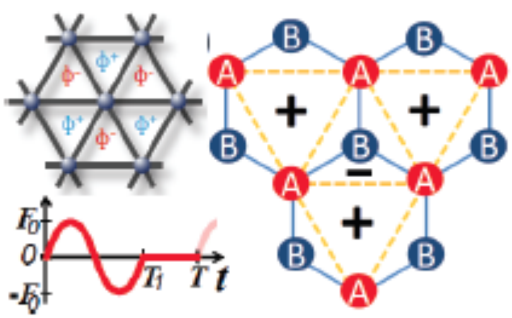
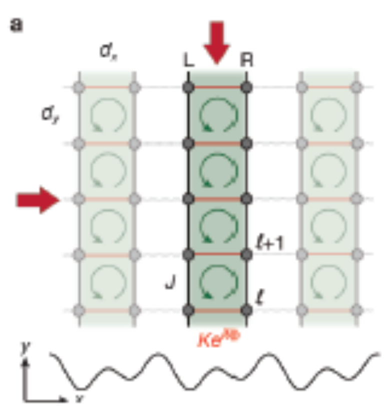
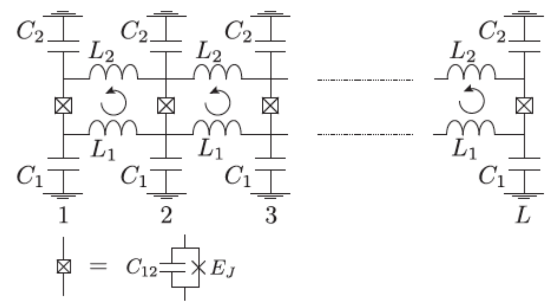
Spin Meissner Effect, Artificial Gauge Fields, Mott Phases

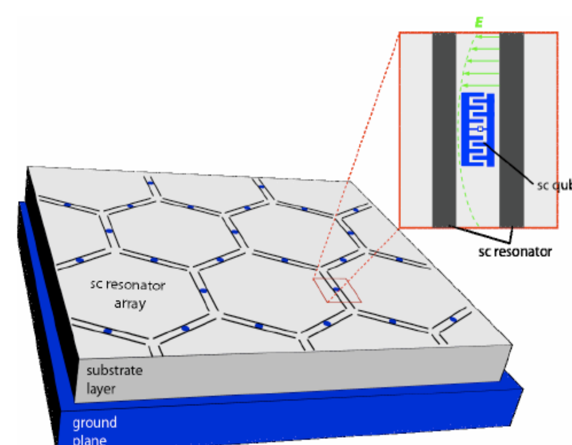
Karyn LE HUR, CPHT Ecole Polytechnique & CNRS

Nordita, August 2014









Topological "quantum numbers"

Tight-Binding models with **Bands** with non-zero Chern number

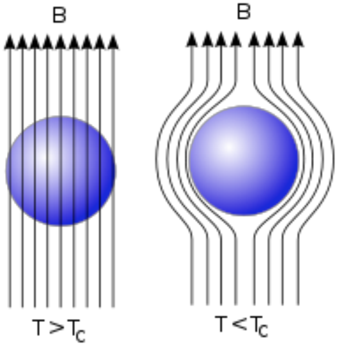
$$C^{(n)} = \frac{1}{2\pi}\int_{\mathrm{BZ}} d^2\mathbf{k}\left(\partial_{\mathbf{k}} \times \mathscr{R}^{(n)}(\mathbf{k})\right),$$

where the vector field $\mathscr{R}^{(n)}(\mathbf{k})$ is the Berry gauge potential associated to the n^{th} Bloch band,

$$\mathscr{R}^{(n)}(\mathbf{k}) = -i\langle n\mathbf{k}|\partial_{\mathbf{k}}|n\mathbf{k}\rangle.$$

Superconductors or bosonic Superfluids: Meissner Currents

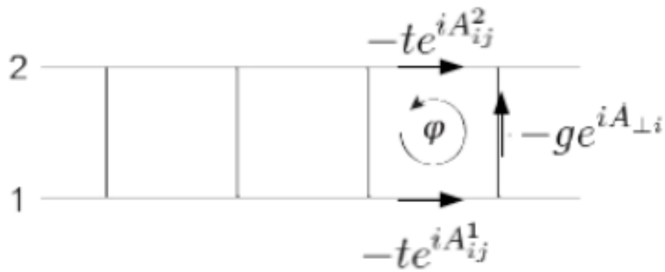
$$\oint \mathbf{A}\cdot d\mathbf{r} = -\frac{\hbar}{2e}\oint \mathrm{grad}S\cdot d\mathbf{r} = n\frac{h}{2e}$$



D. Thouless,
Review from
his web page, 1997
Korea's school

Question: current in Mott phase?

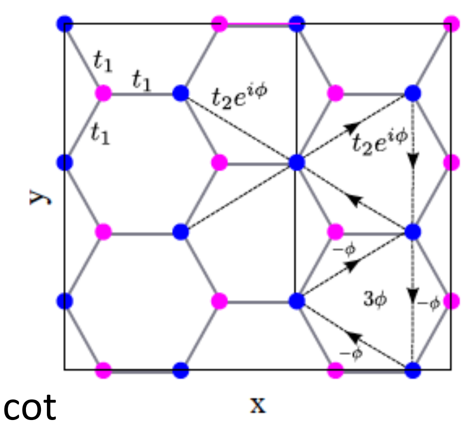
Example 1: Spin Meissner Effect



Work done with A. Petrescu (Yale and Ecole Polytechnique); PRL 2013

Example 2: Plaquette Mott Insulator

bosonic graphene model

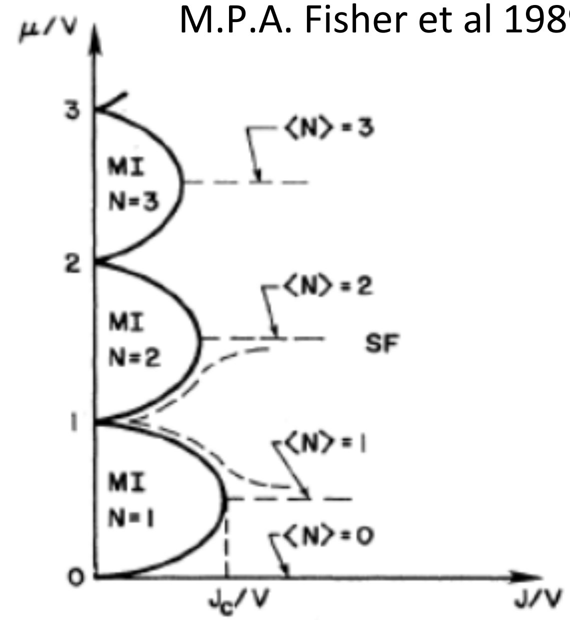


I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411

Doped Honeycomb Mott t-J model: d+id SC chiral state
A. M. Black-Schaffer, Wei Wu, KLH arXiv:1407.2914
W. Wu, M. M. Scherer, C. Honerkamp, KLH PRB (2013)
P-wave state: work in progress with Tianhan Liu, C. Repellin, N. Regnault, B. Doucot
See also T Hyart, A. Wright, G. Khaliullin, B. Rosenow, 2012

Mott Physics in Boson Systems: Lattice Effects

M.P.A. Fisher et al 1989



Bose-Hubbard model of a single lattice boson:

$$H = -t\sum_{<ij>} b_i^\dagger b_j + \sum_i \frac{U}{2} n_i(n_i - 1) - \mu n_i$$

