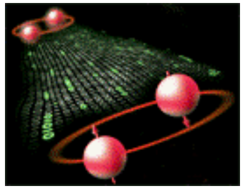
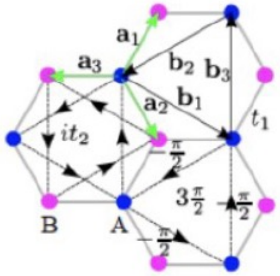
Fractional Topological Numbers and Correlated Entangled Matter



Karyn Le Hur

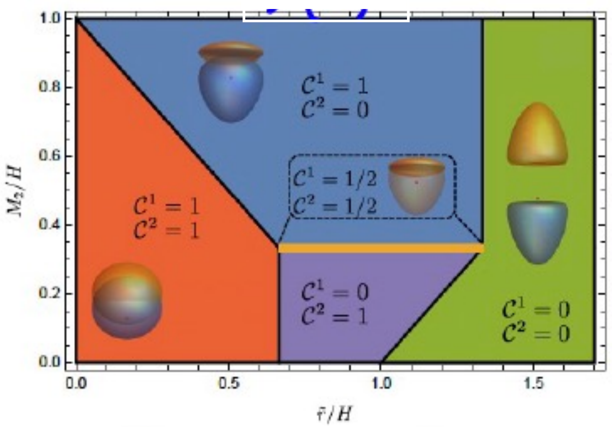
Depuis 80 ans, nos connaissances bâtissent de nouveaux mondes

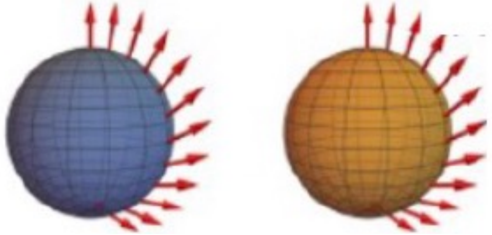


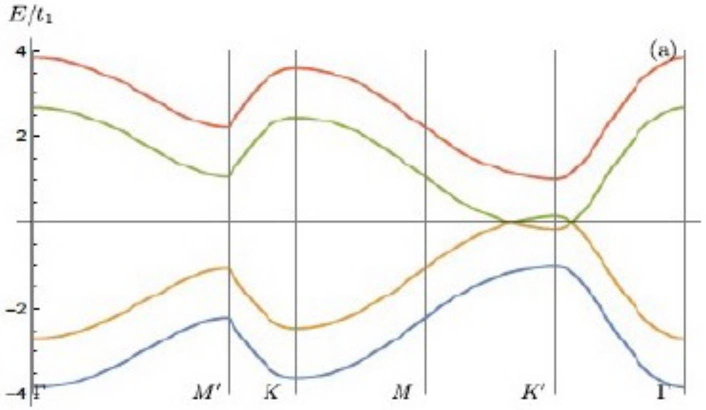
Centre de Physique Theorique, Ecole Polytechnique and CNRS France

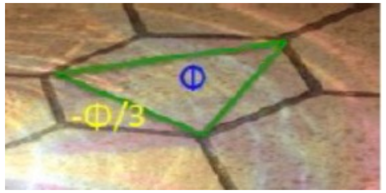
Presentation, 1 hour: CRMath Montreal August 24th 2022

Organized by William Witczak-Krempa, Joseph Maciejko, Virginie LeDuc

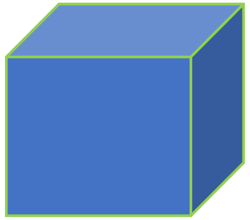








Mathematics of Shapes and Table

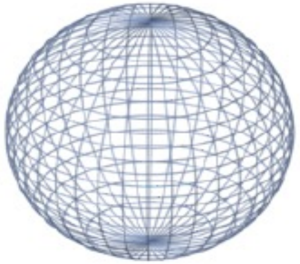


Euler characteristic is defined as

$$\chi = V - E + F$$

where V is the number of vertices (corners), E edges and F faces.

Take a cube. What is the Euler characteristic? Is this non-zero? $\chi = 2$

Sphere		**2**

- In the presence of a Dirac monopole or Topological charge, $\chi = 2 - 2g = 0$
- Agrees with Gauss-Bonnet theorem and integration of Berry curvature on the surface and Poincare-Hopf Theorem; Similar to a cup or donut
- Equivalent to 2 circles from Stokes' theorem

<u>Physics:</u> g related to quantum Hall conductivity, D. Thouless 1982 quantum Hall effect

Quantum Hall Effect and Chern Insulator

K. Von Klitzing, G. Dorda, M. Pepper (1980)

Graphene: A. Geim, K. Novoselov, Ph. Kim, E. Andrei…

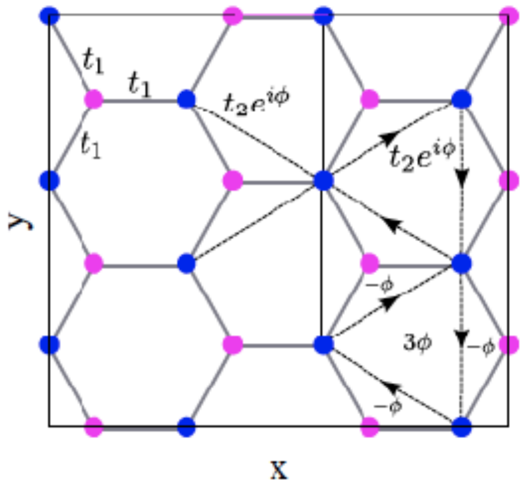
Graphene

Haldane model

$$\mathcal{H}_0 = \sum_i (-1)^i M c_i^\dagger c_i - \sum_{<i,j>} t_1 c_i^\dagger c_j - \sum_{\ll i,j \gg} t_2 e^{i\phi_{ij}} c_i^\dagger c_j$$

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988)

No net flux   M = Semenoff mass   (G. Semenoff, 1984)



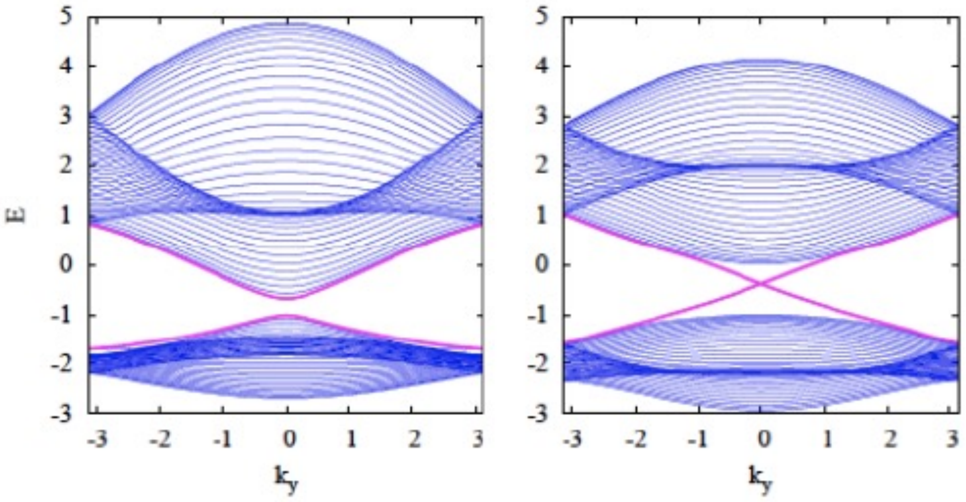
Honeycomb Lattice

Realized in quantum materials, graphene, cold atoms, light systems

Phase diagrams of interacting
Bosonic & Fermionic Models

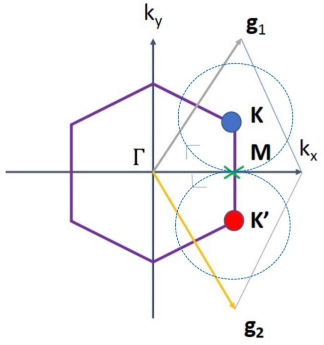
I. Vasic, A. Petrescu, K. Le Hur, W. Hofstetter,
Phys. Rev. B 91, 094502 (2015)

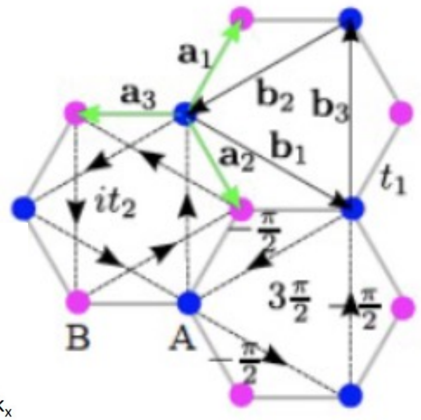
Ph. Klein, A. Grushin, K. Le Hur,
Phys. Rev. B 103, 035114 (2021)



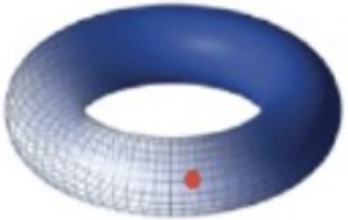
Mott transition and New Methods

Spin-1/2 analogy

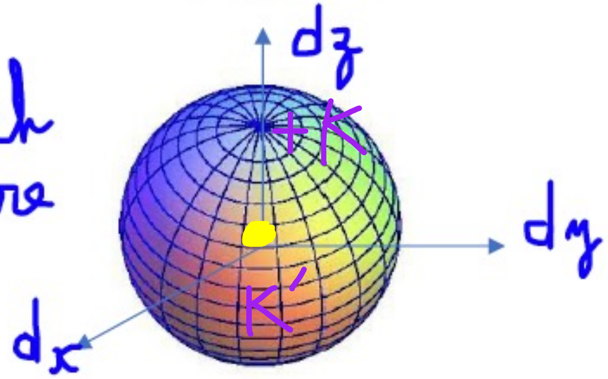




$(k_x, k_y) \mapsto (\theta, \varphi)$



Bloch Sphere



$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\left(\mathbf{k}\right) \cdot \hat{\sigma},$$

Hilbert Space

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for particles on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\,\mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\,\mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\,\mathbf{b_i}\right).$$

The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

$$m = \pm 3\sqrt{2}\, t_2 \quad \begin{matrix} +K, \\ -K' \end{matrix}$$

radial magnetic field

$C = 0, \pm 1$

$\mathbb{Z}$ number

Periodic Table of Topological Invariants: Application of Topology to Physics
Topological Insulators and Superconductors

