Charge Relaxation Resistance as probe of Many-Body Physics

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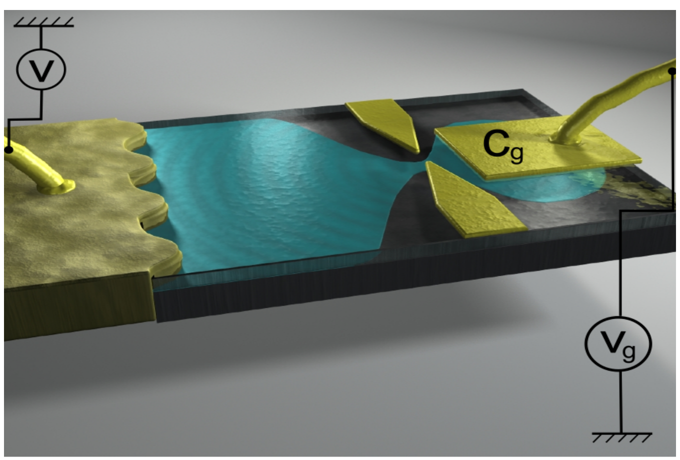
Rutgers, April 23rd 2012



J. Gabelli, et al. Science 313, 499 (2006)
Group LPA at ENS Paris

Collaborators:
-C. Mora & M. Fillipone (ENS, Paris)
-P. Dutt & T. Schmidt (Yale)

-I. Garate (Yale)

Outline

Quantum RC Circuit: C. Mora & KLH Nature Physics **6**, 697 2010

- Charge Relaxation
- Interaction Effects & Many-Body Problem
- Universal resistances in the AC limit
- Larger Class of Models: FQHE-states, TIs

Anderson model: One level Phys. Rev. Lett. **107**, 176601 (2011)

- Giant magnetoresistance in the charge relaxation

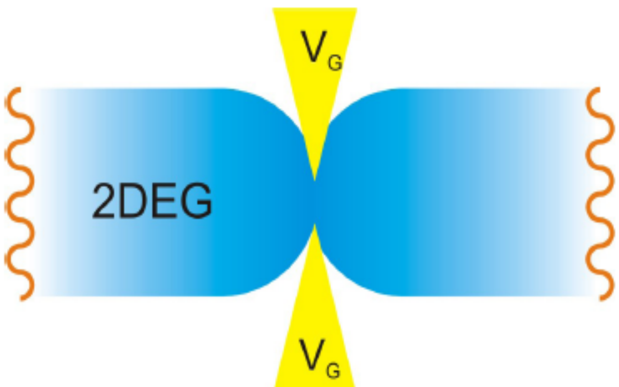
New developments, 2012

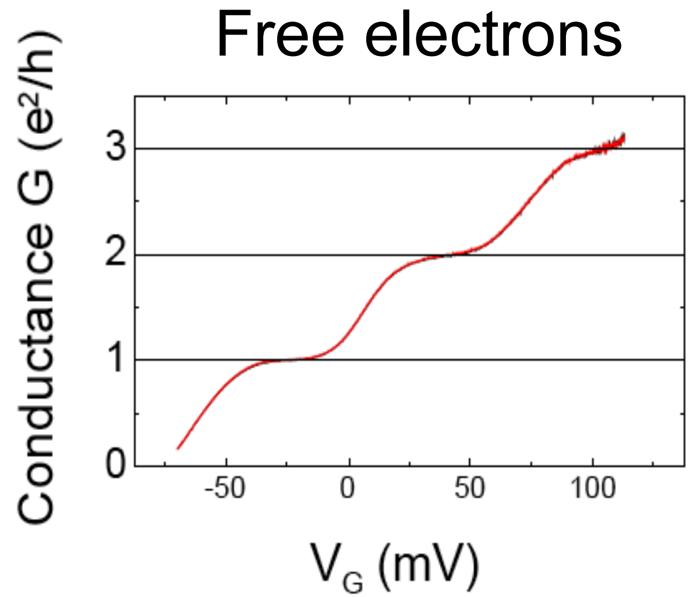
DC Transport

DC transport

conductance = transmission

Quantum point contact



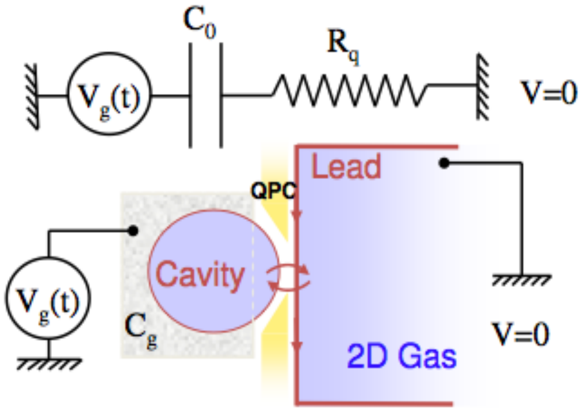


Conductance related
To Transmission

Landauer formula : $G = \frac{e^2}{h}\sum_n D_n$

AC Transport?

