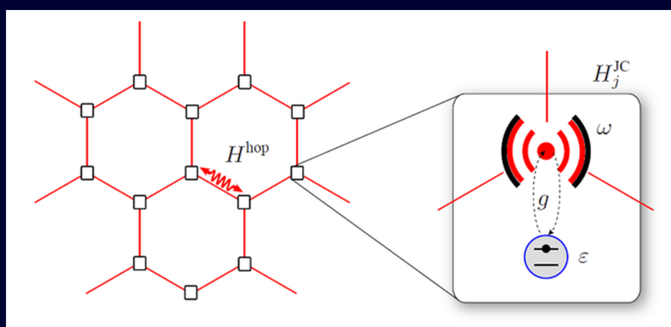
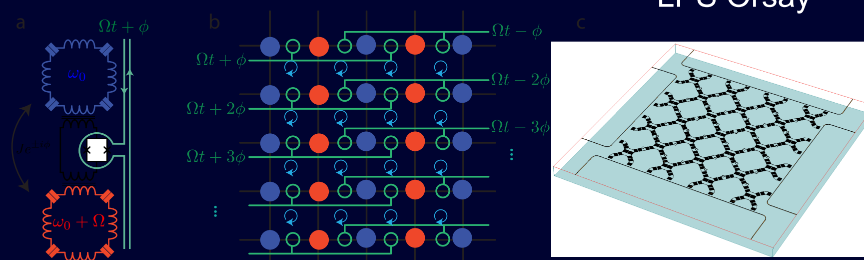
Many-Body Quantum Physics with Photons: Introduction
Grenoble the 23rd of June 2017

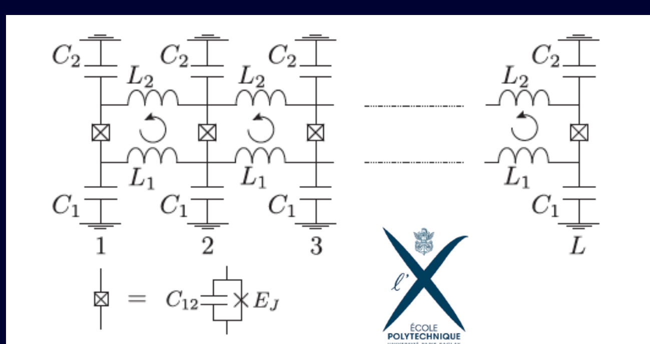
Karyn Le Hur

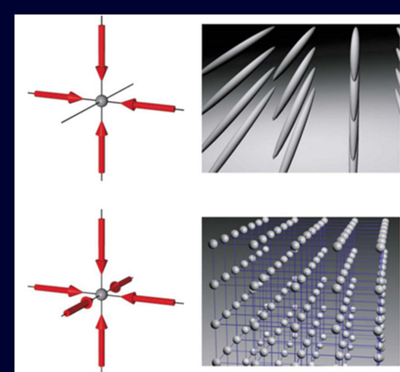
CPHT Ecole Polytechnique & CNRS

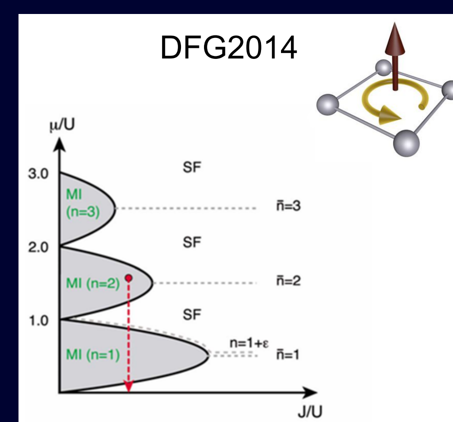


LPS Orsay



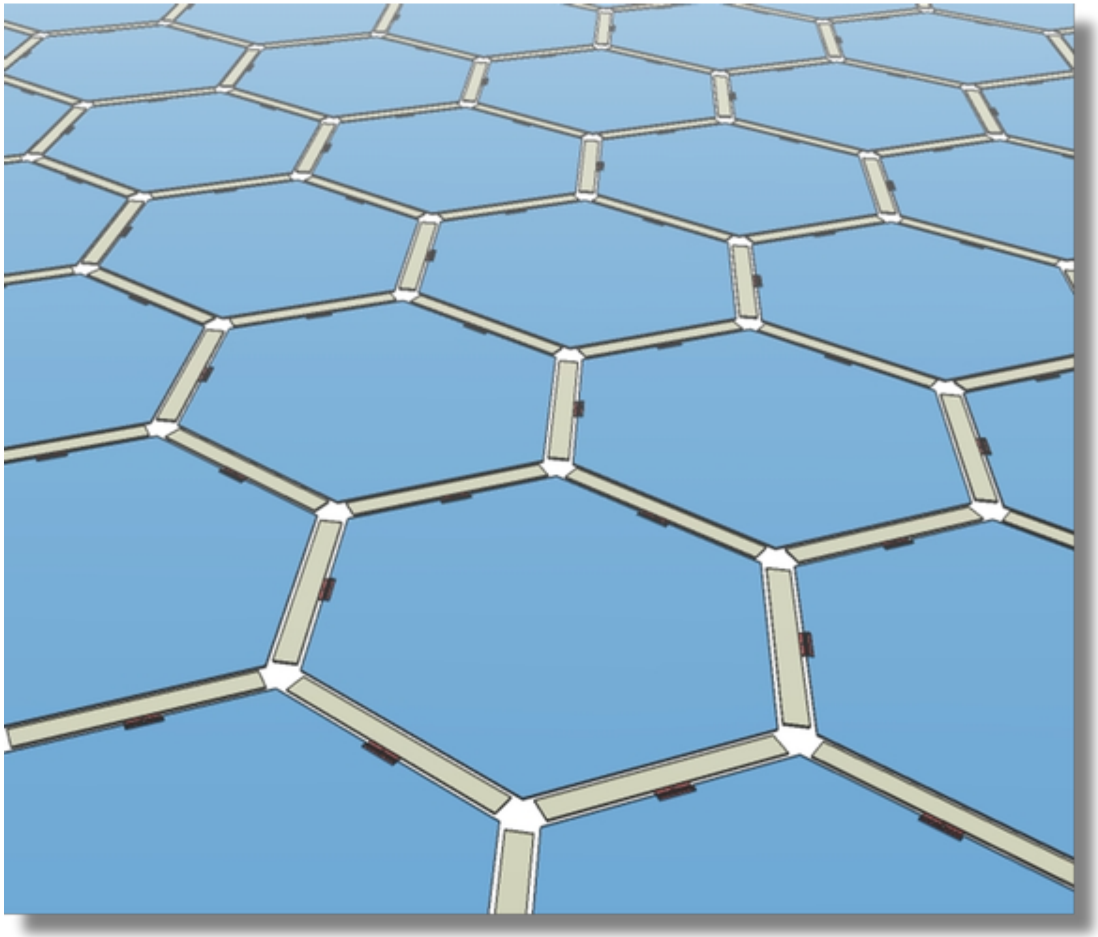






Summary: I will start from older works

Graphene in cQED (precursory work in Grenoble: B. Pannetier and R. Rammal, SC nanowires around 1985;
also Y. Xiao, P. Chaikin and D. Huse et al. 2001)



Jaynes-Cummings lattice model

Jens Koch and KLH, PRA **80**, 023811 (2009)

Artificial Gauge Fields

Jens Koch, A. Houck, KLH, S. M. Girvin
***PRA* 82**, 043811 (2010)

Relation to cold atoms: D. Jaksch & P. Zoller;
I. Spielman; F. Gerbier and J. Dalibard;
T. Stanescu, V. Galitski & S. das Sarma

Topological phases

From old to new results on cQED
and Josephson junctions

Quantum impurities

2 recent reviews

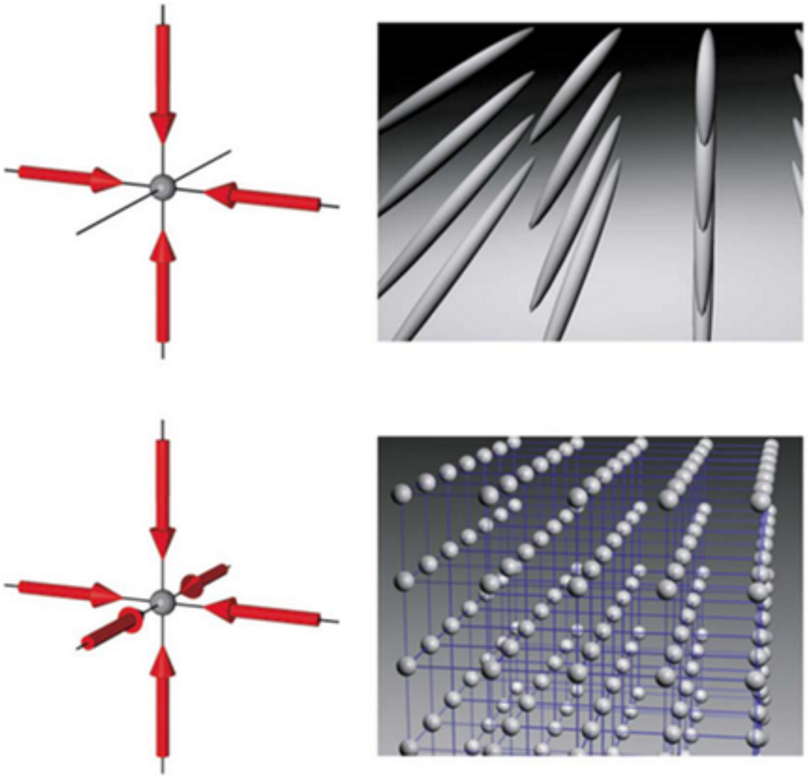
KLH, L. Henriet, L Herviou, K. Plekhanov, A. Petrescu, T. Goren, M. Schiro, C. Mora, P. Orth, arXiv: 1702.05135

