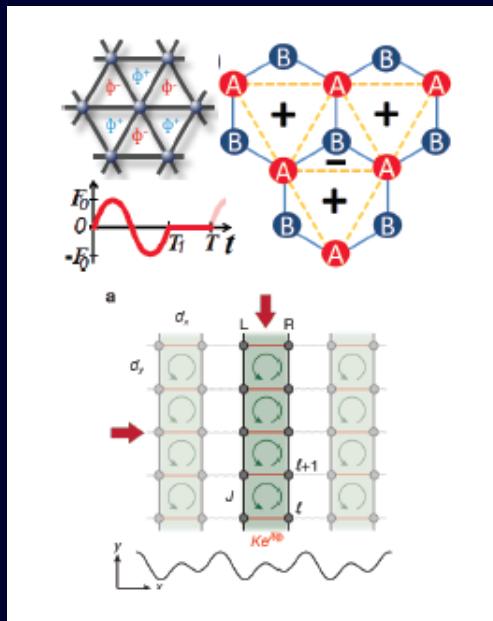
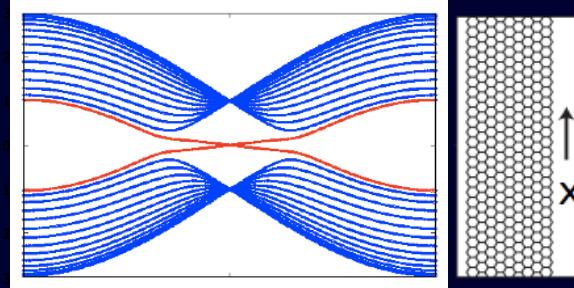


July 2017: conference on Spin-Orbit Materials, Luxembourg
organizers: Thomas Schmidt, Bjorn Trauzettel, Patrik Recher

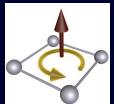
Spin-Orbit Coupling in the Mott State : Quantum Spin Liquids, Superconductivity & Majorana Boxes



Karyn Le Hur
CPHT Ecole Polytechnique, France & CNRS



Funding DFG & Labex Palm



PALM
Laboratoire d'Excellence
Physique : Atomes Lumière Molécule

Topological Phases, Interaction Effects & Gauge Fields

From Materials, to Ultra-Cold Atoms and Photon Simulators

Start from Graphene: Majorana fermions in SC Graphene

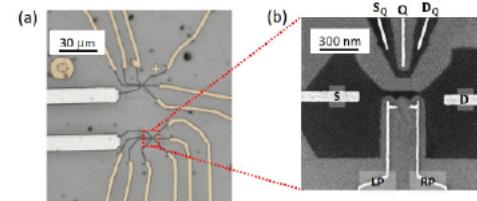
D. Bergman and KLH, PRB 2009

following P. Ghaemi & F. Wilczek

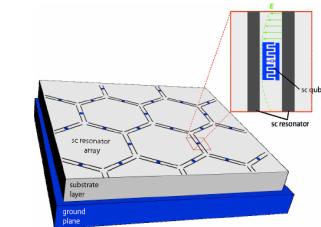
[di Bernardo et al 2017 (p-wave like?); progress in realization]

- Spin-orbit coupling in graphene lattices
Kane-Mele Model, Interaction Effects (fermions, bosons)
Topological Mott Insulators, quantum spin liquids in 2D?
ladder system: QHE of bosons $U=\infty$

- Z2 quantum Majorana liquids
Kitaev chain, Ladders
Majorana Boxes, cQED
SSH model of hole pair

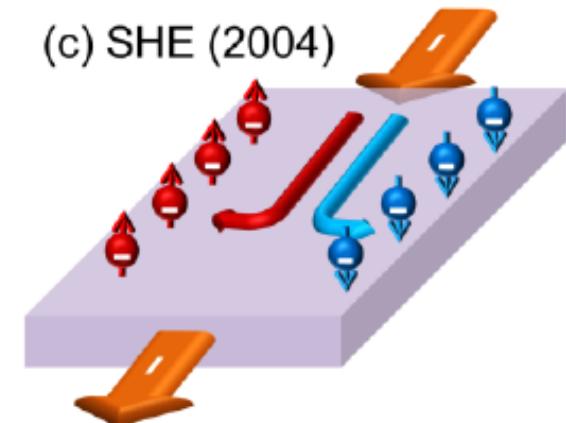
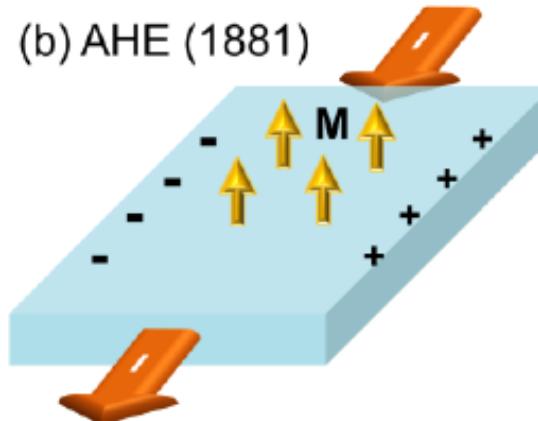
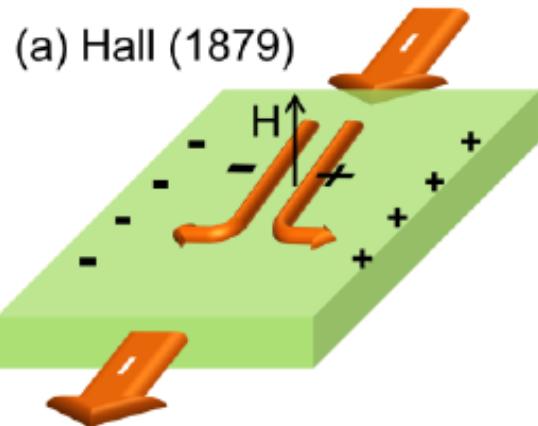


Mesoscopic graphene
cQED, group of G.-P. Guo



J. Koch & KLH, 2009
Artificial graphene

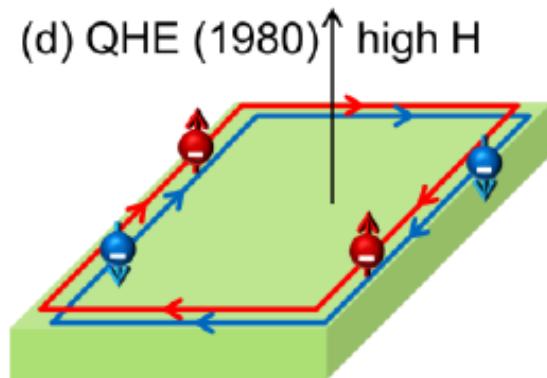
Topological states of matter simple zoology from flat land



Von Klitzing, Dorda, Pepper;
fractional charges (Grenoble, CEA Saclay, Weizmann)

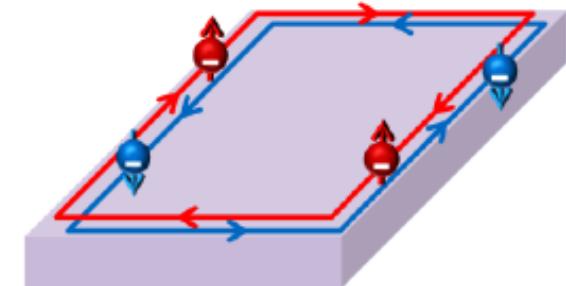
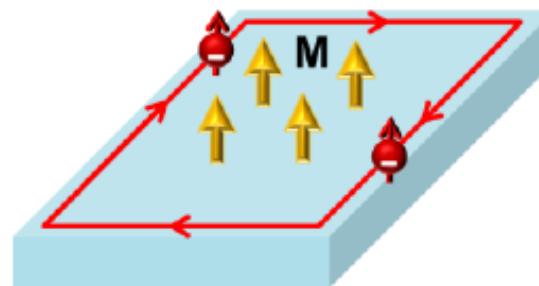
Haldane (1988)

REALIZED AT WURZBURG IN HGTE (Molenkamp)
3D MERCURY ANALOGUES, PRINCETON (Hasan)



(e) QAHE (2013)

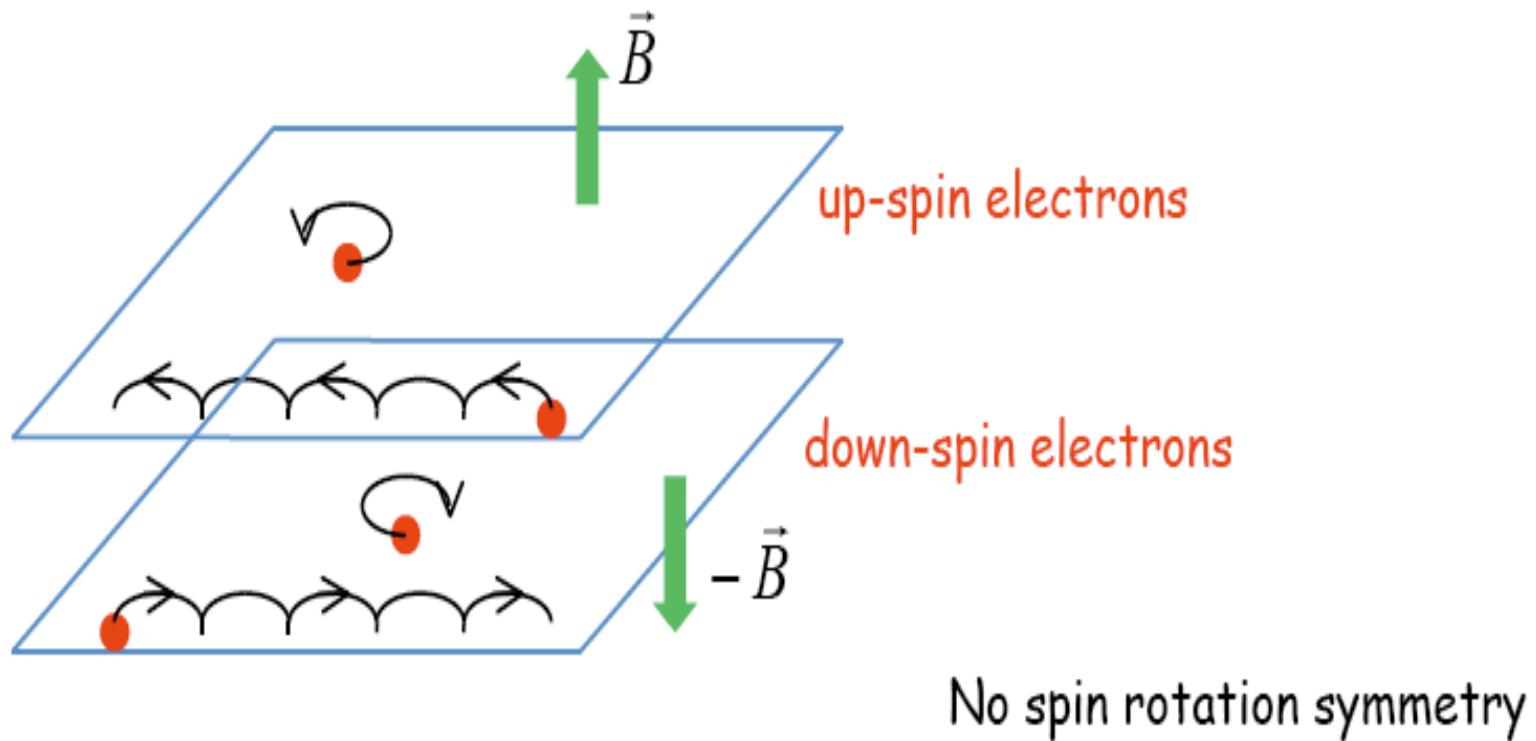
(f) QSHE (2007)



