

# FROM NODAL LIQUID TO NODAL INSULATOR

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**John Hopkinson** (Toronto)

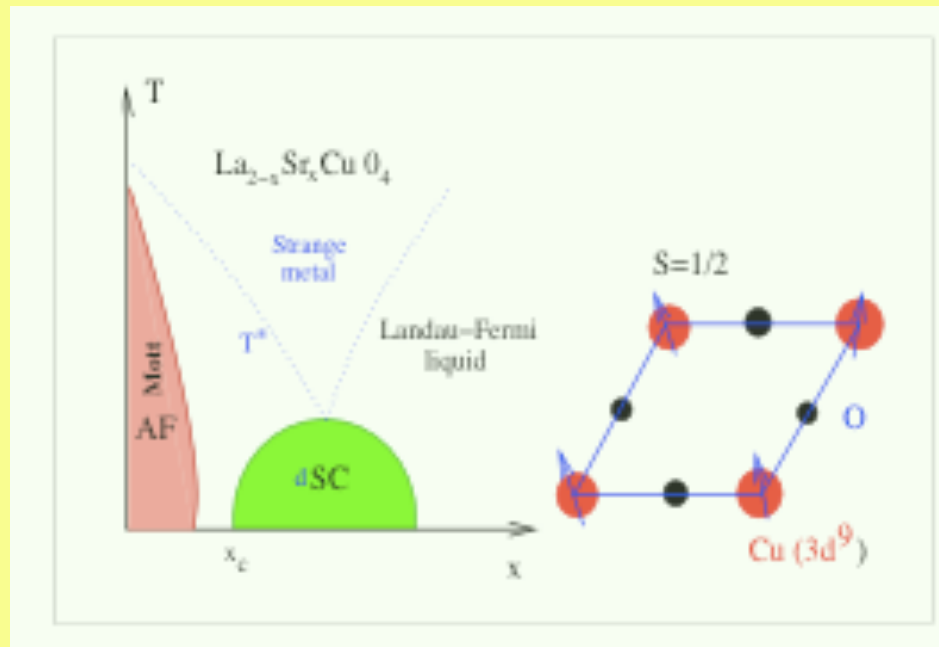
Fonds de recherche  
sur la nature  
et les technologies

Québec 

**GORDON, 2004, Oxford**



# Doped Mott insulator?



**Mott physics:  $U$**

**Antiferro fluctuations:  $J$**

**SC fluctuations above  $T_c$**

Nernst effet?

**Strange metal** at optimal doping:  $\rho(T) \sim T$

**Pseudo-gap phase  $T \leq T^*$ : reduct<sup>o</sup> of  $e^-$ , spin excitat<sup>o</sup> ...;**

possibly, an **RVB spin liquid** “**Small Fermi surface**”  
recent cond-mat of Anderson, Lee, Rice, Zhang, Randeria et Trivedi

# RVB $\leftrightarrow$ BCS nicely connected

$$|\Psi\rangle = \exp(iS) P_G |\text{BCS}\rangle$$

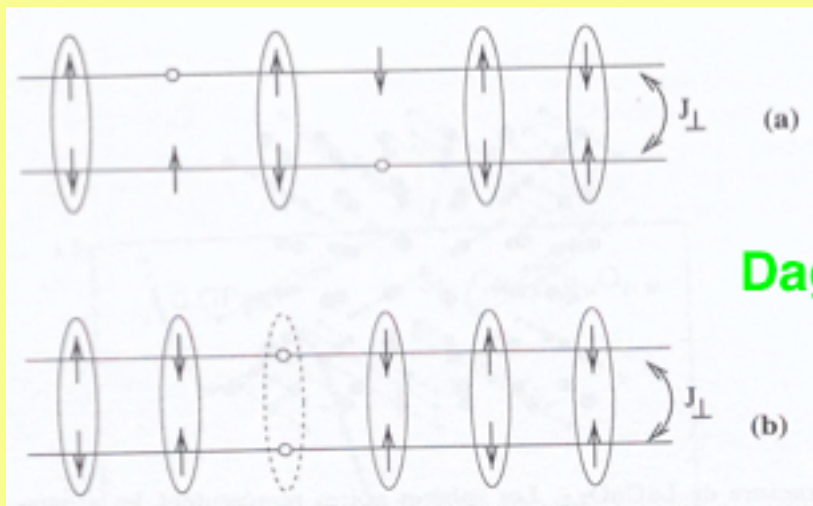
$P_G$  Gutzwiller projector

**BUT  $\langle \psi | O | \psi \rangle$  difficult to compute...**

## EXCITATIONS?

- Gutzwiller approximations: F.C. Zhang, Anderson
- Variational Quantum Monte Carlo: M. Randeria
- two-patch model in 2D: RG+exact diagonalization (A. Laeuchli, K. Honerkamp, and M. Rice)
- quasi-1D or ladder-type approaches: my talk...

# Two-leg ladder superconductor



Shelton and Tsvetlik, 1996

Dagotto & Rice, Science 271, 618 (1996)

*1D system: no long-range order - Bethe, before the 2nd war*

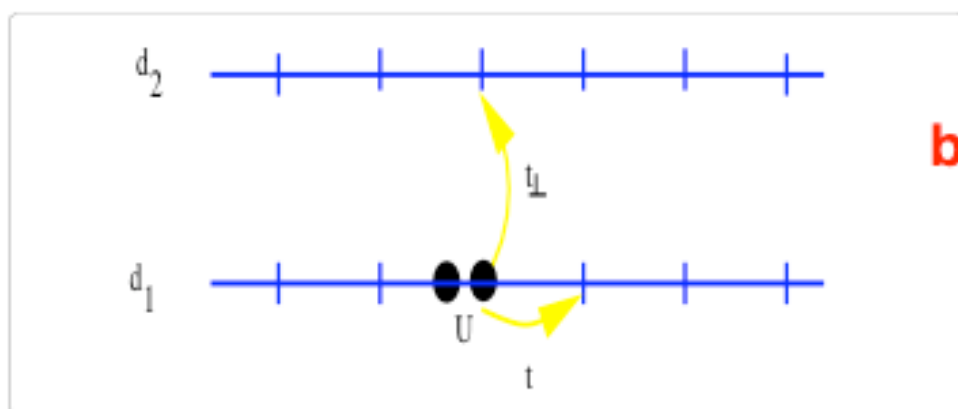
**Undoped case: RVB system with Spin Gap**