RC circuit: Buettiker "free" result

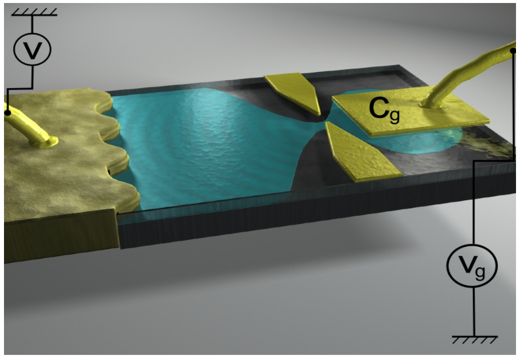


Low Frequency:

$$\frac{Q(\omega)}{V_g(\omega)} = C_0(1 + i\omega C_0 R_q)$$

Buettiker: $R_q = h/2e^2$

<u>Quantum circuit:</u>
The values of the capacitance C_0
& of the charge relaxation resistance
R_Q are "entangled"
Phase coherent Transport



J. Gabelli, et al. Science 313, 499 (2006).

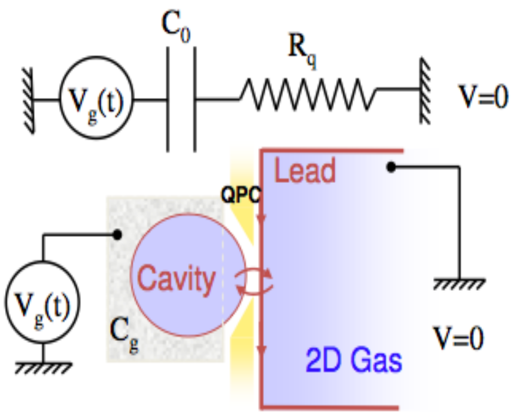
Effect of Interactions?

Hamiltonian: polarized case

C. Mora and KLH, Nature Physics **6**, 697 2010

3 pieces in the Hamiltonian

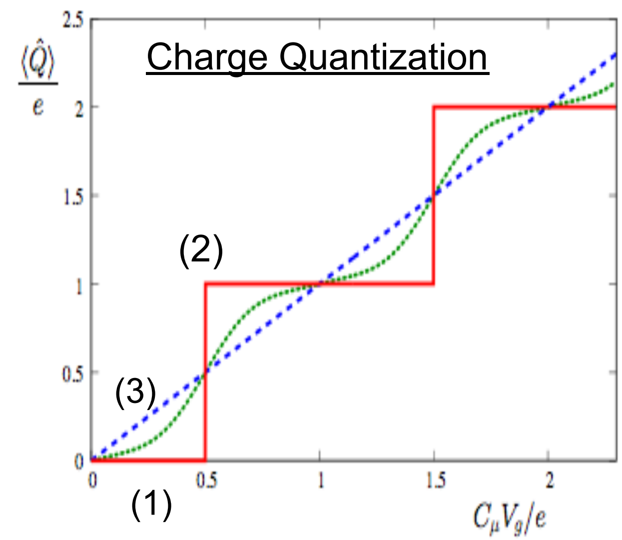
$$
\begin{aligned}
H_0 &= \sum_p \varepsilon_p \, c_p^\dagger c_p + \sum_k \varepsilon_k \, d_k^\dagger d_k, \\
H_c &= E_c(\hat{N} - N_0)^2, \\
H_T &= t \sum_{k,p} \left(d_k^\dagger c_p + c_p^\dagger d_k \right)
\end{aligned}
$$



