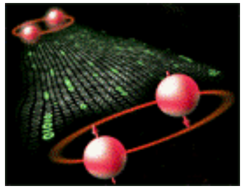
Correlated Topological Matter

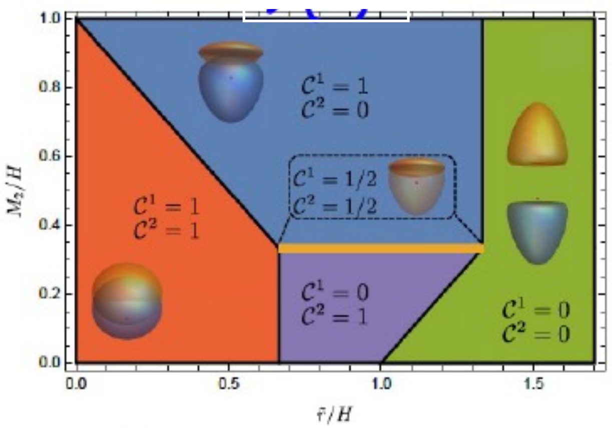


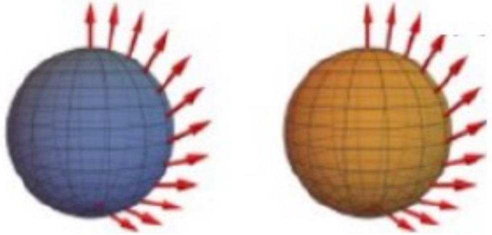
Karyn Le Hur

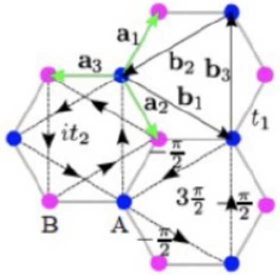
Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

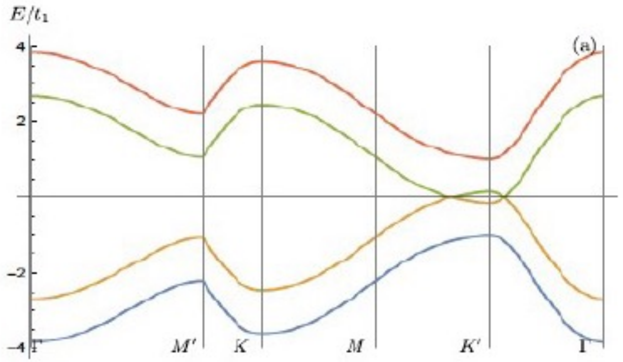
Centre de Physique Theorique, Ecole Polytechnique and CNRS France

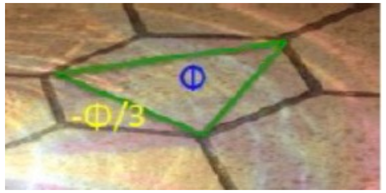
Presentation Dresden Conference “topcorr22 April 2022”











New Topological Phases from Correlations?

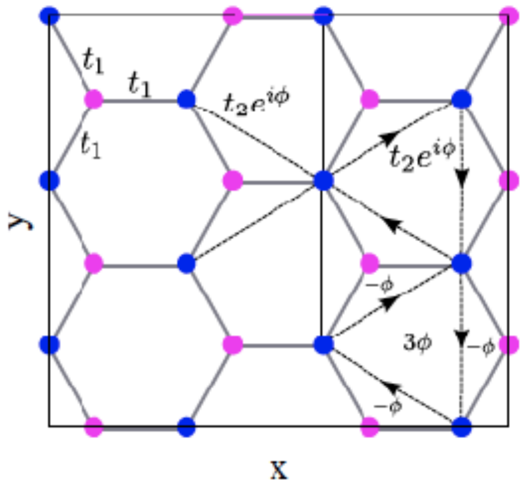
Topological Bloch bands

Haldane model

$$\mathcal{H}_0 = \sum_i (-1)^i M c_i^\dagger c_i - \sum_{<i,j>} t_1 c_i^\dagger c_j - \sum_{\ll i,j \gg} t_2 e^{i\phi_{ij}} c_i^\dagger c_j$$

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988)

M = Semenoff mass



Realized in quantum materials, graphene, cold atoms, light systems

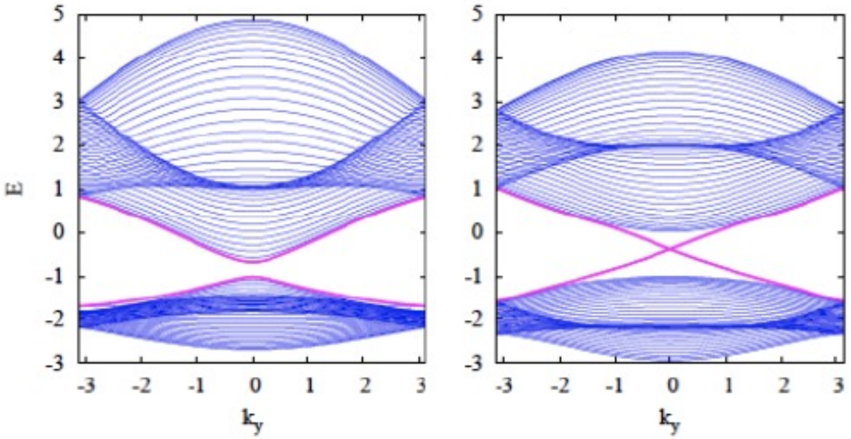
Phase diagrams of interacting Fermionic and Bosonic Haldane & Kane-Mele Models: New Efforts on Mott Transition