*P.W. Anderson, Science 235, 1196 (1987)*

**Doping: d-wave like superconductivity**

$$\langle \Delta^\dagger(x) \Delta(0) \rangle \propto x^{-1/2}$$

# weak-coupling approach for RVB

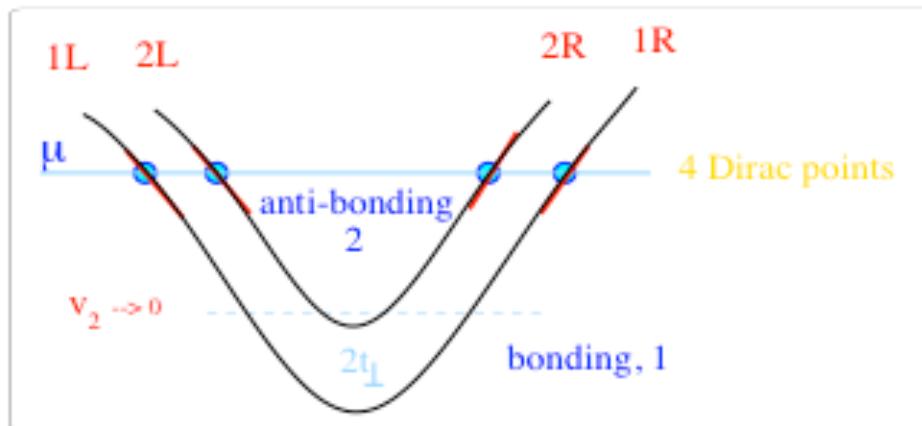


**bonding-antibonding bands  $\Psi_{1,2}$**

$$\Psi_{1,2} = \frac{1}{\sqrt{2}} \{d_1 \pm d_2\}$$

Limit of large  $t_{\perp} \gg U$

Band structure:



$$\epsilon_j(k) = \mp t_{\perp} - 2t \cos(k)$$

Half-filling:  $v_1 = v_2$   
and  $k_{F1} + k_{F2} = \pi$

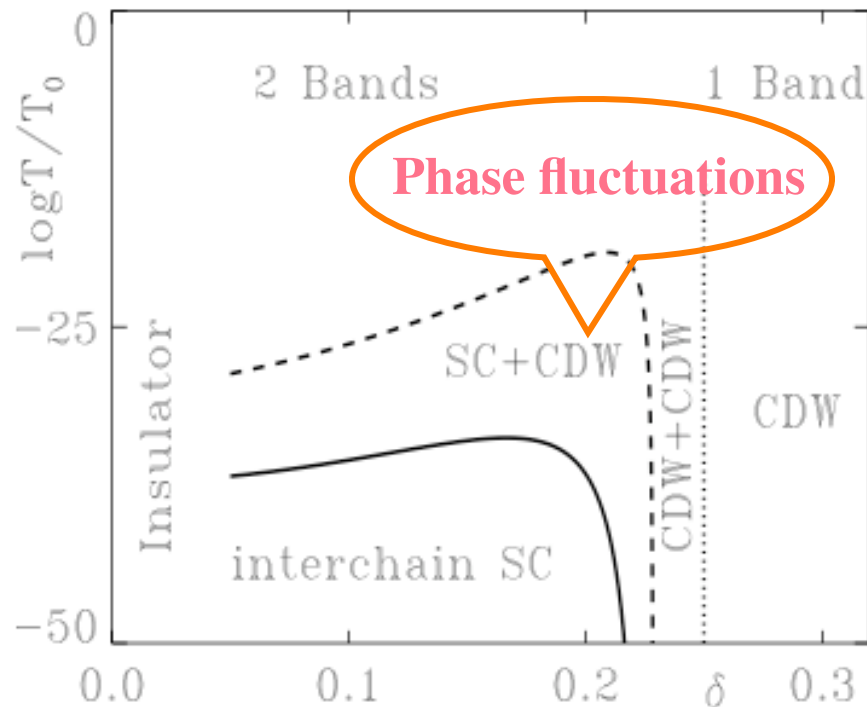
Large Doping:  $v_2 \ll v_1$

Half-filling: 7 Couplings diverge with a fixed ratio

“SO(8) Gross-Neveu Model”

Spin- & Charge Gaps  $\propto \exp -\pi v_1/U$

Short-Range RVB insulating system  
with preformed Cooper pairs



Doping:

Cooper pairs liberated

Symmetry SO(6)