C. Z. Chang and M. Li, Topical Review, arXiv:1510.01754
From material science (spintronics), to cold atoms and photons

Stable towards interactions: exemples S. Rachel & KLH Kane-Mele-Hubbard model 2010 QSH; D. Pesin & L. Balents, 3D (2010)
C. Varney, K. Sun, M. Rigol, V. Galitski (Maryland) 2010 QAH

- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda \vec{L} \cdot \vec{\sigma}$
- Gapless helical edge mode (Kramers pair)



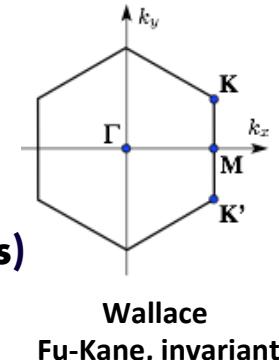
Microscopic Description: Simple Standard Model, Kane-Mele

Time reversal invariant of Haldane model (1988): Kane-Mele model

Kane & Mele, PRL 95, 226801 (2005)

*see also: Bernevig, Hughes, and Zhang, Science 314, 1757 (2006)
+ Molenkamp-experiments (Mercure)*

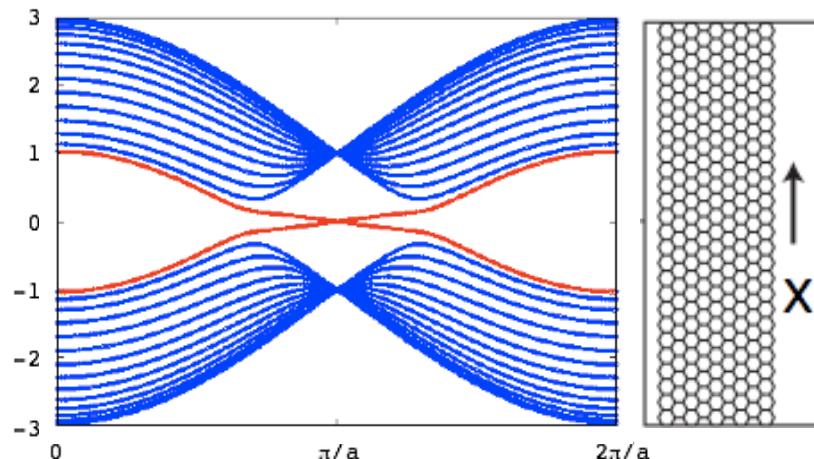
in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)



$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma_{\sigma\sigma'}^z c_{i\sigma}^\dagger c_{j\sigma'}$$

$\nu_{ij} = \pm 1$

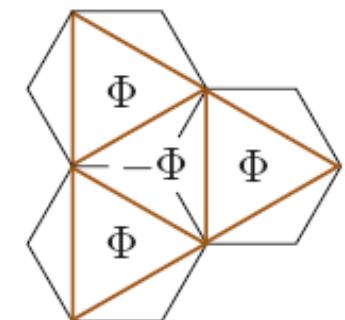
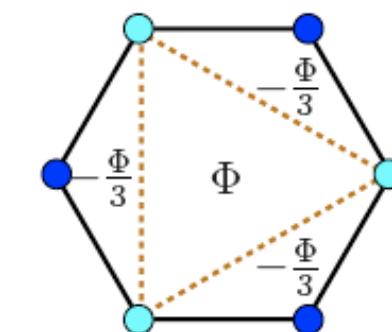
strip geometry:



edge states: Kramers's pair

Transition semi-metal to Mott graphene

$$\mathcal{H} \propto \Psi_k^\dagger \sigma^z \tau^z \Psi_k$$



Transition from topological band insulator to Mott Insulator? Relevant to iridates...

“Kane-Mele-Hubbard”: XY Neel

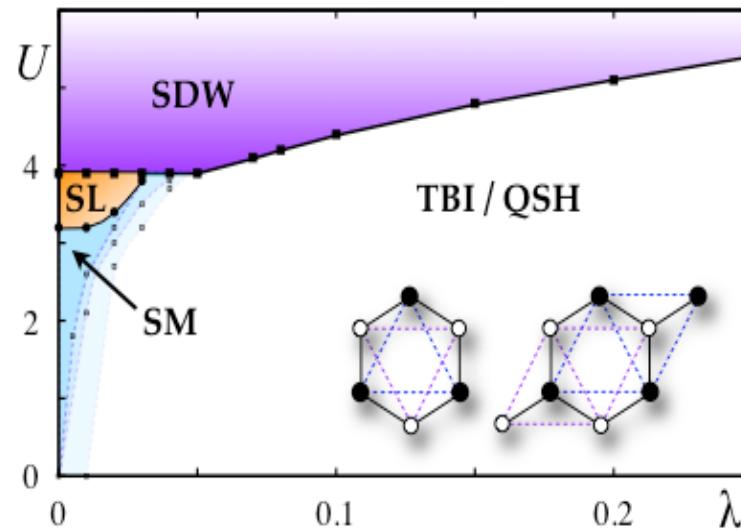
Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

Real-space version
QMC continuous-time
Impurity solver

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)

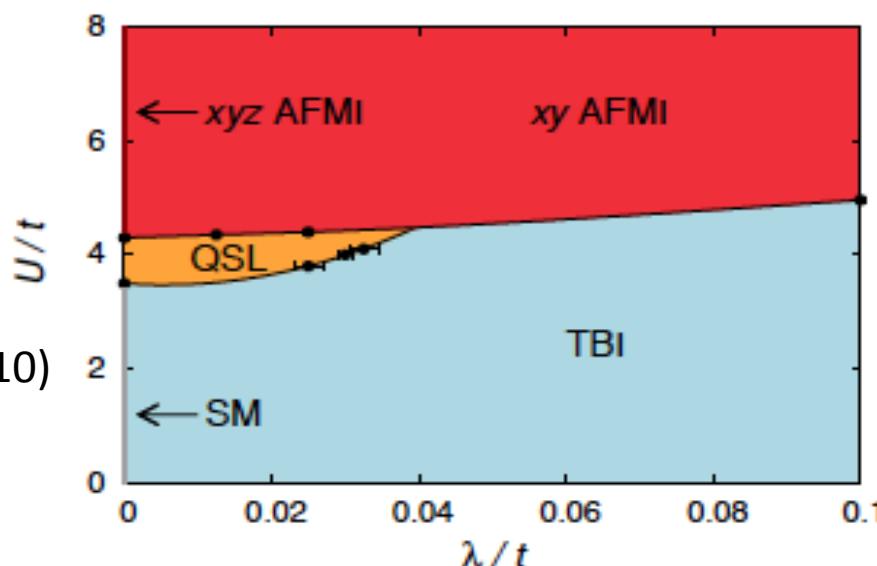


S. Rachel & KLH, PRB 2010

**3D XY (charge model,
Josephson model)**

S. Rachel & KLH, 2010
Griset & C. Xu, 2011

D.-H. Lee, 2011



M. Hohenadler et al.
arXiv:1111.3949

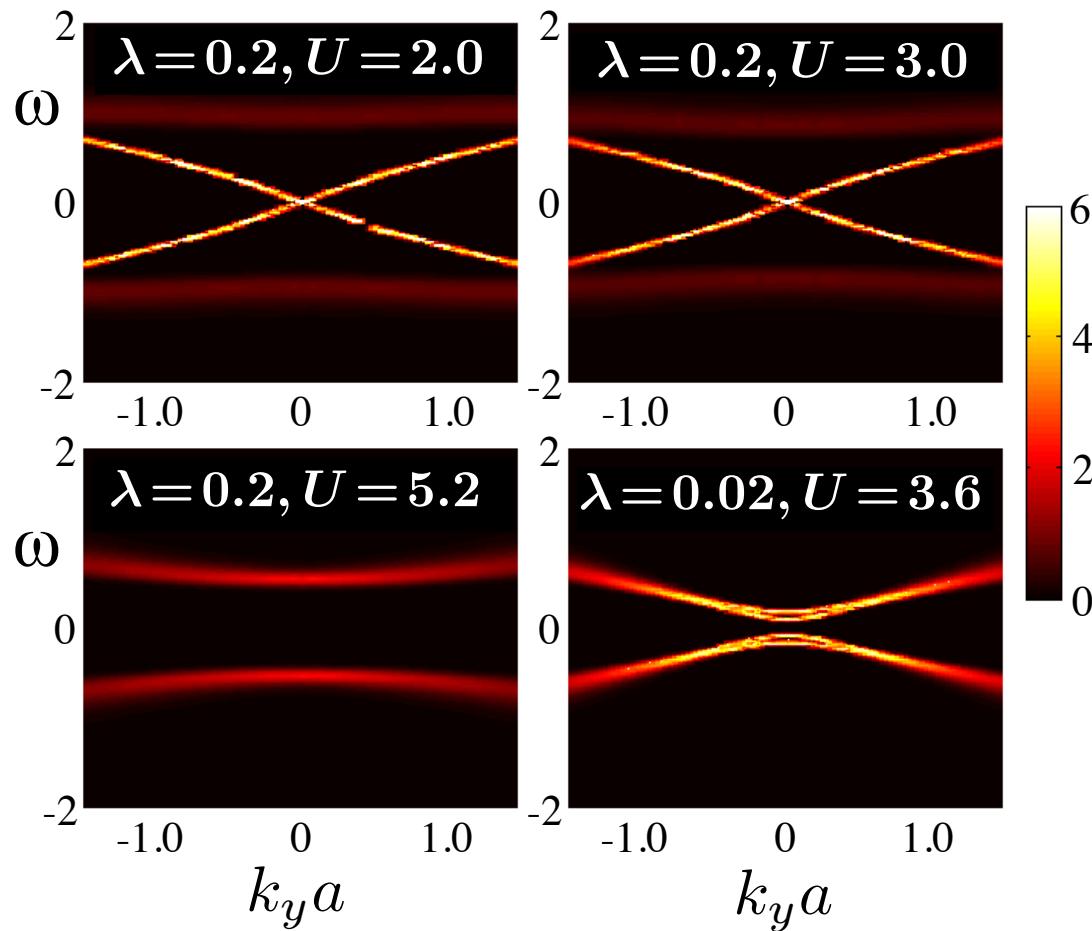
Phys. Rev. Lett. **106**,
100403 (2011)

Absence of spin liquid:

S. Sorella et al. Scientific Reports 2012; S. R. Hassan & D. Senechal PRL 2013 (large N limit, I. Herbut)

No single-particle gap closing at TBI-Mott transition

SDW, breaks time-reversal symmetry: edge states vanish
in agreement with Luttinger theory + Sine Gordon argument



CDMFT

Real-space version
QMC continuous-time
Impurity solver

Some Reviews (not full list):

G. Kotliar et al, RMP 2006
T. Maier et al, RMP 2005
A.-M. Tremblay, B.-S. Kyung,
D. Senechal, 2006

DMFT:

A. Georges, G. Kotliar,
W. Krauth & M. Rozenberg et al.;
Metzner & Vollhardt

Wei Wu, Stephan Rachel, Wu-Ming Liu and KLH, PRB **85**, 205102 (2012)

See also Yamaji & Imada, 2011; Yu, Xie & Li 2011; Zheng, Zhang & C. Wu, 2011

Connection to reality?