H_c represents the charging energy

$$\hat{N} = \sum_k d_k^\dagger d_k$$

$$N_0 = C_g V_g / e$$



R_Q in the tunneling limit: linear response (1)

$$Q(\omega) = e^2 K(\omega) V_g(\omega)$$

$$K(t-t') = i\theta(t-t')\langle[\hat{N}(t), \hat{N}(t')]\rangle$$

Key Steps:

$$\langle T_t[\hat{N}(t)\hat{N}(0)]\rangle = \frac{\langle\phi_{GS}|T_t[\hat{N}(t)\hat{N}(0)U(\infty,-\infty)]|\phi_{GS}\rangle}{\langle\phi_{GS}|T_t[U(\infty,-\infty)]|\phi_{GS}\rangle}$$

and expand the evolution operator

$$U(\infty,-\infty) = \sum_{n=0}^{+\infty} (-i)^n \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \ldots \int_{-\infty}^{t_{n-1}} dt_n \, H_T(t_1) H_T(t_2) \ldots H_T(t_n)$$

in powers of $H_T(t) = e^{i(H_0+H_c)t} H_T e^{-i(H_0+H_c)t}$

Interactions treated Exactly: No Wick Theorem

Results: 2 values for R_Q

- Results for Im K(ω) pushed to 4th order
- Intermediate states have a Coulomb gap $E_{\pm} = E_c(1 \mp 2N_0)$
- Particle-hole pair creation (dissipation)

Small cavities ω<Δ (=level spacing of the box)

$$\mathrm{Im}\, K(\omega) = \pi\omega\,[\mathrm{Re} K(0)]^2$$

R_Q=h/2e^2 (Buettiker et al)

Large cavities ω>Δ

$$\mathrm{Im}\, K(\omega) = 2\pi\omega\,[\mathrm{Re} K(0)]^2$$

R_Q=h/e^2

Recall:

$$\frac{Q(\omega)}{V_g(\omega)} = C_0(1 + i\omega C_0 R_q)$$

$$C_0 = e^2 K(0)$$

Matveev, 1991; measurements R. Ashoori et al.;…

The many-body Problem (2)…

Large cavity: analogy to Kondo model

$\uparrow$ = charge e on the cavity

$\downarrow$ = no (extra) charge

$$\hat{N} = \frac{1}{2} - S^z$$

State on the dot described by Ket: $|\psi\rangle \otimes |Q\rangle$

$$H_T = t \sum_{k,p} \left(d_k^\dagger c_p S^+ + c_p^\dagger d_k S^- \right)$$

$$K(t-t') = \chi_{zz}(t-t') = i\theta(t-t')\langle [S^z(t), S^z(t')] \rangle$$

Korringa-Shiba relation of the (anisotropic) Kondo model

$$\operatorname{Im} \chi_{zz}(\omega) = 2\pi\alpha\omega \left[\operatorname{Re}\chi_{zz}(0) \right]^2$$

α=1

R_Q=h/e^2

Sassetti-Weiss and Saleur et al.

Perfect Transmission (3)

Effective 1D Field Theory:

Problem exactly solvable through bosonization

$$S_0 = \frac{1}{\pi} \sum_n \phi_0(\omega_n)\phi_0(-\omega_n) \left[\frac{|\omega_n|}{1 - e^{-2|\omega_n|L/v_F}} + \frac{E_c}{\pi} \right]$$

The field ϕ_0 is related to the charge on the cavity

$$\hat{N} = C_\mu V_g / e + \phi_0 / \pi$$

$$G_0(\omega) = \frac{\pi^2}{2E_c} \left(A + i \frac{\omega \pi}{E_c} B \right)$$

Weak Backscattering

Expansion to second order (cancellations)