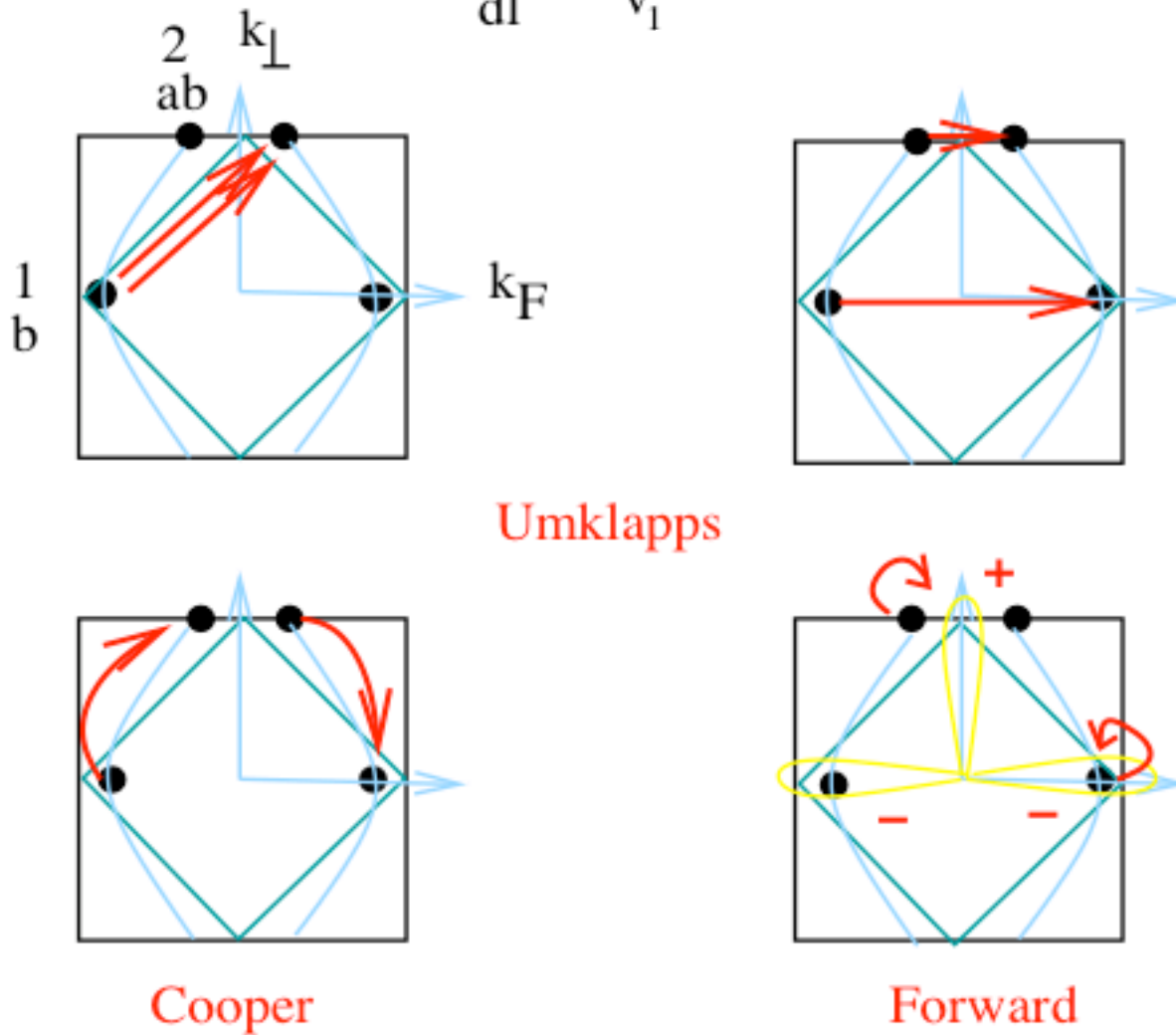
Symmetry SO(5) with  
Coulomb term

Lin-Balents-Fisher  
Schulz

2D: Demler-Zhang

# Some deep connection with the antinodal physics in 2D

$$\frac{dgi}{dl} = \frac{1}{v_1} (\dots) gjgk; \quad l = -\ln(T; \omega)$$



How to include the nodal direction?

# Ladders: Route to High-Tc materials

2-leg ladder, Nice prototype system:

“doping RVB material, d-wave superconductivity”

(Rice *et al.* - Schulz - Balents and MPA Fisher - Emery-Kivelson...)

3-leg ladder, still better:

Focus both at nodal and antinodal points

Possibility of truncation of FS and significance  
in real space (holes in a d-RVB state)

***U. Ledermann, K. Le Hur and T.M. Rice, PRB 62, 16383 2000***

***J. Hopkinson and K. Le Hur, PRB 69, 245105 2004***



# Truncation of the Fermi surface

$$\epsilon_j(k) = -2t \cos(k) - 2t_{\perp} \cos(k_{Fj}^y)$$

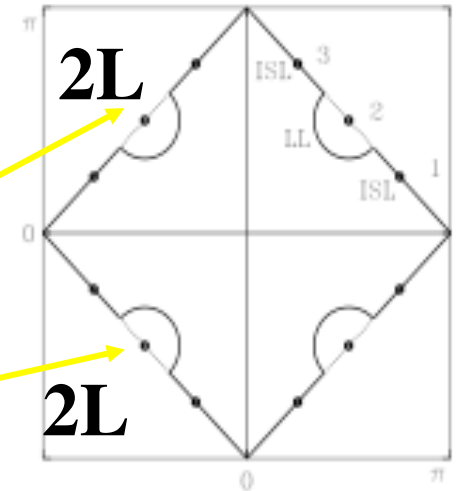
## 2D Mapping

Longitudinal Fermi momentum of the band  $j$ :

$$\mathbf{k}_{Fj} = \pi - \arccos \left[ \frac{t_{\perp}}{t} \cos \left( \frac{\pi j}{N+1} \right) \right]$$

Corresponding transverse Fermi momentum:

$$\mathbf{k}_{Fj}^y = \pm \frac{\pi j}{N+1}$$

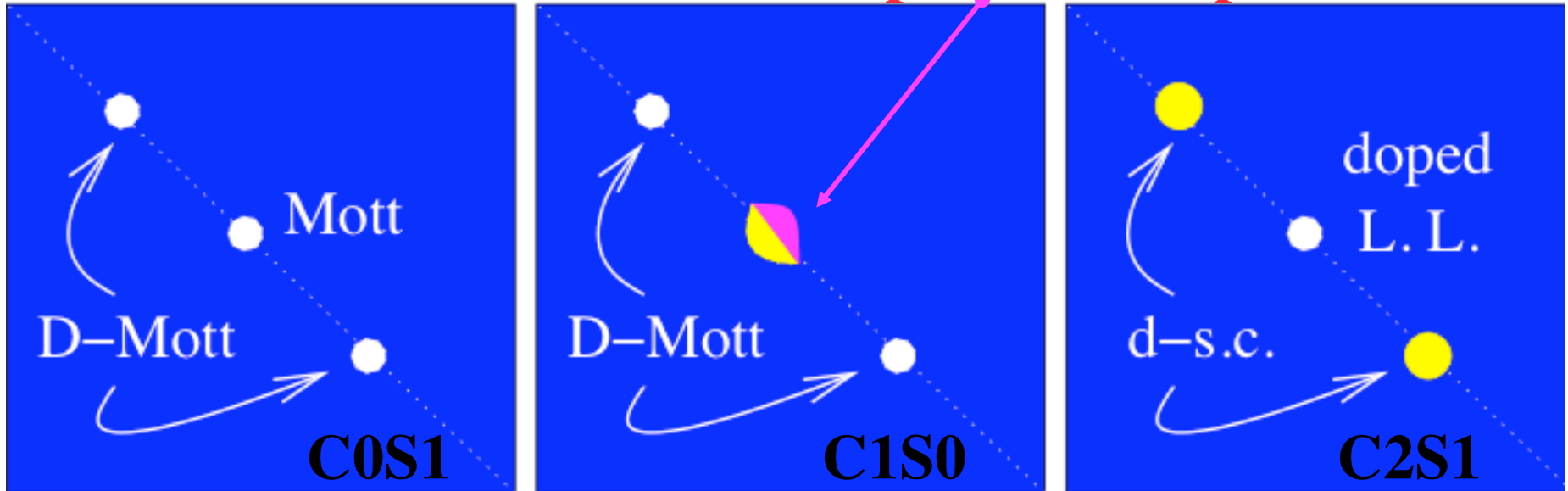


**for open boundaries in y direction**

# HOT spots and COLD spots

*Gapful points and gapless points*

Spectral  $f^0$  = power law



**Nodal liquid**

**D-wave SC**

**& preformed pairs**

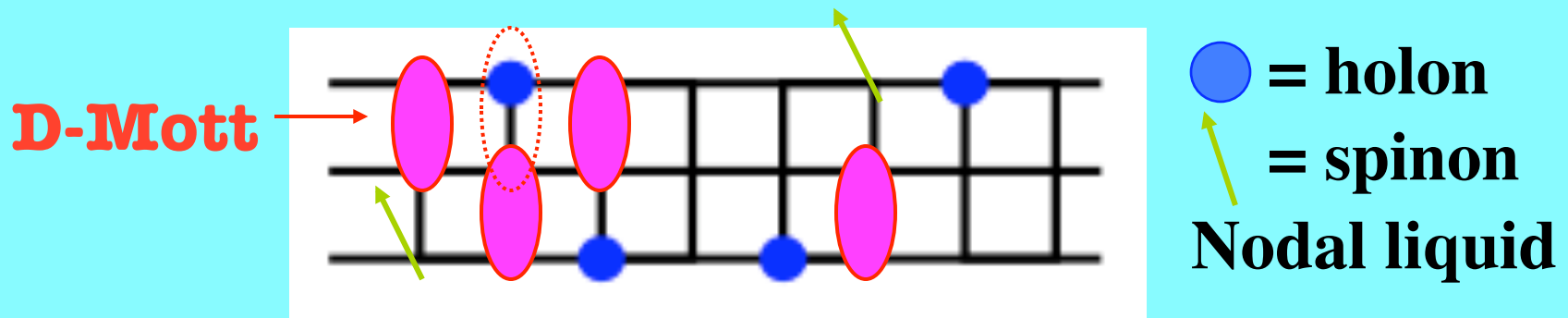
*insulating (2D, Balents et al.)*

***U. Ledermann, K. Le Hur and T.M. Rice, PRB 62, 16383 2000***

**Strong U: T.M. Rice, S. Haas, M. Sigrist and F.C. Zhang, PRB 97**

**DMRG: White & Scalapino**

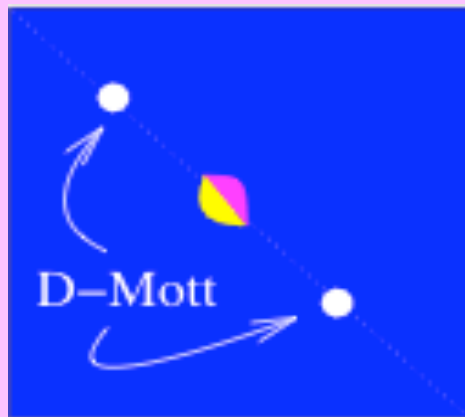
# Picture in real space



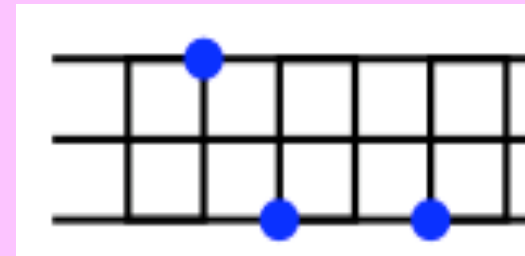
Conduct<sup>o</sup> by holons at the edges: **Cooper pairs frozen but resonating**

**Nodal liquid fragile against disorder**  
**Karyn Le Hur, unpublished**  
**following Giamarchi & Schulz**

# Effect of long-range repulsion?



**Wigner  
crystal of  
holes?**



**cond-mat/0406038**

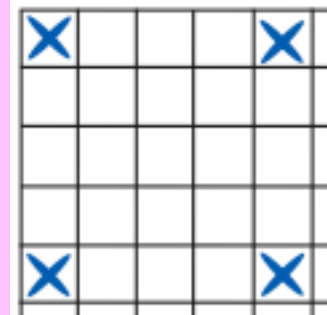
**A suggested  $4 \times 4$  structure in underdoped cuprate superconductors: a Wigner supersolid.**

P. W. Anderson

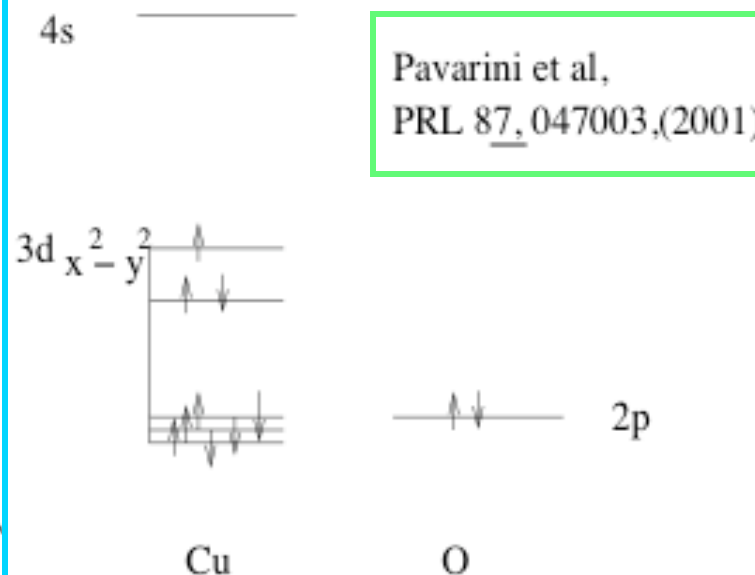
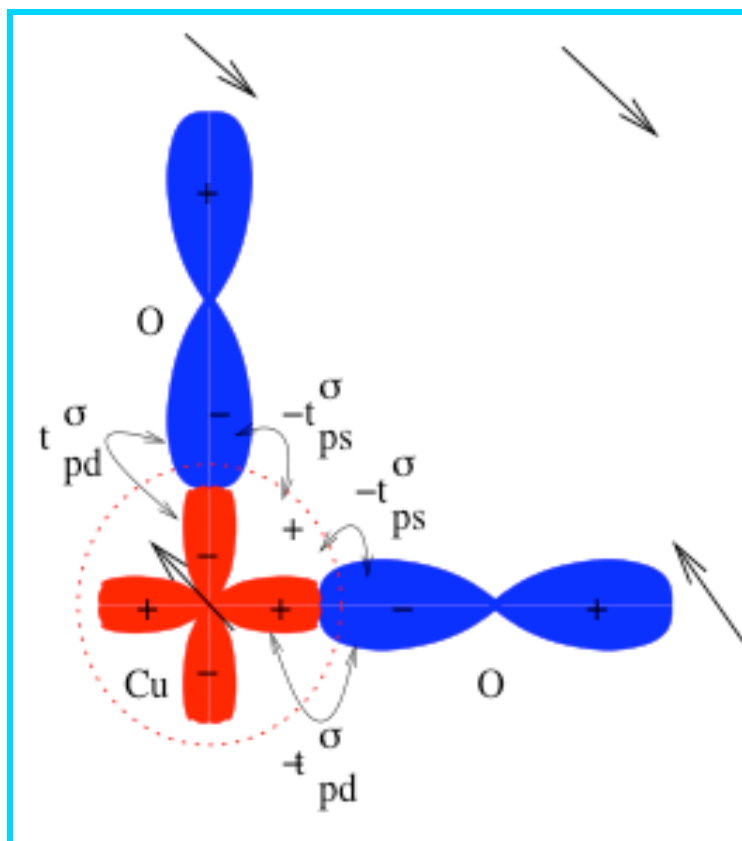
*Department of Physics, Princeton University, Princeton, N.J. 08544, U. S. A.*