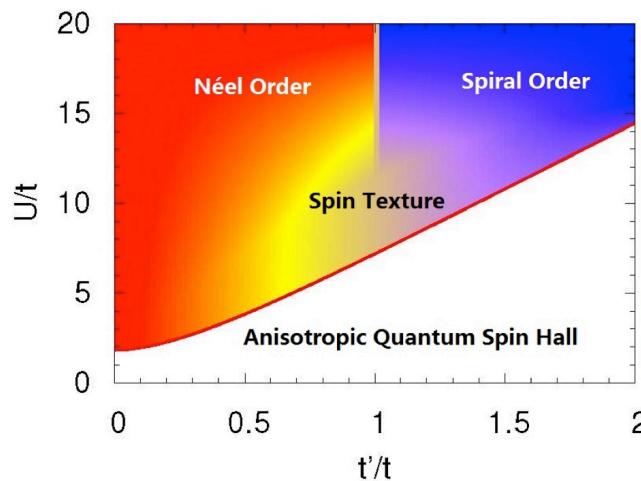
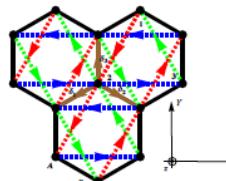
- Na_2IrO_3 : anisotropic spin-orbit coupling (thin films: arXiv:1303:5245, M. Jenderka et al)

Shitade et al. PRL 102 256402 (2009); G. Jackeli & G. Khaliullin, PRL 102, 017205 (2009)

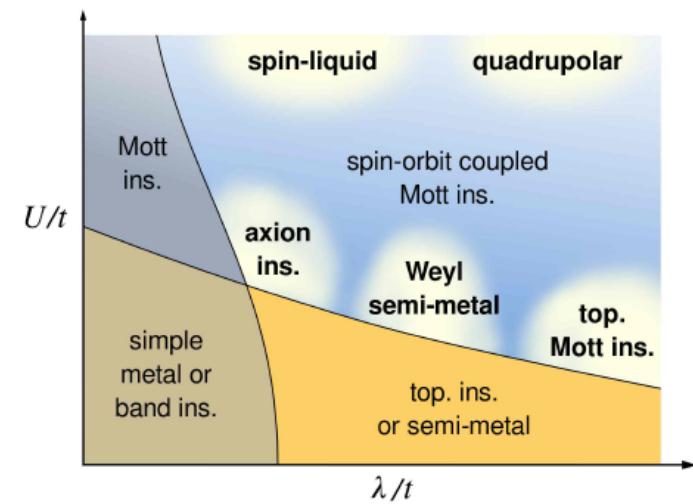
H.-C. Jiang, Z.-C. Gu, X.-L. Qi and S. Trebst, Phys. Rev. B 83, 245104 (2011);

S. Bhattacharjee, Sung-Sik Lee and Yong-Baek Kim, New J. Phys. 14, 073015 (2012)

Y. Singh et al. 2012; Z. Nussinov & J. van den Brink, arXiv:1303.5922 ...



**α Lithium Iridates
and Spiral order**
R. Coldea



Tianhan Liu, Benoit Doucot, Karyn Le Hur, PRB 2013

A. Ruegg and G. Fiete, PRL 2012

J. Reuther, R. Thomale & S. Rachel, PRB 2012

M. Kargarian, A. Langari, G. Fiete PRB 2012

D. Pesin & L. Balents, Nature Phys. 2010
Krempa, Choy, Y.-B. Kim & L. Balents

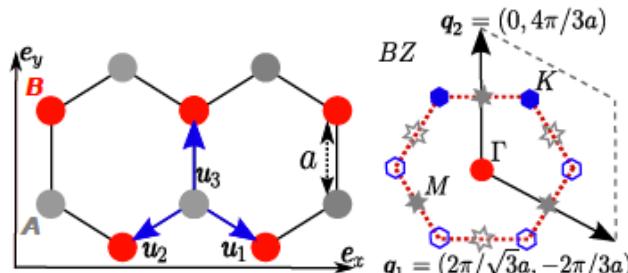
New attempt in 2D: Bosonic KMH model

K. Plekhanov, I. Vasic, A. Petrescu, R. Nirwan, G. Roux, W. Hofstetter & KLH,
arXiv:1707.07037

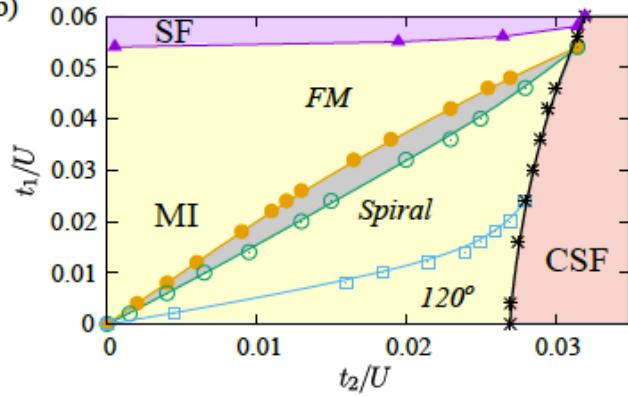
$$S_{\mathbf{r}_i}^z = (n_{\uparrow, \mathbf{r}_i} - n_{\downarrow, \mathbf{r}_i})/2$$

$$S_{\mathbf{r}_i}^x + iS_{\mathbf{r}_i}^y = b_{\uparrow, \mathbf{r}_i}^\dagger b_{\downarrow, \mathbf{r}_i}, S_{\mathbf{r}_i}^- = S_{\mathbf{r}_i}^x - iS_{\mathbf{r}_i}^y = b_{\downarrow, \mathbf{r}_i}^\dagger b_{\uparrow, \mathbf{r}_i}$$

(a)



(b)



BDMFT

CSS; no order in XY
plane (see next slide)

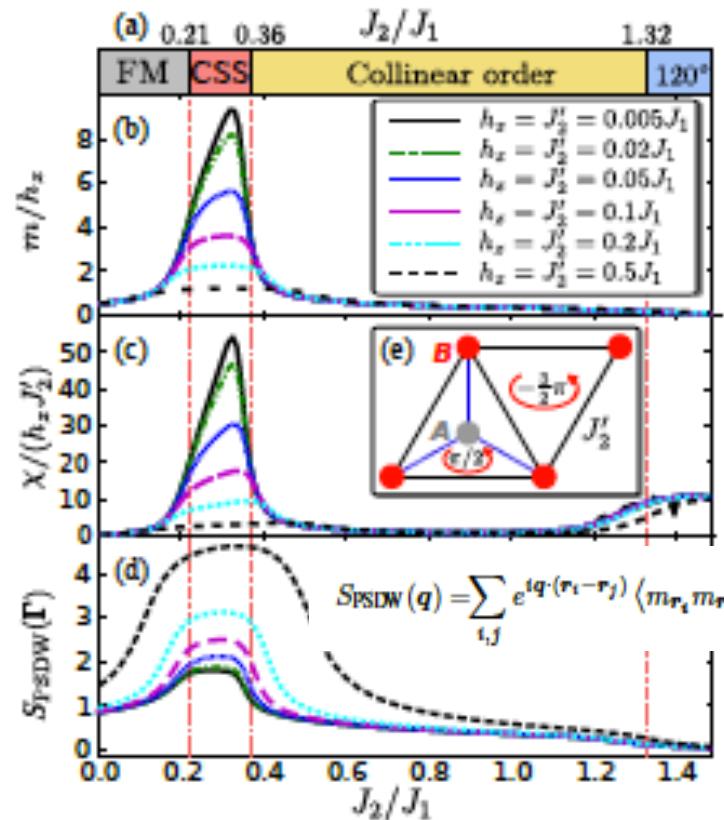
$$H = - \sum_{\langle ij \rangle} \left[J_1 (S_{\mathbf{r}_i}^+ S_{\mathbf{r}_j}^- + \text{h.c.}) - K_1 S_{\mathbf{r}_i}^z S_{\mathbf{r}_j}^z \right] \\ + \sum_{\langle\langle ik \rangle\rangle} [J_2 (S_{\mathbf{r}_i}^+ S_{\mathbf{r}_k}^- + \text{h.c.}) + K_2 S_{\mathbf{r}_i}^z S_{\mathbf{r}_k}^z],$$

MOTT space

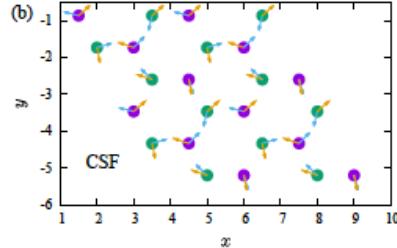
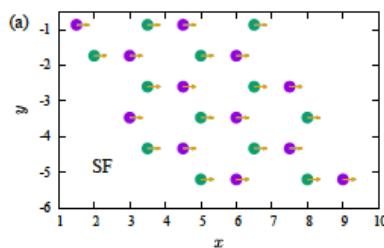
chiral spin state (CSS); Chern-Simons, topological?
Sedrakyan, Glazman, Kamenev

$$\chi = \langle S_{\mathbf{r}_i} \cdot (S_{\mathbf{r}_i+\mathbf{u}_1} \times S_{\mathbf{r}_i+\mathbf{u}_2}) \rangle$$

ED

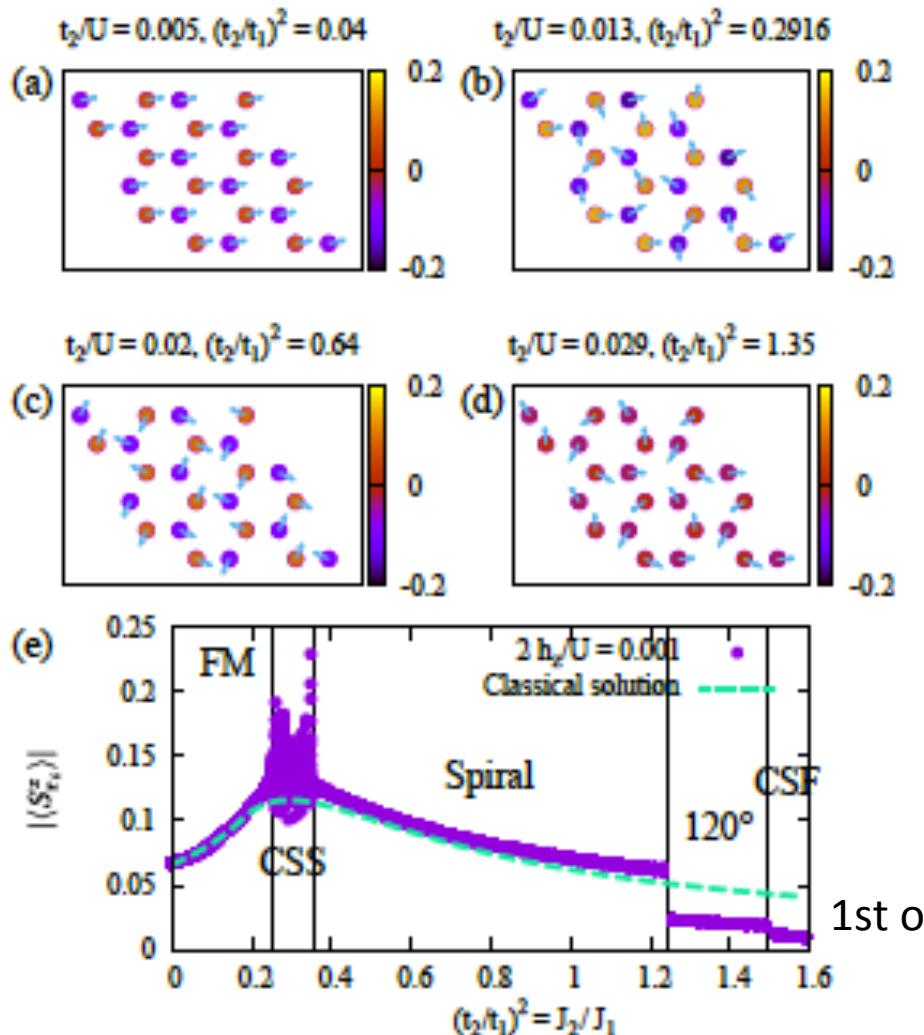


Discuss nature of
Phase transitions



Magnetism & Berry analysis

Real space BDMFT & classical analysis



$$C = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} B(\theta_1, \theta_2) d\theta_1 d\theta_2 .$$

