Interacting Topological Phases and Light

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Conference: ct.qmat, Dresden September 2024

Thanks to the Organizers and Matthias Vojta

Thanks to the members of the team

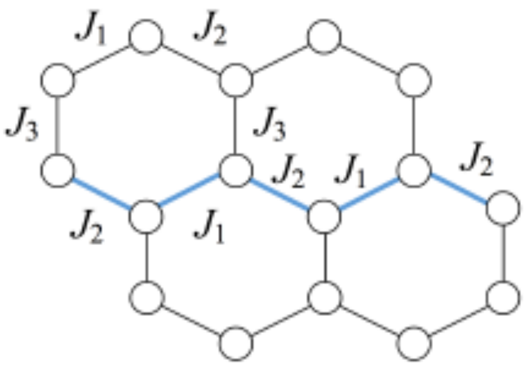
Complexity and Topology
In Quantum Matter

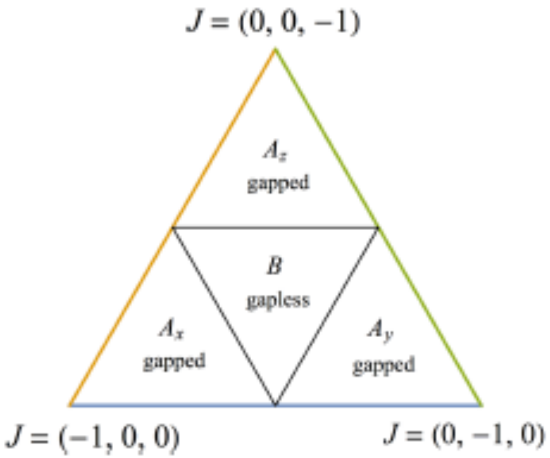
Quantum Spin Liquids

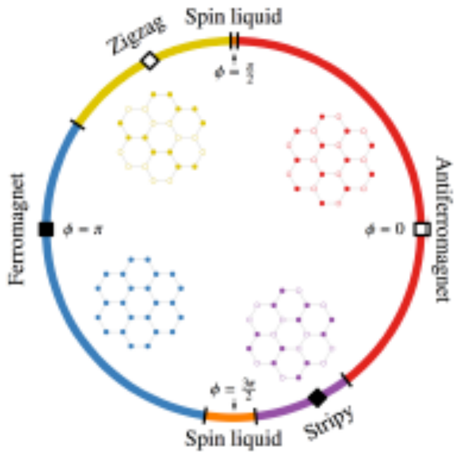
Proposal in cold atoms:
Duan, Demler, Lukin 2003
Kitaev paper 2005 published later in 2006

A. Kitaev
1997 toric code
2005 arXiv

J. Rau & H. Y. Kee, review
S. Trebst, review
Iridate materials, αRuCl3
Mixing angle (Heisenberg & Kitaev)





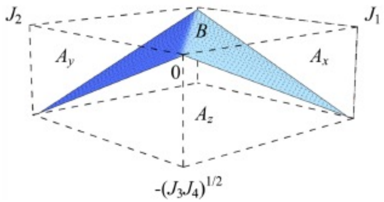


$$H_K = \sum J_\nu \sigma_j^\nu \sigma_k^\nu, \quad \nu = x, y, z$$
$\langle jk \rangle \in \nu$-links in d-dimensional lattice $\mathcal{G}$

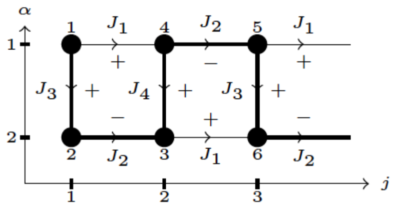
Review L. Savary & L. Balents

Kitaev model on honeycomb lattice: mathematical Majorana representation with 4-Majoranas (2006)
3 gapped A Anderson Resonating Valence Bond States
B gapless phase (gapped with magnetic fields, Z2 Haldane model)
Anyons, Majorana fermions, toric code

Generalization in Tunable Ladder representation with 2 Majoranas
K. Le Hur, A. Soret and F. Yang, 2017 ; see also X.-Y. Feng et al. 2007

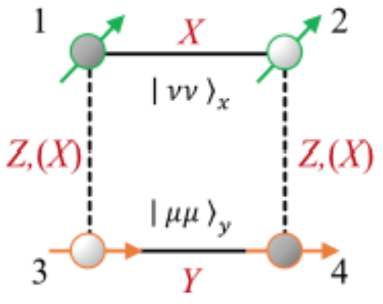


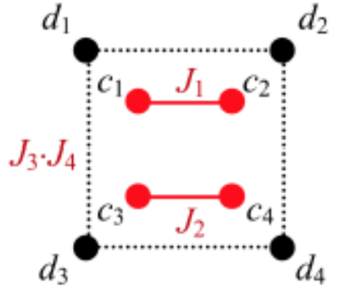
Z2 gauge field



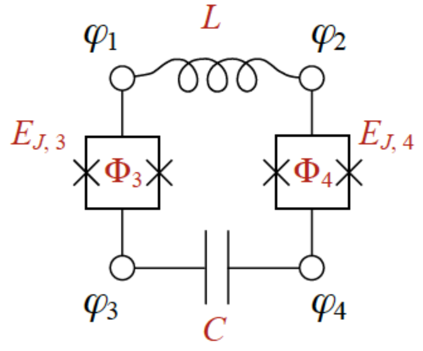
Quantum Technologies: Boxes with SC devices

Fan Yang, Loic Henriet, Ariane Soret, Karyn Le Hur, Phys. Rev. B 98, 035431 (2018)