(Dated: June 3, 2004)

A wave function is proposed for the " $4 \times 4$ " inhomogeneous structures observed on cuprate superconductors. It is based on the Gutzwiller-RVB technique proposed in recent papers, and consists of a Wigner solid of hole pairs embedded in a sea of  $d$ -wave spin singlet pairs. Arguments are given that the nodal quasiparticles may remain unscattered and even superconducting on such a structure.



# The need for *frustrated* hopping: Chemistry

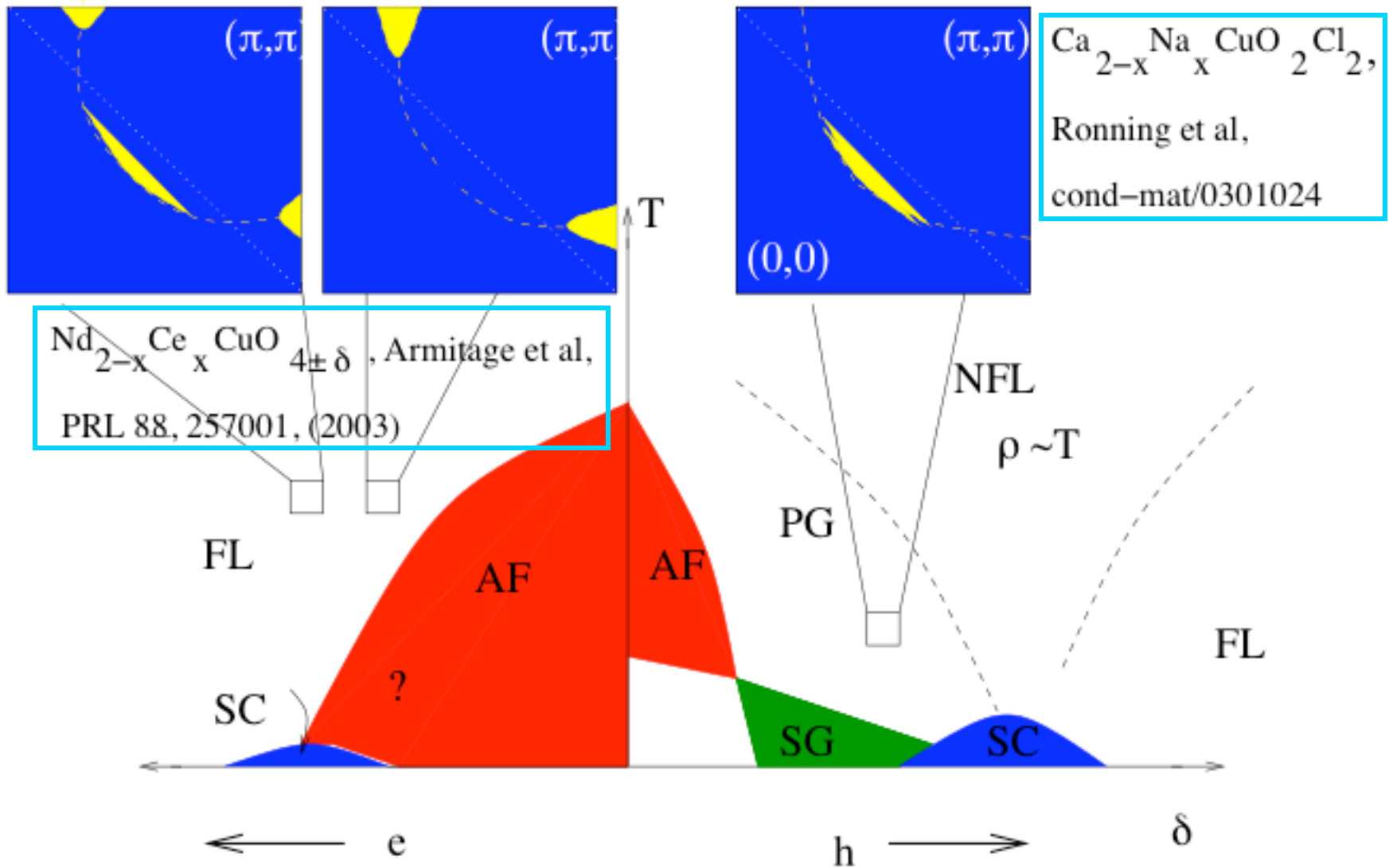


4 band  $\Rightarrow$  1 band +  $4t' \cos(k_x) \cos(k_y) + \dots \Rightarrow$  curved F.S.

