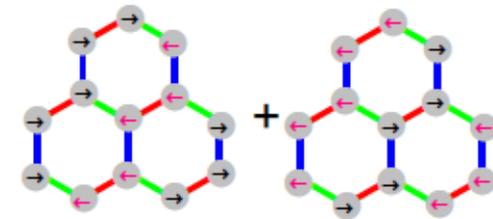
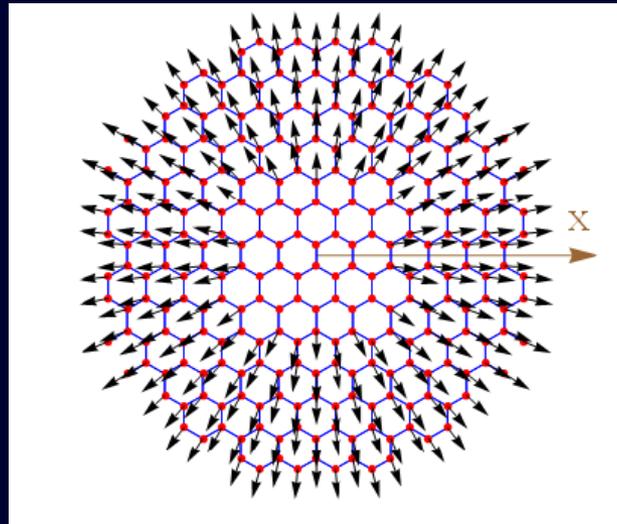
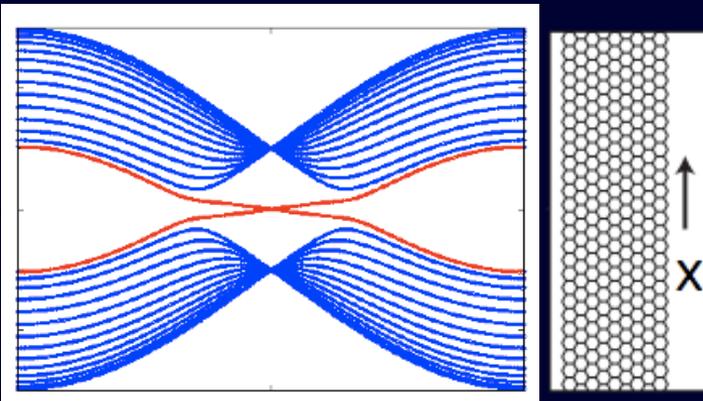


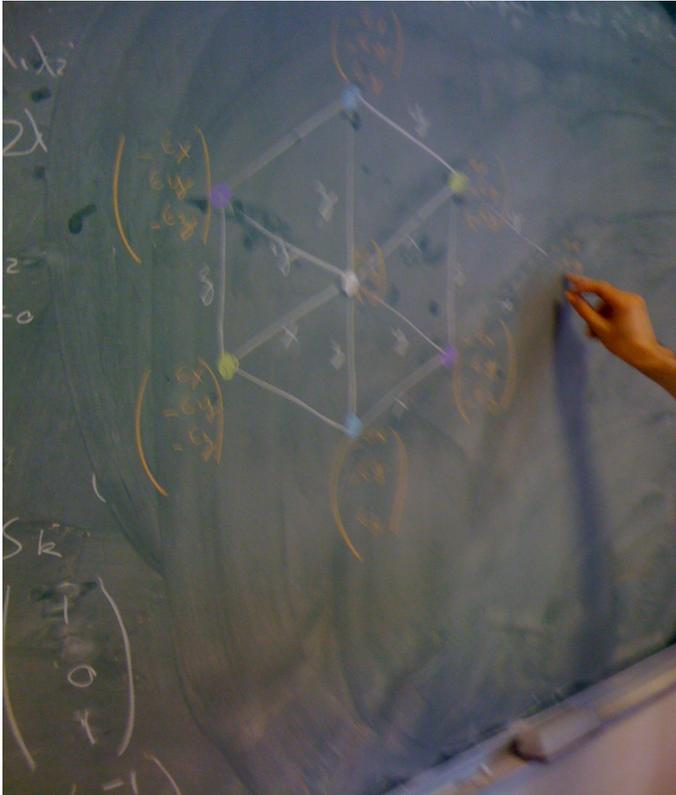
Topological Phases on the Honeycomb Lattice: Spin-Orbit Coupling, Correlations & Superconductivity

Karyn Le Hur

CPHT Ecole Polytechnique (X), France & CNRS



New phases of matter with electrons (fermions)



B. Douçot with the honeycomb lattice...

I) Topological Insulators Interaction Effects

iridate materials, cold atoms

*II) SC on honeycomb lattices:
d+id SC starting from half-filling
2 possible ground states
(related to talk by Y. Ran)*

*New honeycomb materials
Doped iridate materials (?)*

Team Acknowledgements



Wei Wu
now in Paris
(college France)



Stephan Rachel
TU Dresden



Alexandru
Petrescu
Yale & X
PhD 2015



Tianhan Liu, X &
LPTHE **B. Douçot**,
PhD 2015



D. Bergman



Peter Orth (now KIT)



Loic Henriët
X
PhD 2016



Ivana Vasic
Frankfurt
Serbia



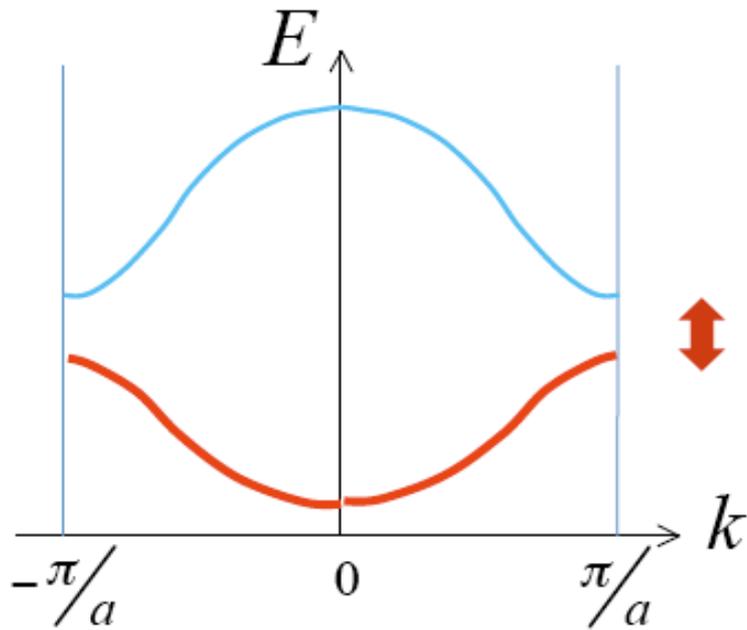
Annica
Black-Schaffer
(prof. Uppsala)

W.-M. Liu (KITP China), Walter Hofstetter (Frankfurt), Cécile Repellin (ENS), Nicolas Regnault (Princeton/ENS)
T. Maurice Rice (ETH), C. Honerkamp (Aachen), M. Scherrer, C.-H. Chung (Taiwan)

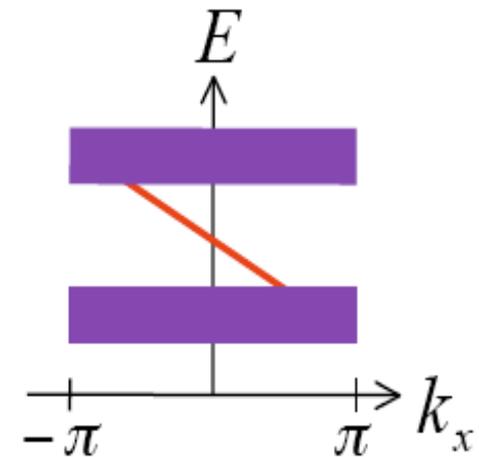
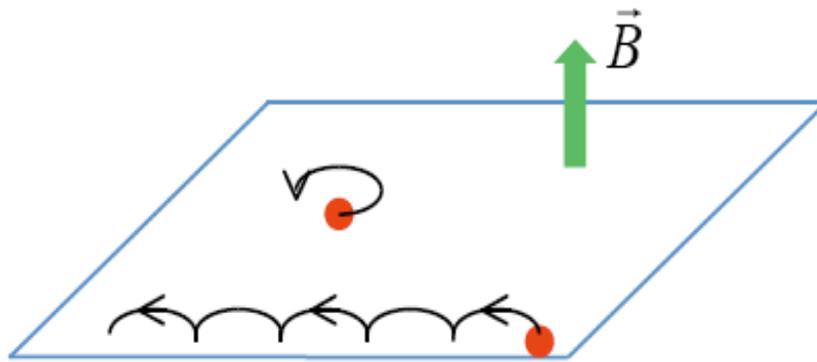
On-going work with **Loic Herviou (PhD, X)** and C. Mora (ENS) on Majoranas in quasi-1D

Work with **A. Petrescu (PRL2013, PRB 2015)**, M. Piraud, **K. Plekhanov**, G. Roux on Topo-phases with bosons

