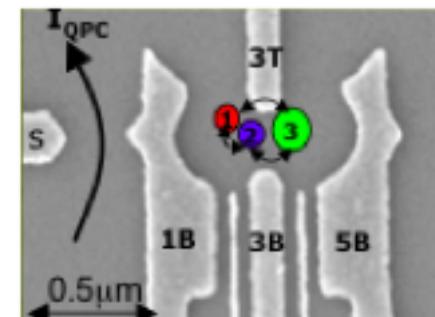
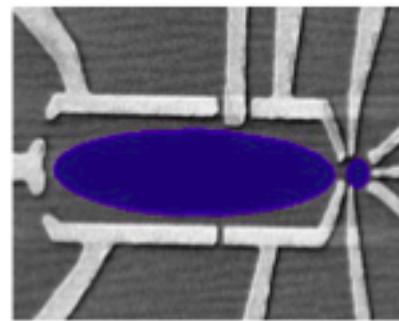
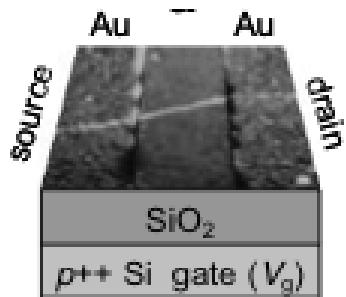


# Quest for Novel Kondo Nano systems

Karyn Le Hur

*Yale University*

Duke, 9th of November 2006



# The resistance minimum...

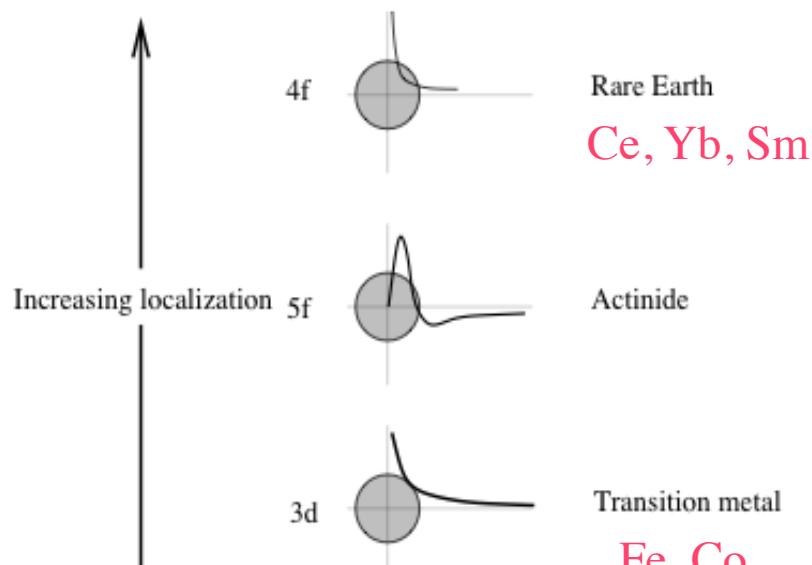


FIGURE 1. Depicting localized  $4f$ ,  $5f$  and  $3d$  atomic wavefunctions.

At Leiden, 1934, the resistance minimum  
de Haas, Von de Boer, Van den Berg  
Physica **1**, 1115 (1934)

1936: Casimir joined Leiden

Casimir and Van den Berg worked on the Kondo effect. In 1938, based on an earlier suggestion by Peierls, Casimir worked on heat conductivity of pure single crystals...

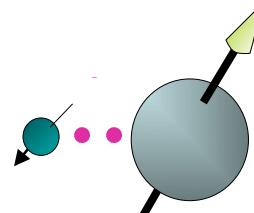
« zero-point fluctuations of phonons »

« The resistance minimum was first observed in noble metals containing small concentration of 3d transition metals (such as iron or manganese) »

Jun Kondo, « 40 years after discovery », JPS of Jpn (2005)

Leiden group, 1930s  
Sarachik, 1960s

# Summary

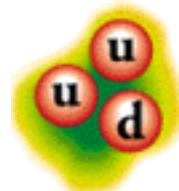


-*Kondo effect definition:*

Involves a magnetic impurity and conduction electrons

A standard « confinement » problem

(meV instead of GeV for quark confinement...)



-Physical observables: universal functions of the confinement scale

-*Why Nanotechnology is useful:*

« Artificial spin qubit », building block for quantum computing, but also

Engineer some « toy » models and check for fundamental physics

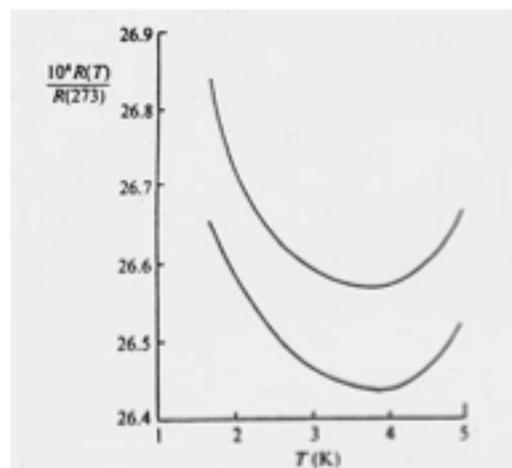
Explore new confinement phenomena

Explore quantum phase transitions through the Kondo effect

# Kondo model: impurity spin in metal

$$\rho_0 \sim 0.1 \Omega \text{cm}$$

Good metal:  
 $\rho \sim 10^{-5} \Omega \text{cm}$



'34: Resistivity minimum of Au

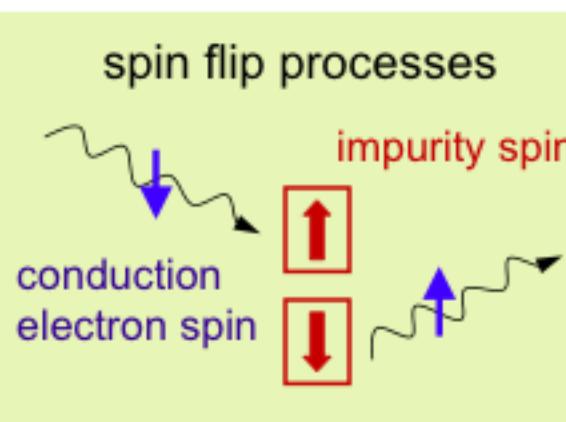
De Haas et al., Physica 1,1115



'62: Explanation  
by J. Kondo

Electron spin operator:

$$\mathbf{S}^e(\mathbf{x}=0) = \sum_{\mathbf{k}, \mathbf{k}', \alpha, \beta} c_{\mathbf{k}\alpha}^\dagger \frac{\sigma_{\alpha\beta} c_{\mathbf{k}'\beta}}{2}$$



$$J_\perp [S_+^{(imp)} S_-^{(e)} + h.c.] + J_z S_z^{(imp)} S_z^{(e)}$$

Kondo singlet  $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

at  $T < T_K$  (Kondo temperature)

# Many-body strong-coupling picture

The renormalization group equations tell us that at scales of the order of  $T_K$  the Kondo coupling becomes of the order of bandwidth

*Anderson, 1970*

Precursor of renormalization group for interacting fermions: *Shankar*, Rev. Mod. Phys. 94

## Strong-coupling stable

*Wilson (1975)*: Numerical Renormalization Group method

$$C_v = \gamma T$$
$$\gamma = \frac{\pi}{3} \frac{0.4128 \pm 0.002}{8 T_K}$$

$$W = \frac{\chi/\chi^0}{\gamma/\gamma^0} = \frac{\chi}{\gamma} \frac{\pi^2 k_B^2}{3 \mu_B^2} = 2$$

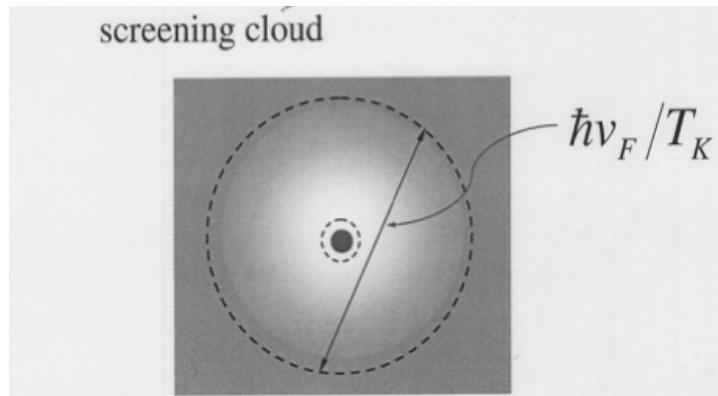
agree with Bethe-Ansatz calculations  
*N. Andrei; P. Wiegmann (1980)*

$T \ll T_K$   
*Renormalized density of states*  
 $N^*(0) = 1/T_K \Rightarrow$  Heavy Mass  $\propto 1/T_K$

