Many-Body Systems and the Bell State

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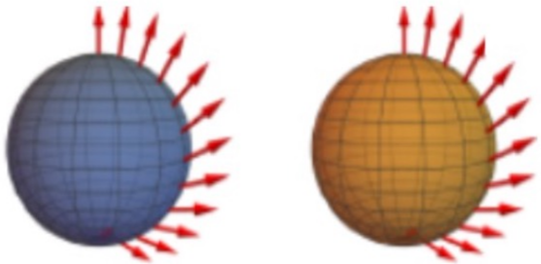
Filip Ronning: Topics “probing entanglement in condensed matter systems and strongly correlated 2D materials”

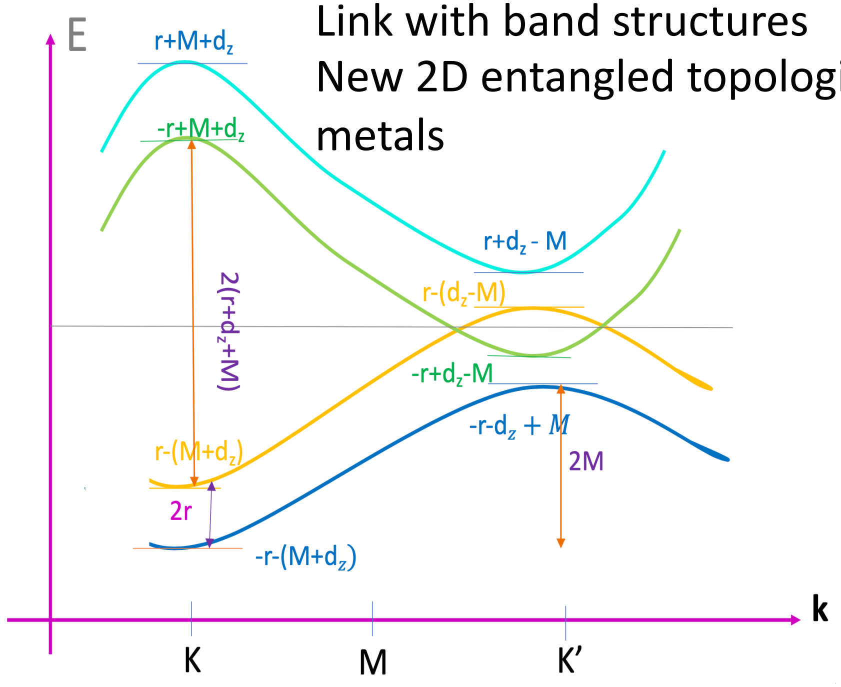
LANL Los Alamos Center of Materials

Quantum Physics



Bell state, Einstein-Podolsky-Rosen pair
Pair of ½ Topological Numbers: measurable

Ring of
5 Entangled
Spheres:
Our version



Link with band structures
New 2D entangled topological metals

Probe of many-body entanglement: bi-partite fluctuations

Also interacting wires…

PHYSICAL REVIEW VOLUME 129, NUMBER 4 15 FEBRUARY 1963

Fluctuation Compressibility Theorem and Its Application to the Pairing Model

J. S. Bell*
University of Washington, Seattle, Washington, and CERN, Geneva, Switzerland
(Received 4 September 1962)

A theorem of statistical mechanics relates density fluctuations to compressibility. A new derivation of this is given. The theorem is violated in the BCS model of a superconductor. The difficulty is resolved by those same improvements in the theory which lead to a gauge-invariant Meissner effect.

$$G(\mathbf{x}-\mathbf{y})=\langle\rho(\mathbf{x})\rho(\mathbf{y})\rangle-\langle\rho(\mathbf{x})\rangle\langle\rho(\mathbf{y})\rangle,$$

$$\int d\mathbf{x}\, G(\mathbf{x})=kT\frac{\partial\rho}{\partial\mu},$$

$$N'=\int_{\Omega'} d\mathbf{x}\,\rho(\mathbf{x})$$

$$\langle N'^2\rangle-\langle N'\rangle^2=\Omega' kT(\partial\rho/\partial\mu),$$

$$\lim_{\mathbf{k}\to 0}\{\lim_{\Omega\to\infty}\tilde{G}(\mathbf{k})\}=kT\frac{\partial\rho}{\partial\mu}.$$

This nice article, that is at the heart of our work, is only cited a few times compared to the article in 1964

Entanglement and Observables in Many-Body Systems: "A small Recap from our side"

Review: *Karyn Le Hur, Annals of Physics 2008*

- Quantum Impurities and Quantum Phase Transitions

Ohmic spin-boson model
Kondo model
NRG and Bethe-Ansatz

Kosterlitz-Thouless
Berezinskii transition

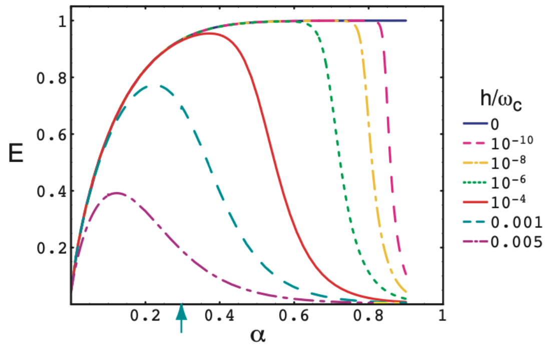
Von Neumann
Entropy
Entanglt entropy

E = S

$$E = -p_+ \log_2 p_+ - p_- \log_2 p_-,$$

$$p_\pm = \left(1 \pm \sqrt{\langle\sigma_x\rangle^2 + \langle\sigma_y\rangle^2 + \langle\sigma_z\rangle^2}\right)/2.$$

probes norm
of the spin & university
class of transitions



- Bi-partite Fluctuations of charge, spin in Luttinger liquids or 1D wires, spin chains

Not yet observed

$$\mathcal{F}_A = \langle(\hat{N}_A - \langle\hat{N}_A\rangle)^2\rangle, \qquad \pi^2 \mathcal{F}_{\rm LL}(x) = K \ln\frac{x}{a},$$

$$\frac{\mathcal{S}(x)}{\pi^2 \mathcal{F}(x)} \sim \frac{c}{3\pi v \kappa},$$

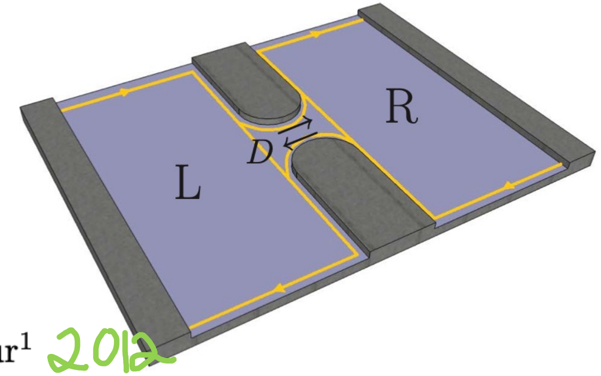
B. Hsu, E. Grosfeld, E. Fradkin, 2009; I. Klich, L. Levitov 2009 ; H. F. Song, S. Rachel, K. Le Hur 2010

- Exact Series for Free Fermions

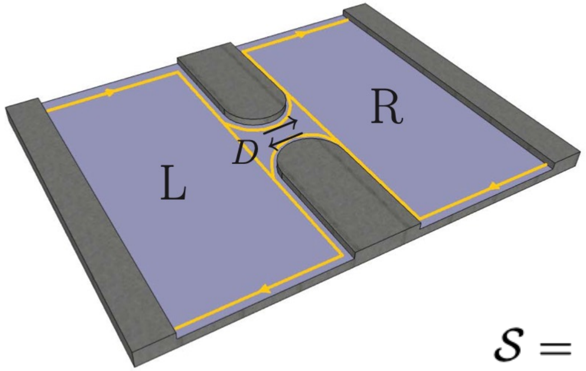
U.S. National Science Foundation

Yale

$$\mathcal{S} = \lim_{K\to\infty} \sum_{n=1}^{K+1} \alpha_n(K) C_n, \qquad \alpha_n(K) = \begin{cases} 2\sum_{k=n-1}^{K} \frac{S_1(k,n-1)}{k!k} & \text{for } n \text{ even,} \\ 0 & \text{for } n \text{ odd,} \end{cases}$$