$$R_q = \frac{h}{e^2}\frac{B}{A^2}$$

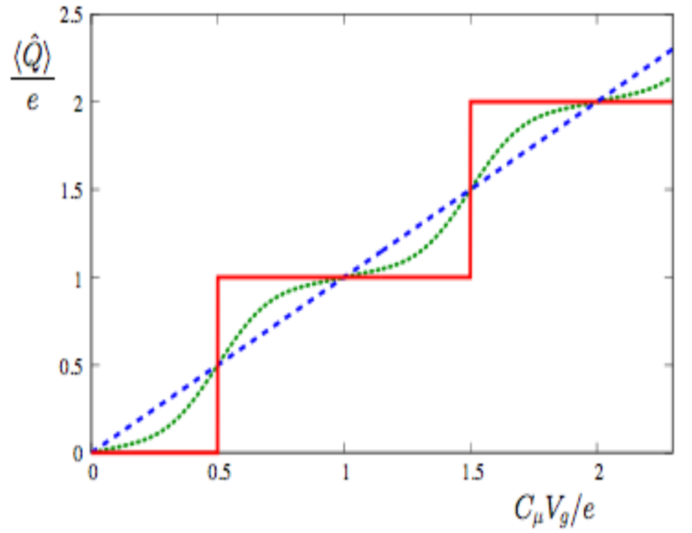
Small Cavity

$$A^{-1} = 1 + \Delta/2E_c$$
$$B^{-1} = 2(1 + \Delta/2E_c)^2$$

$$\Delta = \pi v_F/L$$

Large Cavity $A = B = 1$

Summary I: C. Mora & KLH Nature Physics **6**, 697 2010

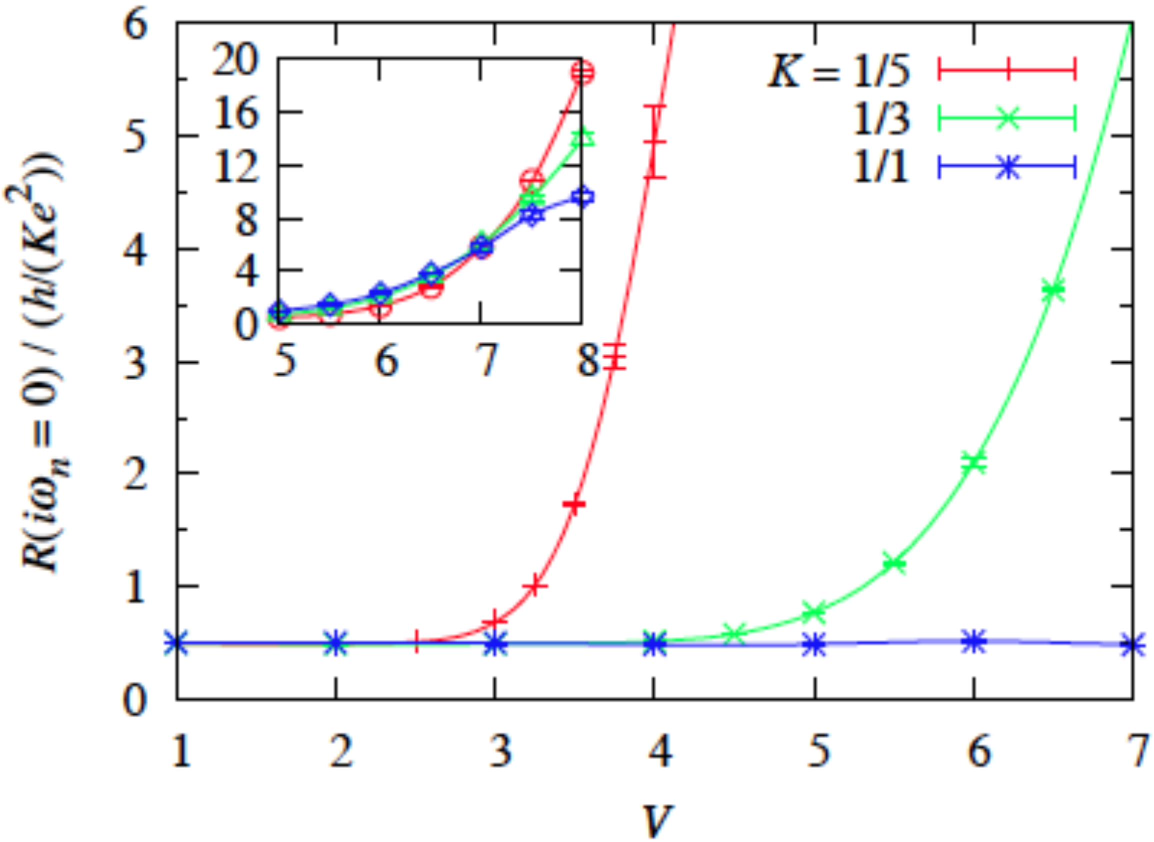


2 Universal Resistances in the charge Relaxation
Large cavity, 2 "contact" resistances in series

Temperature: Thermally incoherent regime

Close to the perfect transmission, temperature can reach E_c for a small and large cavity

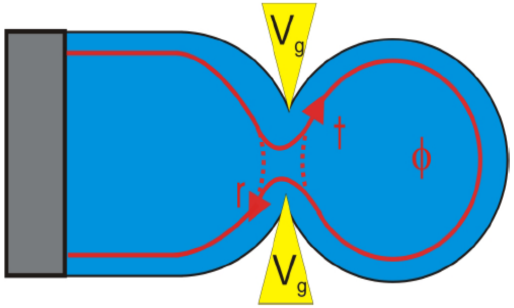
Numerical Work: path integral MC



Hamamoto, Jonckheere, Kato and T. Martin, PRB **81**, 153305 (2010)

