Karyn Le Hur

Centre de Physique Theorique, Ecole Polytechnique and CNRS

4 classes Saclay Lectures Series: 1h30 each

Thanks to Sylvain Ravets, Igor Ferrier-Barbut, Benoit Valiron for invitation
Thanks to be here for the last lecture

Institut d'Optique Graduate School

Geometry and Topology in the Quantum!

- Class I: Quantum Geometry, Information and Topological Physics from Bloch Sphere (June 9)
- Class II: Application in Topological Lattice Models and Quantum Matter (June 16)
- Class III: Applications in Transport and Light-Matter Interaction (June 23)
- Class IV: Entangled WaveFunction and Fractional Topology (June 30) V

Thanks to the team...

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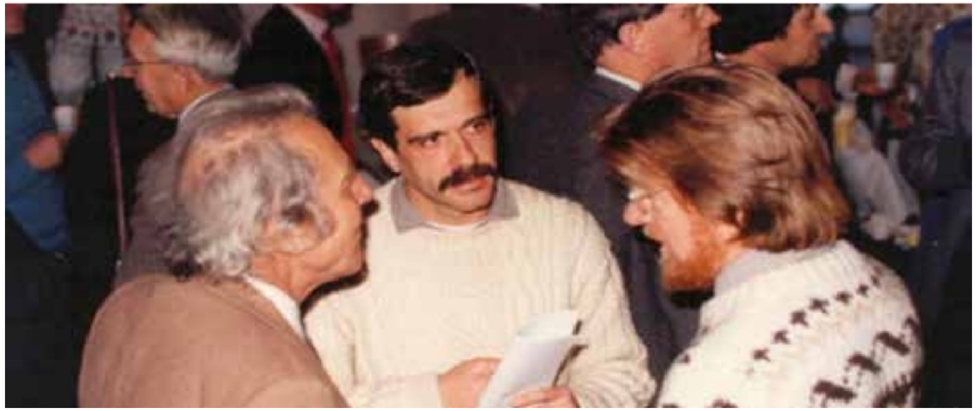
BELL'S THEOREM : THE NAIVE VIEW OF AN EXPERIMENTALIST†

Alain Aspect

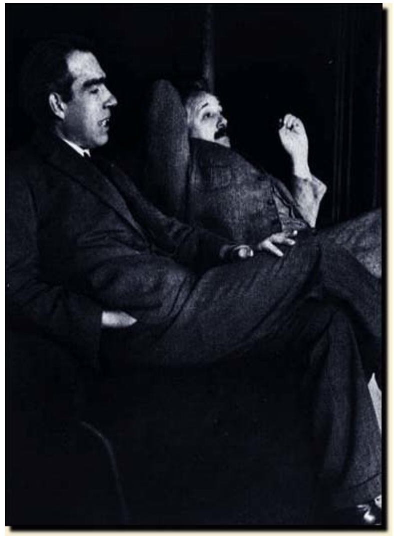
$$P_+(\mathbf{a}) = P_-(\mathbf{a}) = 1/2$$
$$P_+(\mathbf{b}) = P_-(\mathbf{b}) = 1/2$$



A. Aspect,
J. Bell, A. Messiah

1985, conference
Hommage A. Kastler

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+,-\rangle - |-,+\rangle\right)$$



Niels Bohr and
Albert Einstein meditating

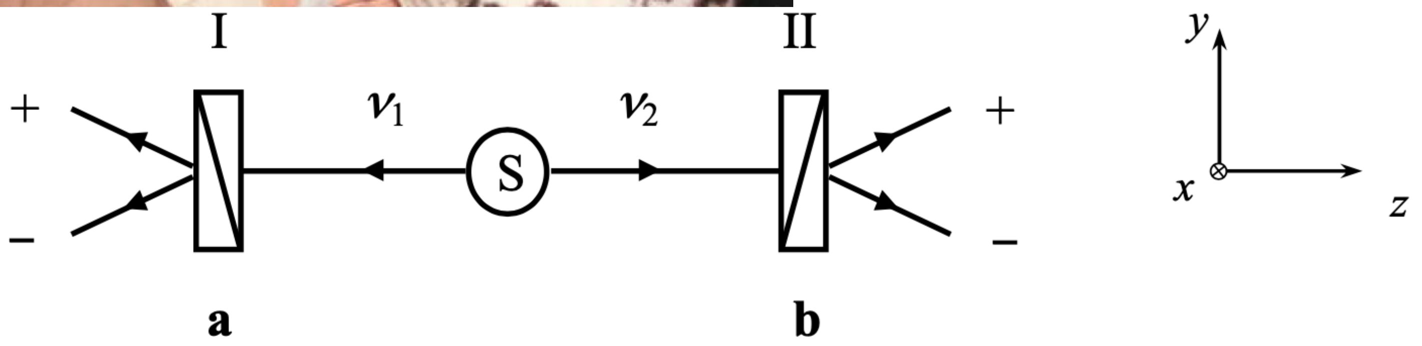


Figure 1. Einstein-Podolsky-Rosen-Bohm Gedankenexperiment with photons*. The two photons ν_1 and ν_2, emitted in the state $|\Psi(1,2)\rangle$ of Equation (1), are analyzed by linear polarizers in orientations* **a** *and* **b**. *One can measure the probabilities of single or joint detections in the output channels of the polarizers.*

Correlation Functions

$$\boxed{E(\mathbf{a},\mathbf{b}) = p_{++}(\mathbf{a},\mathbf{b}) + p_{--}(\mathbf{a},\mathbf{b}) - p_{+-}(\mathbf{a},\mathbf{b}) - p_{-+}(\mathbf{a},\mathbf{b})}$$

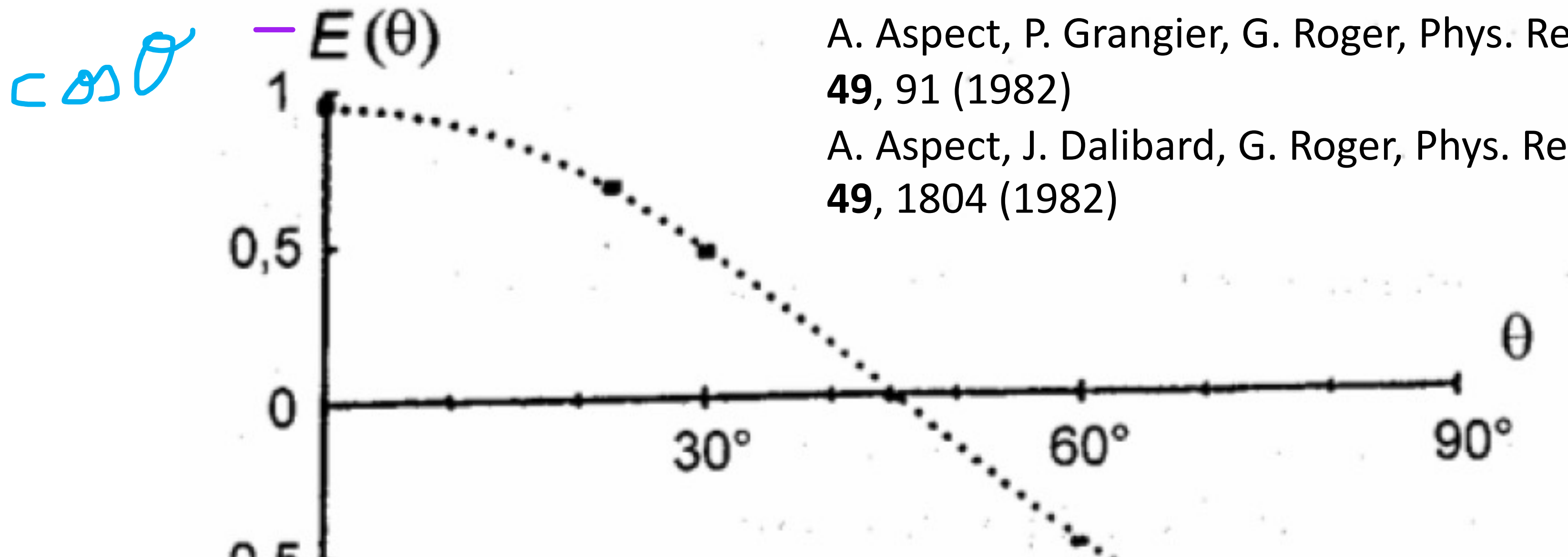
$$\hat{E}(\mathbf{a},\mathbf{b}) = \left(\hat{P}_+(\mathbf{a}) - \hat{P}_-(\mathbf{a})\right) \otimes \left(\hat{P}_+(\mathbf{b}) - \hat{P}_-(\mathbf{b})\right)$$

$$\hat{P}_+(\mathbf{a}) - \hat{P}_-(\mathbf{a}) = \underline{2\hat{O}_3(\mathbf{a})} = (|+\rangle\langle+| - |-\rangle\langle-|)_{\varphi_{\mathbf{a}}}$$

If Alice measures along her axis: same chance to see $\pm\frac{\hbar}{2}$ (spins-$\frac{1}{2}$)

what about Bob?

$$\boxed{\langle\psi|\hat{\vec{S}}_a \otimes \hat{\vec{S}}_b|\psi\rangle = -\frac{\hbar^2}{4}\vec{u}\cdot\vec{v} = -\frac{\hbar^2}{4}P(\vec{u},\vec{v})} = -\frac{\hbar^2}{4}\cos\theta$$



A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. **49**, 91 (1982)
A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 1804 (1982)