See Andrei B. Bernevig book

Quantum Hall system
Quantum Anomalous Hall Effect

Kitaev p-wave superconductor

p+ip superconductor

Topological Insulators

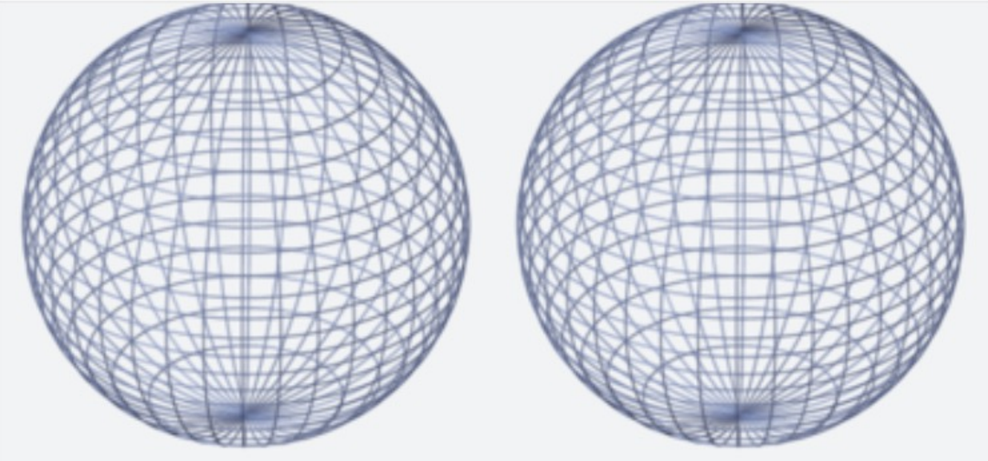
Symmetry Class	Time reversal symmetry	Particle hole symmetry	Chiral symmetry
A	No	No	No
AIII	No	No	Yes
AI	Yes, $T^2 = 1$	No	No
BDI	Yes, $T^2 = 1$	Yes, $C^2 = 1$	Yes
D	No	Yes, $C^2 = 1$	No
DIII	Yes, $T^2 = -1$	Yes, $C^2 = 1$	Yes
AII	Yes, $T^2 = -1$	No	No
CII	Yes, $T^2 = -1$	Yes, $C^2 = -1$	Yes
C	No	Yes, $C^2 = -1$	No
CI	Yes, $T^2 = 1$	Yes, $C^2 = -1$	Yes

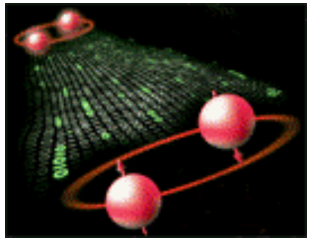
Topological states are characterized by a topological invariant linked to transport
Topology can also be achieved by applying a radial magnetic field on a (Bloch) sphere

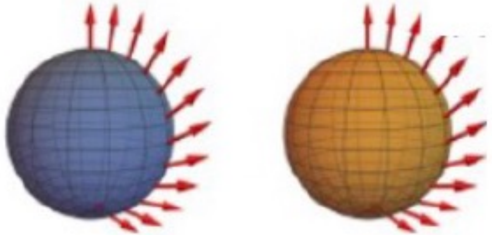
Analogy to Gauss' law for electromagnetism
Realized with current technology, spin-1/2 in curved space

Monopoles

Question:

Two spheres (not connected) *(Disjoint union of two spheres)*		$2 + 2 = \mathbf{4}$



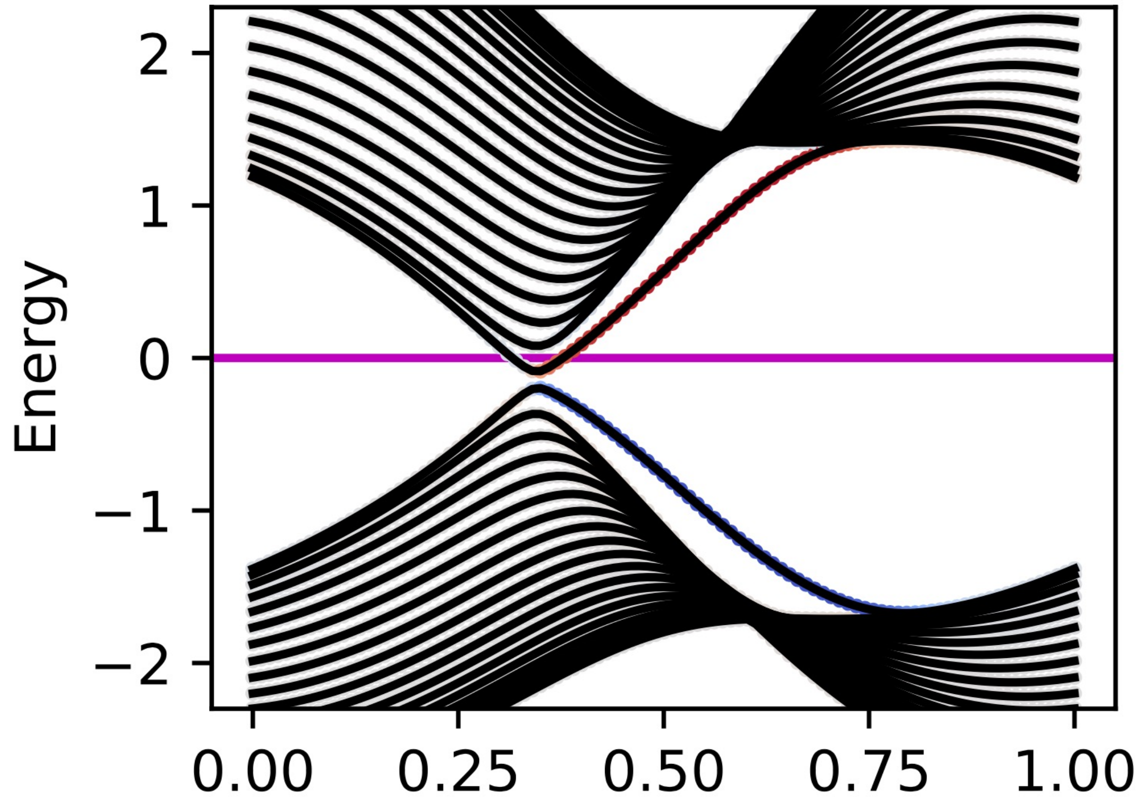


What if the spheres are topological and interacting?

Role of quantum mechanics and Entanglement (Bloch sphere) ?

Relation to physics, many-body physics and condensed matter?

Related Question with M:



From PyThtb Platform adjusting the code

Topological Transition induceed by the Semenoff Mass M: one Dirac point with a π *Berry phase and semimetal* at one Dirac point with a jump of the topological number from C=1 to C=0 (implying C=1/2)

Can we turn this transition point into a “phase” or a line?

Question and Geometry

$\vec{B} = \vec{\nabla} \times \vec{A} = B\vec{e}_z$

Suppose a sphere with a Dirac monopole such that

Solution of this equation on surface with radius r

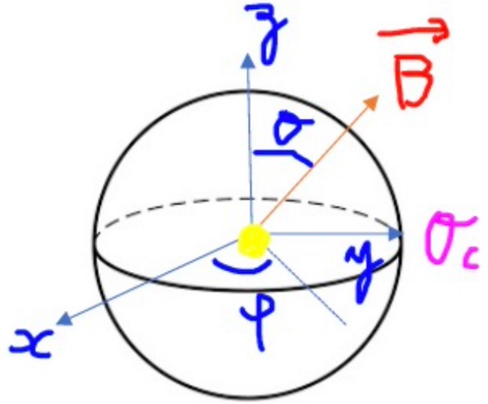
$$A_\theta = 0$$

$$\frac{\partial}{\partial\theta}(A_\varphi \sin\theta) = Br\sin\theta$$

It is useful to introduce $A'_\varphi = A_\varphi \sin\theta$ and integrate this equation on two hemispheres $0 < \theta < \theta_c$ and $\theta_c < \theta < \pi$ (Wu and Yang, 1975)

$$A'_\varphi(\theta < \theta_c) = -Br(\cos\theta - 1) = 2Br\sin^2\frac{\theta}{2}$$

$$A'_\varphi(\theta > \theta_c) = -Br(\cos\theta + 1) = -2Br\cos^2\frac{\theta}{2}$$



Our “simple” Observation

Observation: It is useful then to introduce the field $\tilde{A}_\varphi = -Br\cos\theta$ smooth on the whole surface:

$$A'_\varphi(\theta < \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(0)$$

$$A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\theta) - \tilde{A}_\varphi(\pi)$$

such that

$$\boxed{A'_\varphi(\theta < \theta_c) - A'_\varphi(\theta > \theta_c) = \tilde{A}_\varphi(\pi) - \tilde{A}_\varphi(0)}$$

Integration of Flux on the Surface

$$\Phi = B(r)r^2 \left(\int_0^{2\pi} d\varphi \right) \left(\int_0^{\pi} \sin\theta d\theta \right) = 4\pi r^2 B(r)$$

We can define an effective flat metric such that $F_{\theta\varphi} = B\sin\theta = \frac{1}{r}\partial_\theta A'_\varphi$

$$\Phi = 2\pi r^2 \left(\int_0^{\theta_c^-} F_{\theta\varphi} d\theta + \int_{\theta_c^+}^{\pi} F_{\theta\varphi} d\theta \right)$$

$$\Phi = (2\pi)(A'_\varphi(\theta_c^-) - A'_\varphi(0) + A'_\varphi(\pi) - A'_\varphi(\theta_c^+))$$

$$\boxed{\Phi = (2\pi)(A'_\varphi(\theta_c^-) - A'_\varphi(\theta_c^+)) = (2\pi)(\tilde{A}_\varphi(\pi) - \tilde{A}_\varphi(0))}$$

The topological properties are measurable locally from the poles

Application to Quantum Physics: Bloch Sphere

Vector Potential (momentum) becomes the Berry gauge field

$$A_\varphi = -i\langle\psi|\frac{\partial}{\partial\varphi}|\psi\rangle$$

$$H = -\mathbf{d}\cdot\boldsymbol{\sigma} = -d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)\cdot\boldsymbol{\sigma}$$

Lowest-energy eigenstate $A_\varphi = -\cos\frac{\theta}{2}$ and $A_\theta = 0$ such that

$$A'_\varphi(\theta < \theta_c) = A_\varphi(\theta) - A_\varphi(0) = \sin^2\frac{\theta}{2}$$

$$A'_\varphi(\theta > \theta_c) = A_\varphi(\theta) - A_\varphi(\pi) = -\cos^2\frac{\theta}{2}$$

$C = \frac{\Phi}{2\pi} = 1$

For Berry Curvature

Same Identities as for the classical case with $B = \frac{1}{2}$ if $r = 1$.