$e \leftrightarrow h$  symmetry broken

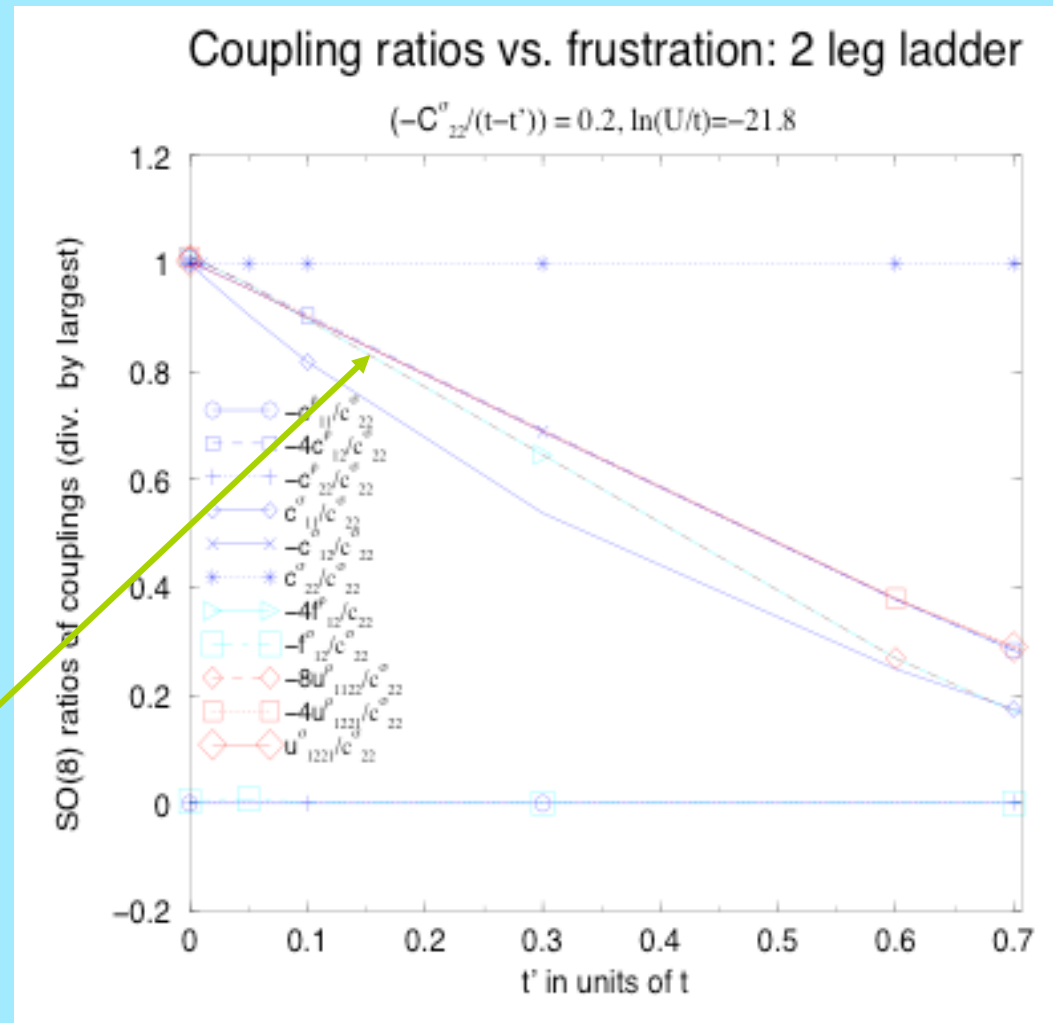
**Opposite sign!**

# High-Tc: doped Mott insulators



# Effect of $t'$ on the 2-leg ladder physics

**John Hopkinson & K. Le Hur, PRB 69, 245105 2004**

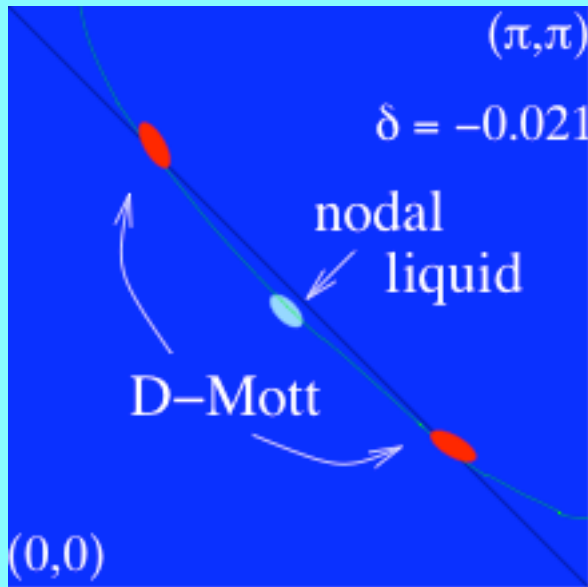


**Link with  
2D-patch  
Model...**

**Breaking SO(8) symmetry but still charge and spin gaps**

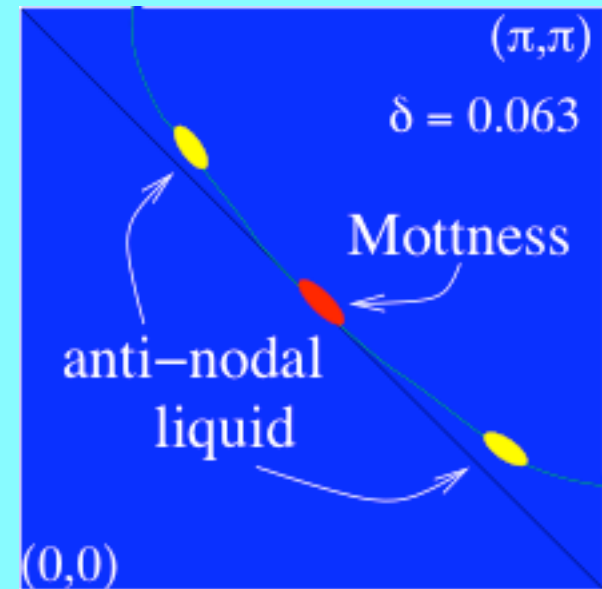
# Revisiting the 3-leg Hubbard ladder with $t'$

Possibility to separate umklapp processes!