# Emergence of heavy fermions

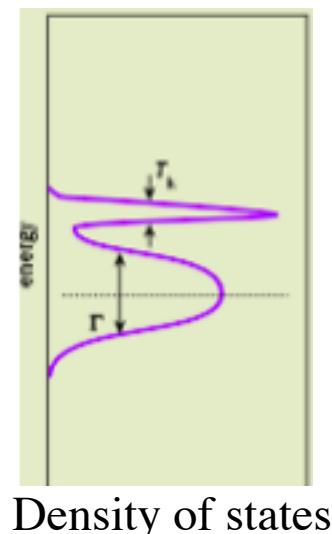
Real-space picture:  
Nozières, 1974

BCFT Spectroscopy:  
Affleck & Ludwig



Key point:  $T \ll T_K$   
Spin induces interactions  
between electrons  $\approx 1/T_K$   
↓  
« Emergence of heavy Mass »

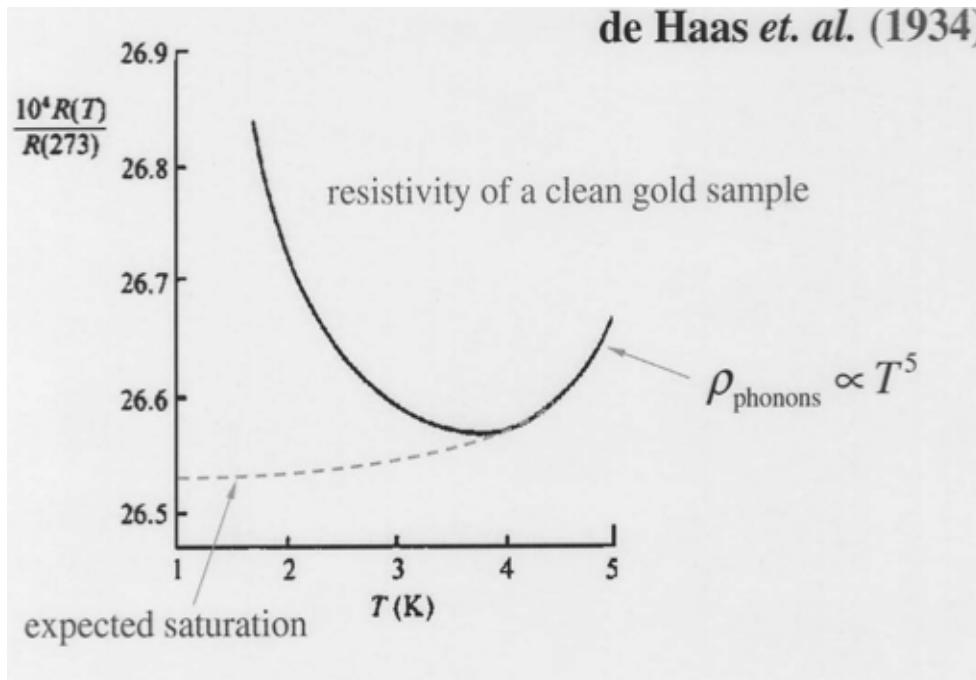
Large-N theory  
(N. Read & Newns 1983)



Another approach:  $T \ll T_K$   
Spin behaves itinerantly &  
formation of an Abrikosov-Suhl  
resonance of width  $T_K$

Landau quasiparticles with  $N^*(0) = 1/T_K \Rightarrow$  Sommerfeld cst,  $\gamma^* \propto 1/T_K$

# Resistivity Summary



Kondo 1964

$$T \approx T_K \quad \Delta\rho \approx \lambda^2 + \lambda^3 \ln(E_F/T)$$

$$T \ll T_K \quad \Delta\rho \approx \rho_o - (T/T_K)^2$$

Nozières 1974

$$T \gg T_K \text{ then } \lambda = JN(0) \ll 1$$

Poor man scaling: Anderson 1970

$$d\lambda/[d\ln(E_F/T)] \approx \lambda^2 - \dots$$

$\lambda[T_K] \approx 1$  Strong coupling realm

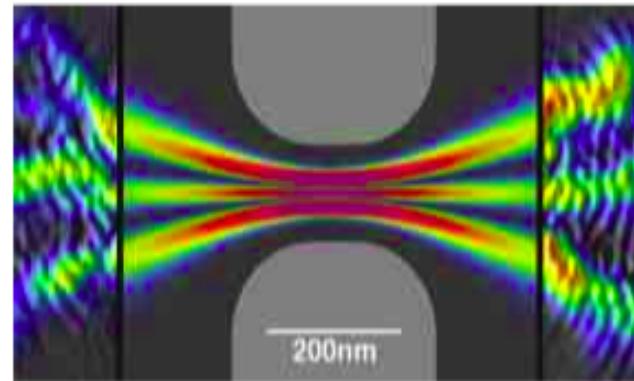
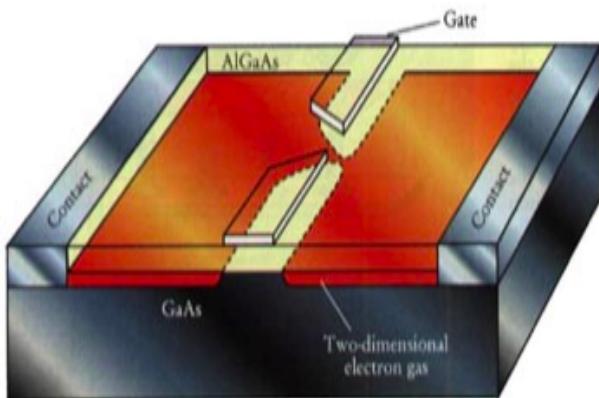
$$\Rightarrow T_K \approx E_F \exp{-1/\lambda}$$

Few K

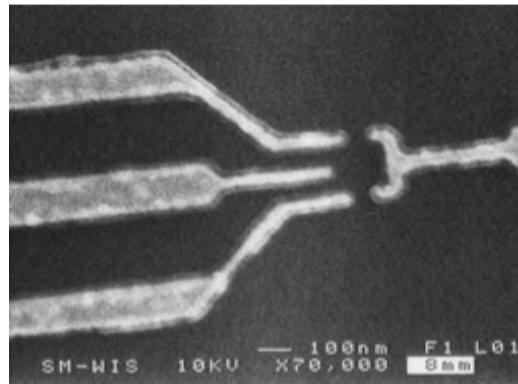
1eV

Energy relaxation in Ag, Cu, Au wires:  
F. Pierre, H. Pothier, D. Esteve, M. Devoret,  
N. Birge, JLTP 118, 437 (2000)

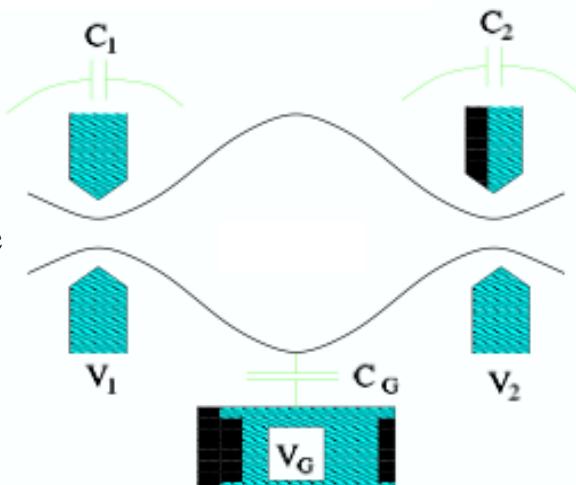
# Dot as an artificial atom



Imaging Coherent Electron Flow  
Westervelt, Heller and Gossard



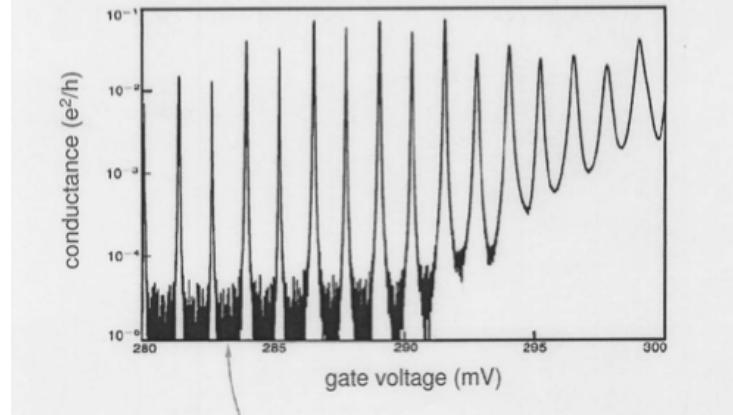
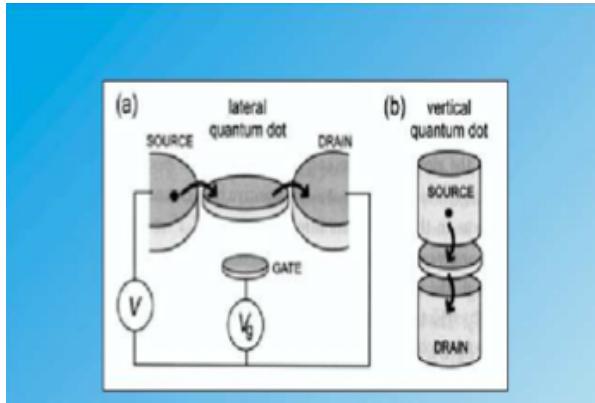
$R \gg R_K$     $R = h/(2e^2 T)$   
Quantum of resistance  
 $h/2e^2 = 12.9 k\Omega s$