Topological first Chern Number

$$C = (A'_\varphi(\theta_c^-) - A'_\varphi(\theta_c^+)) = A_\varphi(\pi) - A_\varphi(0)$$

Geometrical Interpretation:

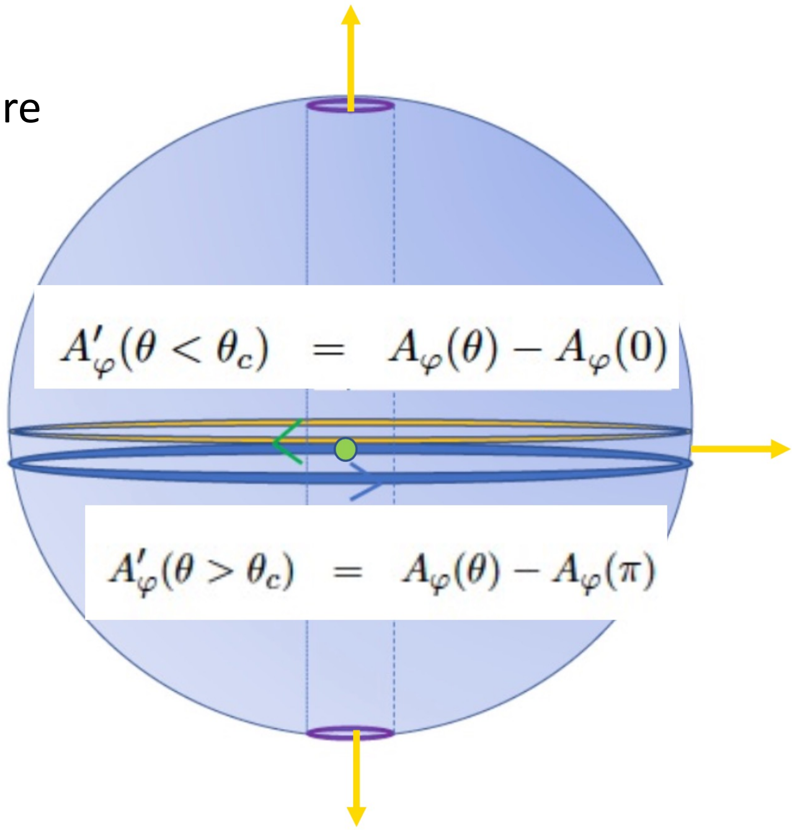
Stokes' theorem with two boundaries per hemisphere

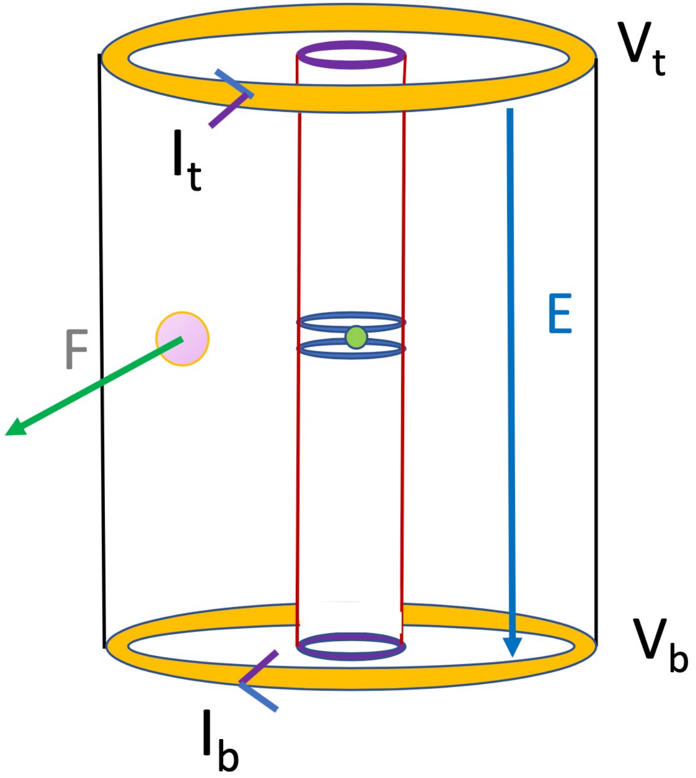
Joel Hutchinson and Karyn Le Hur, Communications Physics **4**, 144 (2021), Nature

Joel Student of Joseph Maciejko at Alberta and NSERC post-doctoral Fellow at CPHT, Ecole Polytechnique

Undergraduate work with Gordon Semenoff at UBC Vancouver and Charlotte Kristjansen Denmark

Riemann, Poincare
Bloch sphere





Sphere can be smoothly deformed as an ellipse or Laughlin cylinder : a thin Dirac handle transporting the charge to the poles

Applications of Quantum TopoMetry

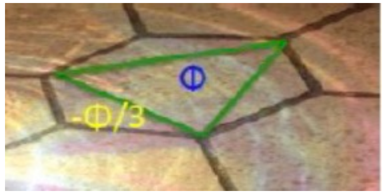
The Berry curvature and topological numbers can be measured when driving from north to south pole
Realization of spheres in mesoscopic cQED systems

D. Schroer et al. PRL 2014 (Boulder, K. Lehnert); P. Roushan et al. Nature (John Martinis, Santa Barbara) 2014
Theory: A. Polkovnikov, V. Gritsev, M. Kolodrubetz

Quantum Dynamo Effect & Energy when rolling the spin coupled to a harmonic oscillator or « quantum bath »
L. Henriet, A. Sclocchi, P. P. Orth, K. Le Hur PRB 2017; E. Bernhardt, C. Elouard, K. Le Hur, 2022

Transport from geometry, Newton mechanics and Quantum Mechanics, Parseval-Plancherel Theorem
Joel Hutchinson and Karyn Le Hur, Communications Physics **4**, 144 (2021)

Introduction of a function $I(\theta)$ *related to quantum distance, measuring* C^2 *from* K, K', M
Measurable in time from circularly polarized light or rotating magnetic field for a spin-1/2
K. Le Hur, Phys. Rev. B **105**, 125106 (2022), arXiv:2106.15665



$$\mathcal{I}(\theta) = \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_x} \right| \psi_+ \right\rangle + \left\langle \psi_+ \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_- \right\rangle \left\langle \psi_- \left| \frac{\partial \mathcal{H}}{\partial p_y} \right| \psi_+ \right\rangle = 2v_F^2 \left(\cos^4 \frac{\theta}{2} + \sin^4 \frac{\theta}{2} \right)$$

$$\frac{\mathcal{I}(\theta)}{2v_F^2} = \left(2\mathcal{A}'_\varphi(\theta < \theta_c)\mathcal{A}'_\varphi(\theta > \theta_c) + C^2\right)$$

Applications of circularly polarized light in cold atoms
L. Asteria, D. T. Tran, T. Ozawa et al. Nature Phys. **15** 449 (2019)
Theory: D.T. Tran, A. Grushin, P. Zoller and N. Goldman, Sciences Advances **3** (2017)
Ph. Klein, A. Grushin, K. Le Hur, Phys. Rev. B **103**, 035114 (2021)

Topological Insulators (TI) & Quantum Spin Hall Effect (QSH)

- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda \vec{L}\cdot\vec{\sigma}$
- Gapless helical edge mode (Kramers pair)

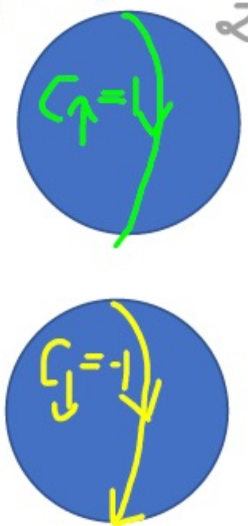
Mercury, Wurzburg
Bismuth, Princeton

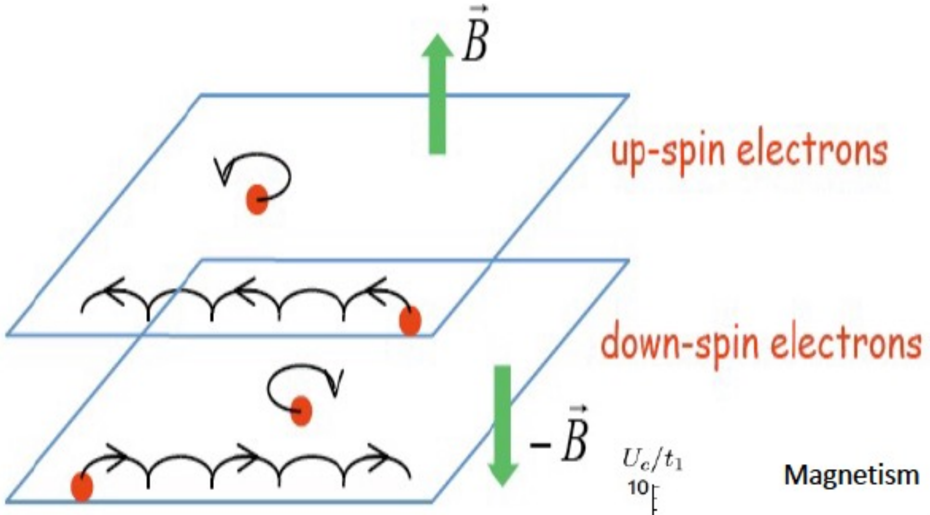
$\lambda L_z \sigma_z$

$L_z \to -L_z,\ \sigma_z \to -\sigma_z$

D. Sheng
& D. Haldane

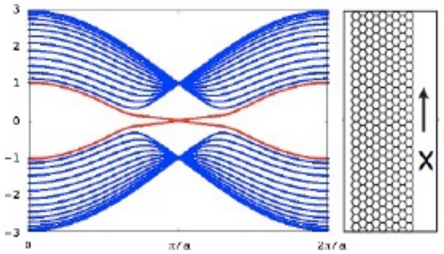
$C_\uparrow - C_\downarrow = 2$ $\mathbb{Z}_2$