can be related to Z2 pump
for bosons

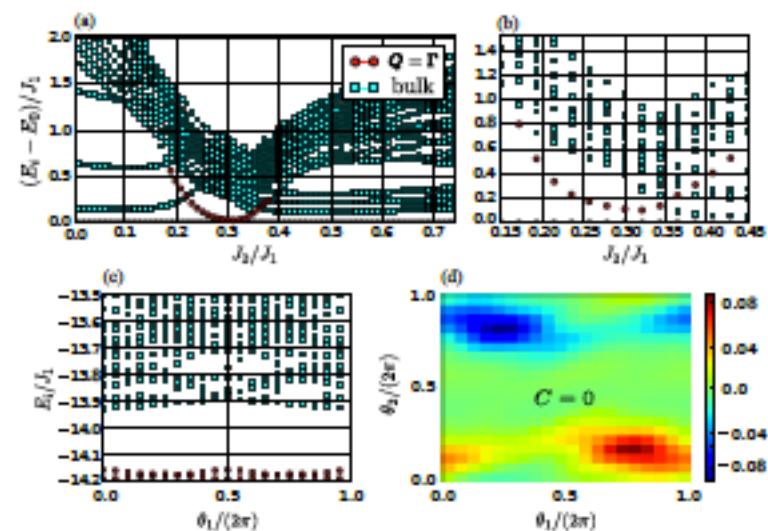


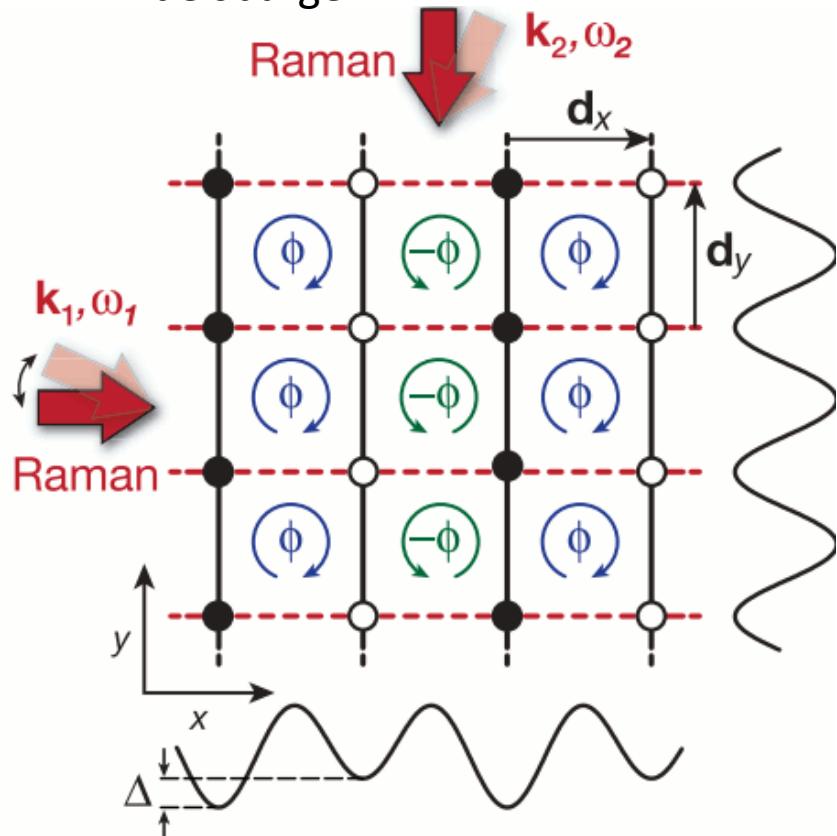
FIG. 4. ED calculations of the low energy spectra as a function of J_2/J_1 (a) on a lattice of 4×3 unit cells for various S_{tot}^z ; (b) on a lattice of 4×4 unit cells in the $S_{\text{tot}}^z = 0$ sector only. (c) Low energy spectrum as a function of the twist angle θ_1 for $J_2/J_1 = 0.3$ and $\theta_2 = 0$ on a lattice of 4×3 unit cells. (d) Typical shape of the Berry curvature calculated using the non-abelian formalism resulting in a vanishing Chern number shown for $J_2/J_1 = 0.3, h_x/J_1 = J_2'/J_1 = 0.02$ on a lattice of 4×3 unit cells.

Cold Atoms & Gauge Fields

Goal: strongly correlated atoms in strong artificial magnetic fields.

Maximal flux per plaquette of order π

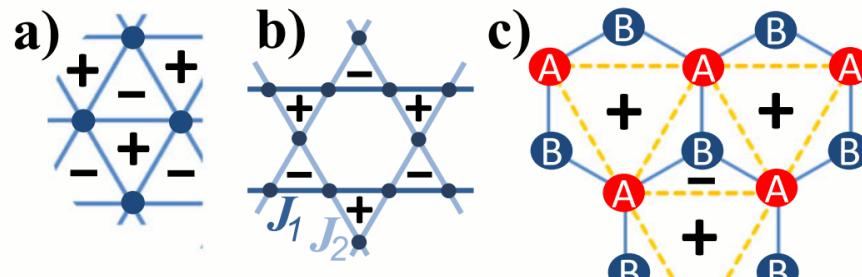
M. Aidelsburger



Laser-assisted tunneling in optical superlattice. PRL 107, 255301 (2011)
(Immanuel Bloch's lab at Muenich)

Non-Abelian Models with interactions

P. P. Orth, D. Cocks et al J. Phys. B: At. Mol. Opt. Phys. 46 (2013) 134004 (review);
M. Scheurer, S. Rachel, P. Orth Sci. Reports 2015



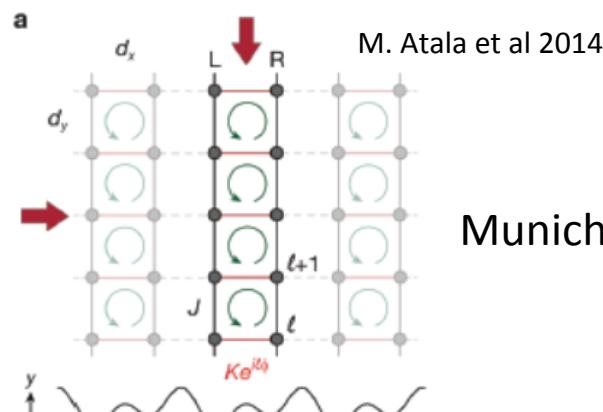
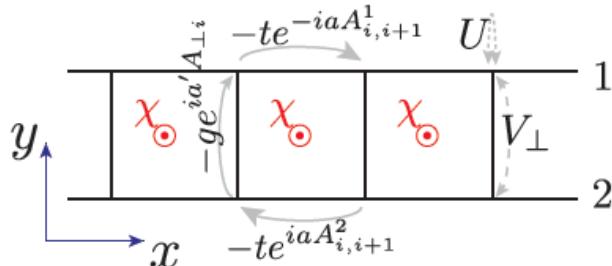
$$\hat{H}(t) = - \sum_{\langle ij \rangle} J_{ij} \hat{a}_i^\dagger \hat{a}_j + \sum_i v_i(t) \hat{n}_i + \hat{H}_{\text{on-site}}$$

K. Sengstock's lab at Hamburg
Realization of Haldane model at Zurich,
Jotzu et al. (Esslinger lab)

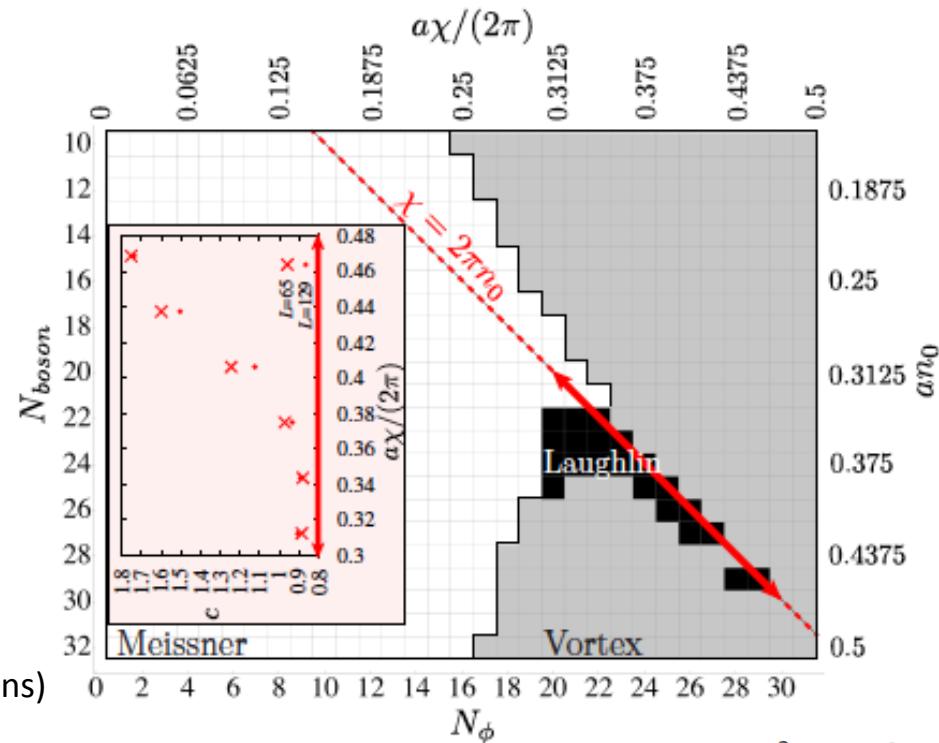
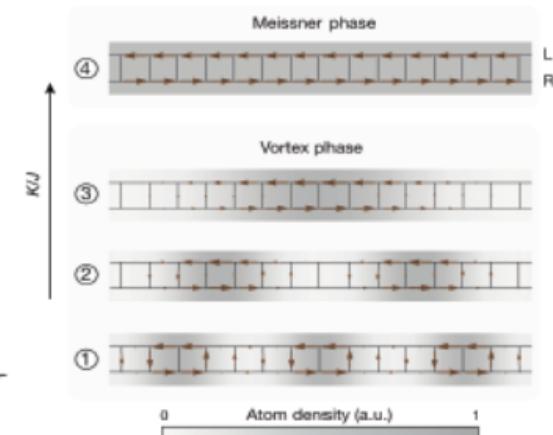
A lot of efforts at NIST, Paris, MIT,...
I. Spielman, W. Philipps (NIST)
J. Dalibard, F. Gerbier (Paris)...

Ladder simulation: bosons

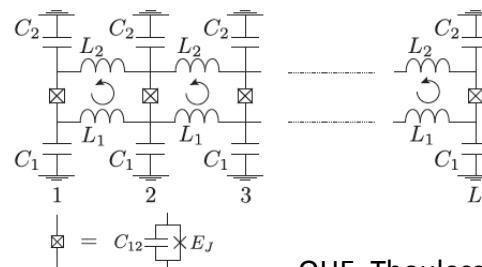
A. Petrescu, M. Piraud, G. Roux, I. McCulloch, KLH 2016 to appear PRB
 M. Calvanese Strinati et al. PRX 2017; classification Teo-Kane



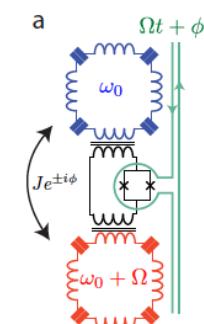
$U=\infty$, $V=0$ (spins)



Neel group: Josephson circuits



J. Gabelli
 J. Esteve
 Orsay



QHE, Thouless pump gives $C=1/2$ (Grusdt-Honing)
 charges at the edges 2/5 (talk by M. Sassetti;
 bipartite fluctuations Talk by Loic Herviou)

See F. Wilczek, Majorana returns, Nat. Physics 2009

They appear accidentally in spin chains: via Jordan-Wigner transformation (1928)

Generalization of Dirac algebra for harmonic oscillators 1925 (group theory)

high energy physics (neutrino...)

