Haldane Model, Chern Insulators

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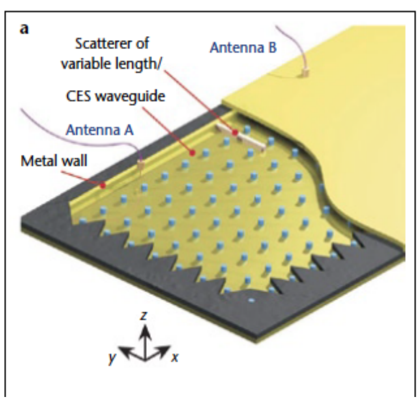
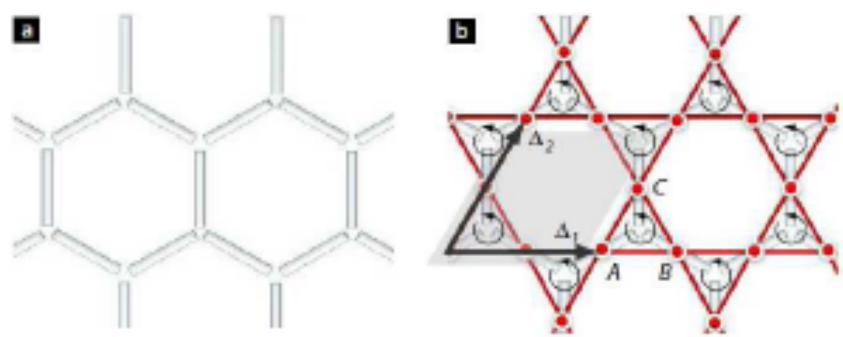
Centre Physique Théorique X and CNRS

Cergy-Pontoise December 18th 2015



(a) A model of the photonic crystal.
The distance between the ferrite rods is 4 cm.



Haldane Model, Chern insulators

Fermions, Bosons

Experimental Realizations: Photons, Atoms, and Materials…

Honeycomb Lattice, Kagome Lattice, analogue Circuit

Interaction Effects

anecdotes

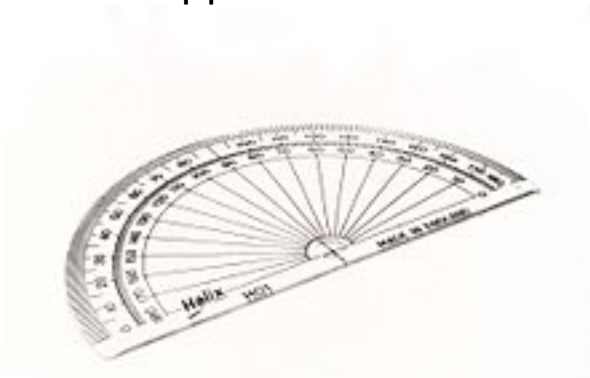
The first discussion with Duncan was in Trieste when I was a PhD student with Bernard Coqblin (1996). I worked on the Kondo model and knew the PhD papers by Duncan on this topic
F. D. M. Haldane, PRL 1978

In Zuerich (2000), with Maurice Rice and Duncan, we discussed ladder systems
(F. D. M. Haldane, bosonization (1981) and spin-1 chain (1983) …)
At that time, Andreas Honecker was also in Zuerich

Over all these years, I have benefitted from nice discussions with Duncan and also from his papers

We have built a connection between entanglement spectrum (Li and Haldane; Regnault-Bernevig) and charge cumulants for the integer quantum Hall effect
A. Petrescu, H. F. Song, S. Rachel, Z. Ristivojevic, C. Flindt, N. Laflorencie, I. Klich, N. Regnault, KLH, JSTAT (2014) P10005

Protractor = Rapporteur in french



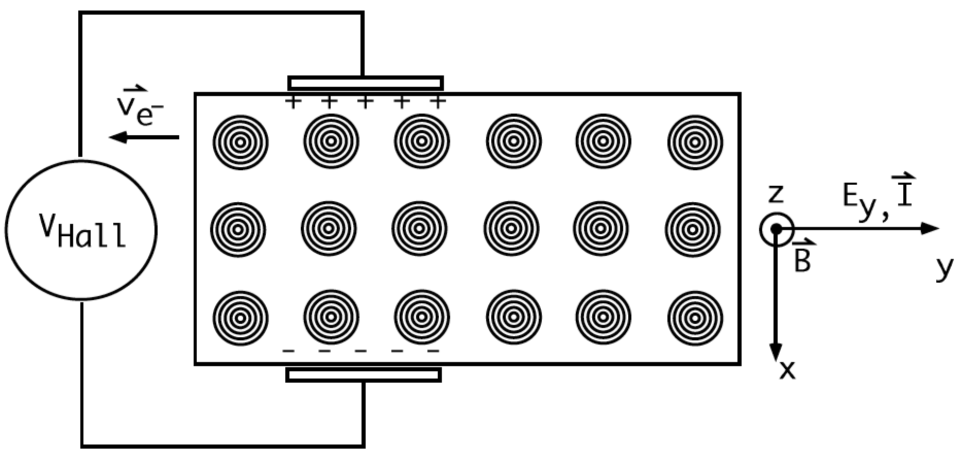
The tour de France in Cargèse was a nice experience (2013)
as well as the english-french traductions, not always rigorous
(Princeton, 2012)

« Centre de guidage » is probably not the correct term

Quantum Hall Effect

Hall, 1878

Von Klitzing Nobel prize 1985



Drude view, classical Hall

$$j_x = \underbrace{\frac{\frac{-\sigma_0^2 B}{ne}}{1 + (\frac{\sigma_0 B}{ne})^2}}_{\sigma_{xy}} E_y$$

$$\text{and} \quad j_y = \underbrace{\frac{\sigma_0}{1 + (\frac{\sigma_0 B}{ne})^2}}_{\sigma_{yy}} E_y \quad .$$

Quantization : $\ell_B = \sqrt{\frac{\hbar}{eB}}$ Lorentz force

levels (Schodinger equation, « deformed » harmonic oscillator in 2D – Centre de guidage)

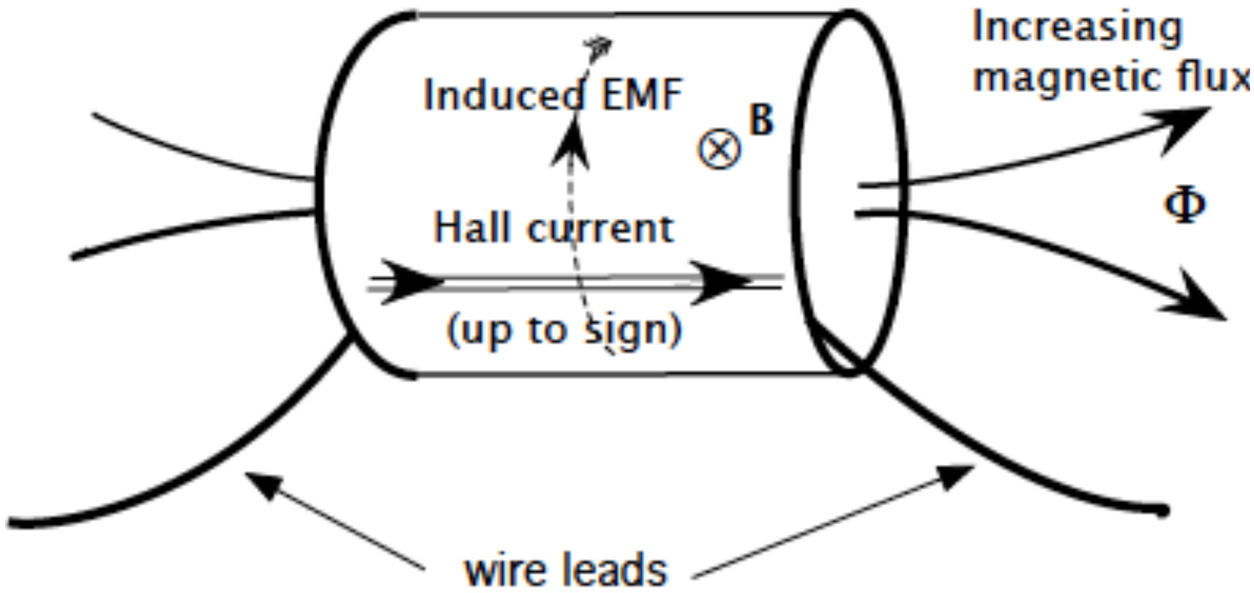
$$nL_xL_y = N\nu_{max} \Longrightarrow n = N\frac{eB}{h},$$

$$\nu_{max} = \frac{L_xL_y}{2\pi\ell_B^2}.$$

$$\sigma_{xy} \to \frac{ne}{B} = N\frac{e^2}{h}$$

N filled Landau levels

Relation with Chern number, edge theories



"By gauge invariance, adding Φ_0 maps the system back to itself, … [which results in] the transfer of n electrons." – R. B. Laughlin

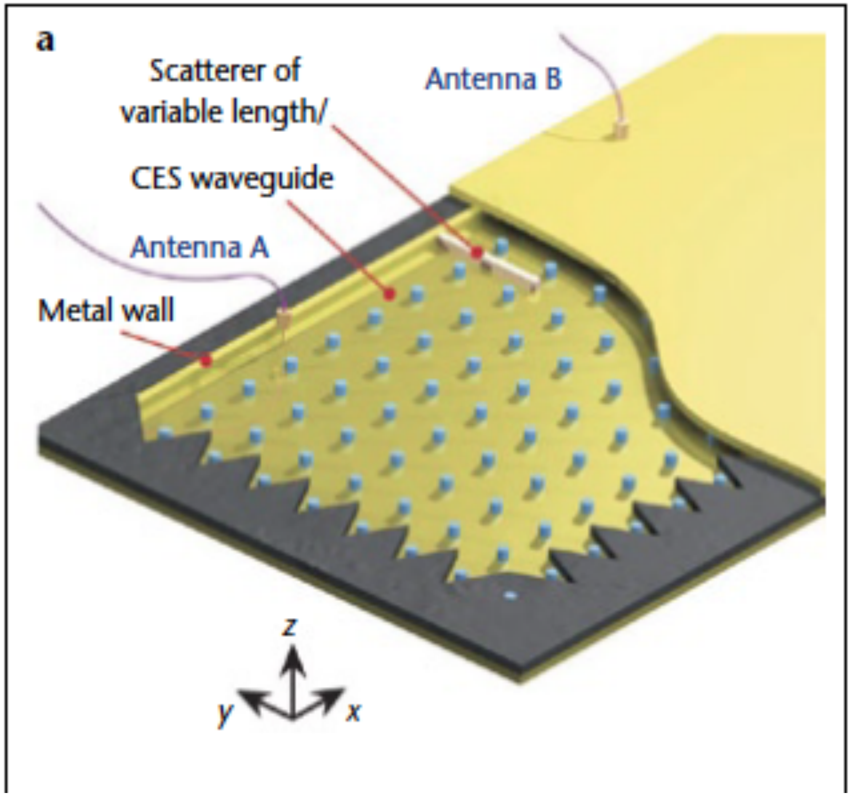
Gauss-Bonnet-Chern $\frac{1}{2\pi}\int_S K\, dA = 2(1-g)$

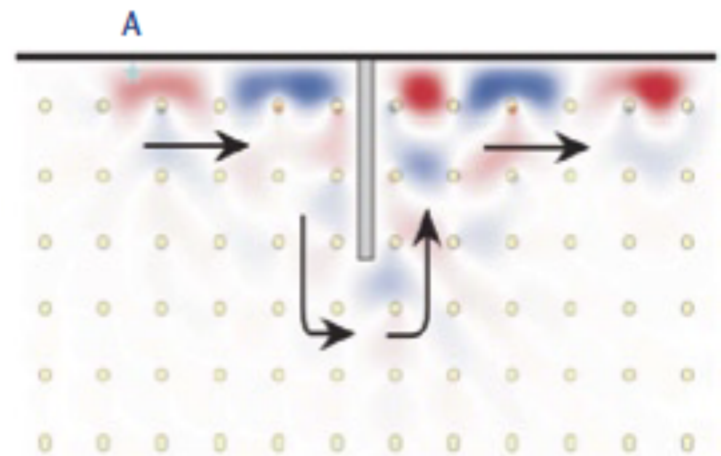
TKNN point of view

$$\sigma_{xy}(m) = \frac{e^2}{2\pi h}\int d\theta_x \int d\theta_y\, 2\,\Im\left\langle \frac{\partial \psi_m}{\partial \theta_x}\middle|\frac{\partial \psi_m}{\partial \theta_y}\right\rangle$$
$$= C_1(m)\frac{e^2}{h}$$

One-Way Road in a Photonic Crystal

Chiral edge states channel light waves in one direction, like electrons in the quantum Hall effect



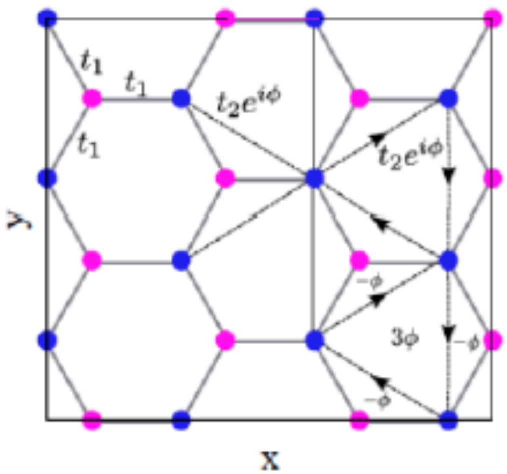


(a) A model of the photonic crystal. The distance between the ferrite rods is 4 cm.