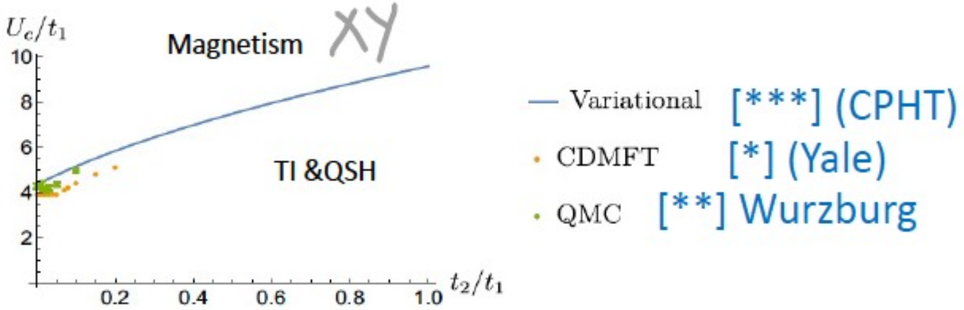




Kane-Mele Model 2005, 2006

B. A. Bernevig & T. Hughes, S.C. Zhang

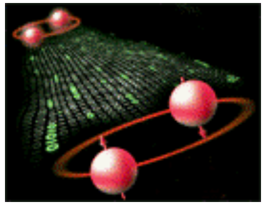




Measurable with light from Dirac points
Relation with zeros of Kane-Mele Pfaffian
K. Le Hur, arXiv:2106.15665

Interaction Effects + Mott : S. Rachel and K. Le Hur (2010) [*]; W. Wu et al. (*, CDMFT, 2012); F. Assaad et al. (**. 2010, QMC)
Analytical Solution of Mott Transition [***] J. Hutchinson, Ph. Klein, K. Le Hur, Phys. Rev. B 104, 075120 (2021) (fluctuations)

Fractional Topological Numbers



J. Hutchinson and K. Le Hur, Communications Physics **4,** 144 (2021)

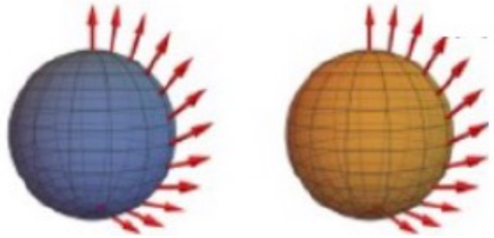
$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2$$

$$\mathbf{H}_i = H(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

Prerequisite from geometry: Ground State $|\uparrow\rangle \otimes |\uparrow\rangle$ at one Pole and Einstein-Podolsky-Rosen State or Bell Pair at the other one Pole $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$

$$C_i = \frac{1}{2}(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle) = \frac{1}{2}$$

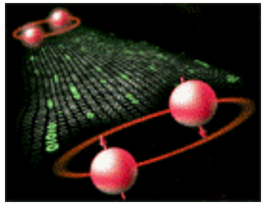
$$C_i = A_\varphi(\pi) - A_\varphi(0) = \frac{1}{2} = \frac{q}{2}$$



Total $C = \sum_i C_i = 1$ in agreement with Triplet State : $S_z = \sigma_{1z} + \sigma_{2z} = 1$ at one pole and $S_z = 0$ at the other one pole $C = \frac{1}{2}(\langle S_z(0)\rangle - \langle S_z(\pi)\rangle) = 1.$

Fractional-1/2 Topological State

J. Hutchinson and K. Le Hur, Communications Physics **4,** 144 (2021)



Coherent Superposition of two halved-regions:

one encircling the topological charge and one entangled region: “Half Surface Radiating”

From Stokes’ theorem, the C=1/2 per sphere is equivalent to a circle (χ=0) on top of a disk (χ=1)

The disk acts as a mirror: Euler characteristic of each sphere, χ = (2-2g) = (2-2C) = 0+1 =1 with C=1/2

½ thin handle relating 1 pole to the topological charge

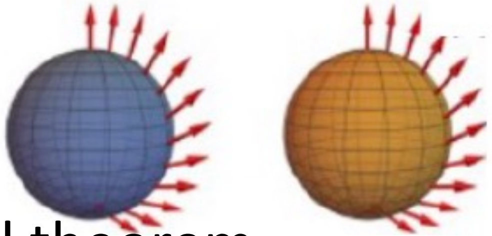
χ =1 has been reported in black holes with a boundary (Gibbons and Kallosh, 1995)

Quantized π Berry phase at one pole per sphere

$$-\oint d\varphi A'_{j\varphi}(0^+) = 2\pi C_j$$

0

$$C_j = \frac{q}{2} = \frac{1}{2}(\langle\sigma_{jz}(0)\rangle - \langle\sigma_{jz}(\pi)\rangle)$$



From Newton mechanics, semi-classical analysis, Parseval-Plancherel theorem

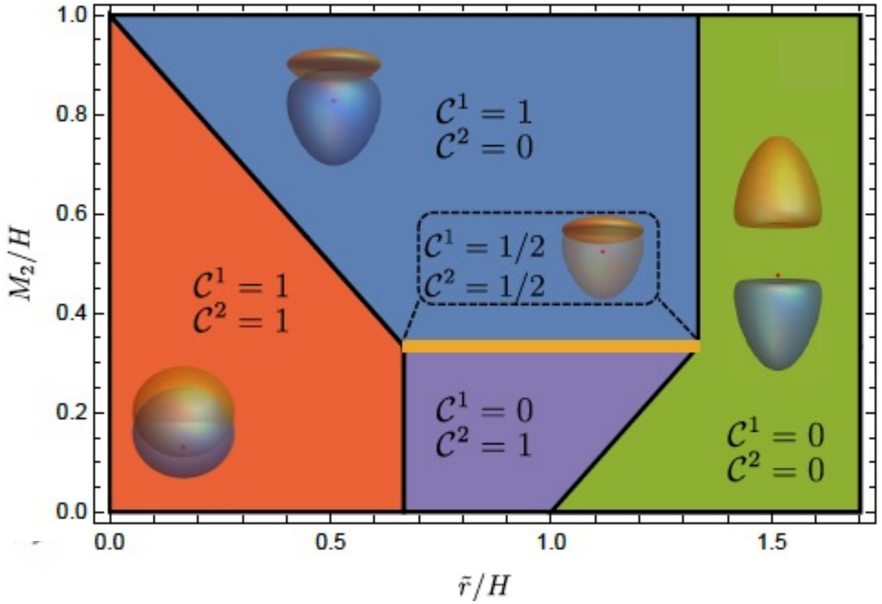
C=1/2 enters in the pumped charge and quantum Hall conductivity on the cylinder geometry

½ : Interpretation in terms of projection on one of the two entangled particles for the measure

Interacting Bloch Spheres' Model

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$



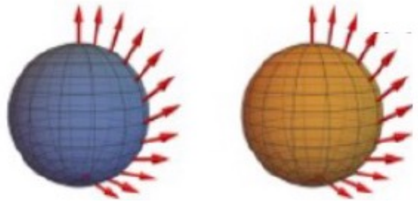
- Phase Diagram obtained from energetics at the poles
- Region C_j=1/2 occurs for various f(θ) $and\ f(\theta) = cst$