D. Goldhaber-Gordon, Nature 1998

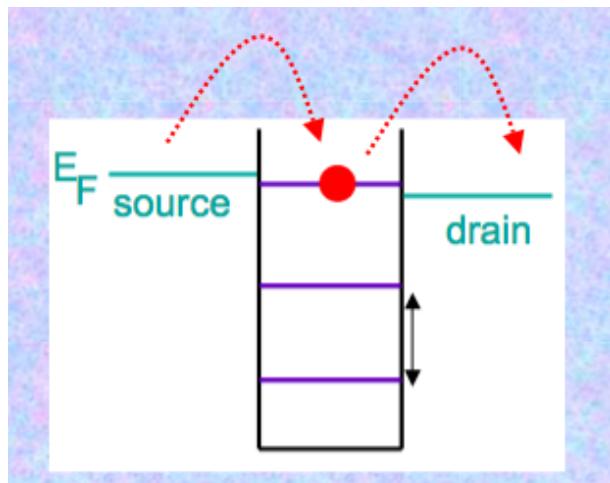
# « Single-Charge Tunneling »

Edited by Hermann Grabert and Michel Devoret, 1992, Plenum Press, NY



M. Kastner, Physics Today (1993)

Resonant tunneling case

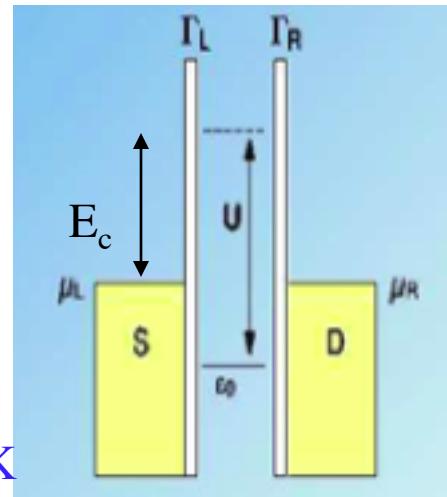


Charging  
energy  $E_c = U + \epsilon_o$   
blocks transport

$$G \propto e^{-E_c/T}$$

Ultraviolet cutoff:  $E_c$  few K

Coulomb valley

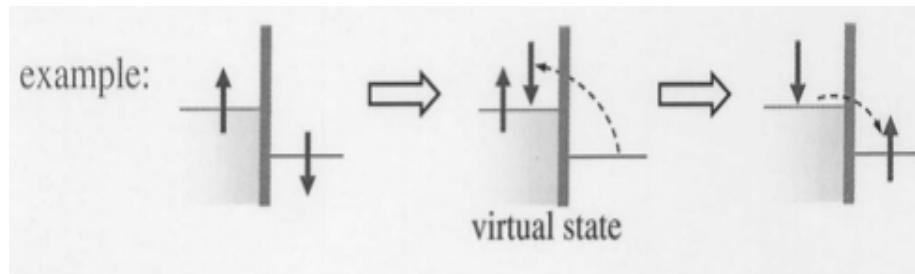
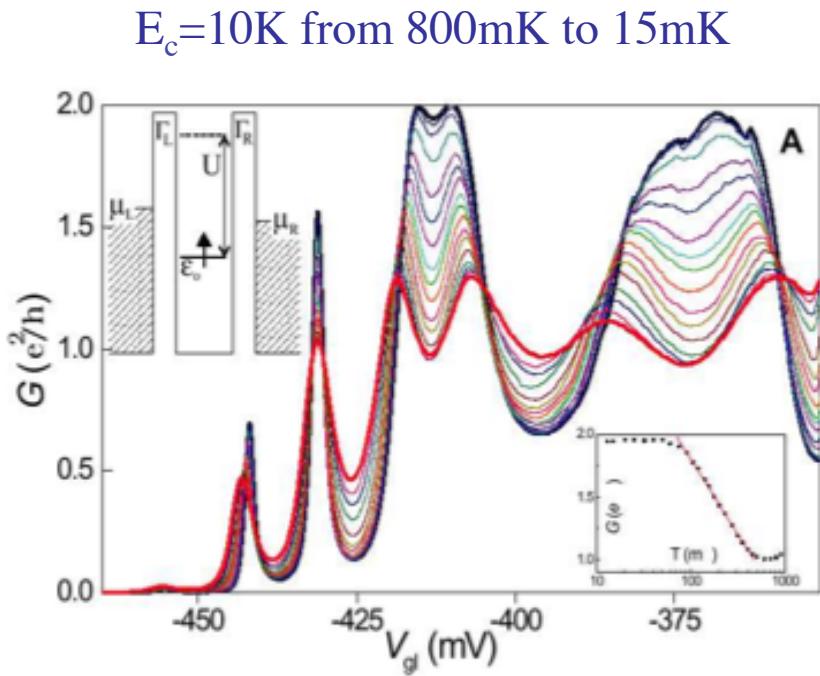


# Universal Kondo physics...

Details of Coulomb peaks depend on shape and size

R. Jalabert, D. Stone, Y. Alhassid Phys. Rev. Lett. **68**, 3468 (1992)

Universal features for small dots (200nm & few tens of electrons)

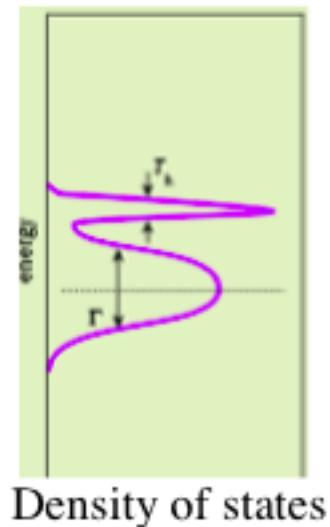


Glazman & Raikh, 1988  
Ng & P. Lee, 1988

$$G(\varepsilon) = (2e^2/h)\sin^2 \delta(\varepsilon)$$

$$\delta(\varepsilon=0)=\pi/N$$

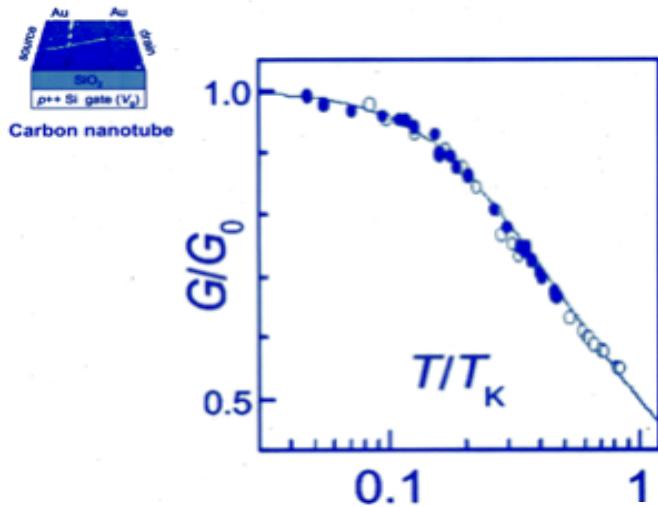
$N=2$  screening electrons



W.G. Van der Wiel et al.,  
Science **289**, 2105 (2000)

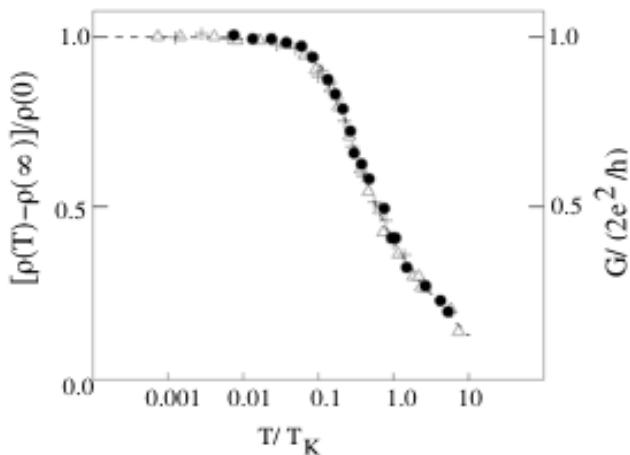
« Friedel sum rule for unitary diffusion, 1952 »

# Interesting remarks



GaAs dot → Carbon nanotube dot  
Features are « dot-independent »