Realizations of AQHE in Photonic crystals: following Haldane & Raghu, PRL 2008 (Dirac points and Faraday effect opens a gap breaking time-reversal symmetry)
Experiment: M. Soljacic et al. Nature **461**, 772 (2009)

Quantum Anomalous Hall Effect

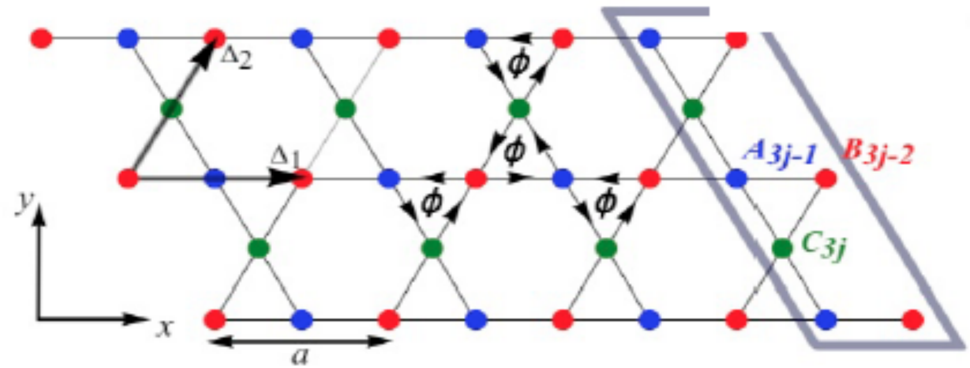
F. D. M. Haldane 1988

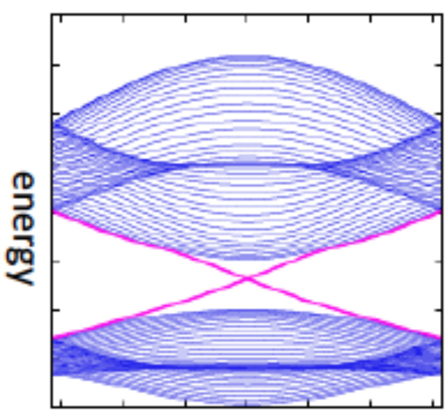


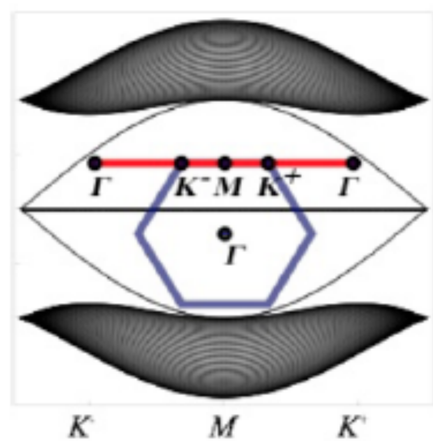
Kagome version:

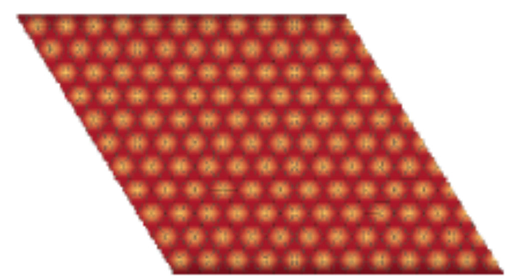
A. Petrescu, A. A. Houck and KLH, 2012

See also J. Koch, A. Houck, KLH, S. Girvin 2010







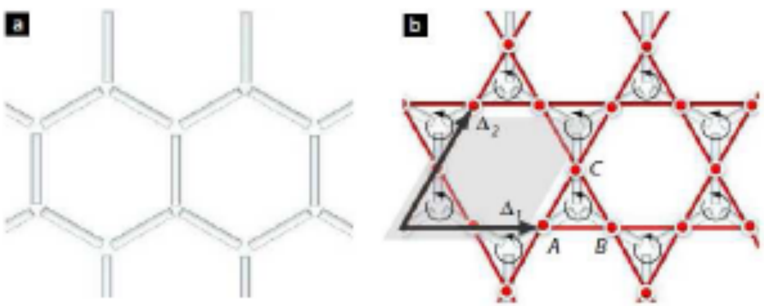


Graphene
+gap

other studies: Julien Vidal, Remi Mosseri, Benoît Douçot

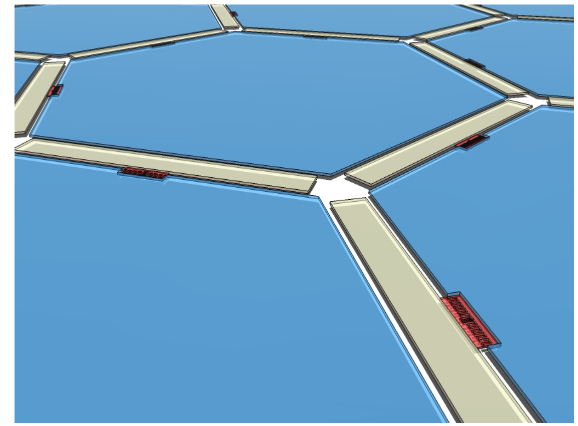
1 connection between lattices

Also in cold atoms (L. Tarruell); polaritons

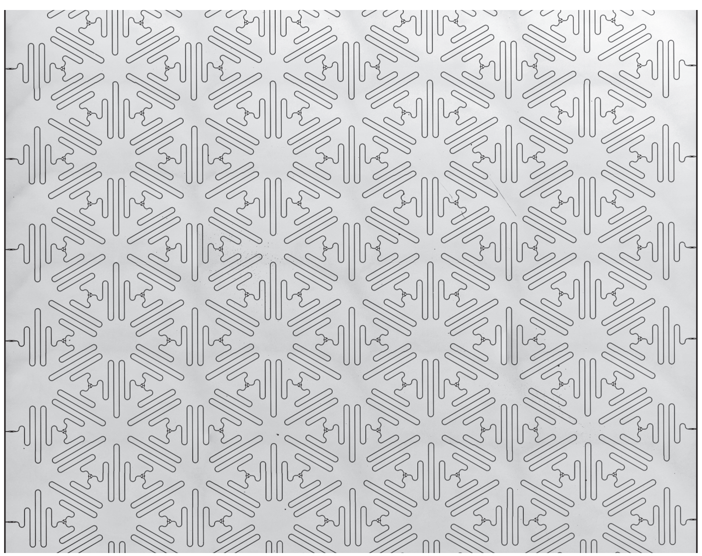


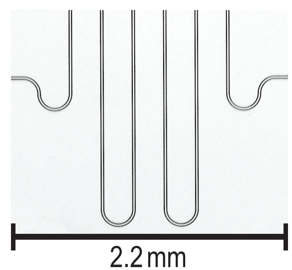
Experiments
Pannetier
Grenoble, 1980'

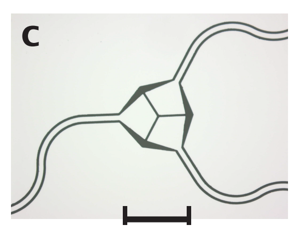
Y. Xiao et al. 1981
Superconducting wires
Aluminium

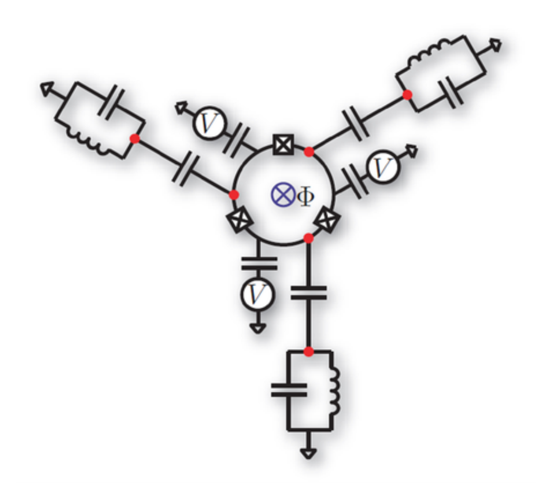


A. Houck lab, Princeton









Superconducting cQED networks (also J. Gabelli; J. Esteve Orsay)
Photon arrays microwave (A. Houck, J. Koch, H. Tureci, Nature Physics)

Progress at Santa Barbara
P. ROUSHAN ET AL. PRIVATE COMMUNICATION (J. MARTINIS)

Cold Atoms:

Jaksch & Zoller 2003

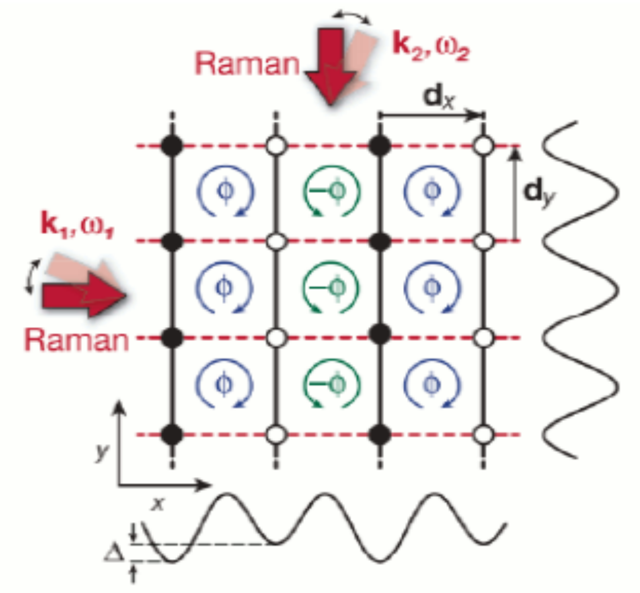
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
I. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- Ways to implement magnetic fields & gauge fields

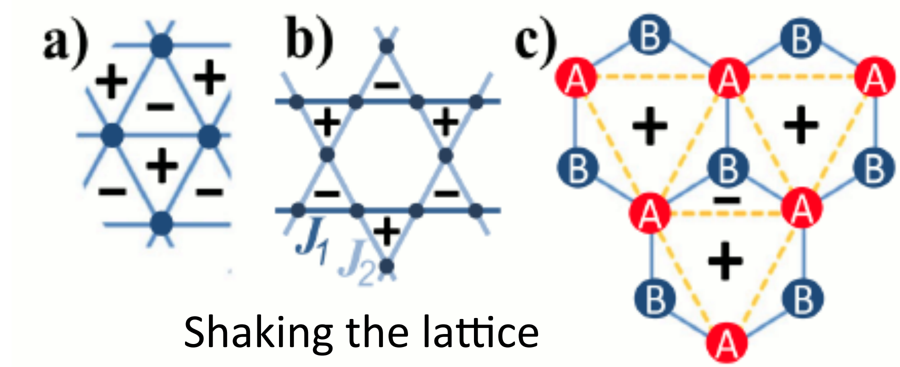
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)



Shaking the lattice

Floquet Topological Insulators:

Reviews: J. Cayssol, B. Dora, F. Simon, R. Moessner, arXiv:1211.5623

N. Goldman, J. Dalibard, PRX 2014

Kagome lattice: why interesting...

Flat band (search for ferromagnetism)

A. Mielke; H. Tasaki; E. Lieb

Exotic Topological Phases: fractional quantum Hall state

E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011

N. Regnault and A. Bernevig, PRB 2012,...

Spin liquid search, classical degeneracies

Experimentally relevant: 2D Materials (Orsay; Princeton;...)

Cold atoms: Berkeley; see D. Stamper-Kurn group, 2011

L. Balents, Nature 464, 199 (2010)

S. Yang, D. Huse and S. White, Science (2011)

Work by Claire Lhuillier and co-authors,...