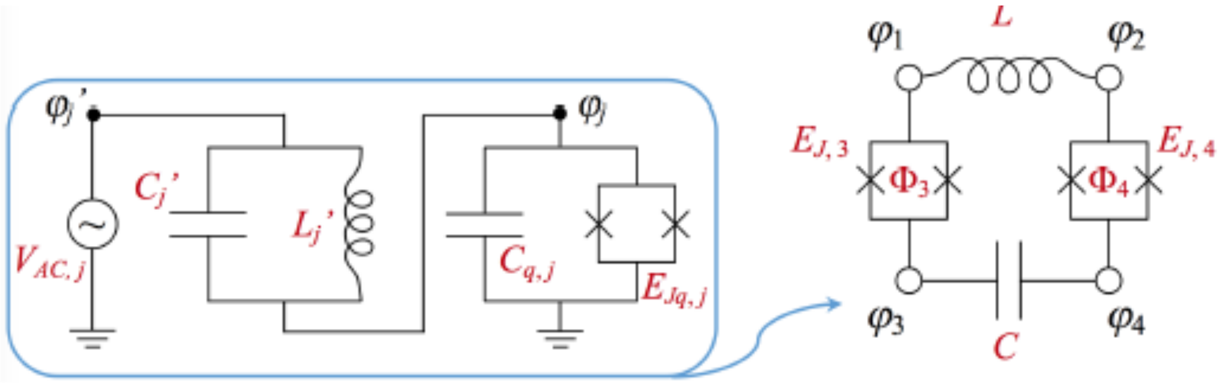




Coupling
4 Majoranas

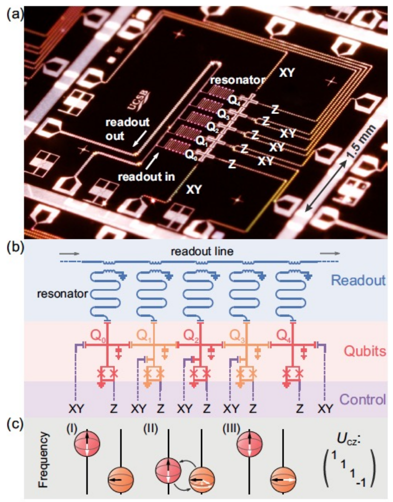


$$H_K = J_1\sigma_1^x\sigma_2^x + J_2\sigma_3^y\sigma_4^y + J_3\sigma_1^z\sigma_3^z + J_4\sigma_2^z\sigma_4^z, \quad H_\perp^x = J_3^x\sigma_1^x\sigma_3^x + J_4^x\sigma_2^x\sigma_4^x,$$



$$H_z = \sum_{j=1}^{4} \frac{\omega_{0,j}}{2}\sigma_j^z$$

J. Martinis group

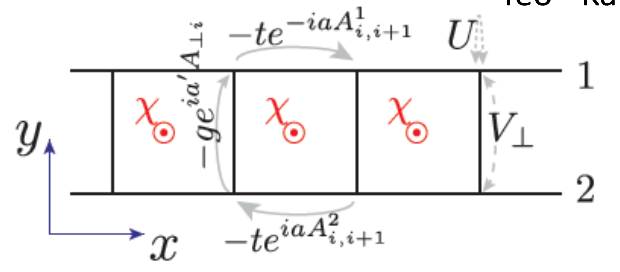


Kitaev spin models **(double Floquet period):** NMR blue box

Applications in quantum codes, implementation of Sachdev-Ye-Kitaev model

A. Petrescu & K. Le Hur, PRB 2015; quantum field theory

Teo - Kane



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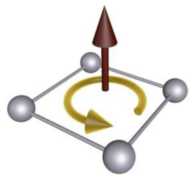
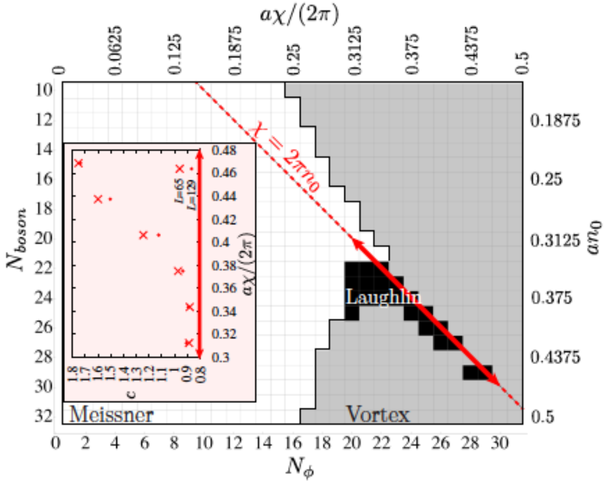




A. Petrescu, M. Piraud, G. Roux, I. McCulloch, K. Le Hur, PRB 2017

DMRG

2D:
F. Yang, V. Perrin, A. Petrescu, I. Garate, K. Le Hur 2020

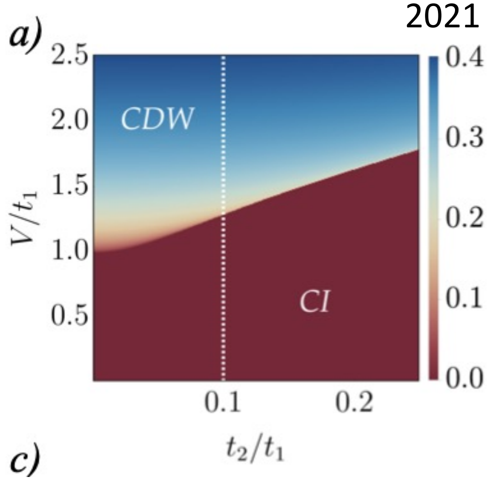
Also, novel DCI phase in Kitaev superconducting ladders
L. Herviou, C. Mora, K. Le Hur 2016 (Majorana liquid)

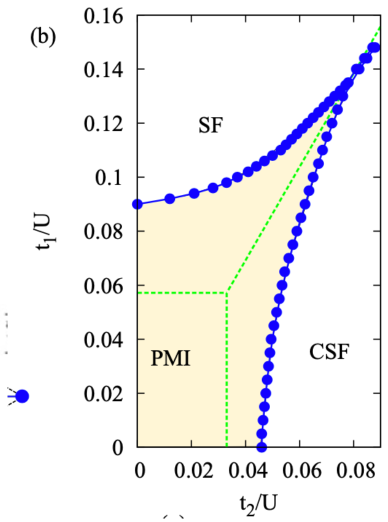
Interactions

Fractional quantum Hall phase at $\nu = \frac{1}{2}$ *in a Ladder*

Interacting Haldane (1988) & Kane-Mele (2005) models fermions and bosons

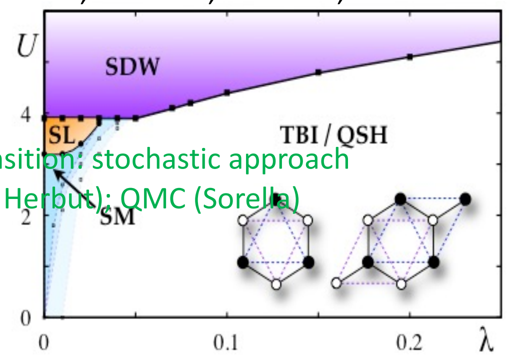
Interacting Haldane fermion model
Stochastic variational approach
Agrees with DMRG in 2D
Ph. Klein, A. Grushin, K. Le Hur PRB 2021





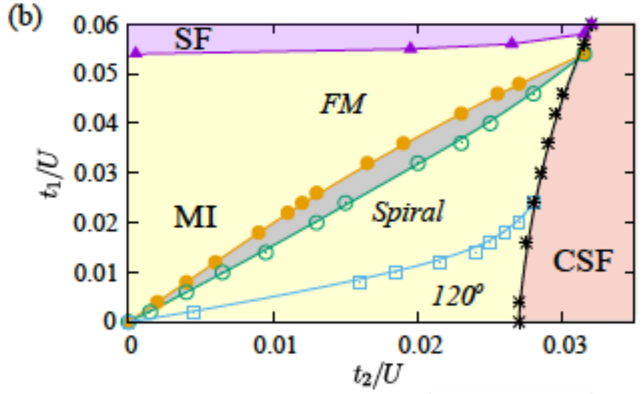
Chiral superfluid (CSF)