KLH, L. Henriet, A. Petrescu, G. Roux, M. Schiro   C. R. Physique 17 (2016) 808-835

Bose-Hubbard model

I. Bloch, J. Dalibard, W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)

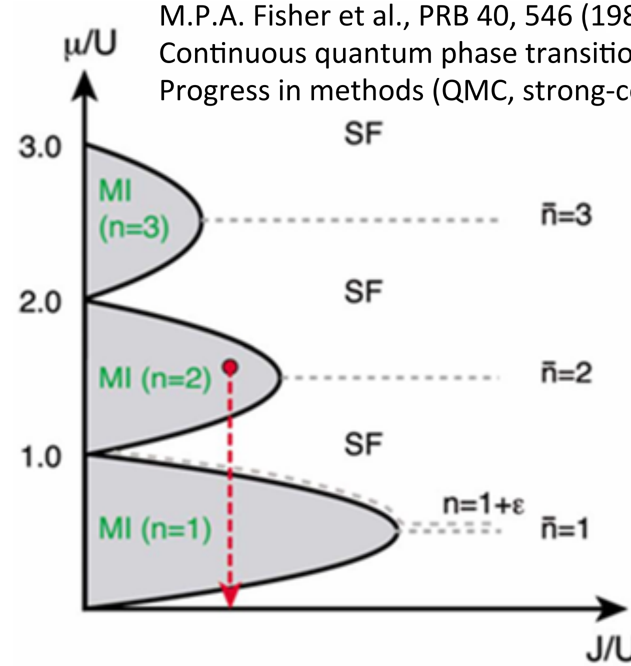
D. Jaksch et al., Phys. Rev. Lett. **81**, 3108 (1998)
M. Greiner et al., Nature **415**, 39 (2002)



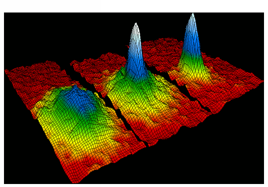
e.g., realization of the Bose-Hubbard model:

$$H = \sum_j [-\mu a_j^\dagger a_j + \frac{1}{2} U n_j(n_j - 1)] - J \sum_{\langle i,j \rangle} (a_j^\dagger a_i + \text{h.c.})$$

M.P.A. Fisher et al., PRB 40, 546 (1989)
Continuous quantum phase transition (second order) in 2D
Progress in methods (QMC, strong-coupling, …)

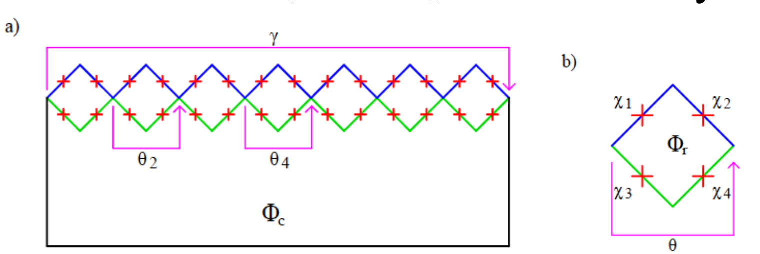


Nobel prize 2001



Bose-Einstein Condensate (microK)

Also Josephson junction arrays

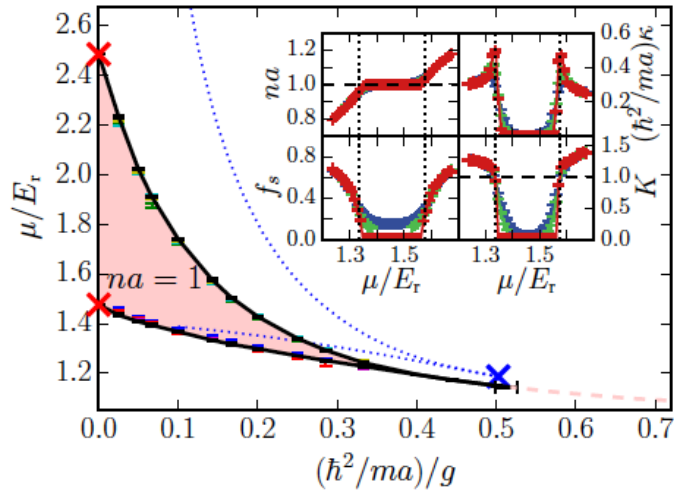


See les Houches lectures: D. Esteve & D. Vion; M. Devoret; J. Martinis & Kevin Osborne 2004
Ioan Pop et al. (Rhombi): Neel Grenoble 2008

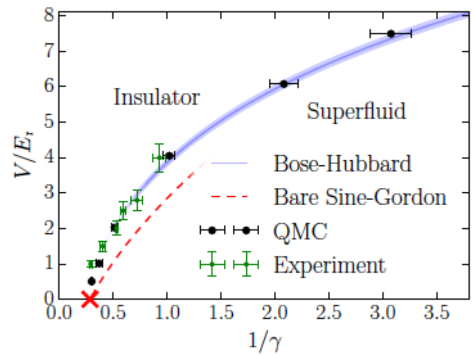
Mott 1D….

disorder ; Alain Aspect,
Vincent Josse, Philippe Bouyer
Juliette Billy…
Anderson localization

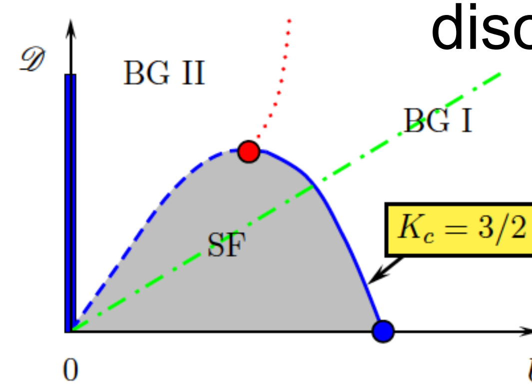
Kosterlitz-Thouless transition



1D: interactions are included
in fluid Luttinger description
K is the Luttinger parameter
Haldane 1981 (K=1 Tonks limit)



disorder



H. J. Schulz & T. Giamarchi 1988
Z. Ristivojevic, A. Petkovic, (Toulouse)
T. Giamarchi, P. Le Doussal (ENS) 2012
E. Altman, Y. Kafri, A. Polkovnikov, G. Refael

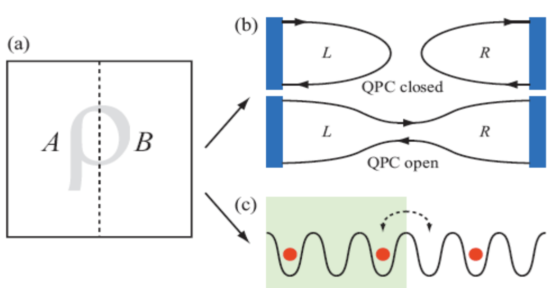
2016
Experiment Modugno, Florence. Theory & numerics T. Giamarchi (Geneva), L. Sanchez Palencia CPHT Ecole Polytechnique

New probes: bi-partite entanglement
Entropies, entanglement spectrum
Linked with conformal field theory
(John Cardy, P. Calabrese)

$$\mathcal{F}_{\mathcal{A}} = \Big\langle \Big(\sum_{i \in \mathcal{A}} \mathcal{O}_i \Big)^2 \Big\rangle - \Big\langle \sum_{i \in \mathcal{A}} \mathcal{O}_i \Big\rangle^2, \qquad \mathcal{F}(L) = \frac{K}{\pi^2} \ln L + \text{cst},$$

Critical coupling strength

Kc=2



Year	Reference	Technique	Observable	Estimate
1991	Krauth [5]	(approximate) Bethe Ansatz		$1/(2\sqrt{3}) \simeq 0.2887$
1992	Batrouni *et al.* [6]	QMC	Superfluid stiffness	0.2100(100)
1994	Elesin *et al.* [7]	Exact Diagonalization	Gap	0.2750(50)
1996	Kashurnikov *et al.* [8]	QMC	Gap	0.3000(50)
1999	Elstner *et al.* [9]	Strong coupling	Gap	0.2600(100)
2000	Kühner *et al.* [10]	DMRG	Correlation function	0.2970(100)
2008	Zakrzewski *et al.* [11]	Time Evolving Block Decimation	Correlation function	0.2975(5)
2008	Laüchli *et al.* [12]	DMRG	von Neuman entropy	0.2980(50)
2008	Roux *et al.* [13]	DMRG	Gap	0.3030(90)
2011	Ejima *et al.* [14]	DMRG	Correlation function	0.3050(10)
2011	Danshita *et al.* [15]	Time Evolving Block Decimation	Excitation spectrum	0.3190(10)
2011	This work	DMRG	Bipartite Fluctuations	0.2989(2)