*Hole doping*

**Nodal liquid+D-Mott**



*Electron doping*

**Nodal insulator+  
d-wave pairing**

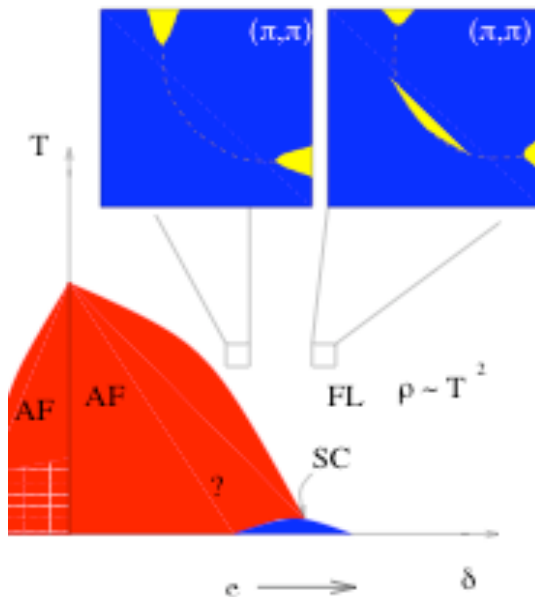
**Interpretation in real space?**



# Main message

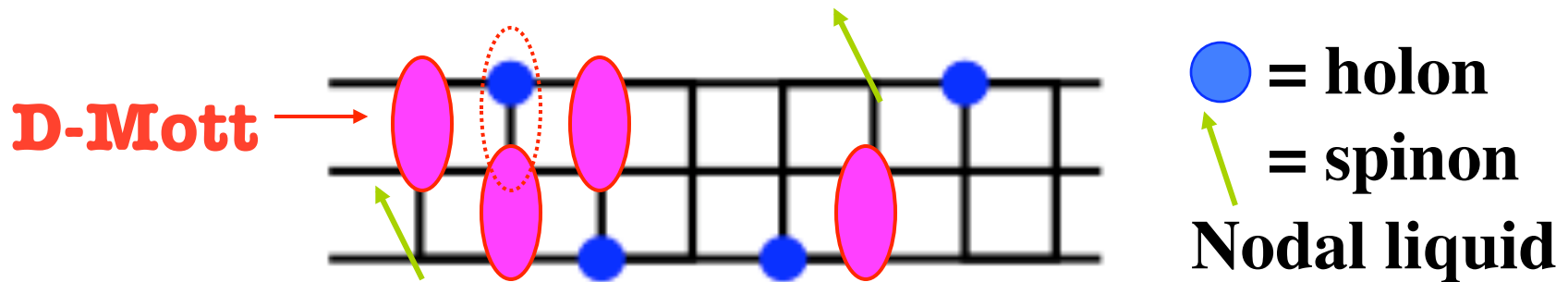
**Hole doped: pseudogap = d-RVB state from corners  
short-range magnetism**

**Electron doped: pseudogap = Mottness at the nodes  
dominant SDW**

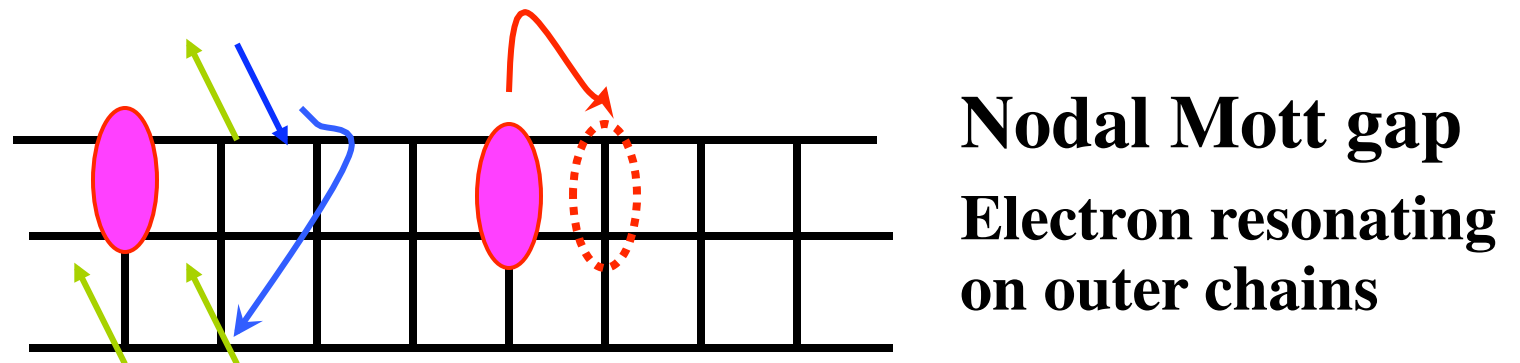


effect on low-T thermal  
conductivity for low doping  
on SC phase

# Pictures in real space



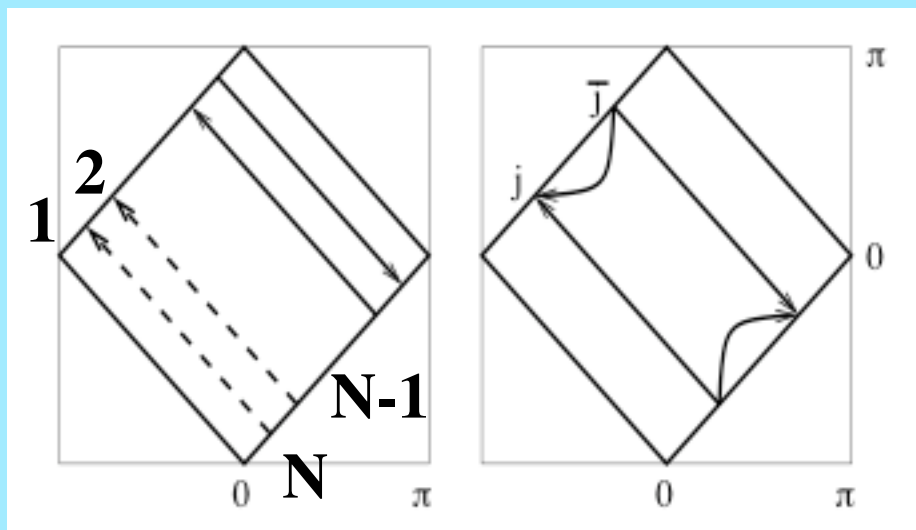
**Conduct<sup>o</sup> by holons at the edges: Cooper pairs frozen but resonating**



**Electron doped case**

**Cooper pairs conducting**

## Close to half-filling: Antiferromagnetism



*Urs Ledermann, PhD*

Spin gap suppression for N even

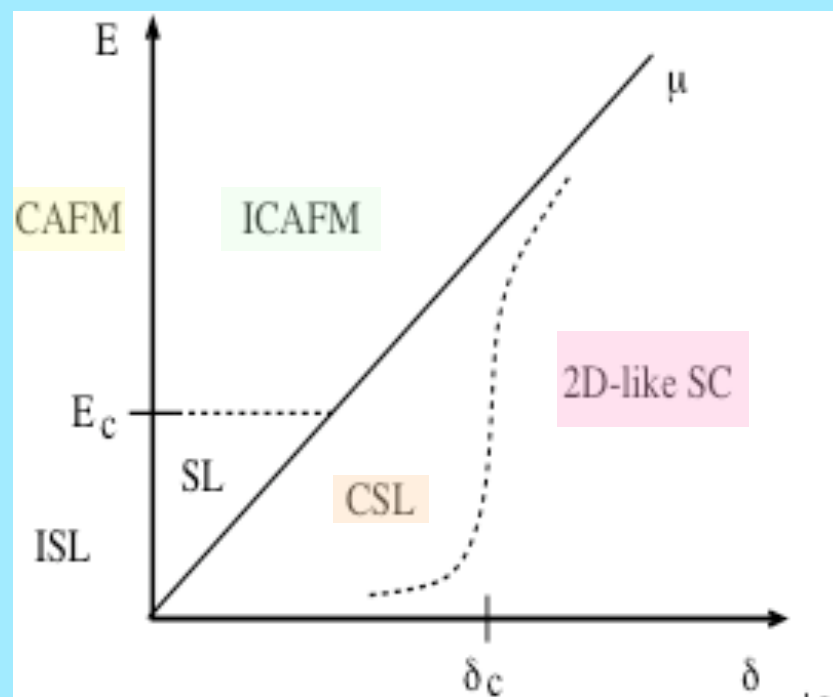
Doping = Decrease N

Expansion of phase coherence towards antinodal points

**Umklapps on whole FS**

Uniform Mott gap

Long-range AF at T=0



**3-leg ladder: beautiful prototype system**  
**Truncated Fermi surface: nodal- liquid or Mott gap**  
**Breaking p-h symmetry with  $t'$  hopping term**