Fractional Quantum Hall state

X. Wen edge theory



$$S_0 = \frac{1}{4\pi\nu}\int_{-\infty}^{+\infty} dx \int_0^{\beta} d\tau \left[i(\partial_\tau\phi)(\partial_x\phi) + v(\partial_x\phi)^2 \right] + E_c \int_0^{\beta} d\tau \left(\frac{\phi(L)-\phi(-L)}{2\pi} - N_0 \right)^2$$

$$S_{BS} = -\frac{v_F r}{\pi a}\int_0^{\beta} d\tau \cos\left[(\phi(L)-\phi(-L))/\nu\right]$$

Again, we single out the mode $\phi_0 = \frac{\phi(L)-\phi(-L)}{2}$

$$S_0 = \frac{1}{\pi}\sum_n \phi_0(\omega_n)\phi_0(-\omega_n)\left[\frac{|\omega_n|/\nu}{1-e^{-2|\omega_n|L/v_F}} + \frac{E_c}{\pi}\right]$$

Small cavity

$$A^{-1} = 1 + \Delta/(2\nu E_c)$$
$$B^{-1} = 2\nu\left[1 + \Delta/(2\nu E_c)\right]^2$$

$$R_q = h/(2\nu e^2)$$

See also T. Martin et al. 2010

Anderson Model: small cavity (fermions with spins)

- Giant Magnetoresistance in the charge Relaxation

The hamiltonian of the Anderson model is given by

$$H = \sum_{\sigma,k} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \varepsilon_d \hat{n}_d + U \hat{n}_\uparrow \hat{n}_\downarrow + t \sum_{k,\sigma} \left(c^\dagger_{k\sigma} d_\sigma + d^\dagger_\sigma c_{k\sigma} \right)$$

Electron number on the dot

$$\hat{n}_d = \hat{n}_\uparrow + \hat{n}_\downarrow \text{ with } \hat{n}_\sigma = d^\dagger_\sigma d_\sigma$$

M. Filippone, K. Le Hur, and C. Mora, PRL **107**, 176601 (2011)

Small cavity: Rq=h/4e^2

Linear response: $\varepsilon_d(t) = \varepsilon_{d,0} + \varepsilon_1 \cos\omega t$

$$\chi_c(t-t') = i\theta(t-t')\langle[\hat{n}_d(t), \hat{n}_d(t')]\rangle$$

$$\mathcal{P} = \frac{1}{2}\varepsilon_1^2\,\omega\,\mathrm{Im}\chi_c(\omega)$$

Construction of a "minimal" low-energy theory (Fermi liquid)

$$H = \sum_{\sigma,k} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{\chi_c}{2\nu_0}\,\varepsilon_1 \cos\omega t \sum_{k,k',\sigma} c^\dagger_{k\sigma} c_{k'\sigma}$$

$$\hat{A} = \sum_{k,k',\sigma} c^\dagger_{k\sigma} c_{k'\sigma} \qquad \mathcal{P} = \frac{1}{2}\varepsilon_\omega^2\,\omega\left(\frac{\chi_c}{2\nu_0}\right)^2 \mathrm{Im}\chi_{\hat{A}\hat{A}}(\omega)$$

$$R_q = h/4e^2$$

At finite magnetic Field

$$\chi_{c\sigma} = -\frac{\partial \langle \hat{n}_\sigma \rangle}{\partial \varepsilon_d}, \qquad \chi_c = \chi_{c\uparrow} + \chi_{c\downarrow}$$

$$\mathrm{Im}\chi_c(\omega) = \pi\omega\left(\chi_{c\uparrow}^2 + \chi_{c\downarrow}^2\right)$$

It is judicious to introduce the charge magneto-susceptibility

$$\chi_m = -\partial_{\varepsilon_d} m$$

$$R_q = \frac{h}{4e^2}\frac{\chi_c^2 + 4\chi_m^2}{\chi_c^2}$$

Revisited low energy theory

$$H_Z = -\frac{H}{2}\left(\sum_{\sigma,k} \sigma\, c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{\sigma} \sigma\, \hat{n}_{\sigma}\right)$$