Philippe Lecheminant, G. Misguish, L. Messio, B. Bernu, Ph. Sindzingre

Haldane model: introduction

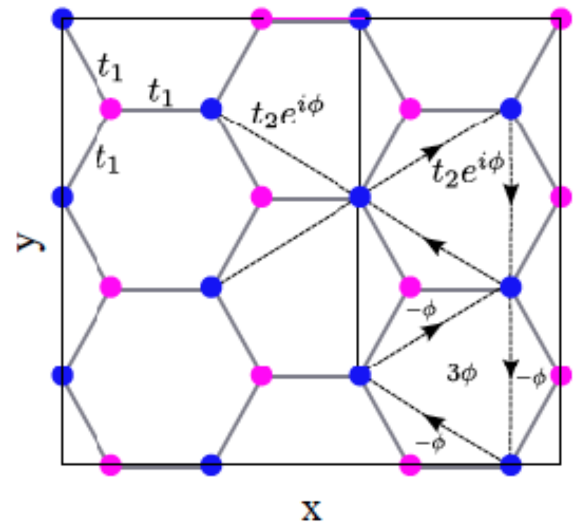
Haldane model

$$\mathcal{H}_0 = \sum_i (-1)^i M c_i^\dagger c_i - \sum_{<i,j>} t_1 c_i^\dagger c_j - \sum_{\ll i,j \gg} t_2 e^{i\phi_{ij}} c_i^\dagger c_j$$

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

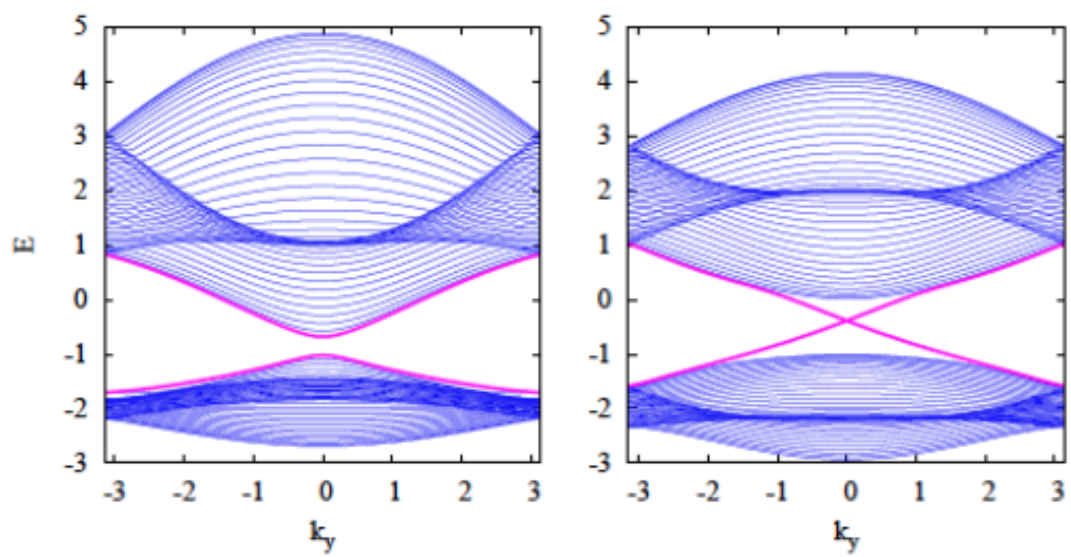
No net flux

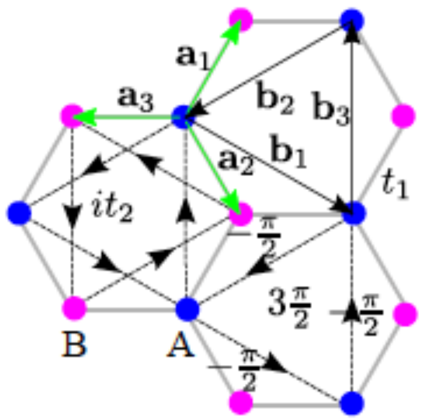
M = Semenoff mass



Spectrum of the non-interacting model

- t_1 only $\Rightarrow$ Dirac cones
- M or t_2 can open the gap
- Non–trivial topological properties if $M < 3\sqrt{3}t_2 \sin\phi$





$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\,(\mathbf{k}) \cdot \hat{\sigma},$$

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\,\mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\,\mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\,\mathbf{b_i} \right).$$

Realized in cold atoms:

Group of T. Esslinger, 2014

arXiv:1406.7874

The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

$$\mathcal{C}_- = \frac{1}{4\pi} \int_{\mathrm{BZ}} d\mathbf{k}\, \hat{\mathbf{d}} \cdot \left(\partial_1 \hat{\mathbf{d}} \times \partial_2 \hat{\mathbf{d}} \right)$$

This is the Chern number of the lower Bloch band, and takes the value $\mathcal{C}_- = 1$. The formula for the upper band is obtained by replacing $\hat{\mathbf{d}}$ by $-\hat{\mathbf{d}}$, and leads to $\mathcal{C}_+ = -1$.

Topology Berry & Chern

Different mathematical equivalent formulations

Chern number of a given band expressed in terms of Berry phase: (curvature)

$$C_n = \nu_n = \iint_{BZ} d^2\vec{k}\,\left[\partial_{\vec{k}} \times \vec{A}_n(\vec{k})\right]$$

TORUS

Berry gauge field associated to a band $\mathcal{A}(k) = -i \int_{\text{unit cell}} d^2\vec{r}\, u^*_{\vec{k}}(\vec{r})\,\partial_{\vec{k}} u_{\vec{k}}(\vec{r})$

Projection on a given band

$$P_\pm(\vec{k}) = |\pm, \vec{k}\rangle\langle \pm, \vec{k}| = \frac{1 \pm \vec{d}\cdot\vec{\sigma}}{2}$$

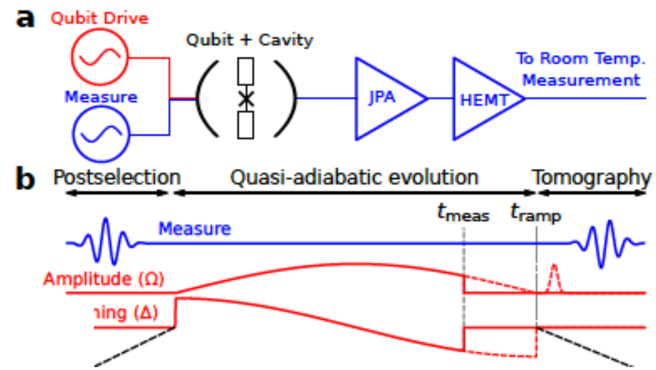
$$C_\pm = -\frac{i}{2\pi} \iint_{BZ} \mathrm{Tr}\left\{ P_\pm(\vec{k}) \left[\partial_{k_1} P_\pm(\vec{k}),\ \partial_{k_2} P_\pm(\vec{k})\right]\right\}$$

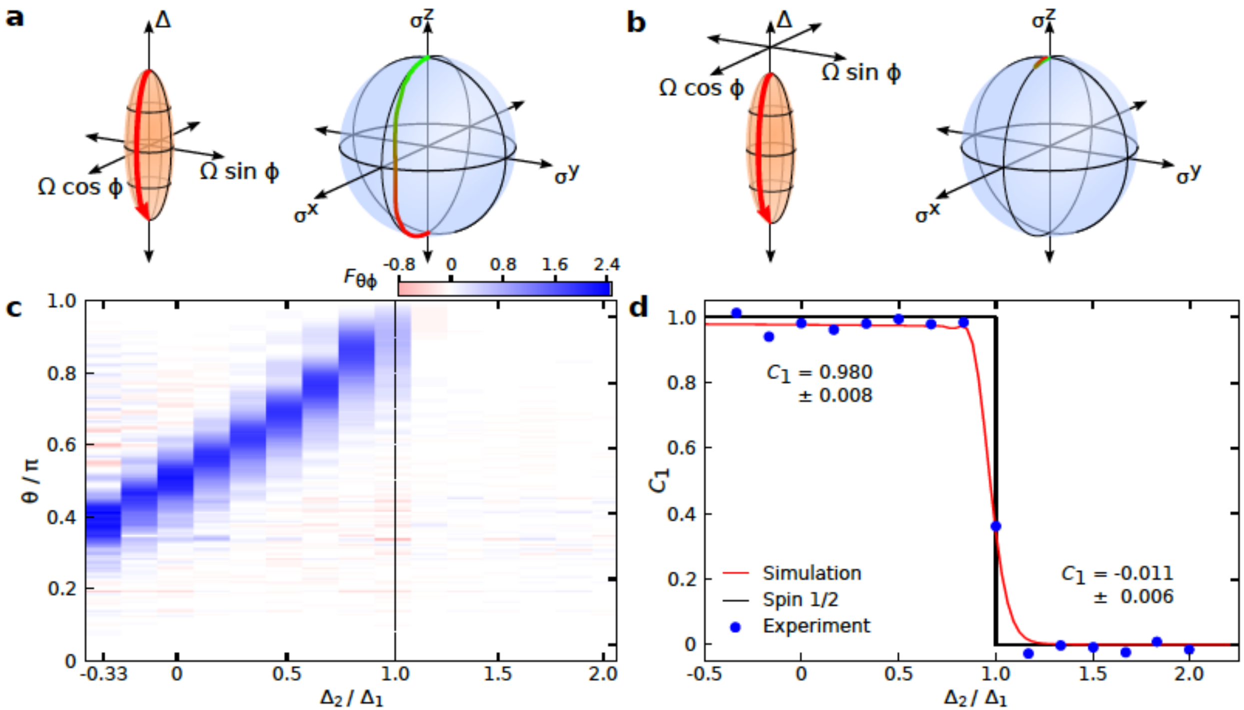
An equivalent formulation is through the Green function

Measurements for spin-1/2 particle
Rotations by driving (sphere)

Konrad Lehnert group (Colorado)
D. Schroer et al. PRL 2014
P. Roushan et al. Nature (John Martinis, Santa Barbara) 2014

$$H/\hbar = \frac{1}{2}\left[\Delta\, \sigma_z + \Omega\, \sigma_x \cos\phi + \Omega\, \sigma_y \sin\phi\right] \;,$$





$$\Delta = \Delta_1 \cos\theta + \Delta_2 \;,\;\; \Omega = \Omega_1 \sin\theta$$

Ramp protocole

$$\theta(t) = \pi t / t_{\rm ramp}$$

$$F_{\theta\phi} = \frac{\langle \partial_\phi H \rangle}{v_\theta} = \frac{\Omega_1 \sin\theta}{2 v_\theta} \langle \sigma^y \rangle,$$

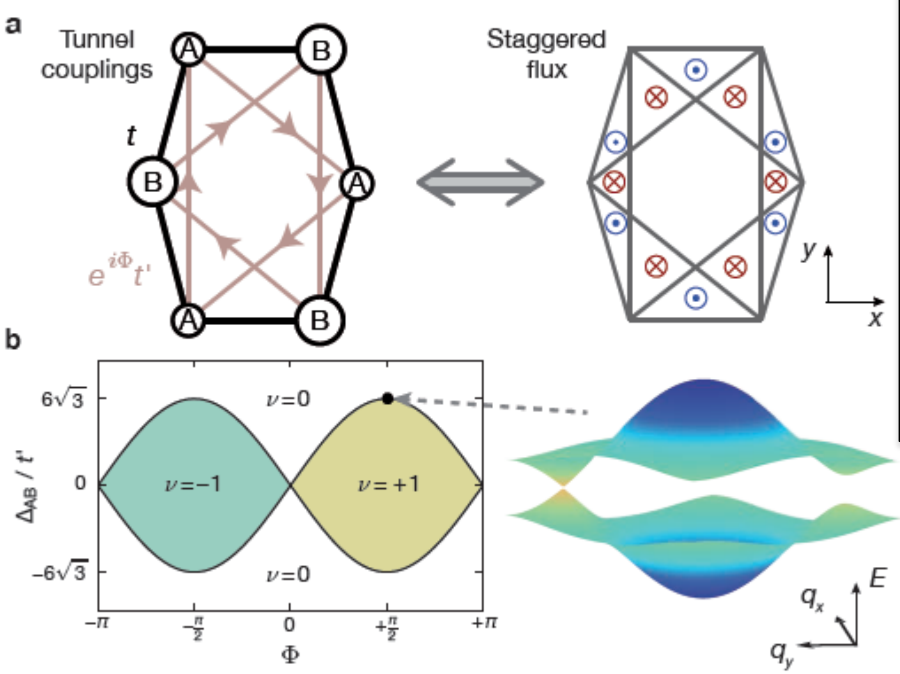
$$C_1 = \int_0^\pi F_{\theta\phi} d\theta \,.$$

Tramp 1micro.s
Theory by Polkovnikov

Stochastic approach + Interaction effects (PhD thesis Loic Henriet (2016))
See also P. P. Orth, A. Imambekov, KLH 2013 and L. Henriet, Z. Ristivojevic, P. P. Orth, KLH PRA 2014

Other Experimental observations

- Ultra-cold atoms – see for example Esslinger's experiment (ETH)
- Ultra-cold atoms: importance of Floquet-type point of view



Rubidium atom

<u>Modulation of optical lattice</u>

$$\mathbf{r}_{\text{lat}} = -A\Big(\cos(\omega t)\mathbf{e}_x + \cos(\omega t - \varphi)\mathbf{e}_y\Big),$$

$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{r}}_{\text{lat}}(t)$$

$$\hat{H}_{\text{lat}}(t) = \sum_{\langle ij \rangle} t_{ij}\hat{c}_i^\dagger \hat{c}_j + \sum_i \left(\boldsymbol{F}(t)\cdot\boldsymbol{r}_i\right)\hat{c}_i^\dagger \hat{c}_i$$

$$\hat{U}(T,t_0) = \mathcal{T}e^{-i\int_{t_0}^{t_0+T}\hat{H}(t)dt} = e^{-iT\hat{H}_{\text{eff}}(t_0)}$$