Interacting Kane-Mele Fermionic model
W. Wu, S. Rachel, W.-M. Liu, KLH 2012 (CDMFT)
S. Rachel & KLH, 2010 (quantum field theory; mean-field)
New stochastic variational analytical approach
Moderate effects of fluctuations (polarization bubbles)
J. Hutchinson, Ph. Klein, K. Le Hur, PRB 2021



Direct Transition: stochastic approach
Large-N (I. Herbut); QMC (Sorella)

New chiral spin state
Kane-Mele bosons
A. K. Plekhanov, I. Vasic, A. Petrescu, N. Rajbir, G. Roux, W. Hofstetter, K. Le Hur PRL 2017



Bosonic Haldane model: I. Vasic, A. Petrescu, K. Le Hur, W. Hofstetter 2015

3D topological Mott state: D. Pesin & L. Balents, 2010

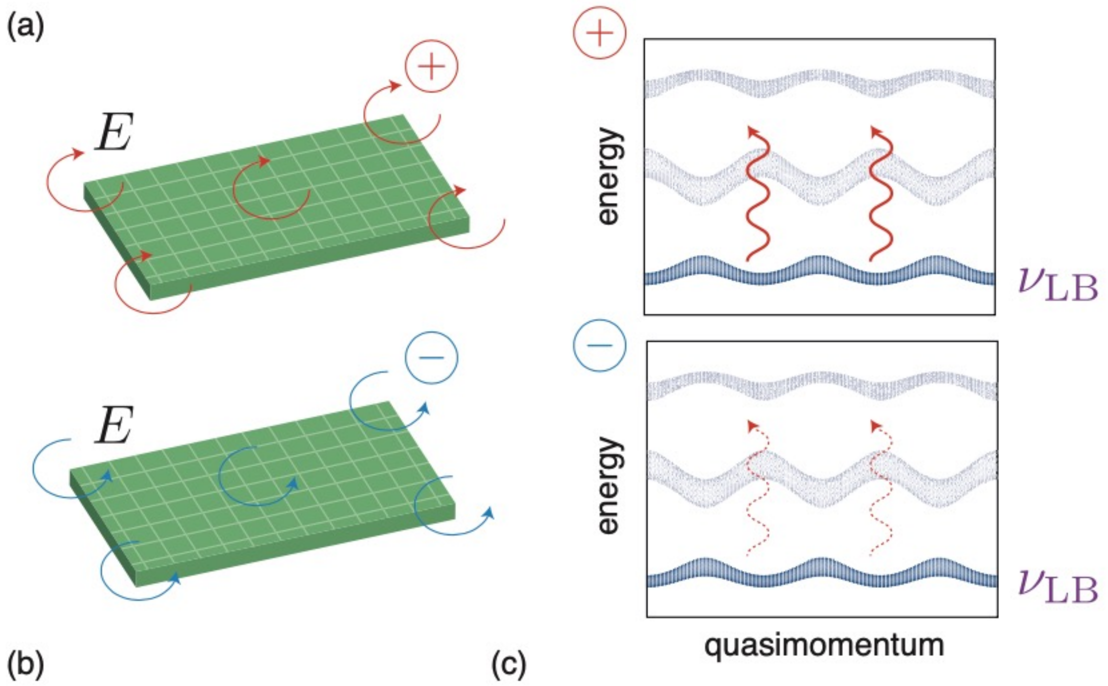
Light
e.g. Classical Light

Probing topology by "heating": Quantized circular dichroism in ultracold atoms

2017

D. T. Tran,[1] A. Dauphin,[2] A. G. Grushin,[3,4] P. Zoller,[5,6,7] and N. Goldman*[1]

L. Asteria, D.-T. Tran, T. Ozawa, M. Tarnowski, B.-S. Rem, N. Flaschner, K. Sengstock, N. Goldman and C. Weitenberg, Nature Physics 15, pages 449-454 (2019).



$$\hat{H}_{\pm}(t) = \hat{H}_0 + 2E\ [\cos(\omega t)\hat{x} \pm \sin(\omega t)\hat{y}]\,,$$

$$\hat{R}_{\pm} = \exp\left\{ i\frac{2E}{\hbar\omega}\,[\sin(\omega t)\hat{x} \mp \cos(\omega t)\hat{y}] \right\}$$

$$\Gamma_{\pm}(\omega) = \sum_{\boldsymbol{k}} \Gamma_{\pm}(\boldsymbol{k};\omega),$$

$$\Gamma_{\pm}(\boldsymbol{k};\omega) = \frac{2\pi}{\hbar}\sum_{n>0} |\mathcal{V}^{\pm}_{n0}(\boldsymbol{k})|^2\,\delta^{(t)}(\varepsilon_n(\boldsymbol{k}) - \varepsilon_0(\boldsymbol{k}) - \hbar\omega),$$

$$|\mathcal{V}^{\pm}_{n0}(\boldsymbol{k})|^2 = (E/\hbar\omega)^2 \left| \left\langle n(\boldsymbol{k}) \middle| \frac{1}{i}\frac{\partial \hat{H}_0}{\partial k_x} \mp \frac{\partial \hat{H}_0}{\partial k_y} \middle| 0(\boldsymbol{k}) \right\rangle \right|^2.$$

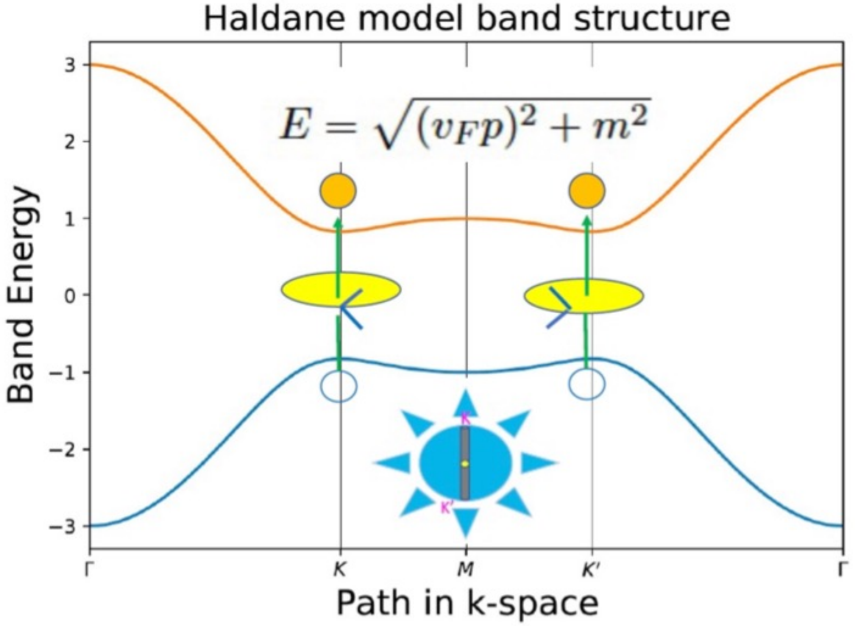
$$\Delta\Gamma^{\rm int}/A_{\rm syst} = \eta_0 E^2, \qquad \eta_0 = (1/\hbar^2)\,\nu,$$