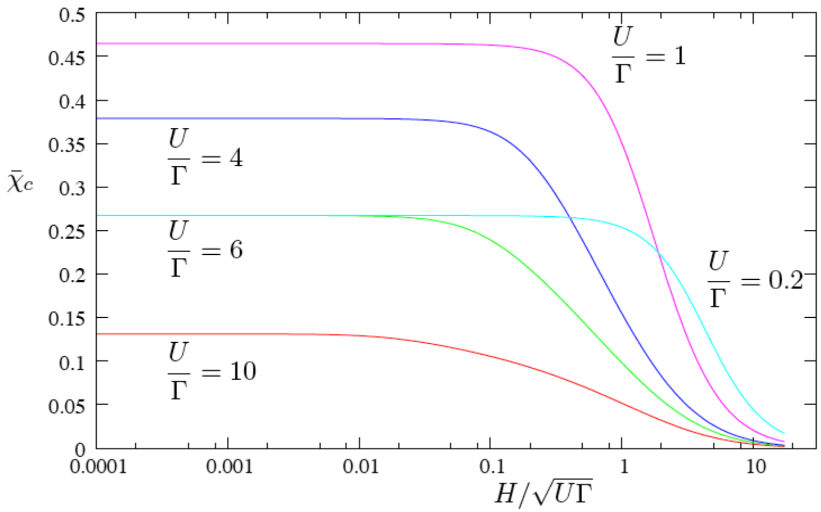
$$H = \sum_{\sigma,k}\left(\varepsilon_k - \frac{H\sigma}{2}\right) c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{\varepsilon_1 \cos\omega t}{\nu_0} \sum_{k,k',\sigma} \chi_{c\sigma}\, c^{\dagger}_{k\sigma} c_{k'\sigma}$$

$$\boxed{R_q = \frac{h}{4e^2}\,\frac{\chi_c^2 + 4\chi_m^2}{\chi_c^2}}$$

Charge Susceptibility χ_c

- Bethe Ansatz results following Tsvelick-Wiegmann, N. Andrei

$$\chi_c = \frac{8\Gamma}{\pi U^2}\left(1 + 6\,\frac{2\Gamma}{\pi U}\right) \qquad U \gg \Gamma$$



$$\chi_c = \frac{8\Gamma}{\pi H^2}$$

Charge susceptibility $\bar{\chi}_c = \chi_c \sqrt{U\Gamma}$ for various ratios of U/Γ.

Application: magnetic field

(Bethe Ansatz: N. Andrei, Wiegmann-Tsvelik)

Kondo "realm" (Ng & P. Lee; Raikh & Glazman):

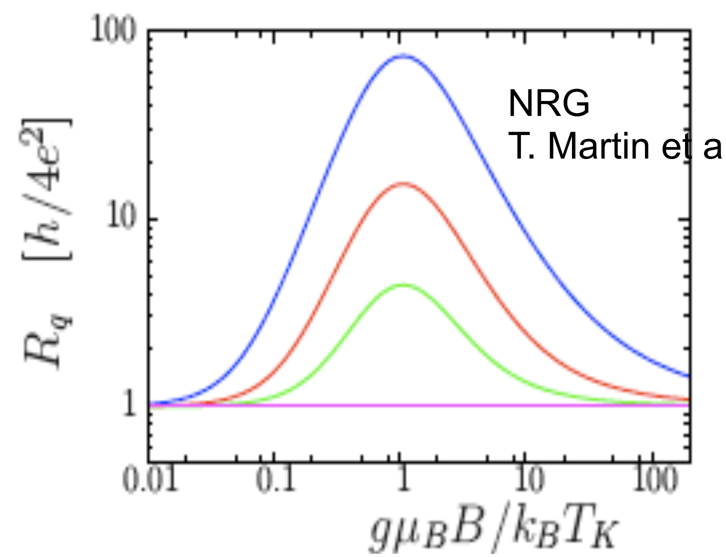
χ_c suppressed

$$\chi_c = \frac{\Gamma}{\pi}\left(\frac{1}{(\varepsilon_d + U)^2} + \frac{1}{\varepsilon_d^2}\right)$$

$$m = \frac{\langle \hat{n}_\uparrow \rangle - \langle \hat{n}_\downarrow \rangle}{2} = f\left(\frac{g\mu_B B}{k_B T_K}\right)$$

$$T_K = 2\sqrt{U\Gamma/\pi e}\,\exp\left[\pi\varepsilon_d(\varepsilon_d + U)/2U\Gamma\right]$$

$$\chi_m = \frac{\pi}{\Gamma}\,\frac{2\varepsilon_d + U}{U}\,\Phi\left(\frac{g\mu_B B}{k_B T_K}\right)$$