T : Hamiltonian periodic in time

$$\hat{H}_{eff} = \hat{H}_0 + \hat{H}_{1\omega} + O\left(\frac{1}{\omega^2}\right)$$

Formally $\hat{H}_0 = \hat{H}_{0\omega}$ time-independant Hamiltonian

$$\hat{H}_{1\omega} = \frac{1}{\omega} \sum_{n=-\infty}^{+\infty} \frac{1}{n} \left[\hat{H}_n , \hat{H}_{-n} \right]$$

Key point

$$\boxed{\hat{H}(t) = \sum_{n=-\infty}^{+\infty} \hat{H}_n e^{in\omega t}}$$

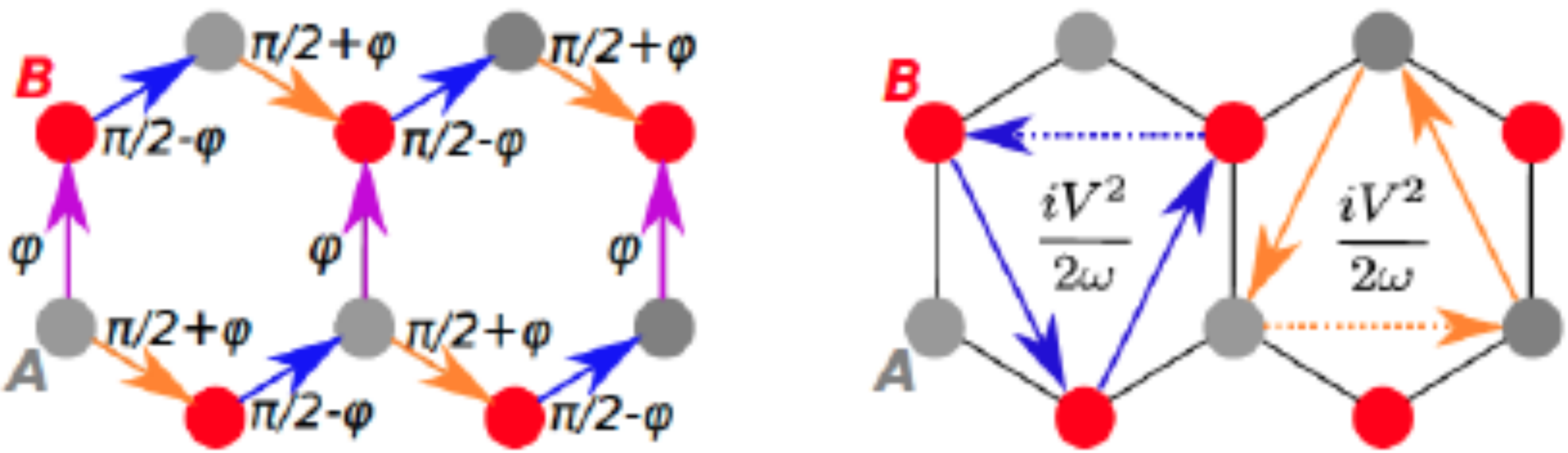
Zürich experiment: after unitary transformation

$$\hat{H}_n = \sum_{\langle ij \rangle} J_n(z_{ij}) e^{in\phi_{ij}} t_{ij} c_i^{\dagger} c_j$$

$\longmapsto$ In our case, t_1 of the form $t_1 (1 + \alpha \cos(\omega t + \varphi_{ij}))(c_i^{\dagger} c_j + h.c.)$

Anisotropic version allowed

Image from KLH, L. Henriet, A. Petrescu, K. Plekhanov, G. Roux and M. Schiro arXiv:1505.00167
To be published, CRAS (review on polaritons, non-equilibrium and topological phases)



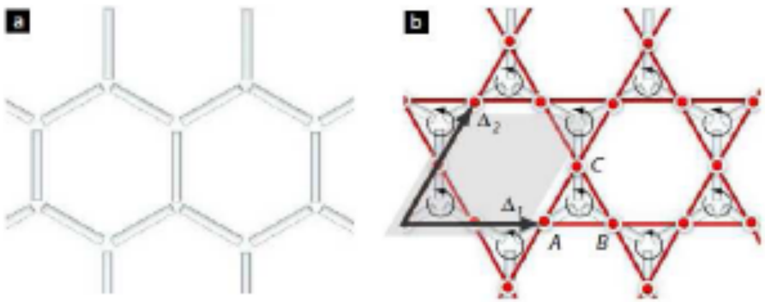
Computation of edge currents and Chern number for free particles

Important point: conservation zero net flux in a unit cell (one can build closed loop with Pi-flux with 4 sites in a unit cell)

Validity of Floquet Hamiltonian + interactions on the honeycomb lattice

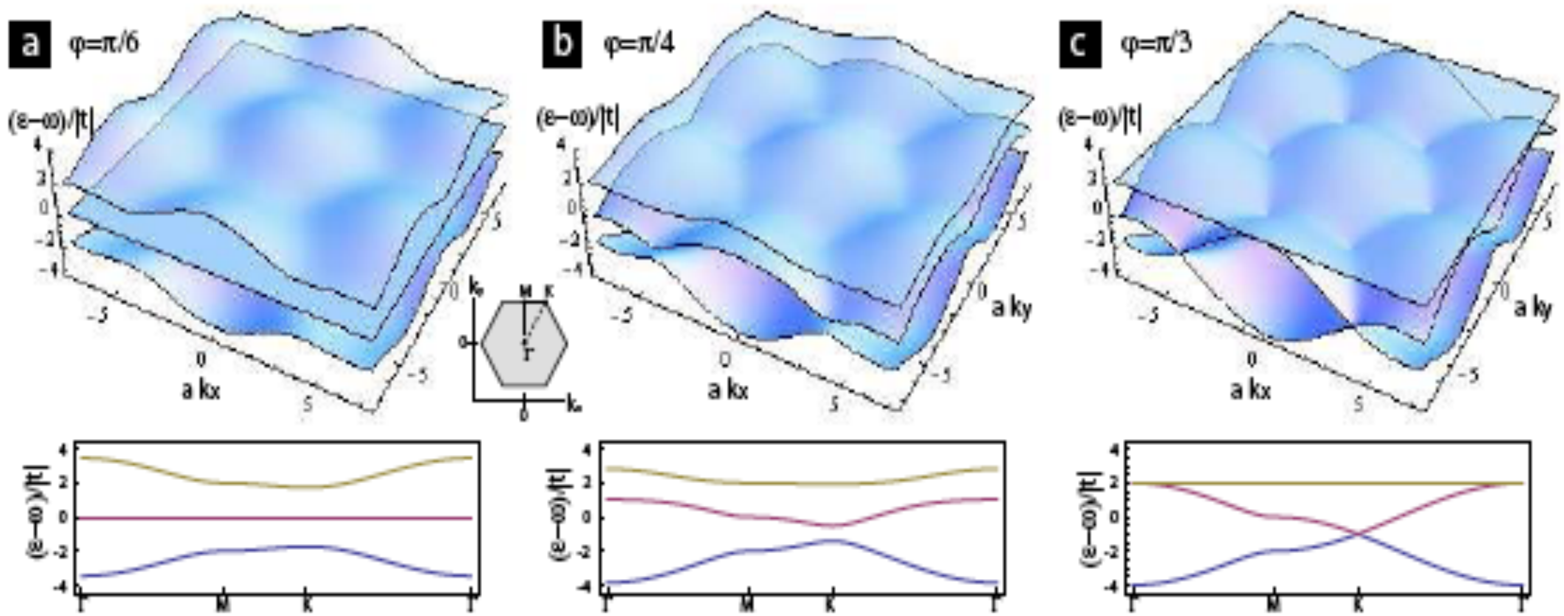
Work in progress with Kirill Plekhanov and Guillaume Roux

Kagome lattice: flat band « fragile »



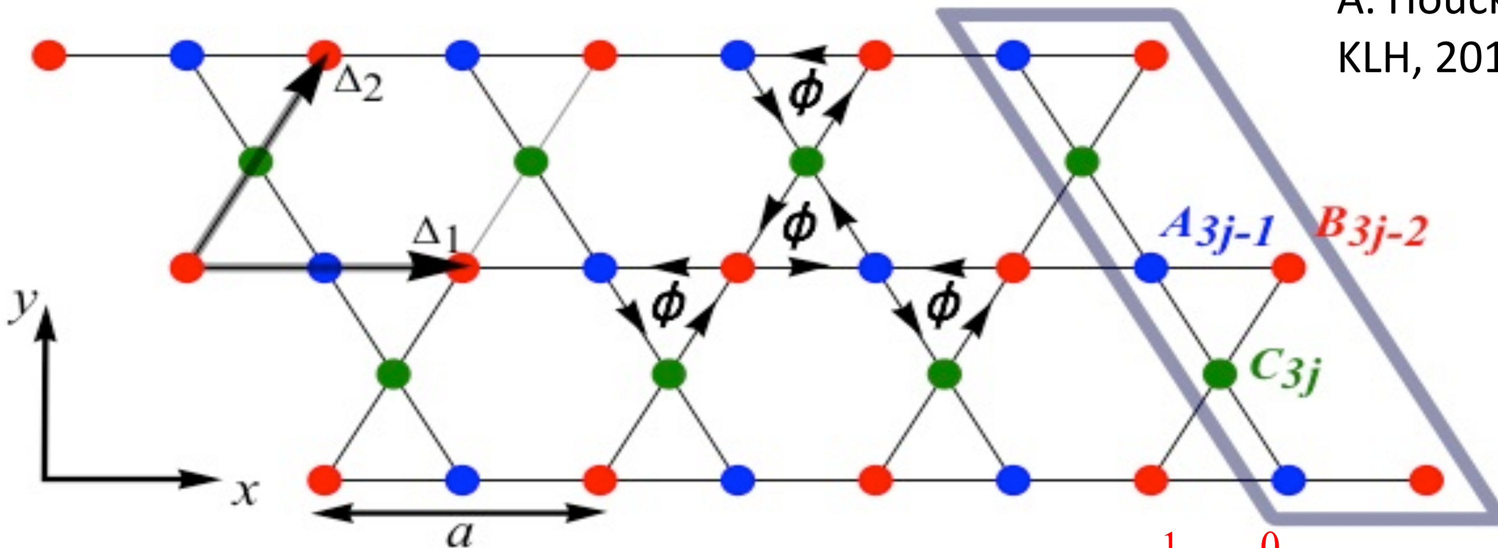
J. Koch, A. Houck, KLH
and S. M. Girvin
PRA **82**, 043811 (2010)

A. Greentree & A. Martin,
Physics 3, **85** (2010)

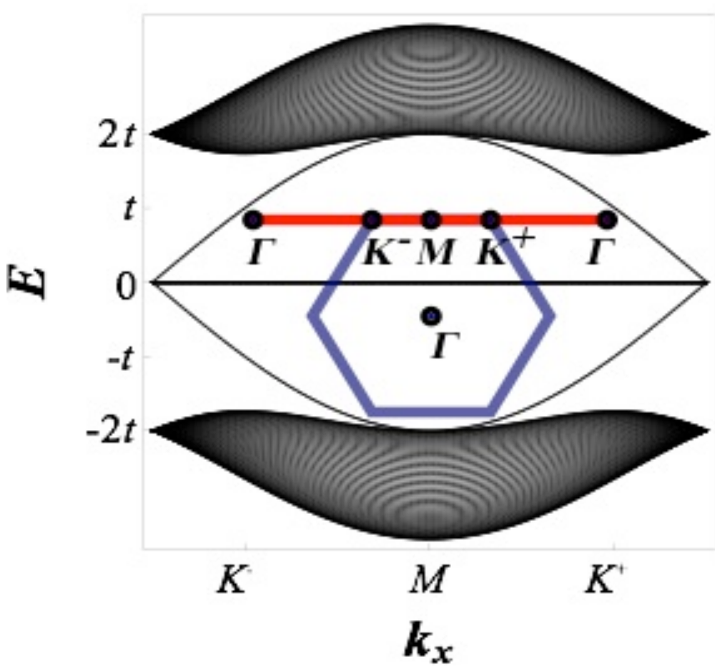


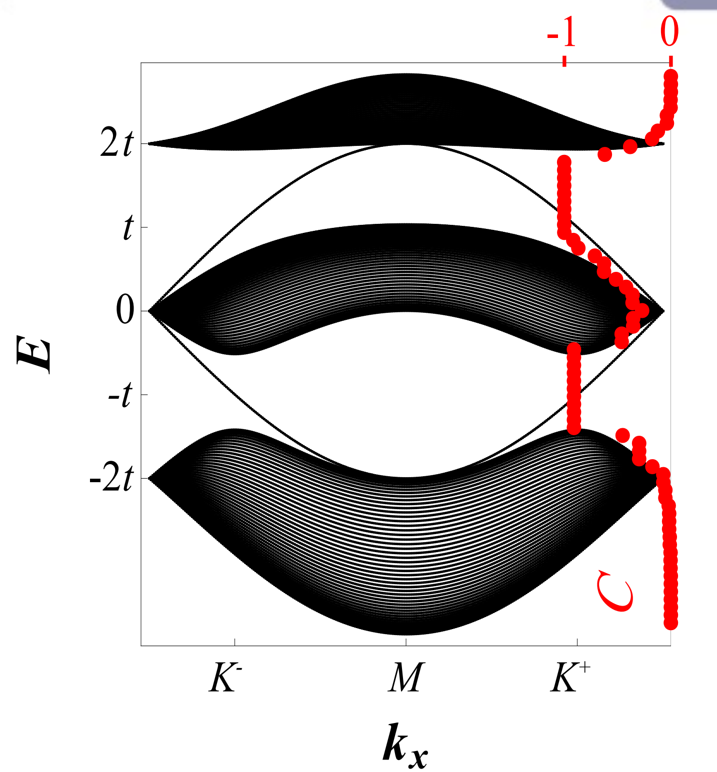
Anomalous Hall Effect

A. Petrescu,
A. Houck &
KLH, 2012



Φ = π/6





Karplus-Luttinger, 1954

D. Haldane, 2004

See also
D. Bergman
& G. Refael, 2010

Φ = π/4

Disorder Effects

J. Bellissard, A. van Elst and H. Schulz-Baldes, arXiv:cond-mat/9411052v1

With disorder, we can re-write $$C(E) = \frac{1}{2\pi} \sum_n \int_{BZ} d^2\vec{k}\, \theta(E - E_n(\vec{k}))\, \partial_{\vec{k}} \times \vec{A}^n(\vec{k})$$

as

$$C(E) = - \lim_{N \mapsto +\infty} \frac{2\pi i}{N} \sum_m \langle \vec{r}_m | P(E) \left[-i[x, P(E)], -i[y, P(E)] \right] | \vec{r}_m \rangle$$