The cat is wondering about the meaning of all this

Observation of entanglement with macroscopic cats

Relation to entanglement measures

Recent measures in quantum

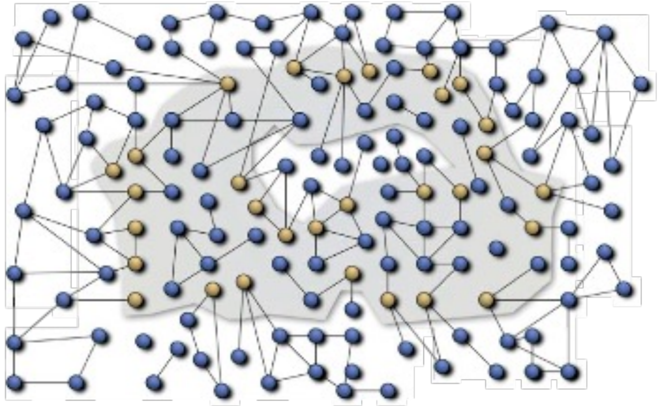
K. J. Satzinger et al. Science (2021)

quantum mechanics:
entropy > 0 without an objective lack of information

non-degenerate
pure ground state

$$\rho_0 = |\psi\rangle\langle\psi|$$

$$\Rightarrow S(\rho_0) = 0$$



(Eisert *et al.*, RMP, 2010)

von Neumann entropy

$$S(\rho) = -\mathrm{tr}\left(\rho \log_2 \rho\right)$$

shaded region A
remainder B

$$\rho_A = \mathrm{tr}_B\left(\rho\right)$$

$$\Rightarrow S(\rho_A) \neq 0$$

Entanglement
Entropy

L. Amico et al. Rev Mod Phys (2008)

Measure of entanglement entropy

PhD Thesis of H. F. Song (Yale 2012), L. Herviou (CPHT Ecole Polytechnique & ENS 2014-2017), A. Petrescu Yale & CPHT 2015

Review: H. Francis Song, S. Rachel, C. Flindt, I. Klich, N. Laflorencie, K. Le Hur, Phys. Rev. B **85**, 035409 (2012)

Entanglement entropy of free fermions

$$\mathcal{S} = \lim_{K\to\infty} \sum_{n=1}^{K+1} \alpha_n(K) C_n,$$

where

$$\alpha_n(K) = \begin{cases} 2\sum_{k=n-1}^{K} \frac{S_1(k,n-1)}{k!k} & \text{for } n \text{ even,} \\ 0 & \text{for } n \text{ odd.} \end{cases}$$

Here $S_1(n,m)$ are **unsigned Stirling numbers of the first kind**. Practically, K is the number of available cumulants and should be taken to be even.

$F = C_2$

$$C_n = (-i\partial_\lambda)^n \ln \chi(\lambda)|_{\lambda=0}$$

$$\chi(\lambda) = \langle e^{i\lambda \hat{N}_A} \rangle$$

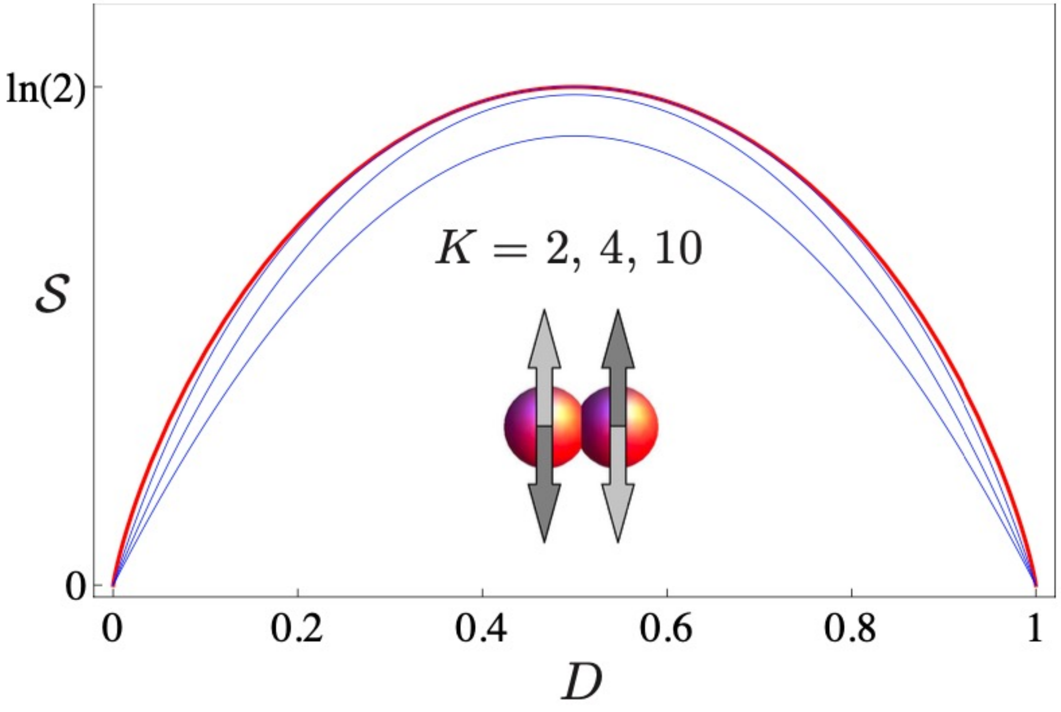
Generating function

J. Bell, 1963 Nice paper on fluctuations in superconductors

F in 1D wires, quantum Hall systems, many-body localization…

Bell or EPR pair

$$|\psi\rangle = \sqrt{1-D}|A_\uparrow\rangle|B_\downarrow\rangle + \sqrt{D}|A_\downarrow\rangle|B_\uparrow\rangle$$



The second cumulant or variance of charge/spin is already very meaningful & reveals similar information as entropy in the sense of “probabilities”

Second cumulant measures entropy

Spin-$\frac{1}{2}$

$$\boxed{F_A(\mathbf{a}) = 4\left[\langle\psi|\hat{O}_3^2(\mathbf{a})\otimes\mathcal{I}_b|\psi\rangle - \left(\langle\psi|\hat{O}_3(\mathbf{a})\otimes\mathcal{I}_b|\psi\rangle\right)^2\right]}.$$

$2\hat{O}_3 = \sigma_z$

$$\sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$F_A(\vec{a}) = F_B(\vec{b}) = 1 - \langle\psi|\sigma_z(\vec{a})\otimes I_b|\psi\rangle^2$$

- Pure state along z-axis: $F_A(\vec{a}) = 0$

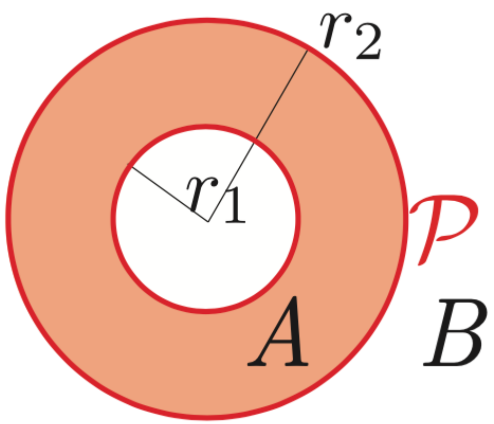
- EPR pair

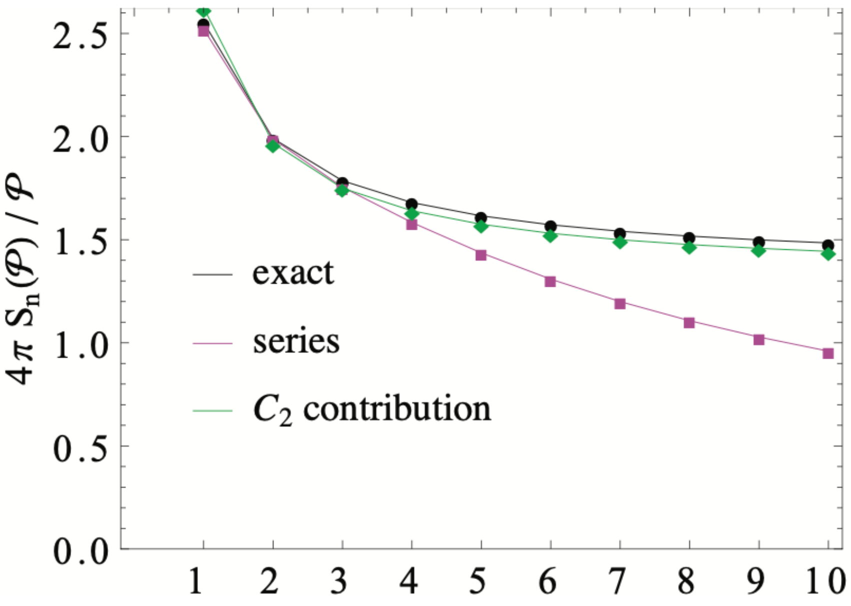
$\frac{1}{2} = P_-^A = P_\downarrow^A \qquad \frac{1}{2} = P_+^A = P_\uparrow^A$

$$\frac{S_A}{\ln 2} = F_A(\vec{a}) = F_B(\vec{b}) = 1$$

Quantum Hall systems

$$S_n = \frac{1}{1-n} \log \mathrm{Tr}_A \left(\rho_A\right)^n$$





Fluctuations and entanglement spectrum in quantum Hall states ‡

Alexandru Petrescu[1,2], H. Francis Song[3], Stephan Rachel[4], Zoran Ristivojevic[2], Christian Flindt[5], Nicolas Laflorencie[6], Israel Klich[7], Nicolas Regnault[8,9], Karyn Le Hur[2]

2014

Laughlin wavefunction

$$L_z = -i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = -i\frac{\partial}{\partial \varphi}$$

$$\boxed{\begin{aligned}\tan\varphi &= y/x \\ r^2 &= x^2+y^2\end{aligned}}$$

$$H = \frac{1}{2m}\left(p_x + \frac{eB}{2}y\right)^2 + \frac{1}{2m}\left(p_y - \frac{eB}{2}x\right)^2 = \frac{\hbar\omega_c}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{4}r^2 - L_z\right)$$

$$H\psi_m = \frac{\hbar\omega_c}{2}\psi_m$$

$m \in [0; N-1]$

$z = x + iy$

$$\psi_m = \mathcal{N} r^m e^{im\varphi} e^{-r^2/4}$$

$$\boxed{\begin{vmatrix} 1 & 1 \\ z_1 & z_2 \end{vmatrix} = z_2 - z_1}$$

N particles

$$\psi = \frac{1}{\sqrt{N!}}\Pi_{j>i}(z_j - z_i)e^{-\frac{1}{4}\sum_{j=1}^{N}|z_j|^2}$$