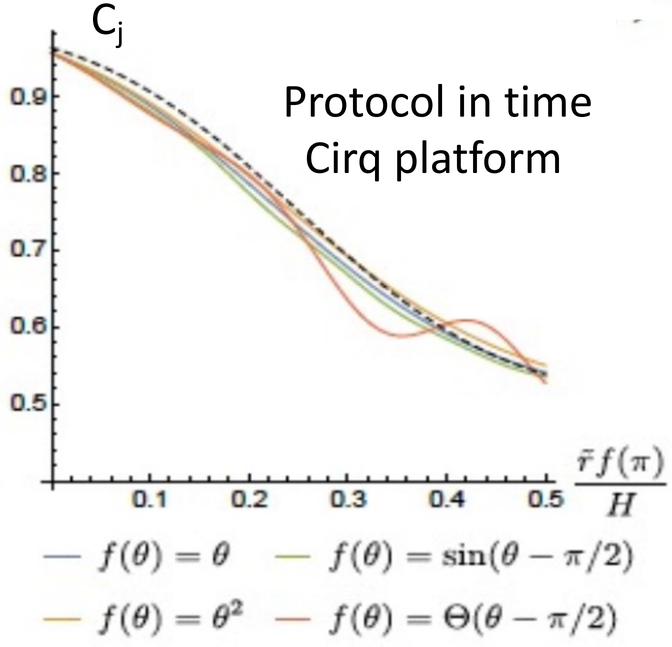
Z$_2$ symmetry

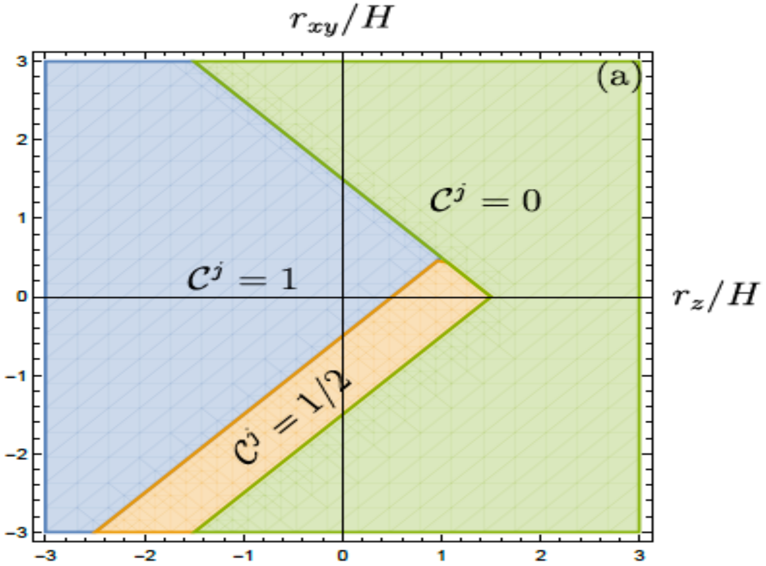
1 <-> 2 spheres

M_1=M_2=M

$$\boxed{H - M < \tilde{r} < H + M,}$$







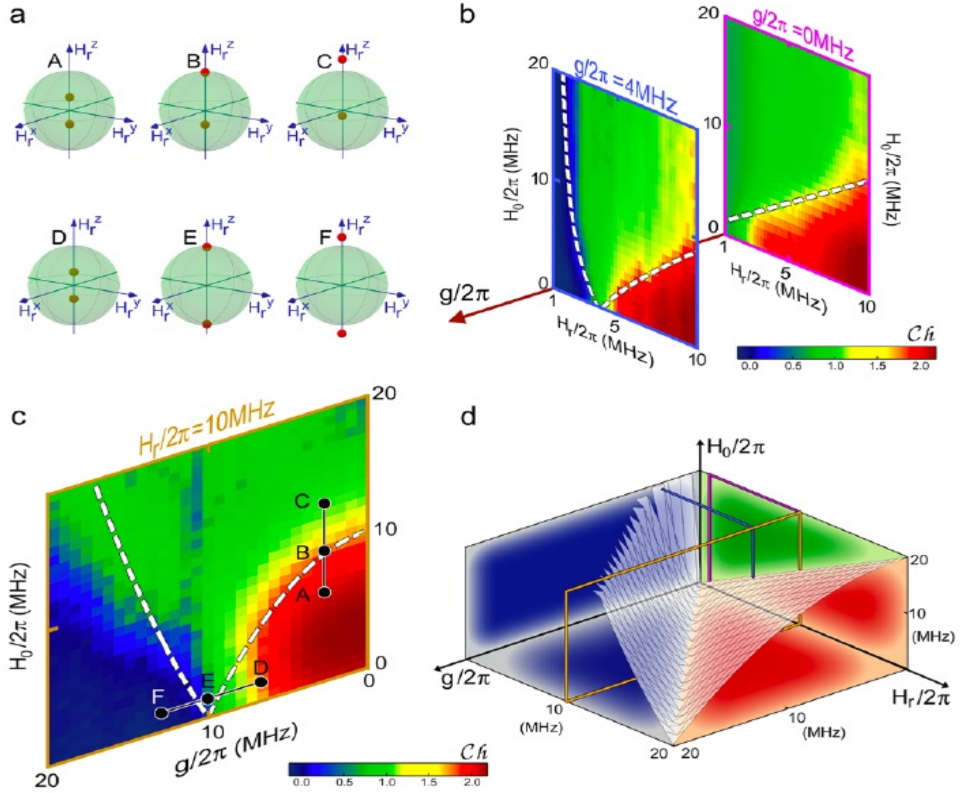
2 spins

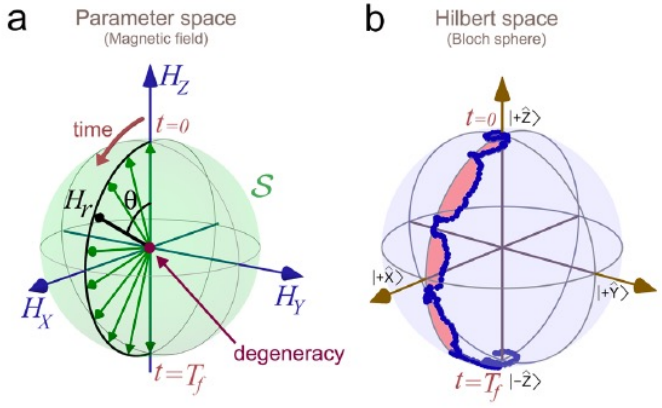
The Hamiltonian of this system is given by

$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)], \tag{5}$$

where 1 and 2 refer to qubit 1 (Q1) and qubit 2 (Q2)

Santa-Barbara “google”:
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)





Application in Energy:

Quantum Dynamo effect in a Bath

L. Henriet, A. Sclocchi, P. P. Orth, K. Le Hur 2017 and quantum phase transitions in curved space

New Developments: Ephraim Bernhardt (CPHT), Cyril Elouard (INRIA & ENS Lyon), Karyn Le Hur arXiv:2208:01707

Applications in many-body physics & condensed-matter

In Progress

- Nodal Ring Topological SemiMetal in Bilayer Graphene

  J. Hutchinson and K. Le Hur, Communications Physics **4,** 144 (2021)

  P. Cheng, P. W. Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg, K. Le Hur,
  Phys. Rev. B **100**, 081107 (2019), “AA-BB” Stacking realizable in ultra-cold atoms

- Application in one layer Honeycomb Graphene, Karyn Le Hur & Sariah Al Saati, in Preparation

- Application in interacting topological superconducting Kitaev wires & Ladders
  Relation between C=1/2 and central charge of CFT models, Matrix Product States

  Frederick del Pozo, Loic Herviou, Karyn Le Hur, in Preparation
  DCI phase : Loic Herviou, Christophe Mora, Karyn Le Hur, Phys. Rev. B **93**, 165142 (2016)

- Application in 3D related to magneto-electric effect on surface of 3D Topological Insulators

  Assemblage of Planks in Cubes’ Geometries and Ramanujan Series

- Smooth Fields and Application to Kagome Lattice, PhD Thesis of Julian Legendre (September 2022)

Proximity Effects with Graphene

$C_1 + C_2 = 0$ $\quad$ $C_1 - C_2 = 2$

$\mathbb{Z}_2$ state with asymmetric masses

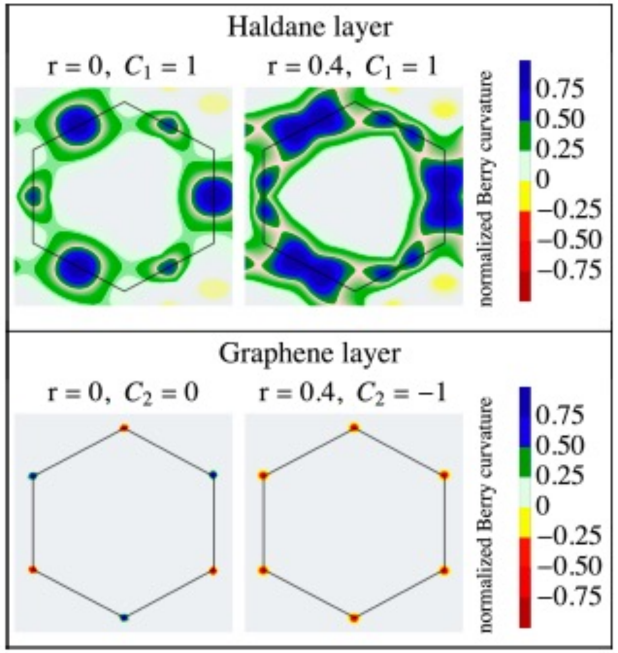
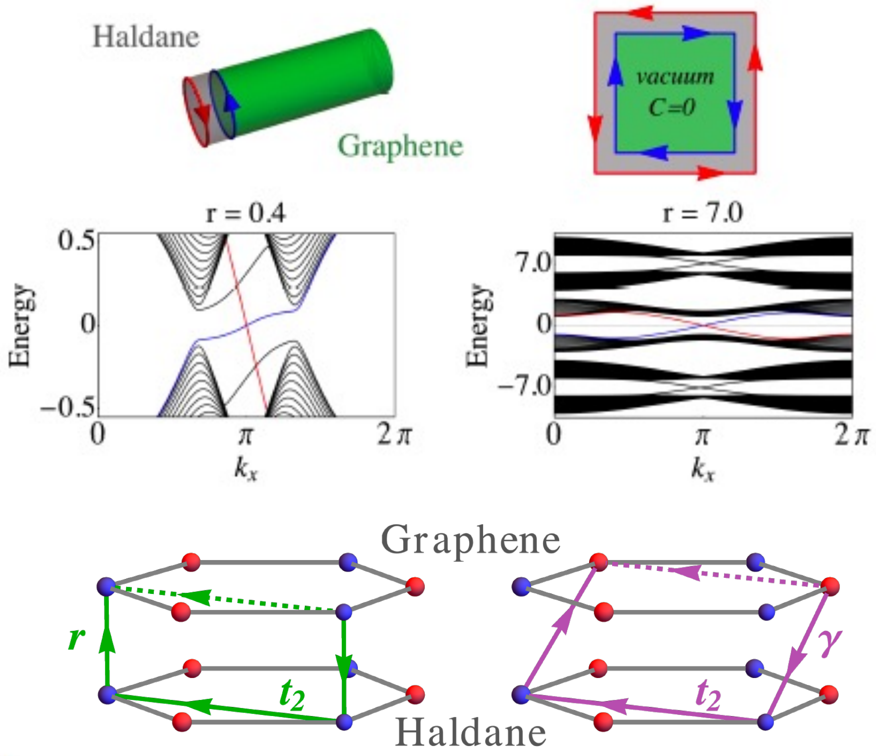


FIG. 1. Berry curvature in the Brillouin zone for the Haldane and graphene layers at $r = 0$ and small r, showing the Berry phase jump effect [35]. Here, $t_1 = 1$ and $t_2 = 1/3$.

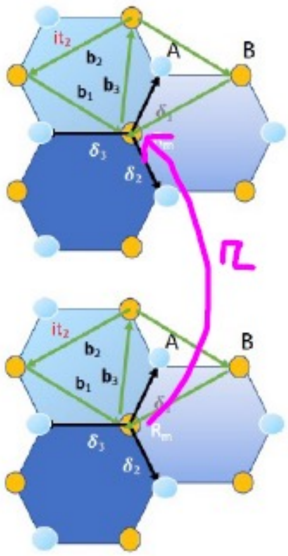


Ising Interaction $\tilde{r}\,\sigma_1^z \sigma_2^z$

interaction in k-space or hopping

Peng Cheng, Philipp Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg and Karyn Le Hur, Phys. Rev. B 100, 08110 (R) (2019). DFG Collaboration with Munich and Hamburg.