Particle and its own antiparticle

Y
Proposals:

Alexei Kitaev

Nick Read

Leonid Levitov

Hans Mooij

Liang Fu

Charles Kane

Carlo Beenakker

Matthew Fisher

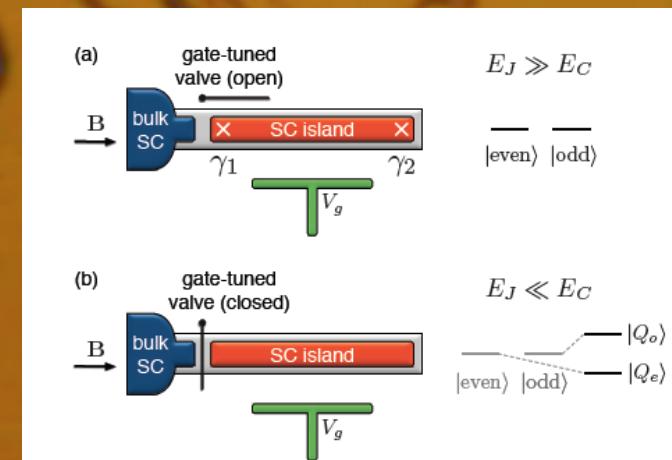
Bert Halperin

Daniel Loss

Pascal Simon

Jelena Klinovaja...

Progress in nano-engineering
to reveal the Majoranas
(talks by F. Nicelle, P. Grunberg)



T. Kontos, A. Cottet (ENS)

D. Aaasen et al. arXiv 2015
Charles Marcus group 2016
Also Ali Yazdani, Princeton

Note: recent work on 2 coupled topological SC chains & Kondo box (B. Beri)

Loic Herviou, Christophe Mora, KLH, 2016

The Majorana fermion states must be occupied in pairs, since the entire physical system can only occupy real fermion states.
So only combinations of Majorana fermions can be occupied

This occupied state is inherently delocalized – it has weight in two spatially separated vortex cores.

$$\hat{c}^\dagger |\Psi_0\rangle = (\hat{\gamma}_1 + i\hat{\gamma}_2) |\Psi_0\rangle$$

Exchange of 1 and 2 $\gamma_1 \rightarrow \gamma_2$
 $\gamma_2 \rightarrow -\gamma_1$

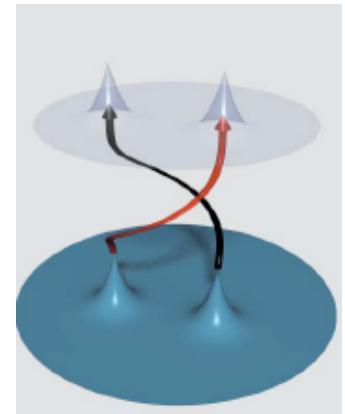
$$(\hat{\gamma}_2 + i\hat{\gamma}_1) |\Psi_0\rangle = i(\hat{\gamma}_1 - i\hat{\gamma}_2) |\Psi_0\rangle = i\hat{c}|\Psi_0\rangle$$

Different final state! – Non-Abelian statistics.

Application qubits : quantum computing

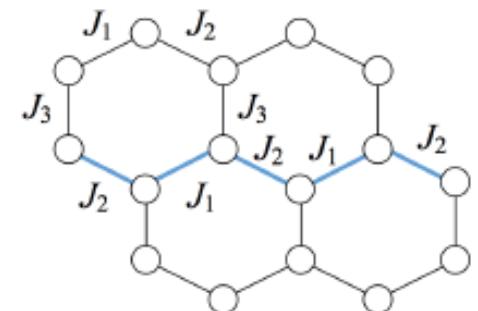
Sankar Das Sarma, Michael Freedman, Chetan Nayak [arXiv:1501.02813](https://arxiv.org/abs/1501.02813)

Talks Frank Pollmann & Simon Trebst



N. Read & D. Green
N. Read & G. Moore
D. Ivanov, Volovik

Kitaev model 2006
Magnetic analogues, solvable
Spin liquids and BCS superconductors

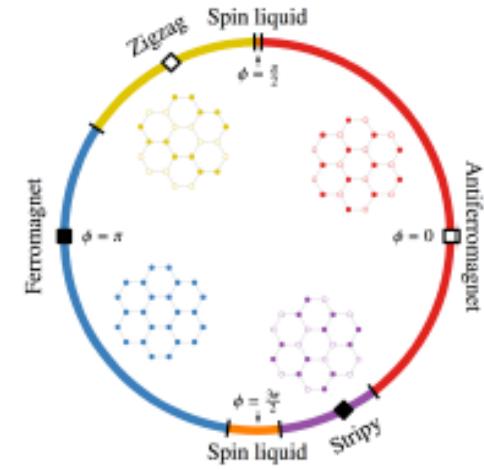
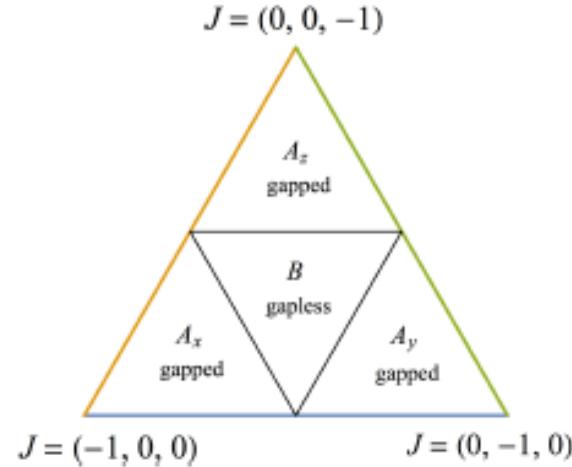
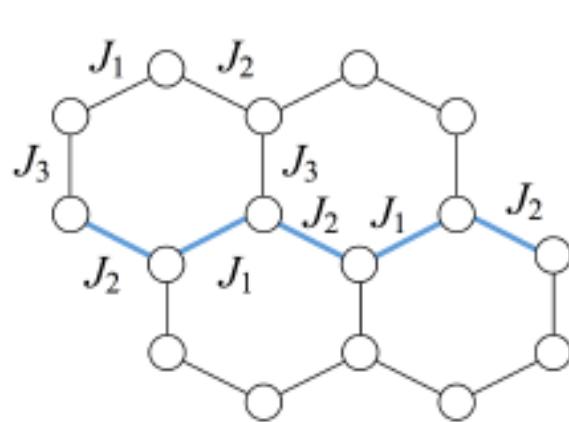


Recent efforts M. Hermanns, S. Trebst
J. Vidal, S. Dusuel,...
T. Liu, B. Douçot, C. Repellin, N. Regnault, KLH

Anisotropic Spin-Orbit Z2 Models

Simulation in cold atoms:
Duan, Demler, Lukin

J. Rau & H. Y. Kee, review
Iridate materials,
Mixing angle (Heisenberg & Kitaev)



Kitaev model on honeycomb lattice: exactly solvable Majorana representation (2006)
3 gapped Anderson Resonating Valence Bond States with short range correlations
B gapless phase

Possible application to quantum materials, iridates and RuCl₃ (J. Banerjee et al. Nature)

Question: Engineering minimal Z2 spin liquid models,
connection to p-wave BCS theory, topology from Mott, doped Mott insulators
Ladders and Boxes in cQED & Josephson junctions; work with Fan Yang & Ariane Soret, see ArXiv 2017

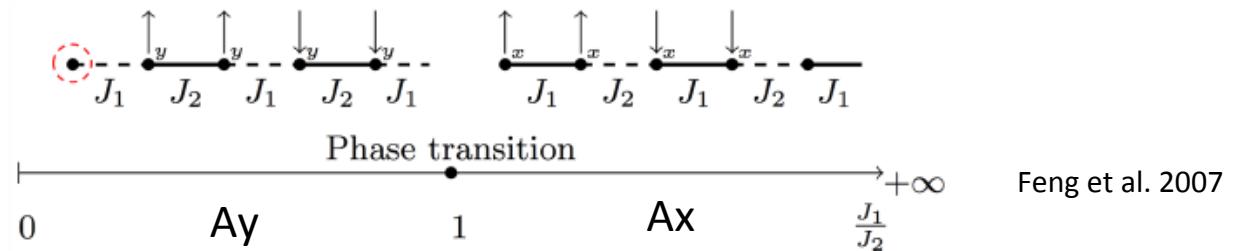
Related works by F. Hassler (talk), B. Terhal; B. Beri; Egger-Altland-Flensberg; L. Fu; ...

RVB picture

work with Fan Yang & Ariane Soret, see ArXiv 2017

$$H = \sum_k (J_1 + J_2) \cos(kl) (a_k^\dagger a_k - a_k a_k^\dagger) + i(J_1 - J_2) \sin(kl) (a_{-k}^\dagger a_k^\dagger + a_{-k} a_k)$$

Phase transition:
Power law decay
(can be probed from
Bipartite fluctuations
Talk by Loic Herviou)



- Pairing term: odd parity $\rightarrow p$ -wave superconductors (topological)
e.g. He^3 , Sr_2RuO_4 , 1D quantum wire etc. Anderson-Morel; Anderson-Brinkmann;
Balian-Werthamer; Leggett (He3)
- $\epsilon(k) = \pm \sqrt{J_1^2 + J_2^2 + 2J_1 J_2 \cos(2kl)}$: invariant under $J_i \rightarrow -J_i$, $i = 1, 2$.

$J_1 \rightarrow 0$; edge modes are spin-1/2

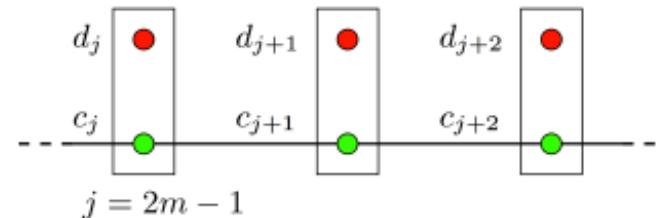
Winding number can be computed (1 or 0)

Perturbation theory in spin space couples edges to order $(J_1/J_2)^{\text{number sites}}$ only
Non-local topological string order parameter similar to spin-1 chain (Feng et al. 2007)

Majorana representation

Spin liquid states in gapped phases

- Pre-formed pairs are similar to spin-1 BCS pairs in He³. RVB states with exponentially decreasing correlation functions.



- Mapping to Majoranas:

$$\begin{cases} c_j = i(a_j^\dagger - a_j), & d_j = a_j^\dagger + a_j, & j = 2m - 1; \\ c_j = a_j^\dagger + a_j, & d_j = i(a_j^\dagger - a_j), & j = 2m. \end{cases}$$

$$H = -i \sum_{j=2m-1} (J_1 c_j c_{j+1} - J_2 c_{j+1} c_{j+2}).$$

The free d-Majorana fermions in the bulk traduce the 2-fold degeneracy of a bond
The c1 Majorana fermion in the Ay phase weakly couples to the bulk
The spin-1/2 edge mode thus turns into a d1 Majorana fermion in the *infinite* time limit
for larger J1 (The d Majorana fermions are not protected against local fields in 1 chain)
See also DeGottardi et al. 2011 and Pedrocchi et al. 2012

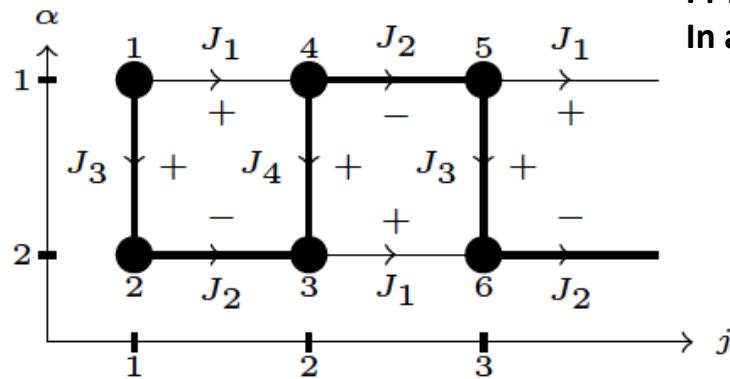
Ladder:

Feng et al. 2007

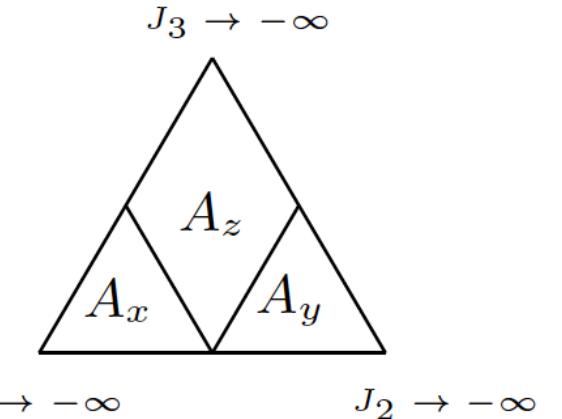
DeGottardi et al. 2011

Pedrocchi et al. 2012 (inhomogeneous ladder)

H. H. Lai and O. Motrunich, PRB 2011



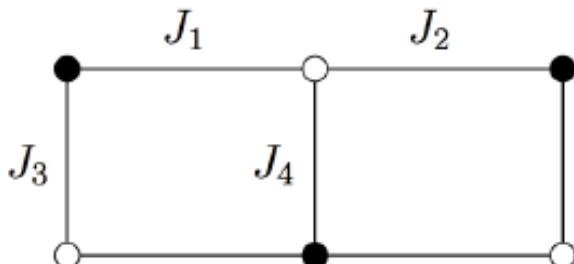
Lieb' theorem
Pi-flux here
In agreement with Kitaev view



$$H = -i \sum_{j=2m-1} [J_1 c_{j,1} c_{j+1,1} - J_2 c_{j+1,1} c_{j+2,1} \\ + J_1 c_{j+1,2} c_{j+2,2} - J_2 c_{j,2} c_{j+1,2} \\ + J_3 D_{j,1} c_{j,1} c_{j,2} + J_4 D_{j+1,1} c_{j+1,1} c_{j+1,2}],$$

$$\tilde{\sigma}_i^\alpha = i b_i^\alpha c_i.$$

$$D_{j,\alpha} = (-i) d_{j,\alpha} d_{j,\alpha+1}$$



KLH, A. Soret and F. Yang, 2017

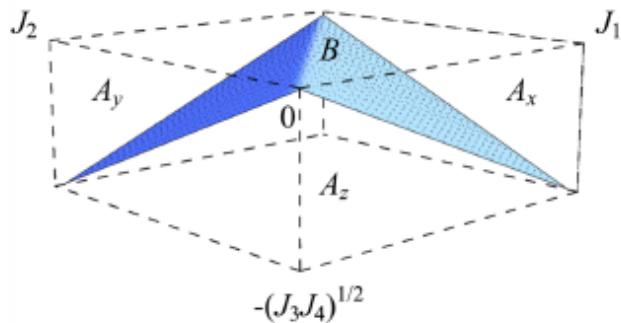
honeycomb or ribbon ladder: J4=0

**0 flux
for J4=0**

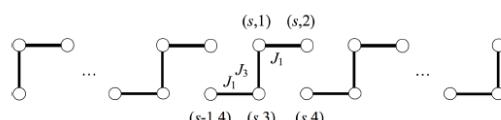
Square ladder, Feng et al 2007
 Az phase all Majoranas paired
 Ax and Ay phase, edge modes

Generality of the phase diagram?
 Honeycomb ladder, etc... gapless phase
 Z2 protection of ground state
 Loop operators

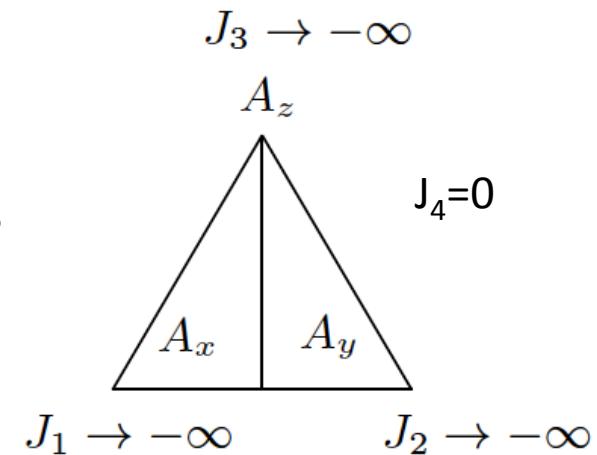
Generalized ladder: arXiv:1703.07322



Cluster solvable model



Adiabatic evolution from X to Z polarization



No flux frustration in the ground state (pi flux; 0 flux J4=0)

honeycomb ribbon

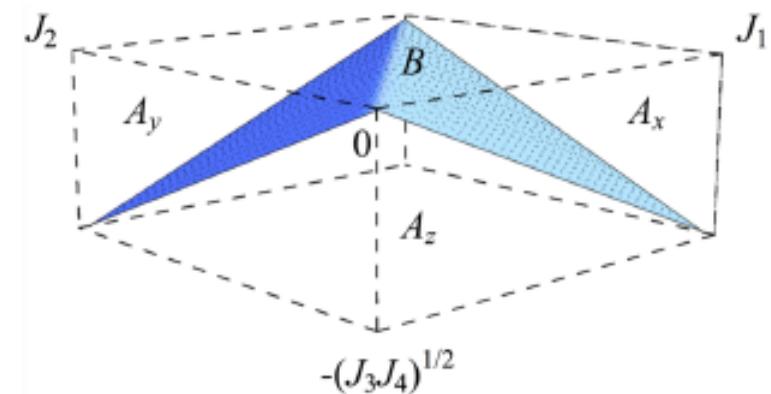
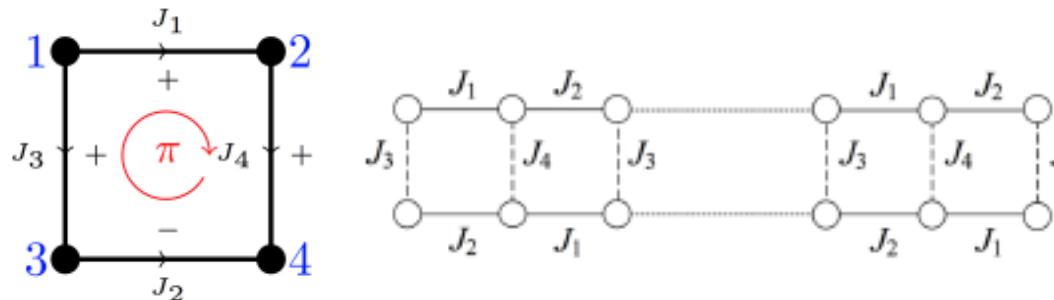
$$H = H_{\text{chain},1} + H_{\text{chain},2} + H_I,$$

$$H_I = \sum_{j=2m-1} J_3 \sigma_{j,1}^z \sigma_{j,2}^z + J_4 \sigma_{j+1,1}^z \sigma_{j+1,2}^z \quad (\text{four-body})$$

$$= -i \sum_{j=2m-1} (J_3 c_{j,1} c_{j,2} + J_4 c_{j+1,1} c_{j+1,2}) \quad (\text{Lieb's theorem, 1994})$$

Line : pre-formed pairs resonating along chains (rung tensor product states; bosonization c=1)

Loop qubit: « Majorana box »



Az phase of the phase diagram: Pi flux ground state protection
the c-Majoranas and d Majoranas are massive (local noisy fields, robustness)

$D_{1,3} = (-i)d_1d_3$ and $D_{2,4} = (-i)d_2d_4$ are fixed to the same value +1 or -1 in the ground state.

one can introduce a four-spin operator $\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z$

The qubit operator can be re-written as $d_1d_2d_3d_4$

**Protocol to braid d1 and d2: operate and measure in spin space
(x,y, z Bell correlation functions for the spins 1 and 2)**

- Braiding: $\delta J_2 \sigma_j^y \sigma_{j+1}^y = i \delta J_2 d_j d_{j+1}$, $\delta J_2 \rightarrow -\delta J_2 \Rightarrow d_j \leftrightarrow d_{j+1}$.

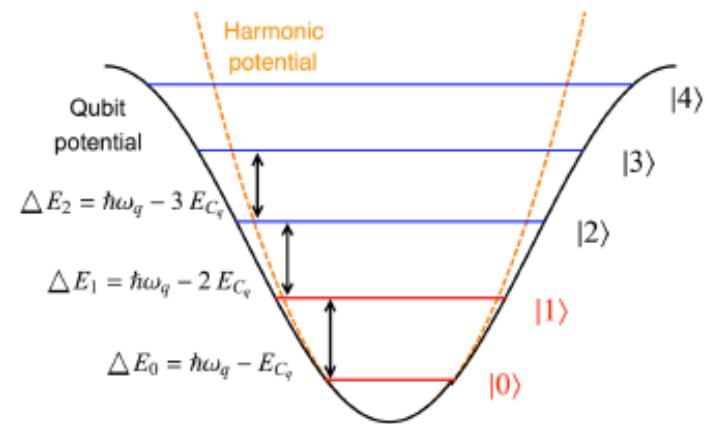
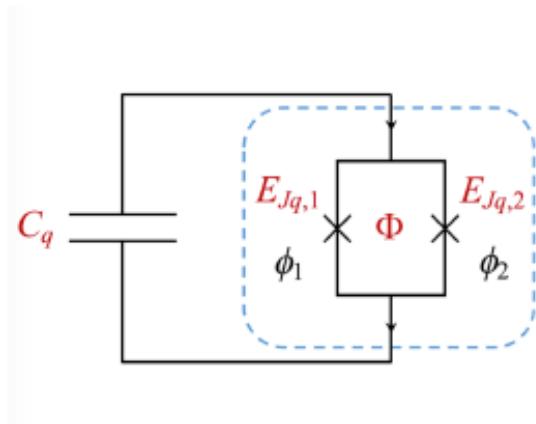
Effective spin representation

Quantization of transmon qubit

- Harmonic oscillators with anharmonicity from Josephson junctions ($t \gg 1$):

$$H_q = \frac{\hat{Q}^2}{2C_q} - E_{Jq} \cos \hat{\varphi} = -E_{Jq} + \hbar\omega_q \left(b^\dagger b + \frac{1}{2} \right) - \frac{E_{Cq}}{12} \left(b^\dagger + b \right)^4$$

$$[\hat{\varphi}, \phi_0 \hat{Q}] = i\hbar, \quad \hat{\varphi} = \frac{1}{t} (b^\dagger + b), \quad \hat{Q} = \frac{-et}{i} (b^\dagger - b), \quad t = \left(\frac{E_{Jq}}{2E_{Cq}} \right)^{1/4}$$



Yale, Santa Barbara (implementation of a small toric code),...