Review: H. Francis Song,[1] Stephan Rachel,[1] Christian Flindt,[2] Israel Klich,[3] Nicolas Laflorencie,[4] and Karyn Le Hur[1] 2012



"Quantum limit"

$$\mathcal{S} = -\frac{eVt}{h}\left[D \ln D + (1-D)\ln(1-D)\right].$$

K=2

$$\mathcal{S} = \frac{1}{3}\ln\frac{t}{\tau_c}.$$

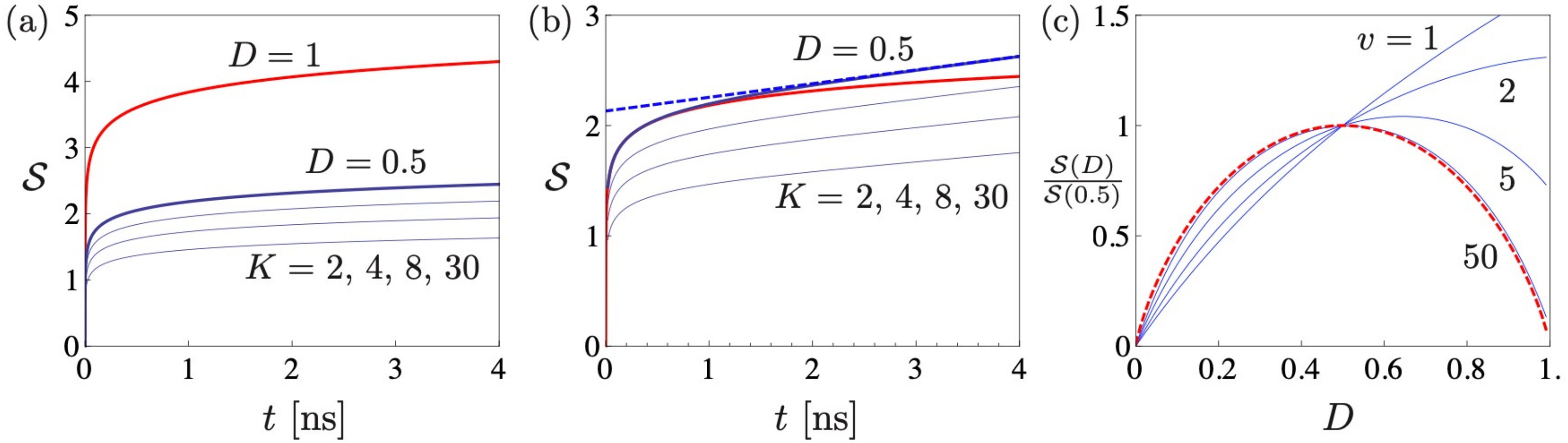


FIG. 5: (color online). Entanglement entropy in a quantum point contact (QPC). (a) Zero-temperature results for the time dependent entanglement entropy with different QPC transmissions D. The $D = 1$ result shown in red is given by Eq. (2.41). For $D = 0.5$, results were obtained from the series (2.1) with increasing cutoff number K from bottom to top. The thick blue line is the converged result for $K = 30$. The ultraviolet short-time cutoff is $\tau_c = 10^{-5}$ ns. (b) Finite-temperature results for $T = 10$ mK (or $\tau_\beta \simeq 1.5$ ns), $D = 0.5$, and $\tau_c = 10^{-5}$ ns. Blue lines show results obtained with increasing cutoff number K and the thick blue line is the converged result. For comparison, zero and high temperature limits are indicated with a red and a blue dashed line, respectively. For short times $t < \tau_\beta$ the time-dependence is logarithmic, while for long time the high-temperature behavior eventually prevails and the entropy grows linearly with time. (c) Results for a biased QPC as a function of the transmission D. Here $v = eV/(2k_BT)$ is the ratio of the applied voltage V over temperature T. In the long-time limit $t \gg \tau_\beta$, the ratio $\mathcal{S}(D)/\mathcal{S}(0.5)$ does not depend on time. The cutoff number is $K = 10$. As the voltage is increased, the entanglement entropy changes from a nearly linear dependence on D to that of a binomial event with success probability D, given by Eq. (2.45) and shown with a red dashed line.

1D Kitaev p-wave SC Wire (transverse Ising spin chain): Insight from Topology

L. Herviou, C. Mora, K. Le Hur, Phys. Rev. B 96, 121113 (2017)

Further work on applications in Weyl semimetals & 2D superconductors Phys. Rev. B 99, 075133 (2019)

Nambu spinor, $\Psi_k^{+} = (c_k^{+}, c_{-k})$

$$\theta_k = \mathrm{Arg}(\varepsilon_k - i\Delta_k)$$

$$m = \oint \frac{d\theta_k}{2\pi},$$

$$H = \frac{1}{2}\sum_k \Psi_k^\dagger(\varepsilon_k\sigma^z + \Delta_k\sigma^x)\Psi_k,$$

$$\hat{Q}_j = \frac{q_e}{2}\Psi_j^\dagger\sigma^z\Psi_j,$$

Bi-Partite Fluctuations

Fejér Kernel

$$F_{\hat{Q}}(A) = q_e l \iint_{\mathrm{BZ}^2} \frac{dk dq}{16\pi^2} f(k-q,l)$$
$$(1 - \cos(\theta_k)\cos(\theta_q) + \sin(\theta_k)\sin(\theta_q)),$$

$$F_{\hat{Q}}(A) = i_{\hat{Q}} l + b\log(l) + \mathcal{O}(1),$$

$c^{+}c \sim i\eta_1\eta_2$

$$b = \ominus\frac{q_e}{2\pi^2}$$

$$i_{\hat{Q}} = \lim_{L\to+\infty}\frac{1}{L}\left\langle \hat{Q}^2\right\rangle_C = q_e\int_{\mathrm{BZ}}\frac{dk}{4\pi}\sin^2(\theta_k),$$

Fisher Information Density (QFID)

Quantum Phase Transition of a Kitaev Wire

F. del Pozo (Phd student), L. Herviou, K. Le Hur Phys. Rev. B 107, 155134 (2023)

$$\vec{S} = \begin{pmatrix} c_k^\dagger c_{-k}^\dagger + c_{-k}c_k \\ -i\left(c_k^\dagger c_{-k}^\dagger - c_{-k}c_k\right) \\ c_k^\dagger c_k - c_{-k}c_{-k}^\dagger \end{pmatrix}$$

Anderson pseudospin

$$H_K = -\mu \sum_j n_j - t\sum_j c_j^\dagger c_{j+1} + \text{h.c.}$$
$$+\Delta e^{i\varphi} \sum_j c_j^\dagger c_{j+1}^\dagger + \text{h.c.}.$$

$$c_j = \frac{1}{2}\left(\gamma_{A,j} + i\gamma_{B,j}\right).$$

$$C = \frac{1}{2}\left(\langle S^z(\vartheta = 0)\rangle - \langle S^z(\vartheta = \pi)\rangle\right).$$

$$\mathcal{H}_k = -\vec{d}_k \cdot \vec{S}_k.$$

$C = 1$
$\mu = 0$

$$\cos(\vartheta_k) = \frac{2t\cos(ka) + \mu}{E(ka)},$$
$$\sin(\vartheta_k)\, e^{-i\tilde{\varphi}} = -\frac{i\Delta e^{i\varphi} 2\sin(ka)}{E(ka)}$$