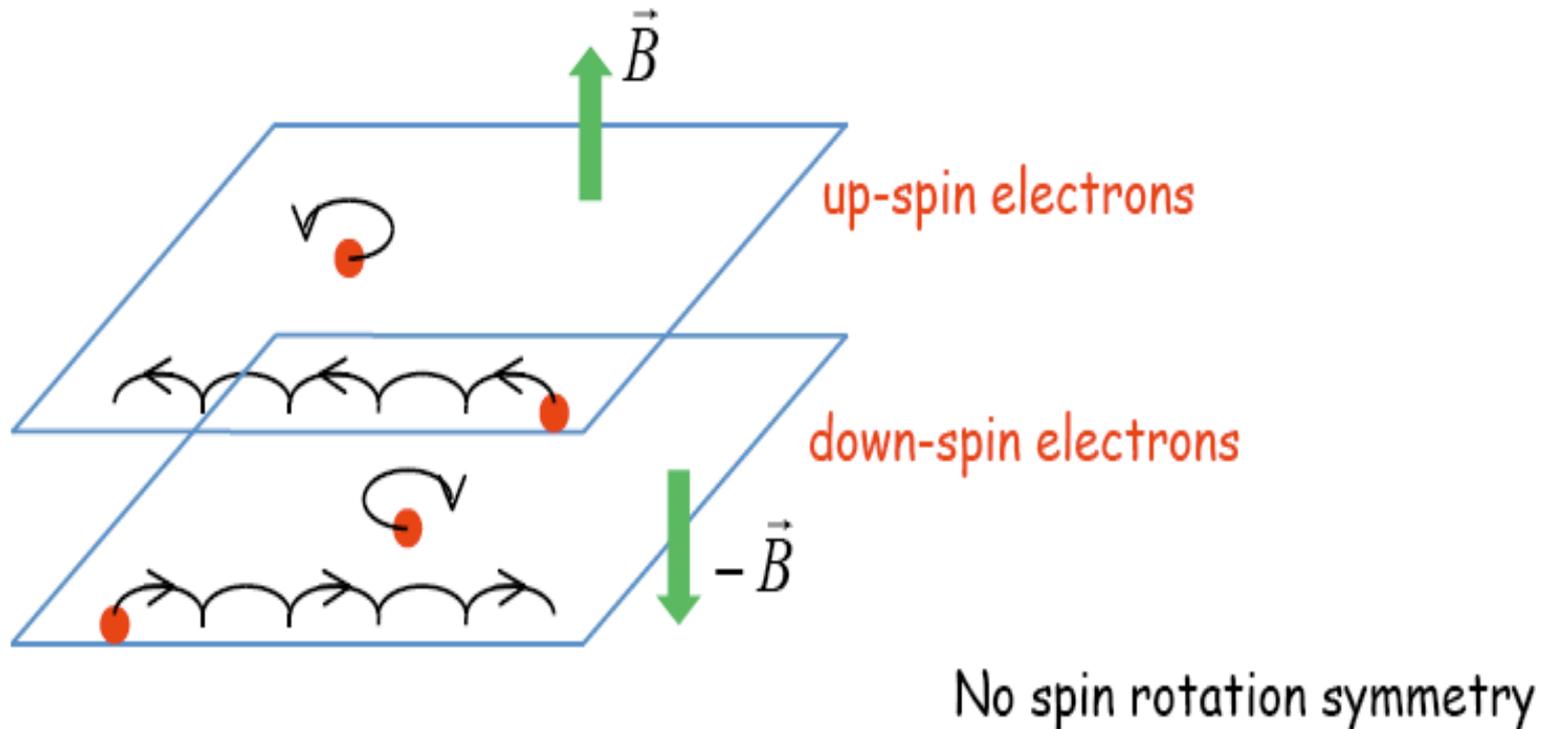
I) Topological Insulators



That's all? No



- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda \vec{L} \cdot \vec{\sigma}$
- Gapless helical edge mode (Kramers pair)



Microscopic Description: Simple Standard Model, Kane-Mele

Time reversal invariant of Haldane model (1988): Kane-Mele model

Kane & Mele, PRL 95, 226801 (2005); Fu-Kane

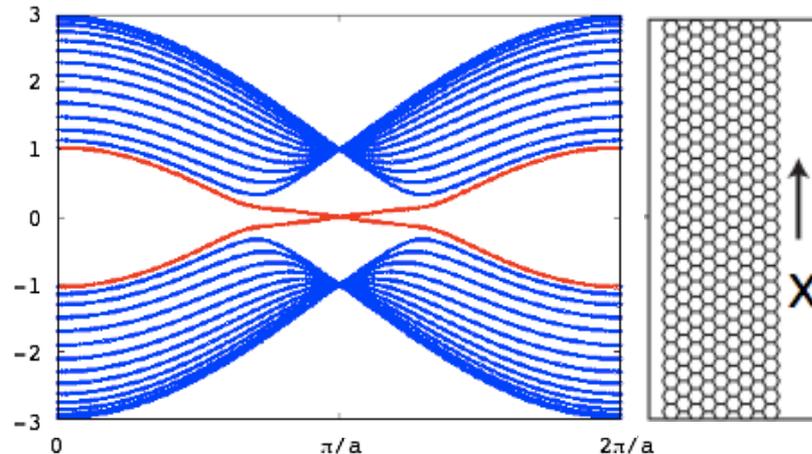
see also: Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)

Also realizations in photon systems for example: [M. Hafezi, S. Mittal, J. Fan, A. Migdall, J. Taylor](#) (2013)

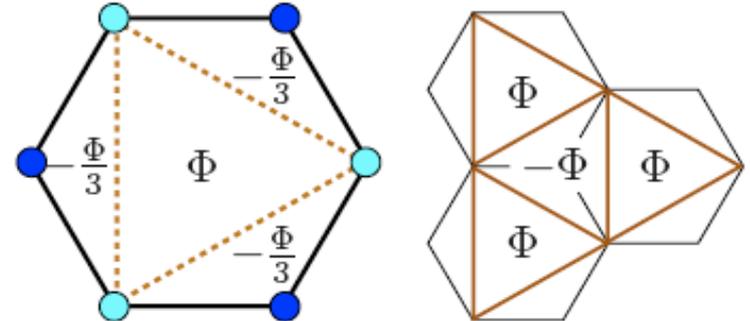
[Mikael C. Rechtsman, Julia M. Zeuner, Yonatan Plotnik, Yaakov Lumer, Stefan Nolte, Mordechai Segev, Alexander Szameit](#) (2013)

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\sigma\sigma'} \nu_{ij} \sigma_{\sigma\sigma'}^z c_{i\sigma}^\dagger c_{j\sigma'}$$

strip geometry:



$$\mathcal{H} \propto \Psi_k^\dagger \sigma^z \tau^z \Psi_k$$



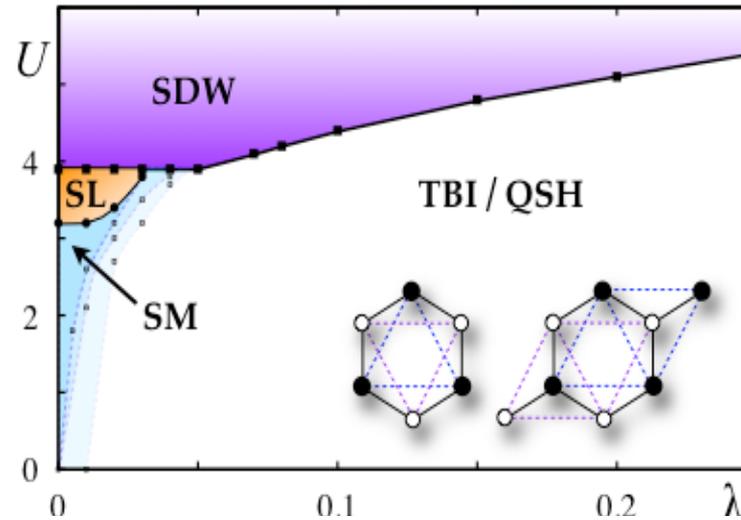
edge states: Kramers's pair

Phase Diagram: “Kane-Mele-Hubbard”

Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

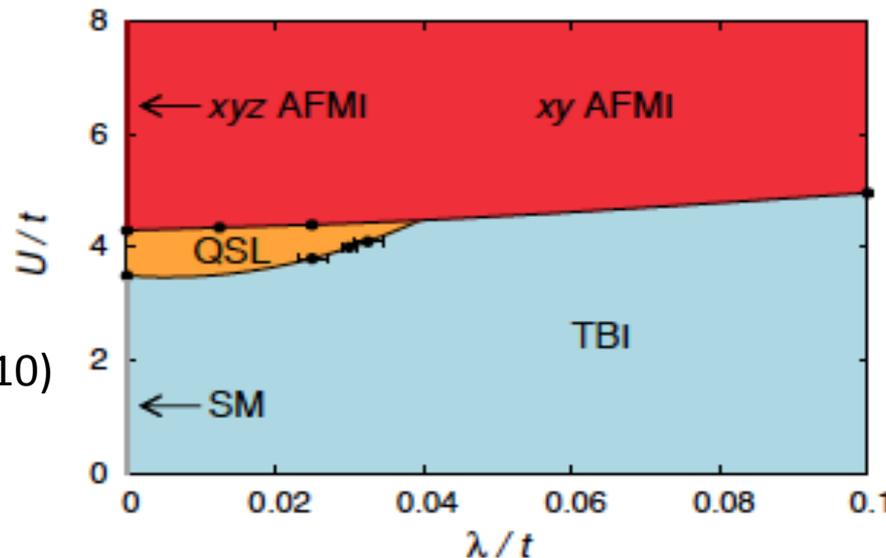
Real-space version
QMC continuous-time
Impurity solver



Transition: 3D XY
S. Rachel & KLH, 2010
Griset & C. Xu, 2011
D.-H. Lee, 2011

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)