PhD H. Francis Song, Yale 2011
PhD Loic Herviou CPHT X and ENS 2017

S. Rachel, N. Laflorencie (Toulouse), H. F. Song, and K. Le Hur 108, 116401 (2012)

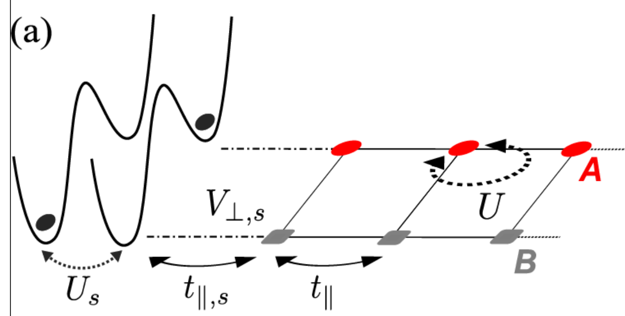
Microscope:
Example of spin-boson model

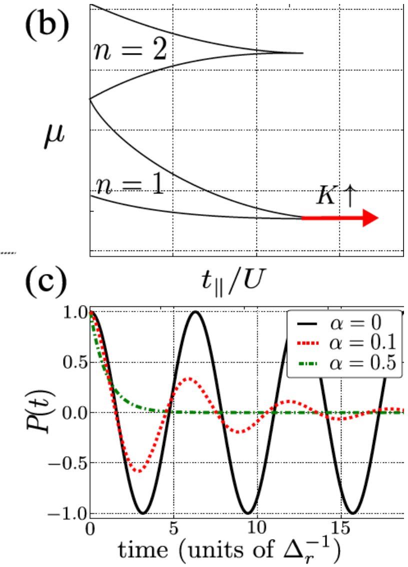
The dynamics of the qubit can be computed using a stochastic schrodinger approach
in the BEC phase (tip of Mott lobe).
In Mott, perfect Rabi oscillations

Work with P. Orth (PhD Yale 2011), A. Imambekov Yale
PhD thesis of Loic Henriet, CPHT 2016

Efforts in Grenoble, S. Florens
Experiments: W. Guichard, N. Roch, O. Buisson, C. Naud⋯

Recent review 1702.05135 (part 3.4)

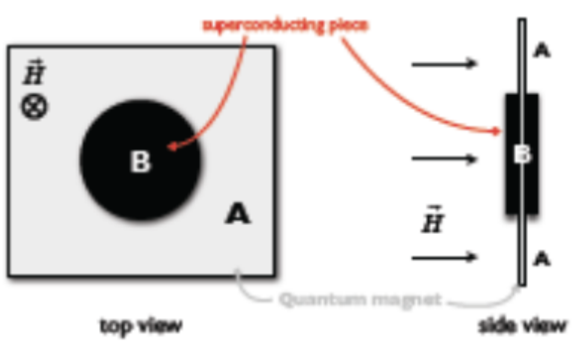




2D Heisenberg antiferromagnet:

Entanglement entropy & F-number from spin wave analysis
H. F. Song, N. Laflorencie, S. Rachel and K. Le Hur, PRB 2011
See also: A. Kallin, R. Melko, M. Hastings et al. 2011; M. Metlitski & T. Grover, 2011;…

S. Rachel, N. Laflorencie (Toulouse), H. F. Song, and K. Le Hur 108, 116401 (2012)



EPR pairs

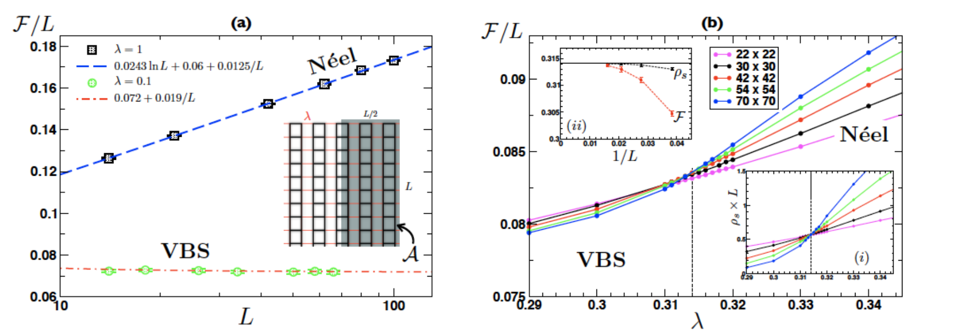


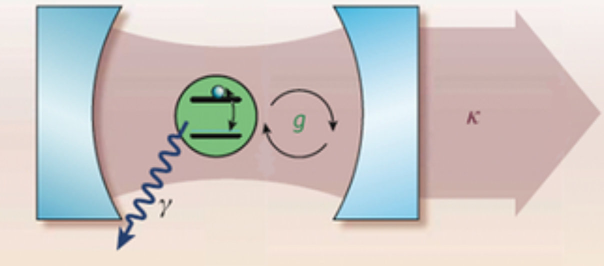
FIG. 4: (color online). Quantum Monte Carlo results for $T=0$ fluctuations $\mathcal{F}$ of the total magnetization in a region $\mathcal{A}$ for 2D coupled spin-$\frac{1}{2}$ ladders [Eq. (5)], depicted in the inset of (a). Left (a): $\mathcal{F}/L$ increases logarithmically with L in the Néel regime (black squares $\lambda=1$) whereas it saturates to a constant in the valence bond state (green circles $\lambda=0.1$). Right (b): $\mathcal{F}/L$, plotted vs. λ for various system sizes, displays a crossing point at λ_c. Insets: (i) crossing of the stiffness $\rho_s\times L$ at λ_c for the same sizes; (ii) $1/L$ convergence of the crossing point for $\mathcal{F}$ (red squares) and ρ_s (black circles) to the critical value (horizontal black line) $\lambda_c=0.31407$ [25].

Cavity & Circuit QED: 1 mode of light …

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity

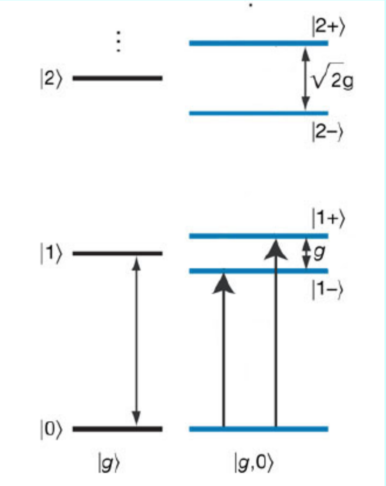
cavity QED, ENS Paris and Collège De France



$2g$ = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

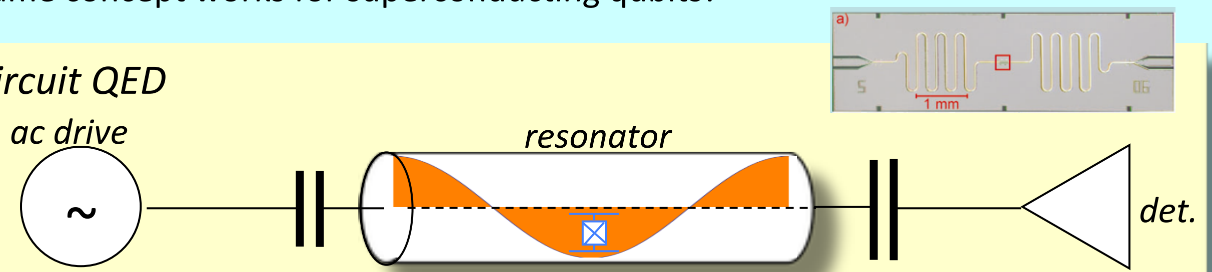
Jaynes-Cummings Hamiltonian

$$H = \tfrac{1}{2}\omega_a \sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\mathrm{drive}} + H_{\mathrm{baths}}\right)$$



- same concept works for superconducting qubits!