Quantum phase transition detectable from classical light: Z_2 invariant

$C = \frac{1}{2}$

F. Del Pozo & K. Le Hur,
Phys. Rev. Lett B 110, L060503 (2024)

$\mu = 2t, \ \sigma \in [0, \frac{\pi}{2}]$

$$H_K = \sum_j i\frac{t-\Delta}{2}\gamma_{B,j}\gamma_{A,j+1} + i\frac{t+\Delta}{2}\gamma_{B,j+1}\gamma_{A,j}$$
$$-i\frac{\mu}{2}\sum_j \gamma_{A,j}\gamma_{B,j}.$$

$$\sqrt{2}\gamma_{L/R}(x = ja) = \gamma_B(x = ja) \pm \gamma_A(x = ja).$$

$$H_K \overset{a\to 0}{\longrightarrow} ita\int_x \mathrm{d}x\, \left(\gamma_L(x)\partial_x\gamma_L(x) - \gamma_R(x)\partial_x\gamma_R(x)\right).$$

$c = \frac{1}{2}$

A pair of ½ Topological Numbers as a Probe of Bell State or EPR pair

ANR BOCA France
NSERC Canada

J. Hutchinson (post-doctoral fellow) and K. Le Hur, Communications Physics 4, 144 (2021), Nature Journal

$$H = -(\mathbf{d}_1 \cdot \boldsymbol{\sigma}_1 + \mathbf{d}_2 \cdot \boldsymbol{\sigma}_2) + r f(\theta)\sigma_{1z}\sigma_{2z}$$

$$\mathbf{d}_i = (d \sin\theta \cos\varphi, d \sin\theta \sin\varphi, d\cos\theta + M_i)$$

$$|\psi\rangle = \sum_{kl} o_{kl}(\theta)|\Phi_k(\phi)\rangle_1 |\Phi_l(\phi)\rangle_2.$$

Berry Formalism

$$A_\varphi = -i\langle\psi|\partial_\varphi \otimes \hat{1}|\psi\rangle$$

$r = 0$

$M_1, M_2 = 0$

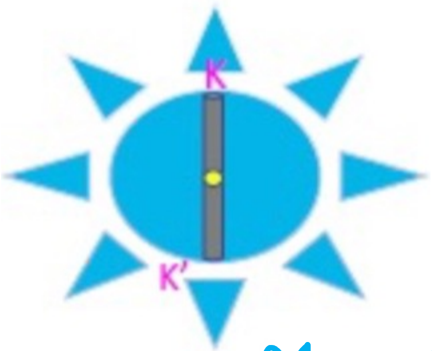
$$C_j = \int_0^\pi \frac{\sin\theta}{2} d\theta = -\frac{1}{2}[\cos\theta]_0^\pi = (\mathcal{A}_\varphi(\pi) - \mathcal{A}_\varphi(0)).$$

$$C_j = (\mathcal{A}_\varphi(\pi) - \mathcal{A}_\varphi(0)) = \frac{1}{2}(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle) = 1$$

1 monopole in each Bloch sphere

K. Le Hur, Review 2209.15381, application in transport and light
See also lectures on my web page: Paris Saclay Lectures Series
Classical Physics

Skyrmions

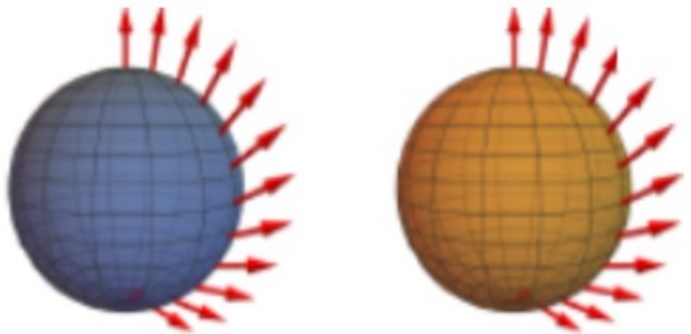


Haldane Model

r>>d+M classical antiferromagnet, C=0

$M_1 = M_2 = M$

$$d - M < r < d + M$$



$$|\psi(0)\rangle = |\Phi_+\rangle_1 |\Phi_+\rangle_2 = |\Phi_+\rangle_1 \otimes |\Phi_+\rangle_2$$ ferro

$$|\psi(\pi)\rangle = \frac{1}{\sqrt{2}}(|\Phi_+\rangle_1 |\Phi_-\rangle_2 + |\Phi_-\rangle_1 |\Phi_+\rangle_2).$$ EPR pair

K. Le Hur, Phys. Rev. B 108, 235144 (2023)

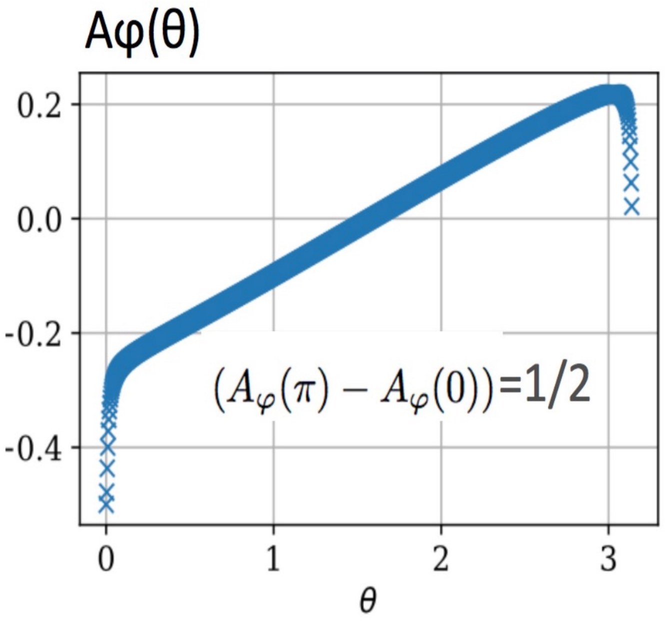
$$\mathcal{A}_{j\varphi}(\pi) = -i\langle\psi(\pi)|\partial_{j\varphi}|\psi(\pi)\rangle = \frac{\mathcal{A}_{j\varphi}(0)}{2} + \frac{\mathcal{A}^{r=0}_{j\varphi}(\pi)}{2},$$

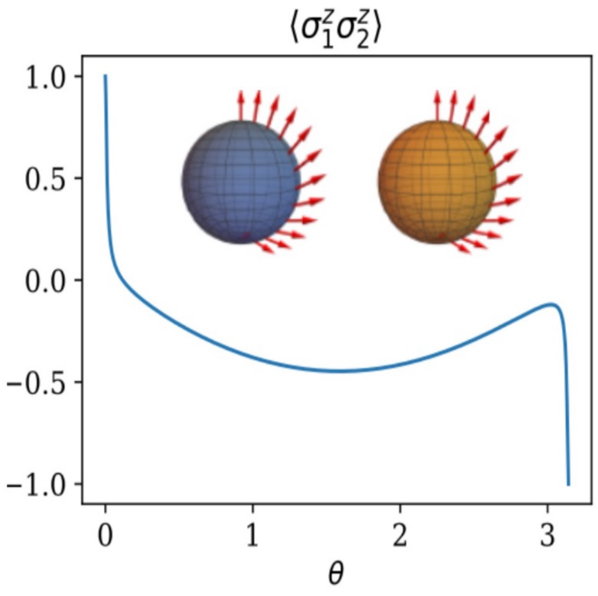
$$\mathcal{A}^{r=0}_{j\varphi}(\pi) - \mathcal{A}_{j\varphi}(0) = q = 1$$

Analogue of ½ flux quantum in superconductor

$$\mathcal{A}_{j\varphi}(\pi) - \mathcal{A}_{j\varphi}(0) = q\frac{1}{2} = C_j,$$

$$\frac{1}{2\pi}\iint_{S^2} \nabla_j \times \mathcal{A}_j \cdot d^2\mathbf{s} = C_j = q\frac{1}{2}$$