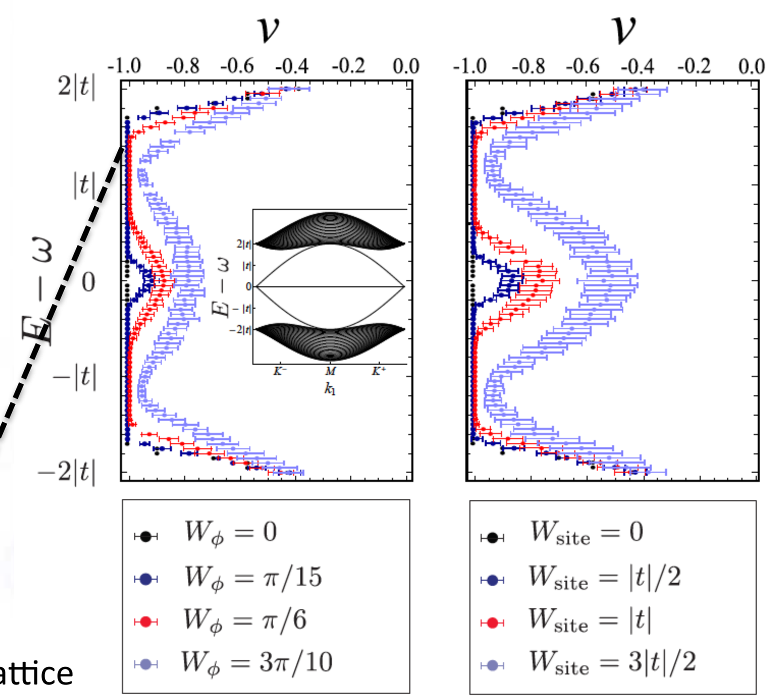
Ensemble average over disorder configurations

$$C(E) = \int d\mu(\delta)\, C_\delta(E)$$

Example Kagome lattice

$$\phi = \frac{\pi}{6}$$

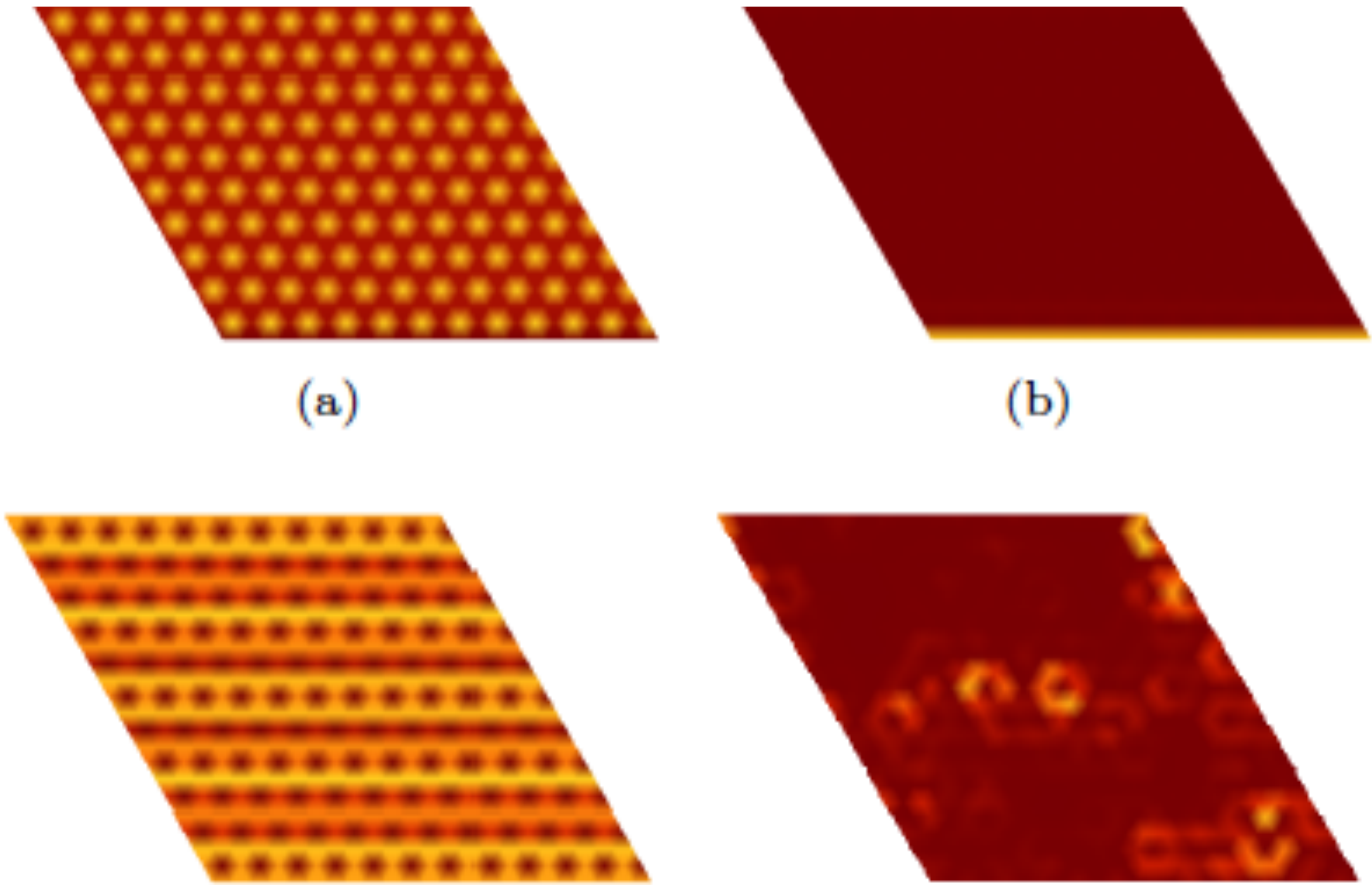
Similar to Honeycomb lattice stability



Uncorrelated white noise
Random number between –W;W

Visible in local density of states

A. Petrescu,
A. Houck &
KLH, 2012

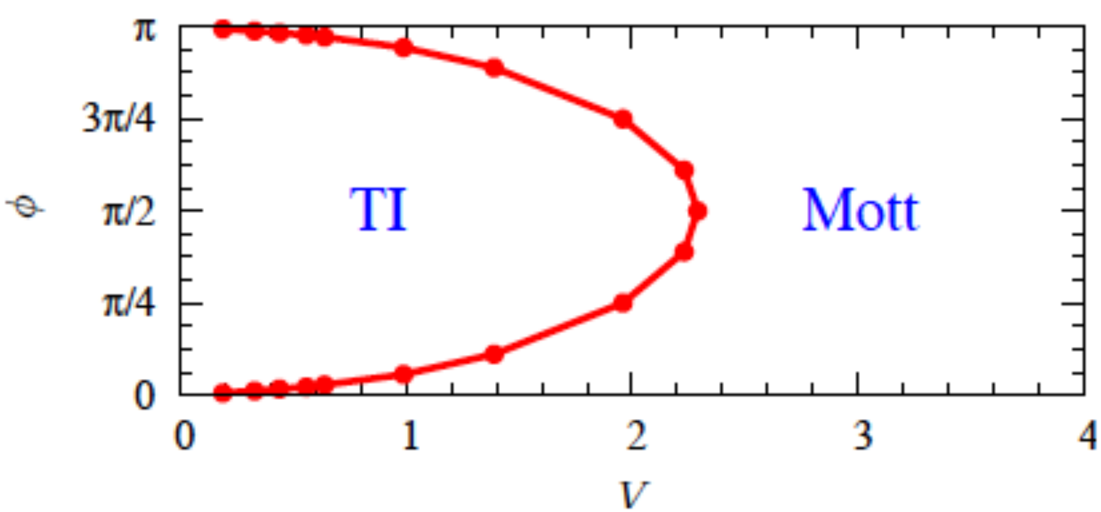


Honeycomb lattice: stability towards disorder

See, for example, E. V. Castro, M. Pilar Lopez-Sancho, M. Vozmediano, 2015

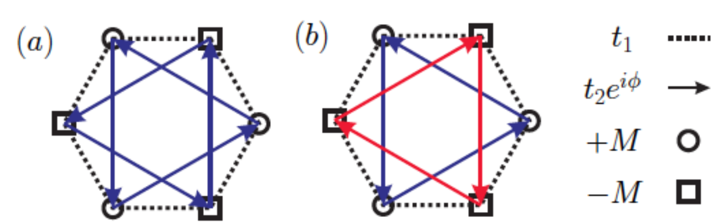
Role of interactions (fermions)

- Stability of the QAH phase (mean-field: interactions give a chemical potential). Of course, one can make the argument more rigorous. Quantum-Field Theory techniques and numerics.



Analytics + exact diagonalisation (Lanczos)

$$H_{\text{int}} = V \sum_{\langle i\,j \rangle} n_i n_j$$



Expected: large interactions
Half-filling localize charges

C. Varney et al. 2010 (Maryland)

Engineering topology with interactions

- Interactions can also mediate a topological phase

(S. Raghu, X.-L. Qi, C. Honerkamp, S.-C. Zhang, 2008 – mean-field
A. C. Grushin et al. 2013 – PRB 87, 085136 (2013))

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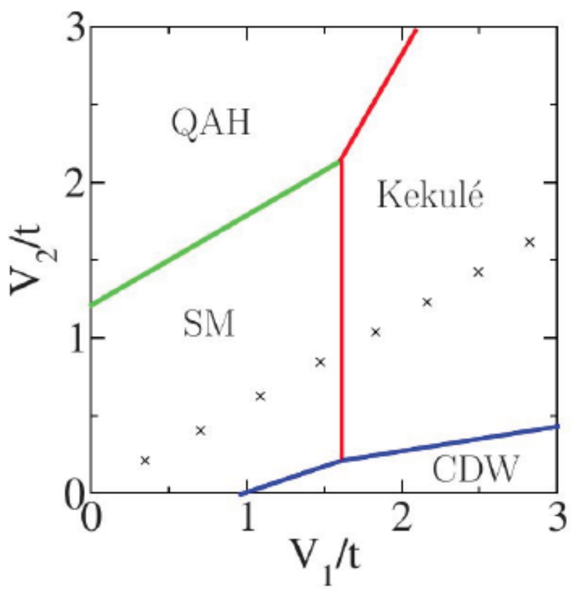
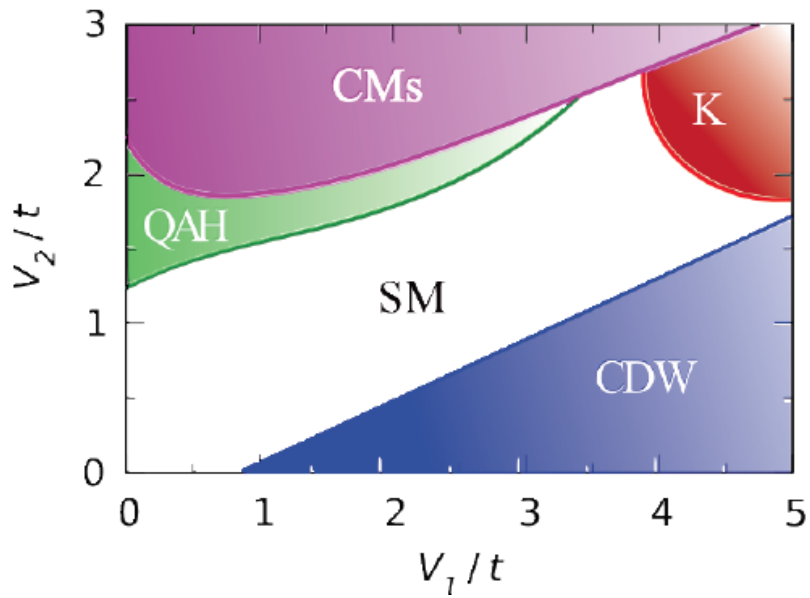




Numerical results contredict mean-field results so far (difficult to frustrate CDW and Kekule with V1>0 and V2>0 only) – Exact Diagonalization studies:

- Maria Daghofer and Martin Hohenadler, PRB **89** 035103 (2014)
- Sylvain Capponi and A. Lauechli, PRB **92** 085146 (2015)
- J. Motruk et al. PRB **92** 085147 (2015).