M. Hohenadler et al.
arXiv:1111.3949

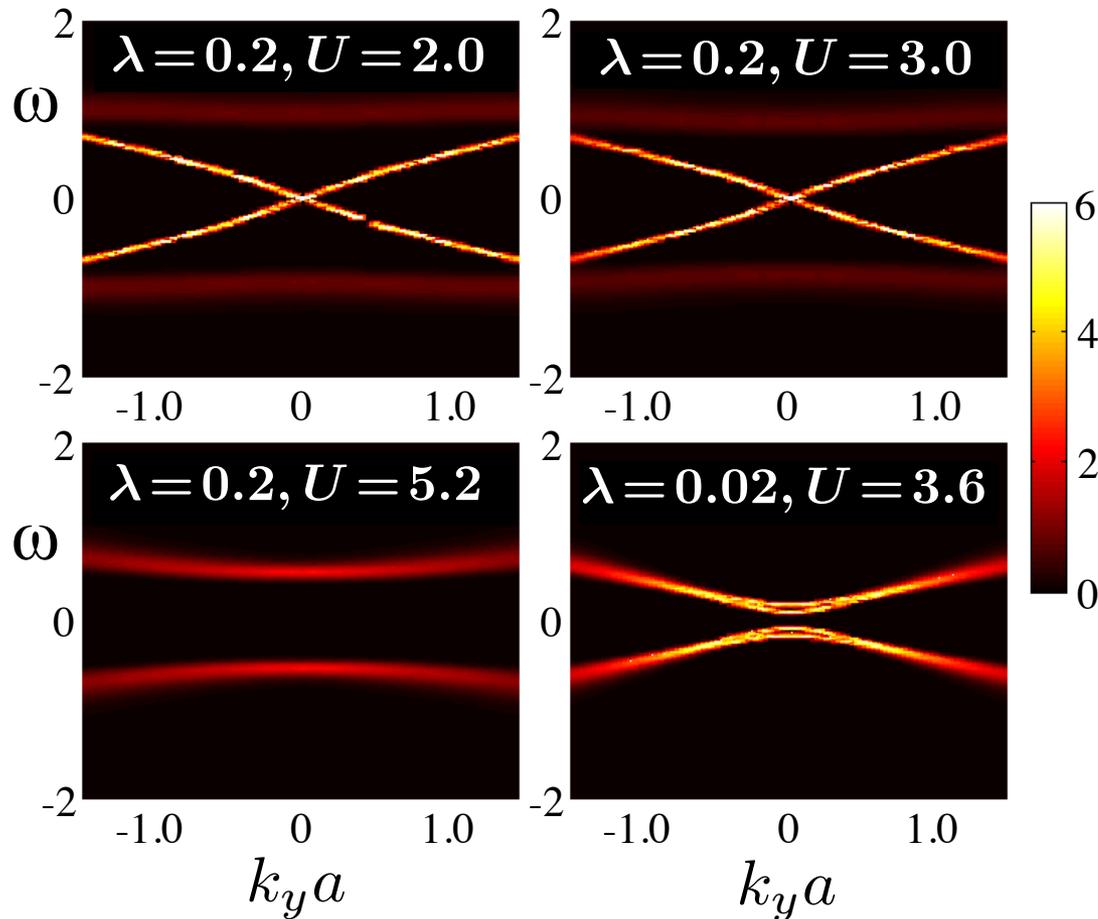
Phys. Rev. Lett. **106**,
100403 (2011)

**Review: Hohenadler
& Assaad, 2013**

Absence of spin liquid for Hubbard (QSL and SL Needs frustration – see later):

S. Sorella et al. Scientific Reports 2012; S. R. Hassan & D. Senechal PRL 2013

Single-particle gap does not close at the Mott transition...



CDMFT

Real-space version
QMC continuous-time
Impurity solver

Some Reviews (not full list):

G. Kotliar et al, RMP 2006

T. Maier et al, RMP 2005

A.-M. Tremblay, B.-S. Kyung,
D. Senechal, 2006

DMFT:

A. Georges, G. Kotliar,
W. Krauth & M. Rozenberg et al.;
Metzner & Vollhardt

Wei Wu, Stephan Rachel, Wu-Ming Liu and KLH, PRB **85**, 205102 (2012)

See also Yamaji & Imada, 2011; Yu, Xie & Li 2011; Zheng, Zhang & C. Wu, 2011

Edge Theory & Mott Transition

C. Xu & J. Moore; C. Wu, A. Bernevig & S.-C. Zhang;...

$$H_0 = v_F \int dx \left(\psi_{R\uparrow}^\dagger i\partial_x \psi_{R\uparrow} - \psi_{L\downarrow}^\dagger i\partial_x \psi_{L\downarrow} \right)$$

$\psi_{R\uparrow}^\dagger \psi_{L\downarrow} + \text{h.c.}$ (**elastic**) Backscattering forbidden

$$H_I = U \int dx \left(\psi_{R\uparrow}^\dagger \psi_{R\uparrow} \psi_{L\downarrow}^\dagger \psi_{L\downarrow} \right)$$

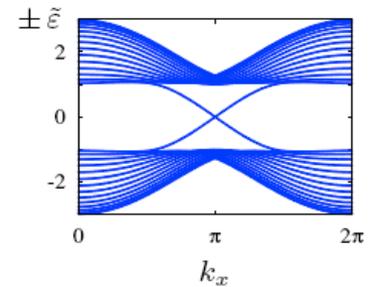
$$H = \int dx \frac{v}{2} \left[\frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \theta)^2 \right] - \frac{Um \sin \sqrt{4\pi} \phi}{(\pi a)^2}$$

Minimal Model
for TBI/QSH phase

with $m = \langle \psi_{R\uparrow}^\dagger \psi_{L\downarrow} \rangle$

Technical Note on Field-Theory techniques...

- ▶ Apply (Slave)-Rotor theory of Florens & Georges, PRB 70, 035114 (2004)
- ▶ See review E. Zhao & A. Paramekanti
- ▶ Rewrite fermions as rotors (charge degrees of freedom) and spinons



$$c_{i\sigma} = e^{i\theta_i} f_{i\sigma}$$

▶ Introduce constraint

$$\sum_{\sigma} f_{i\sigma}^{\dagger} f_{i\sigma} + L_i = 1$$

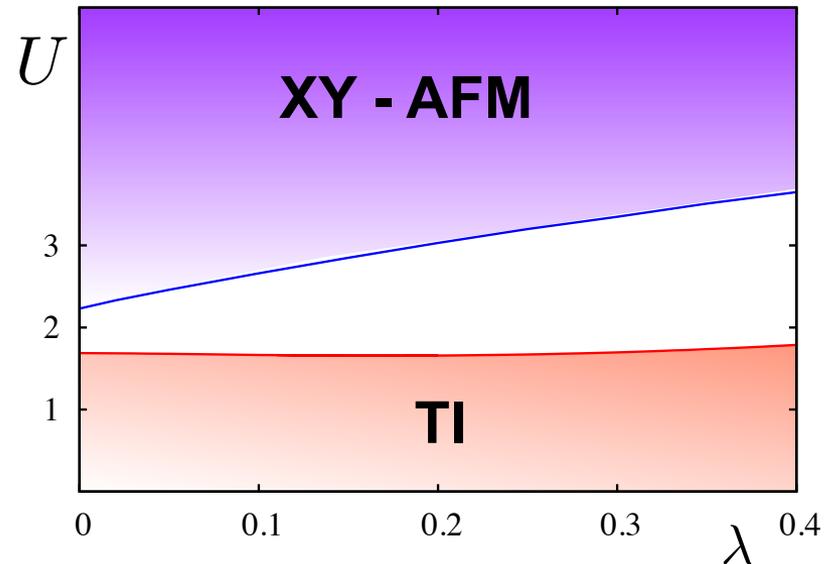
$$\frac{U}{2} \sum_i \left(\sum_{\sigma} n_{i\sigma} - 1 \right)^2 \rightarrow \frac{U}{2} \sum_i L_i^2$$

▶ Hubbard interaction simplifies

$$L = (i/U) \partial_{\tau} \theta.$$

▶ Interaction affects rotor only

▶ weak U : rotor condense, $f_{\sigma} \propto c_{\sigma}$



S. Rachel & KLH, PRB 82, 012405 2010
Application to QSH phenomena

[+ other theory arguments]

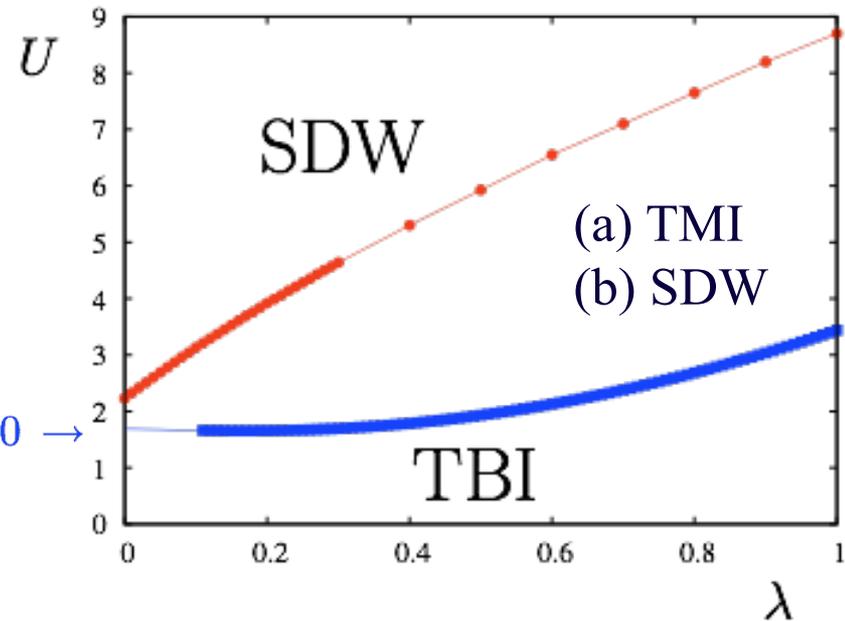
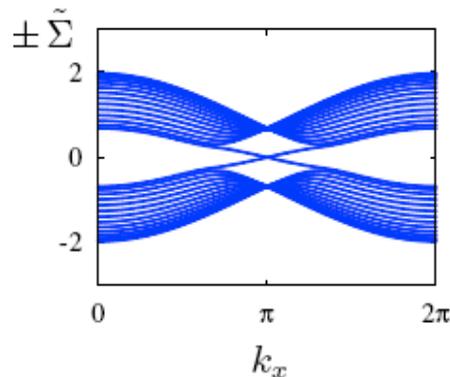
Presence of spin orbit coupling