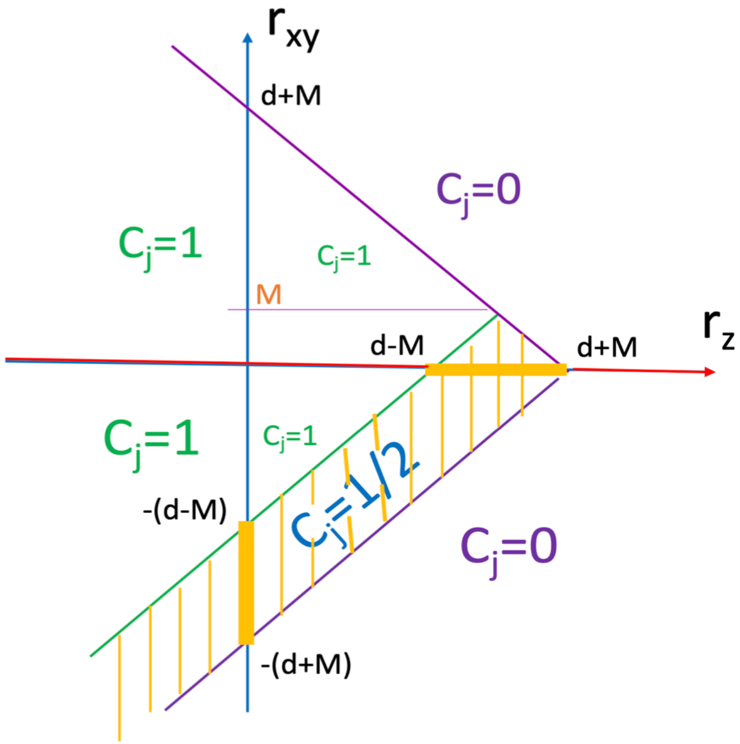
Correlation Functions

$$\langle\psi(\pi)|\sigma_{1z}\sigma_{2z}|\psi(\pi)\rangle = -|o_{++}(0)|^2 = -\frac{2C_j}{q} = -1.$$

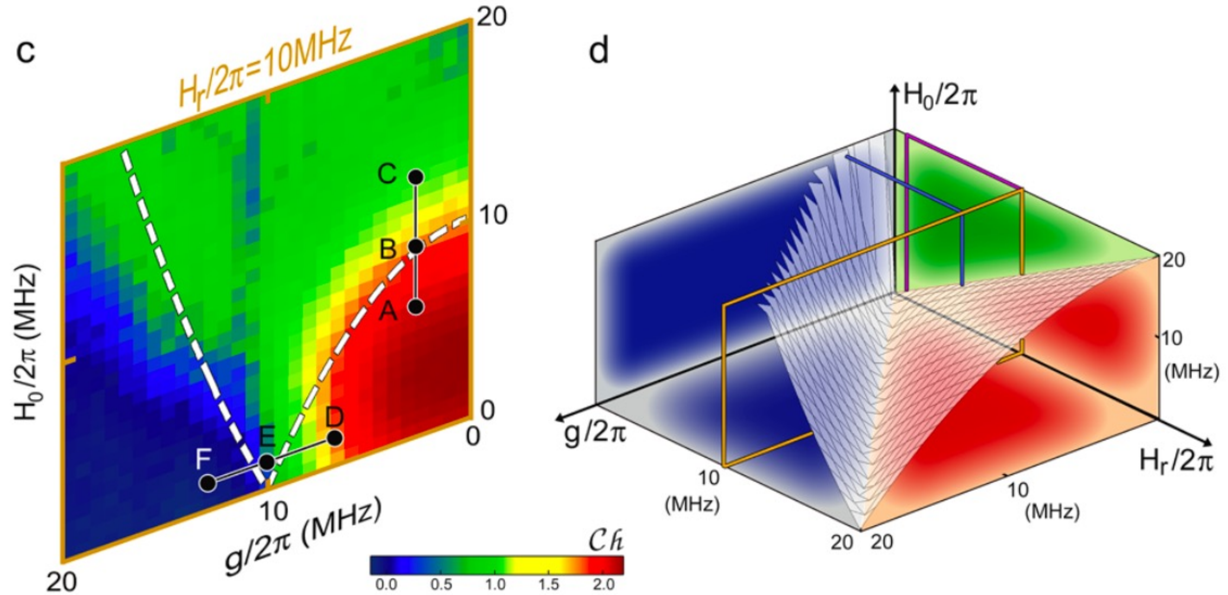
Bi-partite Fluctuations (~ entropy)

$$F(\pi) = \langle\psi(\pi)|\sigma_{1z}^2 \otimes \mathbb{I}|\psi(\pi)\rangle - \langle\psi(\pi)|\sigma_{1z}\otimes\mathbb{I}|\psi(\pi)\rangle^2.$$

$$F(\pi) = 4|o_{+-}(\pi)|^2|o_{-+}(\pi)|^2 = \frac{2C_j}{q} = +1,$$



$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)],$$



P. Roushan et al. Nature 515 241-244 (2015)

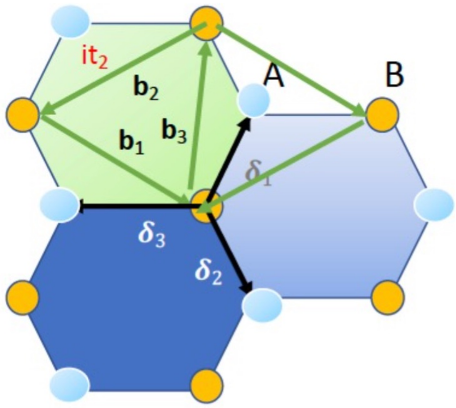
Entangled WaveFunctions in Topological Band Structures

Topological Nodal Ring Semimetal: Topological Half Metal

J. Hutchinson & K. Le Hur, Communications Physics 4, 144 (2021)
P. Cheng, Ph. W. Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg, K. Le Hur, Phys. Rev. B 100, 081107 (2019)

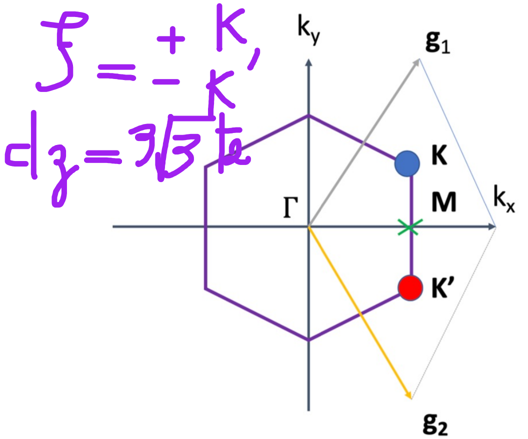
Thanks to DFG FOR2414

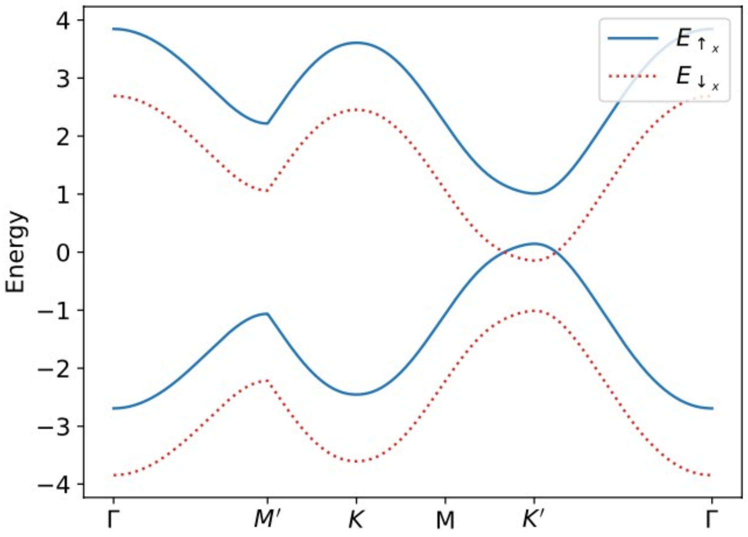
$$H = (\zeta d_z + M)\sigma_z \otimes \mathbb{I} + d_x\sigma_x \otimes \mathbb{I} + d_y\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes s_x$$