Niu-Thouless-Wu formula,

See book of E. Fradkin

$$z = e^{i\varphi} = \sum_{i=1}^{N} z_i = \sum_i e^{i\varphi_i}$$

$$\boxed{\sigma_{xy} = \frac{e^2}{i\hbar}\oint d\varphi_i \langle\psi|\frac{\partial}{\partial\varphi_i}|\psi\rangle = \frac{e^2}{i\hbar}\oint d\varphi \int_0^{L_x} dx \int_0^{L_y} dy i|\psi|^2 = \frac{e^2}{h}}$$

Laughlin wavefunction for fractional QHE, $\frac{1}{p} = \nu = \frac{1}{3} = \frac{N}{N_\phi}$

$z^p = e^{i\tilde{\varphi}} = e^{i\varphi p}$

$$\psi = \frac{1}{\sqrt{N!}} \Pi_{j>i}(z_j - z_i)^p e^{-\frac{1}{4}\sum_{j=1}^{N}|z_j|^2}$$

Modifying $\tilde{\varphi}_i = p\varphi_i$ (1 electron is p fractional charges)

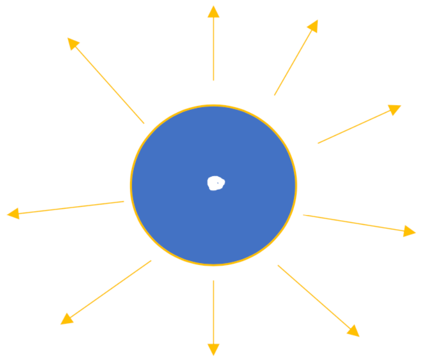
$$\sigma_{xy} = \frac{e^2}{i\hbar}\int_0^{\frac{2\pi}{p}} \frac{d\tilde{\varphi}_i}{p} p\langle\psi|\frac{\partial}{\partial\tilde{\varphi}_i}|\psi\rangle = \frac{e^2}{h}\frac{1}{p}.$$

$$\sigma_{xy} = \frac{e^2}{i\hbar}\int_0^{2\pi} d\tilde{\varphi}_i \frac{1}{p}\langle\psi|\frac{\partial}{\partial\tilde{\varphi}_i}|\psi\rangle$$

For the Berry field $A_{\tilde{\varphi}}$, this is the same as multiplying by a factor $\frac{1}{p}$ (fractional).

Observed in various experiments: H. L. Stormer, D. C. Tsui, A. C. Gossard, Rev. Mod. Phys. 1999
Fractional charges at the edges: L. Saminadayar, D. C. Glattli, Y. Jin, B. Etienne (CEA Saclay, CNRS), 1997

Class I: Topological Sunshine



$$\boxed{H = -\vec{\mathbf{d}}\cdot\vec{\boldsymbol{\sigma}}}$$

$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$
Pauli Matrices

$$\vec{\mathbf{d}}(\varphi,\theta) = d(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta) = (d_x, d_y, d_z)$$

$$E_+ = -|\vec{d}|$$
$$E_- = +|\vec{d}|$$

$$|\psi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}, \quad |\psi_-\rangle = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{i\frac{\varphi}{2}} \end{pmatrix}$$

$$A_\varphi = -i\langle\psi|\partial_\varphi|\psi\rangle = -\frac{\cos\theta}{2}$$
$$F_{\theta\varphi} = \sin\theta/2$$

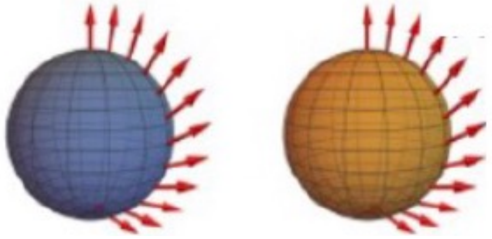
$$C = \frac{1}{2\pi}\iint F_{\theta\varphi}\, d\theta\, d\varphi = \int_0^\pi \frac{\partial}{\partial\theta}A_\varphi\, d\theta \qquad A_\theta = 0$$

$$C = A_\varphi(\pi) - A_\varphi(0) = \frac{1}{2}\left(\langle\sigma_z(0)\rangle - \langle\sigma_z(\pi)\rangle\right) = \langle\sigma_z(0)\rangle$$

Topological Entangled Aspects

$$|\Phi_+\rangle = e^{-i\frac{\varphi}{2}}|\uparrow\rangle$$
$$|\Phi_-\rangle = e^{i\frac{\varphi}{2}}|\downarrow\rangle$$

$$|\psi\rangle = \sum_{kl} c_{kl}(\theta)|\Phi_k(\varphi)\rangle_1|\Phi_l(\varphi)\rangle_2,$$



- Suppose a direct product pure state at one pole $|\psi(0)\rangle = |\Phi_+\rangle_1|\Phi_+\rangle_2$ $\quad \langle \sigma_z^1(0)\rangle = 1$

- An entangled Einstein-Podolsky-Rosen or Bell wavefunction at the other pole

$$|\psi(\pi)\rangle = \frac{1}{\sqrt{2}}(|\Phi_+\rangle_1|\Phi_-\rangle_2 + |\Phi_-\rangle_1|\Phi_+\rangle_2).$$

$$\langle \sigma_z^1(\pi)\rangle = 0$$

Question: can we have

$$C_1 = \frac{1}{2}\left(\langle \sigma_z^1(0)\rangle - \langle \sigma_z^1(\pi)\rangle\right) \stackrel{?}{=} \frac{1}{2}$$

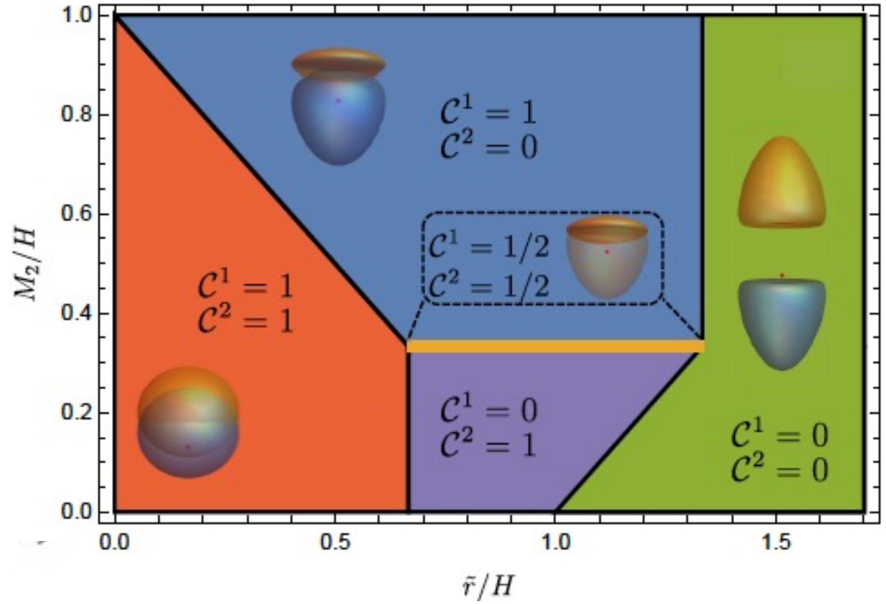
Interacting Bloch Spheres' Model

Thanks to
NSERC Canada
& ANR BOCA

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021), Nature Journal

$$\mathcal{H}^{\pm} = -(\boldsymbol{H}_1 \cdot \boldsymbol{\sigma}^1 \pm \boldsymbol{H}_2 \cdot \boldsymbol{\sigma}^2) \pm \tilde{r} f(\theta) \sigma_z^1 \sigma_z^2.$$

$$\boldsymbol{H}_i = (H \sin\theta \cos\phi, H \sin\theta \sin\phi, H \cos\theta + M_i)$$

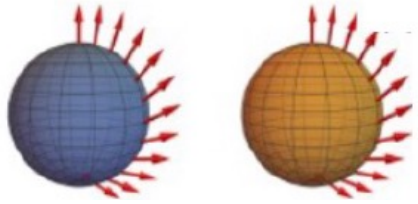


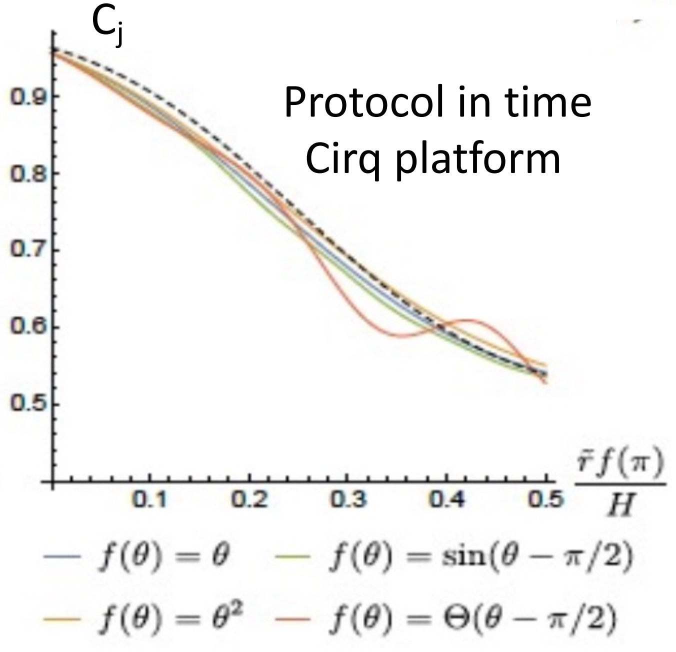
- Phase Diagram obtained from energetics at the poles
- Region C_j=1/2 occurs for various f(θ) $and\ f(\theta) = cst$