$U/\Gamma = 15, 10, 7.$

χ_m vanishes at particle-hole symmetric point

Physical Interpretation

$$R_q = \frac{h}{4e^2}\frac{\chi_c^2 + 4\chi_m^2}{\chi_c^2}$$

Strong Interactions suppress χ_c

Zero magnetic field: Fermi liquid $\chi_m \longrightarrow 0$

Strong magnetic field: no spin physics

In between, $\chi_m >> \chi_c$

Universality of R_q in the **multi-channel** limit?
At a general level, **less obvious**

P. Dutt, T. Schmidt, C. Mora & KLH, in progress

Multi-channel dissipative Field Theory & Landau Zener:
Y. Etzioni, B. Horovitz and P. Le Doussal, PRL **106**, 166803 (2011)

Summary of Results (so far)

- Universal resistances in the AC regime
- Novel quantity & Magnetoresistance in the charge relaxation with strong interactions

More scenarios and geometries to explore

Christophe Mora & Karyn Le Hur, Nature Physics **6**, 697 2010

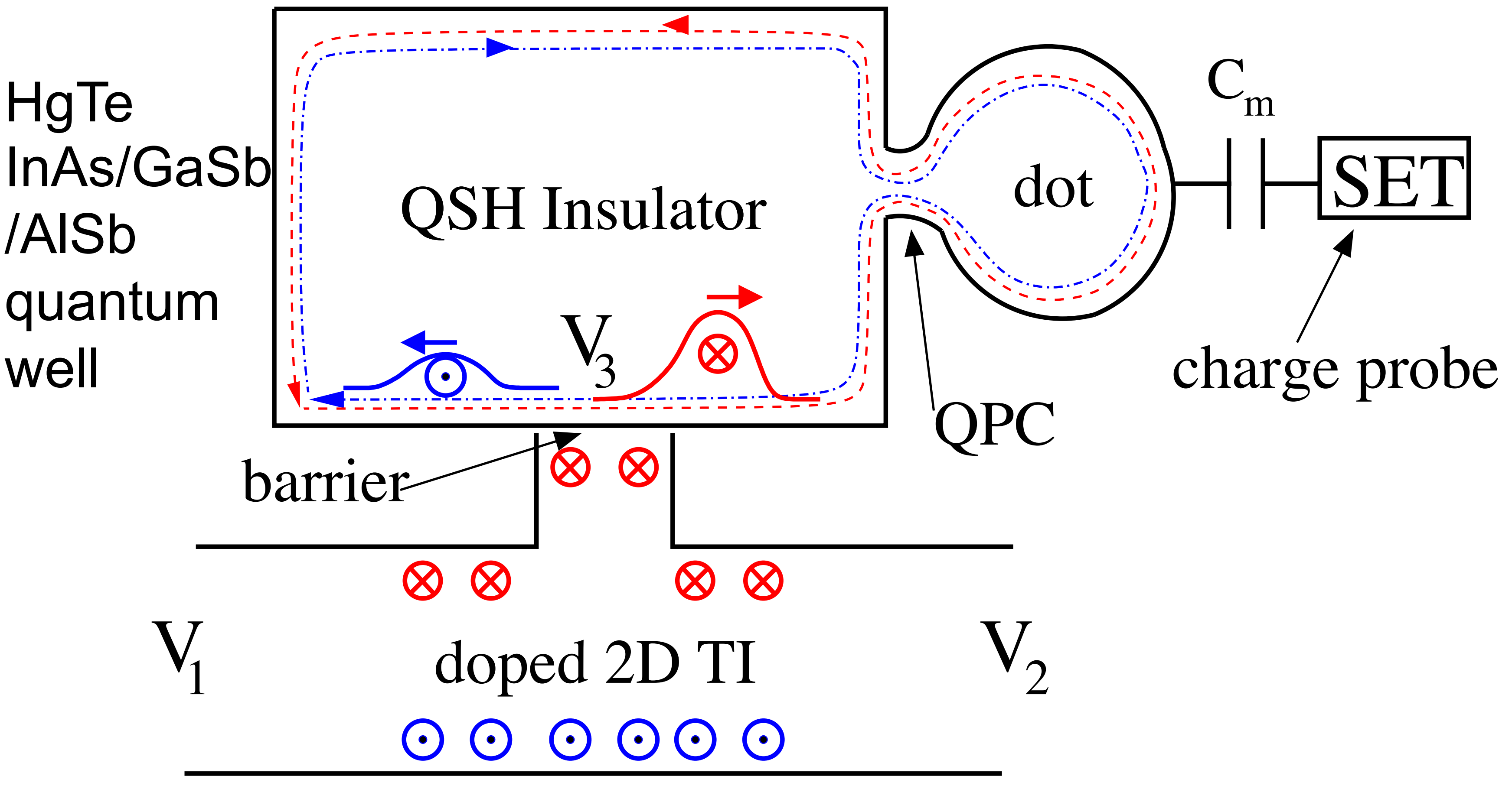
M. Fillipone, K. Le Hur and C. Mora, PRL **107**, 176601 (2011)