- ▶ More Slave-Rotor: use sigma-model representation
- ▶ At the Blue transition, “spin-charge” separation

$$X = e^{i\theta} \quad |X|^2 = 1$$

- ▶ mean-field decoupling or Hubbard Stratonovich
- ▶ Gap of the rotor-field (zero at the transition)

$$\Delta_g = 2\sqrt{U(\rho + \min \xi_k)}$$



S. Rachel & KLH, PRB 82, 075106 (2010);
arXiv:1003.2238, 20 pages

Mean-Field Solution allows TMI phase:

- Mott gap
- Spin degrees of freedom form a topological Kane-Mele phase

**Analogue of S=1 spin Haldane chain
NOT STABLE in 2D, here
3D: Pesin-Balents (OK); WHY?**

See also S.S. Lee & P. Lee PRL 2005
Young, S. S. Lee, C. Kallin, PRB 2008
Pesin & Balents, Nat. Phys. 2010
Y.-B. Kim & et al. 2010 + many recent works

2D: Direct Transition from TBI to XY

Kane-Mele
Spinon model

$$\mathcal{L}_{MF} = m \sum_{a=\pm} \left(f_{\uparrow a}^\dagger \tau^z f_{\uparrow a} - f_{\downarrow a}^\dagger \tau^z f_{\downarrow a} \right)$$

Monopole insertion = “spin flip” operator

Localized $+2\pi$ flux of the gauge field implies that a single extra spin-up spinon will be induced along with the gauge flux, while one spin-down spinon will be depleted

Fermions are gapped:

$$\mathcal{L}_{Maxwell} = (1/2e^2) \sum_{\mu} (\epsilon_{\mu\nu\lambda} \partial_{\nu} a_{\lambda})^2$$

Monopoles only cost a finite action: monopole propagator is long-ranged

Here, this implies magnetic order in the XY plane: $\langle S^+ \rangle$ is finite

Polyakov's gauge field argument: see also S. S. Lee & P. Lee; Y. Ran et al; M. Hermele...

Connection to reality?

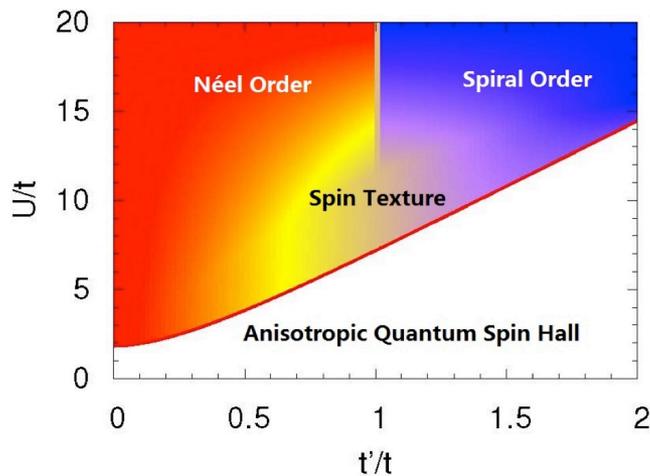
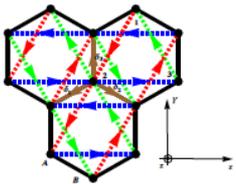
- **Na₂IrO₃**: anisotropic spin-orbit coupling (thin films: arXiv:1303:5245, M. Jenderka et al)

Shitade et al. PRL **102** 256402 (2009); G. Jackeli & G. Khaliullin, PRL 102, 017205 (2009)

H.-C. Jiang, Z.-C. Gu, X.-L. Qi and S. Trebst, Phys. Rev. B 83, 245104 (2011);

S. Bhattacharjee, Sung-Sik Lee and Yong-Baek Kim, New J. Phys. 14, 073015 (2012)

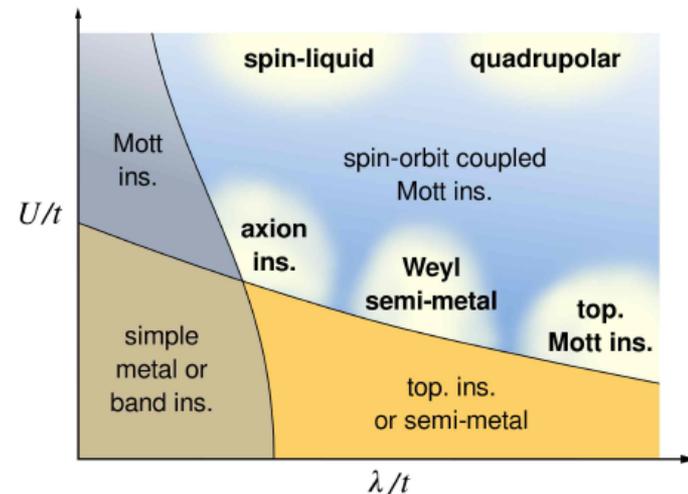
Y. Singh et al. 2012; Z. Nussinov & J. van den Brink, arXiv:1303.5922 ...



α Lithium Iridates
and Spiral order
R. Coldea

Talk next week
G. Cao

Phase diagram 3D: L. Balents



D. Pesin & L. Balents, Nature Phys. 2010

Krempa, Choy, Y.-B. Kim & L. Balents

Spin Ice physics: N. Shannon; S. Onoda

R. Coldea, Titanate Pyrochlores...

Tianhan Liu, Benoit Doucot, Karyn Le Hur, PRB 2013

A. Rugg and G. Fiete, PRL 2012

J. Reuther, R. Thomale & S. Rachel, PRB 2012

M. Kargarian, A. Langari, G. Fiete PRB 2012

Shitade model and Mott transition

A. Shitade, H. Katsura, J. Kunes, X.-L. Qi, S.-C. Zhang, and N. Nagaosa, PRL 2009

$$H_0 = \sum_{\langle i,j \rangle} t c_{i\sigma}^\dagger c_{j\sigma} + \sum_{\langle\langle i,j \rangle\rangle} it' \sigma_{\sigma\sigma'}^w c_{i\sigma}^\dagger c_{j\sigma'}$$

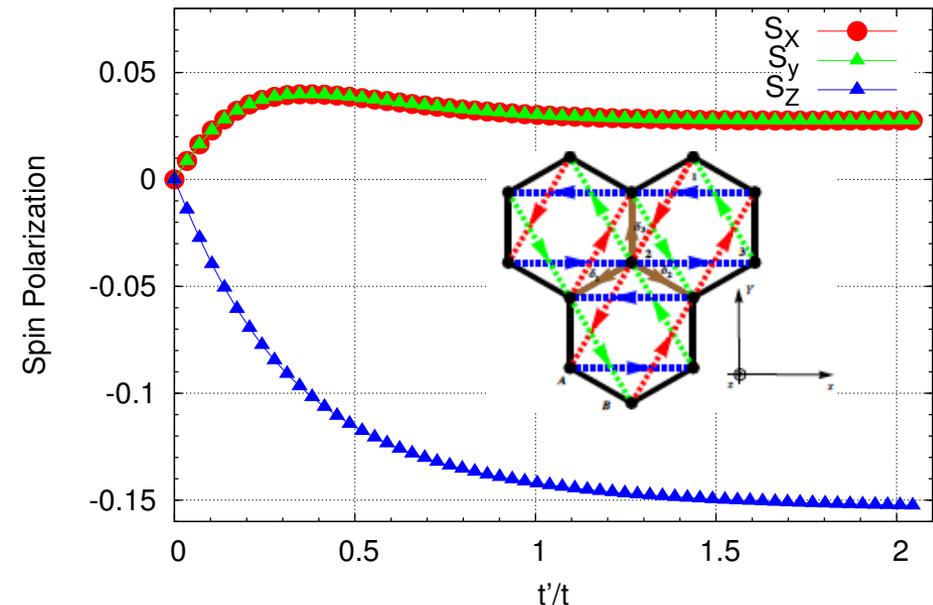
- Critical U identical to Kane-Mele-Hubbard model
- Spin Polarization when inserting a localized Flux

Insertion of monopoles in the slave-rotor $U(1)$ representation: dilute gaz of instantons

Spin Polarization: Linear Response

At one Site

Spin Texture...



Tianhan Liu, Benoit Doucot, Karyn Le Hur, PRB 2013

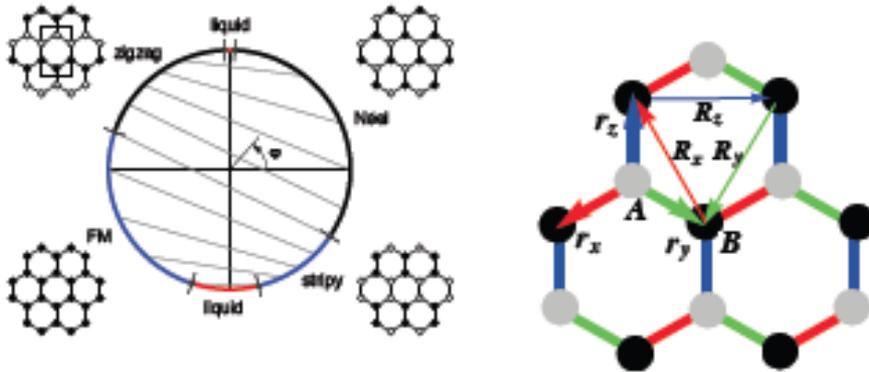
Possible spin liquid?