I. Vasic, A. Petrescu, K. Le Hur, W. Hofstetter,
Phys. Rev. B 91, 094502 (2015)

K. Plekhanov, I. Vasic, A. Petrescu, R. Nirwan, G. Roux, W. Hofstetter,
K. Le Hur, Phys. Rev. Lett. (2018)

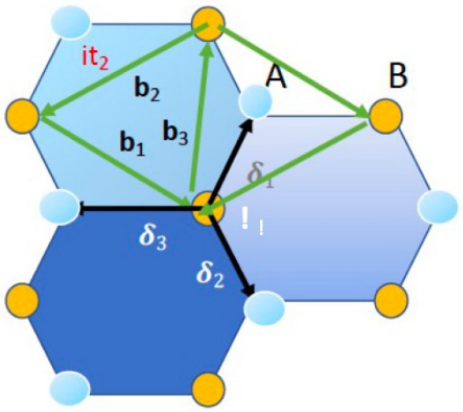
Ph. Klein, A. Grushin, K. Le Hur, PRB 2021

Joel Hutchinson, Ph. W. Klein, K. Le Hur, PRB 2021



Quantum Hall State

1 plane and Haldane Model on Bloch sphere



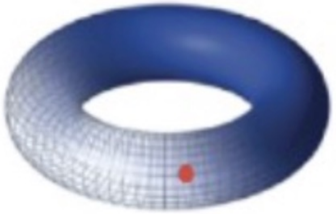
Spin-1/2 Analogy: 2d-Hilbert space of sublattices

$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\left(\mathbf{k}\right)\cdot\hat{\sigma},$$

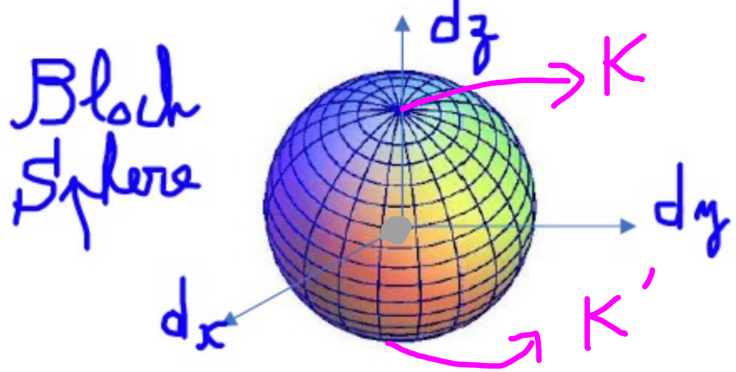
$$\mathbf{d}(\mathbf{k}) = \left(t\sum_{\boldsymbol{\delta}_i}\cos(\mathbf{k}\cdot\boldsymbol{\delta}_i), t\sum_{\boldsymbol{\delta}_i}\sin(\mathbf{k}\cdot\boldsymbol{\delta}_i), 2t_2\sum_{\mathbf{b}_j}\sin(\mathbf{k}\cdot\mathbf{b}_j)\right)$$

$m = \pm 3\sqrt{2}\, t_2 \quad \begin{matrix} + & K \\ - & K' \end{matrix}$

radial magnetic field



$(k_x,k_y) \rightarrow (\theta,\varphi)$



$$\mathbf{d}(\varphi,\theta) = d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta) = (d_x, d_y, d_z).$$

Analogous to a Dirac monopole: Topological number $C = \pm 1$

Gauss-Bonnet Theorem, Poincare-Hopf Theorem

From Berry formalism

$$A_\varphi = -i\langle\psi|\frac{\partial}{\partial\varphi}|\psi\rangle$$

$$\mathbf{F} = \boldsymbol{\nabla}\times\mathbf{A} = \boldsymbol{\nabla}\times\mathbf{A}'$$

Key Results from Geometry

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021)
Ph. Klein, A. Grushin, K. Le Hur, Phys. Rev. B 103, 135114 (2021)

Introduction of smooth Fields

$$C = \frac{1}{2\pi}\int\int_{S^{2\prime}} \boldsymbol{\nabla}\times\mathbf{A}'\cdot d^2\mathbf{s},$$

$$A'_\varphi(\theta<\theta_c) = A_\varphi(\theta) - A_\varphi(0)$$

$$A'_\varphi(\theta>\theta_c) = A_\varphi(\theta) - A_\varphi(\pi)$$

Topological number described through the poles

$$C = (A_\varphi(\pi) - A_\varphi(0)) = \frac{1}{2}(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle)$$

Transport, quantum Hall conductivity and Cylinder Geometry

C^2 as a local topological response observable from light-matter coupling

$$\mathcal{I}(\theta) = \left\langle\psi_+\left|\frac{\partial\mathcal{H}}{\partial p_x}\right|\psi_-\right\rangle\left\langle\psi_-\left|\frac{\partial\mathcal{H}}{\partial p_x}\right|\psi_+\right\rangle + \left\langle\psi_+\left|\frac{\partial\mathcal{H}}{\partial p_y}\right|\psi_-\right\rangle\left\langle\psi_-\left|\frac{\partial\mathcal{H}}{\partial p_y}\right|\psi_+\right\rangle$$

mesurable in time at K,K',M

K. Le Hur, Phys. Rev. B 105, 125106 2022

New Phases from Correlation Effects

Topological Mott physics in 2D (Kitaev model) ?