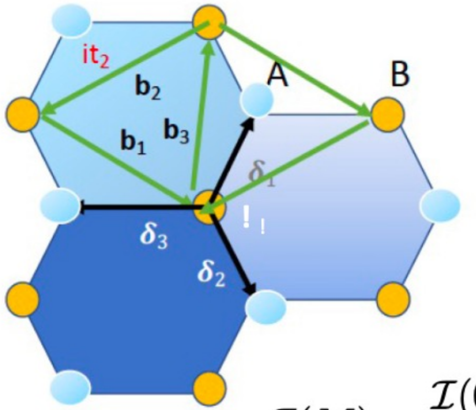
$$\Delta\Gamma^{\rm int} = 4\pi(E/\hbar)^2\,{\rm Im}\sum_{n>0}\sum_{\boldsymbol{k}} \frac{\langle 0|\partial_{k_x}\hat{H}_0|n\rangle\langle n|\partial_{k_y}\hat{H}_0|0\rangle}{(\varepsilon_0 - \varepsilon_n)^2}.$$

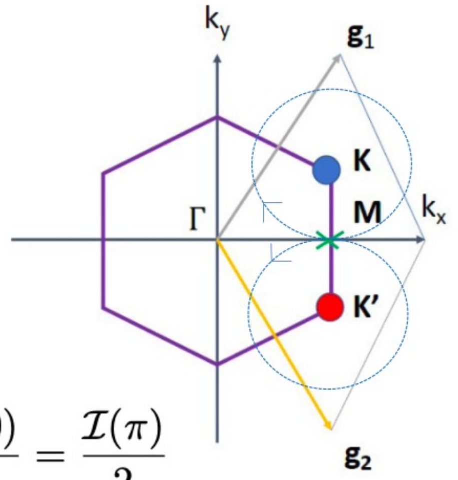
$(\Gamma_+ - \Gamma_-)/2$

Fractional $\nu = \frac{1}{2}$ (C. Repellin and N. Goldman, 2019)

New ways to probe topological states with light





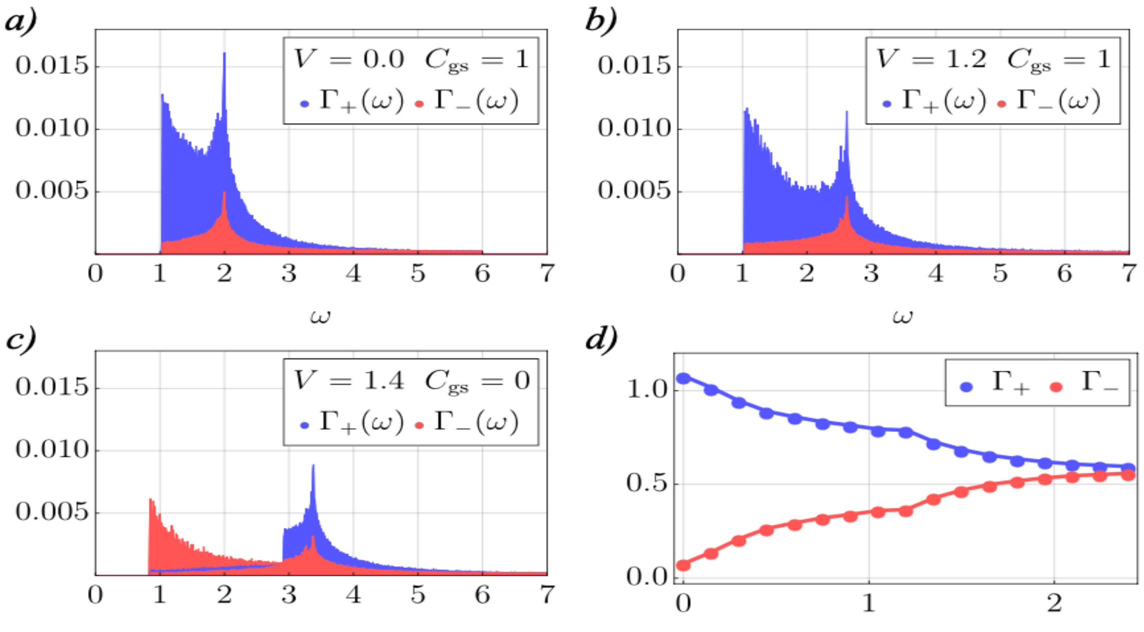


$$\mathcal{I}(M) = \frac{\mathcal{I}(0)}{2} = \frac{\mathcal{I}(\pi)}{2}$$

$$\mathcal{I}(\theta) = \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_+ \right\rangle + \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_+ \right\rangle$$

Stochastic Variational Approach with interactions

Theory Mott transition: 1st order transition



$$\frac{1}{2}\int_0^\infty d\omega \sum_{\boldsymbol{k}=\boldsymbol{K},\boldsymbol{K}'} \left(\Gamma^+_{l\to u}(\omega_{\boldsymbol{k}}, \boldsymbol{k}) - \Gamma^-_{l\to u}(\omega_{\boldsymbol{k}}, \boldsymbol{k})\right) = \rho C$$

Ph. W. Klein, A. G. Grushin, K. Le Hur, Phys. Rev. B 103, 035114 (2021)

probe topology from the Dirac points with light C^2

From a mapping on the sphere; new geometrical function

K. Le Hur, Phys. Rev. B 105, 125106 (2022)

Applications to topological insulators

Kane-Mele, Fu model

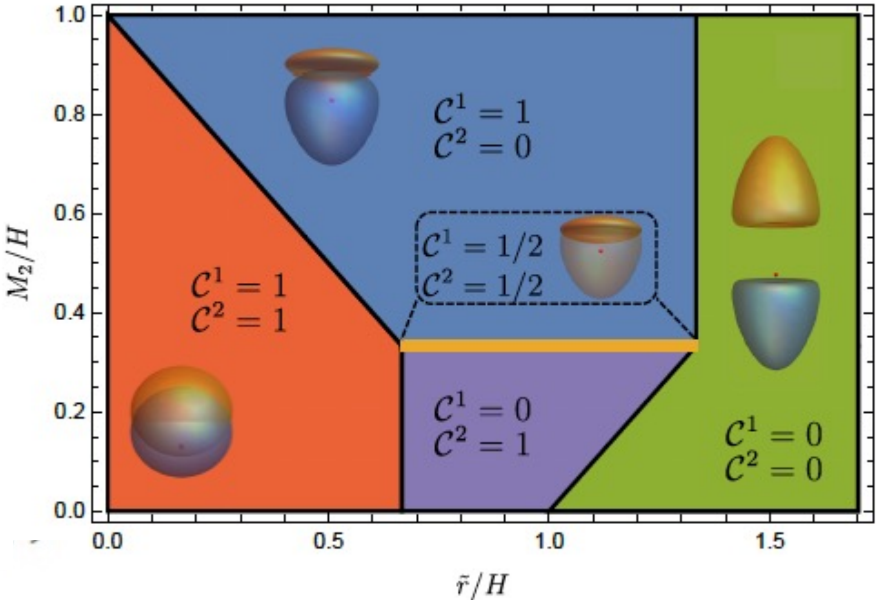
New way to include interactions on sphere