A. Rugg & G. Fiete PRL 2012

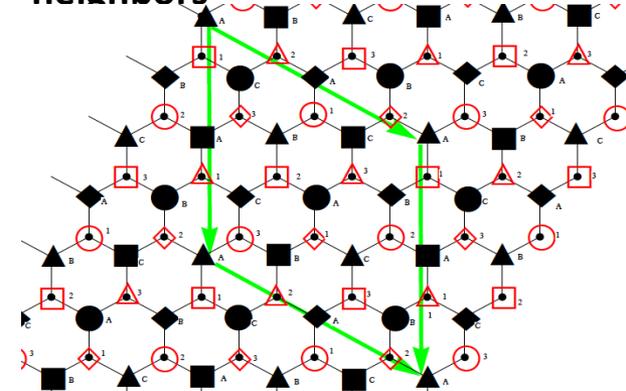
Large-U: Magnetism depends on the Kitaev-Heisenberg models

Talk by N. Perkins

J. Chaloupka, G. Jackeli and G. Khaliullin Phys. Rev. Lett. 105, 027204 (2010).



SPIRAL phase :
 Kitaev term on second-nearest neighbors

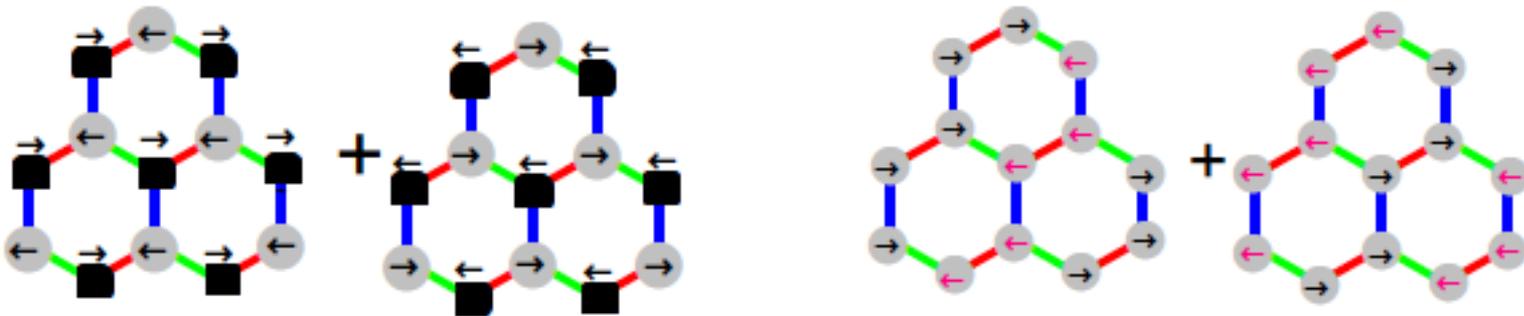


$$H_{KH} = A \sum_{\langle i,j \rangle} (2 \sin \varphi S_i^y S_j^y + \cos \varphi \mathbf{S}_i \cdot \mathbf{S}_j)$$

See also papers H.Y. Kee; experiment by B. J. Kim's group –theory G. Jackeli

Zig-Zag phase (α Na-iridates) : duality to Heisenberg model

Spiral also in J1-J2-J3 model



T. Liu, C. Reppelin, N. Regnault, B. Douçot & KLH (check with ED)

Bosons: possibility of topological excitations in Mott phase

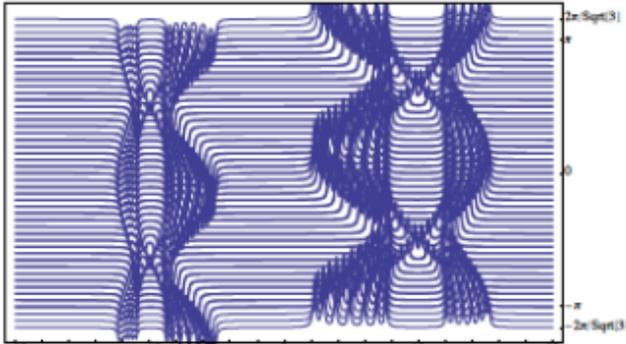
\mathcal{H}_H : Haldane model

$$\mathcal{H} = \mathcal{H}_H + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i,$$

I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB)

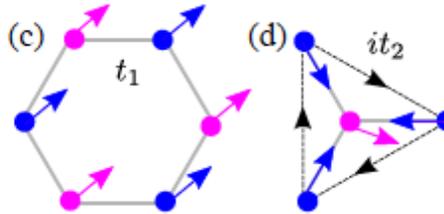
BDMFT+ED
Green MF

Strong-coupling cluster expansion (BDMFT)

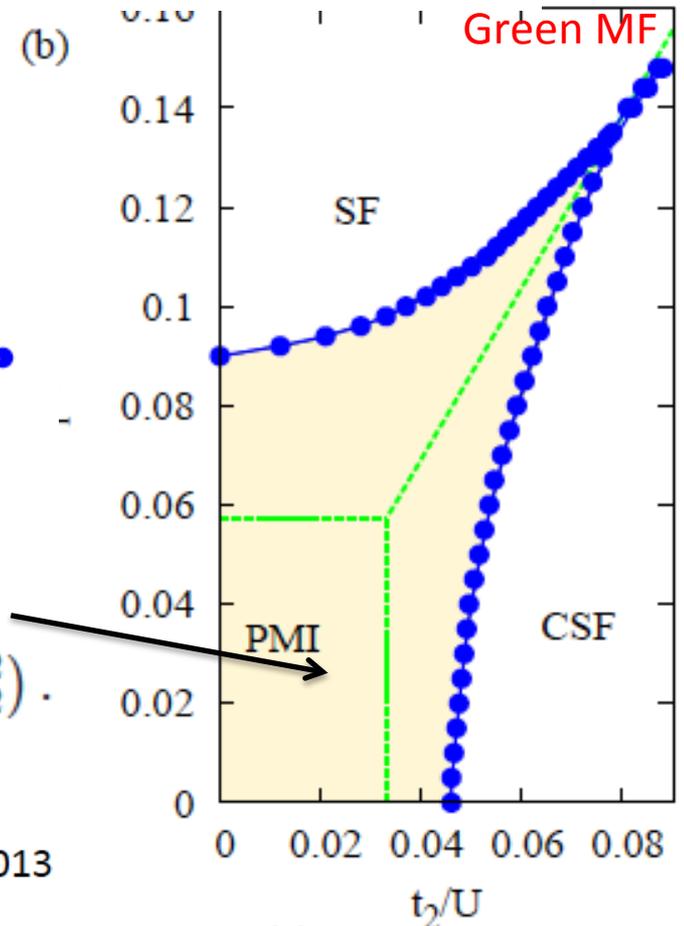


Chiral Excitations in Mott state
Lifetime can be long here

2 Superfluids



$$\frac{J_{AA}^{(2)}}{U} = \frac{36}{U^3} t_2 (t_1^2 - 2t_2^2).$$



Similar effect on triangular lattice and other lattices

M. P. Zaletel, S. A. Parameswaran, A. Rugg and E. Altman, 2013

Cold Atoms:

Ways to implement magnetic fields & gauge fields

A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
M. Aidelsburger et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...

One model proposed by N. Goldman et al. [arXiv:1011.3909 \(PRL\)](#)

[Example of non-Abelian Gauge Fields here on square lattice](#)

$$H_0 = - \sum_j \left\{ t_x c_{j+\hat{x}}^\dagger e^{-i2\pi\gamma\sigma^x} c_j + t_y c_{j+\hat{y}}^\dagger e^{i2\pi\alpha x\sigma^z} c_j \right. \\ \left. + \text{h.c.} \right\} + \lambda_x \sum_j (-1)^x c_j^\dagger c_j ,$$

INTERACTION EFFECTS & PHASE DIAGRAM?

