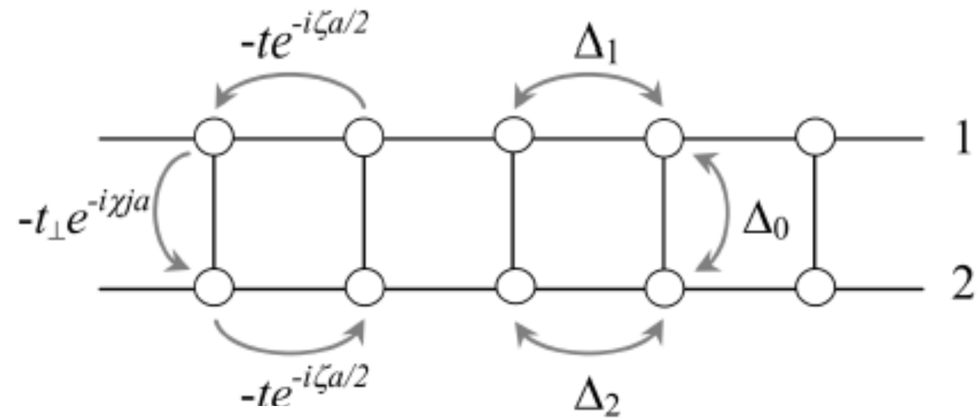
Review: J. Alicea, arXiv:1202.1293

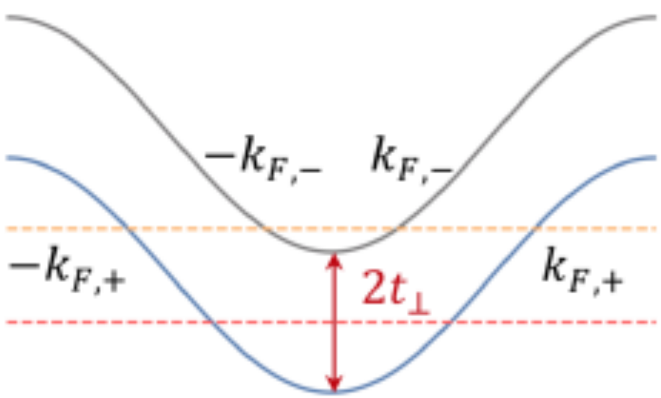


Relation to entanglement measures:
« charge fluctuations »
Loic Herviou, Christophe Mora, Karyn Le Hur (2018)

From one to two wires

Fan Yang, Vivien Perrin, Alexandru Petrescu, Ion Garate, Karyn Le Hur, arXiv:1910.04816, long article Phys. Rev. B 101, 085116 (2020)





$$c_\pm(j) = \frac{1}{\sqrt{2}}\left[c_1(j) \pm c_2(j)\right].$$

$$\mathcal{H}_0 = \mathcal{H}_\parallel + \mathcal{H}_\perp = \sum_{\lambda=\pm}\sum_k \xi_{k,\lambda} c_\lambda^\dagger(k) c_\lambda(k),$$

with an energy dispersion

$k_F^1 = k_F^2$, $\Phi_{tot} = 0$

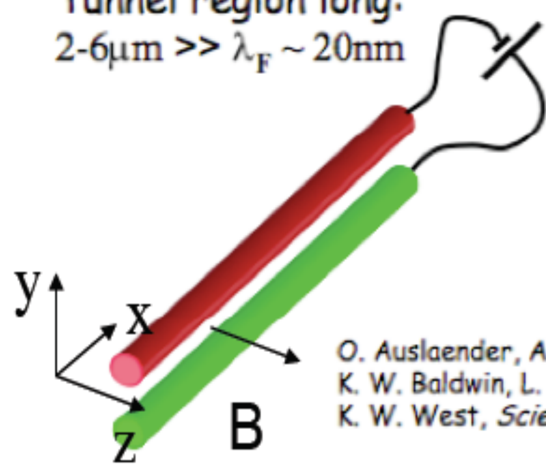
$$\xi_{k,\pm} = -2t\cos(ka) \mp t_\perp - \mu.$$

Regions of 2 and 4 Majorana fermions when tuning the chemical potential: Lifshitz transition

Momentum-resolved tunneling

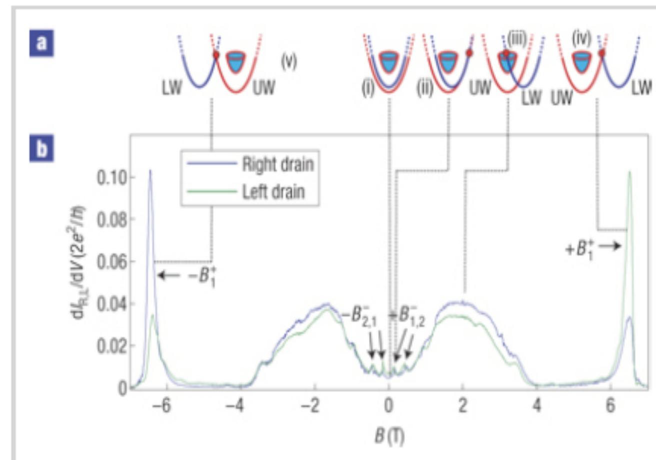
H. Steinberg, G. Barak, A. Yacoby, L. Pfeiffer, K.W. West, B. I. Halperin, K. Le Hur Nature Physics **4**, 116 (2008)

Tunnel region long:
2-6μm >> $\lambda_F \sim$ 20nm



O. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, *Science* **295**, 825 (2002).

Route to quantum Hall state: Munich



B plays the role of momentum
Bias voltage V embodies energy (frequency)
Spectroscopy similar to ARPES

Transverse field

<u>Symmetric gauge</u>:
$\vec{A} = (-By, Bx, 0)/2$
<u>Landau gauge</u>:
$A_y = xB$

Phase accumulated during tunneling xBd
Boost in momentum $\delta k_x \propto eBd = q_B$

Very Recent Progress with fluxes and Kitaev wires: arXiv:2003.13177 (S. Vatiekenas et al.)

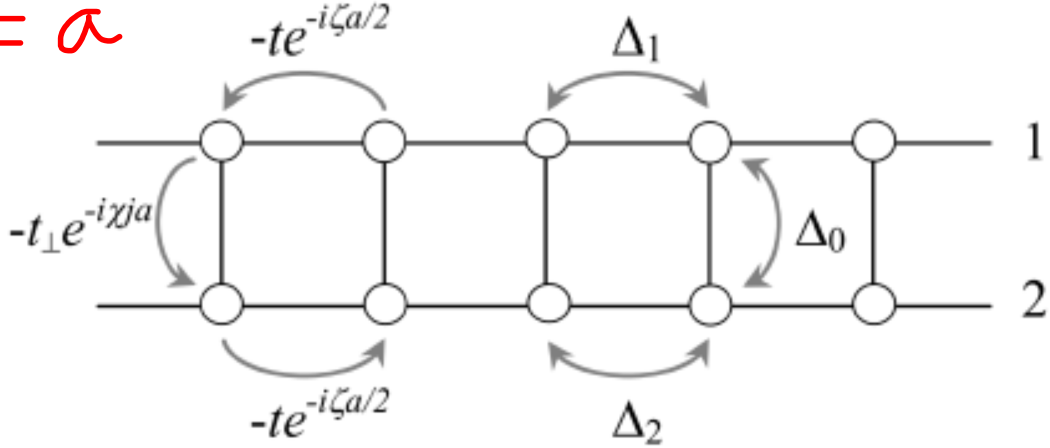
Assembling Wires…

Model with zero net flux in a square

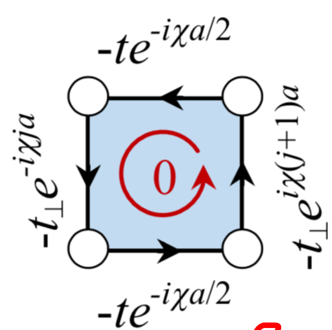
$$B_z = \partial_x A_y - \partial_y A_x = \frac{\chi}{a'} - \frac{\zeta\pi}{2a'}\sin(\pi y/a'),$$