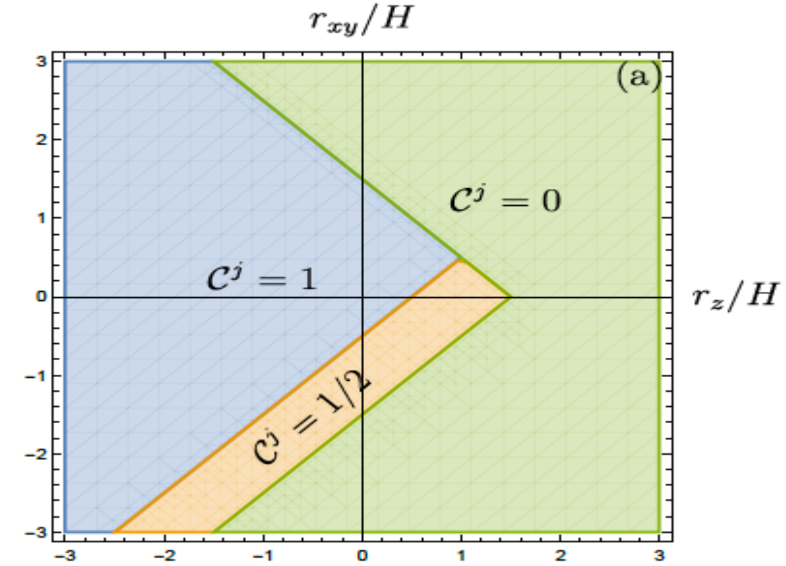
Z_2 symmetry
1 <-> 2 spheres

M_1=M_2=M

$$\boxed{H - M < \tilde{r} < H + M,}$$







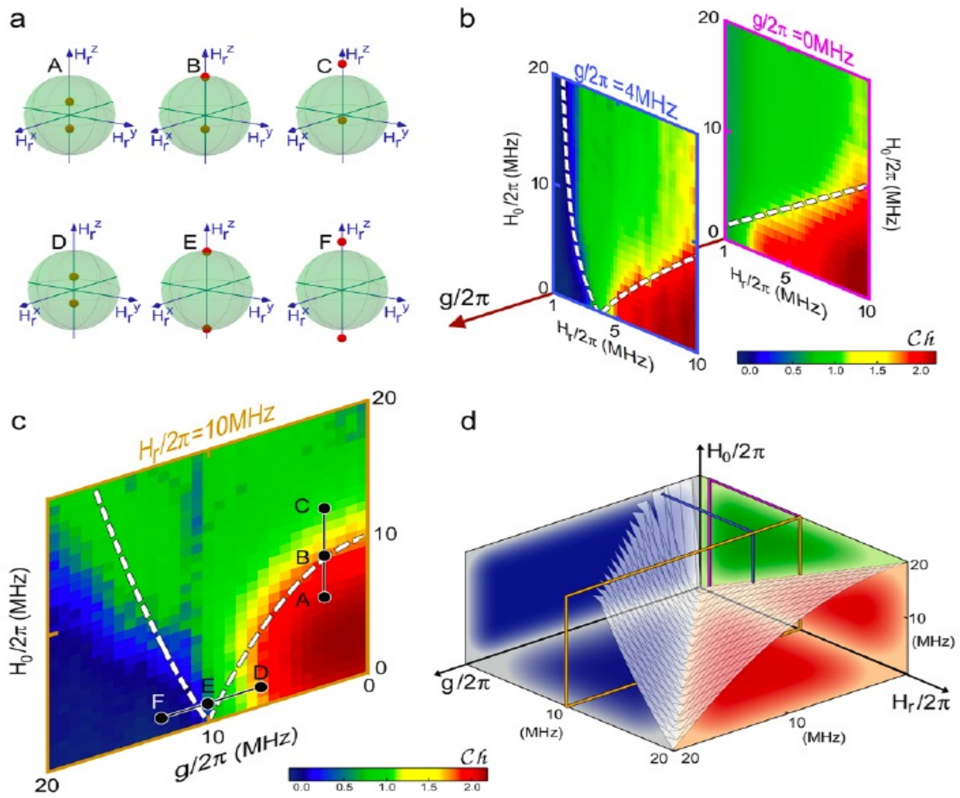
2 spins

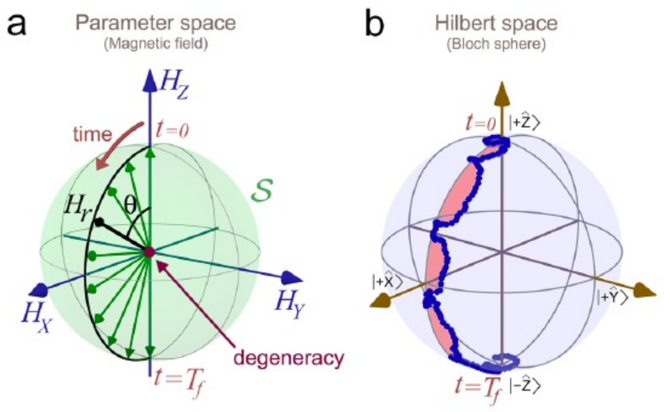
The Hamiltonian of this system reads

$$\mathcal{H}_{2Q} = -\frac{\hbar}{2}[H_0\sigma_1^z + \mathbf{H_1}\cdot\boldsymbol{\sigma_1} + \mathbf{H_2}\cdot\boldsymbol{\sigma_2} - g(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)], \tag{5}$$

where 1 and 2 refer to qubit 1 (Q1) and qubit 2 (Q2)

Santa-Barbara "google":
P. Roushan et al.
arXiv:1407.1585
Nature **515**, 241 (2014)





Application in Energy:
Quantum Dynamo effect in a Bath, class I
L. Henriet, A. Sclocchi, P. P. Orth, K. Le Hur Phys. Rev. B 95, 054307 (2017)
E. Bernhardt, C. Elouard, K. Le Hur, Phys. Rev. A 107, 022219 (2023)

Generalized Geometrical Proof

J. Hutchinson and K. Le Hur, Communications Physics 4, 144 (2021), Nature Journal

$\vec{A}' = \vec{0}$

$\theta = 0, \pi$

$$C = \frac{1}{2\pi} \iint_{S^{2\prime}} \boldsymbol{\nabla} \times \mathbf{A}' \cdot d^2\mathbf{s}$$

$$\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{A}' = \boldsymbol{\nabla} \times \mathbf{A}$$

$$d^2\mathbf{s} = d\varphi d\theta \mathbf{e}_r$$

$$\boldsymbol{\nabla} = (\partial_\theta, \partial_\varphi)$$

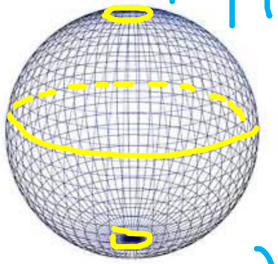
Stokes' 2 boundaries

$S_2' = S_2 - \{0, \pi\}$

$$\iint_{north'} \boldsymbol{\nabla} \times \mathbf{A} \cdot d^2\mathbf{s} = \int_0^{2\pi} \left(A_{N\varphi}(\theta^+, \varphi) - A_\varphi(0)\right) d\varphi$$

$$\iint_{south'} \boldsymbol{\nabla} \times \mathbf{A} \cdot d^2\mathbf{s} = -\int_0^{2\pi} \left(A_{S\varphi}(\theta^-, \varphi) - A_\varphi(\pi)\right) d\varphi$$

$A_\varphi(0)$



$\theta_c = \frac{\pi}{2}$

$A_\varphi(\pi)$

$$\boxed{A'_\varphi(\theta < \theta_c) = A_{N\varphi}(\theta, \varphi) - A_\varphi(0)}$$

$$\boxed{A'_\varphi(\theta > \theta_c) = A_{S\varphi}(\theta, \varphi) - A_\varphi(\pi)}$$

$$\boxed{A_{S\varphi}(\theta \to 0^+) = A_\varphi(0) = A_{N\varphi}(\theta \to 0^+)}$$

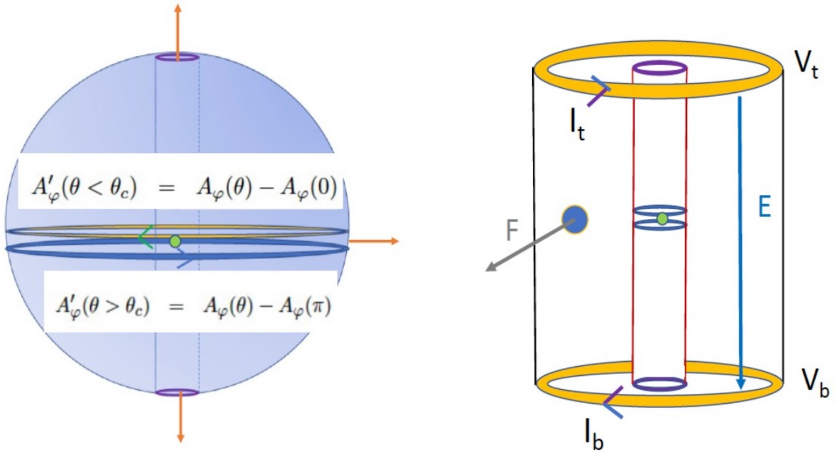
$\theta_c \to 0$

$$\iint_{north'} \nabla \times \mathbf{A} \cdot d^2\mathbf{s} = \int_0^{2\pi} \left(A_{N\varphi}(\theta,\varphi) - A_\varphi(0)\right) d\varphi = 0$$

$$\iint_{south'} \nabla \times \mathbf{A} \cdot d^2\mathbf{s} = -\int_0^{2\pi} \left(A_{S\varphi}(\theta,\varphi) - A_\varphi(\pi)\right) d\varphi$$

$$\boxed{C = A_\varphi(\pi) - A_\varphi(0)}$$

A'_φ introduces a handle



Aφ smooth everywhere

Moving back to the equator, the smoothness of vector fields in each area ensures that $A_{S\varphi}(\theta,\varphi) = A_{N\varphi}(\theta,\varphi) = A_\varphi(\theta)$