P. Orth, D. Cocks, S. Rachel, M. Buchhold, KLH & W. Hofstetter, review 1212.5607

Interacting spinful
Hofstadter-Harper model

$$\gamma = \lambda_x = 0$$

Interaction Effects

At weak U and half-filling,
semi-metal (SM), graphene

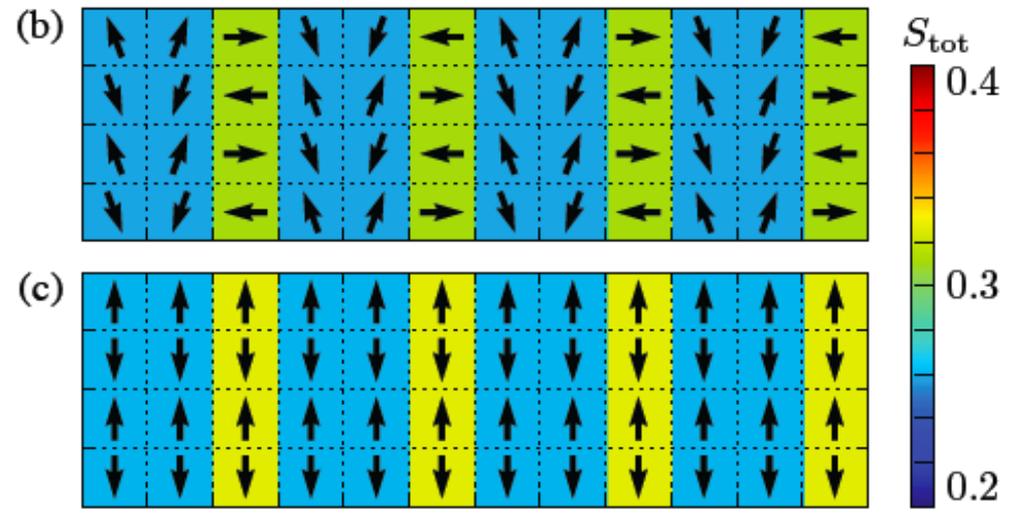
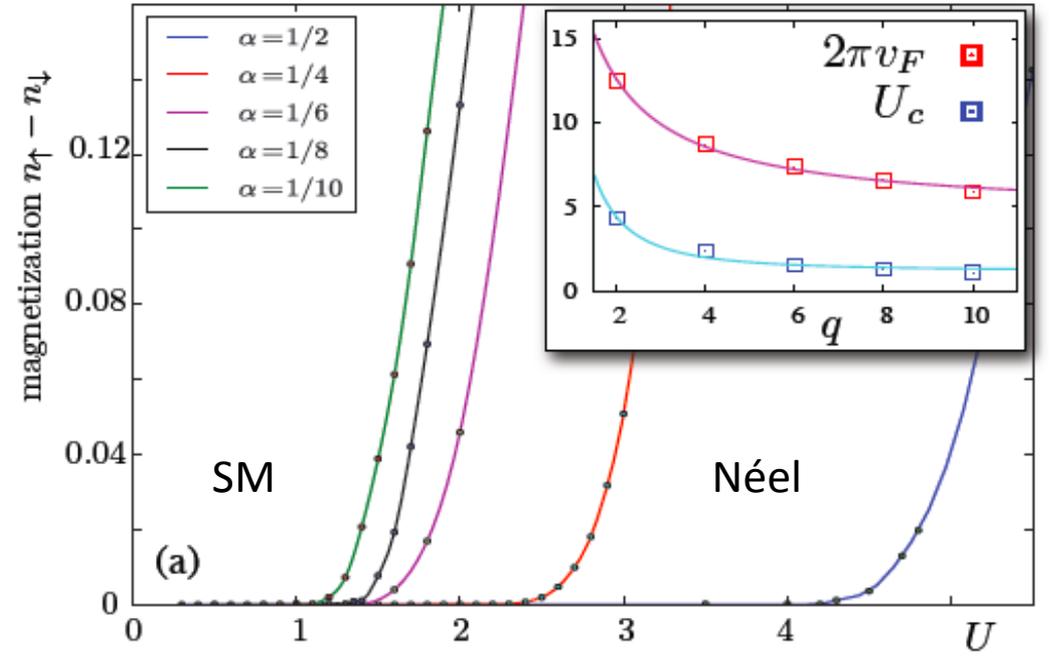
Number of Dirac points
vary with $\alpha = 1/q$ (q even)

Application of I. Herbut's theory
For transition SM to ordered state

The transition occurs for $U_c = 1/q^2$

Magnetism depends on γ

See also M. Scheurer, S. Rachel, P. P. Orth
Sci. Rep. 5, 8386 (2015)



DMFT in Real space

II) BCS superconductor: zero modes

The BCS superconductors can be described using mean field theory (1957)

$$\mathcal{H}_{\text{BCS}} = \sum_{ab} \left[f_a^\dagger T_{ab} f_b - f_b T_{ab} f_a^\dagger + f_a \Delta_{ab} f_b + f_b^\dagger \Delta_{ab}^* f_a^\dagger \right],$$

$$T^\dagger = T \quad \text{Hermitian} \quad \Delta_{ab} = -\Delta_{ba} \quad \text{Anti-symmetric}$$

Quasiparticle spectrum $\gamma^\dagger = u_a f_a^\dagger + v_a f_a$

found as Eigenvalues of Bogoliubov-de-Gennes equations

$$\begin{pmatrix} T & \Delta \\ \Delta^\dagger & -T^T \end{pmatrix} \cdot \psi = E\psi = \mathcal{H}\psi \quad \psi = \begin{pmatrix} u \\ v \end{pmatrix}. \text{ Gurarie, Radzihovsky, 2007}$$

Eigenvalues comes in pairs $\pm E$ since $\omega^x \mathcal{H} \omega^x = -\mathcal{H}^*$ where $\omega^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

This implies $\omega^x \psi^*$ is an eigenstate with $-E$

Zero energy Modes: Majorana fermions

$$\omega \times \psi^* = \psi \rightarrow v = u^*$$

Zero modes have these two states degenerate, and can take a superposition of the form

$$\psi = \begin{pmatrix} u \\ u^* \end{pmatrix}. \quad \gamma^\dagger = u\psi^\dagger + v\psi = u\psi^\dagger + u^*\psi = \gamma$$

The quasiparticle at $E=0$ is a Majorana fermion!

Example: p+ip superconductor or 5/2 FQH state (N. Read & Green)

Topological SCs (Fu-Kane)

1D p-wave Superconductor (A. Kitaev; InAs wires Y. Oreg, G. Refael, F. von Oppen and J. Sau, R. Lutchyn, S. Tewari and S. das Sarma)...

Application for topological quantum computation, quantum error correction codes

C. Nayak, S. Simon, A. Stern, M. Freedman, S. das Sarma RMP 2008

Barbara Terhal, RMP 2015

Example: p+ip SC

excitations (quasiparticles) are described via the Bogoliubov-de-Gennes (BdG) equations

$$\Delta(q) = (q_x + iq_y) \Delta$$

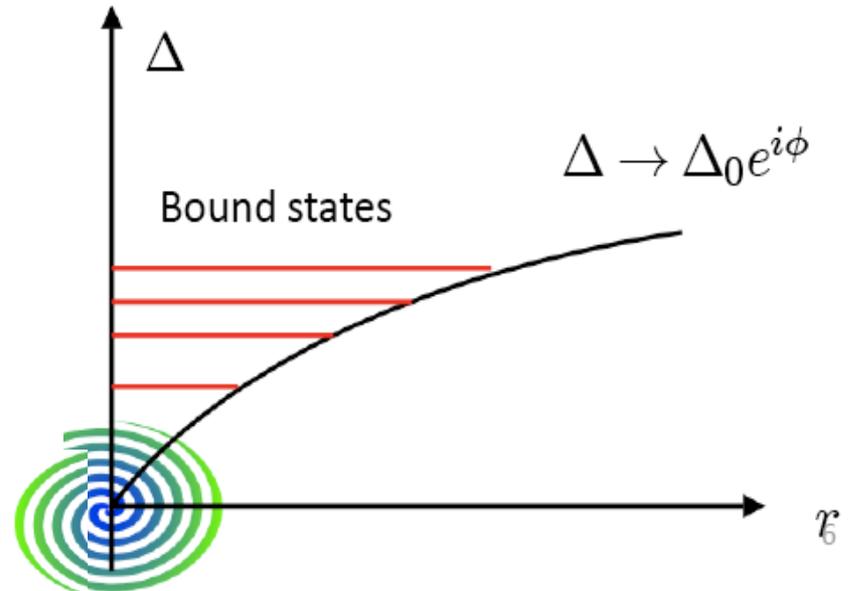
$$\begin{pmatrix} -\frac{\nabla^2}{2m} - \mu & -i\frac{1}{2}\{\Delta, (\partial_x + i\partial_y)\} \\ -i\frac{1}{2}\{\Delta^*, (\partial_x - i\partial_y)\} & \frac{\nabla^2}{2m} + \mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$$

The quasiparticles are gapped in the bulk,

$$E = \pm \sqrt{\left(-\mu + \frac{q^2}{2m}\right)^2 + |\Delta|^2 q^2}$$

but the Gap is reduced and eventually vanishes at the system boundary or in vortices.

Quasiparticle bound states in cores...



Majorana zero energy states

In s-wave condensates, the spectrum of bound states has no zero modes (Caroli, de Gennes, Matricon, 1964).

In p+ip condensate, zero modes exist (Kopnin and Salomaa 1991, G. E. Volovik).

Neglect the quadratic kinetic energy term, and we have the structure of a Dirac equation.

$$\begin{pmatrix} -\mu & -i\frac{1}{2}\{\Delta, (\partial_x + i\partial_y)\} \\ -i\frac{1}{2}\{\Delta^*, (\partial_x - i\partial_y)\} & \mu \end{pmatrix} \begin{pmatrix} u \\ u^* \end{pmatrix} = 0 .$$

Two solutions – exponential decay, and exponentially growing.

In infinite system – only exponential decay normalizable (physical)

so only one bound state zero mode (Green and Read 2000, Tewari et al 2007, Gurarie+Radzihovsky 2006 and others...)

Honeycomb lattice & s-wave case:

BdG equations

Notations J. Alicea & M. Fisher
2006

$$\frac{1}{2} \begin{pmatrix} [\mu - iv (\eta \cdot \nabla)] & +i\Delta_0 \eta^y \sigma^y \tau^y \\ -i\Delta_0^* \eta^y \sigma^y \tau^y & - [\mu + iv (\eta^T \cdot \nabla)] \end{pmatrix} \phi = E\phi$$

Choose states that are spin and “valley” (Dirac node index) eigenstates – 4 fold degeneracy of the entire quasiparticle spectrum.