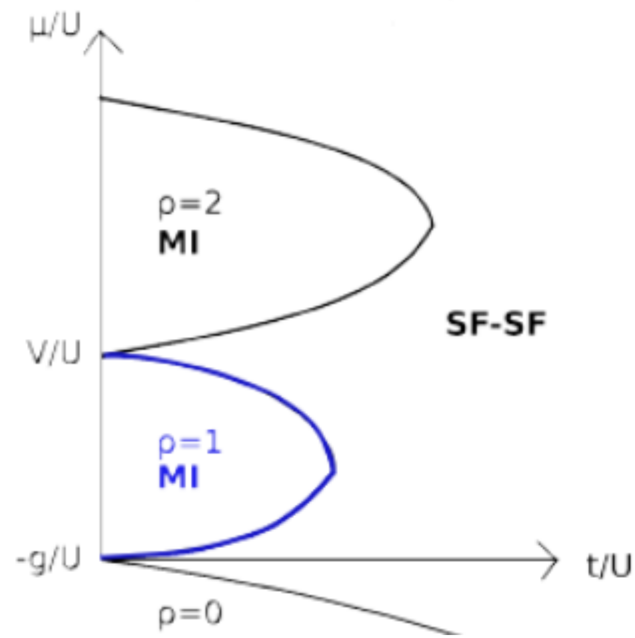
Two-species Bose-Hubbard model:

$$H = -t\sum_{\alpha=1,2}\sum_{<ij>} b_{\alpha i}^\dagger b_{\alpha j} + \sum_{\alpha i} \frac{U}{2} n_{\alpha i}(n_{\alpha i} - 1) - \mu n_{\alpha i}$$

$$+\sum_i V_\perp n_{1i} n_{2i} - g\sum_i b_{1i}^\dagger b_{2i} + H.c.$$

Mott at ρ=1

Interchain coherence: Meissner effect



Multicomponent systems: active field in cold atoms

e.g. E. Altman, W. Hofstetter, E. Demler, M. Lukin 2003

Superfluid Phase, Check

$$j_{\parallel} = it(-e^{iaA^1_{ij}} b^\dagger_{1i} b_{1j} + e^{iaA^2_{ij}} b^\dagger_{2i} b_{2j}) + \text{H.c.},$$
$$j_{\perp} = -2igb^\dagger_{1i} b_{2i} e^{ia' A_{\perp i}} + \text{H.c.}$$

Outside the Mott lobe, the phase-angle representation is justified $b^\dagger_{1,2i} = \sqrt{n} e^{i\theta_{1,2i}}$ (in this reasoning, $n = \rho/2$ represents the mean (superfluid) density in each species).

The conversion takes the form of a Josephson coupling

$$-g\cos(a' A_{\perp i} + \theta_{1i} - \theta_{2i}).$$

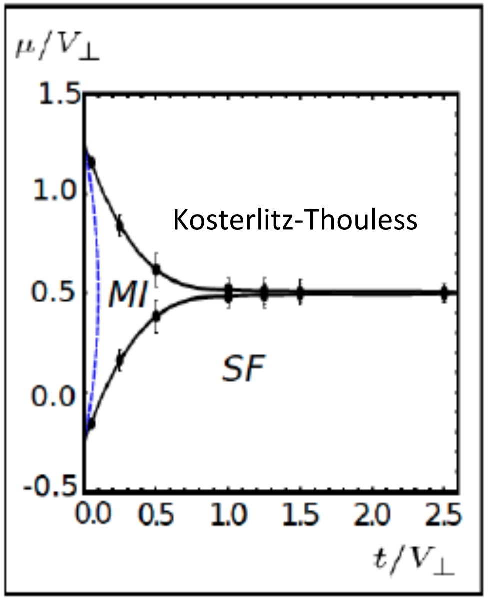
For strong g, the superfluid phases will be pinned by this term such that $a' A_{\perp i} + \theta_{1i} - \theta_{2i} = 0$. Then $j_\perp$ vanishes and furthermore in the small field limit we may expand to obtain the Meissner form of the intraspecies current

$$\langle j_{\parallel} \rangle = -2tn \, \text{phase}_{ij}.$$

Route for Chiral Mott Insulator: Spin Meissner Effect

Mott insulating phase of total density:

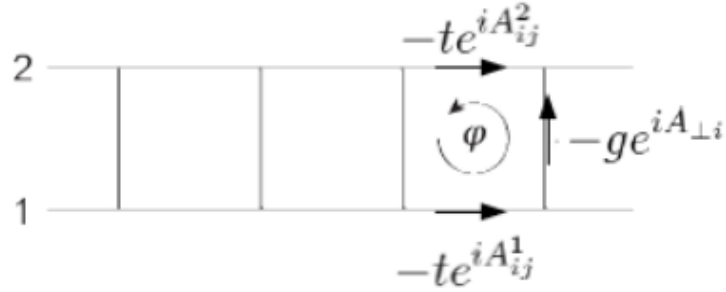
$$\rho = b_1^\dagger b_1 + b_2^\dagger b_2$$



Relative density exhibits fluctuations.

$$\sigma^z = b_1^\dagger b_1 - b_2^\dagger b_2$$

(At ρ=1, spin ½ exchange Hamiltonian)



Example: Ladder System

Mott Regime: Pseudo-spin H

$$H_\sigma = -\sum_{\langle ij\rangle}\left(2J_{xx}(\sigma_i^+\sigma_j^- e^{iaA_{ij}^\sigma} + \text{H.c.}) - J_z\sigma_z^i\sigma_z^j\right)$$

$$-g\sum_i(\sigma_i^x\cos(a'A_{\perp i}) - \sigma_i^y\sin(a'A_{\perp i})),$$

$$\text{with } J_{xx} = \frac{t^2}{V_\perp} \text{ and } J_z = t^2\left(-\frac{2}{U} + \frac{1}{V_\perp}\right)$$

$$j_\parallel = 2J_{xx}\big[\cos(A_{ij}^\sigma)(\sigma_i^y\sigma_j^x - \sigma_i^x\sigma_j^y) + \sin(A_{ij}^\sigma)(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)\big],$$

$$j_\perp = -2g\left[\cos(a'A_{\perp i})\sigma_i^y + \sin(a'A_{\perp i})\sigma_i^x\right].$$

$$\langle j_\parallel\rangle = -2J_{xx}\,\text{phase}_{ij}$$

Meissner currents survive

Spin Meissner Effect

at other filling
A. Tokuno & A. Georges, 2014

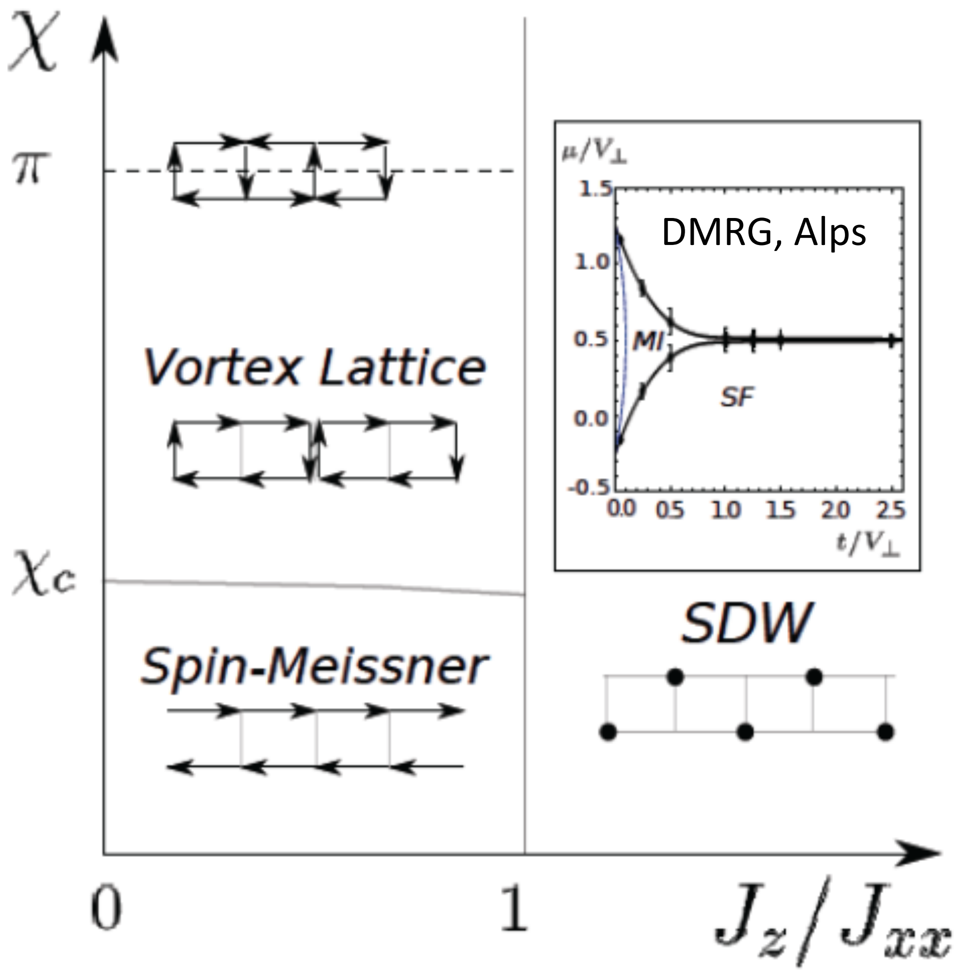
Similar Hamiltonian
different contexts
I. Garate and I. Affleck