Nygard et al. Nature 408, 342 (2000)

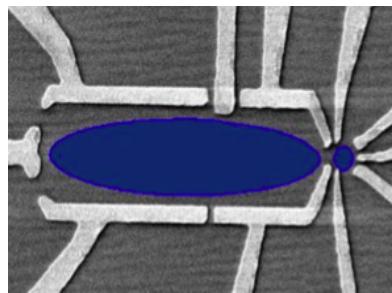


Same Kondo anomaly as in metals:  
 $\ln(E_c/T)$  but ultraviolet much smaller

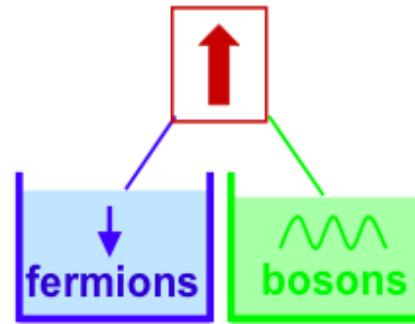
$\text{Nb}_{1-x}\text{Mo}_x$  alloys, Sarachik et al. (1964)  
P. Coleman, cond-mat/0206003

# Quest for novel Nano Kondo liquids

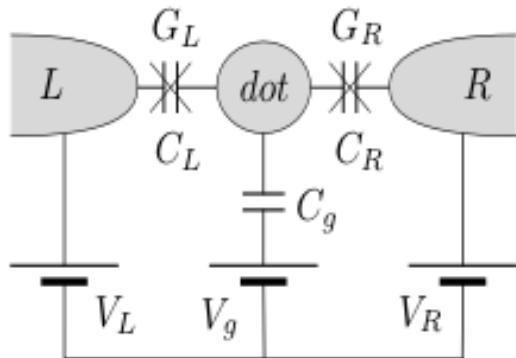
« Competing channels: Destruction of confinement? »



2 channels of fermions  
Intermediate fixed point  
Nozières-Blandin (1980)



D. Goldhaber-Gordon & Y. Oreg



« Out of equilibrium »

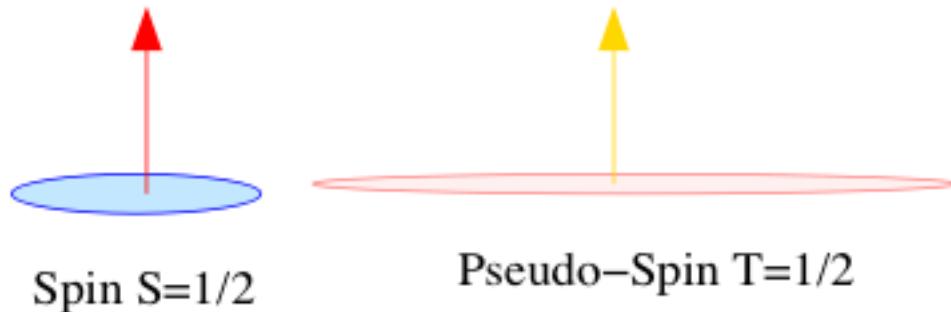
A. Rosch & P. Woelfle

K. Le Hur, Phys. Rev. Lett. 92, 196804 (2004)  
M.-R. Li, K. Le Hur, W. Hofstetter, PRL 95, 086406 (2005)

« Charge-Spin » entanglement  
Quantum Phase Transitions  
Artificial Trimer



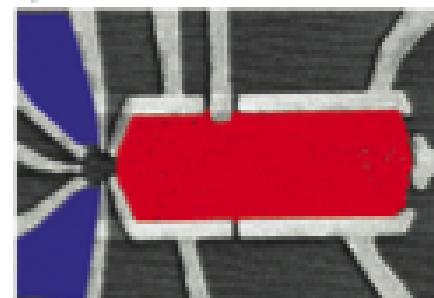
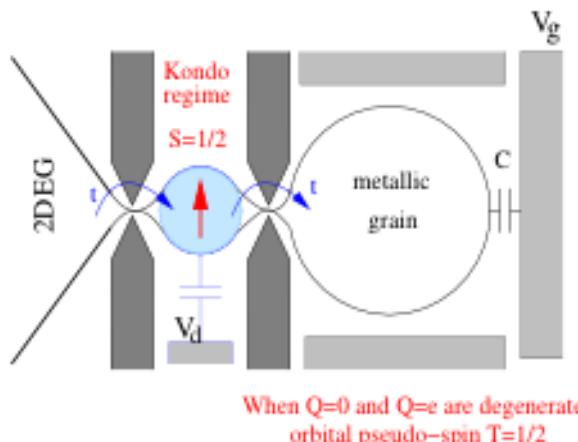
# « Qubits of different types »



Large dot at resonance: « charge object »  
↑ = occupied  
↓ = unoccupied

Spins are coupled through a magnetic interaction: Heisenberg, Ising...  
Charges are coupled through Coulomb forces or Capacitances

*Confinement mechanisms to couple spin & charge? Kondo effect?*



*Courtesy of D. Goldhaber-Gordon*

# Which condition to have entanglement?

SU(4) symmetry

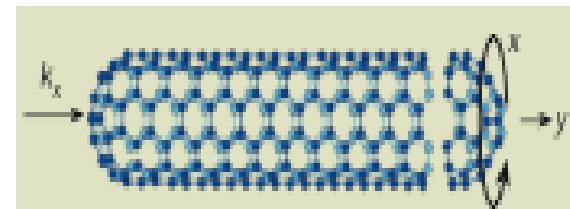
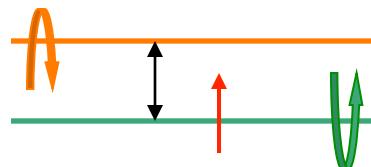
$$\begin{aligned}
 H_K &= J \sum_A \psi^\dagger t^A \left[ \sum_{\alpha,\beta} \left( S^\alpha + \frac{1}{2} \right) \left( T^\beta + \frac{1}{2} \right) \right]^A \psi \\
 &= \frac{J}{4} \sum_A M^A \sum_{\mu,\nu} \psi_\mu^\dagger t_{\mu\nu}^A \psi_\nu,
 \end{aligned}$$

4 screening channels...

Other Proposals for « charge » and « spin »

$$\uparrow = (0,1) : \quad \text{Diagram of two circles connected by a horizontal line, with a magenta dot in the right circle.}$$

$$\downarrow = (1,0) : \quad \text{Diagram of two circles connected by a horizontal line, with a magenta dot in the left circle.}$$



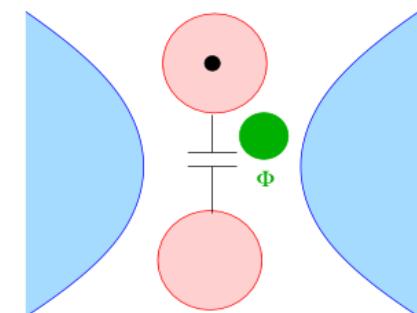
*Orbital & Spin Kondo: Kouwenhoven et al., Nature 2006*

Halperin, et al. PRL 90, 026602 (2003)

M.-R. Li & K. Le Hur, PRL 93, 176802 (2004)

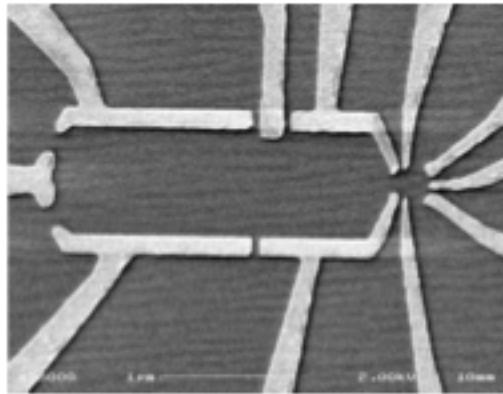
Expts: M. Pioro-Ladrière & A. Sachrajda, PRB (2005)

J. Petta, C. Marcus, et al. PRL (2004)



PRB 71, 115312 (2005)

# Hybrid Large dot - Small dot

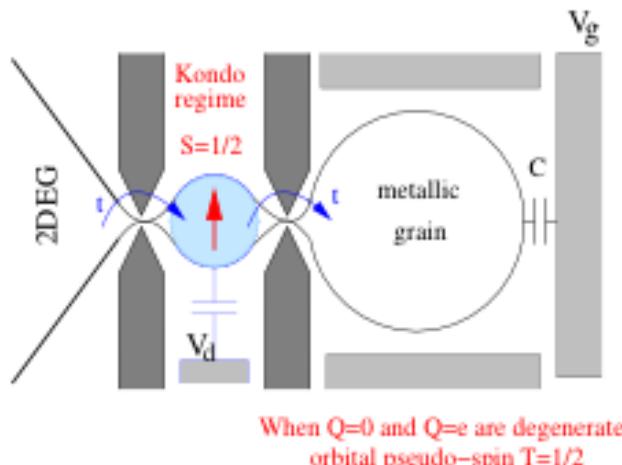


Collaborators: **Pascal Simon & Laszlo Borda**

K. Le Hur & P. Simon, PRB 67, R201308 (2003)

K. Le Hur, P. Simon, L. Borda, PRB 69, 045326 (2004)

**David Goldhaber-Gordon & Ron Potok**



Metallic grain: level spacing  $\Rightarrow 0$   
continuum of fermions  $c_{kga}$