Thanks to
NSERC Canada
& ANR BOCA

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021), Nature Journal

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

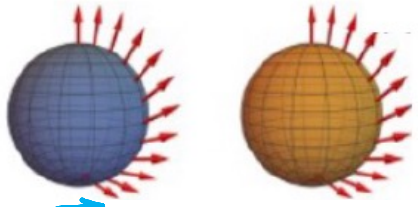


- Phase Diagram obtained from energetics at the poles
- Region C_j=1/2 occurs for various f(θ) $and\ f(\theta) = cst$

Z_2 symmetry
1 <-> 2 spheres

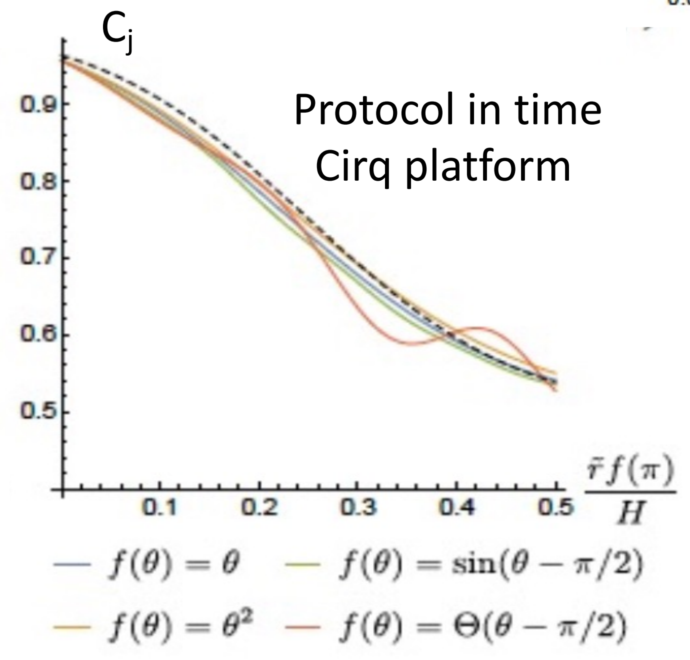
M_1=M_2=M

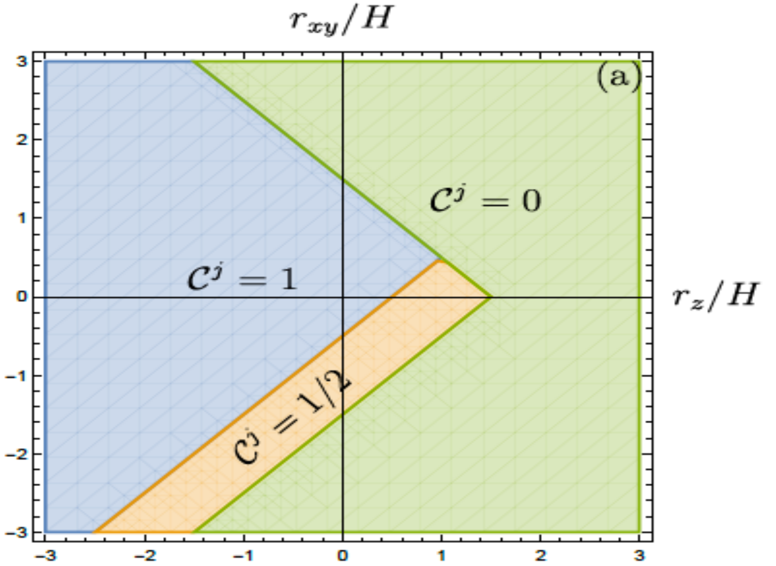
$$H - M < \tilde{r} < H + M,$$



Bell state
EPR pair

Singlet or Triplet state with $\sigma z, tot = 0$

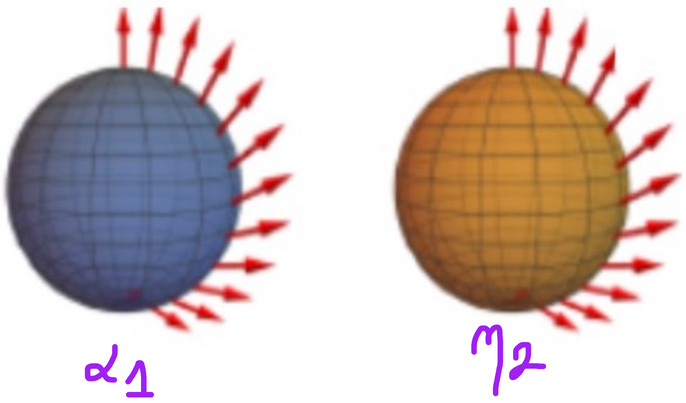




Analogue of ½ flux quantum in superconductor: measure the presence of a EPR pair or Bell state at one pole

$$\mathcal{A}_{j\varphi}(\pi) - \mathcal{A}_{j\varphi}(0) = q\frac{1}{2} = C_j,$$

$$\frac{1}{2\pi}\iint_{S^2} \nabla_j \times \mathcal{A}_j \cdot d^2\mathbf{s} = C_j = q\frac{1}{2}$$



K. Le Hur, Phys. Rev. B 108, 235144 (2023)

Protected Topological Qubits: 1 pair of Majorana fermions at a pole

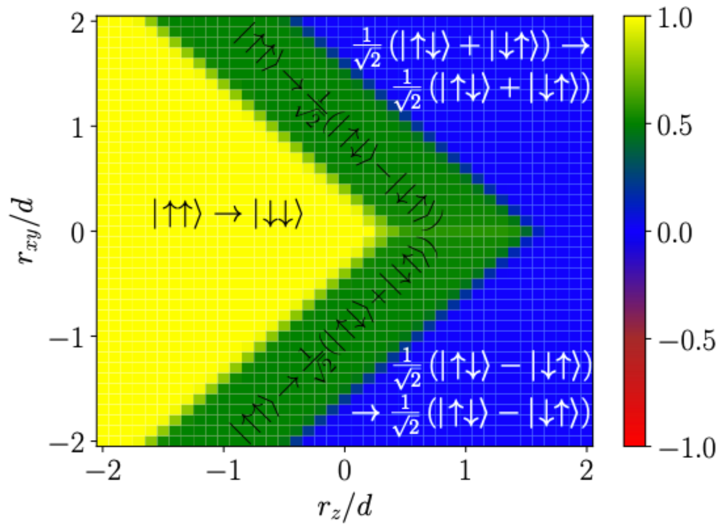
Relation to Kitaev wire for even number of spheres in a ring