Fields uniform in x direction

$a' = a$



$$\Phi_{\text{tot}} = \chi - \zeta. = 0$$



$\chi = \zeta$

Superposition of two staggered magnetic fields in y-direction:

1 staggered-step function and 1 sinus smooth function

One can fix the flux to be $\Phi_{tot}=0$, and change X and ζ (optimum, close to π)

Can also be realized with 2 Peierls phases

Model and Gauge Invariance

$$\mathcal{H}_{\parallel} = -\sum_{j}\sum_{\alpha=1,2}[\mu c_\alpha^\dagger(j)c_\alpha(j) + \overbrace{te^{-i\zeta a/2}}^{at} c_1^\dagger(j)c_1(j+1)$$

$t_\perp(-1)^{x_j}$

$$+ te^{i\zeta a/2}c_2^\dagger(j)c_2(j+1) + \mathrm{H.c.}],$$

$$\mathcal{H}_{\perp} = -\sum_j t_\perp e^{i\chi x_j} c_1^\dagger(j)c_2(j) + \mathrm{H.c.},$$

$\chi = \zeta$

Here, we assume that the 3D Superconducting s-wave reservoir is not affected by orbital magnetic field effects and that all the induced pairing terms are phase coherent:

$$\mathcal{H}_{\Delta} = \sum_{\alpha=1,2}\sum_j \Delta_\alpha c_\alpha^\dagger(j)c_\alpha^\dagger(j+1) + \mathrm{H.c.},$$

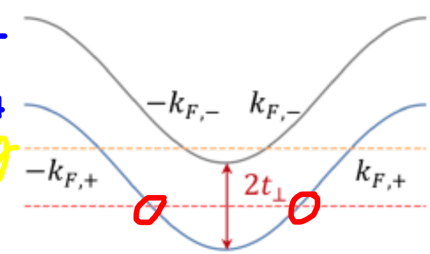
$$\mathcal{H}_{\Delta_0} = \sum_j \Delta_0 c_1^\dagger(j)c_2^\dagger(j) + \mathrm{H.c.}.$$

C_-

C_+

4 May

2 May μ



gauge change: $k_F^1 = k_F^2,\ \Phi_{tot} = 0$

$$c_1(j) = e^{i\chi x_j/2}\tilde{c}_1(j), \quad c_2(j) = e^{-i\chi x_j/2}\tilde{c}_2(j),$$

$$\tilde{c}_\pm(j) = [\tilde{c}_1(j) \pm \tilde{c}_2(j)]/\sqrt{2}.$$

$$\mathcal{H}_{\Delta_0,\mathrm{NN}} = \Delta_0 \sum_j c_1^\dagger(j) c_2^\dagger(j+1) + \mathrm{H.c.}$$

$$\boxed{\chi a = \pi, \quad \widetilde{\Delta} = i\Delta_0/2,}$$

diagonale

ip_x

blue ladder :

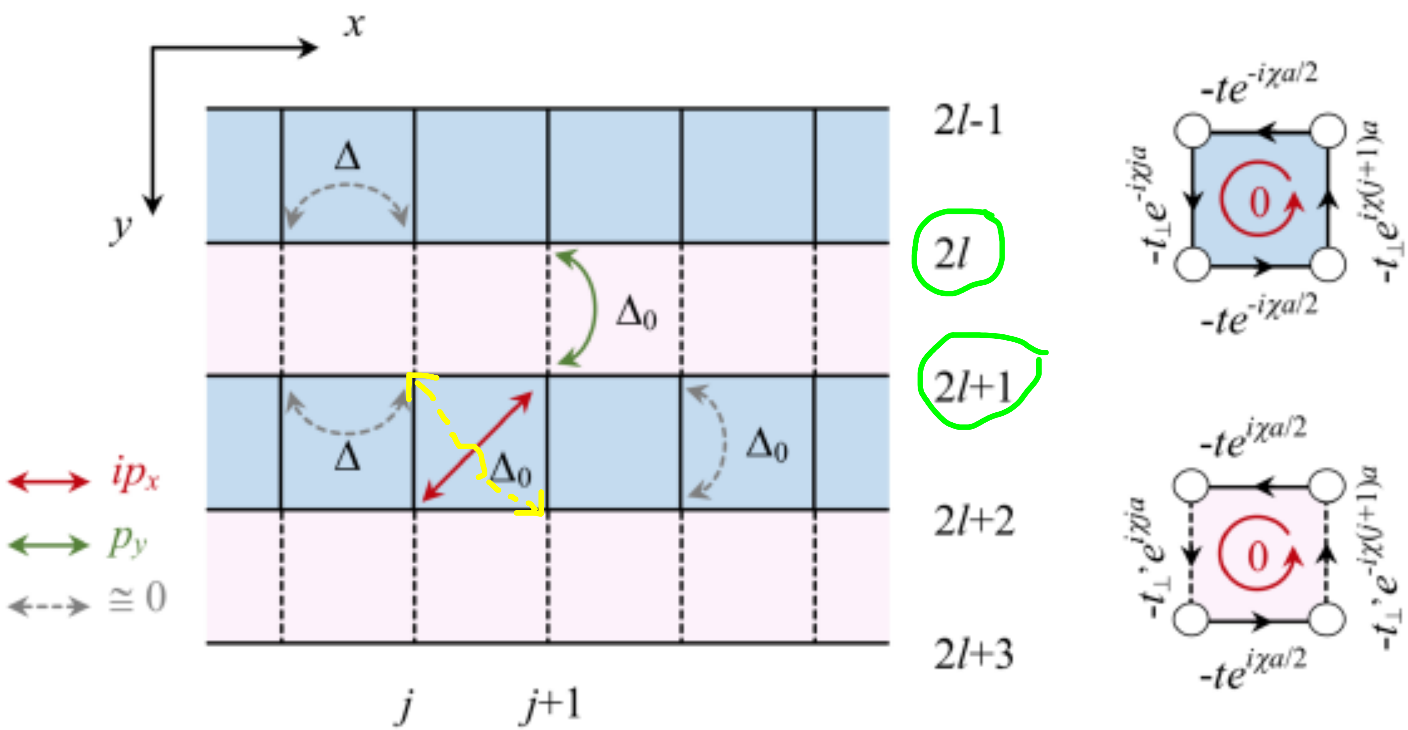
$$= \widetilde{\Delta} \sum_{k_x} i \sin(k_x a) \widetilde{c}_+^\dagger(k_x) \widetilde{c}_+^\dagger(-k_x) + \mathrm{H.c.},$$

pink couplings

$$\mathcal{H}'_\perp = -t'_\perp \sum_{k_y} \cos(k_y a') \widetilde{c}_+^\dagger(k_y) \widetilde{c}_+(k_y),$$

$$\boxed{\mathcal{H}'_{\Delta_0,\mathrm{N}} = \frac{\Delta_0}{2} \sum_{k_y} i \sin(k_y a') \widetilde{c}_+^\dagger(k_y) \widetilde{c}_+^\dagger(-k_y) + \mathrm{H.c.}.}$$

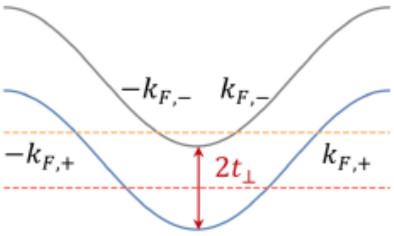
p_y



Majorana Model in 2D

$$\boxed{\widetilde{c}_+(\mathbf{k}) = \frac{1}{2}\left(\gamma_1(\mathbf{k}) + i\gamma_2(\mathbf{k})\right).}$$

$$\begin{aligned}\mathcal{H}_+ &\simeq \mathcal{H}_{0,+} + \mathcal{H}_{\Delta_0,\mathrm{NN}} + \mathcal{H}'_\perp + \mathcal{H}'_{\Delta_0,\mathrm{N}} \\ &= -\frac{1}{4}\sum_{\mathbf{k}} \gamma^T(-\mathbf{k})\mathcal{H}_+(\mathbf{k})\gamma(\mathbf{k}),\end{aligned}$$