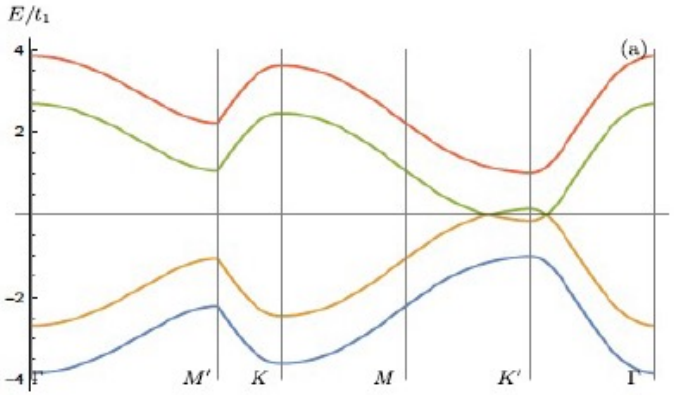
Bilayer system with M_1=M_2



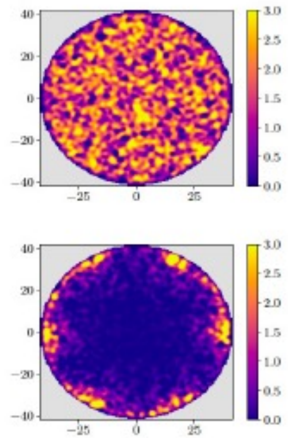
$$\mathcal{H} = (\psi^\dagger_{\boldsymbol{k}1}, \psi^\dagger_{\boldsymbol{k}2})\mathcal{H}(\boldsymbol{k}) \begin{pmatrix} \psi_{\boldsymbol{k}1} \\ \psi_{\boldsymbol{k}2} \end{pmatrix},$$

where $\psi^\dagger_{\boldsymbol{k}i} \equiv (c^\dagger_{\boldsymbol{k}Ai}, c^\dagger_{\boldsymbol{k}Bi})$ and

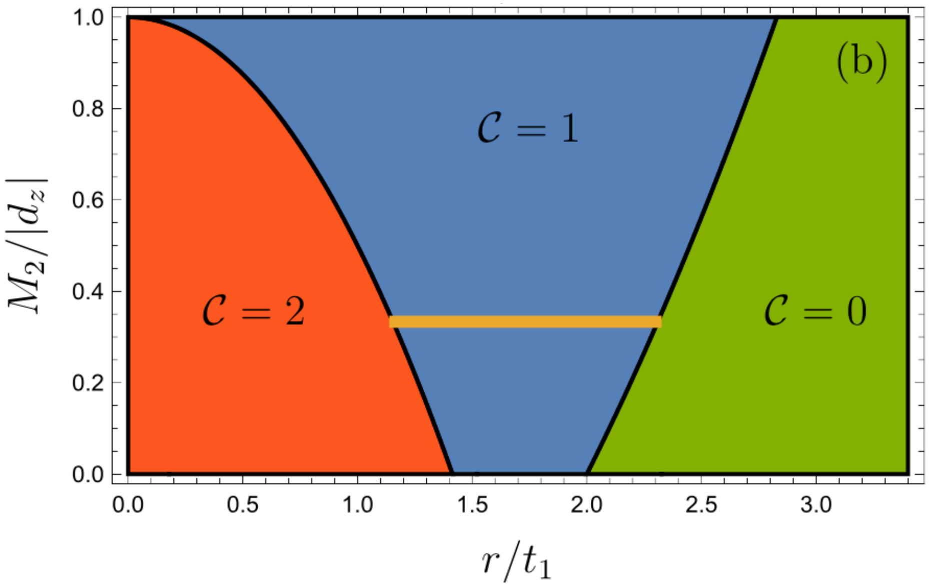
$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} (\boldsymbol{d} + M_1\hat{z}) \cdot \boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\boldsymbol{d} + M_2\hat{z}) \cdot \boldsymbol{\sigma} \end{pmatrix},$$

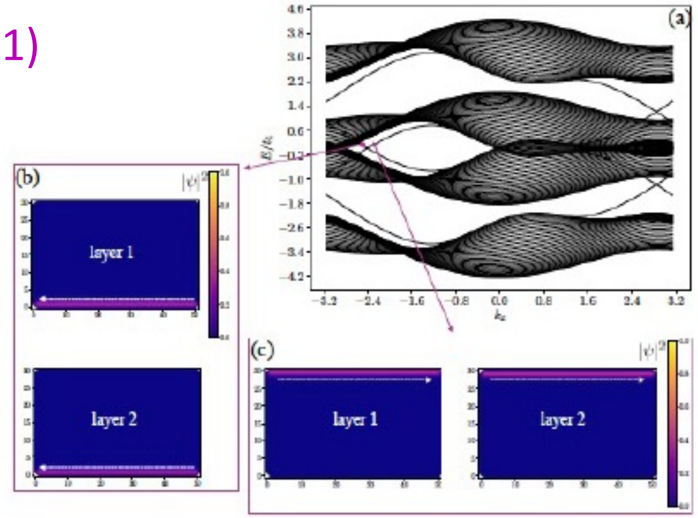


J. Hutchinson and K. Le Hur, Physics 4 144 (2021)



Local density of states





Circular dichroism of light
Jones formalism: average 1 and 0 light responses

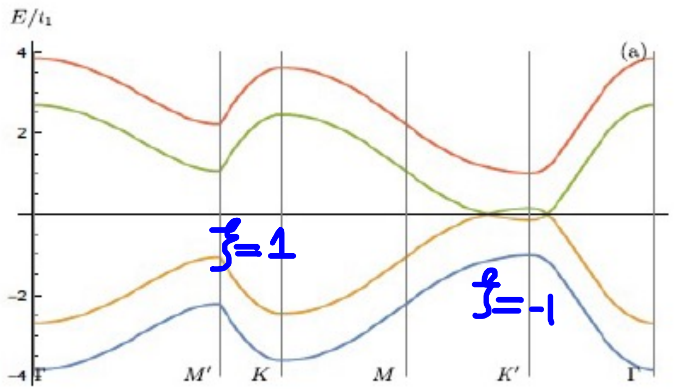
Topological semimetal in two dimensions

Summary of Geometry

$$(J_{xy})^j = C^j \frac{e^2}{h}$$

$C^j = \frac{1}{2}$

This formula is correct and is applicable in a sphere (plane j) from the poles (Dirac points)



$$H = (\zeta d_z + M)\sigma_z \otimes \mathbb{I} + d_1\sigma_x \otimes \mathbb{I} + d_{12}\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes \tau_x$$

$$H^2 = \left(|\mathbf{d}|^2 + r^2\right)\mathbb{I}\otimes\mathbb{I} + 2r\mathbf{d}\cdot\boldsymbol{\sigma}\otimes\tau_x \qquad \mathbf{d} = (d_1, d_{12}, (\zeta d_z + M))$$

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

Fractional
Topological Bloch band

$$c^\dagger_{B1}c^\dagger_{B2}|0\rangle = |\uparrow\uparrow\rangle,\ c^\dagger_{A1}c^\dagger_{A2}|0\rangle = |\downarrow\downarrow\rangle,\ c^\dagger_{B1}c^\dagger_{A2}|0\rangle = |\uparrow\downarrow\rangle,\ c^\dagger_{A1}c^\dagger_{B2}|0\rangle = |\downarrow\uparrow\rangle.$$

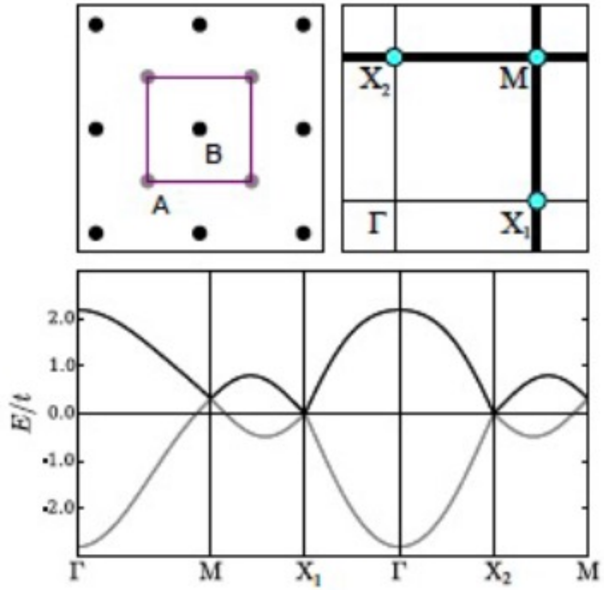
$$\tilde{C}^j = \frac{1}{2}\langle \underbrace{n^j_{KB} - n^j_{KA}}_{\sigma_z} - n^j_{K'B} + n^j_{K'A}\rangle = \frac{1}{2}$$

Table & topological semimetals

Dirac Semimetals in Two Dimensions

S. M. Young & C. L. Kane, PRL 2005

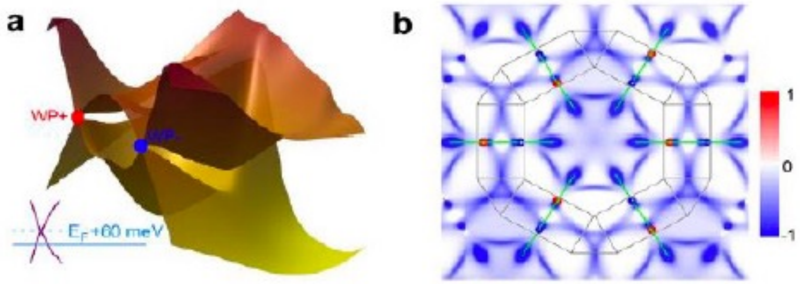
$$H = 2t\tau_x \cos\frac{k_x}{2}\cos\frac{k_y}{2} + t_2(\cos k_x + \cos k_y) + t^{\mathrm{SO}}\tau_z[\sigma_y \sin k_x - \sigma_x \sin k_y],$$

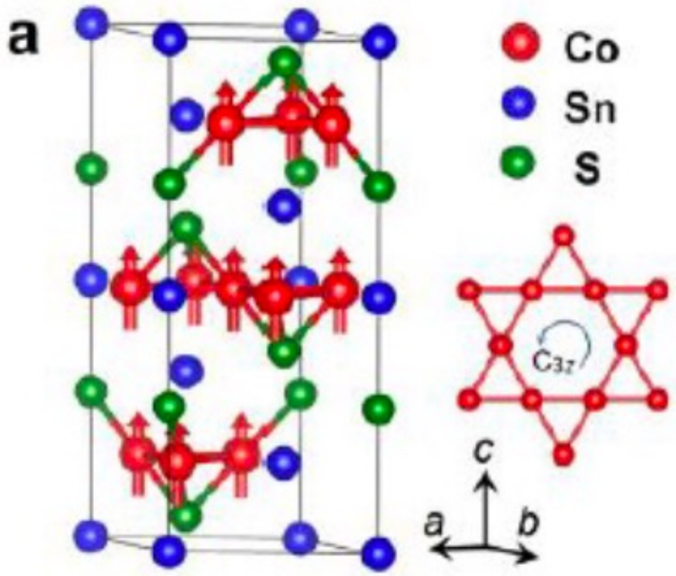


ferro

Magnetic Weyl semimetals in 3D: recent

Z. Guguchia et al., group of Zahid Hasan, Princeton
E. Liu et al., group of Claudia Felser Dresden
Figures from Enke Liu, Berry curvature k_x-k_y plane