Chiral Mott insulator Arya Dhar et al. PRA A 85, 041602 (2012)

Analogy to DDW phase in high-Tc systems

Possible transitions to Laughlin states

Teo – Kane, PRB 2014



PRL 2013

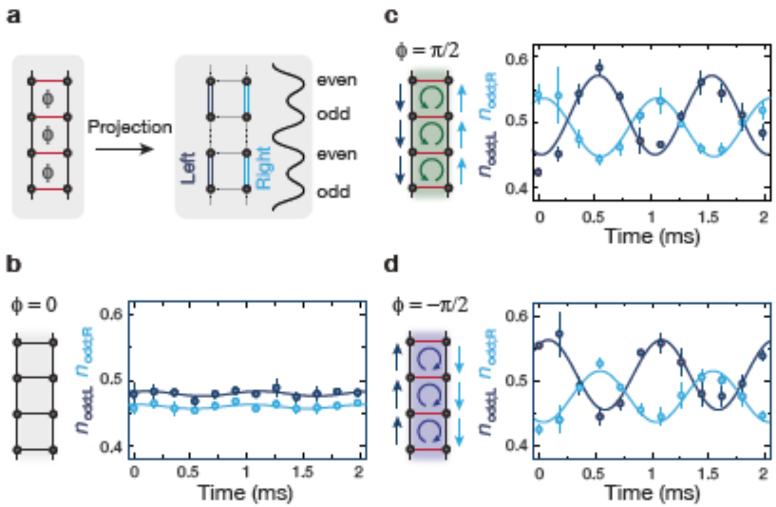
Bosonic Mott insulator with Meissner Currents Alex Petrescu and KLH, arXiv:1306.5986