$k_F^1 = k_F^2,\ \Phi_{tot} = 0$

with $\gamma^T(-\mathbf{k}) = (\gamma_1(-\mathbf{k}), \gamma_2(-\mathbf{k}))$ and close to k=(0,0) bottom of the band

$$\boxed{\mathcal{H}_+(k) = uk_x\tau^z + [\epsilon_0 + \frac{k_x^2}{2m} + T_2\cos(k_y)]\tau^y + R_2\sin(k_y)\tau^x.}$$

$T_2 - T_2 k_y^2/2$

Here τ^i $(i = x, y, z)$ denote Pauli matrices and effective parameters are given by $u = \Delta_0$, $m = 1/(2t)$. The three parameters locating the phase transitions read

$$\epsilon_0 = -2t - t_\perp - \mu, \quad T_2 = -t'_\perp, \quad R_2 = -\Delta_0.$$

$\epsilon_0 = \mp T_2$

Analysis & Stability

ε_0 plays the role of a large chemical potential, which results in a strong-paired phase (zero Majorana mode)

Simple check of the occurrence of 1 chiral Majorana mode in the Moore-Read intermediate phase: $\epsilon_0 = +t'_\perp = +|T_2|$

The energy becomes gapless at the boundary

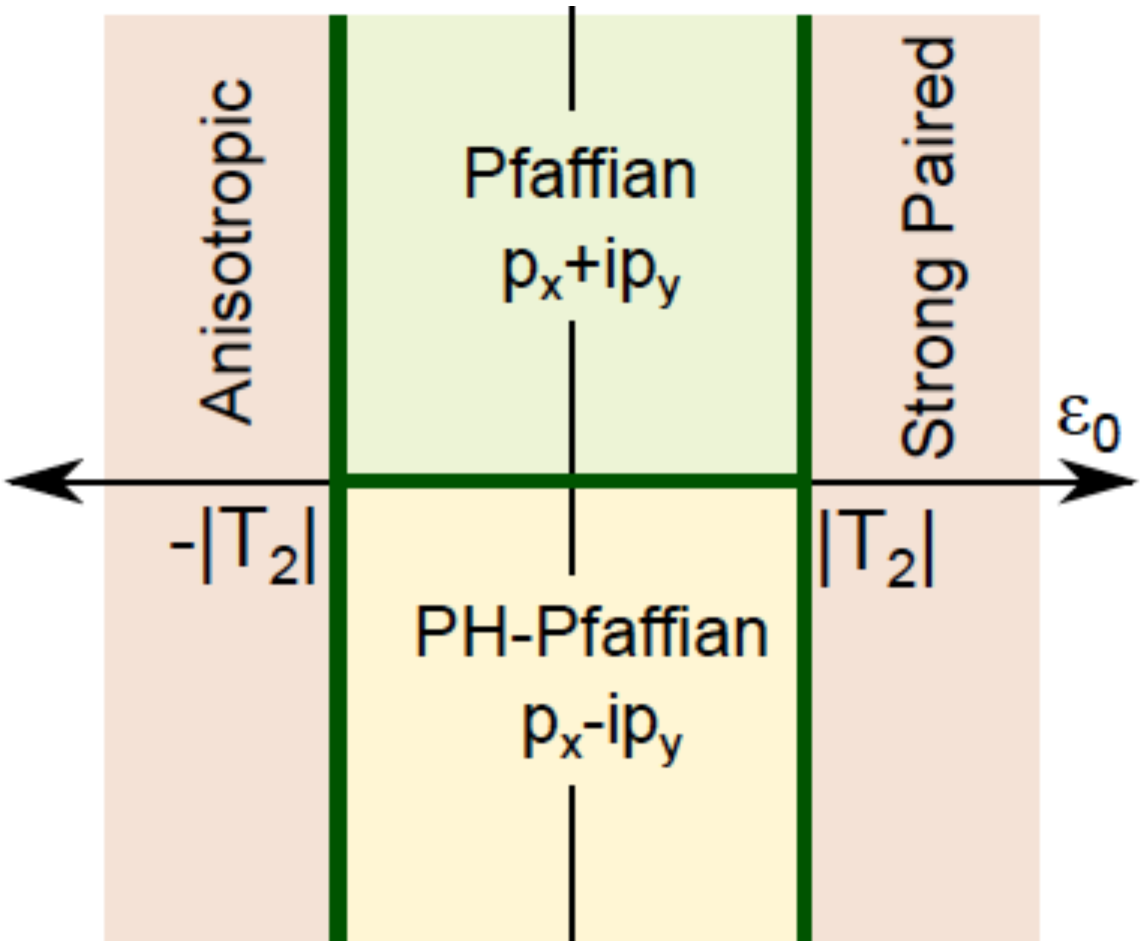
$$-\Delta_0\sqrt{k_x^2 + k_y^2} \text{ close to } k_x = k_y = 0.$$

If one changes the sign of $\chi = \zeta$ then one reverses the sense of propagation of the Majorana mode through R_2 . (This refers to pfaffian and anti-pfaffian in the 5/2 FQHE)

Anisotropic Phase adiabatically connected to N decoupled ladder models: 2N one-dimensional Majorana fermions favorable here!

stability changing (χ, ζ) close to $\frac{\pi}{a}$- values

It’s nice to see some links



C. L. Kane, A. Stern, B. I. Halperin, arXiv:1701.06200 published in Physical Review B.

Hybrid Systems

Karyn Le Hur, PRB RC **64**, 0605002 (2001)
Karyn Le Hur and T. Maurice Rice, Annals of Physics **324**, 1452 (2009) – 98 pages
A. Petrescu & KLH, long paper PRB **91**, 054520 (2015)
Quantum Hall phase at nu=1/2

Andreev coupling

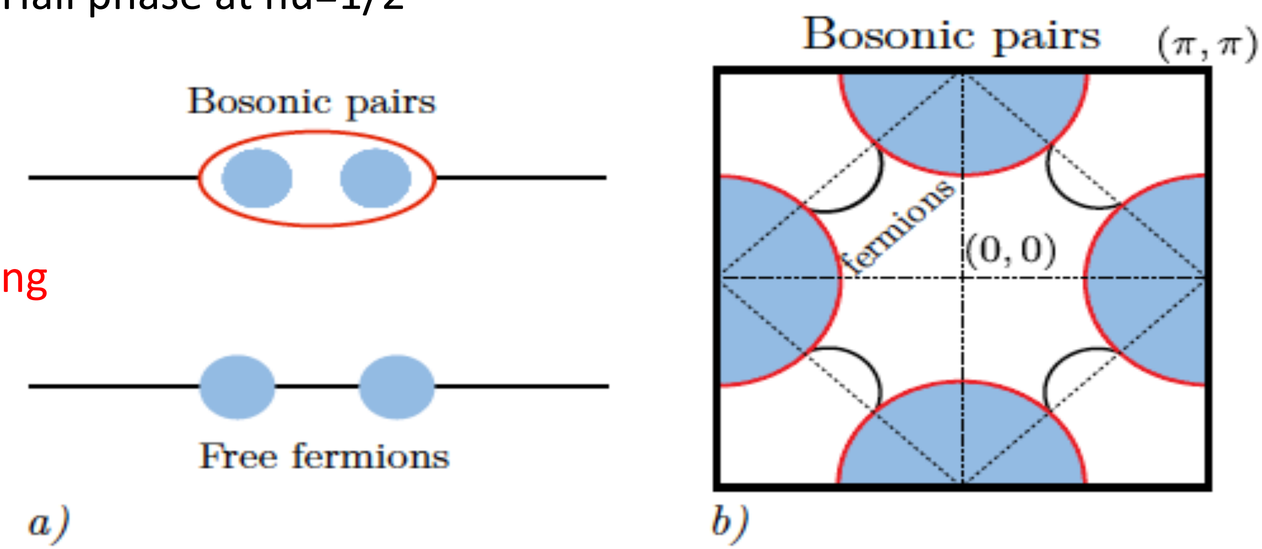


FIG. 5: Hybrid two-leg ladder system introduced in Ref. 65 with preformed Cooper pairs in one chain and repulsive fermions in the other chain. Here, we extend the model by discussing magnetic field effects. This toy model shows certain analogies with the pseudo gap phase of high-Tc cuprates showing hot spots (preformed Cooper pairs) and cold spots (Fermi arcs).[66–68]