- "New Try:" Interactions between two Bloch spheres /topological Bloch bands:

Entanglement may lead to fractional topological numbers

Geometry gives new insight on observables (transport, light-matter coupling)

Realizable in circuit QED and atomic physics

- Application for topological semimetals in 2-planes or bilayers models

- Application in Coulomb-interacting superconducting p-wave wires

- Application for topological systems 3D in cubes, "planks assemblage"

Fractional Numbers

$$|\psi\rangle = \sum_{kl} c_{kl}(\theta)|\Phi_k(\varphi)\rangle_1|\Phi_l(\varphi)\rangle_2,$$

- Suppose a direct product state at one pole $|\Phi_+\rangle_1|\Phi_+\rangle_2$
- an entangled Einstein-Podolsky-Rosen or Bell wavefunction at the other pole

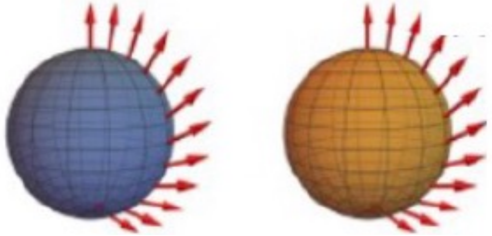
$$|\psi(\pi)\rangle = \frac{1}{\sqrt{2}}(|\Phi_+\rangle_1|\Phi_-\rangle_2 + |\Phi_-\rangle_1|\Phi_+\rangle_2)$$

Geometry: Berry curvature evaluated with the same Hilbert space decomposition for the whole sphere.
Tuning adiabatically r: for r=0, $|\psi(\pi)>$ = $|\Phi_-\rangle_1|\Phi_-\rangle_2$

$$A_{j\varphi}(\pi) = -i\langle\psi(\pi)|\partial_\varphi^j|\psi(\pi)\rangle = \frac{A_{j\varphi}(0)}{2} + \frac{A_{j\varphi}^{r=0}(\pi)}{2}$$

j=1,2

$$A_{j\varphi}^{r=0}(\pi) - A_{j\varphi}(0) = q = 1$$



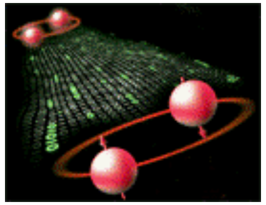
$C_j = \frac{1}{2}$

$$A_{j\varphi}(\pi) - A_{j\varphi}(0) = q\frac{1}{2} = C_j,$$

$$\langle\sigma_{1z}(\pi)\sigma_{2z}(\pi)\rangle = 1 - 2(2C_j)^2 = -1$$

Fractional-1/2 Topological State

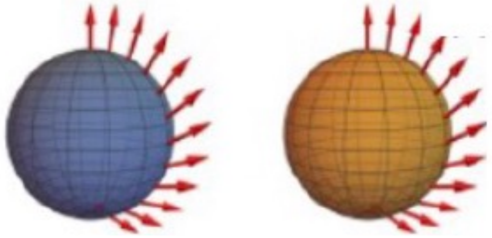
J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021)



Coherent Superposition of two halved-regions: "half-topological" response on the surface
one encircling the topological charge and one entangled region
From Stokes' theorem, the C=1/2 per sphere is equivalent to a circle (χ=0) on top of a disk (χ=1)
Euler characteristics, χ = (2-2g) = (2-2C) = 1+0 =1 with C=1/2
For 1 sphere, 2 circles at the equator χ = (2-2g) = (2-2C) = 0+0=0 with C=1

Quantized π Berry phase at one pole $$-\oint d\varphi A'_{j\varphi}(0^+) = 2\pi C_j$$

$$C_j = \frac{q}{2} = \frac{1}{2}(\langle\sigma_{jz}(0)\rangle - \langle\sigma_{jz}(\pi)\rangle)$$



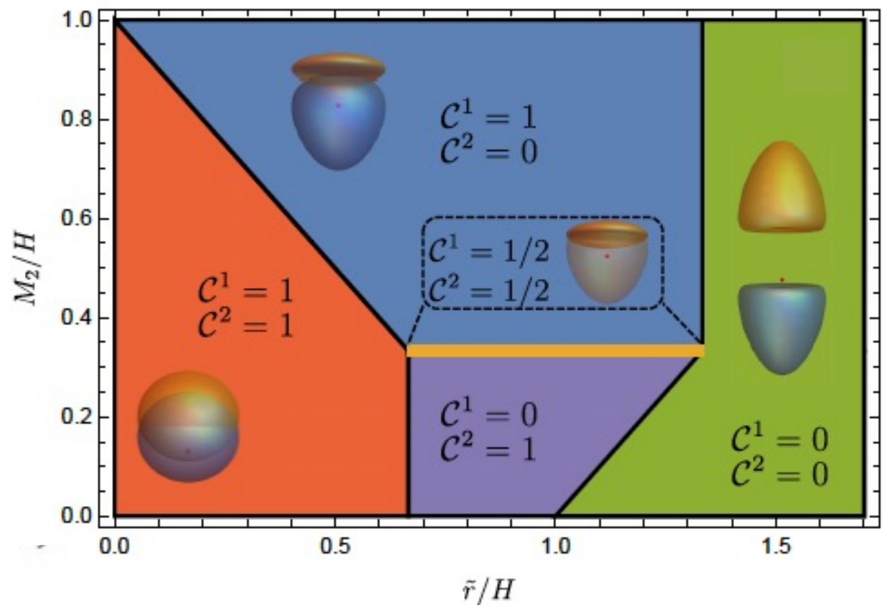
From Newton mechanics, semi-classical analysis, Parseval-Plancherel theorem
C=1/2 enters in the pumped charge and quantum Hall conductivity on the cylinder geometry