Simulation of New Devices with SC devices and Transmons

Anderson RVB states and Majoranas, p-wave SC

KLH, Ariane Soret, Fan Yang (25 pages) : arXiv:1703.07322

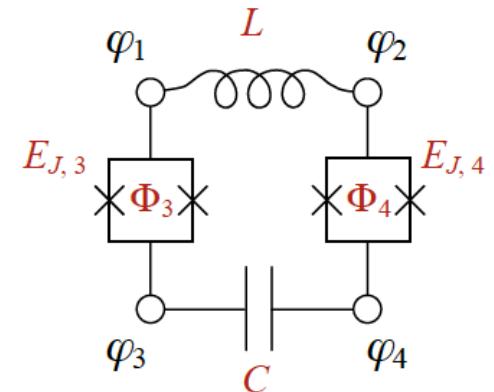
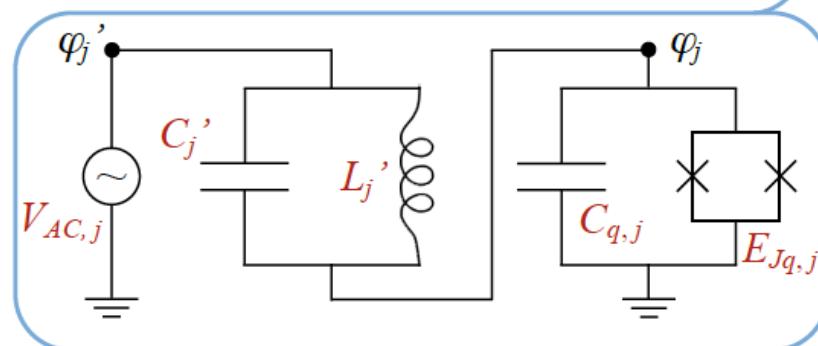
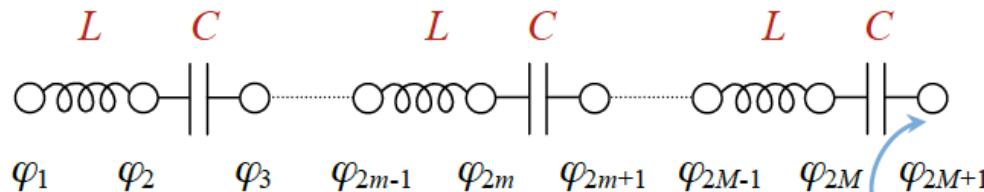
« loop » devices in link with Sachdev-Ye-Kitaev models, ring exchange models

Precise Device engineering in progress, Fan Yang master project M2 below

Su-Schrieffer-Heeger and Rice-Mele model with LC chains

T. Goren, K. Plekhanov, F. Appas, G. Roux, KLH – **in progress**

Probe of topology, Bloch bands, and transport with photons



Coupling
4 Majoranas

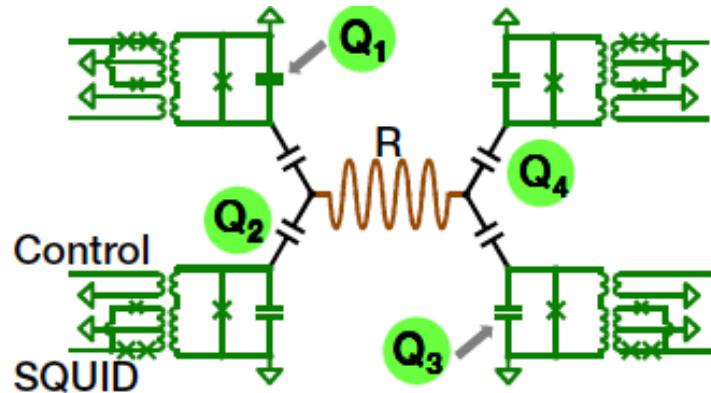
A 4-site toric code
has been initiated
Y. P. Zhong et al
PRL 117, 110501 (2016)

Exemple Realization of a Kitaev spin chain (emergent Majorana chain): NMR device on each port

Loic Herviou, C. Mora and KLH (collaboration with P. Roushan, C. Neill – google Santa Barbara
on generalized quenches and bi-partite fluctuations in XY and Ising quantum spin chains)

Networks in circuit QED and atoms: progress

Zhong et al. arXiv: 1608.04890 (PRL)



Non-Abelian statistics

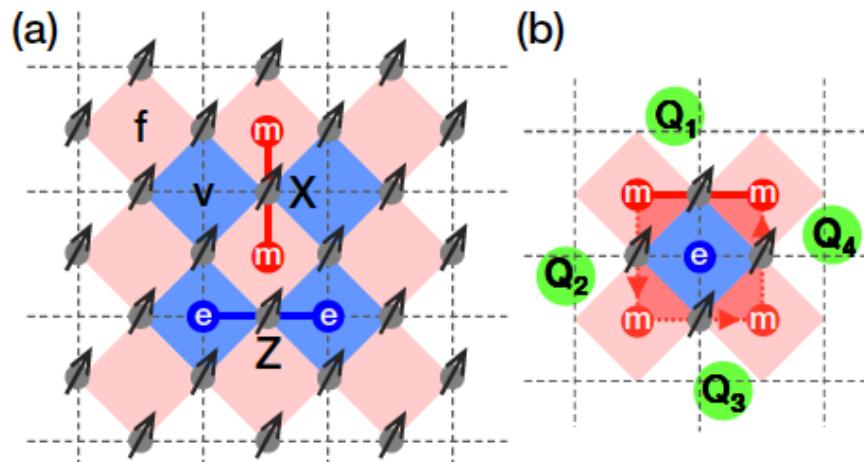
$$H_{\text{exp}}/\hbar = \omega_r a^\dagger a + \sum_{j=1}^4 \omega_j \sigma_j^\dagger \sigma_j + g \sum_{j=1}^4 (\sigma_j^\dagger a + \sigma_j a^\dagger),$$

$$C_{\text{loop}} = X'_4 X'_3 X'_2 X'_1$$

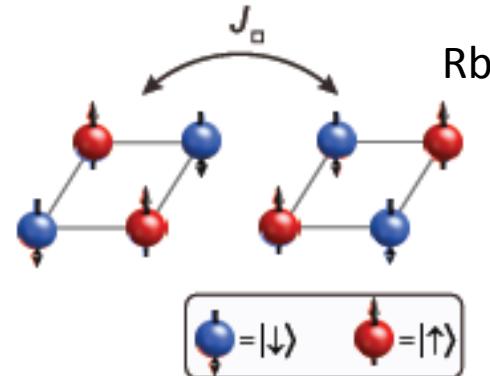
$$C_{\text{loop}} |\psi_g\rangle = |\psi_g\rangle$$

$$Z' C_{\text{loop}} Z' |\psi_g\rangle = -|\psi_g\rangle.$$

H. N. Dai et al. arXiv:1602.05709
Simulation of ring processus

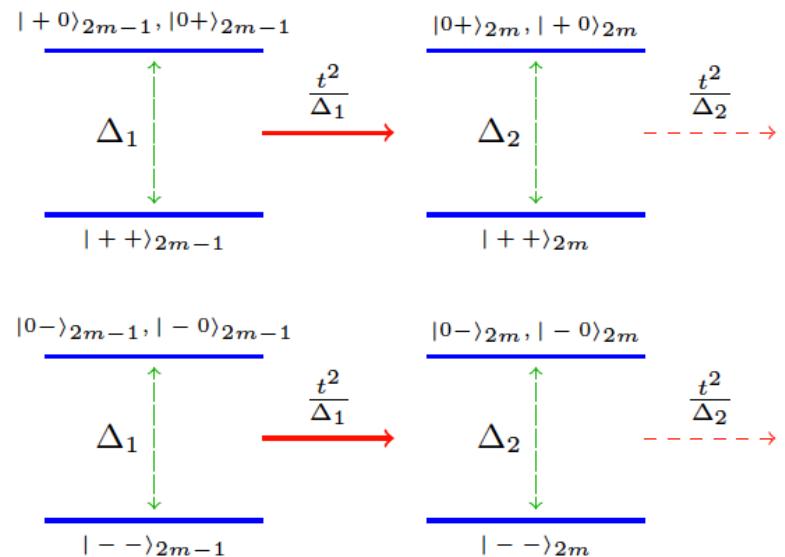
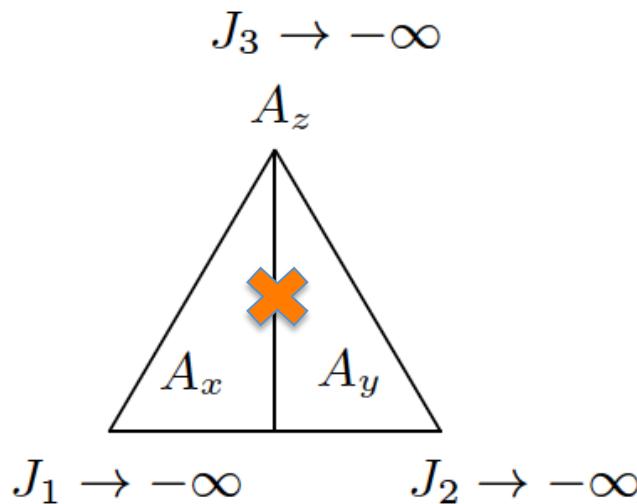


Other proposals to realize ring processus
H. P. Büchler et al. (PRL 2005)



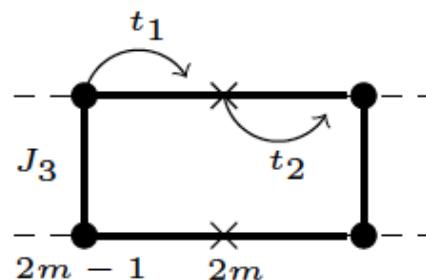
$$\hat{H}_{\text{T}} = -J_{\square} \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x - \frac{J_+}{4} \sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z,$$

Topology of hole pair



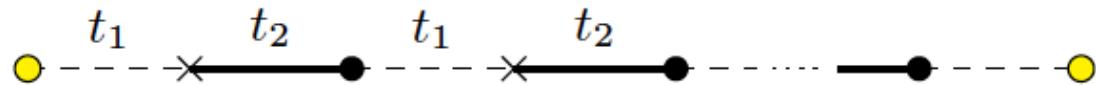
Analogy to SSH model for hole pair:

- Insulating phase : hole pair at boundary
- Superconducting transition at intermediate Coupling (p-wave SC); non topological SC can also be found from Az phase



2D work with T. Liu, C. Reppelin, B. Doucot, N. Regnault; see also B. Rosenow, G. Khaliullin, C. Honerkamp, ...

Doping at the edges



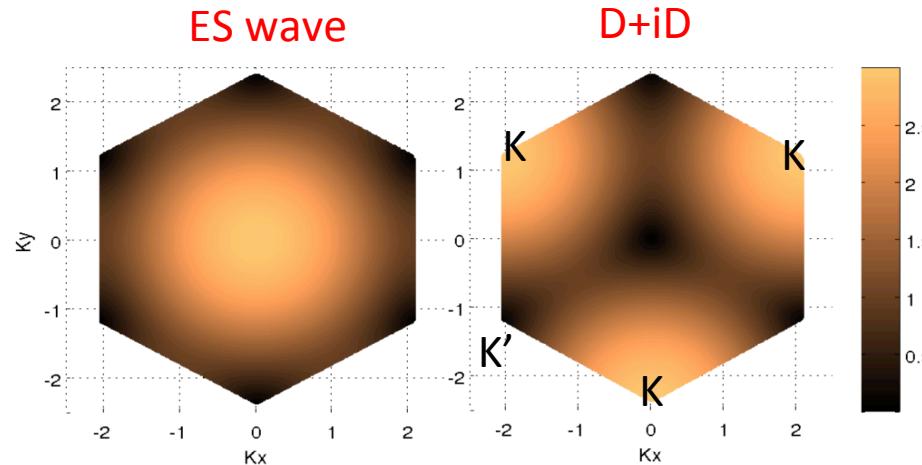
Still Chirality from Hubbard...