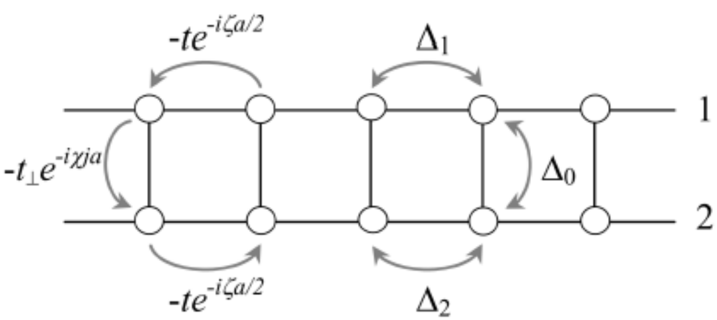
Summary

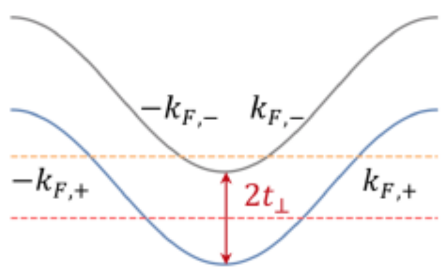
Fan Yang, Vivien Perrin, Alexandru Petrescu, Ion Garate, Karyn Le Hur, Phys. Rev. B 101, 085116 (2020)

Topological Wire Models offer rigorous approaches to realize novel topological phases.
Luttinger paradigm and Renormalization Group arguments allow for a check of stability of such topological phases. Numerical Approaches are also important.

In this presentation, we have addressed the possibility to build a topological p+ip topological superconducting wire network model.

By analogy to the Haldane model (1988) suggesting to build models with zero net flux in a unit cell, yet breaking-time reversal symmetry, we have engineered orbital magnetic field effects with two space-dependent magnetic fields in z-direction.





$k_F^1 = k_F^2,\ \Phi_{tot} = 0$

- Similar ingredients of Berry phases and spin-orbit coupling models have been recently suggested on the honeycomb lattice with superconductivity as well
W. Qin, L. Li, Z. Zhenyu, Nature Physics **15**, 796 (2019).
- A zoo of phases in these wire systems: Quantum Hall, CDW and rung-Mott phases, ...

BOTT Kitaev Classification

Type	TRS	PHS	CS/SLS	Class	d=1	d=2	d=3
AI	+1	0	0	orthogonal	0	0	0
BDI	+1	+1	1	chiral orthogonal	$\mathbb{Z}$	0	0
D	0	+1	0	BdG	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−1	+1	1	BdG	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−1	0	0	symplectic	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−1	−1	1	chiral symplectic	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−1	0	BdG	0	$\mathbb{Z}$	0
CI	+1	−1	1	BdG	0	0	$\mathbb{Z}$

$$TRS = \begin{cases} +1 & \text{if} \quad \mathcal{T}H(\mathbf{k})\mathcal{T}^* = H(-\mathbf{k}),\ \mathcal{T}^2 = +1 \\ -1 & \text{if} \quad \mathcal{T}H(\mathbf{k})\mathcal{T}^* = H(-\mathbf{k}),\ \mathcal{T}^2 = -1 \\ 0 & \text{if} \quad \mathcal{T}H(\mathbf{k})\mathcal{T}^* \neq H(-\mathbf{k}) \end{cases}$$

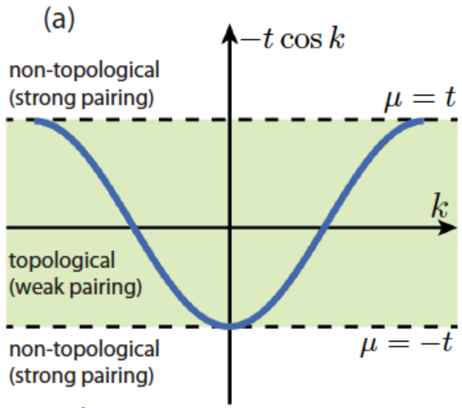
Similarly, the particle hole symmetry (PHS) also gives three classes

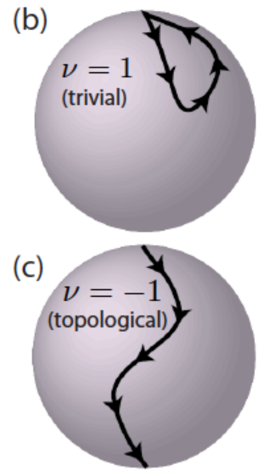
$$PHS = \begin{cases} +1 & \text{if} \quad \mathcal{C}H(\mathbf{k})\mathcal{C}^* = -H(-\mathbf{k}),\ \mathcal{C}^2 = +1 \\ -1 & \text{if} \quad \mathcal{C}H(\mathbf{k})\mathcal{C}^* = -H(-\mathbf{k}),\ \mathcal{C}^2 = -1 \\ 0 & \text{if} \quad \mathcal{C}H(\mathbf{k})\mathcal{C}^* \neq -H(-\mathbf{k}) \end{cases}$$

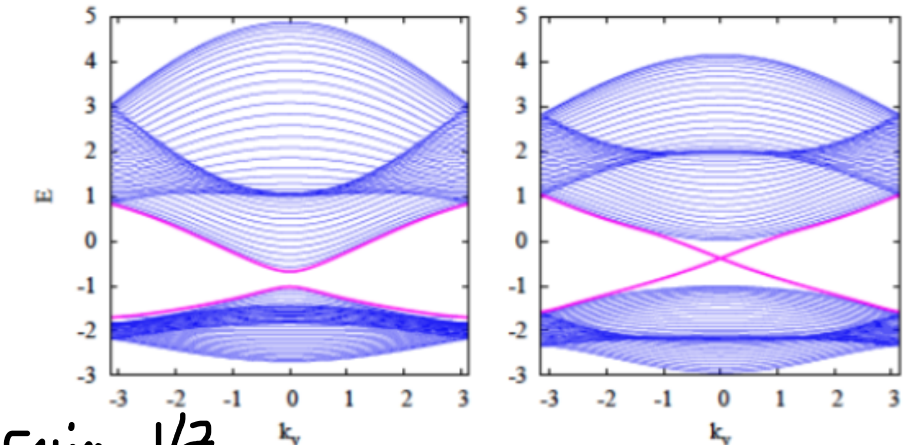
The chiral symmetry (CS) is defined as the product $\mathcal{S} = \mathcal{T} \cdot \mathcal{C}$, sometimes also referred to as the sublattice symmetry (SLS). If both $\mathcal{T}$ and $\mathcal{C}$ are nonzero, then the chiral symmetry is present, i.e., $\mathcal{S} = 1$.

Fractional Topology from curved space

Riemann sphere is the Poincare-Bloch sphere



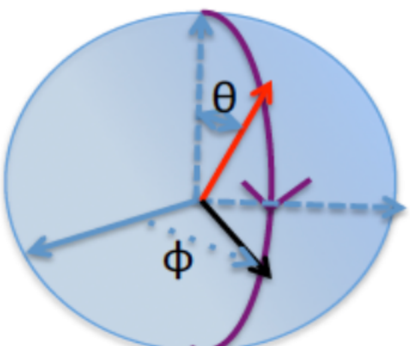


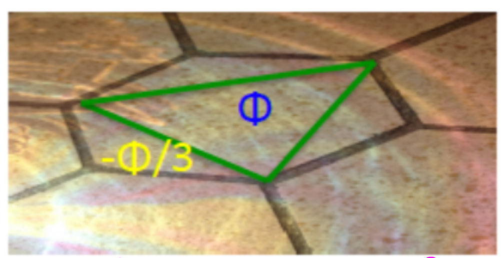


$\phi = \frac{\pi}{2}$

spin-1/2 $\downarrow$ 2×2 matrix

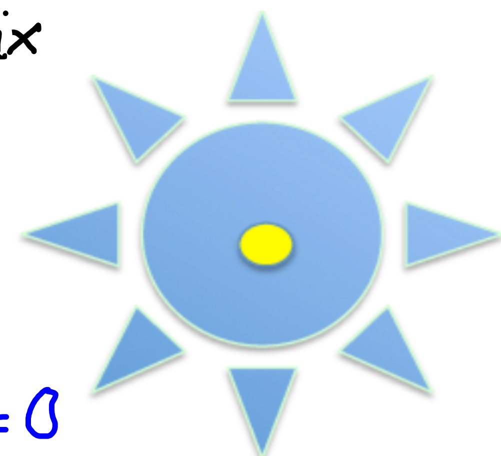
$$\hat{H}_k = -\mathbf{d}_k \cdot \mathbf{S}$$