Interacting Bloch Spheres' Model

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

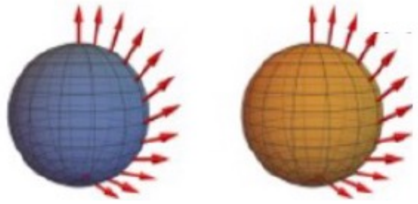
- Phase Diagram obtained from energetics at the poles
- Region C_j=1/2 occurs for various f(θ) $and\ f(\theta) = cst$

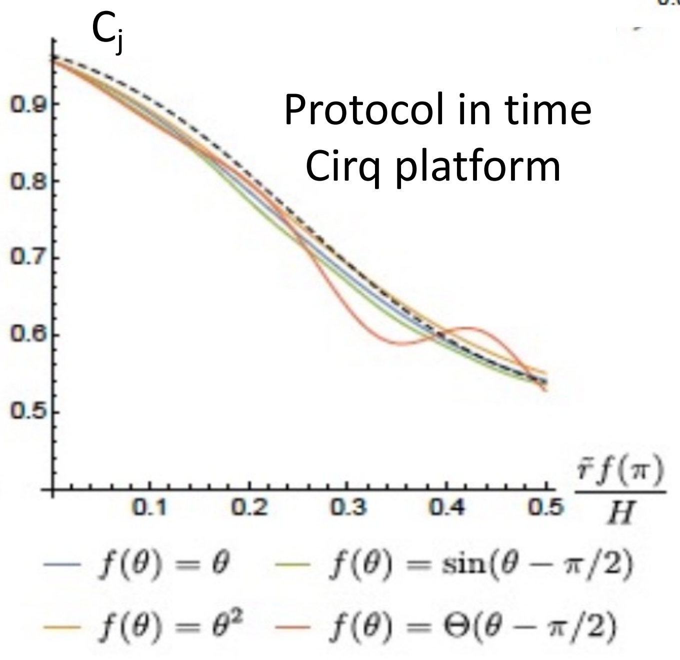


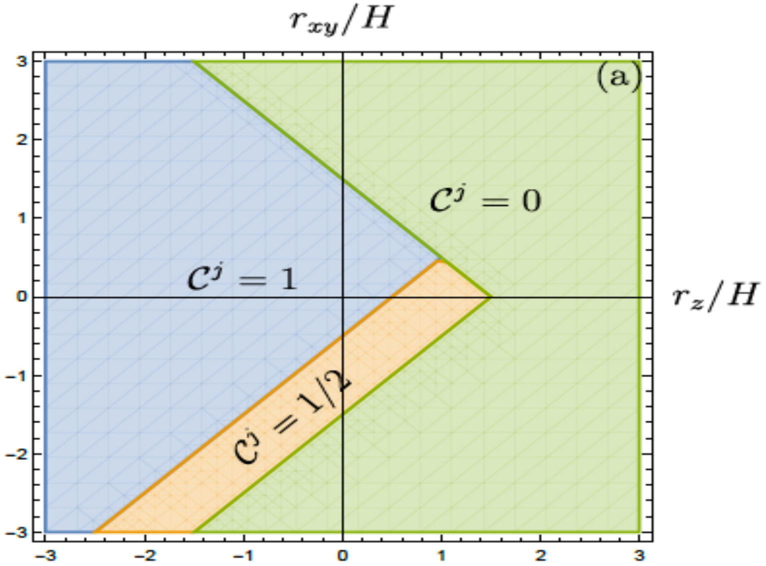
Z_2 symmetry
1 <-> 2 spheres

M_1=M_2=M

$$\boxed{H - M < \tilde{r} < H + M,}$$







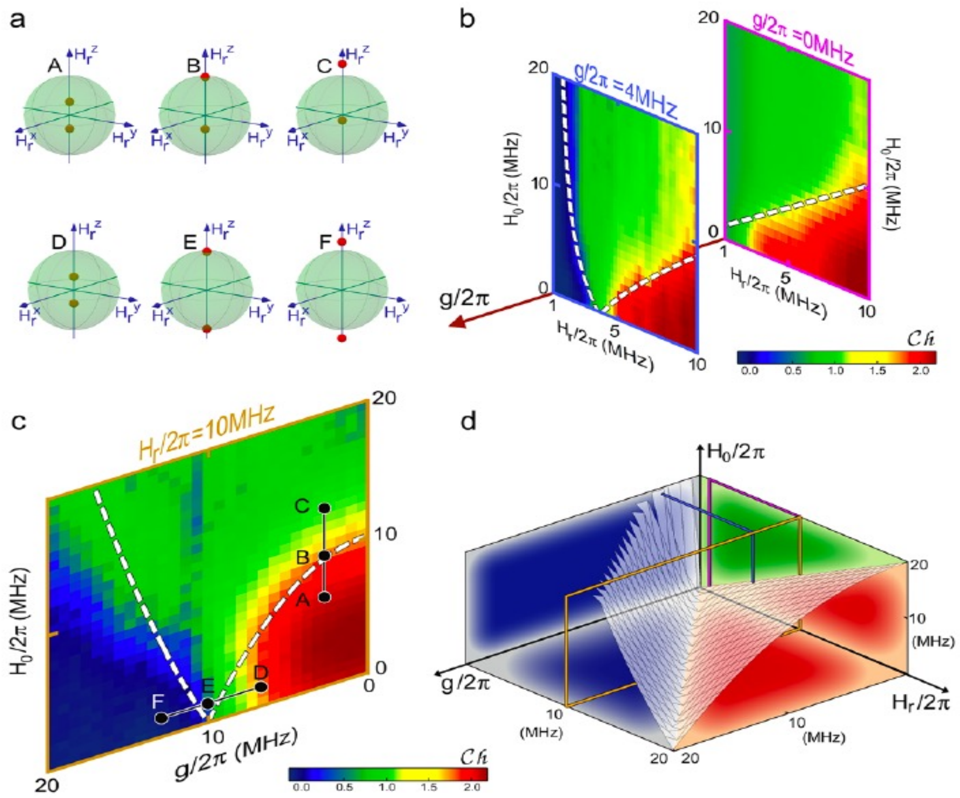
2 spins

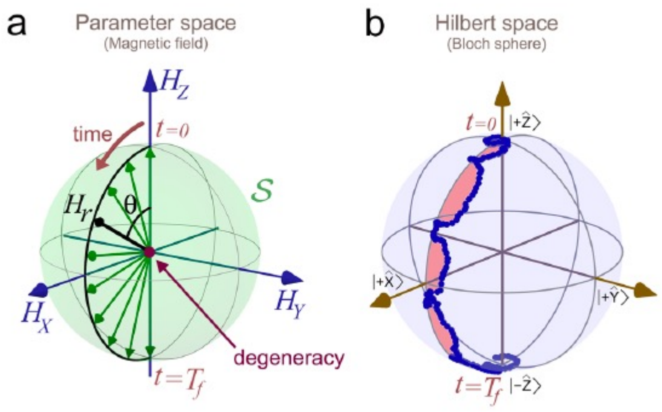
The Hamiltonian of this system is given by

$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)], \tag{5}$$

where 1 and 2 refer to qubit 1 (Q1) and qubit 2 (Q2)

Santa-Barbara “google”:
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)





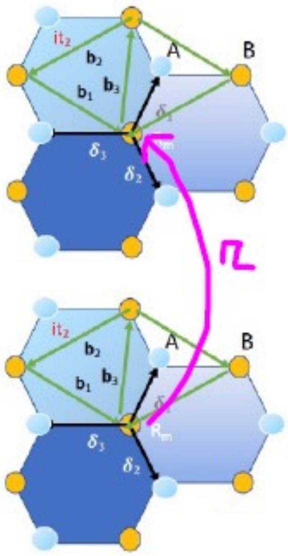
Application in Energy:

Quantum Dynamo effect in a Bath

L. Henriet, A. Sclocchi, P. P. Orth, K. Le Hur 2017 and quantum phase transitions in curved space

New Developments with Ephraim Bernhardt (CPHT) & Cyril Elouard (INRIA & ENS Lyon)

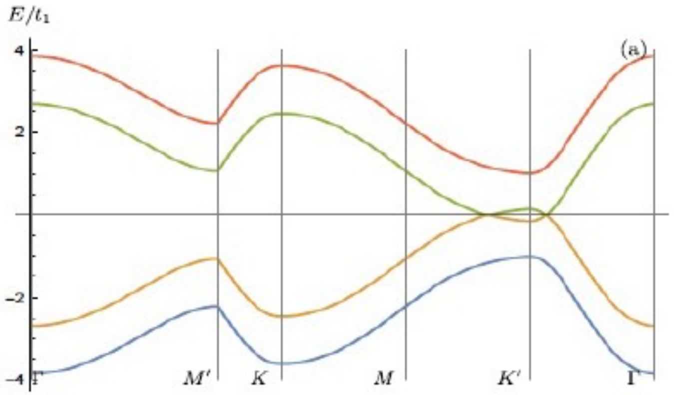
Bilayer system with M_1=M_2



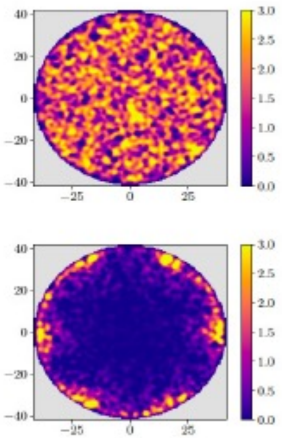
$$\mathcal{H} = (\psi^\dagger_{\mathbf{k}1}, \psi^\dagger_{\mathbf{k}2}) \mathcal{H}(\mathbf{k}) \begin{pmatrix} \psi_{\mathbf{k}1} \\ \psi_{\mathbf{k}2} \end{pmatrix},$$

where $\psi^\dagger_{\mathbf{k}i} \equiv (c^\dagger_{\mathbf{k}Ai}, c^\dagger_{\mathbf{k}Bi})$ and

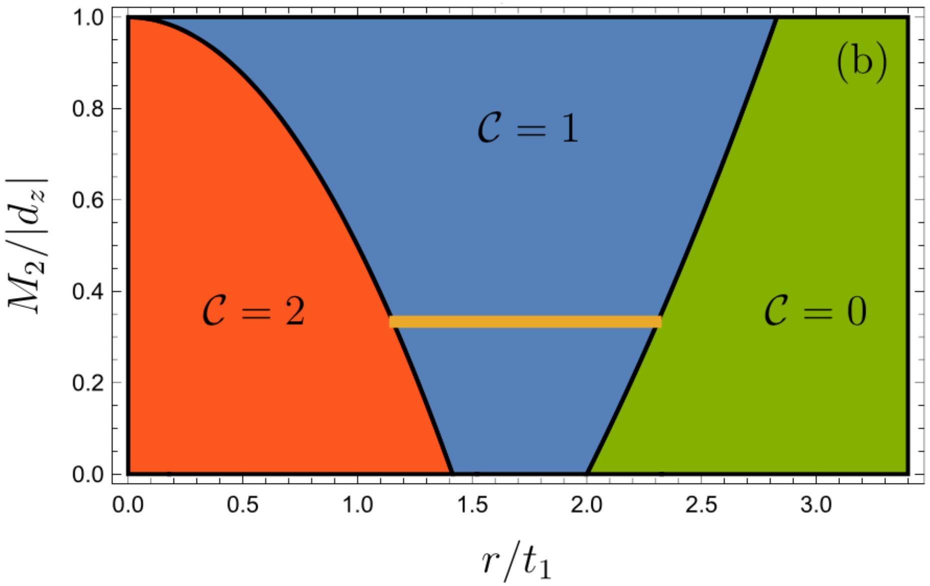
$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} (\mathbf{d} + M_1 \hat{z}) \cdot \boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\mathbf{d} + M_2 \hat{z}) \cdot \boldsymbol{\sigma} \end{pmatrix},$$