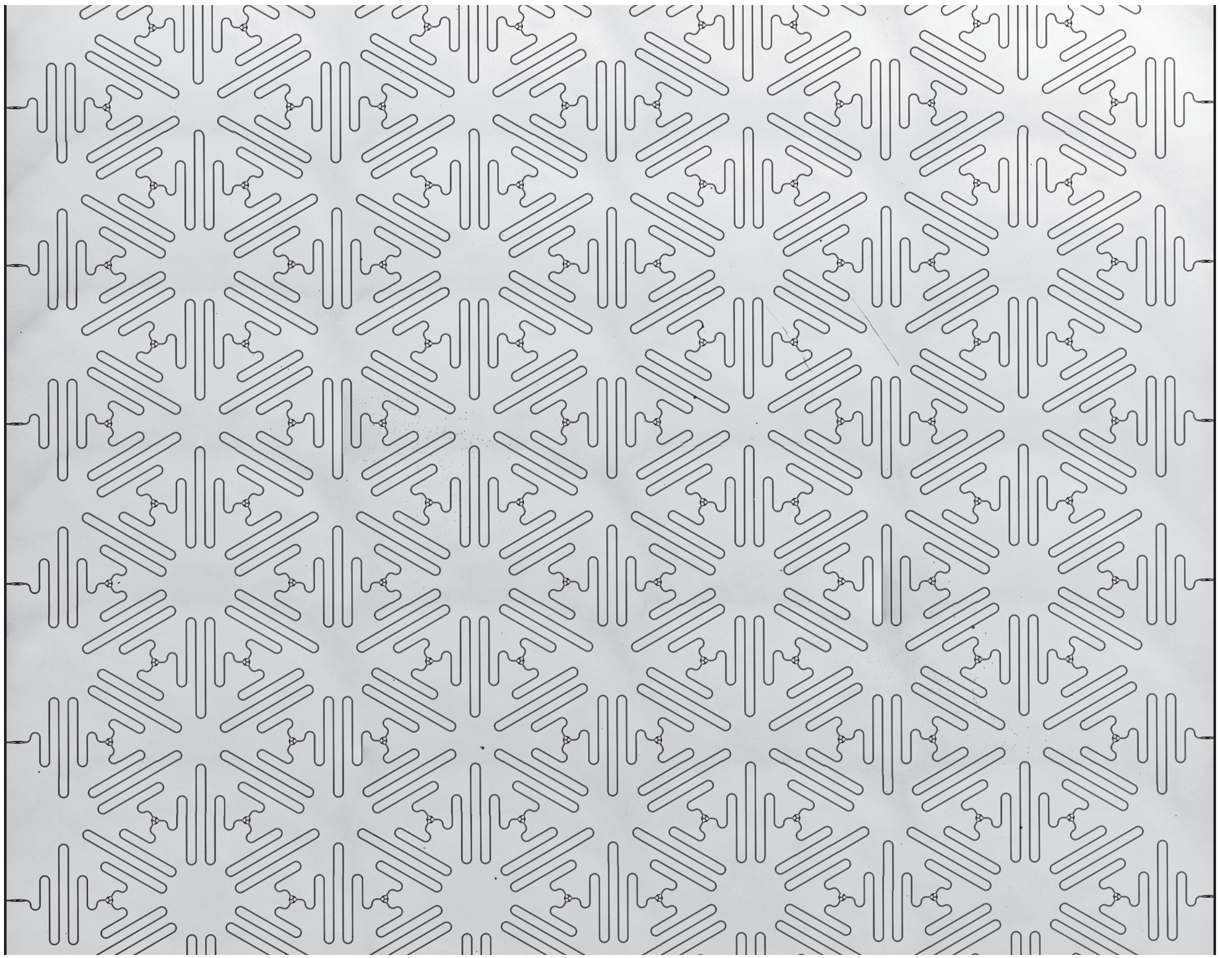
circuit QED

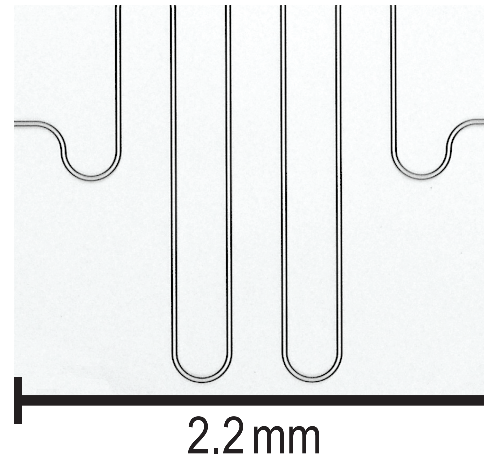


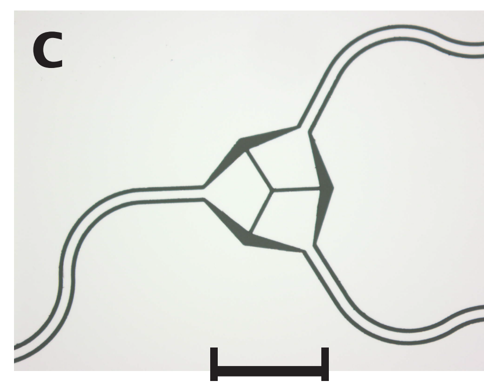
J. M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. **73**, 565 (2001); O. Buisson, F. Hekking
R. J. Schoelkopf, S. M. Girvin, Nature **451**, 664 (2008) and A. Blais et al.; D. Vion et al. (SPEC Saclay) 2002; J. Martinis

A. Houck Lab at Princeton

Niobium or Aluminium (30 and 3K for Tc)

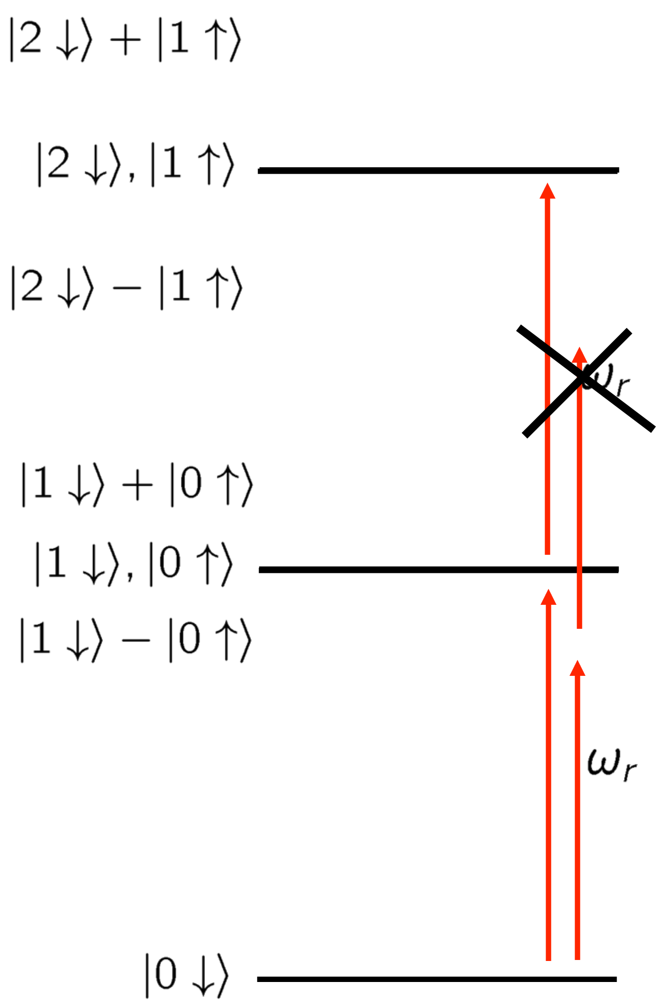






arXiv:1203.5363 (no qubit in this picture… M. Fitzpatrick et al. arXiv:1607.06895
Hopping of photons: capacitive coupling or inductive)

Photon blockade



2-level Qubit or atom — Quantized EM field

$$H = \frac{1}{2}\omega_a \sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right)$$

Dipole coupling

- single atom inside cavity can make spectrum anharmonic!
- hybridized atom/photon object is a *polariton*
- photons have to go one by one!

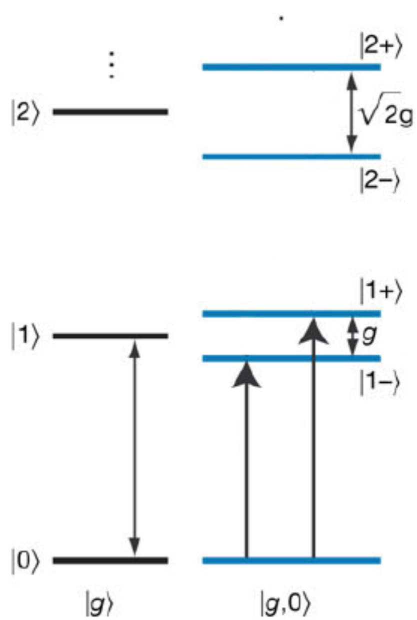
Driven light-matter Systems (AC driving): NMR (Bloch-Siegert Shift included)