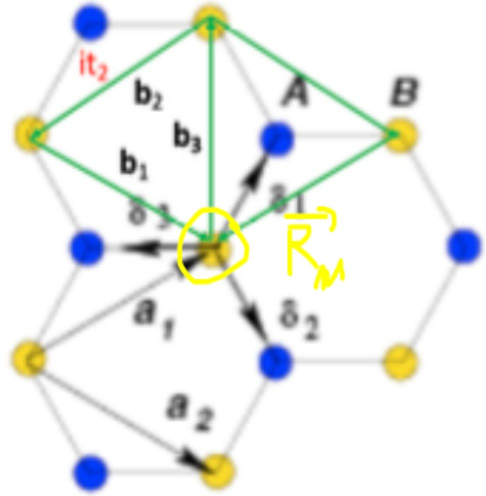
Haldane 1988

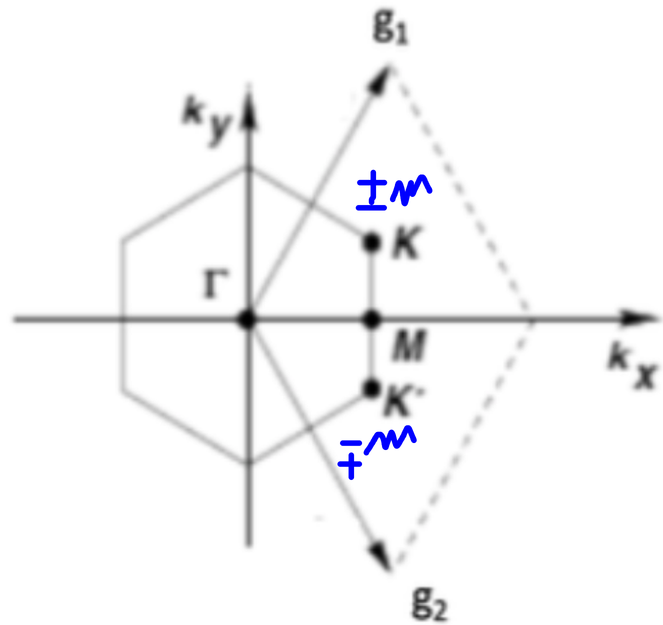
$\phi = 0$



$$\mathbf{d} = H(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta) = (d_x, d_y, d_z)$$

$$\vec{b}_1 = \frac{a}{2}\left(3, -\sqrt{3}\right)$$
$$\vec{b}_2 = -\frac{a}{2}\left(3, \sqrt{3}\right)$$
$$\vec{b}_3 = \left(0, \sqrt{3}\,a\right)$$





6 yellow

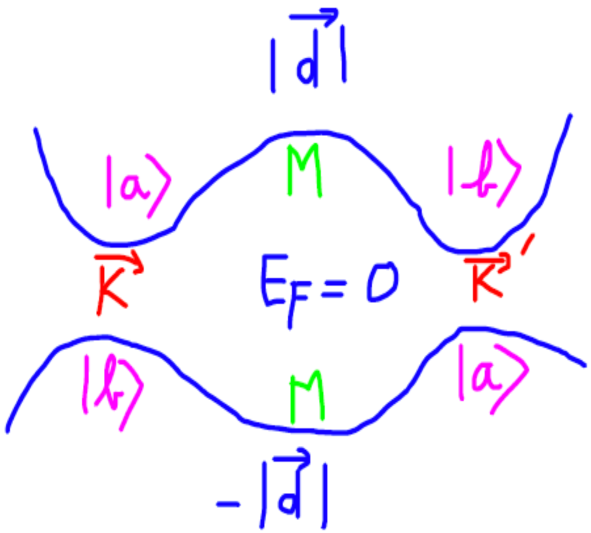
$$H_{t_2}^{\vec{R}_m} = \sum_{\vec{b}_j} t_2 e^{i\phi} \left|\phi_B(\vec{R}_m + \vec{b}_j)\right\rangle\left\langle \phi_B(\vec{R}_m)\right| + h.c.$$

$$\phi = \frac{\pi}{2} \qquad H_{t_2}^{\vec{R}_m} = \sum_{\vec{b}_j} 2t_2 \sin(\vec{k}\cdot\vec{b}_j) \left|\psi_{B\vec{k}}\right\rangle\left\langle \psi_{B\vec{k}}\right|$$

A sites $\vec{b}_j \to -\vec{b}_j$

$$d_z = -2t_2 \sum_{\vec{b}_j} \sin(\vec{k}\cdot\vec{b}_j)$$

Correspondence $(k_x,k_y) \to (\theta, \varphi)$



$$\mathcal{H}(\mathbf{k}) = -\mathbf{d}\cdot\boldsymbol{\sigma} = |\mathbf{d}| \begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

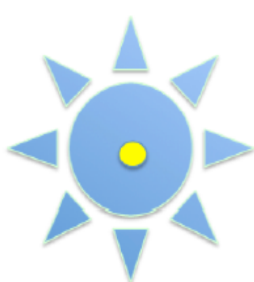
$$\begin{aligned} |u\rangle &= e^{-i\varphi/2}\cos\theta/2|a\rangle + e^{i\varphi/2}\sin\theta/2|b\rangle \\ |l\rangle &= -e^{-i\varphi/2}\sin\theta/2|a\rangle + e^{i\varphi/2}\cos\theta/2|b\rangle \end{aligned}$$

topology on S^2

$\theta = 0$ north pole $\vec{K}$

$|u\rangle = e^{-i\varphi/2}|a\rangle$

$|l\rangle = e^{i\varphi/2}|b\rangle$

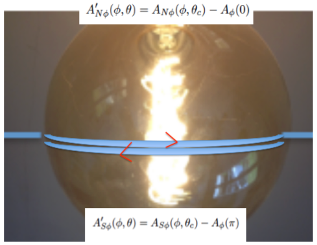


M $\theta = \frac{\pi}{2}$ similar as graphene

$\theta = \pi$ south pole $\vec{K}'$

$|l\rangle = -e^{-i\varphi/2}|a\rangle$ $|u\rangle = e^{i\varphi/2}|b\rangle$

Geometry in the quantum

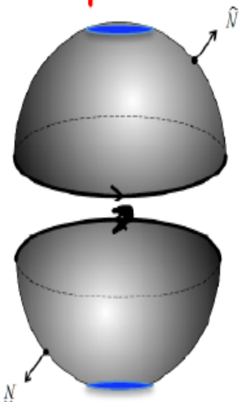


The surface $S^{2'}$ can be decomposed as a north (north') hemisphere and south (south') hemispheres and the fields $\mathbf{A}$ are smooth on $S^{2'}$, such that $\vec{A} = i\langle\psi|\vec{\nabla}|\psi\rangle$ Berry

$$C = -\frac{1}{2\pi}\int_{S^{2'}} \nabla \times \mathbf{A} d^2\mathbf{n}.$$

$$C = -\frac{1}{2\pi}\int_{north'} \nabla \times \mathbf{A}_N d^2\mathbf{n} - \frac{1}{2\pi}\int_{south'} \nabla \times \mathbf{A}_S d^2\mathbf{n}.$$

$A' = 0$



θ_c

On north', we have from Stokes' theorem:

$$-\frac{1}{2\pi}\int_{north'} \nabla \times \mathbf{A}_N d^2\mathbf{n} = -\frac{1}{2\pi}\int_0^{2\pi} d\varphi A_{N\varphi}(\varphi,\theta_c) + \frac{1}{2\pi}\int_0^{2\pi} d\varphi A_\varphi(0).$$