Indeed, this is equivalent to say that the relation $C = A_\varphi(\pi) - A_\varphi(0)$ is valid for any choice of angle θ

Fractional Topological Number

Karyn Le Hur, Review arXiv: 2209.15381

$$|\psi(\pi)\rangle = \frac{1}{\sqrt{2}}(|\Phi_+\rangle_1|\Phi_-\rangle_2 + |\Phi_-\rangle_1|\Phi_+\rangle_2)$$

$$A_{j\varphi}(\pi) = -i\langle\psi(\pi)|\partial_{j\varphi}|\psi(\pi)\rangle = \frac{A_{j\varphi}(0)}{2} + \frac{A^{r=0}_{j\varphi}(\pi)}{2},$$

$$A_{j\varphi}(0) = -i\langle\psi(0)|\partial_{j\varphi}|\psi(0)\rangle$$

$$|\psi(0)\rangle = |\Phi_+\rangle_1|\Phi_+\rangle_2$$

$$A^{r=0}_{j\varphi}(\pi) = -i_{r=0}\langle\psi(\pi)|\partial_{j\varphi}|\psi(\pi)\rangle_{r=0}$$

$$|\psi(\pi)\rangle_{r=0} = |\Phi_-\rangle_1|\Phi_-\rangle_2$$

$$\boxed{A^{r=0}_{j\varphi}(\pi) - A_{j\varphi}(0) = q = 1}$$

$$A_{j\varphi}(\pi) - A_{j\varphi}(0) = q\frac{1}{2} = C_j$$

$$\frac{1}{2\pi}\iint_{S^2} \boldsymbol{\nabla}_j \times \mathbf{A}_j \cdot d^2\mathbf{s} = \frac{q}{2}$$

$\frac{1}{2}$-Skyrmion

$$A_{j\varphi}(\pi) = -i\langle\psi(\pi)|\partial_{j\varphi}|\psi(\pi)\rangle = \frac{A_{j\varphi}(0)}{2} + \frac{A_{j\varphi}^{r=0}(\pi)}{2},$$

$$A_{j\varphi}^{r=0}(\pi) - A_{j\varphi}(0) = q = 1 = \langle\sigma_{jz}(0)\rangle$$

$$A_{j\varphi}(\pi) - A_{j\varphi}(0) = -\frac{A_{j\varphi}(0)}{2} + \frac{A_{j\varphi}(0)}{2} + \frac{\langle\sigma_{jz}(0)\rangle}{2}$$

$$\boxed{C_j = \frac{q}{2} = \frac{1}{2}(\langle\sigma_{jz}(0)\rangle - \langle\sigma_{jz}(\pi)\rangle)}$$

$$\langle\sigma_{jz}(\pi)\rangle = 0$$

$$A_{1\varphi}(\theta) = \sum_l |c_{+l}(\theta)|^2 A_{1\varphi}(0) + \sum_l |c_{-l}(\theta)|^2 A_{1\varphi}^{r=0}(\pi),$$

$$= -\frac{1}{2}\langle\sigma_{1z}(\theta)\rangle$$

$$A_{1\varphi}(0) = -\frac{1}{2}$$

$$A_{1\varphi}^{r=0}(\pi) = \frac{1}{2}$$

Fractional Topology as measure of EPR pair at 1 pole

$$\langle \sigma_{1z}(0)\sigma_{2z}(0)\rangle = |c_{++}(0)|^2 = \langle \sigma_{iz}(0)\rangle^2 = (2C_j)^2 = q^2 \text{ with } q = 1$$

$$\langle \sigma_{1z}(\pi)\sigma_{2z}(\pi)\rangle = |c_{++}(\pi)|^2 + |c_{--}(\pi)|^2 - |c_{+-}(\pi)|^2 - |c_{-+}(\pi)|^2$$

$$= -|c_{+-}(\pi)|^2 - |c_{-+}(\pi)|^2$$

$$= -|c_{++}(0)|^2$$

$$= -(2C_j)^2 = -q^2 = -1$$

$$\langle \sigma_{1x}(\pi)\sigma_{2x}(\pi)\rangle = c^*_{+-}c_{-+} + c^*_{-+}c_{+-}$$