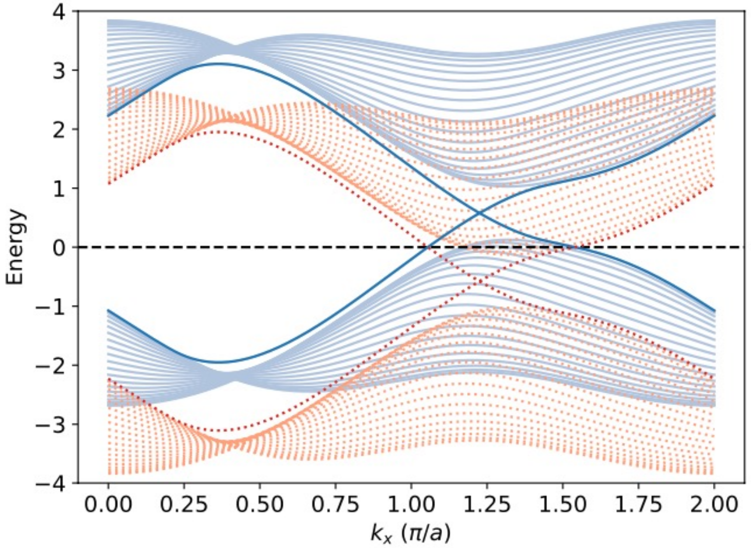


Bilayer model: σ *acts on sublattice and* s *on plane*
Graphene model: σ *acts on sublattice and* s *on spin*
M charge density wave substrate; r Zeeman effect in plane

$3\sqrt{3}t_2 - M < r < 3\sqrt{3}t_2 + M$







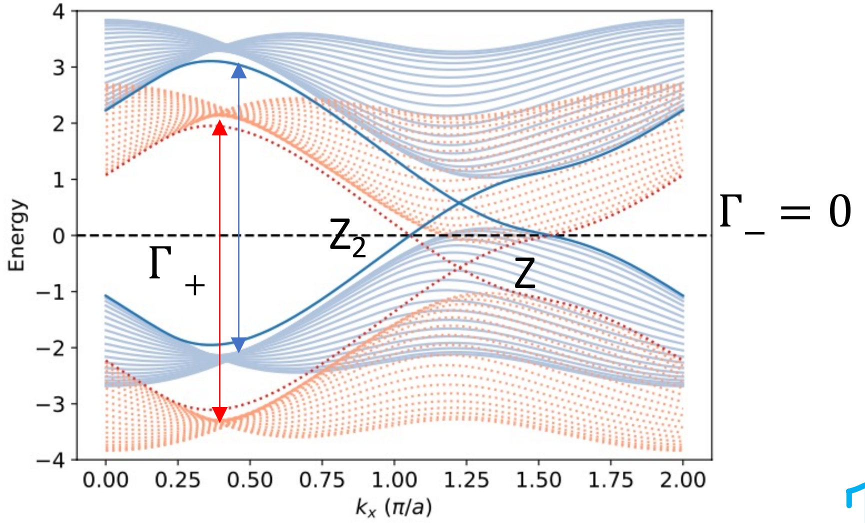
Stable towards Disorder interactions

K. Le Hur and S. Al Saati (PhD student), PRB 107, 165407 (2023) and arXiv:2311.13922

Topological Markers

K. Le Hur and S. Al Saati, arXiv:2311.13922

The spin polarized blue edge mode shows a quantized conductance at the edges



cylinder

Quantum Hall response can be evaluated for the different bands, domains with Kubo formula

$$C_{\downarrow x,-} = \left(\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi\right)\int_0^{\pi} \mathrm{d}\theta \left(-\frac{1}{2}\right)\sin\theta = -1,$$

Red lowest band

Haldane

Z_2 Kane-Mele

$$\hat{C}_{\uparrow x,-} - \tilde{C}_{\downarrow x,+} = C_{\downarrow x,-}.$$

Haldane model

Topological semimetal

Pair of bands (1,3) or (2,4)

$$\left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar} A_0^2 \frac{1}{2} |\langle\sigma_z(0)\rangle|$$

$$\sum_{j=\uparrow,\downarrow} \left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar} A_0^2 |C_{\downarrow x,-}|$$

Response to circularly polarized light: resonance with Dirac points

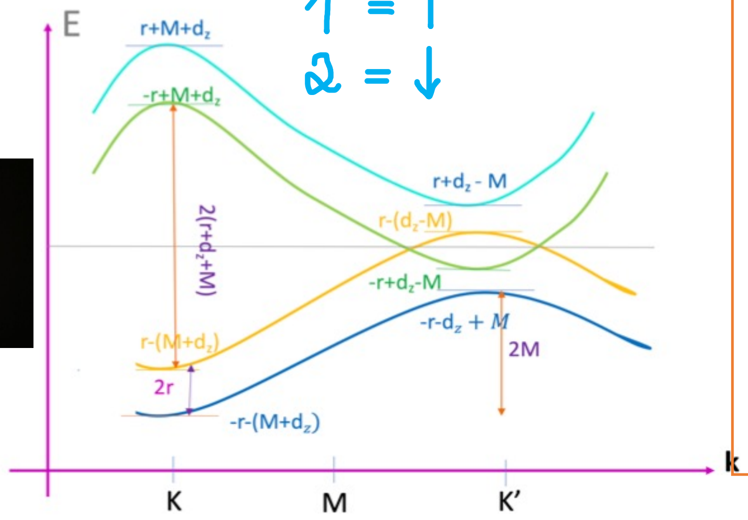
$$\left|\frac{\Gamma_+ - \Gamma_-}{2}\right| = \frac{2\pi}{\hbar} A_0^2 \frac{1}{2} |(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle)| = \frac{2\pi}{\hbar} A_0^2 |\langle\sigma_z(0)\rangle|.$$

$$\Gamma_\zeta(\omega) = \frac{2\pi}{\hbar}\frac{A_0^2}{2}\left|\langle +|\left(\frac{\partial \mathscr{H}}{\hbar\partial(\zeta p_y)} + \zeta i \frac{\partial \mathscr{H}}{\hbar \partial p_x}\right)|-\rangle\right|^2 \times \delta(E_l(\mathbf{p}) - E_u(\mathbf{p}) - \hbar\omega).$$

K. Le Hur, PRB 105, 125106 (2022)
Ph. W. Klein, A. Grushin, K. Le Hur, PRB 103, 035114 (2021)
Tran, Dauphin, Grushin, Zoller, Goldman 2017

Introduction of a stochastic variational approach for interacting topological systems

Relation to the 2 spheres model, entangled wavefunction, and ½ local Topological Marker



$1 = \uparrow$
$2 = \downarrow$

K Dirac point $\quad |GS\rangle = e^{i\pi} c^\dagger_{B\uparrow} c^\dagger_{B\downarrow} |0\rangle.$ ferro

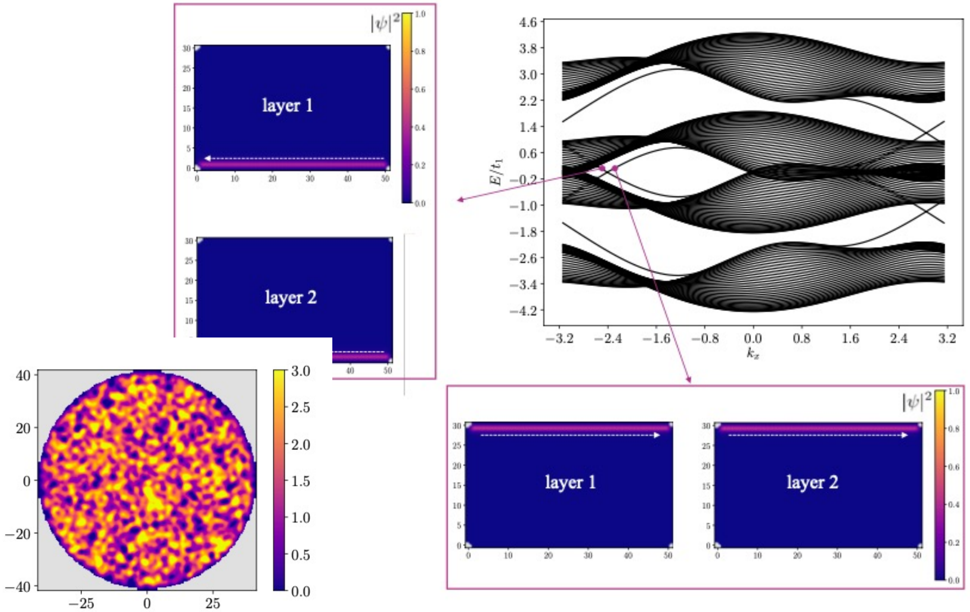
K' Dirac point $\quad |\psi_g\rangle = \frac{1}{2}(c^\dagger_{A\uparrow}c^\dagger_{B\uparrow} - c^\dagger_{A\uparrow}c^\dagger_{B\downarrow} - c^\dagger_{A\downarrow}c^\dagger_{B\uparrow} + c^\dagger_{A\downarrow}c^\dagger_{B\downarrow})|0\rangle.$

$$|GS\rangle_{K'} = \frac{1}{\sqrt{2}}|GS\rangle_{\theta=\pi^-} + \frac{1}{2}\left(c^\dagger_{A\uparrow}c^\dagger_{B\uparrow} + c^\dagger_{A\downarrow}c^\dagger_{B\downarrow}\right)|0\rangle.$$