Observation of the Meissner effect with ultracold atoms in bosonic ladders

M. Atala[1,2], M. Aidelsburger[1,2], M. Lohse[1,2], J. T. Barreiro[1,2], B. Paredes[3] & I. Bloch[1,2]

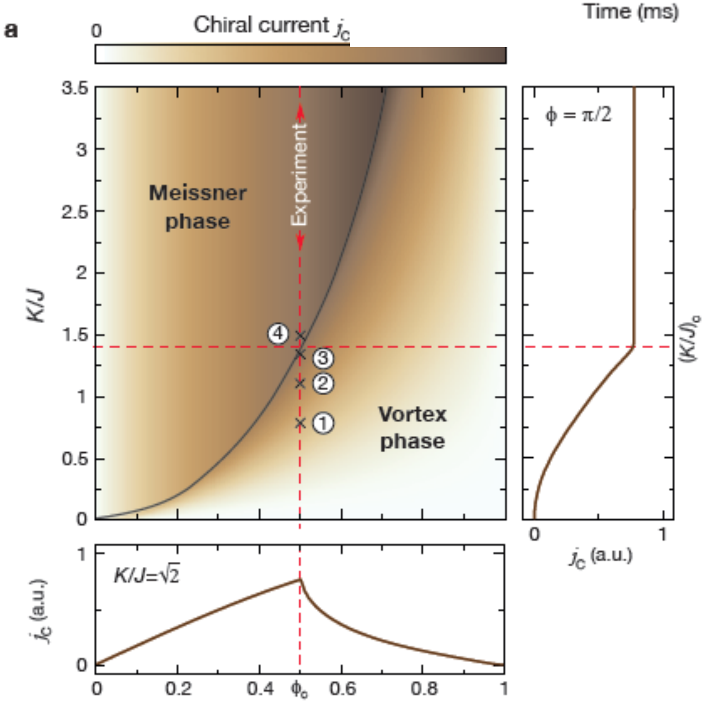
1402.0819.pdf

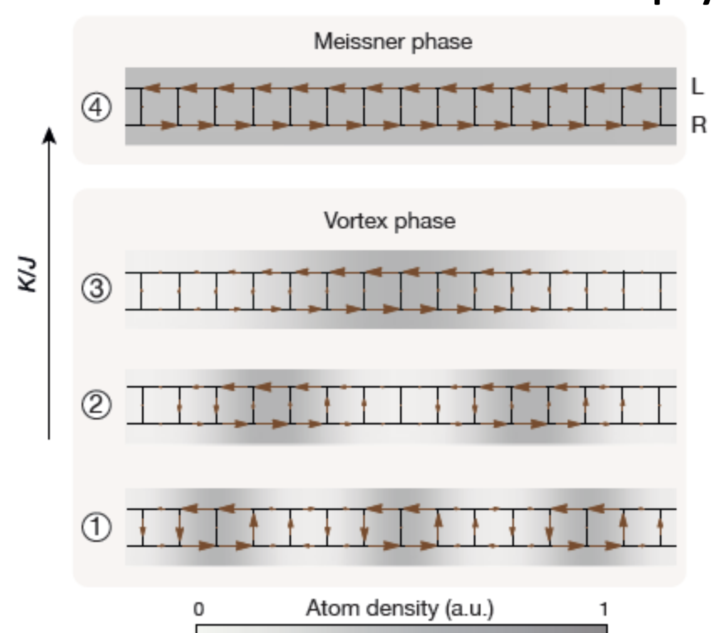
Scheme of Jaksch-Zoller



Theory by E. Orignac & T. Giamarchi 2001

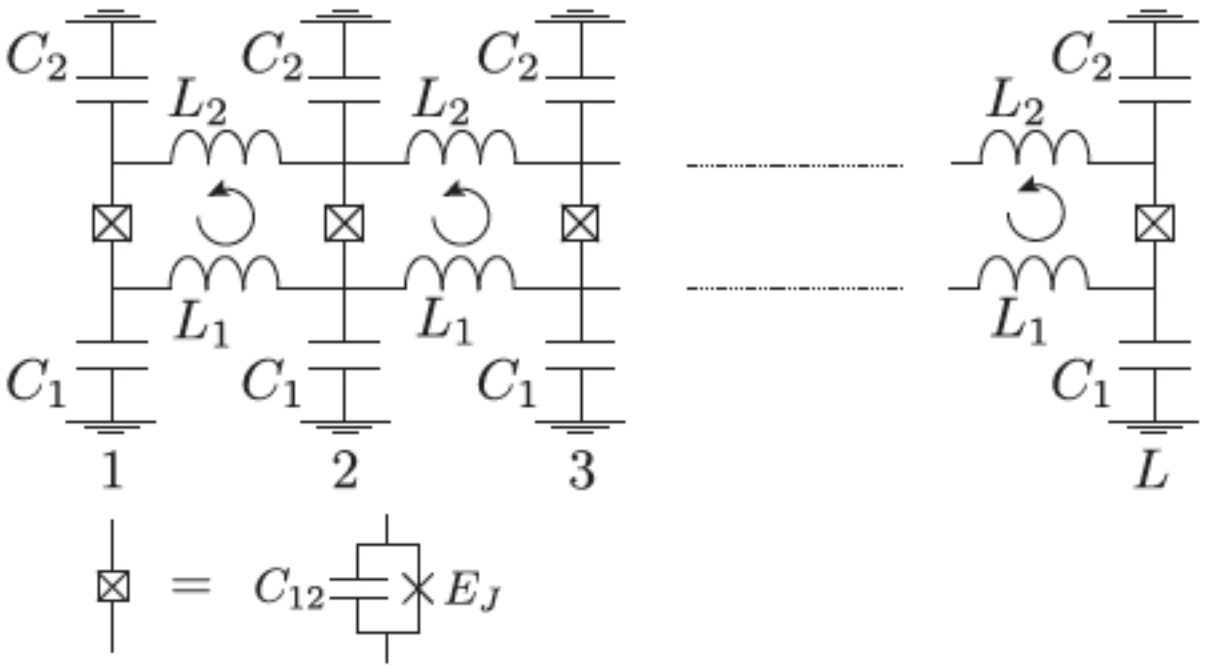
No Mott physics





Other Possible Realization: Josephson Junction ladder

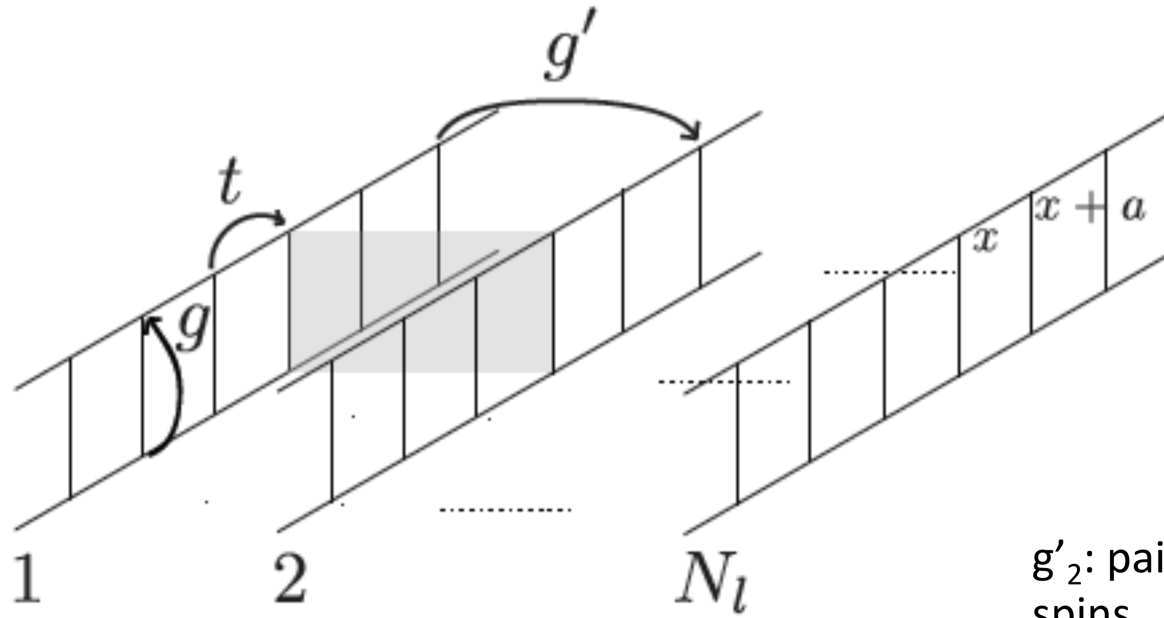
A. Petrescu and Karyn Le Hur, Phys. Rev. Lett. **111**, 150601 (2013)



Two transmission Lines coupled via a capacitive and Josephson couplings

In Ultra-cold atoms, the repulsion between species could be engineered via Rydberg atoms, an extra species (mixtures)
One can also use ultra-cold molecules.

Going to Two Dimensions



g'_2: pair hopping of spins

$$H_c^{eff} \sim -g_2' n \sum_{l=1}^{N-1} \int dx \cos \sqrt{2}(\theta_{\sigma,l} - \theta_{\sigma,l+1} + aA_{\sigma l})$$

Coupled ladders as coupled Planes

Solution

- 1D: The Hamiltonian can be bosonized
- Generalization in higher dimensions

Pseudo-spin coherent states

$$|\psi\rangle = \prod_i (\cos\phi_{\sigma i}\,|\uparrow\rangle_i + e^{i\theta_{\sigma i}}\sin\phi_{\sigma i}\,|\downarrow\rangle_i)$$

$$\begin{aligned} H_\sigma[\theta_\sigma,\phi_\sigma] = \frac{1}{2}\int \frac{d^d x}{a^{d-2}} J_{xx}\left(\nabla\theta_\sigma - A^\sigma\right)^2 \\ - \int \frac{d^d x}{a^d} g\cos\left(\theta_\sigma + a' A_\perp\right) \end{aligned}$$

Coupled Ladder Models give the same conclusion

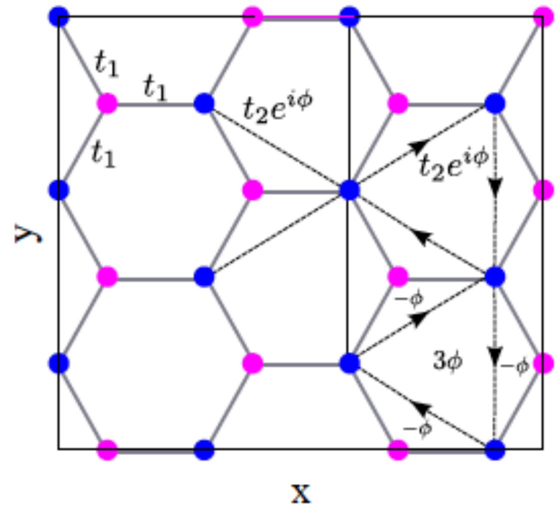
Haldane Model on the Graphene Lattice

Haldane model

$$\mathcal{H}_0 = \sum_i (-1)^i M c_i^\dagger c_i - \sum_{<i,j>} t_1 c_i^\dagger c_j - \sum_{\ll i,j \gg} t_2 e^{i\phi_{ij}} c_i^\dagger c_j$$

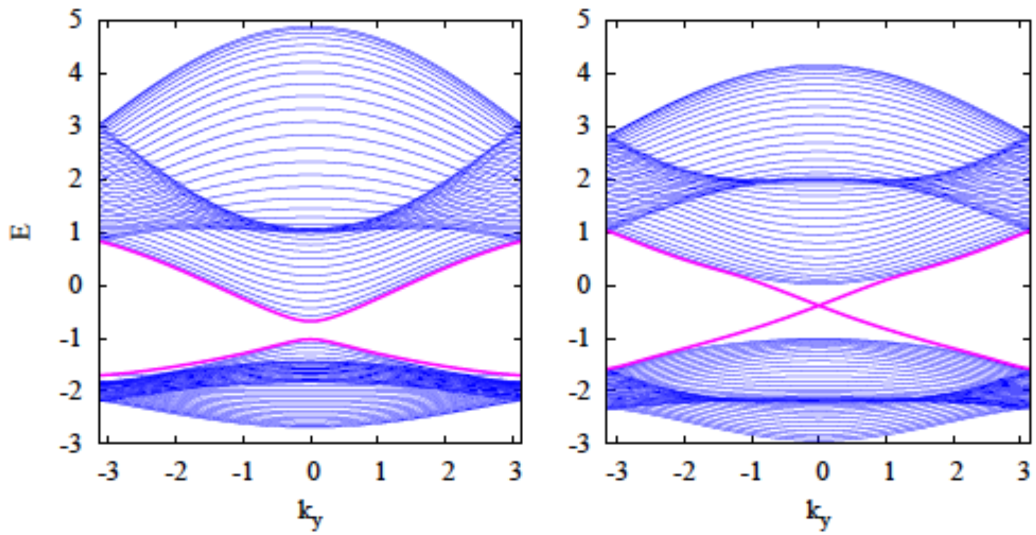
F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