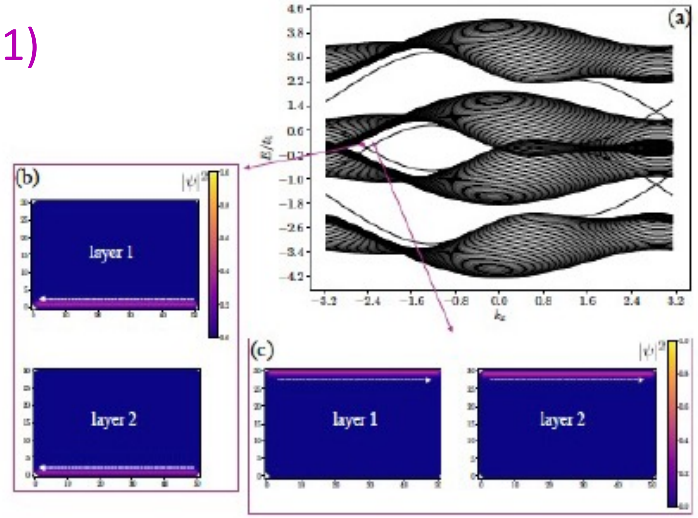


J. Hutchinson and K. Le Hur, Physics 4 144 (2021)



Local density of states





Circular dichroism of light
Jones formalism: average 1 and 0 light responses

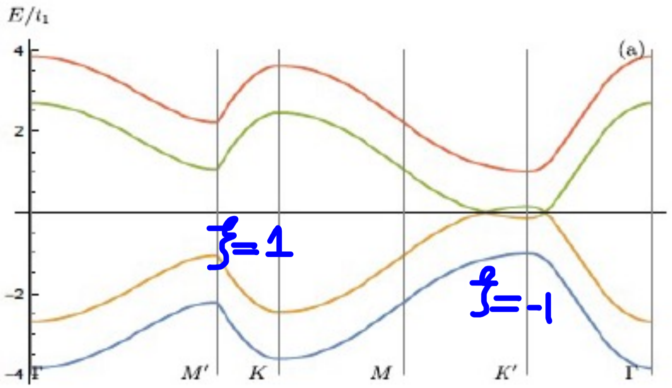
Topological semimetal in two dimensions

Summary of Geometry

$$(J_{xy})^j = C^j \frac{e^2}{h}$$

$$C^j = \frac{1}{2}$$

This formula is correct and is applicable in a sphere (plane j) from the poles (Dirac points)



$$H = (\zeta d_z + M)\sigma_z \otimes \mathbb{I} + d_1\sigma_x \otimes \mathbb{I} + d_{12}\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes \tau_x$$

$$H^2 = \left(|\mathbf{d}|^2 + r^2\right)\mathbb{I} \otimes \mathbb{I} + 2r\mathbf{d} \cdot \boldsymbol{\sigma} \otimes \tau_x \qquad \mathbf{d} = (d_1, d_{12}, (\zeta d_z + M))$$

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

Fractional
Topological Bloch band

$$c^\dagger_{B1}c^\dagger_{B2}|0\rangle = |\uparrow\uparrow\rangle,\ c^\dagger_{A1}c^\dagger_{A2}|0\rangle = |\downarrow\downarrow\rangle,\ c^\dagger_{B1}c^\dagger_{A2}|0\rangle = |\uparrow\downarrow\rangle,\ c^\dagger_{A1}c^\dagger_{B2}|0\rangle = |\downarrow\uparrow\rangle.$$

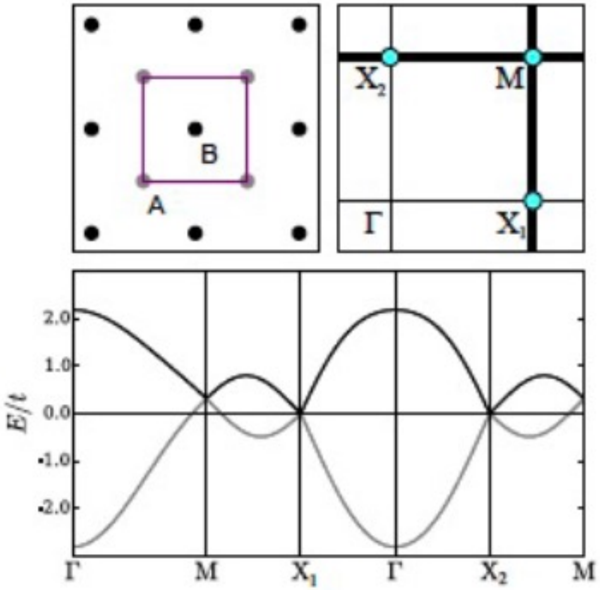
$$\tilde{C}^j = \frac{1}{2}\langle \underbrace{n^j_{KB} - n^j_{KA}}_{\sigma_z} - n^j_{K'B} + n^j_{K'A}\rangle = \frac{1}{2}$$