$\langle \sigma_{jz}(K') \rangle = 0$

$$\mathcal{A}_{j\varphi}(K') = \mathcal{A}_{j\varphi}(K) + \frac{1}{2}q.$$

j = measure of electron with spin along z direction



Blue edge Mode $\uparrow_x = \frac{1}{\sqrt{2}}(\uparrow_z + \downarrow_z)$

$$C_j = \mathcal{A}_{j\varphi}(K') - \mathcal{A}_{j\varphi}(K) = \frac{1}{2}q$$

π Berry Phase

K. Le Hur, Phys. Rev. B 108, 235144 (2023)
K. Le Hur, Review arXiv:2209.15381

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021)

Majorana Fermions

K. Le Hur, Phys. Rev. B 108, 235144 (2023)

Jordan-Wigner transformation

$$c_1^\dagger c_1|\psi(0)\rangle = +|\psi(0)\rangle \qquad \sigma_{1z} = 2c_1^\dagger c_1 - 1$$

$\sigma = \pi$

$$\sigma_{1z} = \frac{1}{i}(c_1^\dagger - c_1),\ \sigma_{1x} = (c_1^\dagger + c_1),\ \sigma_{2z} = \frac{1}{i}(c_2^\dagger - c_2)e^{i\pi c_1^\dagger c_1}$$

$$\sigma_{2x} = (c_2^\dagger + c_2)e^{i\pi c_1^\dagger c_1}$$

$$\frac{1}{\sqrt{2}}(c_j + c_j^\dagger) = \eta_j^\dagger \text{ and } \alpha_j = \frac{1}{\sqrt{2}i}(c_j^\dagger - c_j) = \alpha_j^\dagger$$

$$\begin{aligned} H_{eff} &= r\sigma_{1z}\sigma_{2z} - \frac{d^2\sin^2\theta}{r}\sigma_{1x}\sigma_{2x} \\ &= -2ri\eta_1\alpha_2 - \frac{2id^2}{r}\sin^2\theta\alpha_1\eta_2. \end{aligned}$$

$C_j = \frac{1}{2}$

$$\langle\sigma_{1z}(\pi)\sigma_{2z}(\pi)\rangle = \langle 2i\alpha_2\eta_1\rangle = -1 = -(2C_j)^2.$$

$\sigma = \pi$

$\{\alpha_1, \eta_2\}$ free

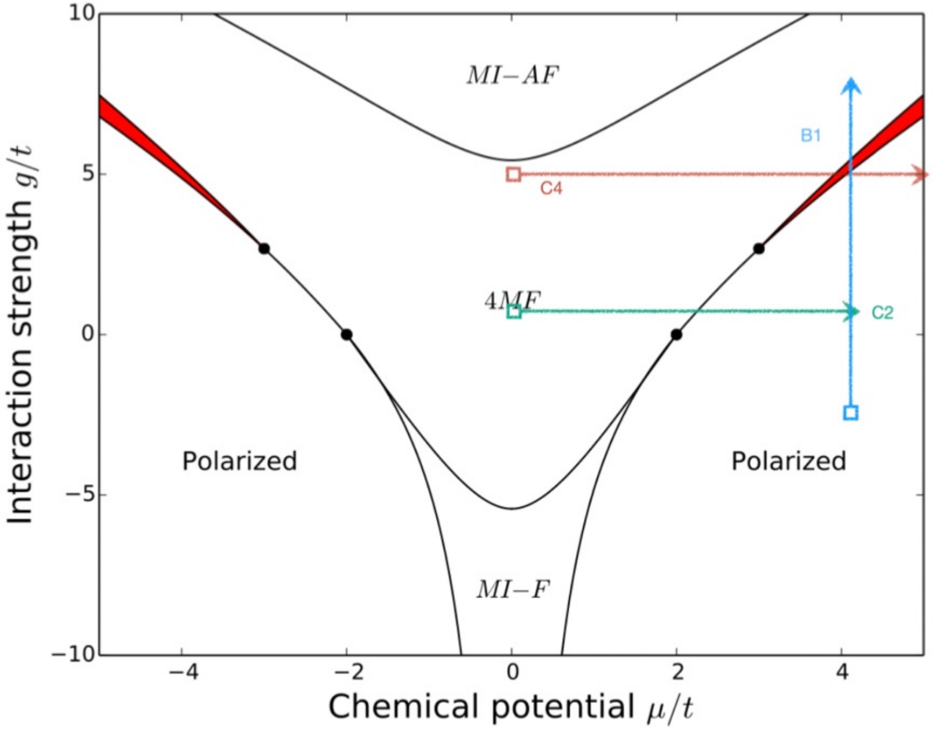
2 Kitaev wires coupled to a Coulomb interaction

L. Herviou, C. Mora, K. Le Hur, Phys. Rev. B 93, 165142 (2016) ; F. del Pozo, L. Herviou, K. Le Hur Phys. Rev. B 107, 155134 (2023)

Majorana liquid: equivalent to pair of Majorana modes at one pole or 2 Ising models

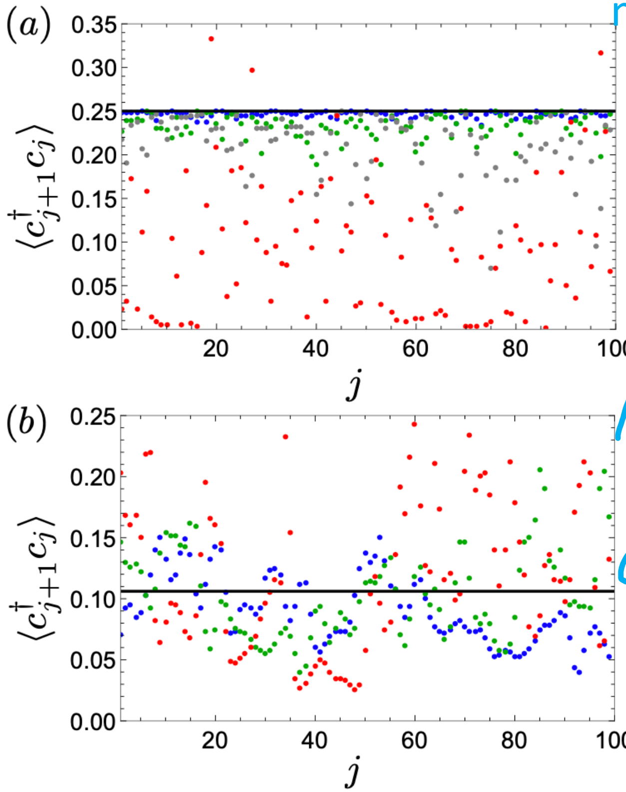
$$\frac{1}{2}\left\langle\left(S^z_{k=0} - S^z_{k=\frac{\pi}{a}}\right)\right\rangle \equiv C = \frac{1}{\sqrt{N}}\sum_{i=0}^{N/2}\langle \mathfrak{S}^z_{2i+1}\rangle$$

$$\mathfrak{S}^z_i = \frac{1}{\sqrt{N}}\sum_j \left(c_j^\dagger c_{j+i} - c_j c_{j+i}^\dagger\right)$$