Realizing Topological Mott Insulators from RKKY Interaction

Tianhan Liu, Benoît Douçot, KLH arXiv:1409.6237 (more complete analysis now, **Green function**)

Analogy to KONDO lattices:

2 types of particles

- **Fast**
- **Slow**

$$H = -t_f \sum_{\langle i,j \rangle} f^\dagger_{i\sigma} f_{j\sigma} + \mu_f \sum_{j,\sigma=\uparrow,\downarrow} f^\dagger_{j\sigma} f_{j\sigma} + U_f \sum_i f^\dagger_{i\uparrow} f_{i\uparrow} f^\dagger_{i\downarrow} f_{i\downarrow}$$

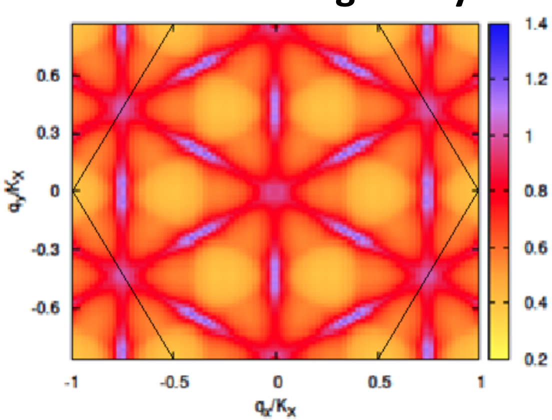
$$- t_c \sum_{\langle i,j \rangle} c^\dagger_i c_j + \mu_c \sum_j c^\dagger_j c_j + \sum_{j,\sigma=\uparrow,\downarrow} g_{cf} f^\dagger_{j\sigma} f_{j\sigma} c^\dagger_j c_j.$$

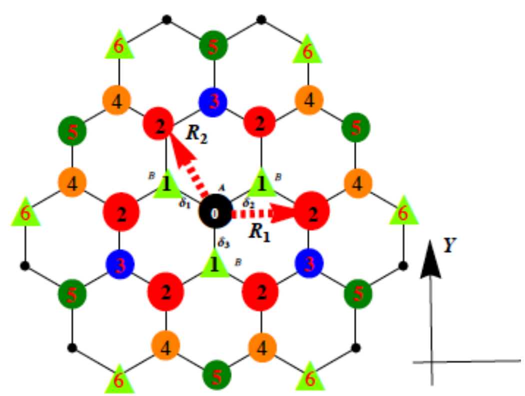
Topological Kondo lattices; see M. Dzero, K. Sun, V. Galitski, P. Coleman

$$H_{int}(\mathbf{r}_{iI} - \mathbf{r}_{jJ}, \mu_c) = \sum_{\mathbf{p},\mathbf{k},i,j,\sigma,\sigma'} \frac{g^2_{cf}\{f[\epsilon_c(\mathbf{p})] - f[\epsilon_c(\mathbf{p}-\mathbf{k})]\}}{N^2(\epsilon_c(\mathbf{p}) - \epsilon_c(\mathbf{p}-\mathbf{k}) + i\eta)} e^{i(\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j))} \alpha_{IJ}(\mathbf{k}) f^\dagger_{iI\sigma} f_{iI\sigma} f^\dagger_{jJ\sigma'} f_{jJ\sigma'},$$

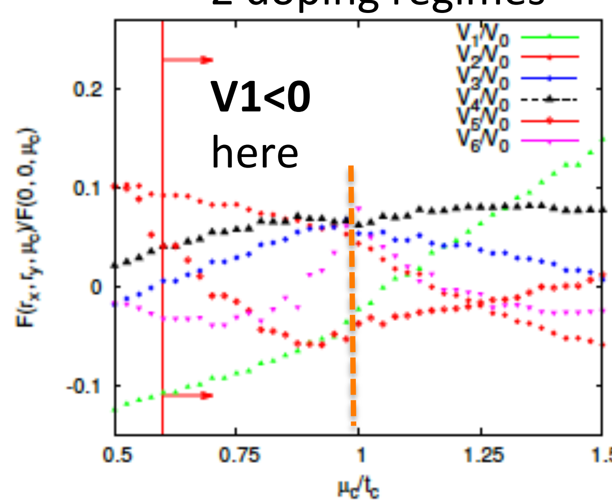
$$F(r_x, r_y) = \sum_{\mathbf{p},\mathbf{k}} \frac{t_c\{f[\epsilon_c(\mathbf{p})] - f[\epsilon_c(\mathbf{p}-\mathbf{k})]\}}{N^2(\epsilon_c(\mathbf{p}) - \epsilon_c(\mathbf{p}-\mathbf{k}) + i\eta)} e^{i(k_x r_x + k_y r_y)} \alpha_{IJ}(\mathbf{k}).$$

Van Hove Singularity





2 doping regimes



Chiral Bosonic Phases on the Haldane Honeycomb Lattice

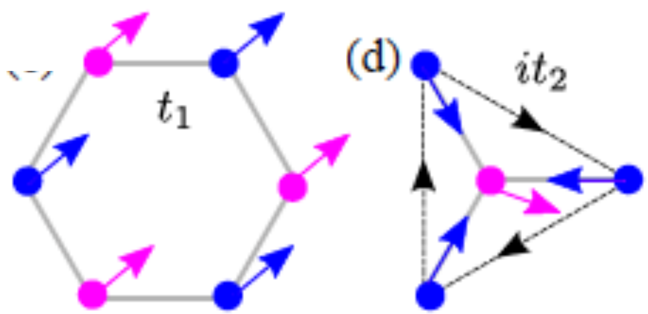
I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB 2015)

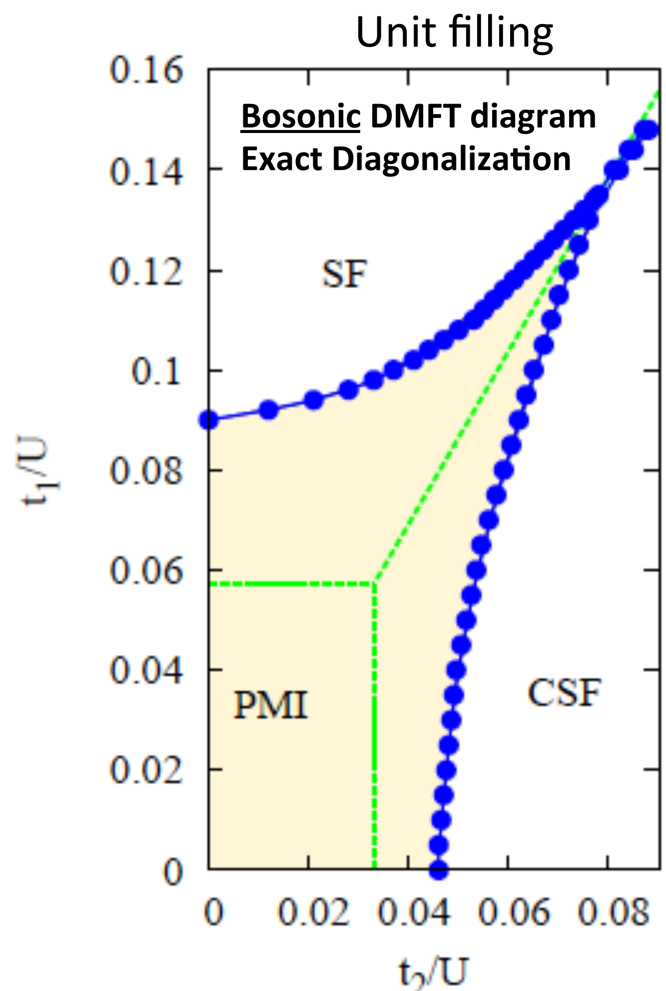
$$\mathcal{H} = \mathcal{H}_{\mathrm{H}} + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu \sum_i \hat{n}_i,$$

Phase-angle variables $b_i^\dagger = \sqrt{n} e^{i\theta_i}$

chiral SF:
nonuniform phase,
plaquette currents

SF:
uniform phase,
"Meissner current"

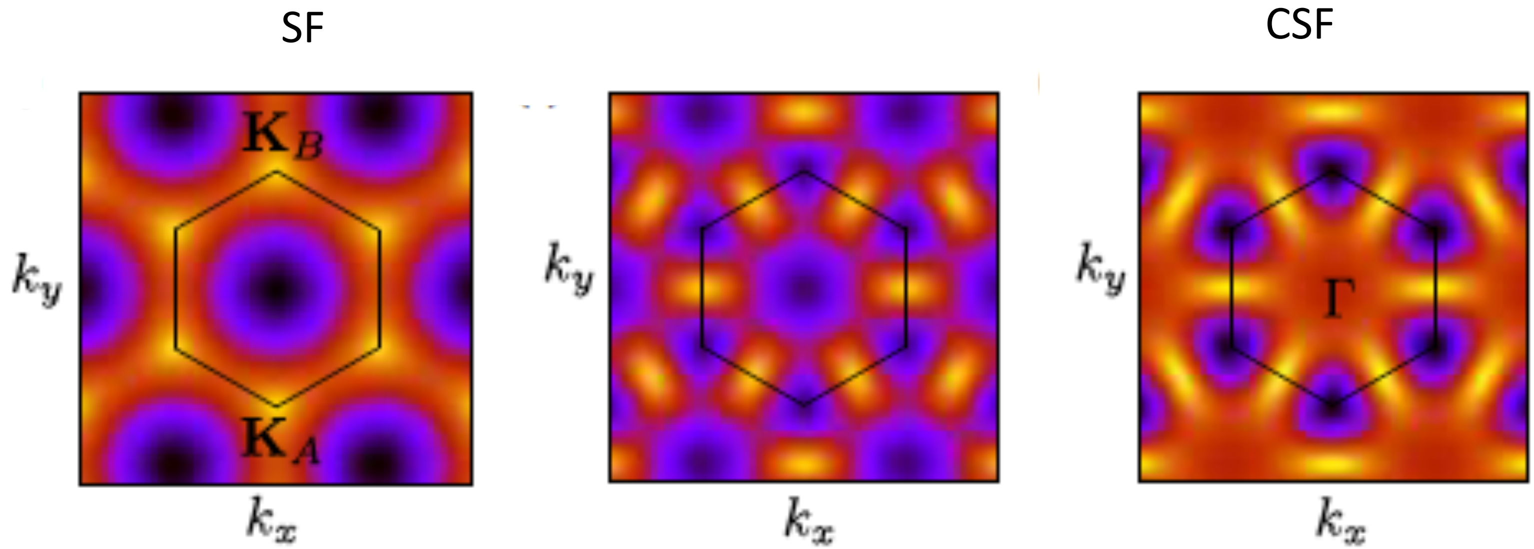




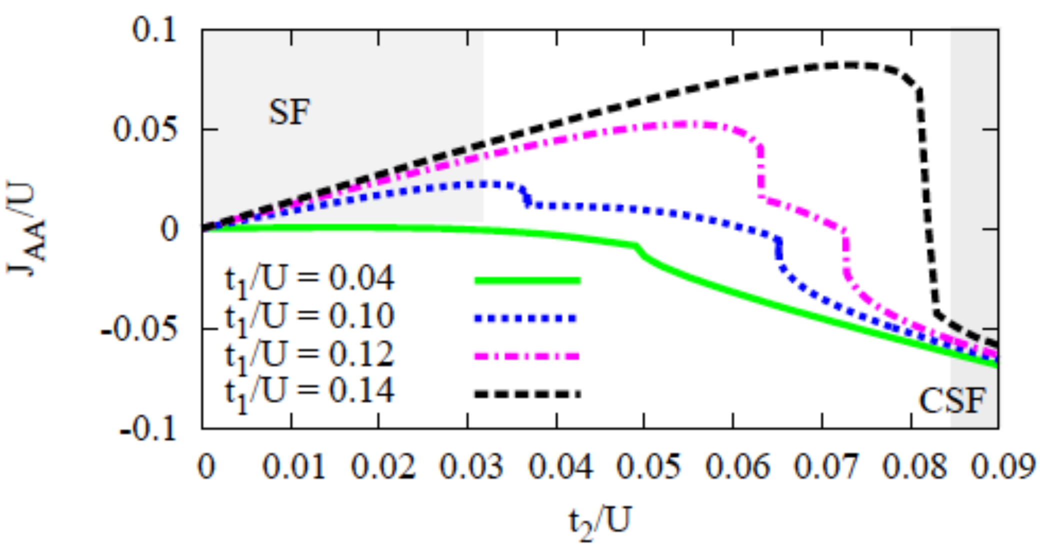
Similar models on square lattice:
L. K. Lim, C. M. Smith and A. Hemmerich,
Phys. Rev. Lett. 100, 130402 (2008) and PRA 2010