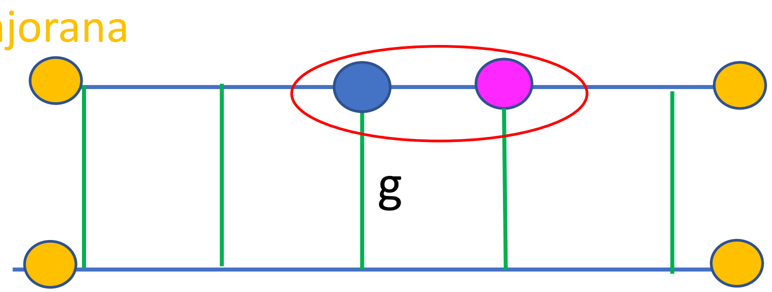


Anomalous Hall conductivity

2D model

Julian Legendre & Karyn Le Hur
Phys. Rev. Research, 2020

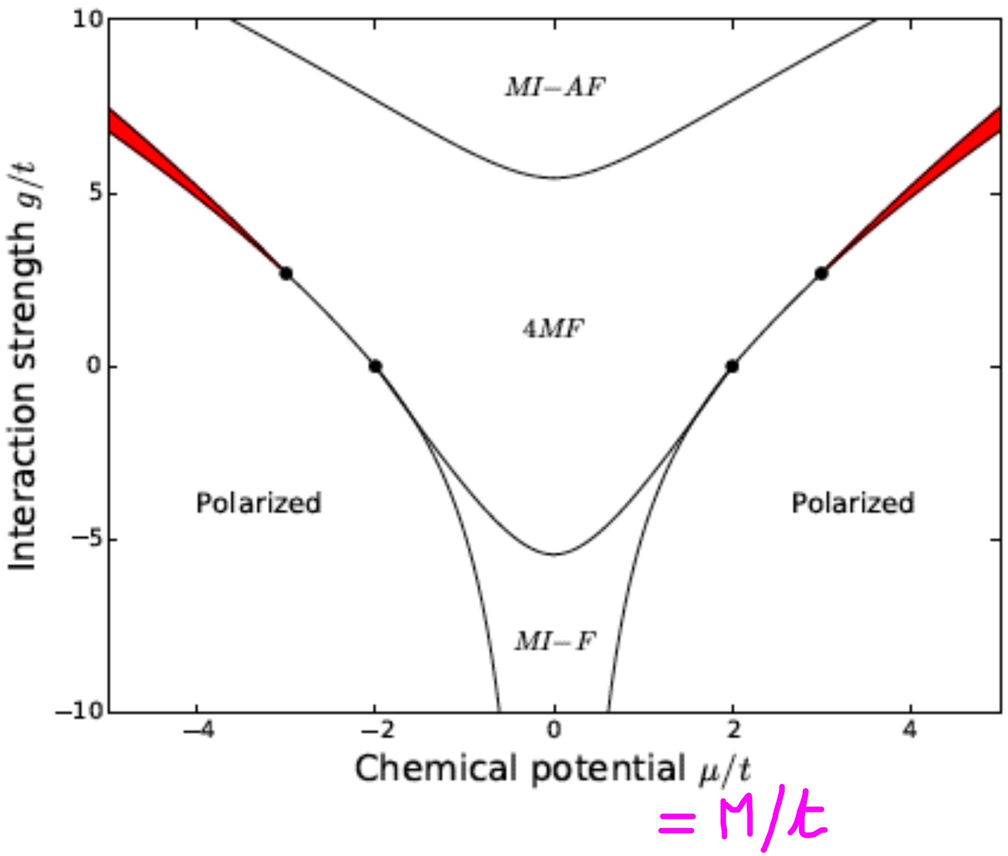
Superconductivity, Topological Aspects and Coulomb Interaction



2 interacting p-wave Kitaev Superconducting wires

Loic Herviou, Christophe Mora, Karyn Le Hur 2017

New Efforts with Frederick del Pozo, CPHT: relation two c and C=1/2



Kitaev wire is a BdG Hamiltonian $(c_k, c^\dagger_{-k})^T$

$$H(k) = \frac{1}{2}\begin{pmatrix} \epsilon(k) & 2i\Delta e^{-i\tilde{\varphi}}\sin(ka) \\ -2i\Delta e^{i\tilde{\varphi}}\sin(ka) & -\epsilon(k) \end{pmatrix}$$

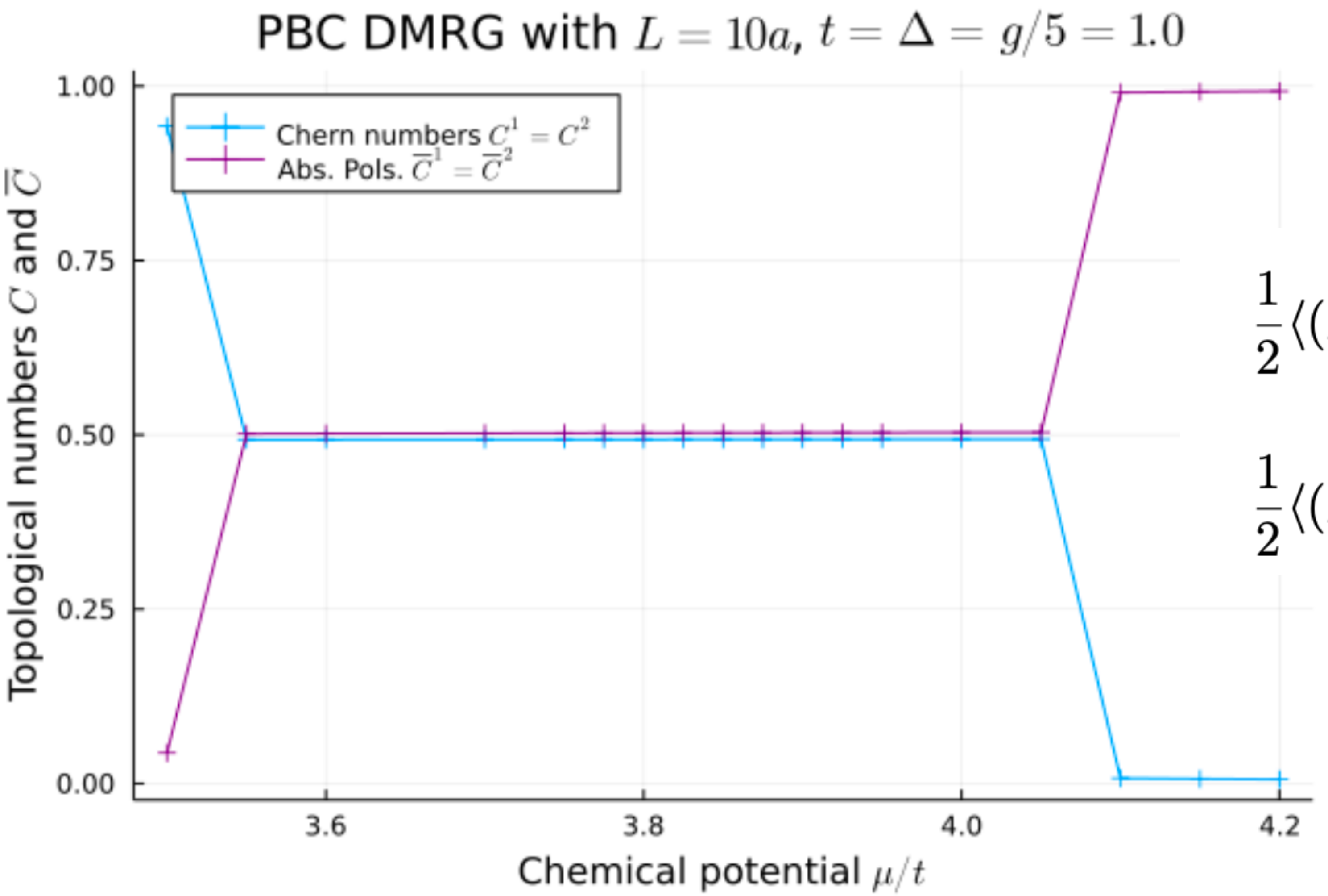
$$H_{\text{int}} = g\sum_j (n_{j,1} - \frac{1}{2})(n_{j,2} - \frac{1}{2}),$$

Quantum field theory tools (Luttinger approach, RG equations)
Density Matrix Renormalization Group Approach
Quantum Information Tools (Entanglement Entropy, Bipartite Fluctuations)

Results from QFT, Geometry and DMRG

DCI phase means Double Critical Ising (2 c=1/2 models or 2 gapless bulk Majorana modes)

Frederick del Pozo, Loic Herviou, Karyn Le Hur, in Preparation



$$\frac{1}{2}\langle(S^z_{k=0} - S^z_{k=\pi})\rangle \equiv C$$

$$\frac{1}{2}\langle(S^z_{k=0} + S^z_{k=\pi})\rangle \equiv \overline{C}$$

Measures in progress

Relation to Fractional Quantum Hall Phases?

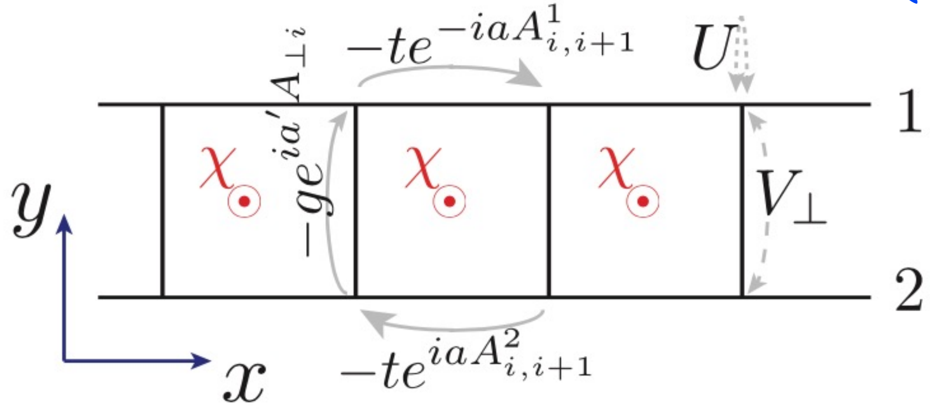
Teo and Kane Wires' Networks and Laughlin phases