*Many open questions in 2D:*

**2D: nodal liquid or Fermi arcs above  $T_c$ ?**

Thermal conductivity: d-wave symmetry above  $T_c$

ARPES: Fermi arcs

**Recent efforts from theory to understand high- $T_c$**

TPSC approach (Tremblay)

Cluster DMFT (Kotliar)

Variational Quantum Monte Carlo

(Paramekanti, Trivedi, Randeria)

3-patch model in 2D?

# N-leg ladder?

- Nband model with N Fermi velocities  $v_j$ :

$$\text{At half-filling: } v_j = v_{\bar{j}} = 2\sqrt{t^2 - \{t_{\perp} \cos[\pi j/(N+1)]\}^2}$$
$$\bar{j} = N + 1 - j$$

$$v_1 = v_N < v_2 = v_{N-1} < \dots$$

**These Fermi velocities will lead to a hierarchy of energy scales**

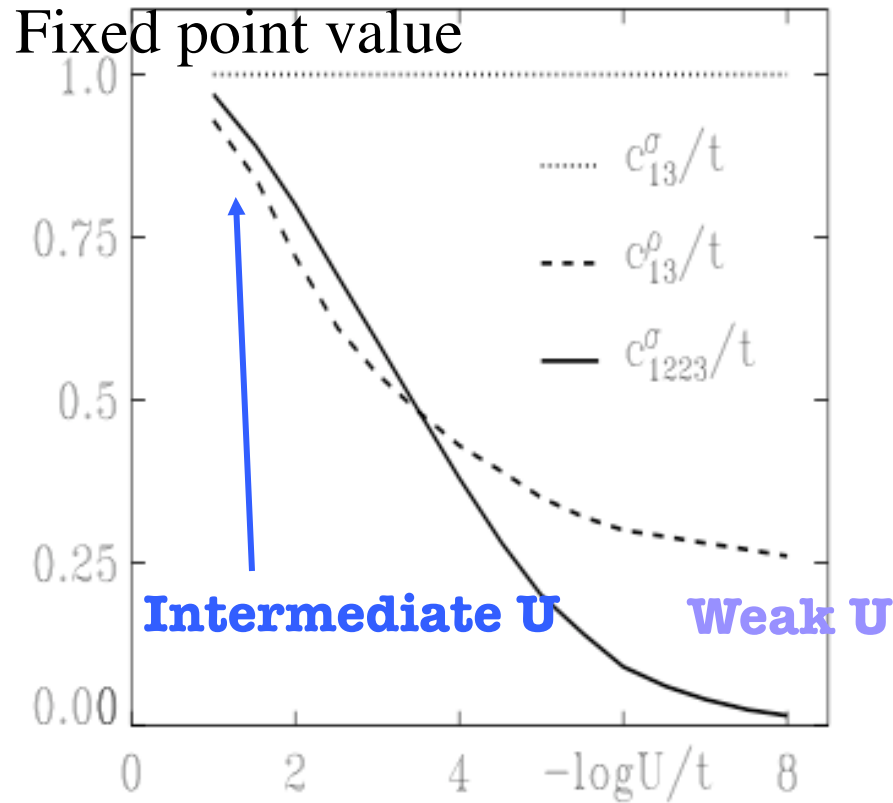
hierarchy of energy-scales (insulating band pairs)  $\mathbf{T}_j \sim e^{-v_j/U}$

*Band pairs flow towards a two-leg ladder*

*Rigorous decoupling at weak U*

*U. Ledermann, K. Le Hur, and T.M. Rice PRB 62, 16383 2000*

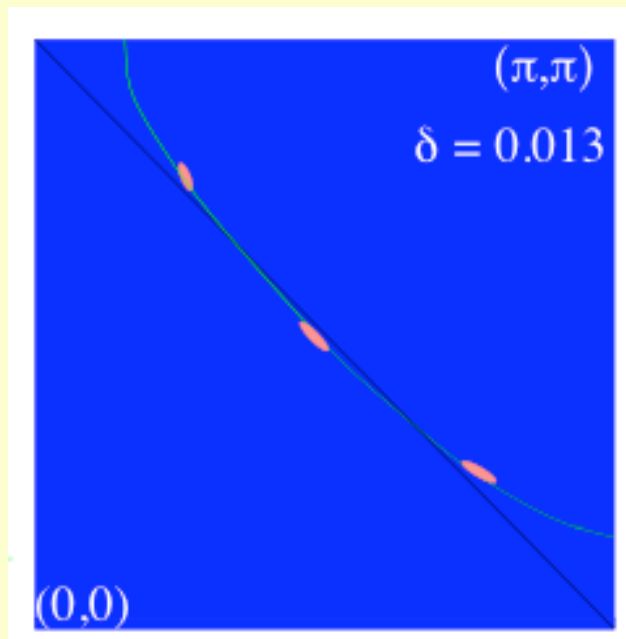
21 coupling constants!



- **N even: Spin gap**  
“disordered d-wave superconductor”: 2-chain
- **N odd: No spin gap**  
“deconfined spinons”: Insulating Single chain

**Even-Odd effect like for spin ladders**  
*SrCu<sub>2</sub>O<sub>3</sub>* Spin gap / *Sr<sub>2</sub>Cu<sub>3</sub>O<sub>5</sub>* No spin gap

# Very-close to half-filling



**3-band umklapp relevant:  
tendency to uniform Mott gap**

**Difficult to tackle theoretically  
Precursor of AF in 2D**

*Pathology of our weak-U treatment: umklapps relevant  
only for specific dopings*

*Larger U: Variational Quantum Monte Carlo, DMRG*

# Expectation in 2D with $t'$

velocity  $v_2$   
changes with  $\delta$