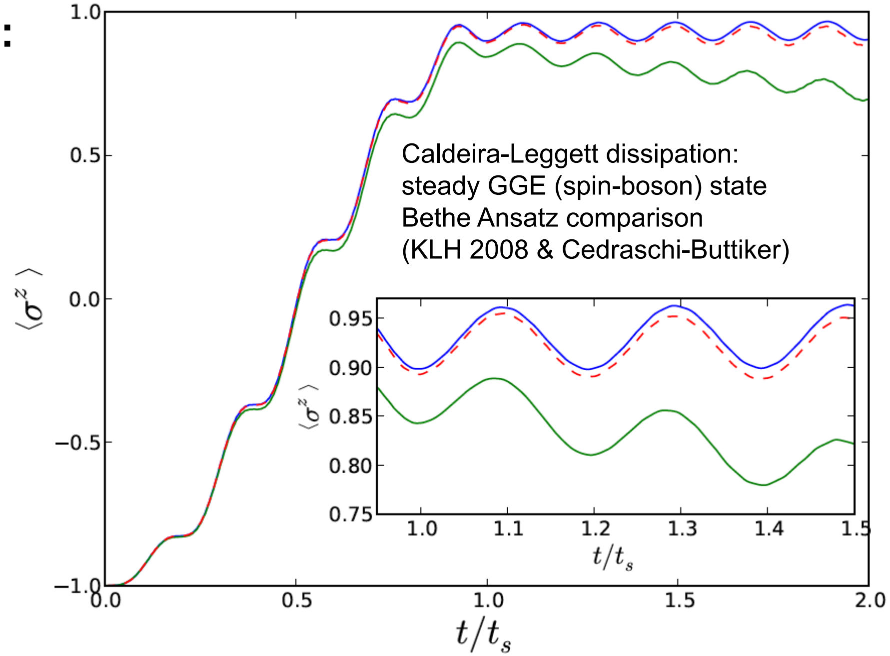
Π rotation to achieve 1 Mott polariton

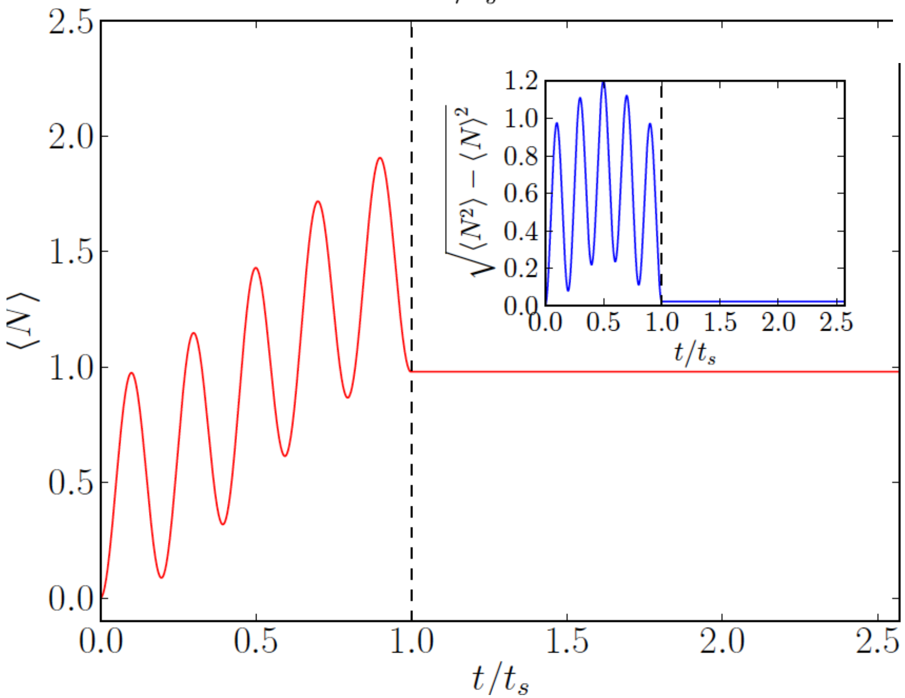
$$N = a^\dagger a + \frac{1}{2}(\sigma^z + 1)$$

Small g limit:

Jaynes-Cummings ladder

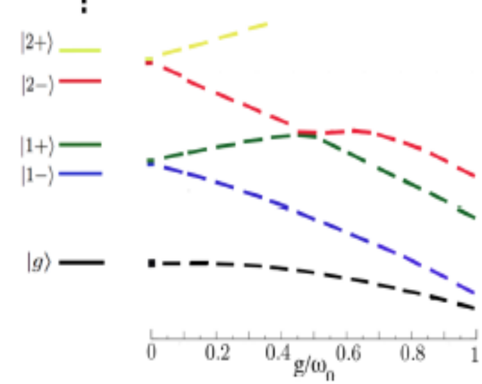






L. Henriet, PhD 2016
Z. Ristivojevic
P. P. Orth, KLH, 2014
(arXiv:1401.4558)
Stochastic approach

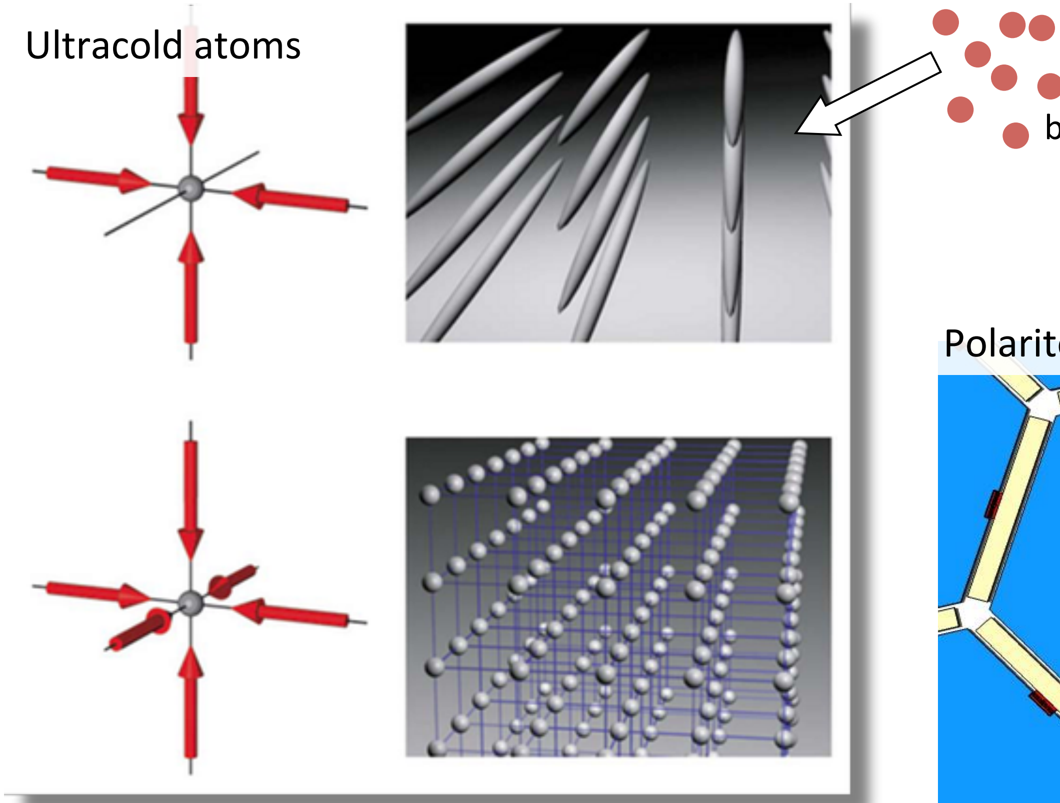
Progress in Rabi model
Braak, Moroz, Batchelor
Integrability; Marco Schiro and Hakan Tureci



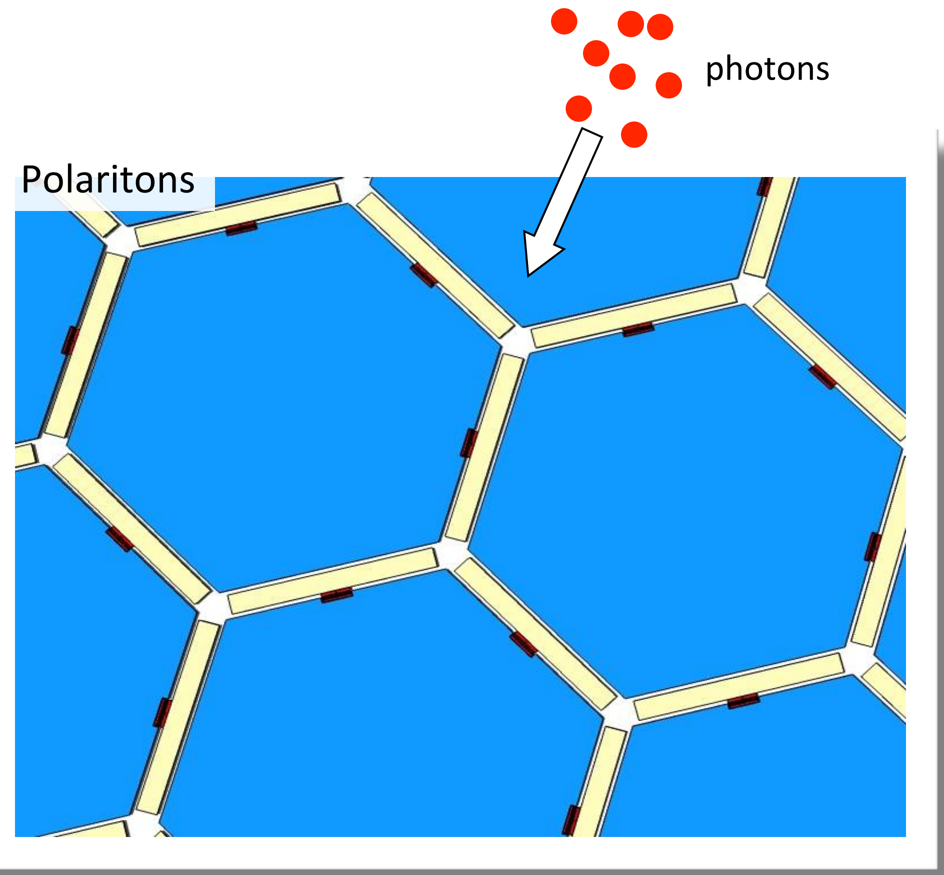
Π rotation also useful to measure Berry phase
L. Henriet, A. Sclocchi, KLH and P. Orth 2017

Related Experiments
Zuerich, 2007 (Ramsey)
Boulder, 2014
Santa Barbara 2014
Saclay