At the resonant condition:

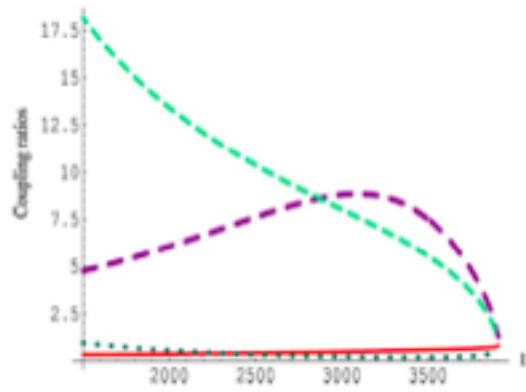
T matrix embodies the two-given charge states

$\psi_\mu$  with  $\mu=(\tau,\alpha)$

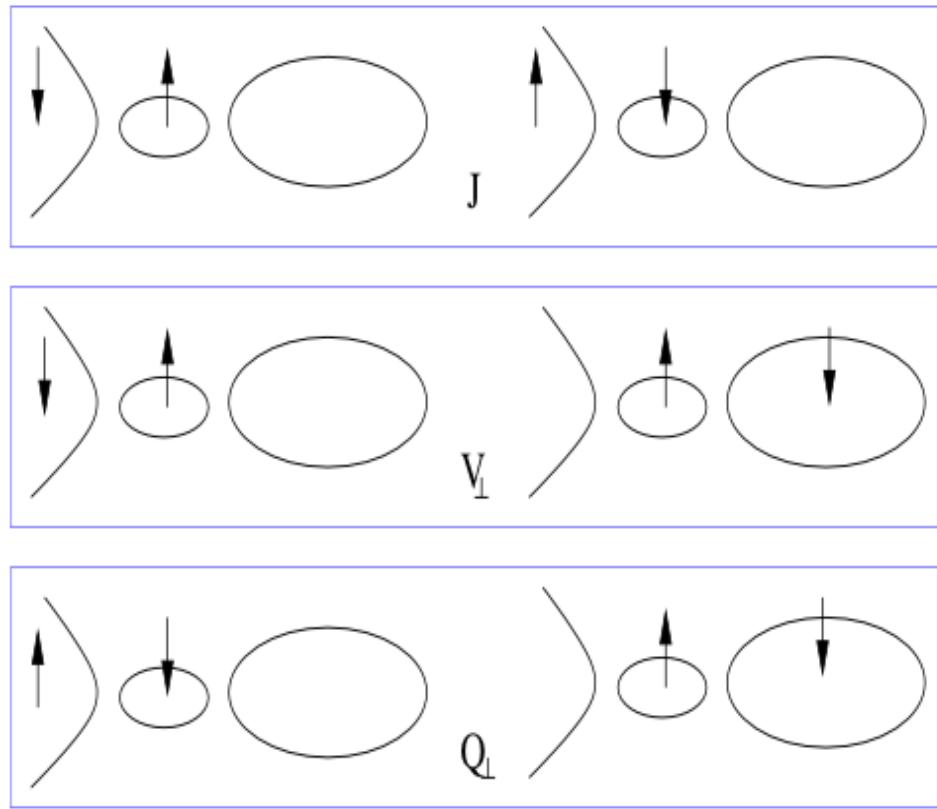
$\tau$ : position g or l  
 $\alpha$ : spin  $\uparrow$  or  $\downarrow$

-Asymmetric bare values

-But for symmetric barriers  
poor man scaling  
+ Wilson NRG approach: **SU(4)**



$$\begin{aligned}
 H_K &= \frac{J}{2} \vec{S} \cdot (\psi^\dagger \vec{\sigma} \psi) \\
 &+ \frac{V_z}{2} T^z (\psi^\dagger \tau^z \psi) + \frac{V_\perp}{2} [T^+ (\psi^\dagger \tau^- \psi) + h.c.] \\
 &+ Q_z T^z \vec{S} \cdot (\psi^\dagger \tau^z \vec{\sigma} \psi) + Q_\perp \vec{S} \cdot [T^+ (\psi^\dagger \tau^- \vec{\sigma} \psi) + h.c.],
 \end{aligned}$$

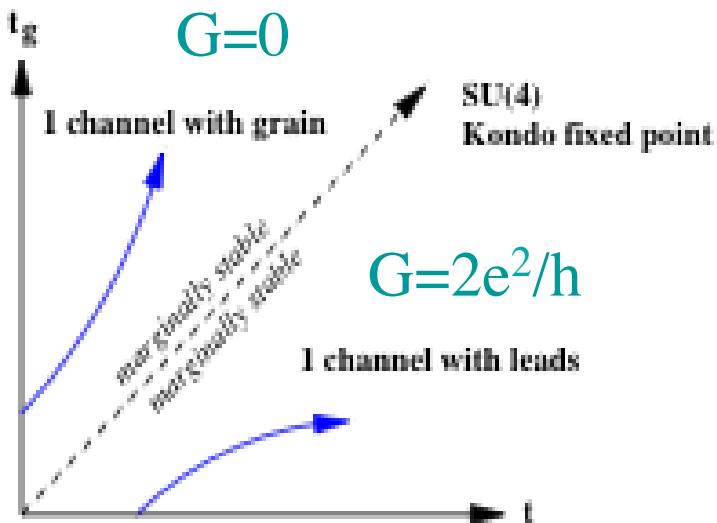


# Experimental consequences

SU(4) strong-coupling fixed point: Marginally stable towards perturbations such as magnetic field or orbital field (grain gate V)

Theory:  $\delta(T=0)=\pi/4$  (here, 4 screening channels!)  
so  $G=e^2/h$  at the fixed point ( $V, T=0$ )

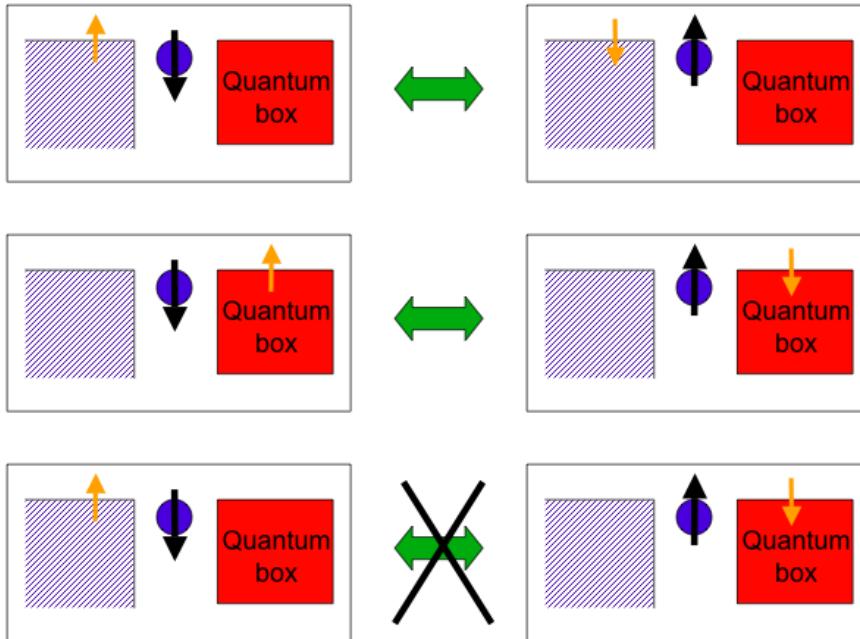
$$G = (2e^2/h)\sin^2 \delta$$



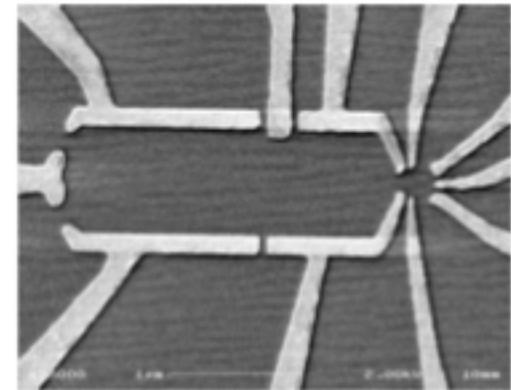
Linear scaling in  $V/T_K$ :  
Abrikosov-Suhl resonance at  $\omega=T_K$

- K. Le Hur & P. Simon, PRB 67, R201308 (2003)*  
*K. Le Hur, P. Simon, L. Borda, PRB 69, 045326 (2004)*  
*K. Le Hur, P. Simon, D. Loss, cond-mat/0609298*