This form assumes that the field is uniquely defined on the boundary path at the north pole with $A_\varphi(0) = A_{N\varphi}(\varphi, 0)$. The right-hand side then corresponds to the two boundary paths encircling *north'*. Similarly, we have for *south'*

$$-\frac{1}{2\pi}\int_{south'} \nabla \times \mathbf{A}_S d^2\mathbf{n} = +\frac{1}{2\pi}\int_0^{2\pi} d\varphi A_{S\varphi}(\varphi,\theta_c) - \frac{1}{2\pi}\int_0^{2\pi} d\varphi A_\varphi(\pi).$$

$S^{2'}$

The field is uniquely defined on the boundary path at the south pole with $A_\varphi(\pi) = A_{S\varphi}(\varphi,\pi)$. We can then define the smooth fields as

$$A'_{N\varphi}(\varphi,\theta) = A_{N\varphi}(\varphi,\theta_c) - A_\varphi(0)$$

$$A'_{S\varphi}(\varphi,\theta) = A_{S\varphi}(\varphi,\theta_c) - A_\varphi(\pi)$$

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Karyn Le Hur
arXiv 2002.11823

$C = A_\varphi(0) - A_\varphi(\pi)$

$A_{N\varphi}(\theta_c) = A_{S\varphi}(\theta_c)$
$= A_\varphi(\varphi, \theta_c)$

Topological proximity Effect

$\vec{F} = \vec{\nabla} \times \vec{A} \qquad \vec{A} = i\langle\psi|\vec{\nabla}|\psi\rangle$

$C_1 + C_2 = 0 \qquad C_1 - C_2 = 2$

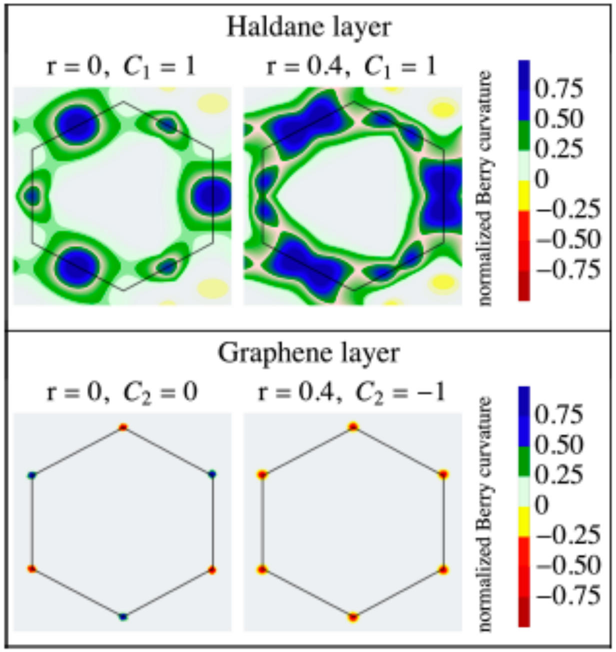
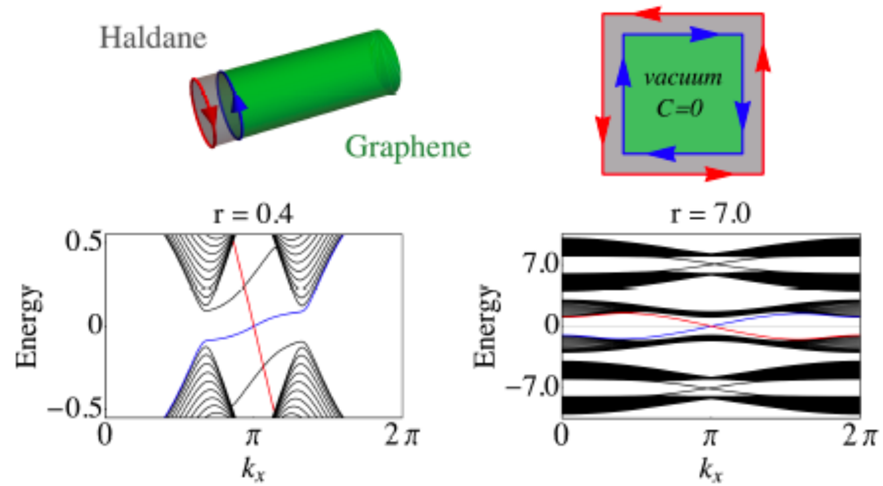
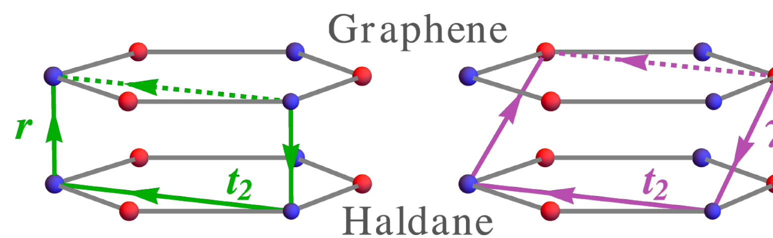


FIG. 1. Berry curvature in the Brillouin zone for the Haldane and graphene layers at $r = 0$ and small r, showing the Berry phase jump effect [35]. Here, $t_1 = 1$ and $t_2 = 1/3$.





Ising Interaction $\tilde{r}\,\sigma_1^z \sigma_2^z$

Peng Cheng, Philipp Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg and Karyn Le Hur, Phys. Rev. B 100, 08110 (R) (2019).

Entanglement From curved space

- for 1 spin-1/2, we find

$$C = A_\varphi(0) - A_\varphi(\pi) = \pm 1, 0$$

- This is also equivalent to

$$C = -\frac{1}{2\pi}\int \vec{\nabla}\times\vec{A} = \frac{1}{2}\left(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle\right)$$

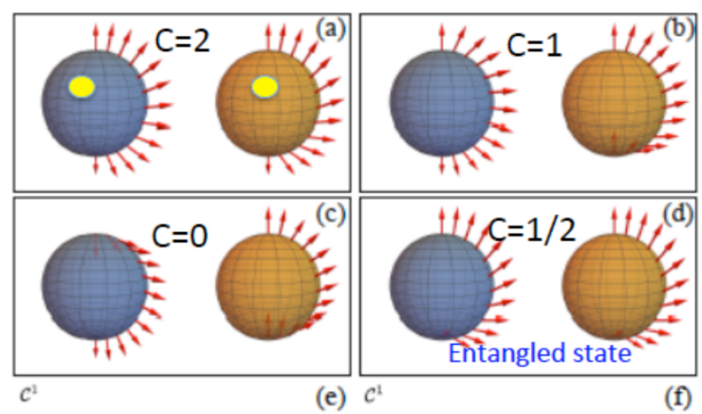
entangled state at 1 pole

$\langle\sigma_z(\pi)\rangle = 0$

$C = 1/2$ possible?

$C = A_\varphi(0) - A_\varphi(\pi)$ possible

model?



Kane-Mele model $\mathcal{H}^-$ with $\tilde{r}=0$

Model

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$\vec{H}_1 = -\vec{H}_2$

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$\mathcal{H}_+$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

$f(\theta) = cst$

$\theta = 0$ $\quad |\uparrow\uparrow\rangle$

$$\begin{aligned}
E_{\uparrow\uparrow} &= -2H - M_1 - M_2 + \tilde{r} f(0) \\
E_{\uparrow\downarrow} &= -M_1 + M_2 - \tilde{r} f(0) \\
E_{\downarrow\uparrow} &= M_1 - M_2 - \tilde{r} f(0) \\
E_{\downarrow\downarrow} &= 2H + M_1 + M_2 + \tilde{r} f(0),
\end{aligned}$$

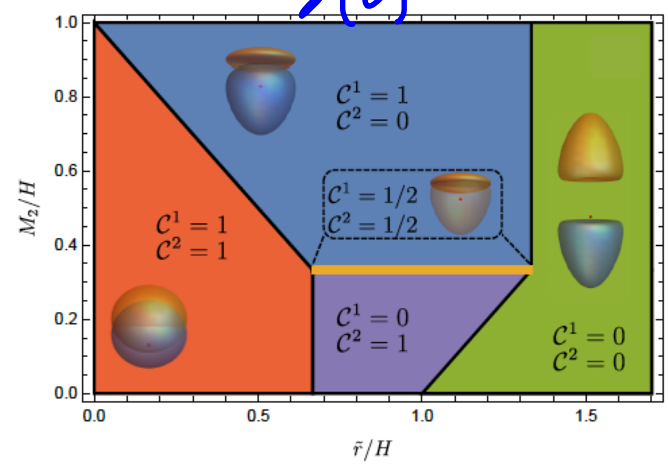
$\theta = \pi$ $\quad \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