$$= -\langle \sigma_{1z}(\pi)\sigma_{2z}\rangle$$

$$= (2C_j)^2 = +q^2 = +1$$

$F_j(0)$

$= 1 - |c_{++}(0)|^2 = 0$

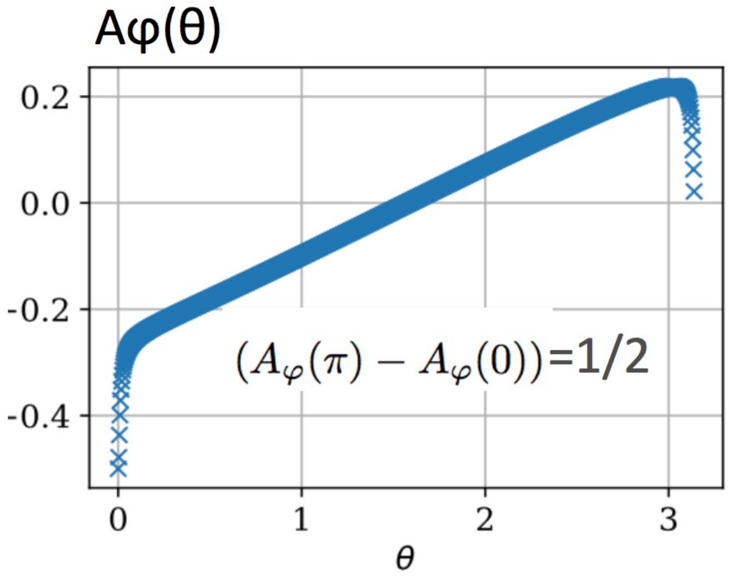
$F_j(\pi) =$

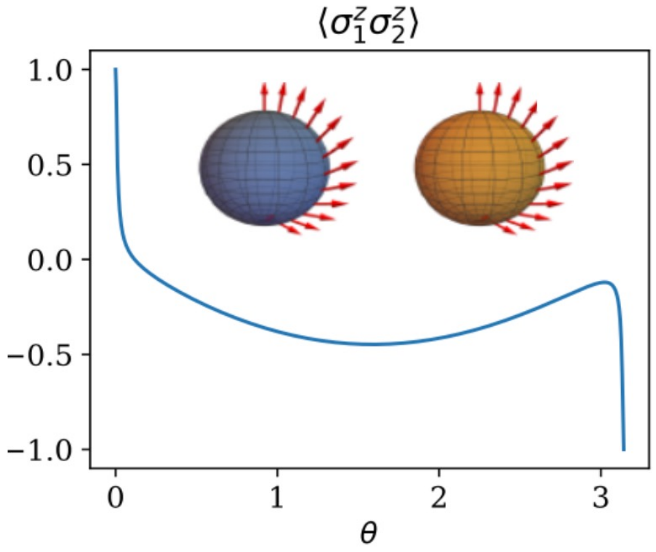
$|c_{++}(0)|^2 = (2C_j)^2$

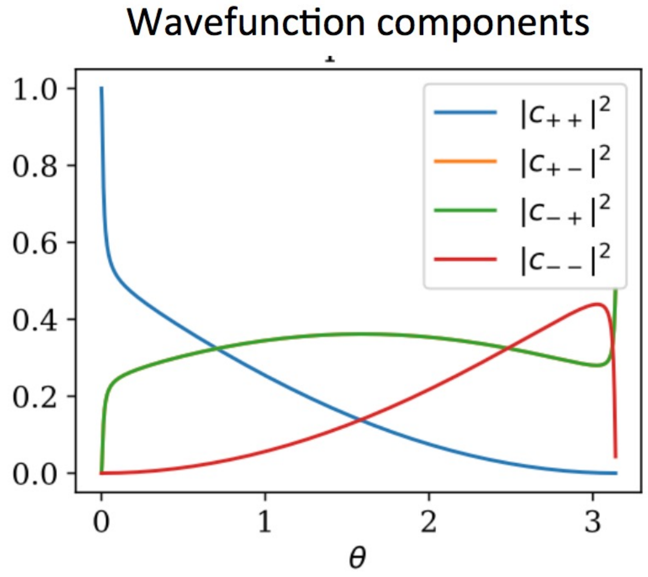
$= q^2 = 1$

Mathematical Formula/Code

$$F_1 = F_2 = |\langle \sigma_{1z}(\pi)\sigma_{2z}(\pi)\rangle| = (2C_j)^2 = 1$$





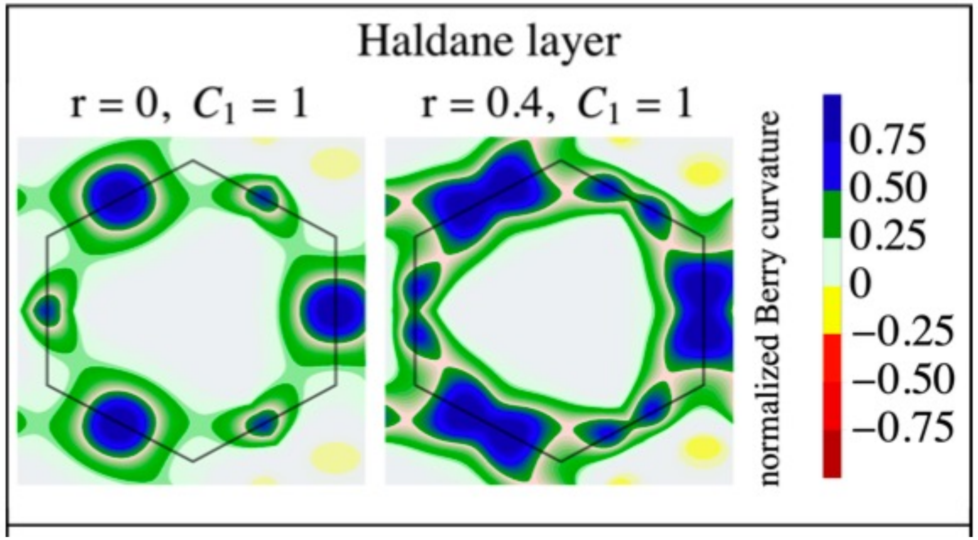


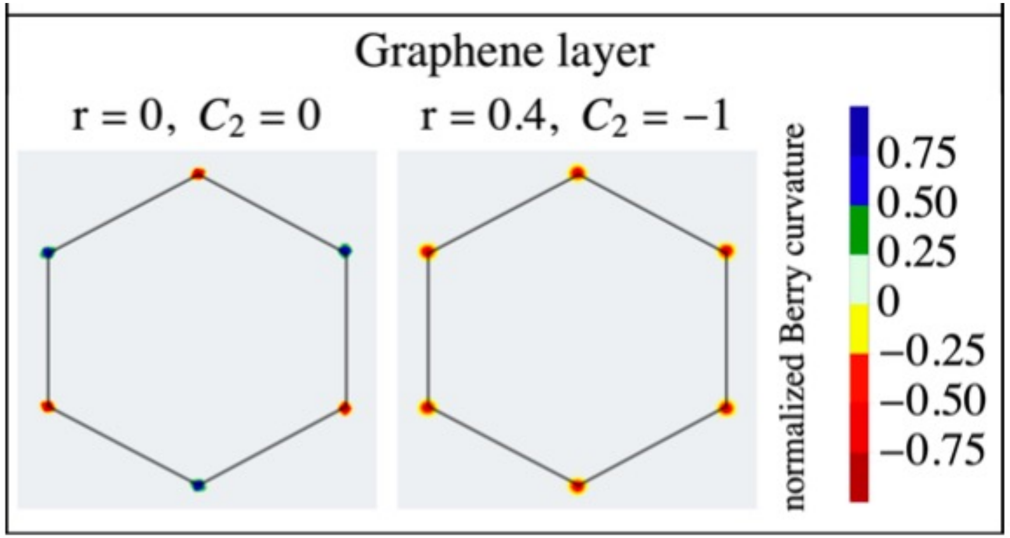
Multi-spheres analysis and disorder effects: work together with Ephraim Bernhardt (PhD Thesis, September 18th 2023)

Topological Proximity Effects

Bulk topological proximity effect: T. H. Hsieh, I. Ishuzika, L. Balents, T. Hughes, Phys. Rev. Lett. 086802 (2016)
J. Panas, B. Irsigler, J.-H. Zheng, W. Hofstetter, Phys. Rev. B 102, 075403 (2020)
Applications Material 3D : Y. Ando

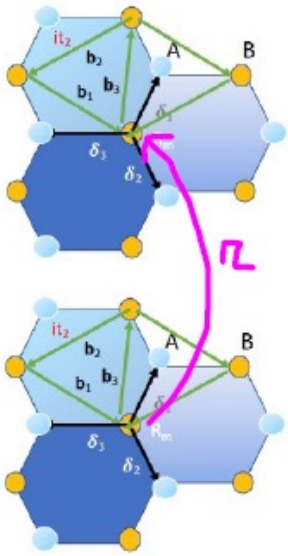
class II: Topological Proximity effect in graphene coupled to an Haldane Model (AA-BB stacking):
(thanks to DFG FOR2414 Germany for funding)





P. Cheng, Ph. W. Klein, K. Plekhanov, K. Sengstock, M. Aidelsburger, C. Weitenberg, K. Le Hur, Phys. Rev. B 100, 081107 2019

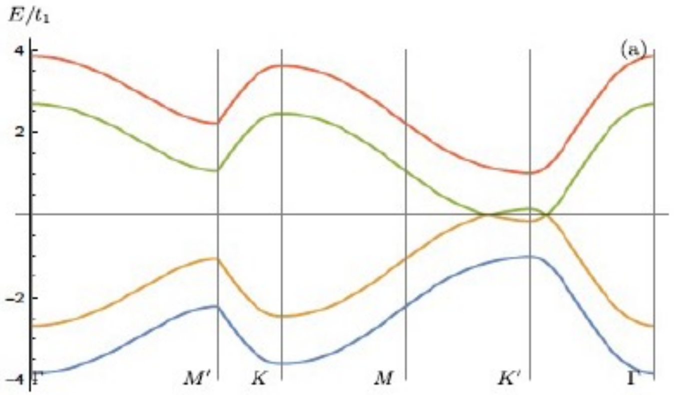
Bilayer system with M_1=M_2



$$\mathcal{H} = (\psi^\dagger_{\boldsymbol{k}1}, \psi^\dagger_{\boldsymbol{k}2})\mathcal{H}(\boldsymbol{k}) \begin{pmatrix} \psi_{\boldsymbol{k}1} \\ \psi_{\boldsymbol{k}2} \end{pmatrix},$$

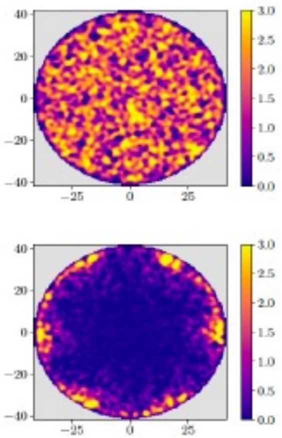
where $\psi^\dagger_{\boldsymbol{k}i} \equiv (c^\dagger_{\boldsymbol{k}Ai}, c^\dagger_{\boldsymbol{k}Bi})$ and

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} (\boldsymbol{d} + M_1\hat{z})\cdot\boldsymbol{\sigma} & r\mathbb{I} \\ r\mathbb{I} & (\boldsymbol{d} + M_2\hat{z})\cdot\boldsymbol{\sigma} \end{pmatrix},$$

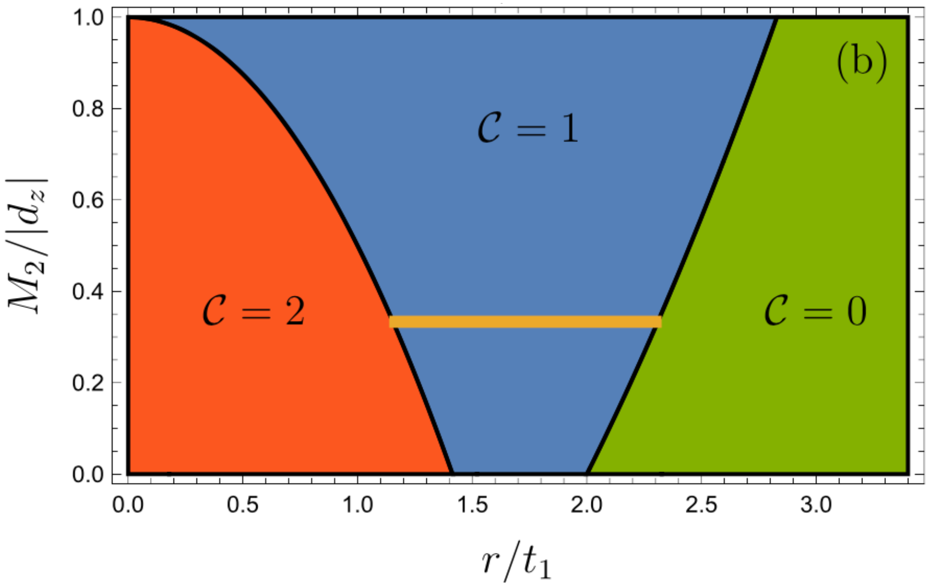


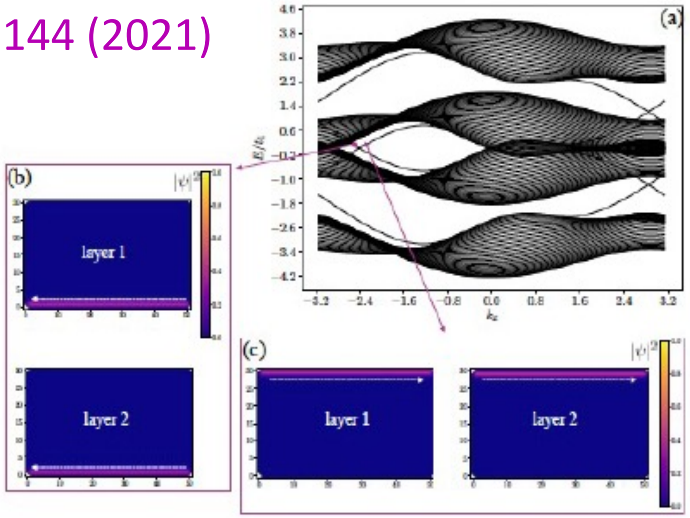
$M_1 = M_2 = M$

J. Hutchinson and K. Le Hur, Communications Physics 4 144 (2021)



Local density of states





Circular dichroism of light

Jones formalism: average 1 and 0 light responses

Topological semimetal in two dimensions

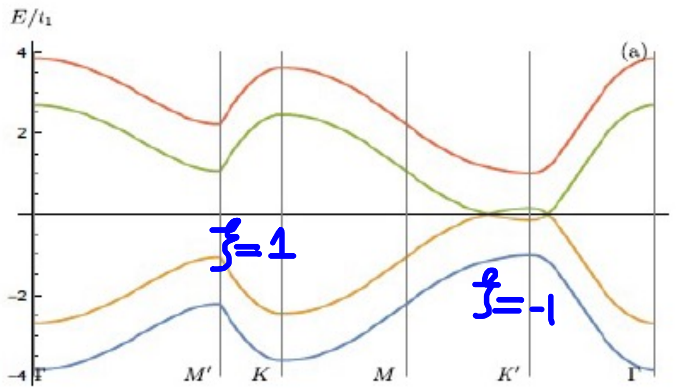
class I :