# 2-channel Kondo physics?



*D. Goldhaber-Gordon & Y. Oreg  
PRL 90, 136602 (2003)*



*R. Potok, D. Goldhaber-Gordon et al*

$$\delta(T=0)=\pi/4 \text{ so } G=e^2/h$$

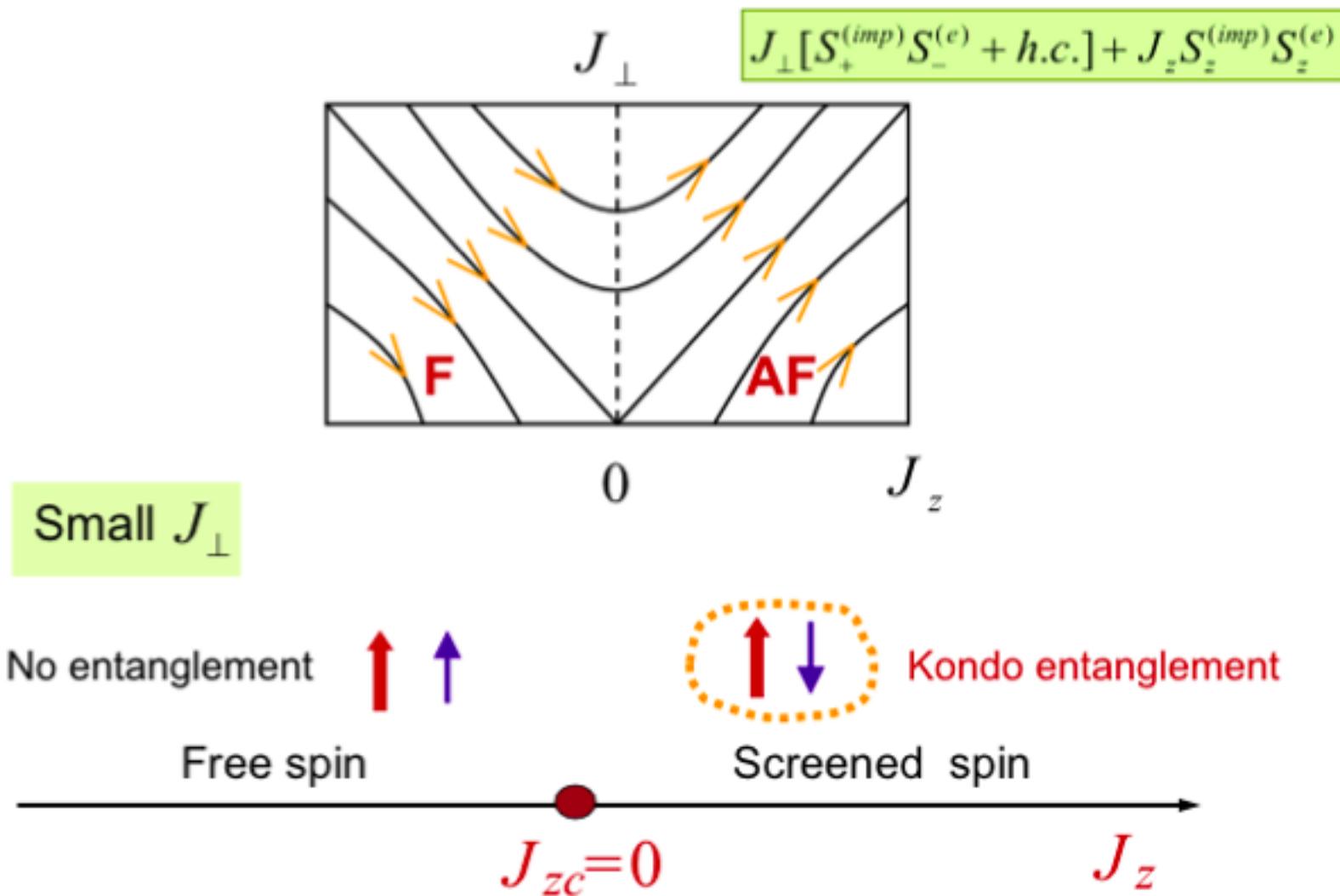
scaling in  $(T/T_K)^{1/2}$

$$H_K = \frac{J_l}{2} \vec{S} \cdot (\psi_l^\dagger \vec{\sigma} \psi_l) + \frac{J_g}{2} \vec{S} \cdot (\psi_g^\dagger \vec{\sigma} \psi_g)$$

*Nozières & Blandin, 1980  
Affleck & Ludwig*

*Other 2-channel proposals:*  
 -5/2 plateau of the FQHE  
*P. Fendley, M. Fisher, C. Nayak*  
 -Orbital Kondo effect  
*K. Le Hur & G. Seelig (2002), ...*

# Quantum phase transition in Kondo model

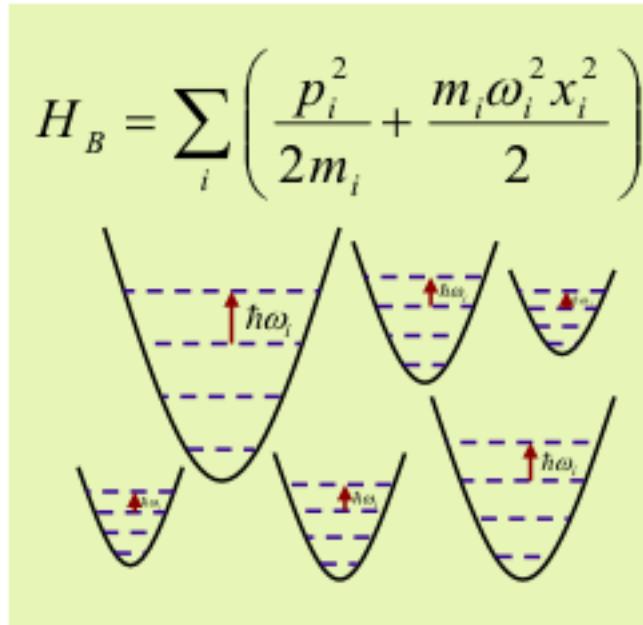


Observability?

# Hidden « Caldeira-Leggett » model

*S. Chakravarty, PRL 49, 681 (1982)*

*A.J. Bray & I.A. Moore, PRL, 49, 1545 (1982)*



$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

*A.J. Leggett et al., Rev. Mod. Phys. 59, 1(1987)*

$$\boxed{J_z \propto 1 - \sqrt{\alpha}} \\ J_{\perp} \propto \Delta$$

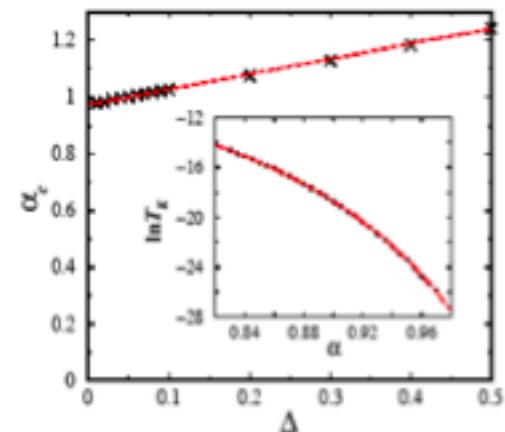
$$\frac{1}{2} \left\langle \sum_i \lambda_i x_i(t) \cdot \sum_i \lambda_i x_i(0) \right\rangle_{\omega} = \hbar J(\omega) \coth(\omega/2k_B T)$$

Ohmic dissipation  
 $J(\omega) = \alpha \pi \hbar \omega / 2$

↑  
Dissipation strength

Numerical RG for bosons:

*M.-R. Li, K. Le Hur, W. Hofstetter  
PRL 95, 086406 (2005)*

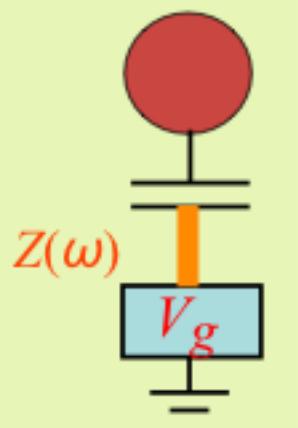


$$\alpha_c = R_c/R_K = 1 + 0.5\Delta/\Lambda$$

# Application: Noisy Qubits

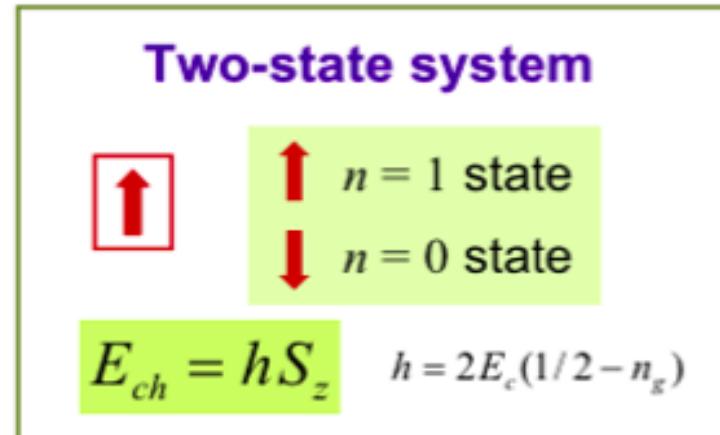
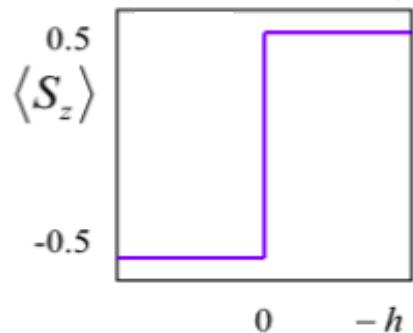
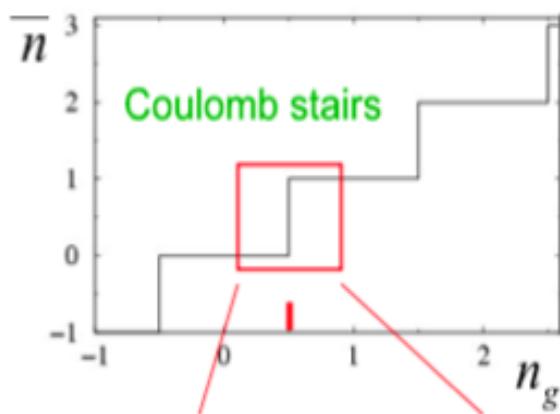
Noisy SC « charge » qubits close to resonance

Schoelkopf, Clerk, Girvin, Lehnert, Devoret, cond-mat/0210347

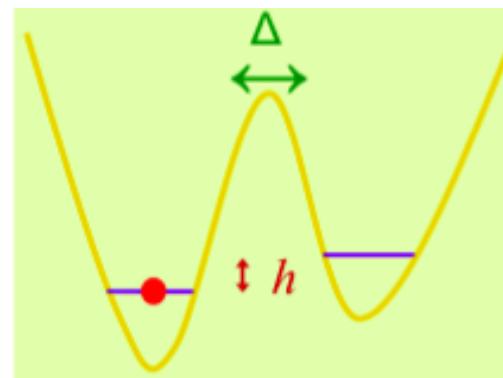


$$E_{ch} = E_c(n - n_g)^2$$

$$n_g = C_g V_g / e$$



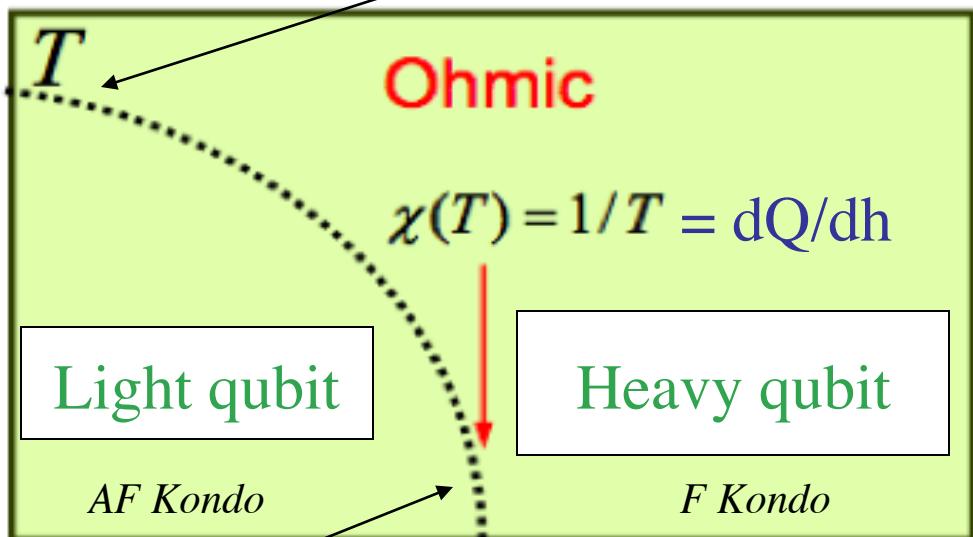
$$H_S = hS_z + \Delta(S_+ + S_-)$$



In the SC:  $\psi = \sqrt{n} \exp i\theta$

constant

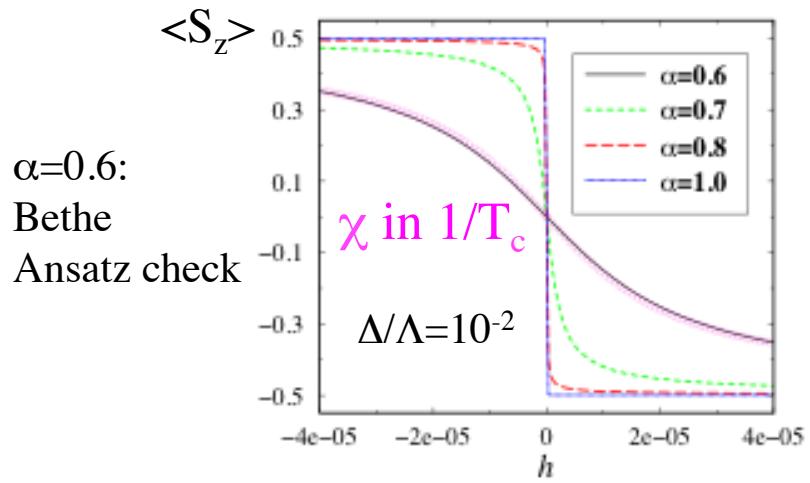
$$T_c = \Delta(\Delta/\Lambda)^{\alpha/(1-\alpha)}$$



$$\ln T_c / \Lambda \approx 1/(\alpha_c - \alpha)$$

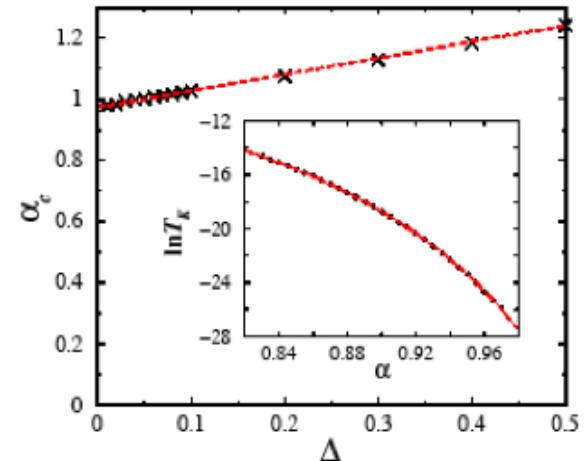
$$KT$$

$$\alpha$$



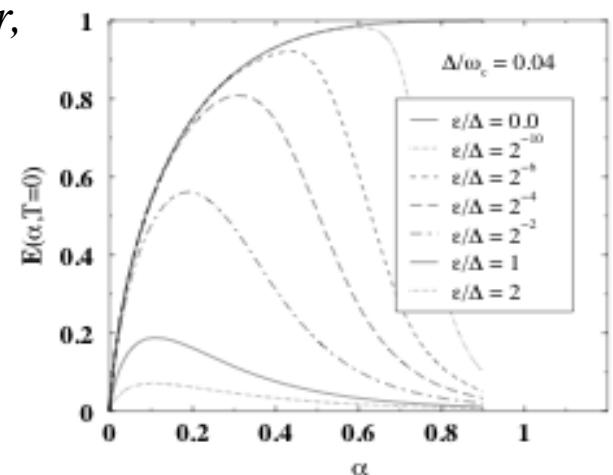
Size of the Jump =  $1 - (\Delta/\Lambda)^2 [2(2\alpha-1)(2\alpha-2)]^{-1}$  for  $\alpha \gg 1$