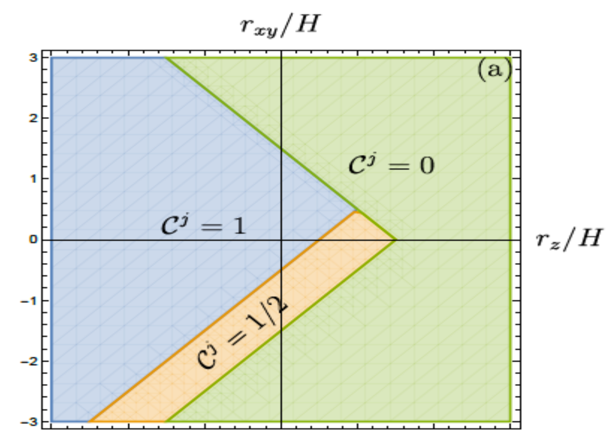
$$\begin{aligned}
E_{\uparrow\uparrow} &= 2H - M_1 - M_2 + \tilde{r} f(\pi) \\
E_{\uparrow\downarrow} &= -M_1 + M_2 - \tilde{r} f(\pi) \\
E_{\downarrow\uparrow} &= M_1 - M_2 - \tilde{r} f(\pi) \\
E_{\downarrow\downarrow} &= -2H + M_1 + M_2 + \tilde{r} f(\pi).
\end{aligned}$$

$M_1 = M_2$

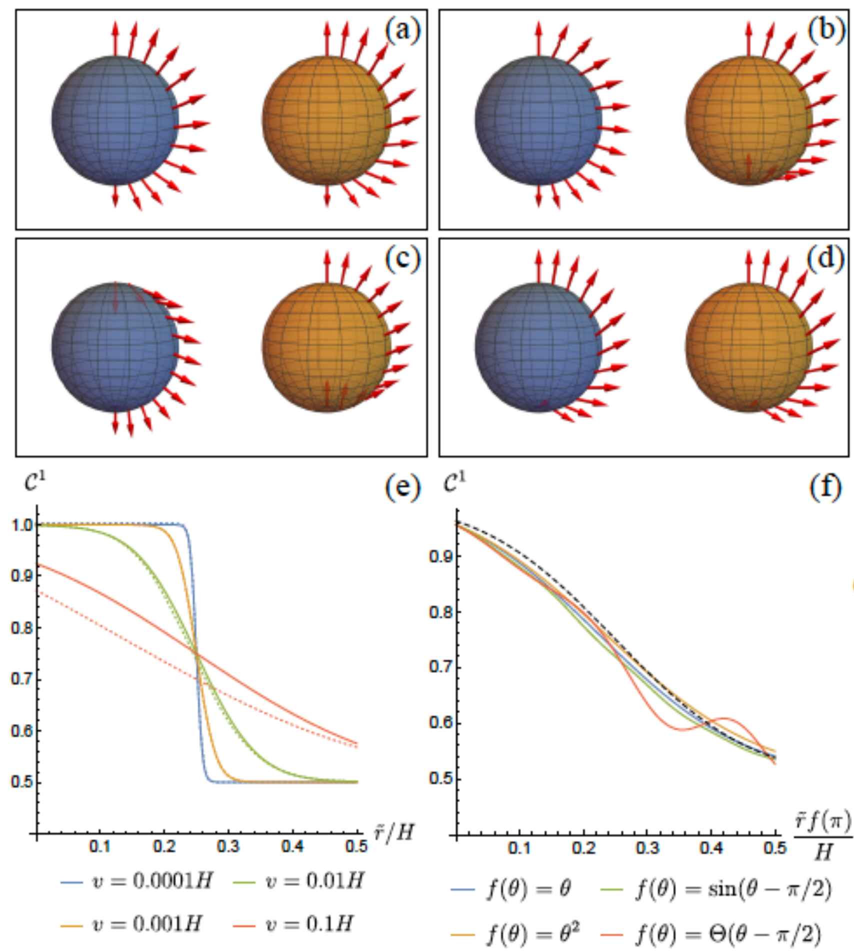
$\mathbb{Z}_2$ symmetry

$$H - M < \tilde{r} < H + M,$$





Time-dependent protocol



dynamics robust to various forms of interactions

$$\mathcal{C}^j \approx \frac{3}{4} + \frac{\pi}{4} \mathrm{Re} \left(e^{i3\pi/4} e^{-\gamma\pi/4} \frac{\mathrm{sgn}(\Delta)\sqrt{\gamma}}{\Gamma(1/2 + i\gamma/4)\Gamma(1 - i\gamma/4)} \right), \tag{117}$$

$$\mathcal{C}^j \to \frac{3}{4} - \frac{1}{4} \mathrm{sgn}(\Delta),$$

$$\mathcal{H}^+_{\text{eff}} \to \sqrt{2} H v t \sigma_z + [\tilde{r} f(\pi) - H + M] \sigma_x.$$

$$\lambda \equiv \sqrt{2} H v, \quad \Delta = \tilde{r} f(\pi) - H + M,$$

Effective Landau-Zener model

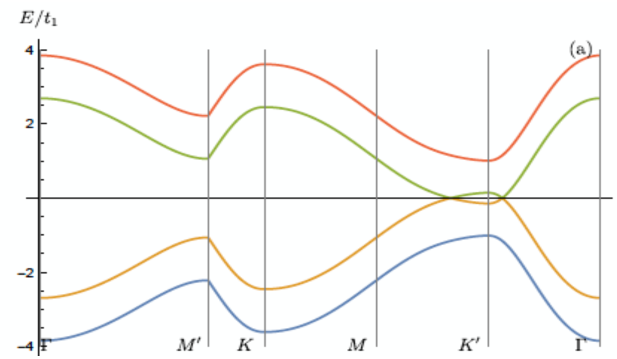
$$t \to t - \pi/v.$$

New bilayer systems

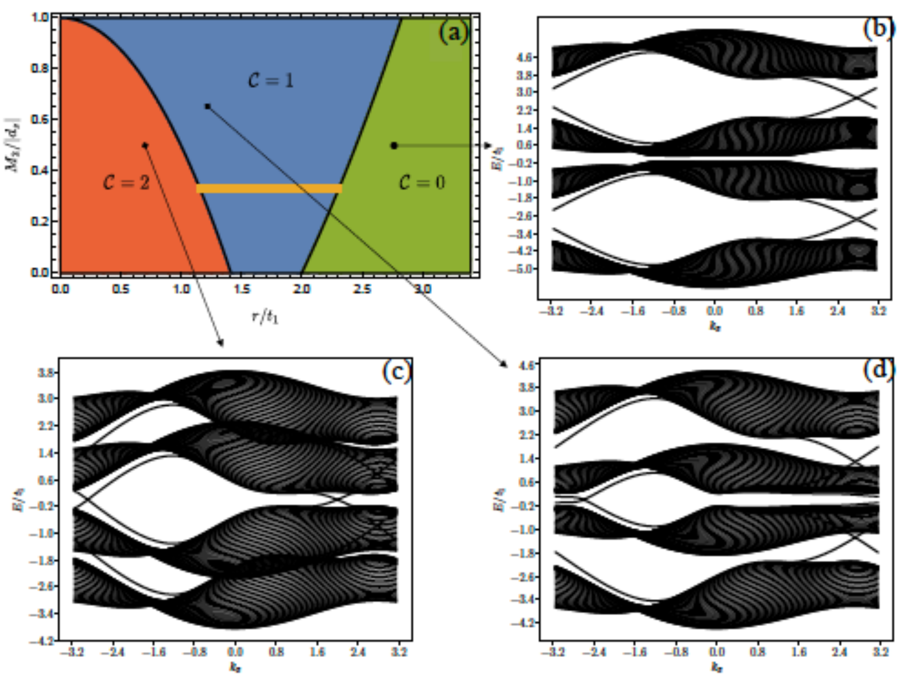
$$\mathcal{H} = (\psi^\dagger_{\boldsymbol{k}1}, \psi^\dagger_{\boldsymbol{k}2})\mathcal{H}(\boldsymbol{k}) \begin{pmatrix} \psi_{\boldsymbol{k}1} \\ \psi_{\boldsymbol{k}2} \end{pmatrix},$$