Condensation of Bosons

I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB 2015)



$$J_{AA}^{SF} = -2\,n\,t_2\,\mathrm{Im}\exp(-i\pi/2) = 2nt_2$$



$$J_{AA}^{CSF} = -2\ \mathrm{Im}\left(t_2 e^{i\phi}\left\langle \hat{b}_{Ai}^{\dagger}\hat{b}_{Aj}\right\rangle\right)$$
$$= -2t_2 n \sin\left[\phi - \mathbf{K}_A\cdot(\mathbf{r}_i - \mathbf{r}_j)\right] = -nt_2$$

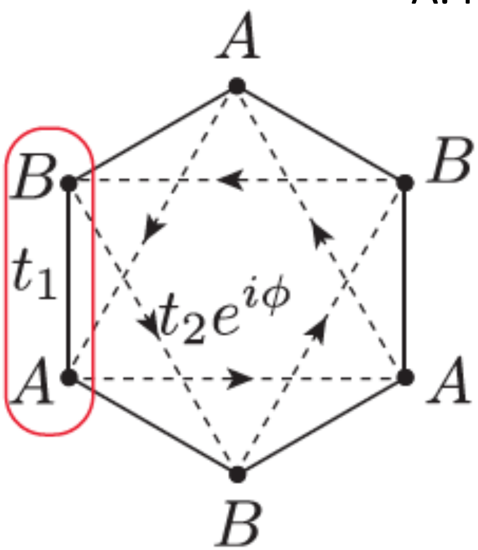
FFLO analogue in Heisenberg-Kitaev doped models
Tianhan Liu, Cécile Repellin, Benoît Douçot, Nicolas Regnault
Karyn Le Hur, submitted to PRL

Excitations in the Mott Phase

Open question: quantitative estimation of lifetime (DMFT & SC-perturb. Theory by hand so far)

Strong coupling perturbation theory

A. Petrescu & KLH

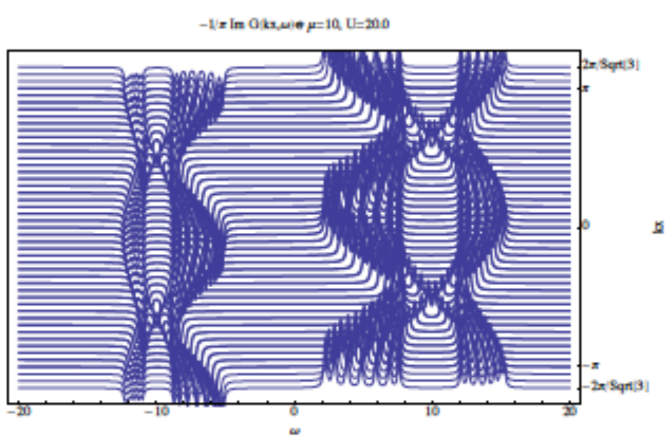


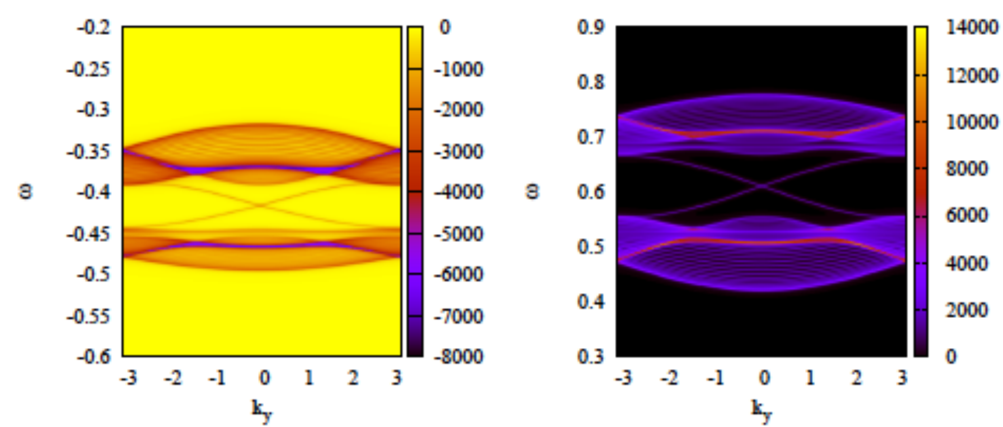
$$G^{-1}(i\omega,k)=g^{-1}(i\omega)-h_k.$$

$g(i\omega)$ = local cluster Green's function

$G(i\omega,k)$ = approximate Green's function

DMFT (I. Vasic & W. Hofstetter)

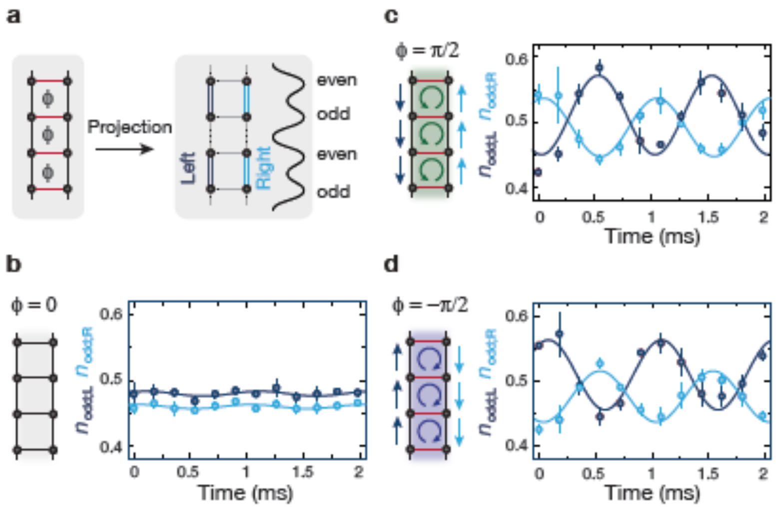




Observation of the Meissner effect with ultracold atoms in bosonic ladders

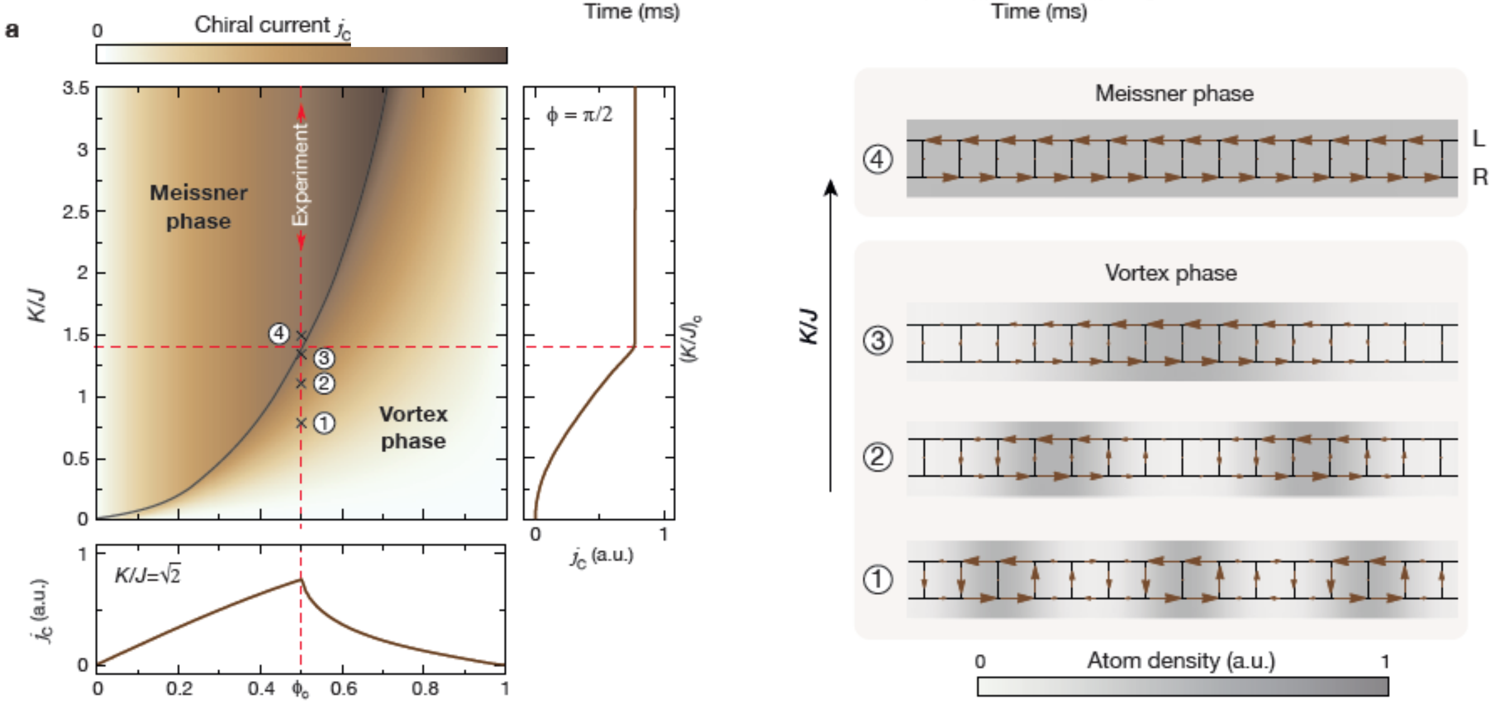
M. Atala[1,2], M. Aidelsburger[1,2], M. Lohse[1,2], J. T. Barreiro[1,2], B. Paredes[3] & I. Bloch[1,2]

Nature Physics 2014



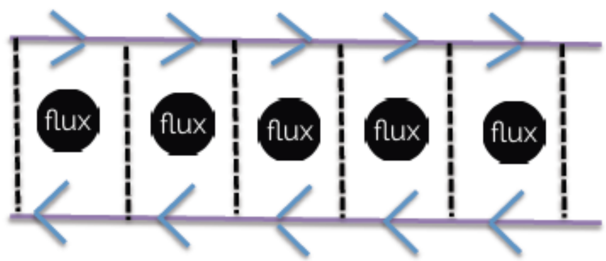
Original Theory by E. Orignac & T. Giamarchi 2001

No Mott physics here



Ladder phases, Interactions

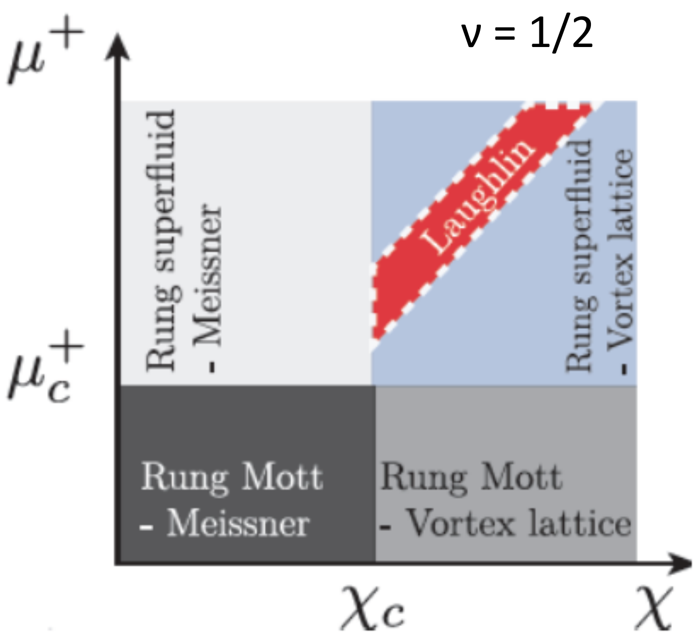
Bosonization (Haldane 1981)