Interacting bosons on a lattice



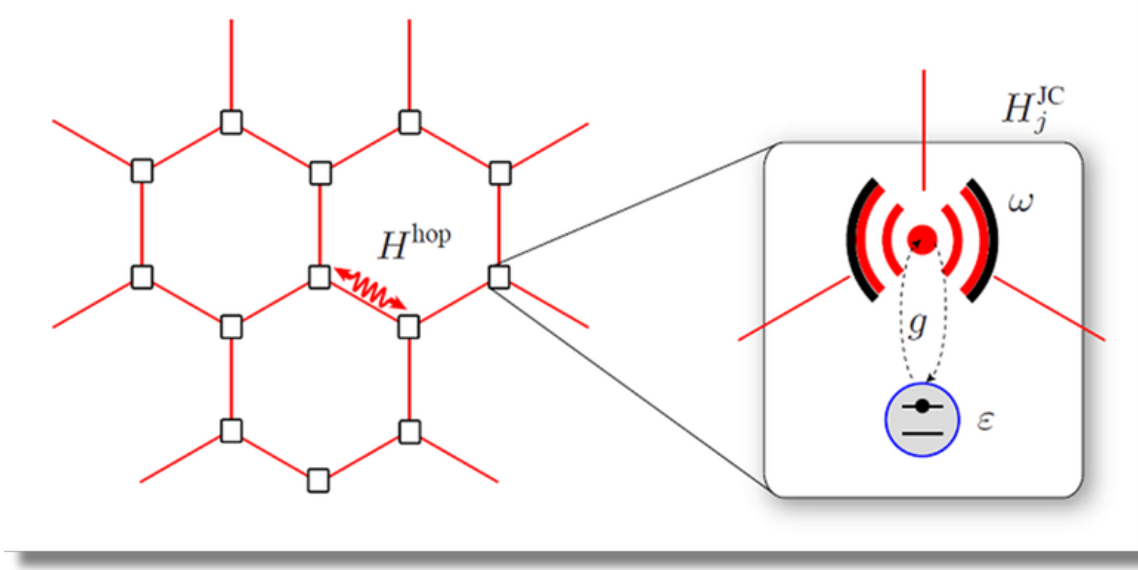
D. Jaksch et al., Phys. Rev. Lett. **81**, 3108 (1998)
M. Greiner et al., Nature **415**, 39 (2002)
I. Bloch et al., Rev. Mod. Phys. **80**, 885 (2008)



Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and Karyn Le Hur, PRA **80**, 023811 (2009)…

Can also be achieved
with cavities & Rydberg atoms

The Jaynes-Cummings "Lattice" Model



Jaynes-Cummings model: 1963
(famous model in quantum optics)

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)

Other groups: R. Fazio, G. Blatter, H. Tureci, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik,… C. Ciuti, I. Carusotto,⋯

Jaynes-Cummings lattice model $H = \sum_j H_j^{\rm JC} + H^{\rm hop} - \mu N$ "chemical potential"

- *Jaynes-Cummings:* $H_j^{\rm JC} = \omega a_j^\dagger a_j + \varepsilon \sigma_j^+ \sigma_j^- + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j)$
- *nearest-neighbor photon hopping:* $H^{\rm hop} = -\kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i)$
- *polariton number:* $N = \sum_j (a_j^\dagger a_j + \sigma_j^+ \sigma_j^-)$

Bath: engineering of chemical potential M. Hafezi, Adhikari, Taylor, 2015
Effective description based on Floquet theory (possibility to achieve such states via driving)

Jaynes-Cummings Model

"Atomic" limit $\kappa \to 0$

$$H = \sum_j (H_j^{JC} + \mu N_j)$$

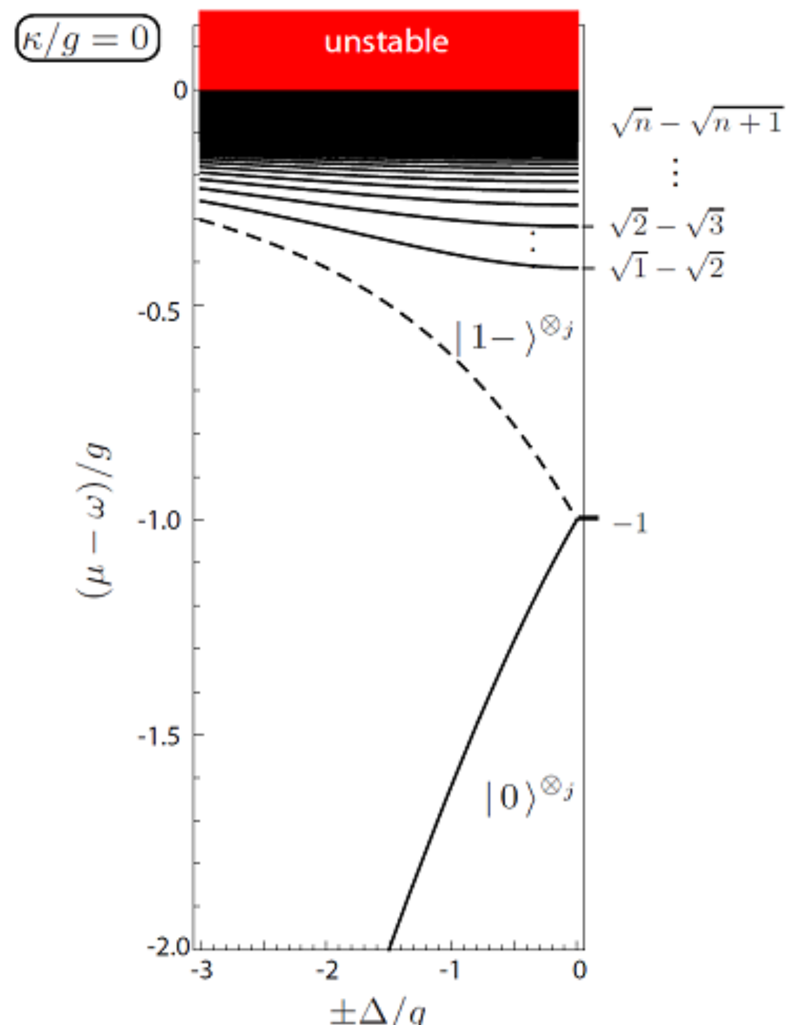
$\Delta = \varepsilon - \omega$

<u>*eigenenergies:*</u> $E^{\mu}_{n\pm} = E_{n\pm} - \mu n$

$$\begin{cases} E_0 = 0 \\ E_{n\pm} = n\omega + \Delta/2 \pm [(\Delta/2)^2 + ng^2]^{1/2} \quad (n \geq 1) \end{cases}$$

<u>*ground state:*</u> $E^{\mu}_{n\alpha} = \min\{E^{\mu}_0, E^{\mu}_{1\pm}, E^{\mu}_{2\pm}, \ldots\}$

- *fixed polariton number on each site*
- *extra polariton on site j does not propagate to other sites*
- *MOTT-INSULATING STATE (gapped, incompressible)*



detuning

$$|n+\rangle = \sin\theta_n |n, g\rangle + \cos\theta_n |(n-1), e\rangle$$
$$|n-\rangle = \cos\theta_n |n, g\rangle - \sin\theta_n |(n-1), e\rangle$$

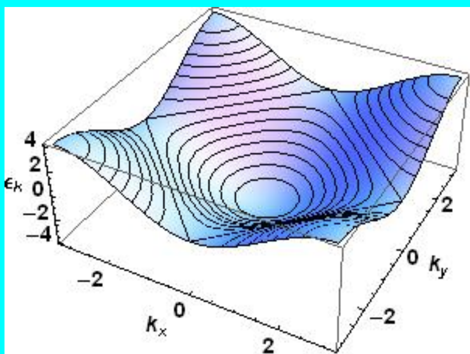
Other simple limit

In contrast to polariton semiconductor systems (J. Bloch, A. Amo, ⋯), the superfluid state of photons has not been shown yet in circuit QED

Review I. Carusotto and C. Ciuti

Hopping dominated limit $\kappa/g \gg 1$

$$H^{\text{tb}} = (\omega - \mu)\sum_i a_i^\dagger a_i - \kappa \sum_{\langle i,j\rangle} \left(a_i^\dagger a_j + a_j^\dagger a_i\right)$$



dispersion of 2d cubic lattice:

$$\epsilon_k = -2\kappa \sum_i \cos(k_i a)$$

...