Possibility of gapped spin liquid

Let's start from Néel half-filled phase and dope the system

Difficulty to solve
this model exactly
Start from Mean-Field

Atomic picture



Doping leads to
Interesting pairing

$$\Delta_{\mathbf{k}}^{d+id} = \cos\left(\frac{\pi}{3}\right)\Delta_{x^2-y^2} + i\sin\left(\frac{\pi}{3}\right)\Delta_{xy}.$$

d+id SC state

A. Black-Schaffer &
S. Doniach

C. Honerkamp

R. Nandkishore, L.
Levitov, A. Chubukov
¼ filling ...

- 1) Wei Wu, Michael M. Scherer, Carsten Honerkamp & KLH, PRB 2013 ([RMFT, fRG: t-J1-J2](#))
- 2) A. Black-Schaffer, W. Wu and KLH, PRB 2014 ([symmetries, RMFT t-J, QMC on Hubbard](#))
- 3) A. Black-Schaffer & KLH, arXiv:1503.02509 ([topological phase transition due to SDW](#))

d+id Pairing: RMFT & QMC

Renormalized mean-field theory

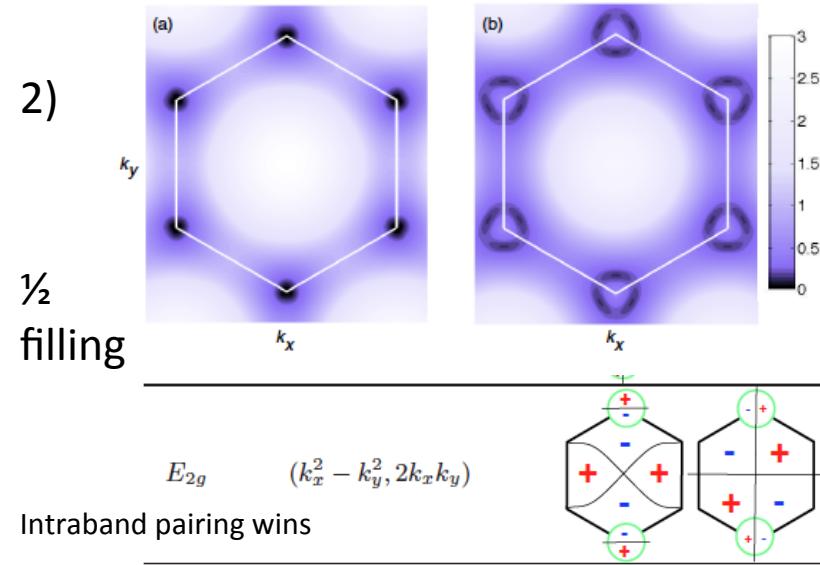
F. C. Zhang, C. Gros, T. M. Rice, and H. Shiba, Supercond. Sci. Tech. **1**, 36 (1988).

P. W. Anderson, P. A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, J. Phys.: Condens. Matter **16**, R755 (2004).

B. Edegger, V. N. Muthukumar, and C. Gros, Adv. Phys. **56**, 927 (2007).

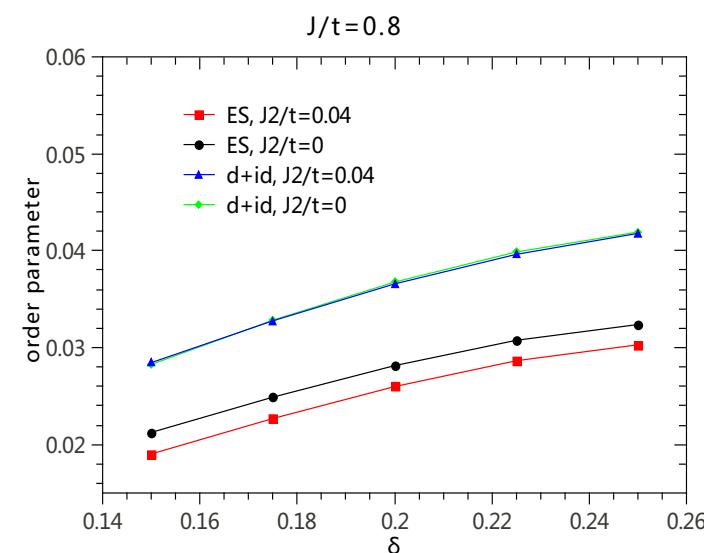
M. Ogata and H. Fukuyama, Rep. Prog. Phys. **71**, 036501 (2008).

K. Le Hur and T. M. Rice, Ann. Phys. **324**, 1452 (2009).

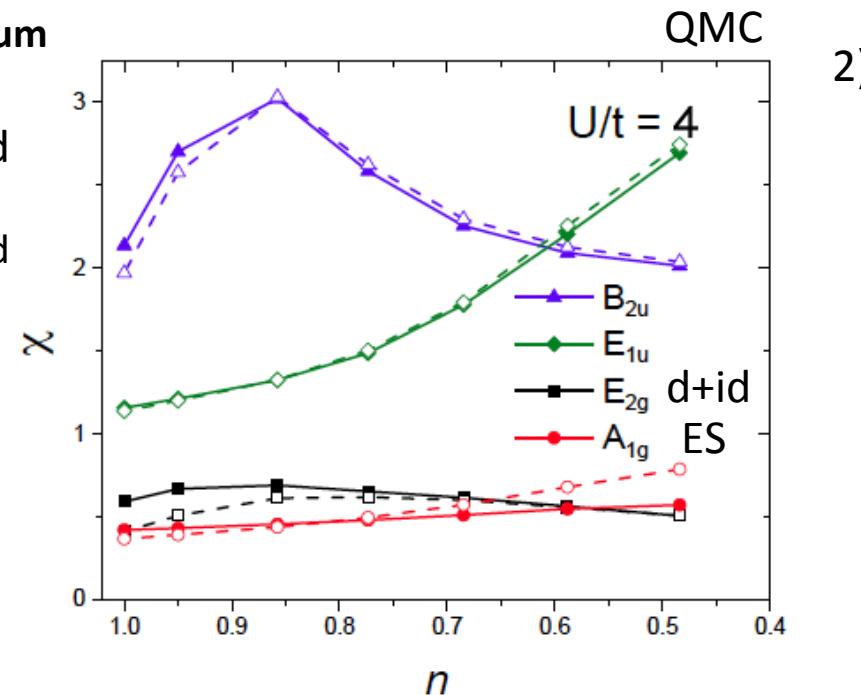


BdG
spectrum

Doped
Fully
gapped



1)

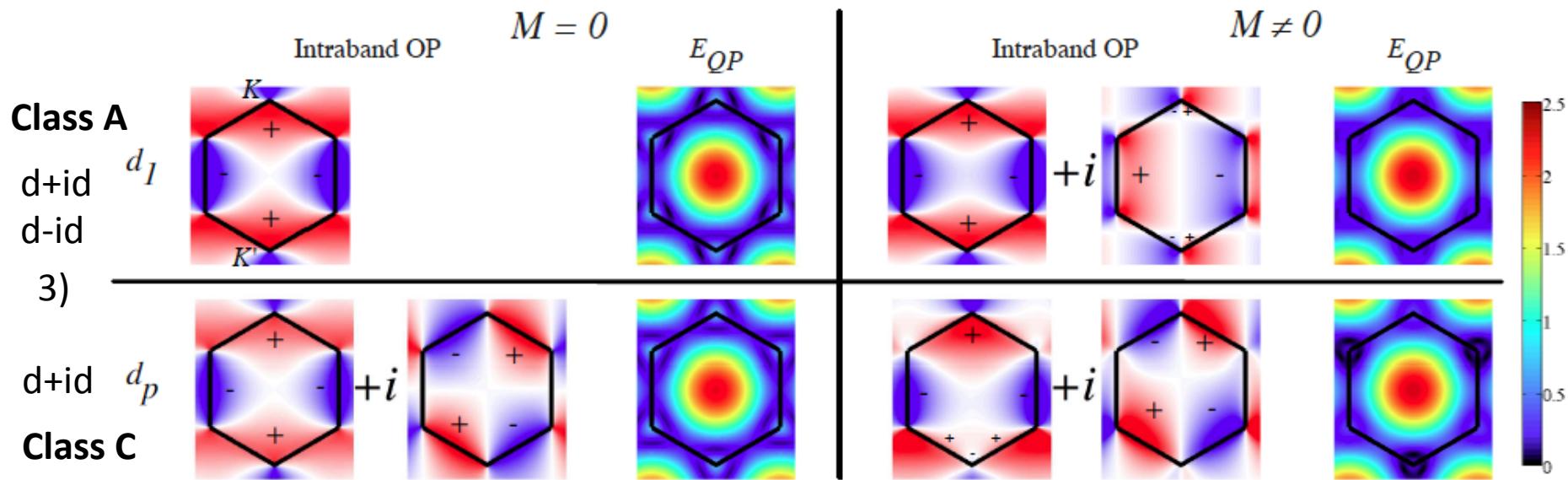
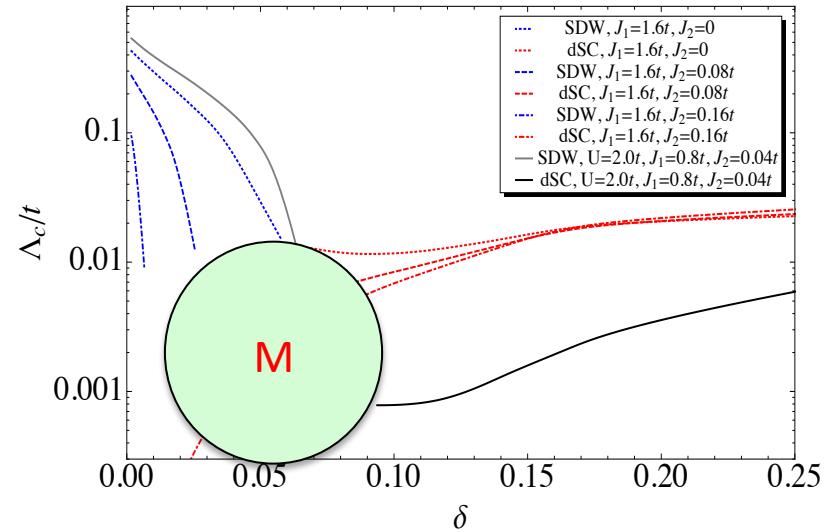
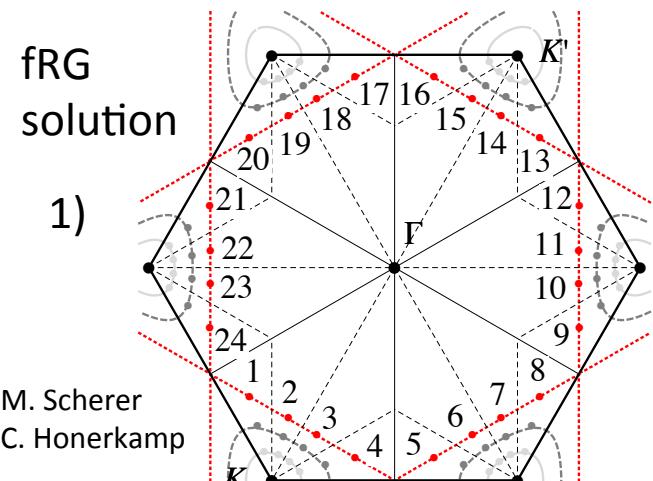


See also results from L. Balents, X. G. Wen :

Z. G. Gu et al. Phys. Rev. B **88**, 155112 (2013)

Grassmann Tensor Network Approach, DMRG

SDW and SC: Two chiral Superconductors



Universality class C=2 (2 co-propagating edge modes)

Counter-propagating helical edge modes
(class A): analogy to Kane-Mele model

Topological Phases, Interaction Effects & Gauge Fields

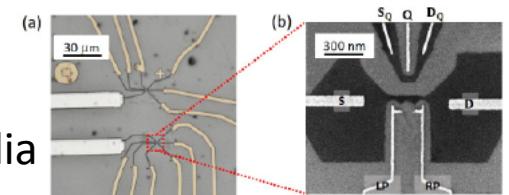
From Materials, to Ultra-Cold Atoms and Photon Simulators

- Spin-orbit coupling in graphene lattices
Kane-Mele Model, Interaction Effects (fermions and Mott, XY Neel)
bosons yield a new CSS (chiral spin state) phase

ladder system: FQHE of bosons U=infinity

- Z2 quantum Majorana liquids
Kitaev chain, Ladders (linking RVB states with BCS theory exactly)
Majorana Box, cQED
SSH model of hole pair

Progress on stochastic dynamics and simulation in dissipative media
KLH, L. Henriet, L. Herviou, K. Plekhanov, A. Petrescu, T. Goren
M. Schiro, C. Mora and P. P. Orth, arXiv:1702.05135



Mesoscopic graphene Kondo cQED, group of G.-P. Guo