Ephraim Bernhardt, Brian Cheung Hang Chung, Karyn Le Hur

Physical Review Research 6, 023221 (2024)

Application to Majorana fermions

τ	Majorana fermions	d-operators	sphere operators
τ^z	$2i\alpha_1\eta_2$	$2d^\dagger d - 1$	$\sigma_1^x\sigma_2^x$
τ^y	$-\sqrt{2}\eta_2$	$-i(d^\dagger - d)$	$\sigma_2^x\sigma_1^y$
τ^x	$\sqrt{2}\alpha_1$	$d^\dagger + d$	σ_1^z



Generalization of fractions for odd number of spheres

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021), Nature Journal

Small noise

M term favors singlet-triplet transitions, qubit cQED Platform

2 spins

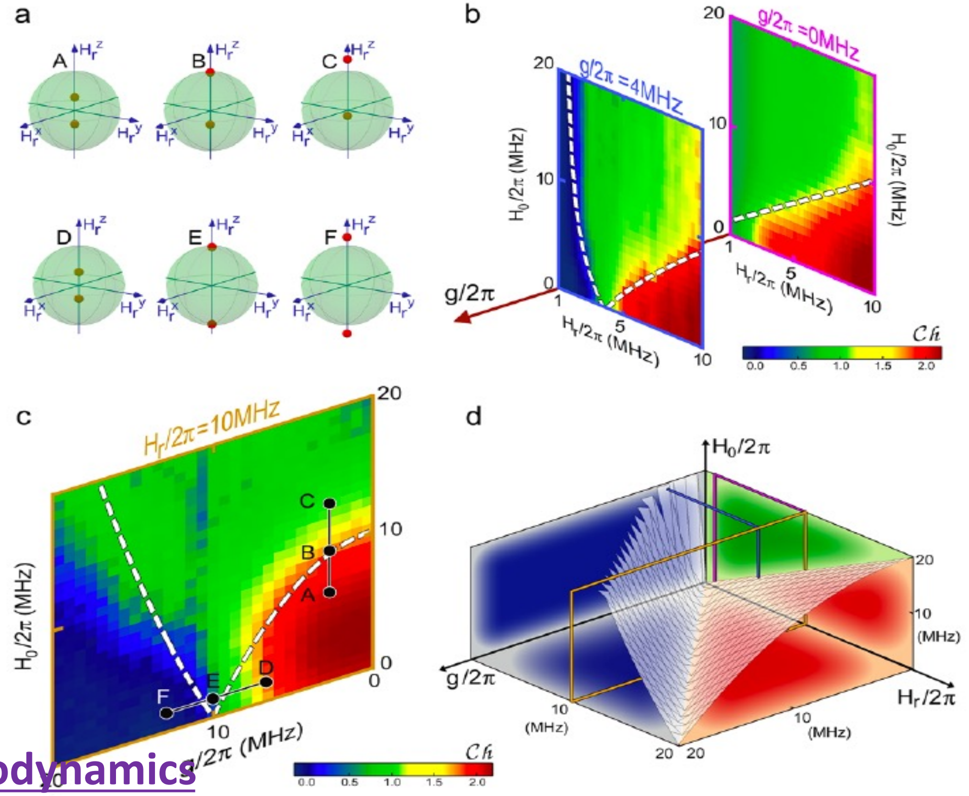
The Hamiltonian of this system reads

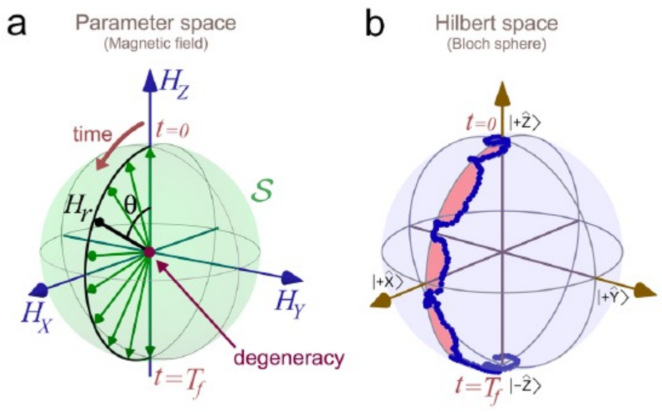
$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)], \quad (5)$$

where 1 and 2 refer to qubit 1 (Q1) and qubit 2 (Q2)

Here M=Ho just acts on 1 spin

Santa-Barbara “google”:
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)





Application in Energy & quantum thermodynamics

Quantum Dynamo effect in a Bath (1 spin coupled to cQED or 1D ohmic boson bath, transmission line)

L. Henriet, A. Sclocchi, P. P. Orth, K. Le Hur Phys. Rev. B 95, 054307 (2017)

E. Bernhardt, C. Elouard, K. Le Hur, Phys. Rev. A 107, 022219 (2023)

New Insights on new states of matter

Topological semimetals and liquids

- 3D Weyl semimetals : Relation to axion physics and photogalvanic effect (*active field since the last decade*)

See e.g. : F. de Juan, A. G. Grushin, T. Morimoto, J. Moore, Nature Communications 2017

- Topological Semimetals in 2D? Young & Kane 2015 Dirac Semimetals in 2D (only a few theory papers, why?)

  Topological Quantization associated to Fermi surface? F. D. M. Haldane 2004

Proposals for quantum anomalous Hall semimetal in 2D: Relation to ½ topological numbers

Realization in bilayer honeycomb lattices (J. Hutchinson and K. Le Hur Comm. Phys. 2021 (arXiv:2020); K. Le Hur PRB 2023)

See also article B. Fu, J.-Y. Zou, Z.-A. Hu, H.-W. Wang, S.-Q. Shen, npj Quantum Materials 7, 94 2022

And Graphene, Useful application of circularly polarized light: K. Le Hur and S. Al Saati, PRB 107, 165407 (2023)

And K. Le Hur & S. Al Saati, arXiv:2311.13922 (Comptes Rendus Academie des Sciences 2024)

Generalization to Topological quantum Spin Hall Semimetal (next time)

- Majorana fermions in 1D & DCI phase F. Del Pozo, L. Herviou, K. Le Hur 2023

  F. del Pozo, L. Herviou, O. Dmytruk, K. Le Hur 2024

Fractional numbers in DMRG

Insight on probing the Z and Z_2 topological numbers of topological p-wave Kitaev superconductors with cQED & classical light