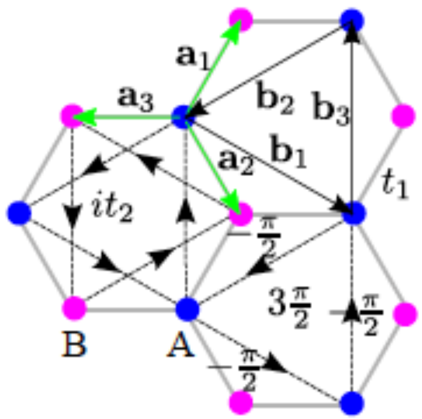
No net flux



Spectrum of the non-interacting model

- t_1 only $\Rightarrow$ Dirac cones
- M or t_2 can open the gap
- Non–trivial topological properties if $M < 3\sqrt{3} t_2 \sin\phi$





$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\left(\mathbf{k}\right)\cdot\hat{\sigma},$$

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\,\mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\,\mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\,\mathbf{b_i}\right).$$

Realized in cold atoms:
Group of T. Esslinger, 2014
arXiv:1406.7874

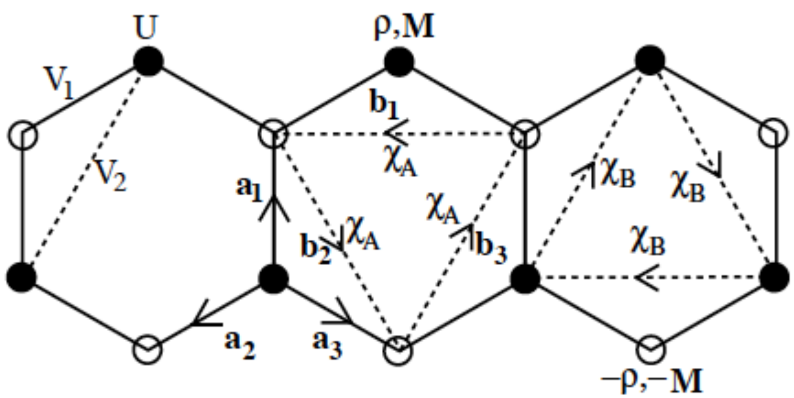
The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

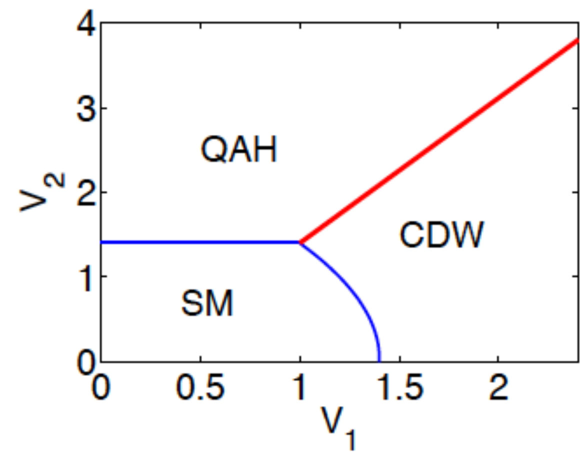
$$\mathcal{C}_- = \frac{1}{4\pi}\int_{\mathrm{BZ}} d\mathbf{k}\,\hat{\mathbf{d}}\cdot\left(\partial_1\hat{\mathbf{d}}\times\partial_2\hat{\mathbf{d}}\right)$$

This is the Chern number of the lower Bloch band, and takes the value $\mathcal{C}_- = 1$. The formula for the upper band is obtained by replacing $\hat{\mathbf{d}}$ by $-\hat{\mathbf{d}}$, and leads to $\mathcal{C}_+ = -1$.

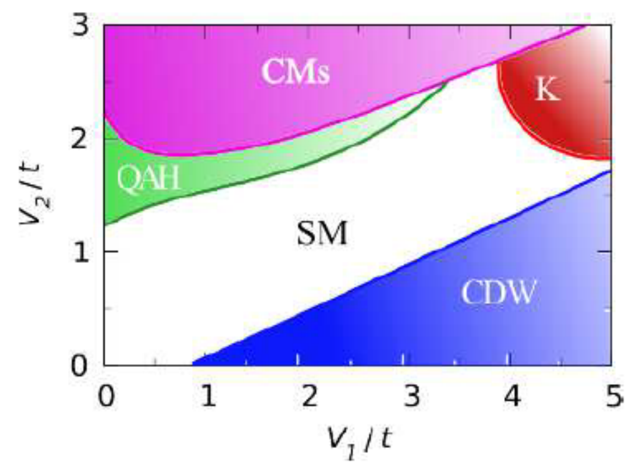
Topological Mott Insulators

S. Raghu[1], Xiao-Liang Qi[1], C. Honerkamp[2], and Shou-Cheng Zhang[1]
[1] *Department of Physics, McCullough Building, Stanford University, Stanford, CA 94305-4045 and*
[2] *Theoretical Physics, Universität Würzburg, D-97074 Würzburg, Germany*
(Dated: February 2, 2008)





How to realize a Large V_2 (Mott) coupling >> V_1 ?
Tianhan Liu, Benoît Douçot, KLH, in progress



Other 3D topological Mott insulators:
M. W. Young, S.S. Lee, K. Kallin 2008
D. Pesin and L. Balents, 2010
S. Rachel and KLH, 2010
W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents 2013
1D: Spin-1 chain (Haldane, Affleck, …)

A.G. Grushin et al. 2013
T. Duric et al. 2014 (ED & EPS)

Chiral Bosonic Phases on the Haldane Honeycomb Lattice

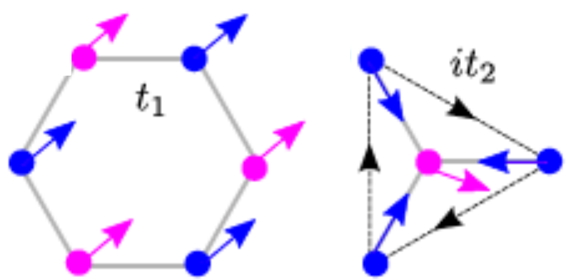
I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411

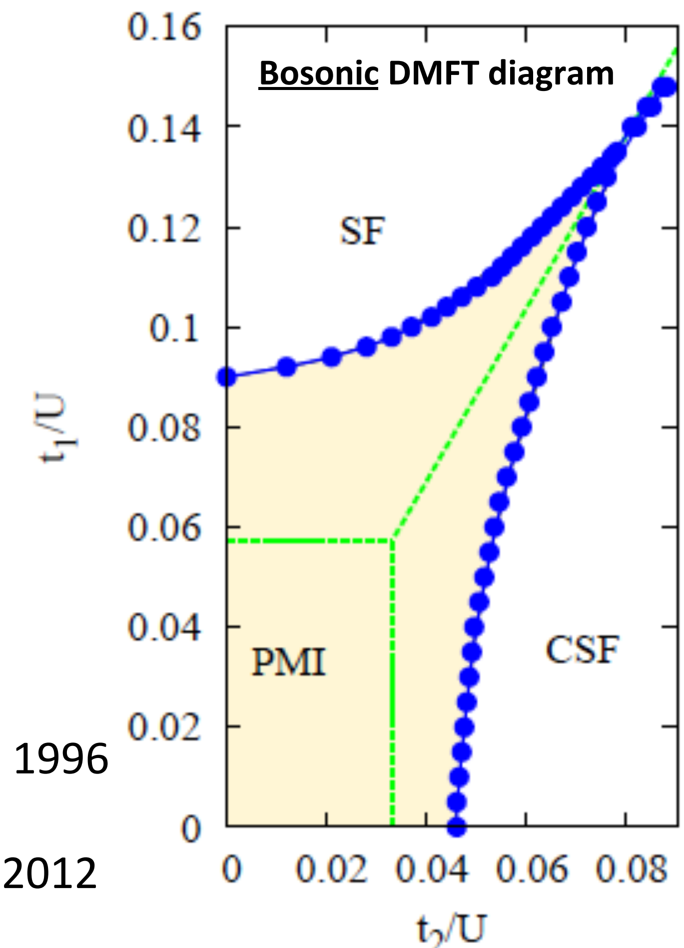
$$\mathcal{H} = \mathcal{H}_{\mathrm{H}} + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu \sum_i \hat{n}_i,$$