M.-R. Li, K. Le Hur, W. Hofstetter,  
PRL 95, 086406 (2005)



$$\alpha_c = R_c/R_K = 1 + 0.5\Delta/\Lambda$$

A. Kopp & K. Le Hur,  
In preparation

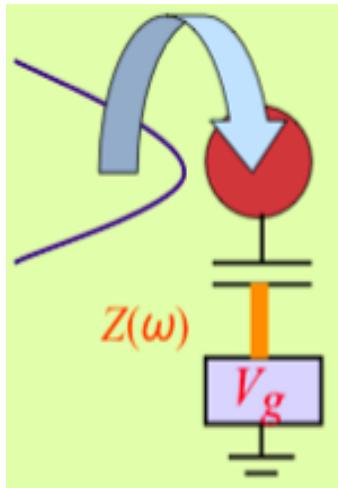


T. Costi & R. McKenzie,  
Phys. Rev. A 68, 034301 (2003)

# Metallic qubit: Bose-Fermi Kondo model!

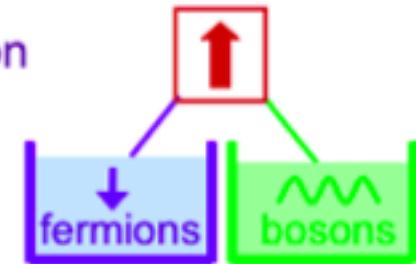
*K. Le Hur, PRL 92, 196804 (2004)*

*M.-R. Li & K. Le Hur, PRL 93, 176802 (2004)*



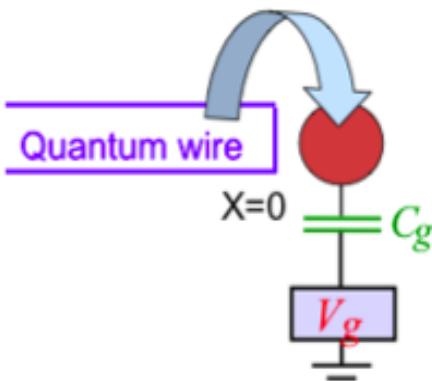
$$H_t = \frac{t c_{dot}^+ c_{lead} S_- + h.c.}{\text{Spin-fermion}} + \frac{\delta V_g S_z}{\text{Spin-boson}} + H_{noise}$$

Link to heavy-fermion physics



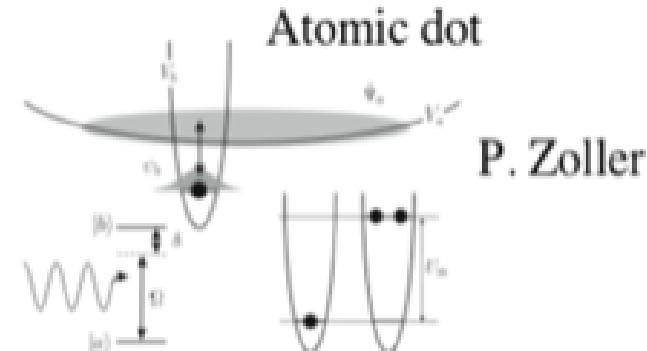
Exact mapping onto Caldeira-Leggett model:  
All noisy qubits belong to the same universality class

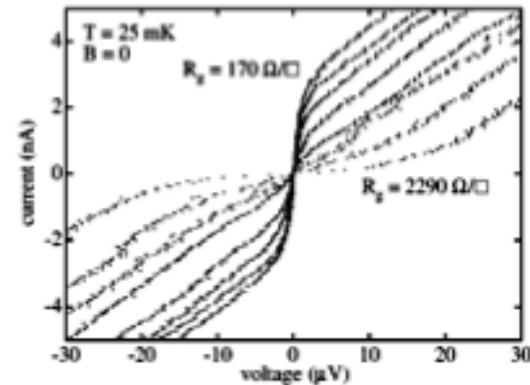
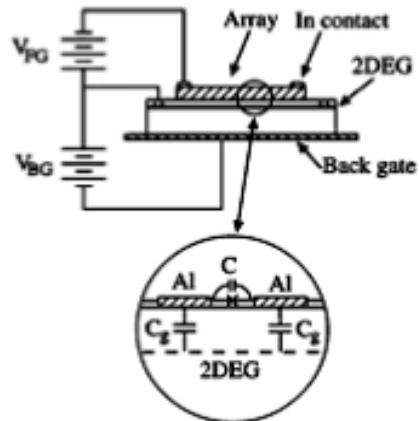
*M.-R. Li, K. Le Hur, W. Hofstetter, PRL 95, 086406 (2005)*



*Karyn Le Hur and Mei-Rong Li  
PRB 72, 073305 (2005)*

Unification Luttinger physics & em noise





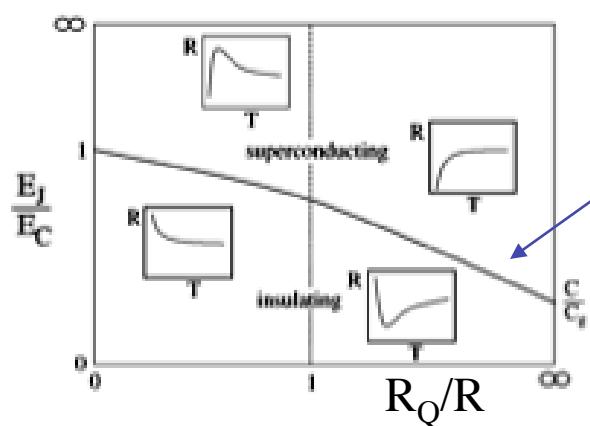
*A. Rimberg, J. Clarke et al. PRL 78, 2632 (1997)*

*N. Mason & A. Kapitulnik PRL 82, 5341 (1999) (MoGe films)*

## Bose-Hubbard model in a dissipative environment

The islands are sufficiently large:  $C_g \gg C$  and  $E_c = e^2/C_{\text{tot}} = 2.4 E_J$

$$H = \sum_i -E_J \cos(\theta_i - \theta_{i+1}) + E_c (\hat{n}_i - n_o)^2$$

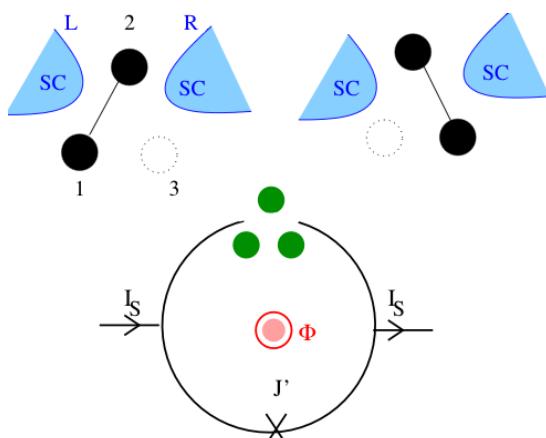
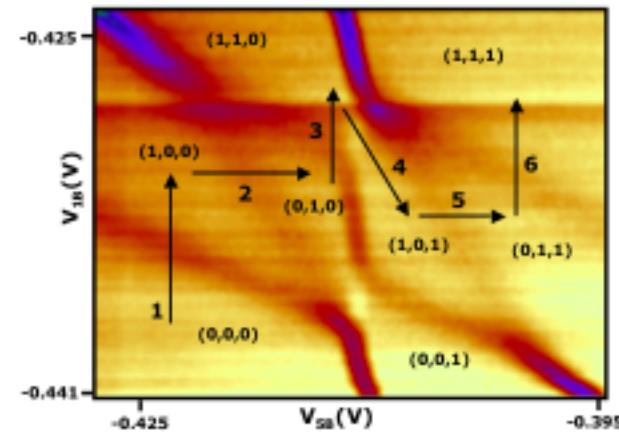
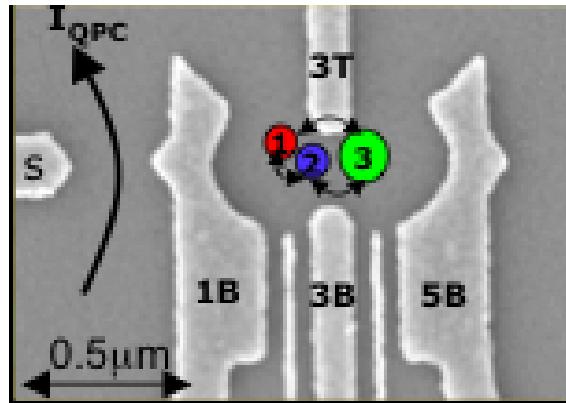


*S. Doniach, PRB 24, 5063 (1981)*

A noise-induced quantum phase transition

*G. Schön et al. PRL 79, 2730 (1997)*

# Futurist Ideas: Beyond double dot



« A Mesoscopic Pendulum »  
K. Le Hur, P. Recher, É. Dupont, D. Loss,  
Phys. Rev. Lett. **96**, 106803 (2006)

« Kondo effect with a trimer »  
K. Ingersent, A. Ludwig, I. Affleck, PRL **95**, 257204 (2005)

# Novel Nano-Kondo systems

- Possibility of exploring novel phases: SU(4) Fermi liquid
- Ferro-Antiferro transition of the Kondo model: noisy charge qubits
- Route towards triple dot: Mesoscopic Pendulum, Kondo with Trimer

## Acknowledgments:

Students: *G. Seelig (Caltech), É. Dupont, M. Pioro-Ladrière (Tarucha, Japan), L. Gaudreau*

Post-Docs: *P. Recher (Stanford), P. Vitushinsky*

Collaborators: *D. Goldhaber-Gordon (Stanford), A. Sachrajda (Ottawa), M. Büttiker (Geneva), A. Clerk (McGill), L. Glazman (Minnesota), D. Loss (Basel), P. Simon (Grenoble)*

*Thank you for your attention !*