Summary of Geometry

$$(J_{xy})^j = C^j \frac{e^2}{h}$$

$C^j = \frac{1}{2}$

This formula is correct and is applicable in a sphere (plane j) from the poles (Dirac points)



$$\mathcal{H}(\mathbf{k}) = (d_z(\mathbf{k}) + M)\sigma_z \otimes \mathbb{I} + d_x(\mathbf{k})\sigma_x \otimes \mathbb{I} + d_y(\mathbf{k})\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes s_x.$$

$$|\psi_g\rangle \equiv \frac{1}{2}(c^\dagger_{A1}c^\dagger_{B1} - c^\dagger_{A1}c^\dagger_{B2} - c^\dagger_{A2}c^\dagger_{B1} + c^\dagger_{A2}c^\dagger_{B2})|0\rangle$$

$= |\psi_1(K')\rangle|\psi_2(K')\rangle$

Fractional
Topological Bloch band

$$c^\dagger_{B1}c^\dagger_{B2}|0\rangle = |\uparrow\uparrow\rangle,\ c^\dagger_{A1}c^\dagger_{A2}|0\rangle = |\downarrow\downarrow\rangle,\ c^\dagger_{B1}c^\dagger_{A2}|0\rangle = |\uparrow\downarrow\rangle,\ c^\dagger_{A1}c^\dagger_{B2}|0\rangle = |\downarrow\uparrow\rangle.$$

$$\tilde{C}^j = \frac{1}{2}\langle \underbrace{n^j_{KB} - n^j_{KA}}_{\sigma_z} - n^j_{K'B} + n^j_{K'A}\rangle = \frac{1}{2}$$

Classification & Realization in graphene with light class III

K. Le Hur and S. Al Saati, Phys. Rev. B 107, 165407 2023
S. Al Saati and K. Le Hur, Quantum Transport within Band Theory

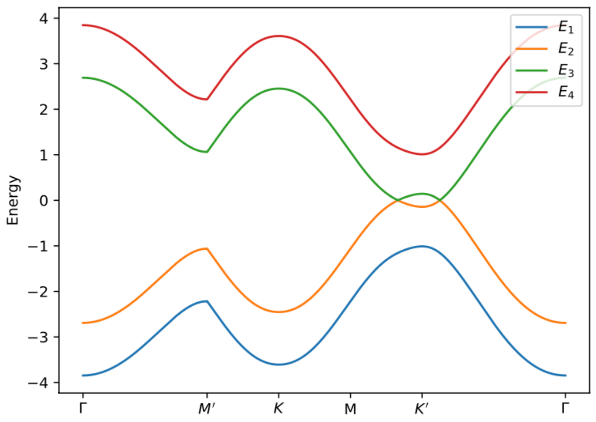
Honeycomb lattice

$$H = \sum_{\mathbf{k}} \psi^\dagger(\mathbf{k})\mathcal{H}(\mathbf{k})\psi(\mathbf{k})$$

$$\psi(\mathbf{k}) = (c_{A\mathbf{k}\uparrow}, c_{B\mathbf{k}\uparrow}, c_{A\mathbf{k}\downarrow}, c_{B\mathbf{k}\downarrow})$$

$\uparrow = 1$
$\downarrow = 2$

$$\mathcal{H}(\mathbf{k}) = (d_z(\mathbf{k}) + M)\sigma_z \otimes \mathbb{I} + d_x(\mathbf{k})\sigma_x \otimes \mathbb{I} + d_y(\mathbf{k})\sigma_y \otimes \mathbb{I} + r\mathbb{I} \otimes s_x.$$



$$\boxed{\mathcal{H}(\mathbf{k})^2 = (|\mathbf{d}(\mathbf{k})|^2 + r^2)\mathbb{I} \otimes \mathbb{I} + 2r\mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma} \otimes s_x}$$

$$|\psi_1\rangle = |\psi_-\rangle \otimes |-\rangle_x$$
$$E_1 = -(r + |\mathbf{d}|)$$

$$|\psi_4\rangle = |\psi_+\rangle \otimes |+\rangle_x$$
$$E_4 = (r + |\mathbf{d}|)$$

$H^2 = 0$

$$\tilde{d}_z = 3\sqrt{3}t_2$$

$$v_F^2|\mathbf{p}|^2 = r^2 - (\tilde{d}_z - M)^2$$

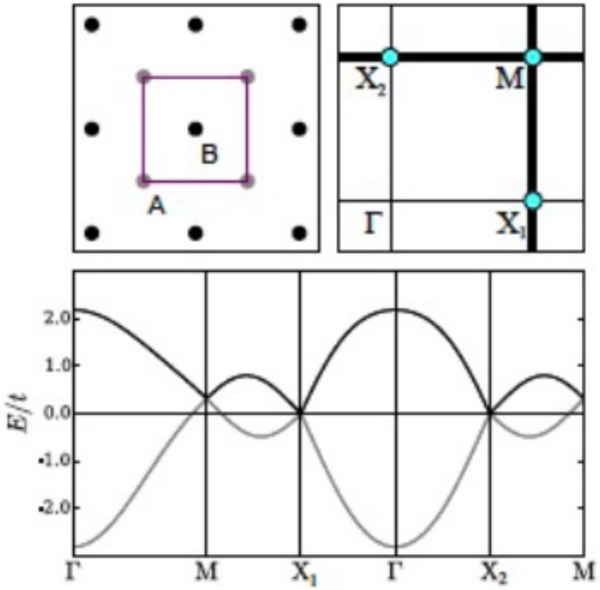
	band 2	band 3
K	$E_2(K) = r - \lvert\mathbf{d}(K)\rvert$ $\lvert\psi_2(K)\rangle = \lvert\psi_-\rangle \otimes \lvert+\rangle_x$	$E_3(K) = -r + \lvert\mathbf{d}(K)\rvert$ $\lvert\psi_3(K)\rangle = \lvert\psi_+\rangle \otimes \lvert-\rangle_x$
K'	$E_2(K') = -r + \lvert\mathbf{d}(K')\rvert$ $\lvert\psi_2(K')\rangle = \lvert\psi_+\rangle \otimes \lvert-\rangle_x$	$E_3(K') = r - \lvert\mathbf{d}(K)\rvert$ $\lvert\psi_3(K')\rangle = \lvert\psi_-\rangle \otimes \lvert+\rangle_x$

Table & topological semimetals

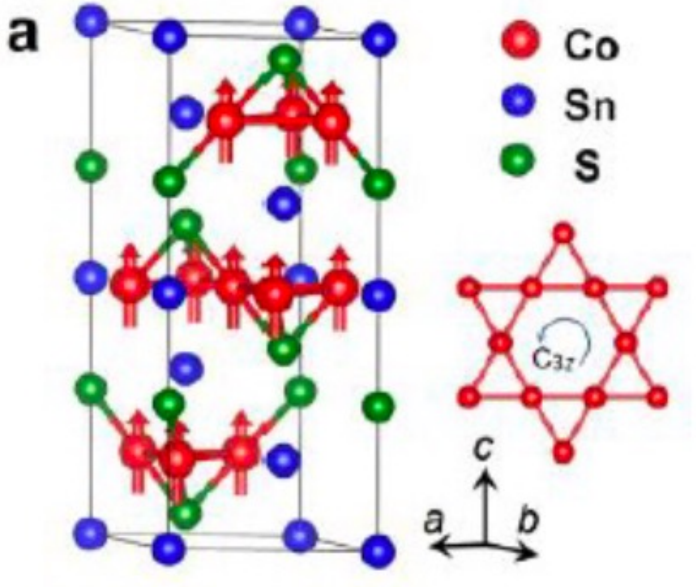
Dirac Semimetals in Two Dimensions

S. M. Young & C. L. Kane, PRL 2005

$$H = 2t\tau_x \cos\frac{k_x}{2}\cos\frac{k_y}{2} + t_2(\cos k_x + \cos k_y) + t^{\mathrm{SO}}\tau_z[\sigma_y \sin k_x - \sigma_x \sin k_y],$$

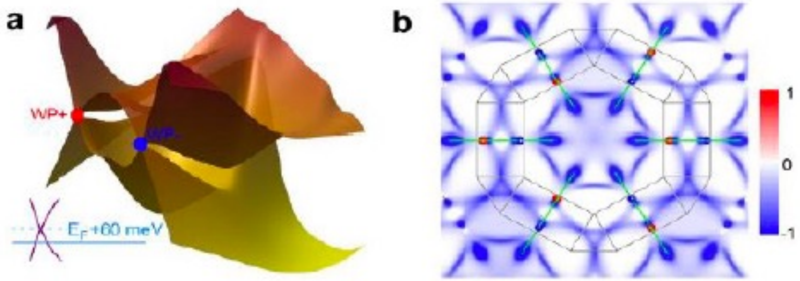


ferro



Magnetic Weyl semimetals in 3D: recent

Z. Guguchia et al., group of Zahid Hasan, Princeton
E. Liu et al., group of Claudia Felser Dresden
Figures from Enke Liu, Berry curvature k_x-k_y plane



Anomalous Hall conductivity

2D model

Julian Legendre & Karyn Le Hur
Phys. Rev. Research, 2020

Quantum Hall conductivity linked to magnetism

2001

Kitaev p-wave Superconductor

Class II

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2}\sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.),$$

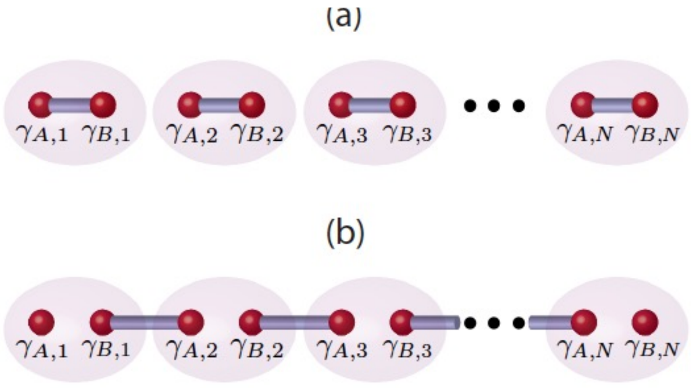
$$\boxed{\mathcal{H}_k = \mathrm{h}(k) \cdot \boldsymbol{\sigma}}$$