Phase-angle variables $b_i^\dagger = \sqrt{n}e^{i\theta_i}$

chiral SF:
nonuniform phase,
plaquette currents

SF:
uniform phase,
"Meissner current"

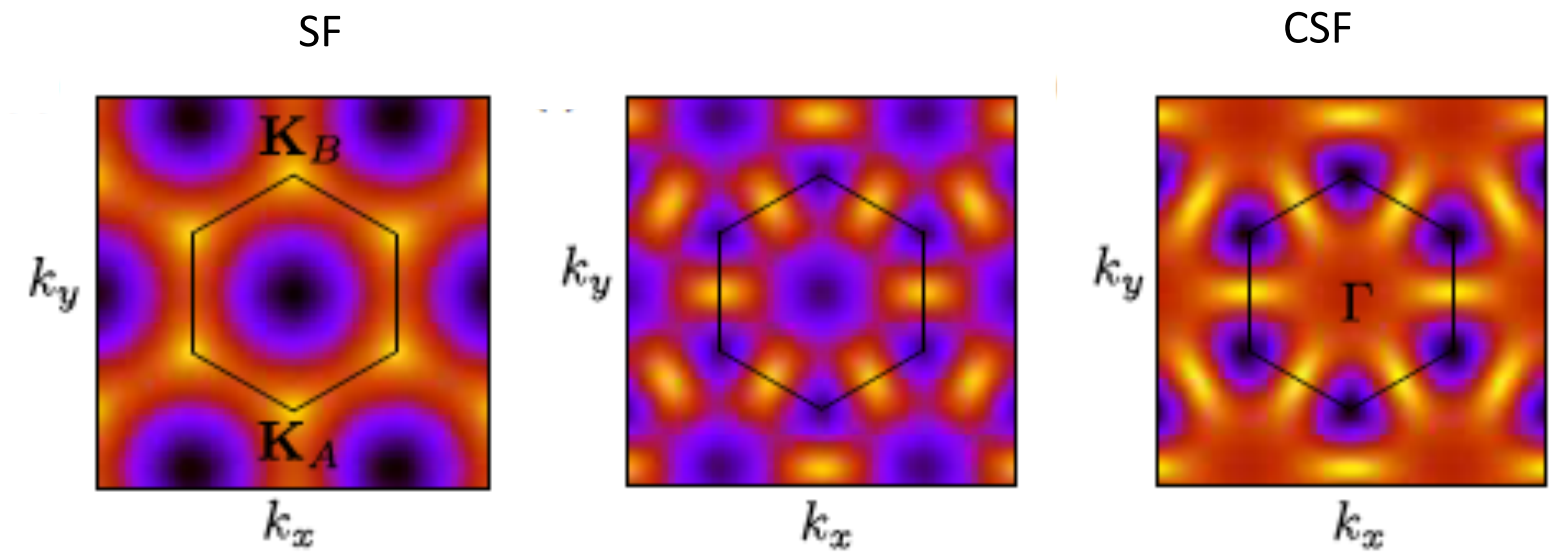




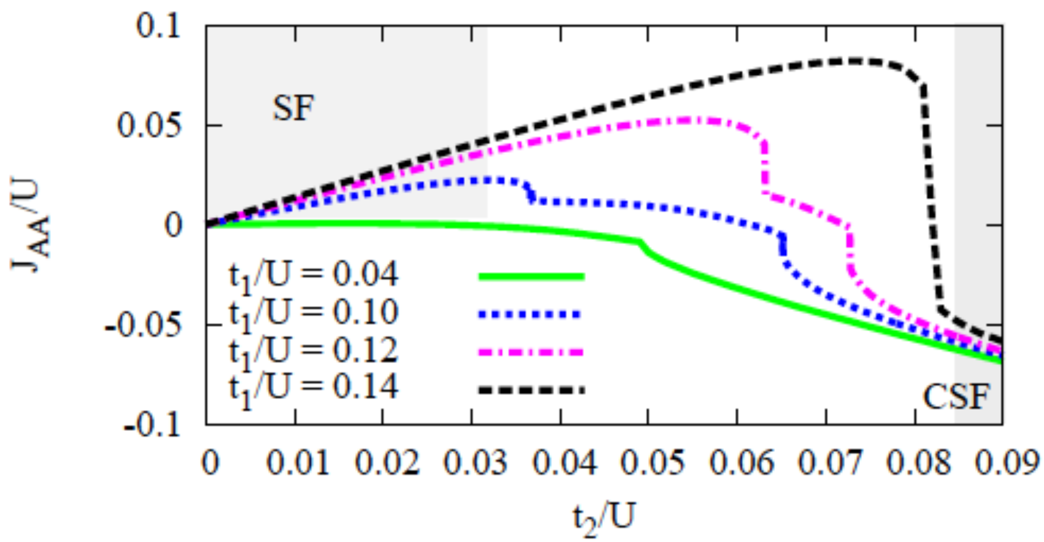
DMFT: A. Georges, G. Kotliar, W. Krauth, M. Rozenberg 1996

Kane-Mele-Hubbard: W. Wu, S. Rachel, W.-M. Liu, KLH 2012
Non-Abelian model: D. Cocks et al. 2012

Condensation of Bosons



$$J_{AA}^{SF} = -2\,n\,t_2\,\mathrm{Im}\exp(-i\pi/2) = 2nt_2$$



$$J_{AA}^{CSF} = -2\ \mathrm{Im}\left(t_2 e^{i\phi}\left\langle \hat{b}_{Ai}^{\dagger}\hat{b}_{Aj}\right\rangle\right)$$
$$= -2t_2 n \sin\left[\phi - \mathbf{K}_A\cdot(\mathbf{r}_i - \mathbf{r}_j)\right] = -nt_2$$

(local) Plaquette Currents subsist in the Mott phase

No long-range order here

Start from Mott picture and compute Green's functions to second-order

$$\langle b_i^\dagger b_j \rangle = -\frac{1}{\beta} \sum_n \exp(i\omega_n 0^+) G_{ji}(i\omega_n),$$

$$\frac{J_{ij}^{(2)}}{U} = -2\mathrm{Im}\, t_{ij} \left(\sum_k t_{jk} t_{ki} \right) \frac{3n(n+1)(2n+1)}{U^3} + \ldots$$

In our case, we obtain the following perturbative result:

$$\frac{J_{AA}^{(2)}}{U} = \frac{36}{U^3} t_2 \left(t_1^2 - 2t_2^2 \right).$$

Similar effect on triangular lattice and other lattices

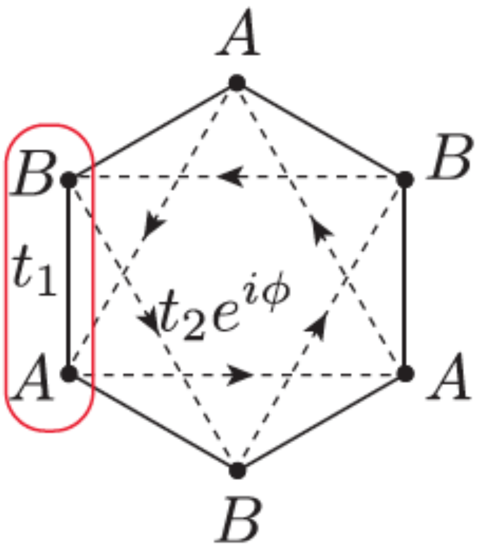
M. P. Zaletel, S. A. Parameswaran, A. Ruegg and E. Altman, 2013