Exact solution can be built: prove the existence of 4 zero energy modes

$$e^{-\frac{1}{v} \int_0^r f(r') dr'} \Delta = -ie^{i\phi} f(r)$$

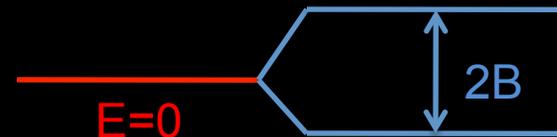
Near-zero energy modes

At exactly half filling, Ghaemi and Wilczek 2007 showed explicitly that zero modes exist, and invoked a result from high energy physics – the existence of zero modes bound to vortex cores in this case is guaranteed by an index theorem (E. Weinberg 1981)

Problem: As pointed out by Ghaemi+Wilczek

Zeeman splitting from a magnetic field splits the spin-degeneracy, and endows these states with non-zero energy (therefore named near-zero modes)

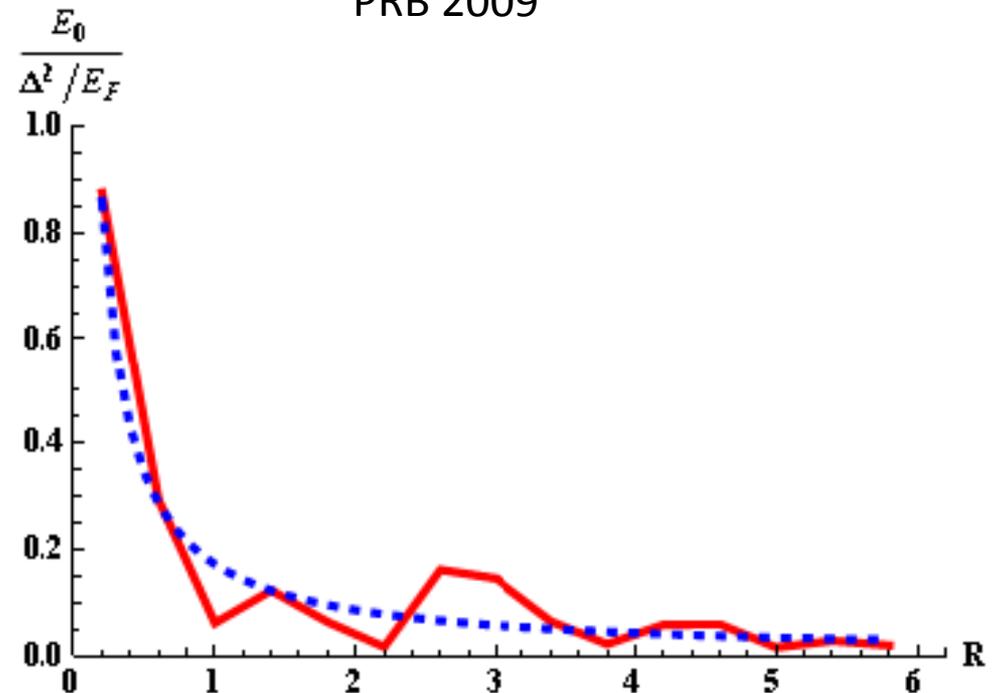
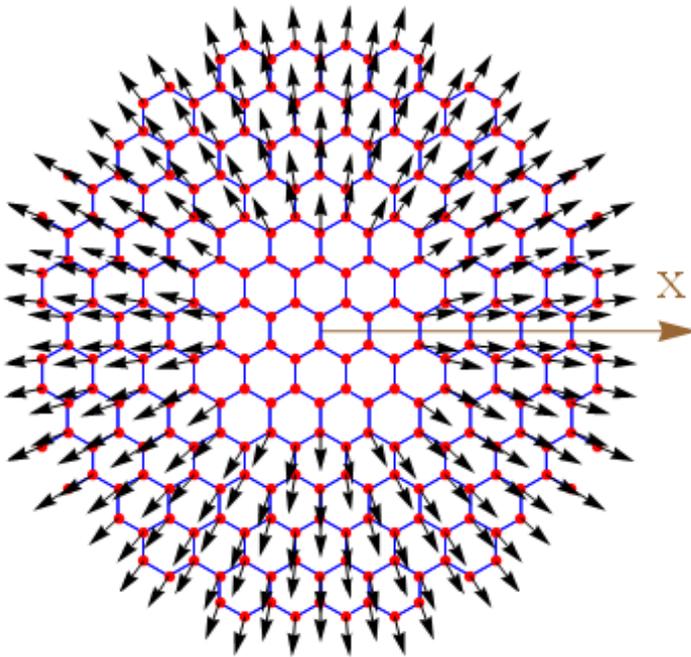
$$(\mathcal{H} + B\sigma^y) \phi = \pm B\phi = E\phi$$



Exact Diagonalization: No protection

Beyond Dirac approximation

D. Bergman & KLH,
PRB 2009



In general : Fully Gapped...

One needs one-1/4 graphene: Topological Insulators (Fu-Kane; Beenakker)
to ensure Majorana protection

Still Chirality from Hubbard...

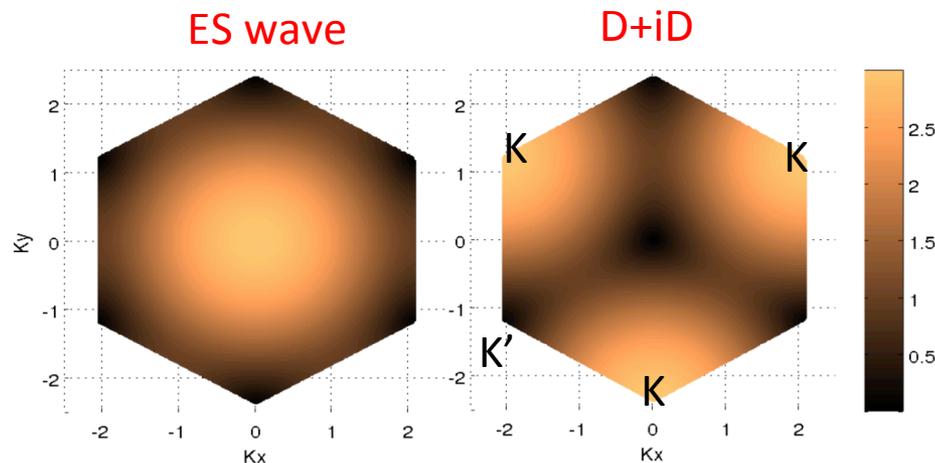
Possibility of gapped spin liquid

J_1 - J_2 model; Tran & Kim, arXiv:1011.1700 & Clark, Abanin & Sondhi PRL 2010;...

Other variants of spin liquid: C. Xu & S. Sachdev; Vaezi et al; ...

Let's start from Néel half-filled phase and dope the system

Difficulty to solve
this model exactly
Start from Mean-Field
Atomic picture



d+id SC state

A. Black-Schaffer &
S. Doniach

C. Honerkamp

R. Nandkishore, L.
Levitov, A. Chubukov
 $\frac{1}{4}$ filling ...

Doping leads to
Interesting pairing

$$\Delta_{\mathbf{k}}^{d+id} = \cos\left(\frac{\pi}{3}\right)\Delta_{x^2-y^2} + i \sin\left(\frac{\pi}{3}\right)\Delta_{xy}$$

- 1) Wei Wu, Michael M. Scherer, Carsten Honerkamp & KLH, PRB 2013 ([RMFT, FRG: t-J1-J2](#))
- 2) A. Black-Schaffer, W. Wu and KLH, PRB 2014 ([RMFT t-J, QMC on Hubbard](#))
- 3) A. Black-Schaffer & KLH, arXiv:1503.02509 ([topological phase transition due to SDW](#))

d+id Pairing: RMFT & QMC

Renormalized mean-field theory

F. C. Zhang, C. Gros, T. M. Rice, and H. Shiba, *Supercond. Sci. Tech.* **1**, 36 (1988).

P. W. Anderson, P. A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, *J. Phys.: Condens. Matter* **16**, R755 (2004).

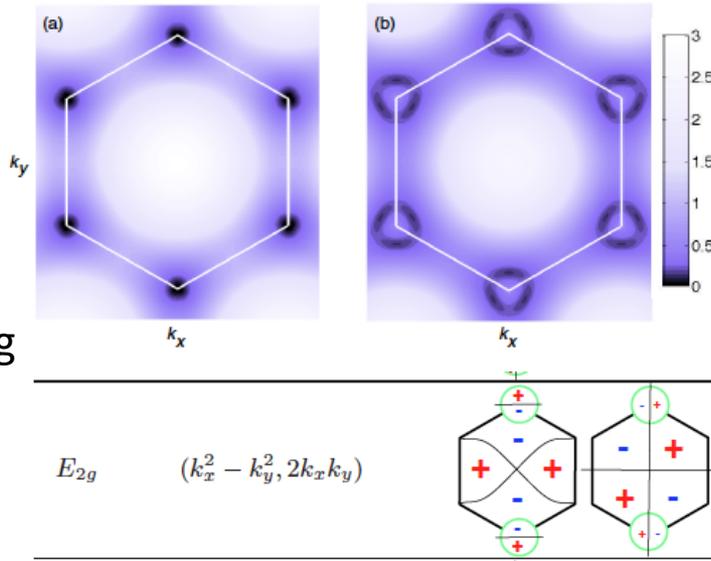
B. Edegger, V. N. Muthukumar, and C. Gros, *Adv. Phys.* **56**, 927 (2007).