$C_k^{\dagger} = (c_k^{\dagger}, c_{-k})$

$$H = \frac{1}{2}\sum_{k \in BZ} C_k^\dagger \mathcal{H}_k C_k, \quad \mathcal{H}_k = \begin{pmatrix} \epsilon_k & \tilde{\Delta}_k^* \\ \tilde{\Delta}_k & -\epsilon_k \end{pmatrix},$$

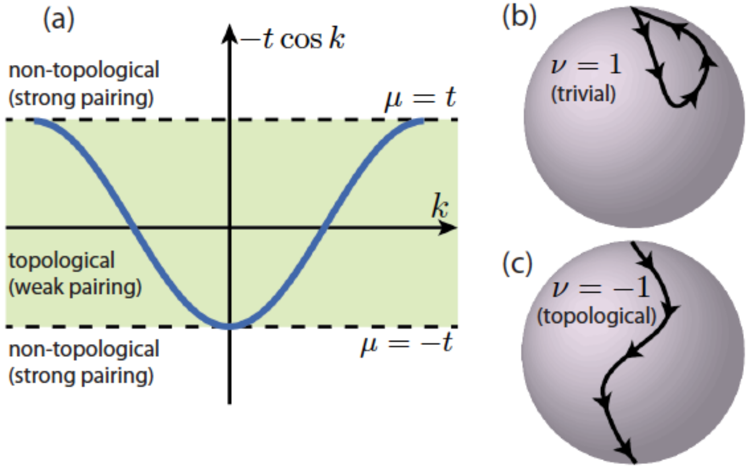
with $\epsilon_k = -t\cos k - \mu$ the kinetic energy and $\tilde{\Delta}_k = -i\Delta e^{i\phi}\sin k$ the Fourier-transformed pairing potential. The

$$c_x = \frac{e^{-i\phi/2}}{2}(\gamma_{B,x} + i\gamma_{A,x}).$$



1937

Majorana fermions



Review: J. Alicea, arXiv:1202.1293

For two spheres's model, one Majorana fermion free at a pole, K. Le Hur, Review, arXiv:2209.15381 Section IXC

Two superconducting p-wave wires coupled through a Coulomb force

Loic Herviou, Christophe Mora, Karyn Le Hur 2017

Frederick del Pozo, Loic Herviou, Karyn Le Hur, Phys. Rev. B 107, 155134 (2023)

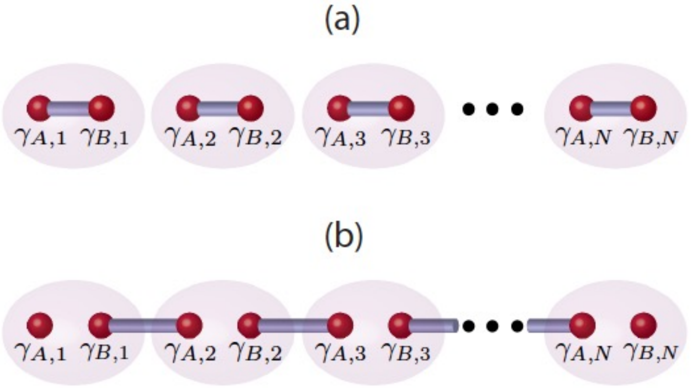
$$H_{int} = gA^{-1}\sum_{\triangle p}\left[\sum_k c^{1\dagger}_{k+\triangle p}c^1_k\right]\left[\sum_q c^{2\dagger}_{q-\triangle p}c^2_q\right]$$

$S^z_k = c^\dagger_k c_k - c_{-k} c^\dagger_{-k}$

$$H_{int} = \frac{gA^{-1}}{4}\sum_{q,k} S^{z,1}_k S^{z,2}_q$$

(2)

$$= \frac{gA^{-1}}{4}\left(\boxed{\sum_{q,k}\delta_{k,q}S^{z,1}_k S^{z,2}_q} + \sum_{q\neq k} S^{z,1}_k S^{z,2}_q\right).$$



(2) $\sum_{k\neq q} = \frac{1}{2}\sum_k \sum_{q\neq k} + \frac{1}{2}\sum_q \sum_{k\neq q}$

chemical potential

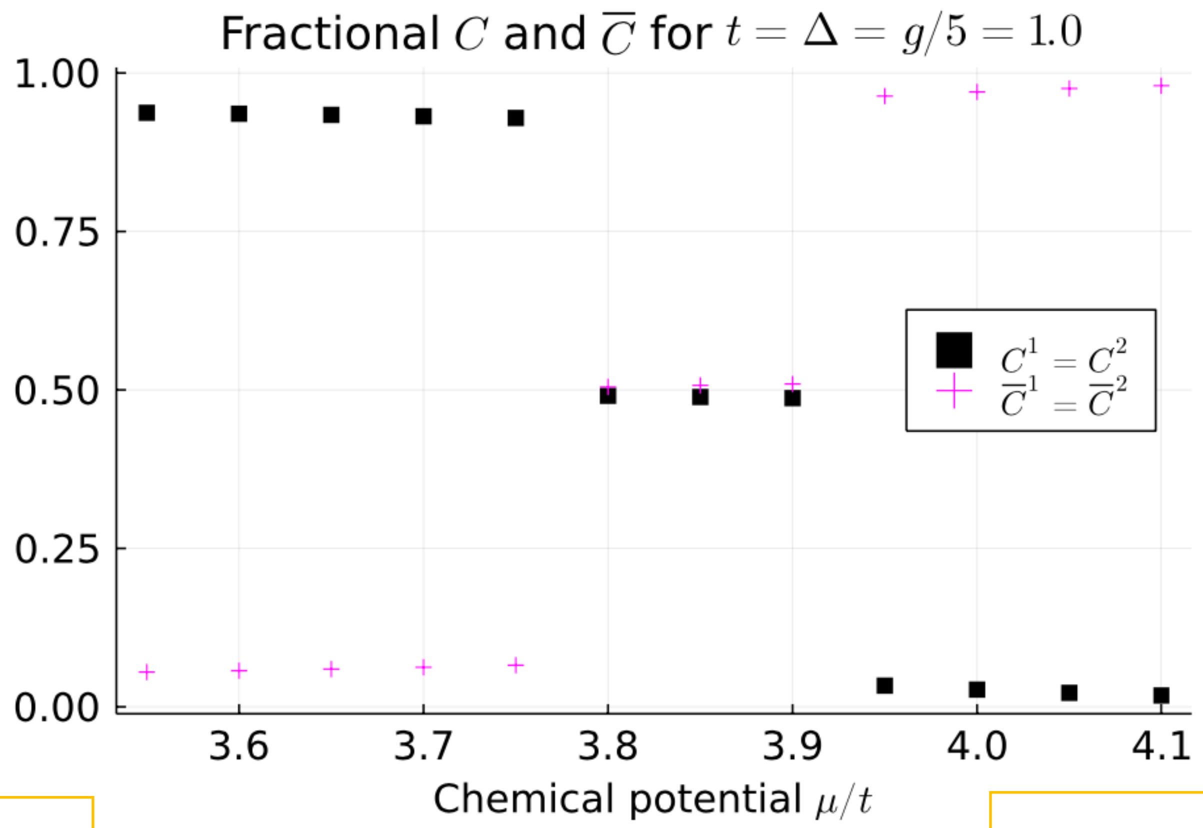
Mapping : from two spheres to two wires (weak interactions)

$$\mathfrak{S}_i^z = \frac{1}{\sqrt{N}} \sum_j \left(c_j^\dagger c_{j+i} - c_j c_{j+i}^\dagger \right)$$

DMRG Results

$$|GS\rangle = \chi_{01} |\mathrm{GS}\rangle_{01} + \chi_{10} |\mathrm{GS}\rangle_{10},$$

Frederick del Pozo, Loic Herviou, Karyn Le Hur, Phys. Rev. B 107, 155134 (2023)



Disorder analysis in wires:
Work together with
Frederick del Pozo
Olesia Dmytruk
At CPHT, Ecole Polytechnique

$$S^z_{ka=0} = \frac{1}{\sqrt{N}} \sum_i \mathfrak{S}_i^z$$

$$S^z_{ka=\pi} = \frac{1}{\sqrt{N}} \sum_i (-1)^i \, \mathfrak{S}_i^z$$

$$\frac{1}{2} \left\langle \left(S^z_{k=0} - S^z_{k=\frac{\pi}{a}} \right) \right\rangle \equiv C = \frac{1}{\sqrt{N}} \sum_{i=0}^{N/2} \langle \mathfrak{S}^z_{2i+1} \rangle$$

$$\frac{1}{2} \left\langle \left(S^z_{k=0} + S^z_{k=\frac{\pi}{a}} \right) \right\rangle \equiv \overline{C} = \frac{1}{\sqrt{N}} \sum_{i=0}^{N/2} \langle \mathfrak{S}^z_{2i} \rangle \, .$$

Karyn Le Hur

Centre de Physique Theorique, Ecole Polytechnique and CNRS

4 classes Saclay Lectures Series: 1h30 each

Thanks to Sylvain Ravets, Igor Ferrier-Barbut, Benoit Valiron for invitation

Institut d'Optique Graduate School

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- Class I: Quantum Geometry, Information and Topological Physics from Bloch Sphere (June 9)
- Class II: Application in Topological Lattice Models and Quantum Matter (June 16)
- Class III: Applications in Transport and Light-Matter Interaction (June 23)
- Class IV: Entangled WaveFunction and Fractional Topology (June 30) √

https://www.cpht.polytechnique.fr/cpht/lehur/Karyn.LeHur.html

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