Table & topological semimetals

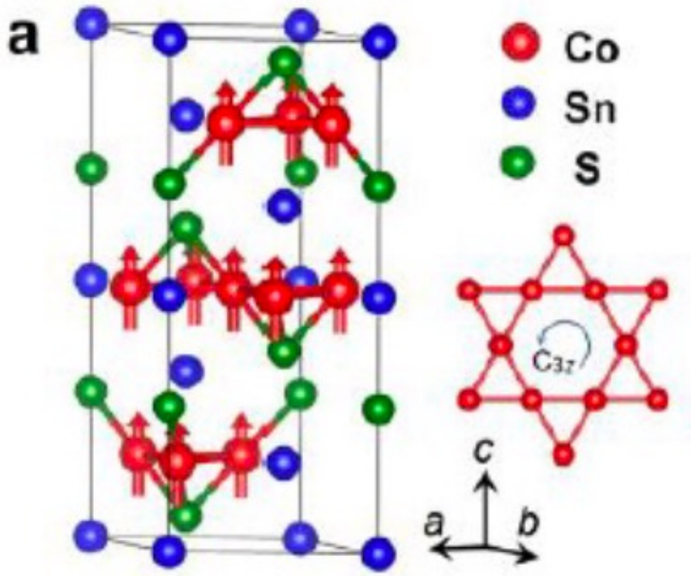
Dirac Semimetals in Two Dimensions

S. M. Young & C. L. Kane, PRL 2005

$$H = 2t\tau_x \cos\frac{k_x}{2}\cos\frac{k_y}{2} + t_2(\cos k_x + \cos k_y) + t^{\mathrm{SO}}\tau_z[\sigma_y \sin k_x - \sigma_x \sin k_y],$$

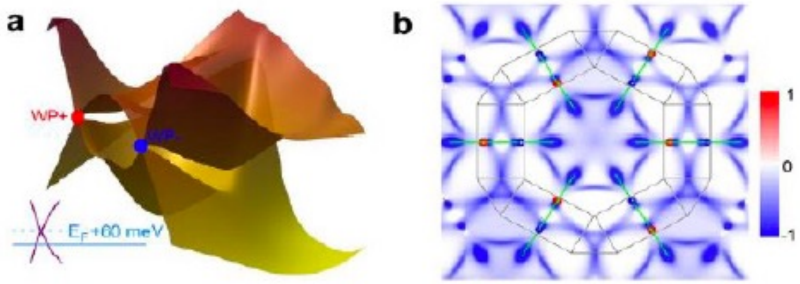


ferro



Magnetic Weyl semimetals in 3D: recent

Z. Guguchia et al., group of Zahid Hasan, Princeton
E. Liu et al., group of Claudia Felser Dresden
Figures from Enke Liu, Berry curvature k_x-k_y plane



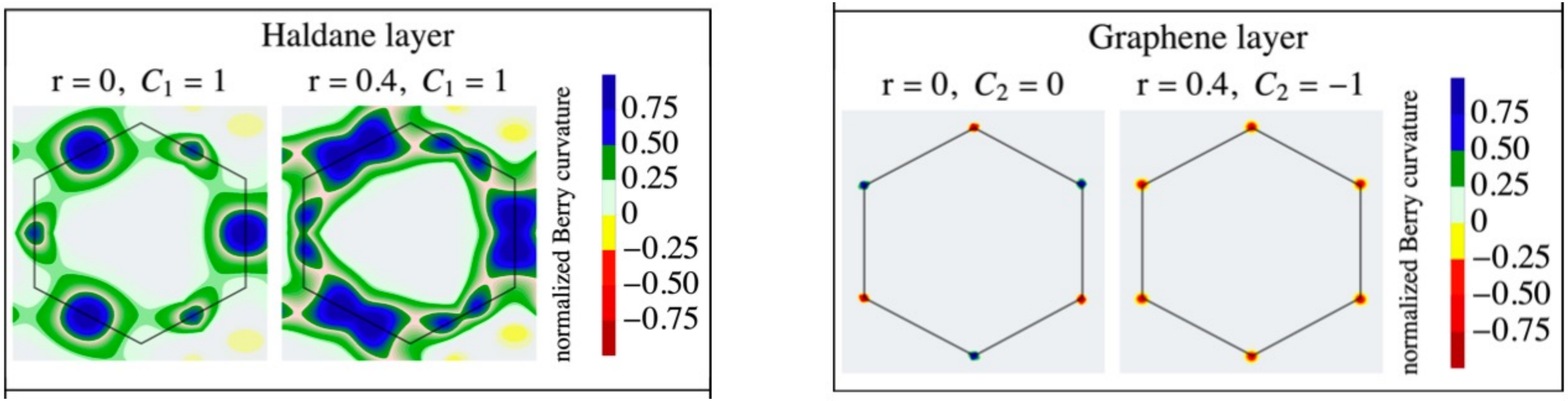
Anomalous Hall conductivity

2D model
Julian Legendre & Karyn Le Hur
Phys. Rev. Research, 2020

Topological Proximity Effects

Bulk topological proximity effect: T. H. Hsieh, I. Ishuzika, L. Balents, T. Hughes, Phys. Rev. Lett. 086802 (2016)
J. Panas, B. Irsigler, J.-H. Zheng, W. Hofstetter, Phys. Rev. B 102, 075403 (2020)