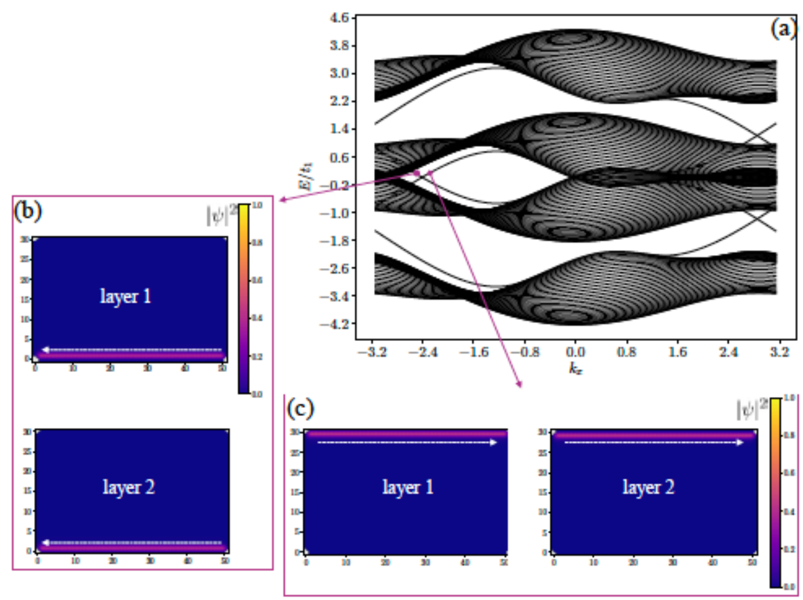
where $\psi^\dagger_{\boldsymbol{k}i} \equiv (c^\dagger_{\boldsymbol{k}Ai}, c^\dagger_{\boldsymbol{k}Bi})$ and

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} (\boldsymbol{d} + M_1\hat{z}) \cdot \boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\boldsymbol{d} + M_2\hat{z}) \cdot \boldsymbol{\sigma} \end{pmatrix},$$



analogy between r & $\tilde{r}$:





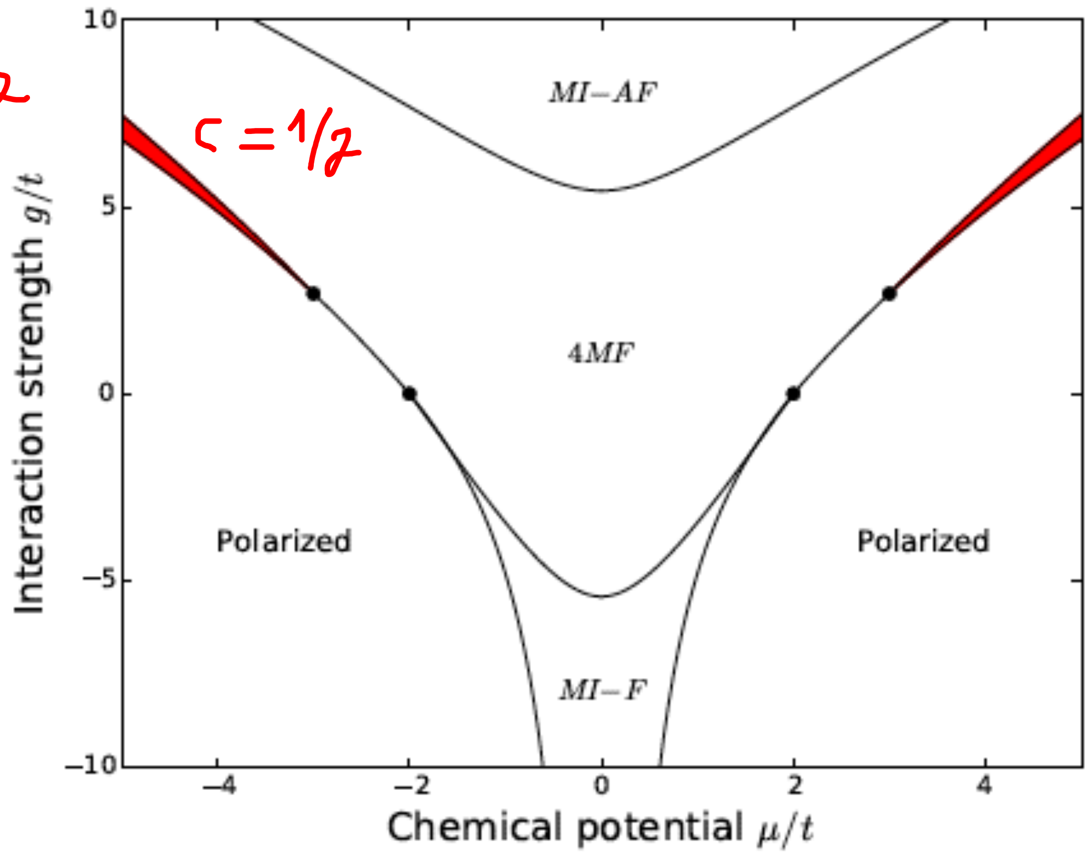
P-wave SC coupled wires and Mott physics

model also related to: + a Coulomb interaction:

$$H_{\text{int}} = g \sum_j (n_{j,1} - \frac{1}{2})(n_{j,2} - \frac{1}{2}),$$

DCI phase double critical Ising phase

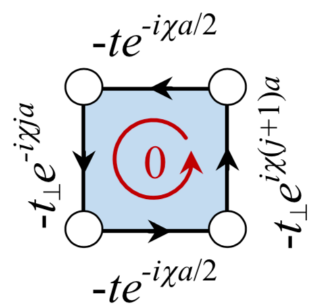
$c = 1/2$



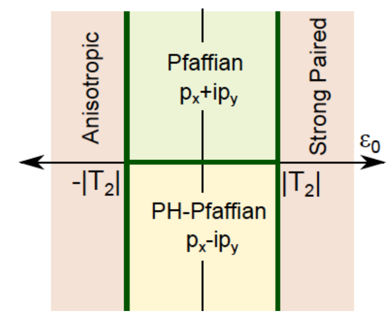
DMRG+
bosonization
+ CFT mappings
+ entanglement probes
(fluctuations, entropy)

L. Herviou, C. Mora, Karyn Le Hur,
arXiv:1601.02998, PRB **93**, 165142 (2016)

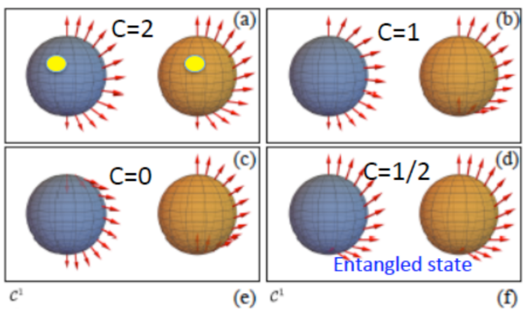
Thanks to students, postdoctoral associates and collaborators



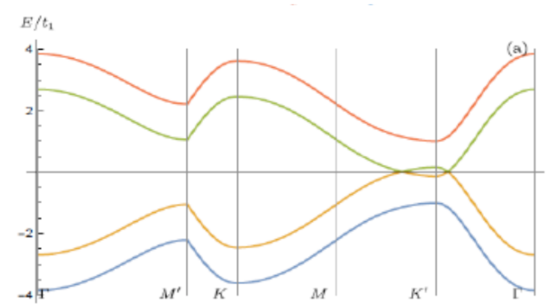
Fan Yang, Vivien Perrin, Alexandru Petrescu, Ion Garate, Karyn Le Hur, Phys. Rev. B 101, 085116 (2020), long article



C. L. Kane, A. Stern, B. I. Halperin, 2017



Joel Hutchinson & Karyn Le Hur arXiv 2002.11823



Thanks for your attention