P. Dutt, T. Schmidt, C. Mora and KLH, in progress

I. Garate and K. Le Hur, arXiv:1111.4581 **(<u>case of pulses and not AC</u>)**

On-Demand Fractional-Charge Source

I. Garate and K. Le Hur, arXiv:1111.4581



All Electrical injection: C. Brune et al. (2010,2011)

Probe of Charge Fractionalization

- TIs: Ideal Platform

No plasmon & no-single particle

Backscattering (tuning gate voltage)

Gapless Edges: conservation Laws

Charge Breakup: (1+g)/2 and (1-g)/2

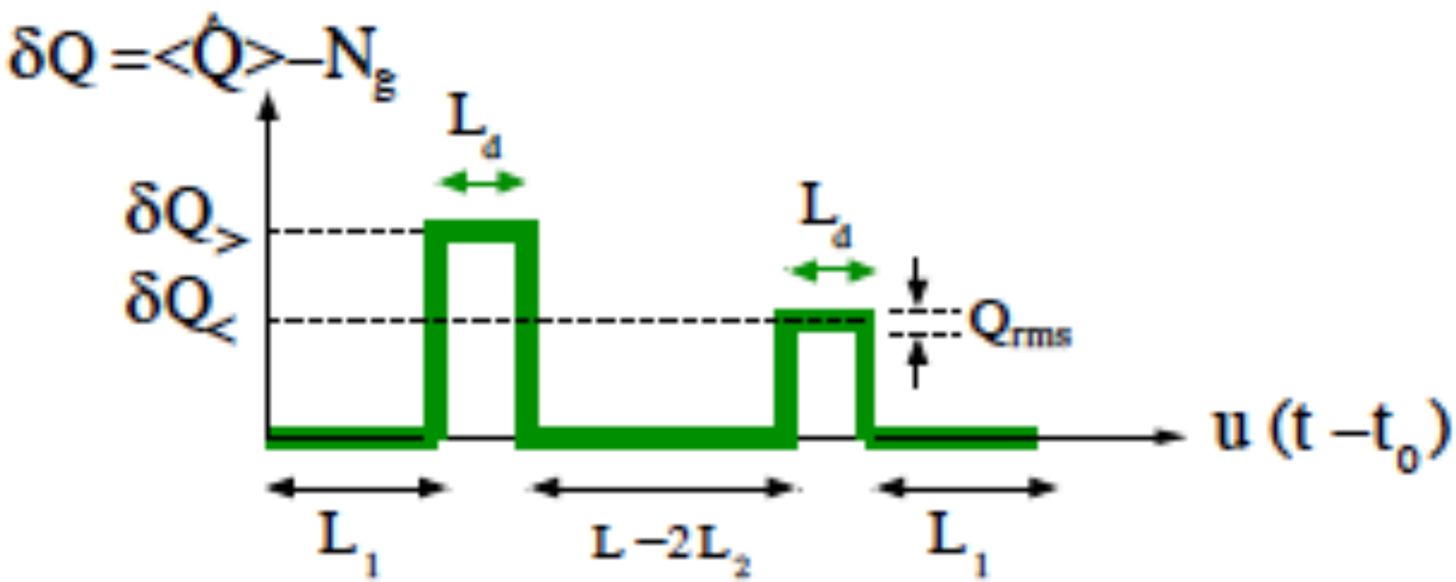
(Safi-Schulz; Pham et al.; Le Hur …)

Experiment: H. Steinberg, G. Barak; A. Yacoby; L. Pfeiffer, K. West; B. Halperin & K. Le Hur, Nature Phys. **4** 116-119 (2008))

DC injection does not work, here

Case of a Pulse

Results:



$$\delta Q_{<}/\delta Q_{>} = (1 - gP)/(1 + gP)$$

P is the polarization