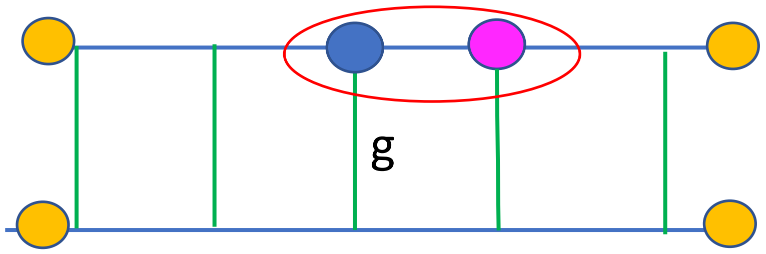
Topological Proximity effect in graphene coupled to an Haldane Model:
(thanks to DFG FOR2414 for funding)



P. Cheng, Ph. W. Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg, K. Le Hur, Phys. Rev. B 100, 081107 2019

New Efforts with Sariah Al Saati and Julian Legendre at CPHT

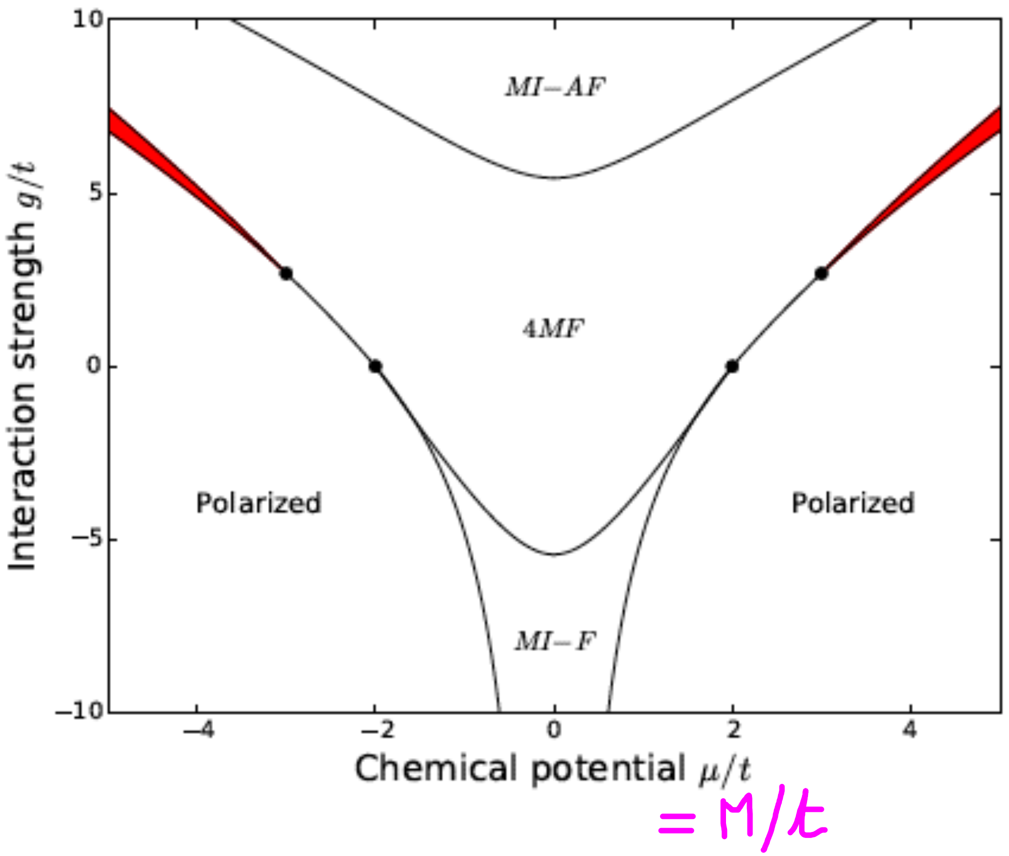
Superconductivity, Topological Aspects and Coulomb Interaction



2 interacting p-wave Kitaev Superconducting wires

Loic Herviou, Christophe Mora, Karyn Le Hur 2017

New Efforts with Frederick del Pozo, CPHT: relation two c and C=1/2



Kitaev wire is a BdG Hamiltonian $(c_k, c^\dagger_{-k})^T$

$$H(k) = \frac{1}{2}\begin{pmatrix} \epsilon(k) & 2i\Delta e^{-i\tilde{\varphi}}\sin(ka) \\ -2i\Delta e^{i\tilde{\varphi}}\sin(ka) & -\epsilon(k) \end{pmatrix}$$

$$H_{\text{int}} = g\sum_j (n_{j,1} - \frac{1}{2})(n_{j,2} - \frac{1}{2}),$$

Quantum field theory tools (Luttinger approach, RG equations)
Density Matrix Renormalization Group Approach
Quantum Information Tools (Entanglement Entropy, Bipartite Fluctuations)

Conclusion

New Insight On Correlated Topological Matter from Reciprocal Space and Bloch Spheres

- Application in quantum Transport

- Response to Circularly Polarized Light quantized

- Stochastic Approach to include Interaction Effects

Fractional Entangled Topology from the curved space, interactions between spheres
Real Applications: mesoscopic & atomic systems, topological semimetals, superconducting wires,
coupled planes & 3D models in cubes

Thanks to the group members and new developments soon …

Funding: Thanks to Ecole Polytechnique, ANR BOCA, DFG FOR2414 and NSERC

Thanks to the organizers and for your Attention