DCI

1 wire ED



math

$\frac{1}{4}$

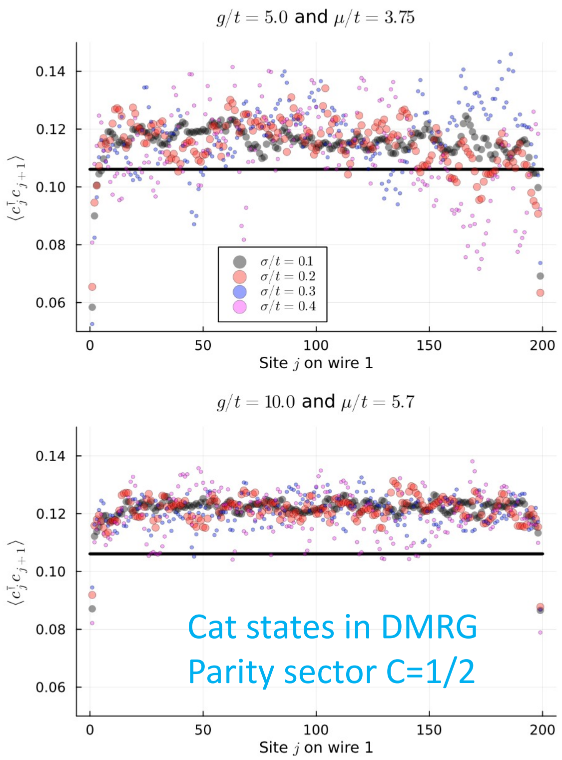
$\mu = 0$

$t = \Delta$

$C \frac{2}{3\pi}$

$\mu = 2t$

DCI : DMRG



Cat states in DMRG
Parity sector C=1/2

F. Del Pozo, L. Herviou, O. Dmytruk, K. Le Hur
arXiv: 2408.02105

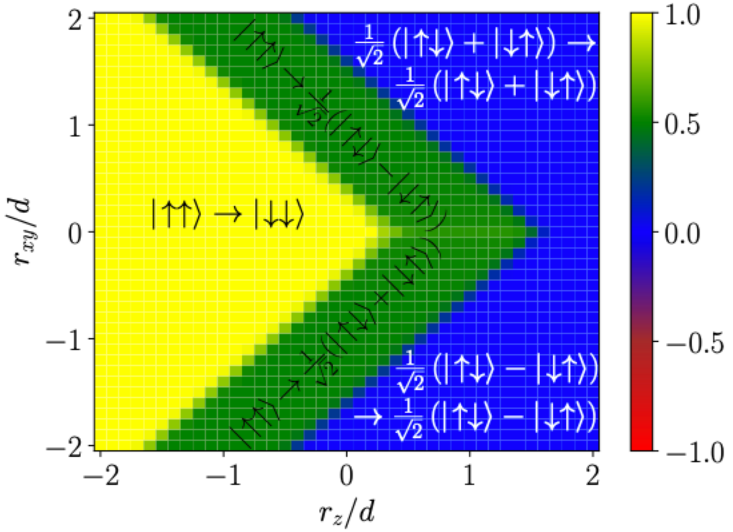
disorder

Majorana Fermions and Protected Quantum Information

Ephraim Bernhardt (Phd student, graduation 2023), Brian Cheung Hang Chung (Master), Karyn Le Hur, Physical Review Research 6, 023221 (2024)

$$H_{\text{eff}}^{|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle}(\theta=\pi^-) = -2ir_z\eta_1\alpha_2 - \frac{r_z d^2\sin^2\theta}{r_z^2-(d-M)^2}2i\alpha_1\eta_2$$

τ	Majorana fermions	d-operators	sphere operators
τ^z	$2i\alpha_1\eta_2$	$2d^\dagger d - 1$	$\sigma_1^x\sigma_2^x$
τ^y	$-\sqrt{2}\eta_2$	$-i(d^\dagger - d)$	$\sigma_2^x\sigma_1^y$
τ^x	$\sqrt{2}\alpha_1$	$d^\dagger + d$	σ_1^z



$$C_{1/2} = \frac{1}{2} \mp \frac{|\delta M_2 - \delta M_1|}{4r_{xy}}.$$

$$\mathcal{H}_\tau = (\delta M_2 - \delta M_1)\tau^x + 2r_{xy}\tau^z.$$

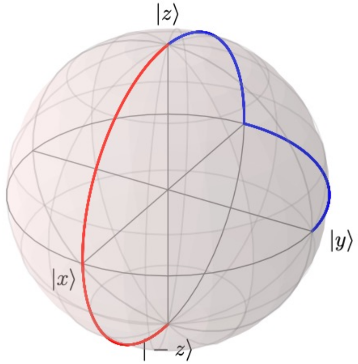
$$\mathcal{H}_\tau = (\delta M_2 - \delta M_1)\sqrt{2}\alpha_1 + 2r_{xy}(2i\alpha_1\eta_2).$$

Circularly polarized light

$$\tilde{V}_{\text{eff}} = B_0\mathbb{I}_2 + \mathbf{B}\cdot\boldsymbol{\tau},$$

Caldeira-Leggett model

$$B_x = \frac{(d-M)(|A_1|^2-|A_2|^2)}{2r_z^2-2(d-M)^2} + \frac{\omega_2-\omega_1}{2},$$
$$B_y = \frac{-r_z}{r_z^2-(d-M)^2}\text{Im}A_2^*A_1,$$
$$B_z = \frac{-r_z}{r_z^2-(d-M)^2}\text{Re}A_2^*A_1.$$



$$i\frac{d}{dt}(\alpha_1\eta_2) = -\Delta\sqrt{2}\eta_2.$$

Topological protection

Generalization to a Ring of Spheres: Relation to Kitaev Wire

$$\sigma_j^z = \frac{1}{i}(c_j^\dagger - c_j)e^{i\pi \sum_{k<j} c_j^\dagger c_j}$$
$$\sigma_j^x = (c_j + c_j^\dagger)e^{i\pi \sum_{k<j} c_j^\dagger c_j}.$$

$$\sum_{j=1}^{N}(d - M)\sigma_j^z = 0.$$

$$\mathcal{H}_{\text{eff}} = \sum_{j=1}^{N} r_z \sigma_j^z \sigma_{j+1}^z - \frac{r_z d^2 \sin^2\theta}{r_z^2 - (d-M)^2}\sigma_j^x \sigma_{j+1}^x$$
$$\mathcal{H}_{\text{eff}} = \sum_{j=1}^{N} -r_z 2i\eta_j \alpha_{j+1} - \frac{r_z d^2 \sin^2\theta}{r_z^2 - (d-M)^2} 2i\alpha_j \eta_{j+1}.$$

Progress in spin array with Majoranas
Xiao Mi et al. Science 378, 785 (2022)

$\sigma = \tau$

$$e^{i\pi \sum_{1\le j<N} c_j^\dagger c_j} = \prod_{1\le j<N} e^{i\pi c_j^\dagger c_j}$$
$$= \left(\prod_{1\le j\le N} e^{i\pi c_j^\dagger c_j}\right) \times e^{i\pi c_N^\dagger c_N}$$
$$= (1 - 2c_N^\dagger c_N).$$

$$\tau^x = \sqrt{2}\alpha_1 = \sigma_1^z$$
$$\tau^y = -\sqrt{2}\eta_N = -\sigma_N^x \sigma_N^y$$
$$\tau^z = 2i\alpha_1\eta_N = \sigma_1^z \sigma_N^z.$$

$$r_z'(t)\sigma_{N+1}^z \sigma_N^z$$

½ number of spheres: GHZ states at south pole and topological number C_j=1/2 stable when adjusting parameters
Joel Hutchinson and Karyn Le Hur, Communications Physics 4, 144 (2021)

Generalizations of Fractions for odd number of spheres in a ring

Generalized Resonating Valence Bonds and Ferromagnet

Classification of states with 1 domain wall in a classical antiferromagnet (stable "gap")