- *polaritons condense into **k**=0 state*
  SUPERFLUID STATE (not gapped)
- *polaritons delocalize over lattice*
- *something bad happens for*
  (instability) $\omega - \mu < \kappa z_c$

This simple reasoning implies a quantum phase transition

Mean-field theory: idea Ψ^4 theory

Thus, don't expect *quantitatively* correct results from MFT – *qualitative* predictions turn out to be correct for d=2 in the present case.
d=1 later. Also this starting point allows to combine with exact stochastic theories for driving and dissipation, L. Henriet et al. PRA 2014
other efforts by Jonathan Keeling (analogy DMFT)

Objective: *find effective single-site description*

general idea: replace coupling of site j to its nearest neighbors by coupling to a *mean field*

e.g. $\langle c_{k=0} \rangle$ BEC

$\langle c_{k\uparrow} c_{-k\downarrow} \rangle$ BCS

$$H = AB \quad \rightarrow \quad H^{\mathrm{mf}} = A\langle B \rangle + \langle A \rangle B - \langle A \rangle \langle B \rangle$$

coupling to mean field serves as order parameter

correction ensures $\langle H^{\mathrm{mf}} \rangle = \langle A \rangle \langle B \rangle$

MFT for the Jaynes-Cummings lattice

$$H^{\mathrm{hop}} = -\kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) = \kappa \sum_i \sum_{j \in \mathrm{nn}(i)} a_i^\dagger a_j \quad \rightarrow \quad \kappa \sum_i \sum_{j \in \mathrm{nn}(i)} \left[\langle a_i^\dagger \rangle a_j + a_i^\dagger \langle a_j \rangle - \langle a_i^\dagger \rangle \langle a_j \rangle \right]$$

SF order parameter $\psi = z_c \kappa \langle a_j \rangle$

$$\Rightarrow \boxed{h_j^{\mathrm{mf}} = \frac{1}{2}(\varepsilon - \mu)\sigma_j^z + (\omega - \mu) a_j^\dagger a_j + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j) - (a_j \psi^* + a_j^\dagger \psi) + \frac{1}{z_c \kappa} |\psi|^2}$$

MFT: How, to go beyond: $\psi = z_c \, \kappa \, \langle a_i \rangle$

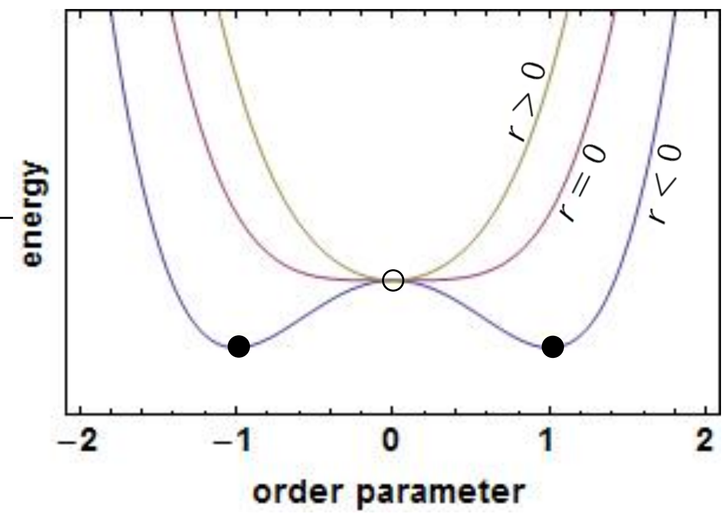
$$h_j^{\mathrm{mf}} = \frac{1}{2}(\varepsilon - \mu)\sigma_j^z + (\omega - \mu)a_j^\dagger a_j + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j) - (a_j \psi^* + a_j^\dagger \psi) + \frac{1}{z_c \kappa}|\psi|^2$$

treat perturbatively!

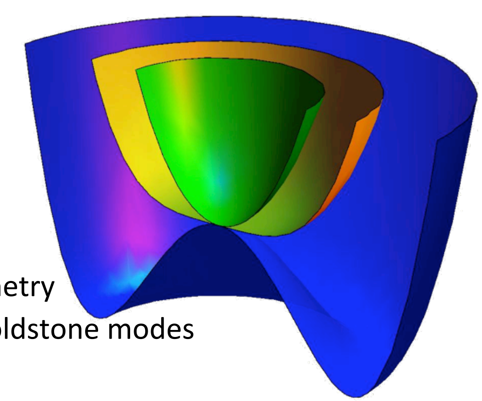
Sufficiently close to the phase boundary: $\psi \sim \langle a_j \rangle \ll 1$

Expansion in orders of

$$E_0(\psi) = E_0^{\mathrm{mf}} + r|\psi|^2 + \tfrac{1}{2}u|\psi|^4 + \mathcal{O}(|\psi|^6)$$



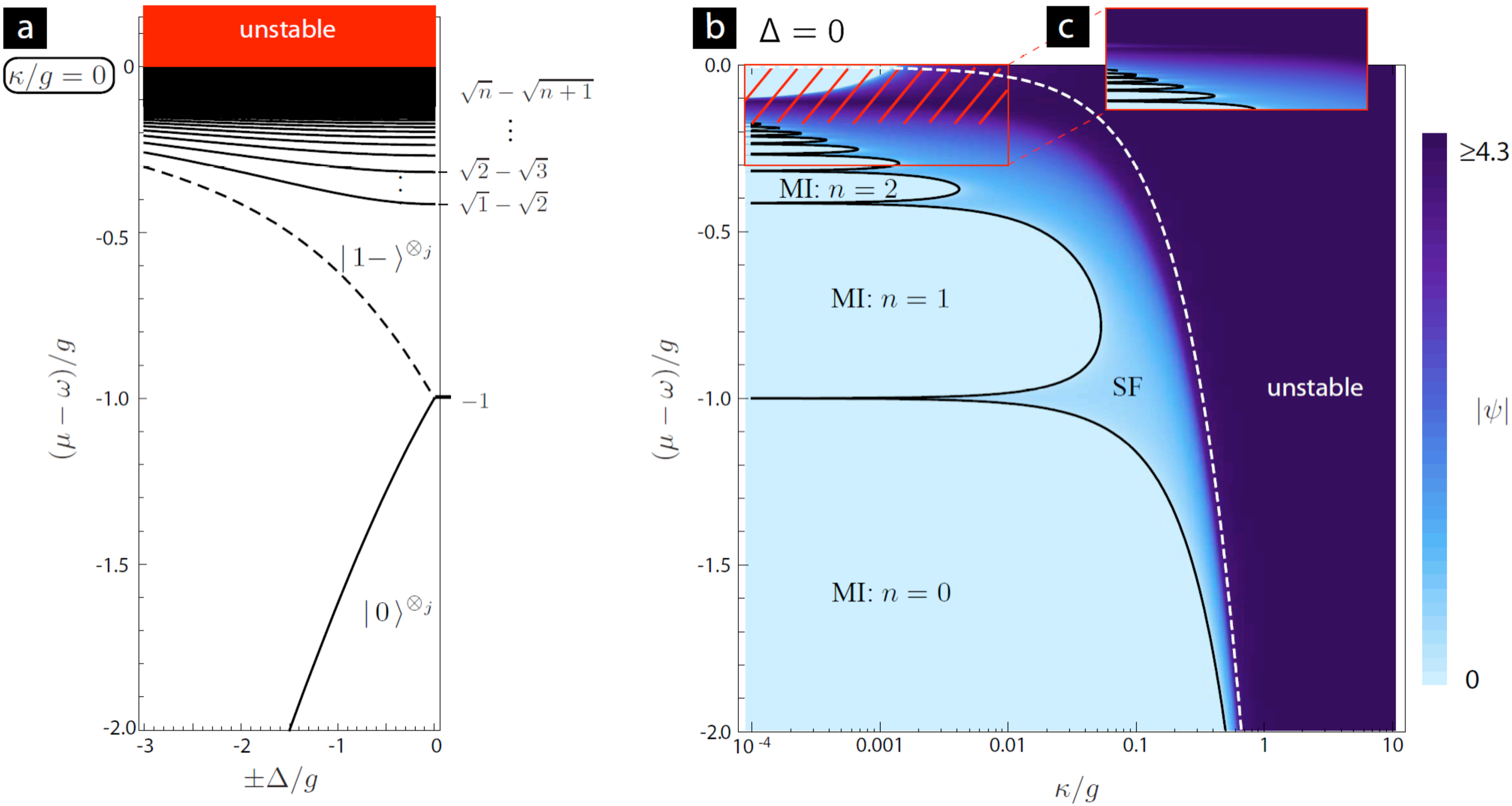
- ▶ standard situation for a MFT phase transition: for *u>0* transition occurs at *r=0*
- ▸ perturbation theory gives analytical expressions for phase boundary!



breaking of **U(1)** symmetry
gapless excitations: Goldstone modes

MFT results for the JC lattice

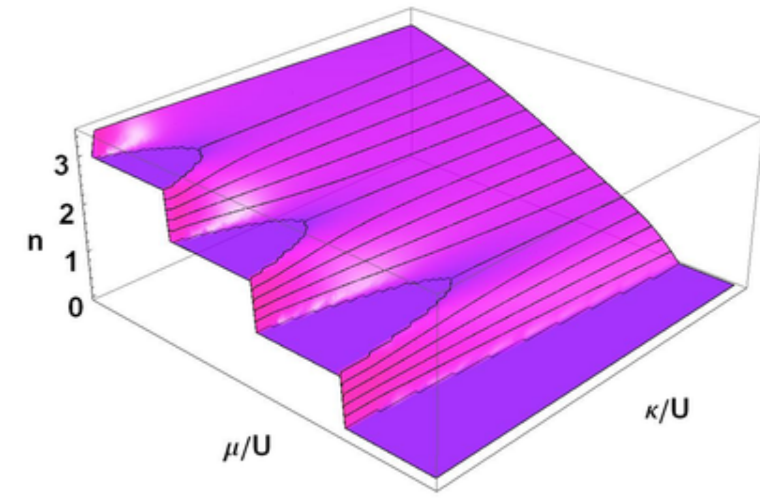
Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)



Jens Koch and KLH, PRA **80** 023811 (2009)