D. Huerga, Jorge Dukelsky, Nicolas Laflorencie, G. Ortiz PRB 2014

Excitations in the Mott Phase

Strong coupling perturbation theory

Open question: quantitative estimation of lifetime

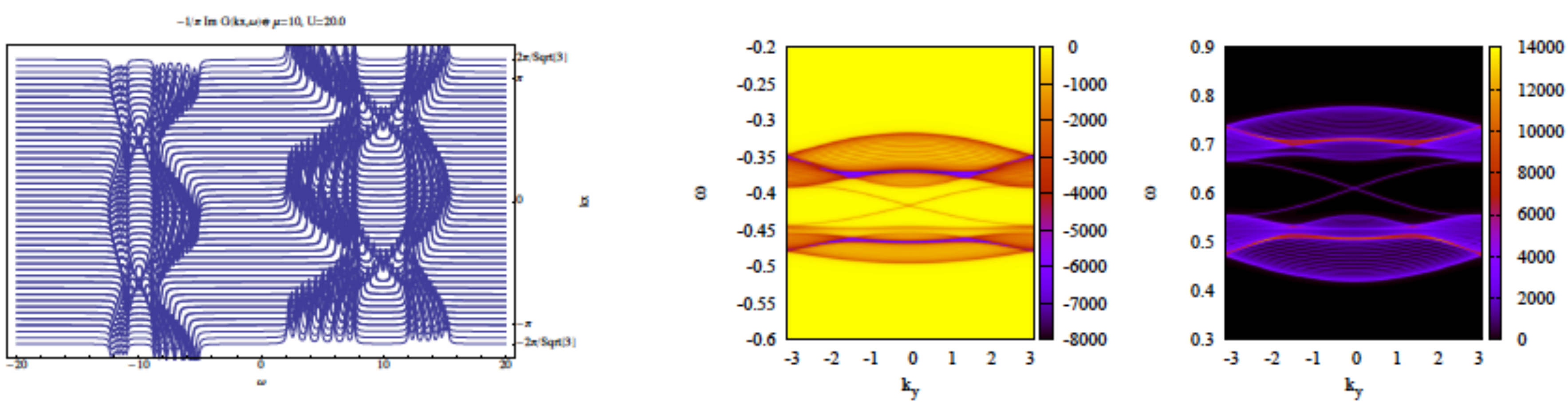


$$G^{-1}(i\omega, k) = g^{-1}(i\omega) - h_k.$$

$g(i\omega)$ = local cluster Green's function

$G(i\omega, k)$ = approximate Green's function

DMFT



Topological Index: In total 0

- Topological index of the interacting model

K. Ishikawa and T. Matsuyama, Nucl. Phys. B **280**, 523 (1987)

$$\sigma_{xy} = \frac{1}{2}\frac{\epsilon_{\mu\nu}}{4\pi^2}\int d\mathbf{k}\int \frac{d\omega}{2\pi} e^{i\omega 0+}\mathrm{Tr}\left[\frac{\partial G}{\partial \omega}\frac{\partial G^{-1}}{\partial k_\mu} G \frac{\partial G}{\partial k_\nu}\right]$$

- It can be expressed as a winding number of a vector corresponding to G^{-1} $(G^{-1}((\mathbf{k},\omega) = \hat{\sigma}\cdot\mathbf{g}(\mathbf{k}) + \mathbf{I}_2\beta(\omega))$

$$\sigma_{xy} = -\frac{1}{2\pi}\sum_\alpha \alpha \int_{BZ} d\mathbf{k}\, \mathbf{g}(\mathbf{k})\cdot(\partial_{\mathbf{k}_1}\mathbf{g}(\mathbf{k}) \times \partial_{\mathbf{k}_2}\mathbf{g}(\mathbf{k}))\,,$$

where the sum is over the negative poles of the Green's function

R. Shindou, L. Balents, Phys. Rev. Lett. **97**, 216601 (2006); Phys. Rev. B **77**, 035110 (2008)

C. H. Wong and R. Duine, Phys. Rev. Lett. **110**, 115301 (2013); Phys. Rev. A **88**, 053631 (2013)

Cold Atoms:

Jaksch & Zoller 2003

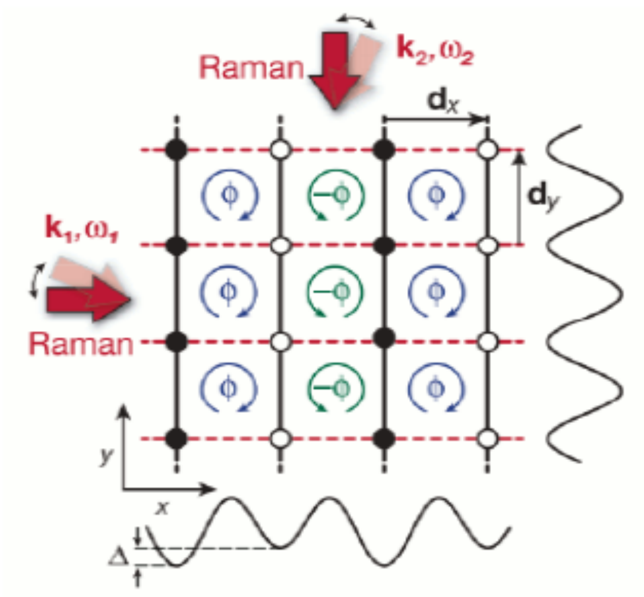
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
I. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- Ways to implement magnetic fields & gauge fields

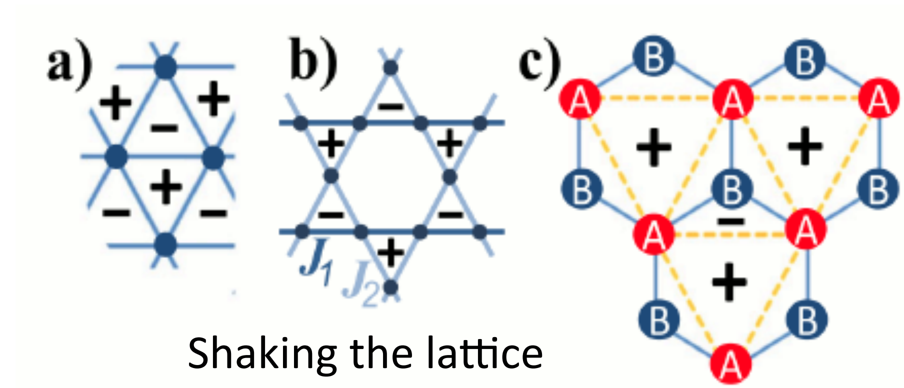
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)



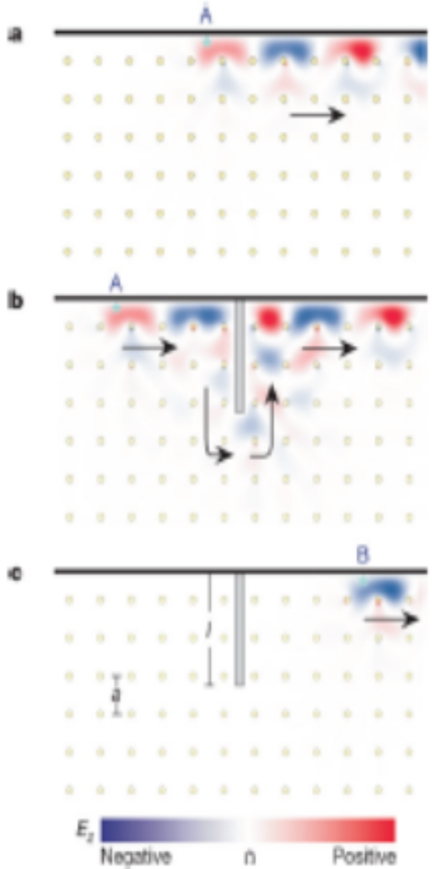
Shaking the lattice

Floquet Topological Insulators:
Recent review J. Cayssol, B. Dora, F. Simon, R. Moessner, arXiv:1211.5623