F. del Pozo and K. Le Hur, Phys. Rev. B 110, L060503 (2024)

F. del Pozo, graduation, December 2024

Entangled WaveFunctions in Topological Band Structures

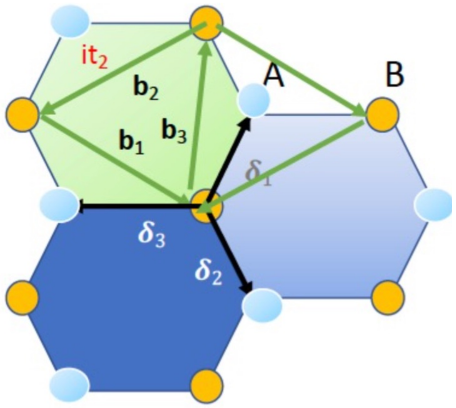
Topological Nodal Ring Semimetal: Topological Half Metal

Thanks to DFG FOR2414

J. Hutchinson & K. Le Hur, Communications Physics 4, 144 (2021)

P. Cheng, Ph. W. Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg, K. Le Hur, Phys. Rev. B 100, 081107 (2019)

$$H = (\zeta d_z + M)\sigma_z \otimes \mathbb{I} + d_x\sigma_x \otimes \mathbb{I} + d_y\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes s_x$$

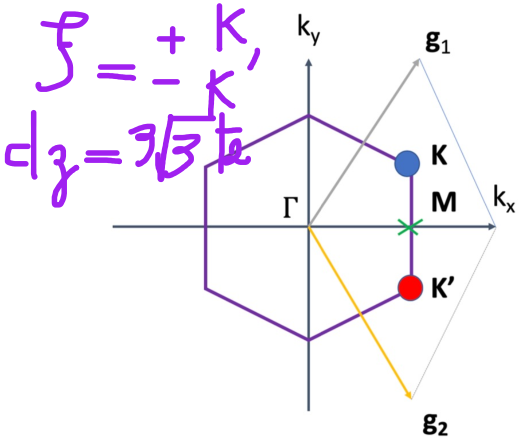


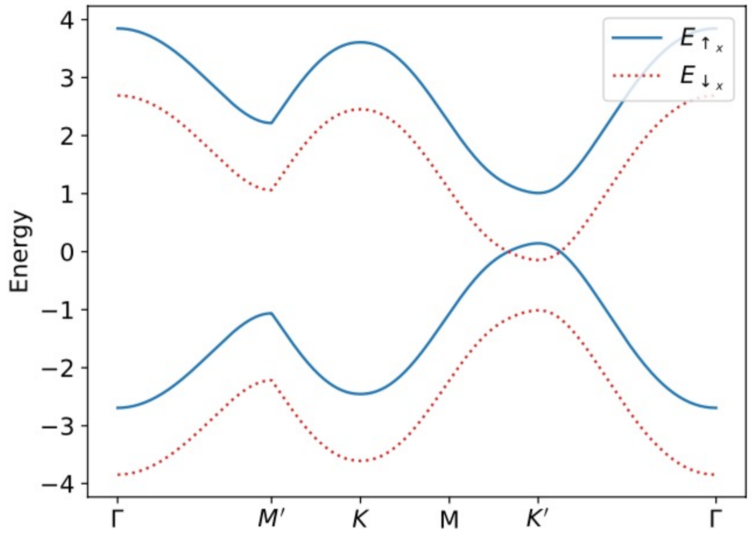
Bilayer model: σ *acts on sublattice and s on plane*

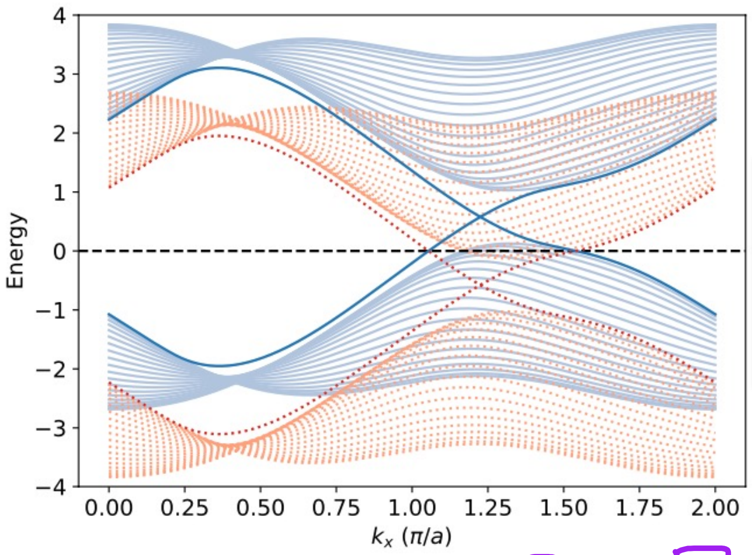
Graphene model: σ *acts on sublattice and s on spin*

M charge density wave substrate; r Zeeman effect in plane

$$3\sqrt{3}t_2 - M < r < 3\sqrt{3}t_2 + M$$







Stable towards
- Disorder
- Interactions

Stochastic Approach

K. Le Hur and S. Al Saati, PRB 107, 165407 (2023)

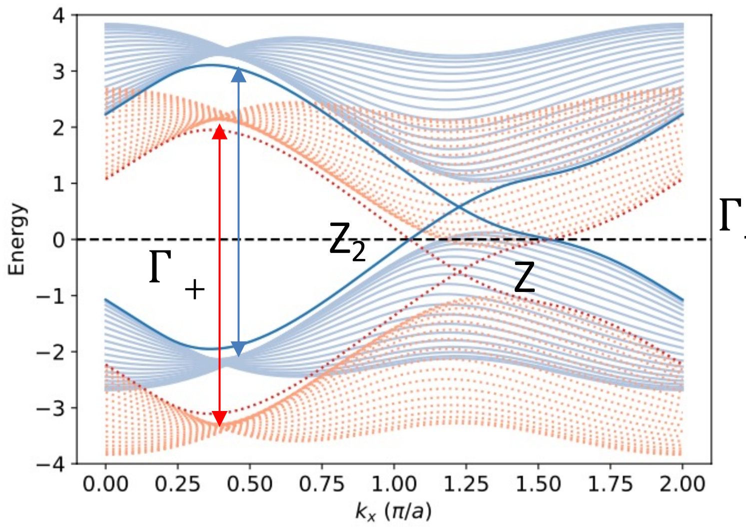
arXiv:2311.13922 (Comptes Rendus Academie des Sciences 2024)

See Poster

Topological Markers

Topological half metal

The spin polarized blue edge mode shows a quantized conductance at the edges



$\Gamma\_ = 0$

cylinder

K. Le Hur and S. Al Saati, arXiv:2311.13922, Comptes Rendus Physique Academie des Sciences in press

Quantum Hall response can be evaluated for the different bands, domains with Kubo formula

$$C_{\downarrow x,-} = \left(\frac{1}{2\pi}\int_0^{2\pi} d\varphi\right)\int_0^{\pi} d\theta \left(-\frac{1}{2}\right)\sin\theta = -1,$$

Red lowest band

Z_2 Kane-Mele ← $\hat{C}_{\uparrow x,-} - \tilde{C}_{\downarrow x,+} = C_{\downarrow x,-}$. ← Haldane