M. Ogata and H. Fukuyama, *Rep. Prog. Phys.* **71**, 036501 (2008).

K. Le Hur and T. M. Rice, *Ann. Phys.* **324**, 1452 (2009).

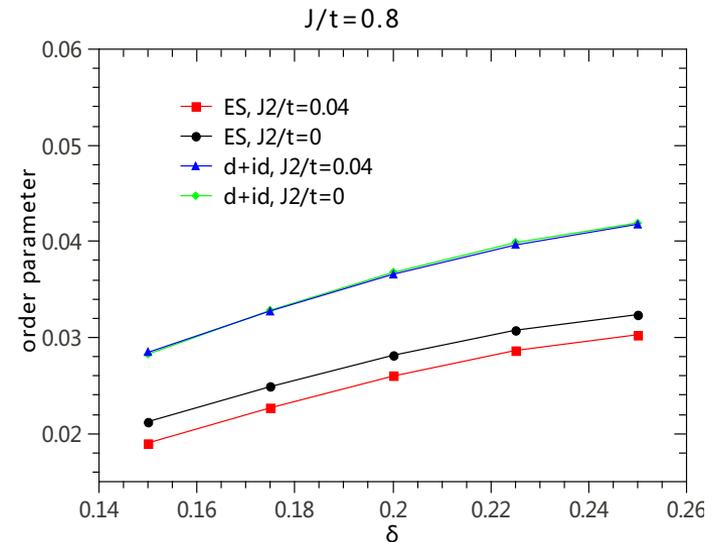
2)

$\frac{1}{2}$
filling



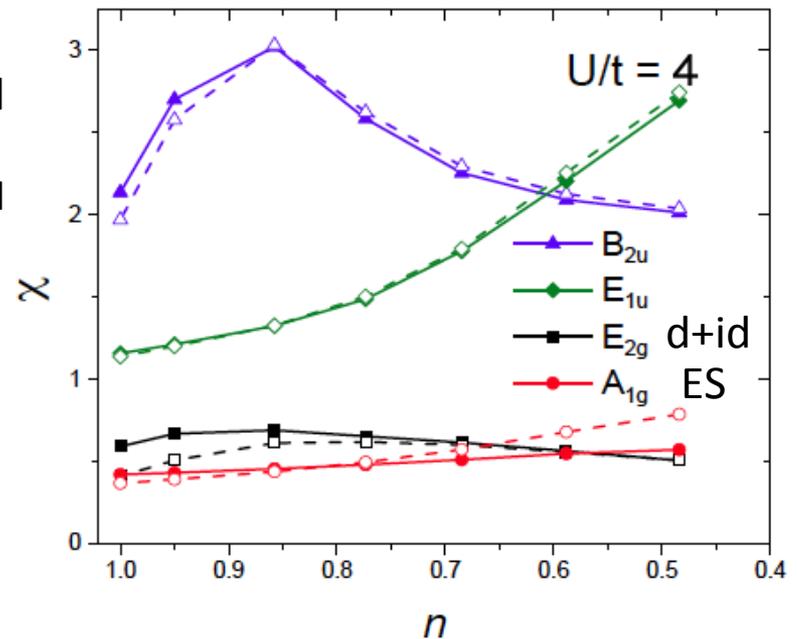
**BdG
spectrum**

Doped
Fully
gapped



1)

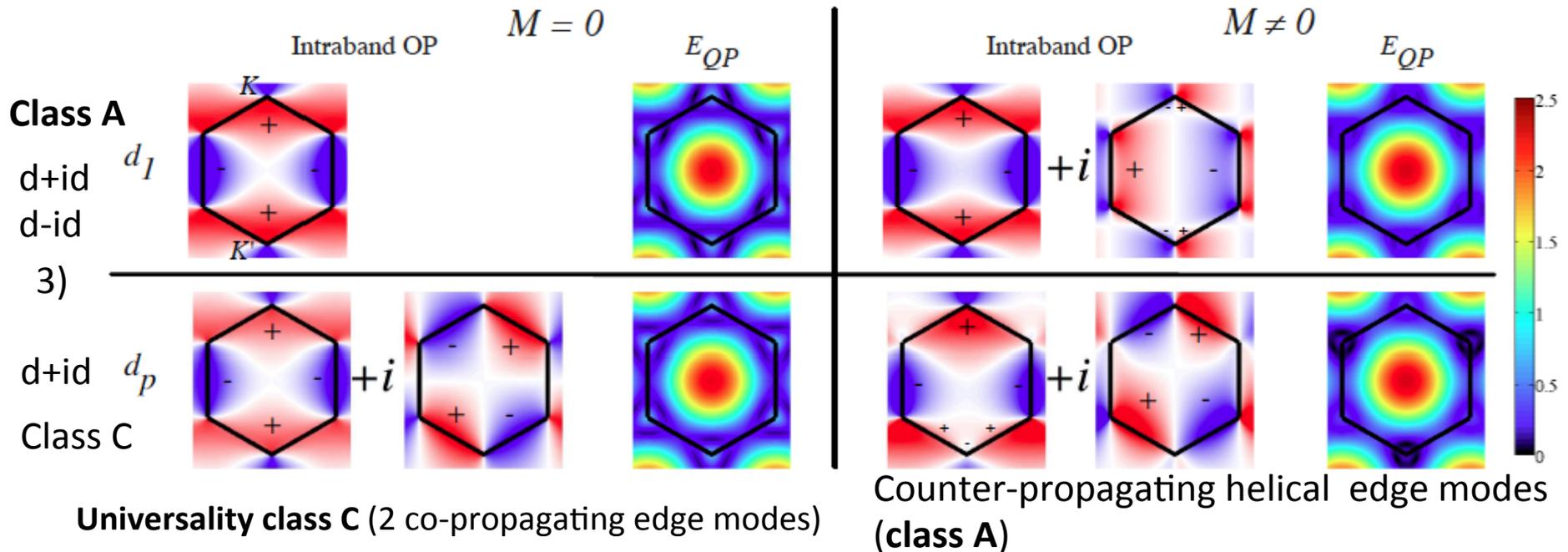
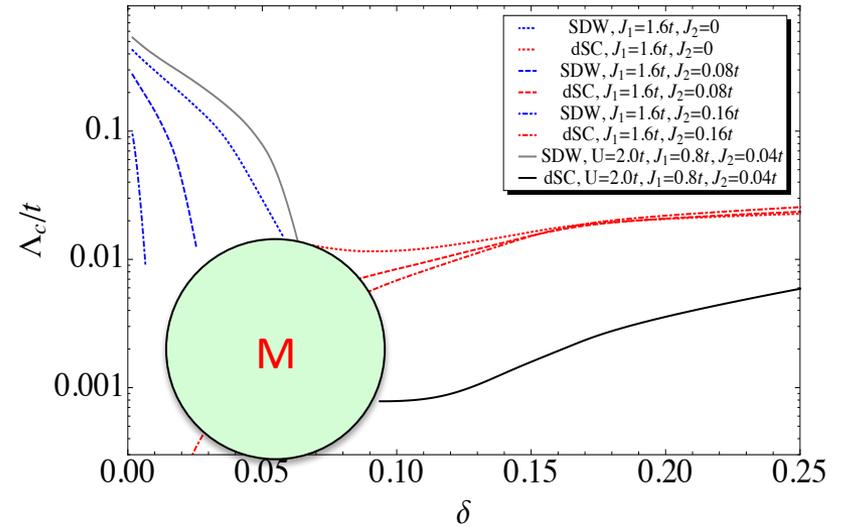
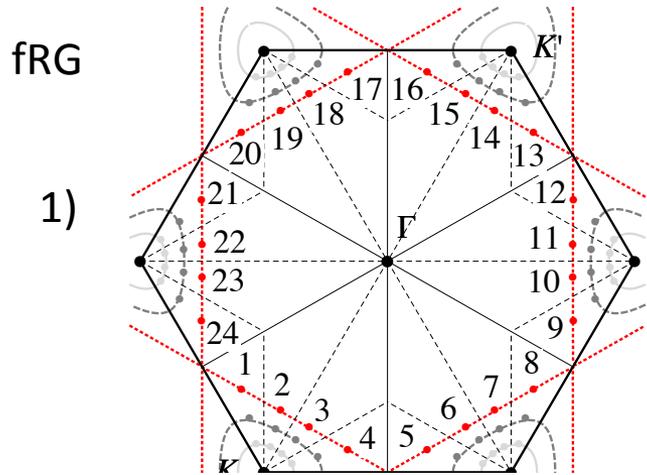
QMC



2)

See also results from L. Balents, X. G. Wen :
Z. G. Gu et al. *Phys. Rev. B* **88**, 155112 (2013)
Grassmann Tensor Network Approach, DMRG

SDW and SC: Two chiral Superconductors



chiral d-wave SCs: honeycomb

Doping $\text{In}_3\text{Cu}_2\text{VO}_9$ possible with Zn (Copper gives spin-1/2)

A. Möller, U. Löw, T. Taetz, M. Kriener, G. André, F. Damay, O. Heyer, M. Braden, and J. A. Mydosh, *Phys. Rev. B* **78**, 024420 (2008).

Y. J. Yan, Z. Y. Li, T. Zhang, X. G. Luo, G. J. Ye, Z. J. Xiang, P. Cheng, L. J. Zou, and X. H. Chen, *Phys. Rev. B* **85**, 085102 (2012).

D.-Y. Liu, Y. Guo, X.-L. Zhang, J.-L. Wang, Z. Zeng, H.-Q. Lin, and L.-J. Zou, *EPL* **103**, 47010 (2013).

Experimental works

Chiral d-wave SCs have also been suggested in SrPtAs, MoS₂, graphene and silicene

Possibility of Majorana fermions adding spin-orbit coupling

S.-J. Sun, C.-H. Chung, Y.-Y. Chang, W.-F. Tsai and F.-C. Zhang, arXiv:1506.02584

SC in doped Heisenberg-Kitaev models: exotic p+ip triplet states

On-going work T. Liu, C. Repellin, N. Regnault, B. Douçot, KLH (PhD thesis of T. Liu)

See also Y. Z. You, I. Kimchi & A. Vishwanath, PRB 2012; Daniel D. Scherer et al PRB 2014

Talk next week G. Khaliullin

Summary

Two-dimensional correlated TBIs

Not yet confirmed in experiments (effort in material aspects)

Magnetism in Mott phase depends on the Kitaev-Heisenberg models (relevant for 2D iridates); **not Kitaev alone ...**

Boson Systems are also interesting (cold atoms; photons)

Review on photons (see refs inside); **arXiv May 2015** – submitted to CRAS

Karyn Le Hur¹, Loïc Henriot¹, Alexandru Petrescu^{2,1}, Kirill Plekhanov¹, Guillaume Roux³, Marco Schiró⁴

Another Workshop in parallel at KITP on « photons » next Fall 2015

Topological SC close to Mott state:

d+id chiral (class C) – stay until $\frac{1}{4}$ filling at large U...

Mixed d+id/d-id (class A) possible when coexistence with SDW