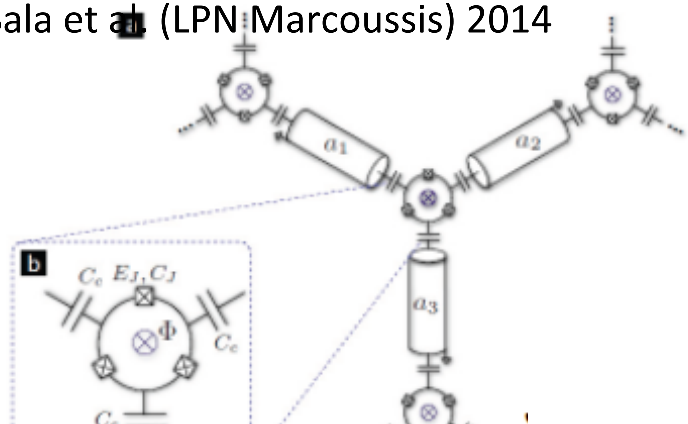
Artificial Gauge Fields with Light

See also M. C. Rechstmann et al Nature 2013
D. Schuster and J. Simon lab Chicago
V. G. Sala et [redacted] (LPN Marcoussis) 2014

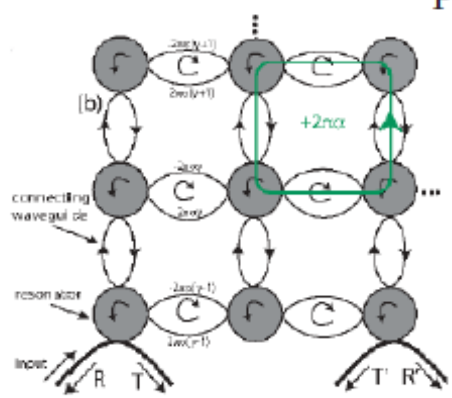
Review:
I. Carusotto
& C. Ciuti
RMP 2012



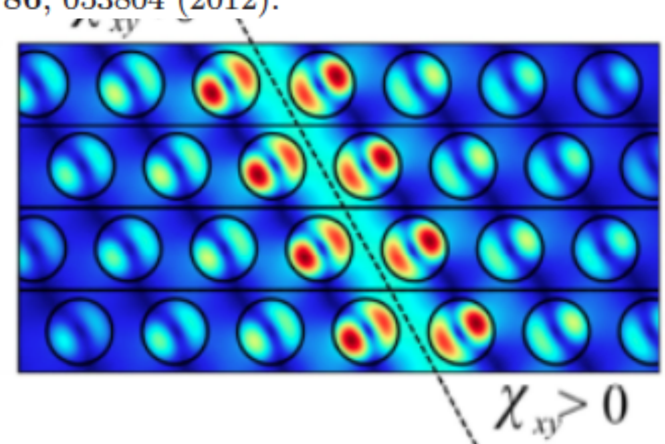
Haldane-Raghu, PRL 2008
Z. Wang et al. Nature 2009



J. Koch, A. A. Houck, K. Le Hur, and S. M. Girvin, Phys. Rev. A **82**, 043811 (2010).
A. Petrescu, A. A. Houck, and K. Le Hur, Phys. Rev. A **86**, 053804 (2012).



M. Hafezi, E. Demler, M. Lukin, J. Taylor 2011



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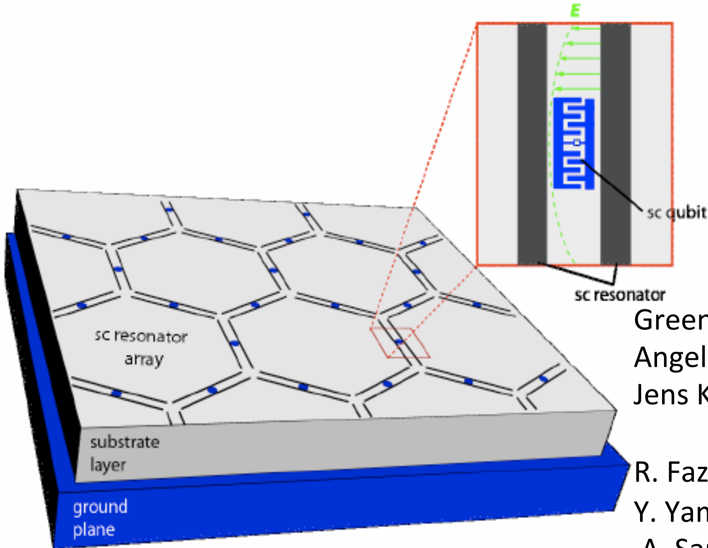
Array cQED Systems



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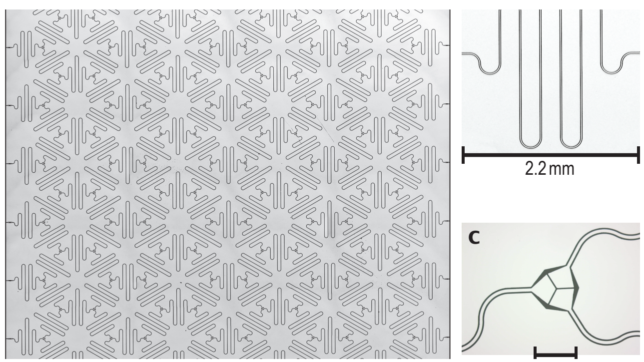

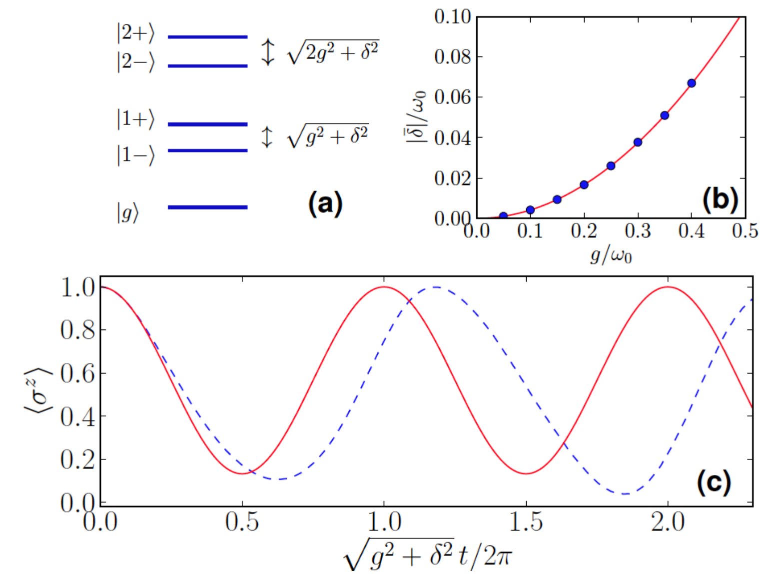

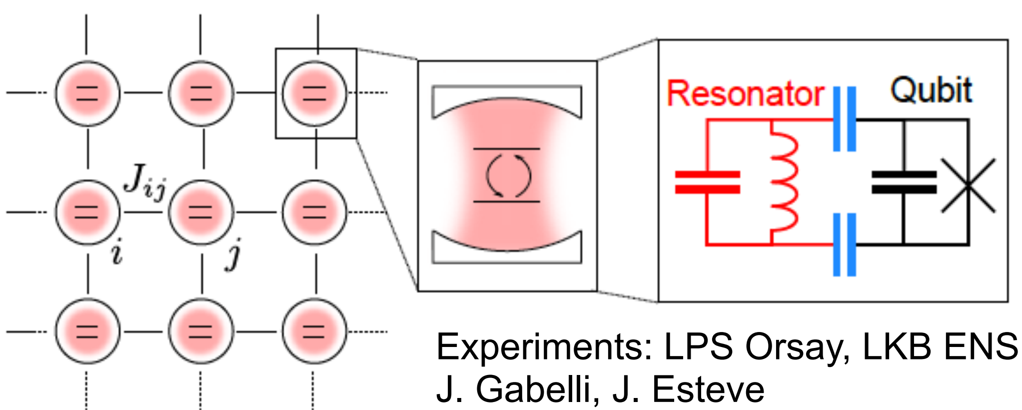

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Thanks for Your Attention