$\sigma = \pi$

$$|GS\rangle = \frac{1}{\sqrt{5}}(\downarrow\uparrow\downarrow\uparrow\downarrow + \uparrow\downarrow\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow\downarrow\uparrow + \uparrow\downarrow\downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow\downarrow\uparrow)$$

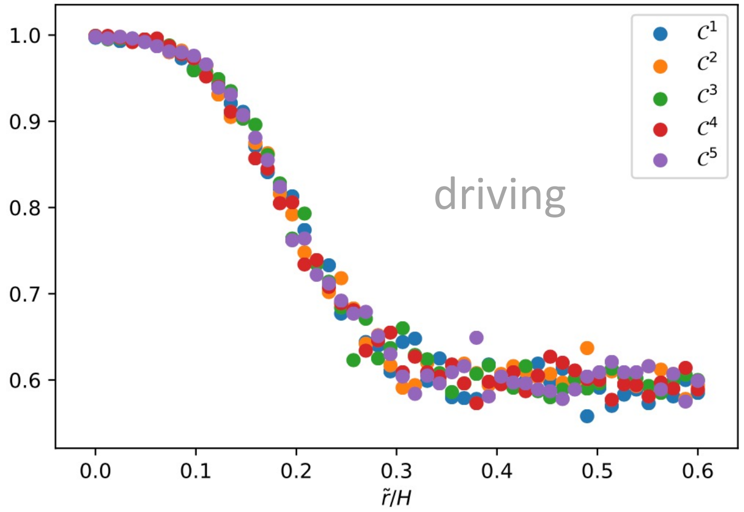
N odd

$$\langle GS|\sigma_{iz}|GS\rangle = -\frac{1}{N} \text{ for } N \text{ odd}$$

. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021)

$$C_j = \frac{1}{2}(\langle\sigma_{jz}(0)\rangle - \langle\sigma_{jz}(\pi)\rangle) = \frac{1}{2}\left(1 + \frac{1}{N}\right).$$

K. Le Hur, Review arXiv:2209.15381



$$C_j = \mathcal{A}_{j\varphi}(\pi) - \mathcal{A}_{j\varphi}(0) = \frac{N+1}{2N}q,$$

$N \to +\infty \quad C_j = \frac{1}{2}$

even = odd

Link with the wire model on a ring
Majorana fermions

Superconductor and Bardeen-Cooper-Schrieffer WaveFunction

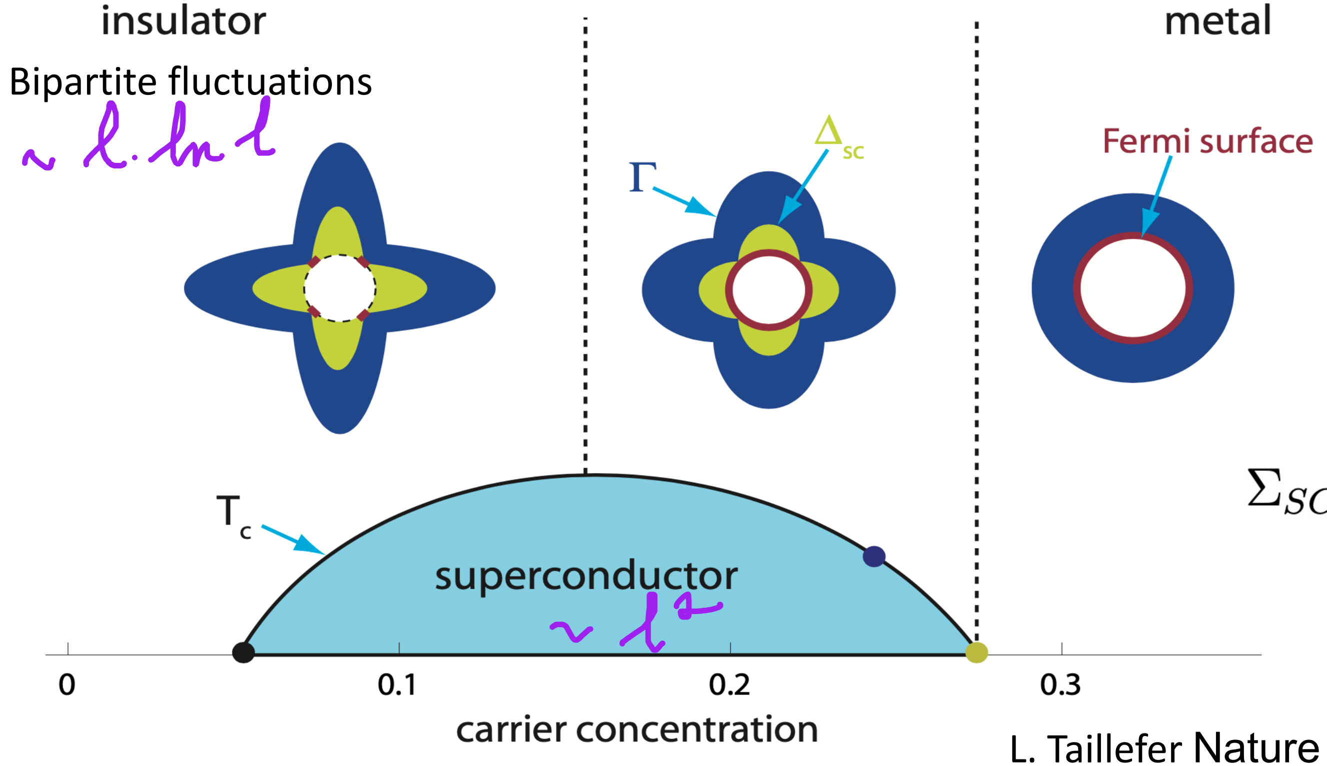
$$|BCS\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} d^{\dagger}_{\vec{k}\uparrow} d^{\dagger}_{-\vec{k}\downarrow})|Vac\rangle$$

K. Le Hur and T. Maurice Rice, Annals of Physics 324 (2009) 1452
Applications in ARPES and cold atoms (time of flight, noise…)

$$S = -z_{\vec{k}} \ln z_{\vec{k}} - (1 - z_{\vec{k}}) \ln(1 - z_{\vec{k}})$$

$$z_{\vec{k}} = |u_{\vec{k}}|^2 = 1 - |v_{\vec{k}}|^2$$

$$\Gamma = \Gamma_0 + aT^2 + bT\cos^2(2\phi),$$



L. Taillefer Nature Physics 2, 810 (2006)

Pseudo-gap: Yang, Rice, Zhang (2006)

$$G^{s}_{coh}(\vec{k},\omega) = \frac{g_t}{\omega - \xi(\vec{k}) - \Sigma_{SC}(\vec{k},\omega)}$$

$$\Sigma_{RVB}(\vec{k},\omega) = |\Delta_{\vec{k}}|^2/(\omega + \xi_0(\vec{k}))$$

$$\Sigma_{SC}(\vec{k},\omega) = \Sigma_{RVB}(\vec{k},\omega) + \frac{|\Delta_{SC}(\vec{k})|^2}{\omega + \xi(\vec{k}) + \Sigma_{RVB}(\vec{k},-\omega)},$$

N leg Hubbard ladder: See Section 4.5 review above
(see also U. Ledermann, K. Le Hur, T. M. Rice 2000)

Summary

Progress on many-body entanglement and probes: our own try “bi-partite fluctuations”
Measurable in quantum wires, e.g. capacitance and charge fluctuations
Ln of Luttinger liquids and 1D fluids (spin chains)
Superconductors linked to quantum Fisher information and negative Ln correction for Kitaev 1D wire

Bell state or EPR pair and Model of Two-Spins on Bloch Spheres: pair of ½ Topological numbers
New way to characterize topological states from the poles and quantum mechanics
New Platform for quantum information and Majorana fermions

Application to Correlated Matter

- BCS wavefunction and entropy of a Cooper pair in a superconductor
- Phase transition of a topological Kitaev superconducting wire and one-half Skyrmion
- Two interacting wires and DCI Double-Critical-Ising phase
- 2D Topological semimetal in bilayer or one-layer graphene

Thanks to Students, post-docs and Collaborators. Thanks for your kind attention.