Haldane model

Topological semimetal

Pair of bands (1,3) or (2,4)

$$\left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar}A_0^2\frac{1}{2}|\langle\sigma_z(0)\rangle|$$

$$\sum_{j=\uparrow,\downarrow}\left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar}A_0^2|C_{\downarrow x,-}|$$

Response to circularly polarized light: resonance with Dirac points

$$\Gamma_\zeta(\omega) = \frac{2\pi}{\hbar}\frac{A_0^2}{2}\left|\langle +|\left(\frac{\partial\mathcal{H}}{\hbar\partial(\zeta p_y)} + \zeta i\frac{\partial\mathcal{H}}{\hbar\partial p_x}\right)|-\rangle\right|^2 \times \delta(E_l(\mathbf{p}) - E_u(\mathbf{p}) - \hbar\omega).$$

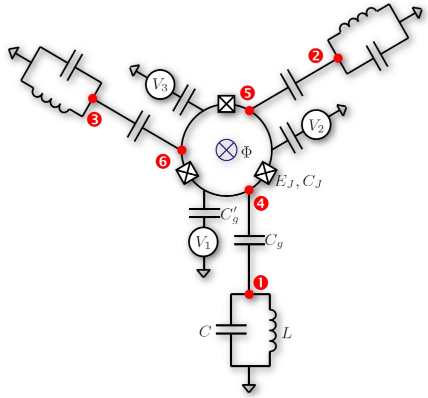
$$\left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar}A_0^2\frac{1}{2}|(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle)| = \frac{2\pi}{\hbar}A_0^2|\langle\sigma_z(0)\rangle|.$$

K. Le Hur, PRB 105, 125106 (2022)
Ph. W. Klein, A. Grushin, K. Le Hur, PRB 103, 035114 (2021)
Tran, Dauphin, Grushin, Zoller, Goldman 2017

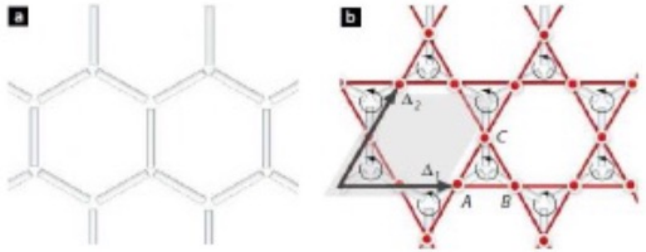
Also, equivalent to a pair of π *Berry phases or ½ Skyrmions (along z direction)*

New Insights on cQED & sensing topological physics

Thanks to ANR BOCA



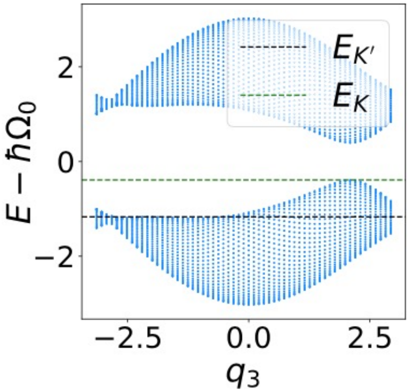
J. Koch, A. A. Houck, K. Le Hur, S. M. Girvin 2010
A. Petrescu, A. A. Houck, K. Le Hur, 2012

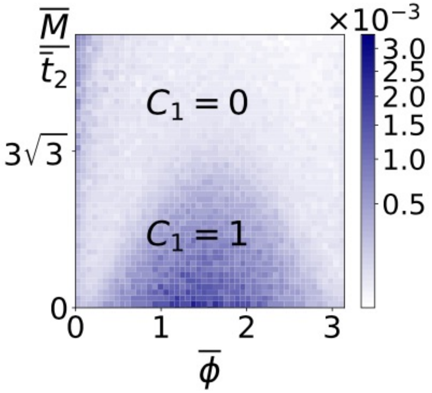


Review: K. Le Hur, L. Henriet, A. Petrescu, K. Plekhanov, G. Roux, M. Schiro, Comptes Rendus de Physique Academie des Sciences 17, 808-835 (2016)

Probing Haldane topological number in 2D lattice with microwave microscope
J. Legendre and K. Le Hur, Phys. Rev. A **109, L021701 2024**

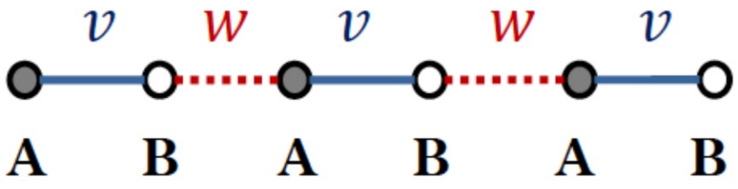
$$C_i = \frac{(-1)^i}{2}[\mathrm{sgn}\, h_2(\mathbf{K}) - \mathrm{sgn}\, h_2(\mathbf{K}')].$$

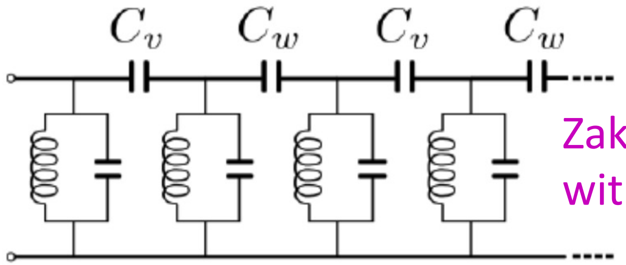




T. Goren, K. Plekhanov, F. Appas, K. Le Hur 2018

Su-Schrieffer-Heeger model in circuits





Zak phase
with cQED probe

Various measurements of LDOS and edge wave functions : weak-coupling
P. St Jean et al. 2017 (Jacqueline Bloch & Alberto Amo)
C. Poli, M. Bellec et al. Nature 2015 (Lancaster and Nice)
E. J. Meier et al. (Brice Gadway's group), Nature comm. 2016

$$\langle m \rangle = \oint \frac{dk}{2\pi}\frac{\partial \theta_k}{\partial k} = \begin{cases} 1 & \text{if } \alpha < 1/2 \\ 0 & \text{if } \alpha > 1/2 \end{cases}$$

E. Rosenthal et al. (Boulder), measure: arXiv:1802.02243

Summary

"Interacting Topological Phases and Light": Review, Karyn Le Hur arXiv:2209.15381 (under review, shorter version)

Main Results shown in the presentation

Quest for new interacting and topological quantum Matter

- Fractional Topological Numbers and New Applications
- Methods and Algorithms: Variational stochastic approach for interacting topological states

Application to quantum spin Hall on Kagome Lattice: I. Titvindze, J. Legendre, K. Le Hur, W. Hofstetter, Phys. Rev. B 105, 235102 (2022)

- New Probes through circularly and linearly polarized light and new insight from cQED ("microscopes")

Thanks to students, post-doctoral fellows and colleagues: https://www.cpht.polytechnique.fr/cpht/lehur/Karyn.LeHur.html
Presentation slides accessible

Thanks for your kind attention and to the organizers