Ladder of Interacting bosons with a Josephson term shows a FQHE with filling factor $\nu = 1/2$

$$\boxed{\nu = \frac{n_0/2}{\chi/(2\pi)},}$$

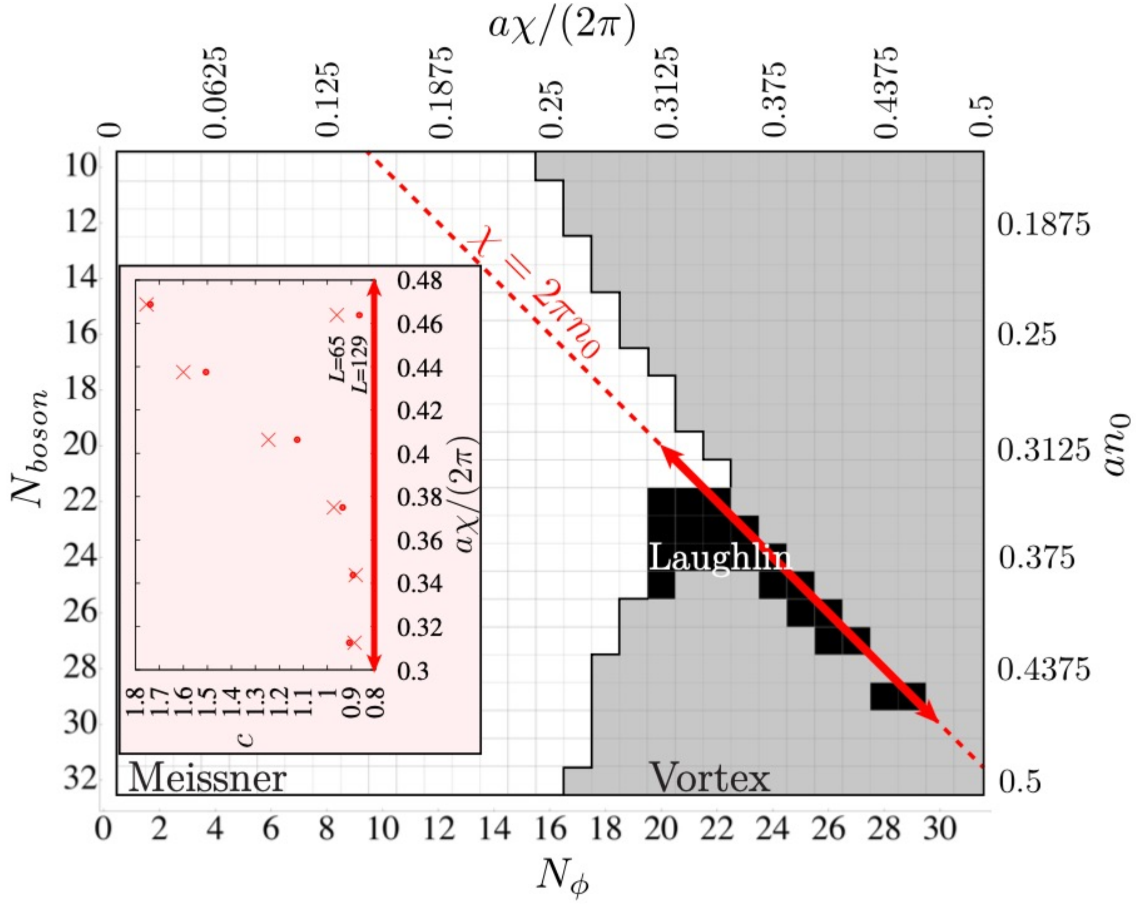
A. Petrescu, M. Piraud, G. Roux, I. P. McCulloch and K. Le Hur, Phys. Rev. B 96, 014524 (2017)

QFT & DMRG



a)

Ground state	Meissner	Vortex	Laughlin
c	1	2	1
N_V	1	≥ 1	> 1



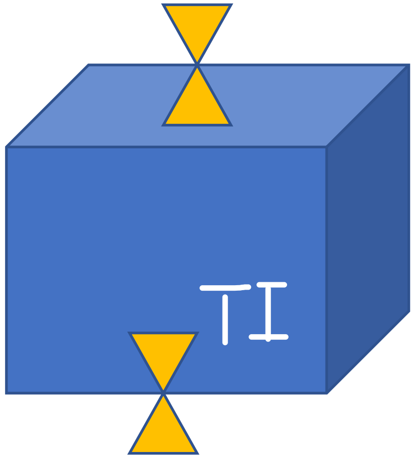
C=1/2

Relation to Magnetoelectric effect in 3D TIs

Experiment: R. Toshimi et al. Nature Communications 2015, $(Bi_{1-x}Sb_x)2Te_3$

Axion electrodynamics in 3D topological materials, recent review, A. Sekine and K. Nomura, arXiv:2011.13601

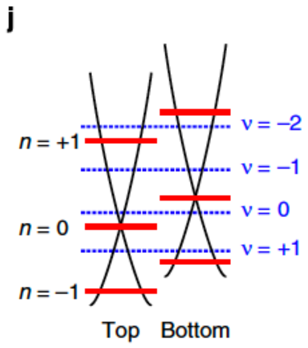
X. Qi, T. Hughes, S.-C. Zhang Phys. Rev. B **78**, 195424 (2008)



$$\sigma_{xy} = \mathrm{sgn}(m_z)\frac{e^2}{2h}$$

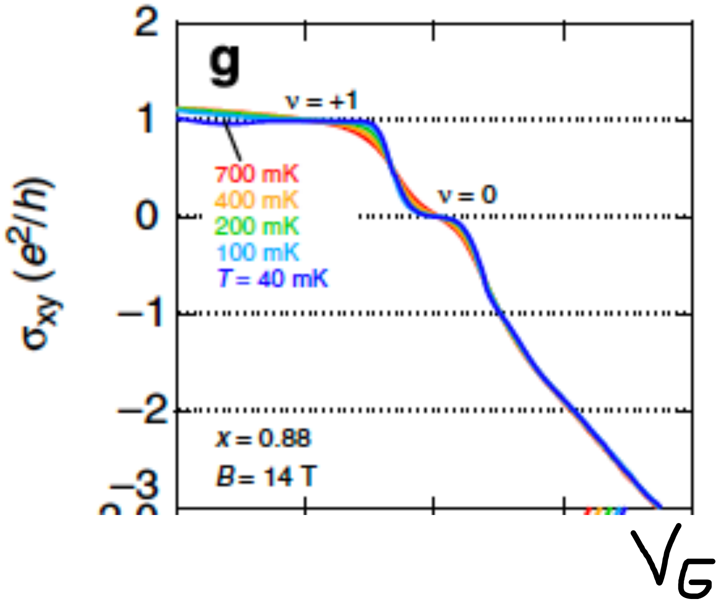
electron density

$$n_e = \frac{\sigma_{xy} B}{e}$$

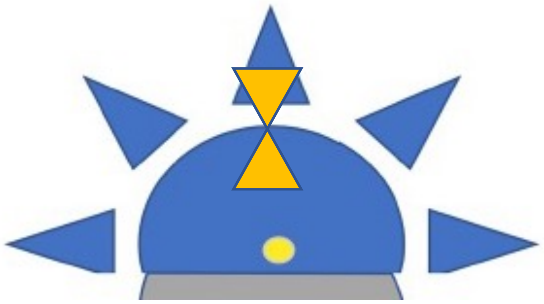


$$\rho_{Top} = \frac{1}{2}\frac{Be^2}{h}$$

$$\rho_{Bott} = -\frac{1}{2}\frac{Be^2}{h}$$



half hemisphere = 1 Dirac cone



$$C = \frac{1}{2}$$

Rational Numbers also occur for spin arrays

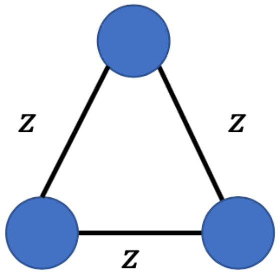
J. Hutchinson and K. Le Hur, Communications Physics **4,** 144 (2021)

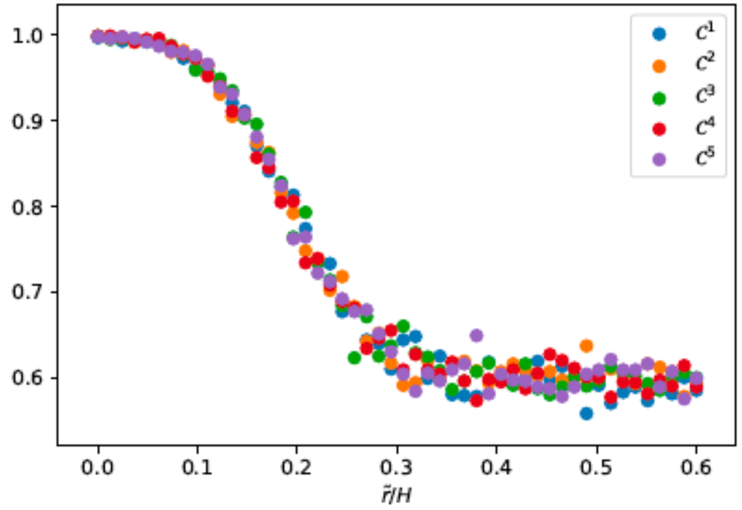
Resonating Valence Bond States

$$C^j = \frac{N+1}{2N},$$

$N = 5$
odd

N even
$C^j = \frac{1}{2}$





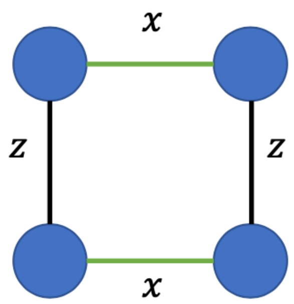


Figure 12: Partial Chern numbers as a function of the coupling $\tilde{r}$ measured in a five-spins quantum circuit simulation with nearest-neighbour Ising interactions and periodic boundary conditions. To time-evolve the spins (qubits), we use a Trotter decomposition with 800 time steps and sweep velocity $v = 0.03H$. The bias field for all qubits is fixed to $M = 0.6H$. For $\tilde{r} \geq H - M \sim 0.4$ we verify the presence of the topological phase with $\mathcal{C}^j = 3/5 = 0.6$ in agreement with Eq. (114).

4 spins: Relation to Z_2 Kitaev spin liquids in a box
K. Le Hur, A. Soret & F. Yang (2017)

Conclusion

Geometry of the sphere is also useful to understand topology of spin-1/2 models

- Application in quantum Transport

- Response to Circularly Polarized Light quantized

- Stochastic Approach to include Interaction Effects

Fractional Topology from the curved space, interactions between spheres
Applications: mesoscopic & atomic systems, topological semimetals, topological matter…

Thanks to the group members and collaborators, CNRS, new developments soon …
Thanks to ANR BOCA, DFG FOR2414, NSERC, Ecole Polytechnique for Funding Support
Thanks to M. Pepper and S. Bose at UCL for Collaborative Grant

Thank You for your Attention, Questions