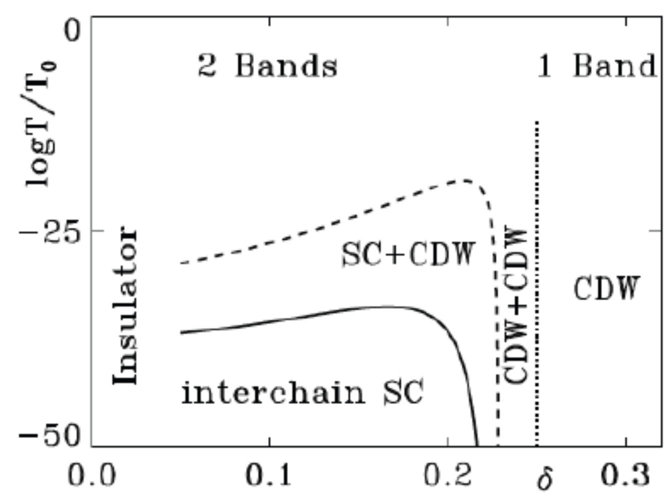


2 coupled Kitaev superconductors or coupled quantum Ising chains

L. Herviou, C. Mora and KLH, to appear
Novel c=1 phase

ν = 1/2





U. Ledermann & KLH, 1999

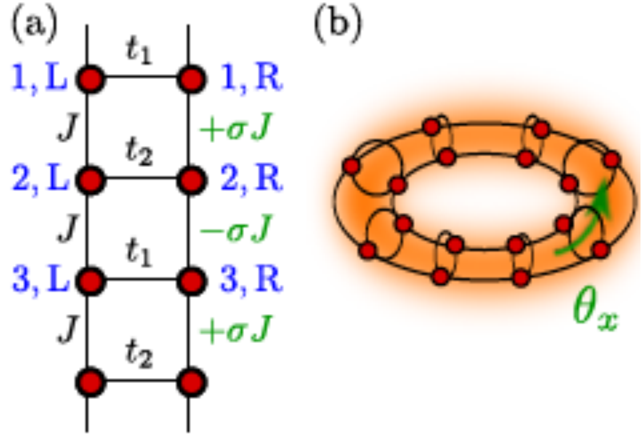
A. Petrescu & KLH, 2013 , 2015

M. Piraud, F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua, U. Schollwoeck 2015

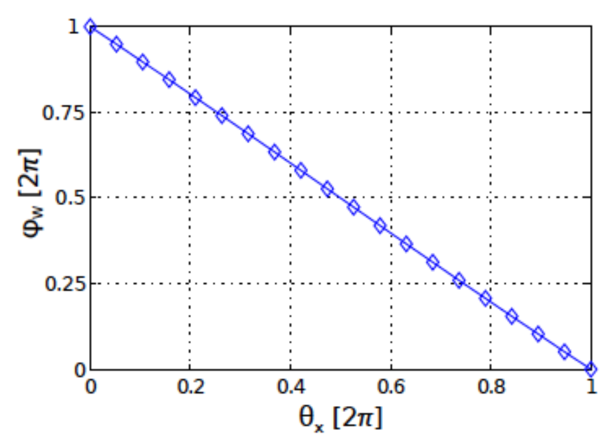
Example of topology Ladder, C=1/2

F. Grusdt & M. Honing, 2014 (density 1/8)
A. Petrescu, M. Piraud, I. McCulloch, G. Roux, KLH in preparation

Bosonic ladder with CDW ground state and 2 degenerate ground states



Topological invariant related to degeneracy and to the Wilson loop phase (generalized Zak)



Two controllable parameters ϑ_x and k_y

$$C = \frac{1}{N_{\text{deg}}} \times \frac{1}{2\pi} \int_0^{2\pi} d\theta_x \; \partial_{\theta_x} \underbrace{\text{Im} \log \det \hat{W}(\theta_x)}_{=\varphi_W} .$$

Experiment Muenich, Thouless pump: M. Lohse et al. arXiv:1507.02225

Spin-1/2 fermions: do not break TR symmetry

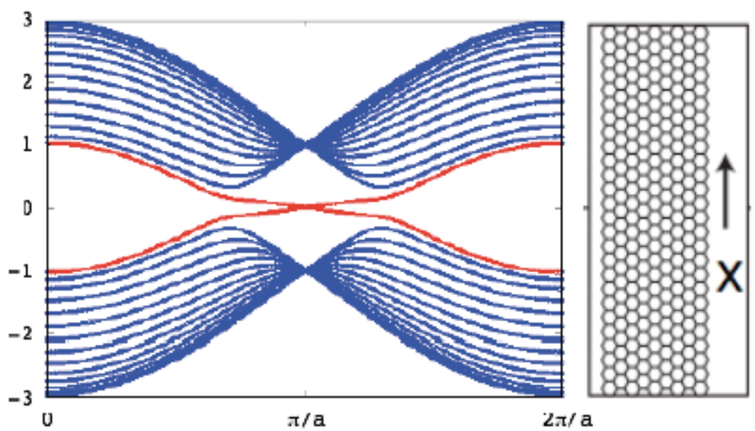
Kane & Mele, PRL 95, 226801 (2005); Fu-Kane

see also: **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)**

Also realizations in photon systems for example: M. Hafezi, S. Mittal, J. Fan, A. Migdall, J. Taylor (2013)
Mikael C. Rechtsman, Julia M. Zeuner, Yonatan Plotnik, Yaakov Lumer, Stefan Nolte, Mordechai Segev, Alexander Szameit (2013)

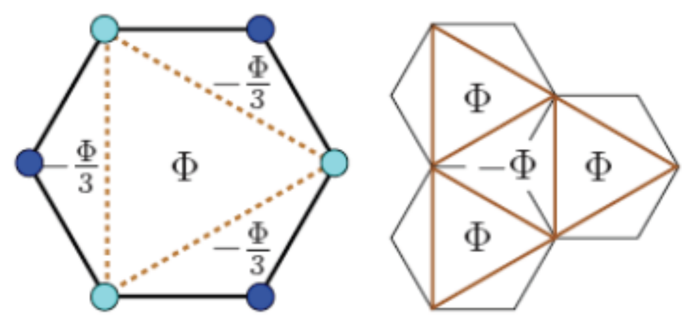
$$\mathcal{H} = -t \sum_{<ij>\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^{z}_{\sigma\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'} \quad \nu_{ij} = \pm 1$$

strip geometry:



edge states: Kramers's pair

$$\mathcal{H} \propto \Psi^{\dagger}_{k} \sigma^{z} \tau^{z} \Psi_{k}$$



Half-filling

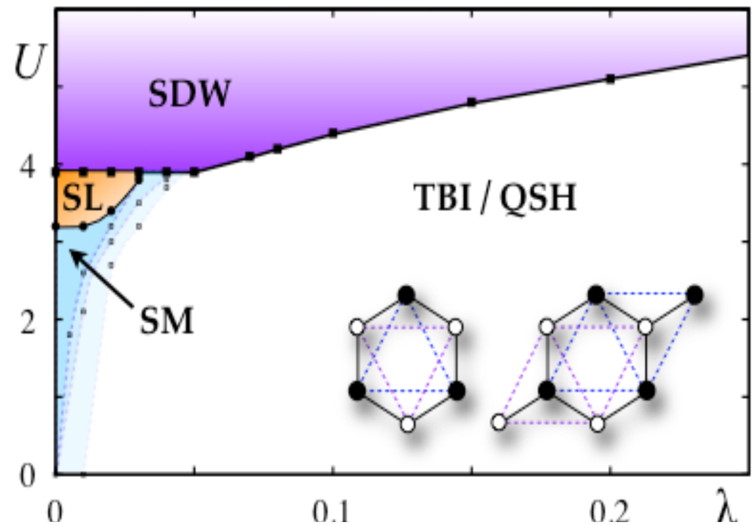
Quantum Spin Hall Effect

Phase Diagram: "Kane-Mele-Hubbard"

Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

A. Georges, G. Kotliar
O. Parcollet, …



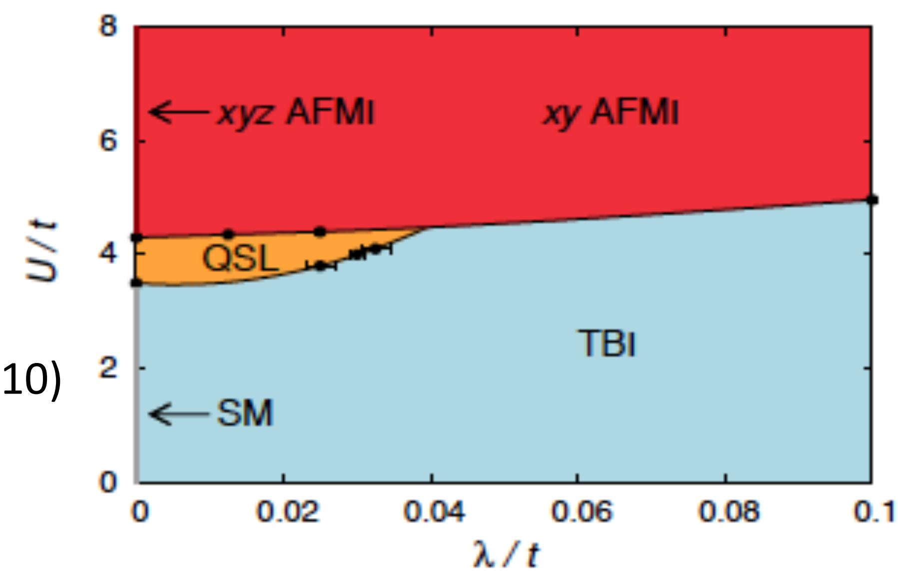
Analytics:
Young, Lee, Kallin 2008
S. Rachel & KLH, 2010
Griset & C. Xu, 2011
D.-H. Lee, 2011 …

M. Hohenadler et al.
arXiv:1111.3949

Phys. Rev. Lett. **106**,
100403 (2011)

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)



Reviews: Hohenadler & Assaad, 2013
Maciejko-Fiete, 2015

Absence of spin liquid for Hubbard (QSL and SL Needs frustration – see later):
S. Sorella et al. Scientific Reports 2012; S. R. Hassan & D. Senechal PRL 2013

Connection to reality?

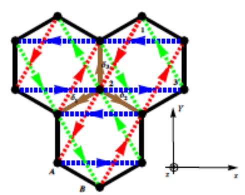
- Na_2IrO_3: anisotropic spin-orbit coupling (thin films: arXiv:1303:5245, M. Jenderka et al)

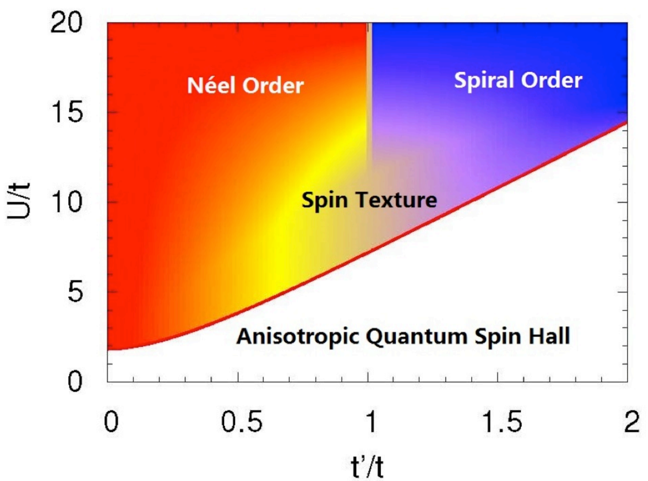
Shitade et al. PRL **102** 256402 (2009); G. Jackeli & G. Khaliullin, PRL 102, 017205 (2009)

H.-C. Jiang, Z.-C. Gu, X.-L. Qi and S. Trebst, Phys. Rev. B 83, 245104 (2011);

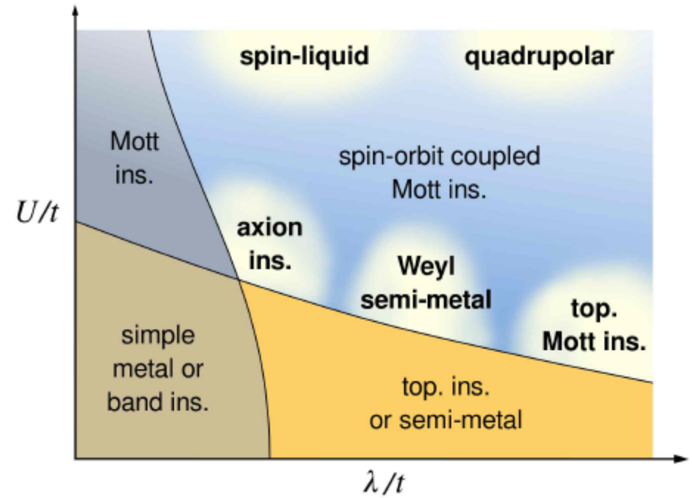
S. Bhattacharjee, Sung-Sik Lee and Yong-Baek Kim, New J. Phys. 14, 073015 (2012)

Y. Singh et al. 2012; Z. Nussinov & J. van den Brink, arXiv:1303.5922 …





α Lithium Iridates and Spiral order R. Coldea



Tianhan Liu, Benoit Doucot, Karyn Le Hur, PRB 2013
A. Ruegg and G. Fiete, PRL 2012
J. Reuther, R. Thomale & S. Rachel, PRB 2012
M. Kargarian, A. Langari, G. Fiete PRB 2012

D. Pesin & L. Balents, Nature Phys. 2010
Krempa, Choy, Y.-B. Kim & L. Balents
Spin Ice physics: N. Shannon; S. Onoda
R. Coldea, Titanate Pyrochlores…

Summary

Haldane Chern insulator

Prototype model : Quantum Hall effect with no

Net flux in a unit cell

Our contribution:

Interaction Effects and exploration of new phases

Thanks to students, collaborators