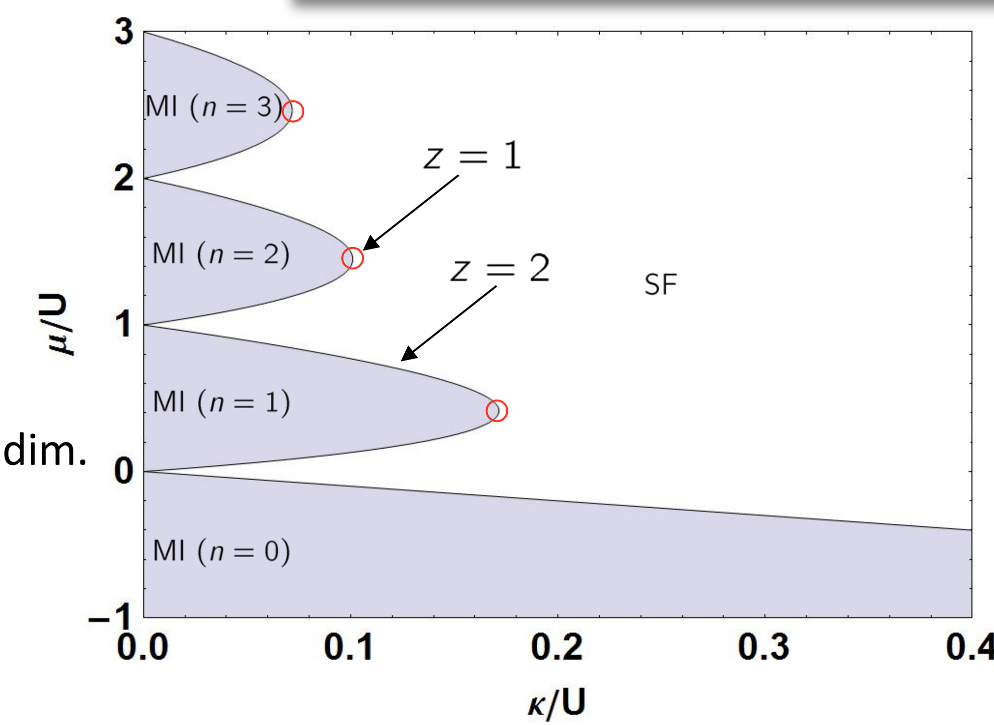
MI n=0
is a vacuum Mott

Multicritical points (BHM)



The tips of lobes are special!

- ▶ generically, crossing the phase boundary is associated with a *change in boson density*
- ▸ at lobe tips, density remains **constant**
- ▸ lobe tips are **multicritical points, universality class differs**



dynamical critical exponent

QPT in d dim. ↔ classical PT in (d+1) dim.

$\xi \sim |\kappa - \kappa_c|^{\nu}$ diverging length scale (correlation length)

$\xi_\tau \sim \xi^{z}$ diverging time scale

dynamical critical exponent

Field theory in terms of coherent state path integrals

exact

partition function: $$Z = \int \prod_j \mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)\delta(\mathbf{N}_j^2 - 1)e^{-S[a_j^*, a_j, \mathrm{N}_j]}$$

action: $$S[a_j^*, a_j, \mathbf{N}_j] = \int_0^\beta d\tau \left\{ \sum_j \langle \mathbf{N}_j(\tau)| \frac{d}{d\tau} |\mathbf{N}_j(\tau)\rangle + \sum_j a_j^* \frac{\partial a_j}{\partial \tau} + \sum_j H_j^{\mathrm{JC}}(a_j^*, a_j, \mathbf{N}_j) + H^{\mathrm{hop}}(a_j^*, a_j) \right\}.$$

w/ $a_j \to a_j(\tau), \quad a_j^\dagger \to a_j^*(\tau), \quad \sigma_j^\alpha \to N_{j,\alpha}$

use *Hubbard-Stratonovich transformation* to decouple hopping term:

$$\exp\left[\int_0^\beta d\tau \sum_{j,j'} a_j^* \kappa_{jj'} a_{j'}\right] = \int \prod_j \mathcal{D}\psi_j^*(\tau)\psi_j(\tau) \exp\left[-\int_0^\beta d\tau \sum_{j,j'} \psi_j^* \kappa_{jj'}^{-1} \psi_{j'}\right] \exp\left[\int_0^\beta d\tau \sum_j \{\psi_j^* a_j + \psi_j a_j^*\}\right]$$

aux. field ψ_j plays role similar to order parameter

integrate out ($\mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)$)

$$Z = \int \prod_j \mathcal{D}\psi_j^*(\tau)\mathcal{D}\psi_j(\tau)\mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)\delta(\mathbf{N}_j^2 - 1)\exp\left(-S'[\psi_j^*, \psi_j, a_j^*, a_j, \mathbf{N}_j]\right)$$

Effective field theory in the critical region

Exact relations via symmetries

$$Z = \int \prod_j \mathcal{D}\psi_j^*(\tau)\mathcal{D}\psi_j(\tau)\mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)\delta(\mathbf{N}_j^2 - 1)\exp\left(-S'[\psi_j^*, \psi_j, a_j^*, a_j, \mathbf{N}_j]\right)$$

$$= \int \prod_j \mathcal{D}\psi^*(x,\tau)\mathcal{D}\psi(x,\tau)\exp\left(-S_{\text{eff}}[\psi^*, \psi]\right)$$

DIFFICULT STEP:

Gradient expansion for effective action:

$$S_{\text{eff}}[\psi^*, \psi] = \int_0^\beta d\tau \int d^d x \left[K_0 + K_1 \psi^* \frac{\partial \psi}{\partial \tau} + K_2 \left|\frac{\partial \psi}{\partial \tau}\right|^2 + K_3 |\nabla \psi|^2 + \tilde{r}|\psi|^2 + \frac{\tilde{u}}{2}|\psi|^4 + \cdots\right]$$

proportional to MFT params. *r*, *u*

S' invariant under:

$$a_j \to a_j e^{i\varphi(\tau)}, \quad \psi_j \to \psi_j e^{i\varphi(\tau)},$$

$$\mathbf{N}_j \to \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{N}_j,$$

$$(\omega - \mu) \to (\omega - \mu) - i\frac{d\varphi}{d\tau}$$

$\Rightarrow$

Coefficients K_1, K_2

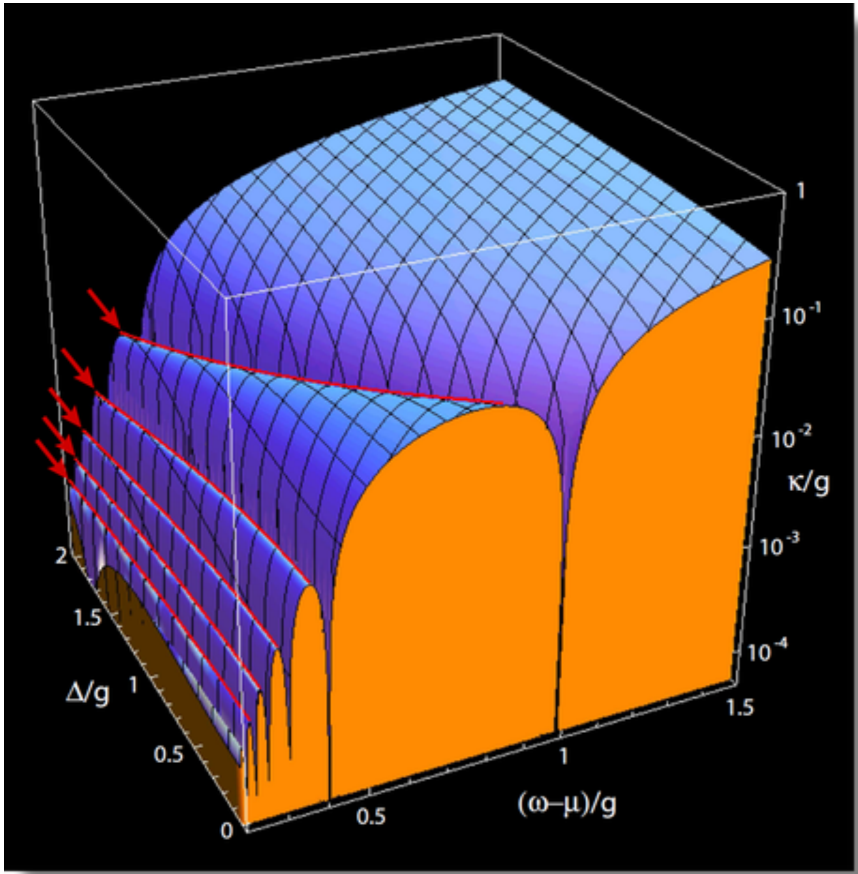
$$K_1 = \frac{\partial \tilde{r}}{\partial(\omega - \mu)}, \qquad K_2 = -\frac{1}{2}\frac{\partial^2 \tilde{r}}{\partial(\omega - \mu)^2}$$

$K_1 \neq 0$ generic case, *z=2*

$K_1 = 0$ multicrit. curves, *z=1*

Multicritical curves for the JC lattice model:

Important, there is a physical THIRD axis…



$K_1 \neq 0$ generic case, *z=2*

$K_1 = 0$ multicrit. curves, *z=1*

and QMC:
M. Hohenadler, M. Aichorn,
S. Schmidt & L. Pollet
arXiv: 1106.0801

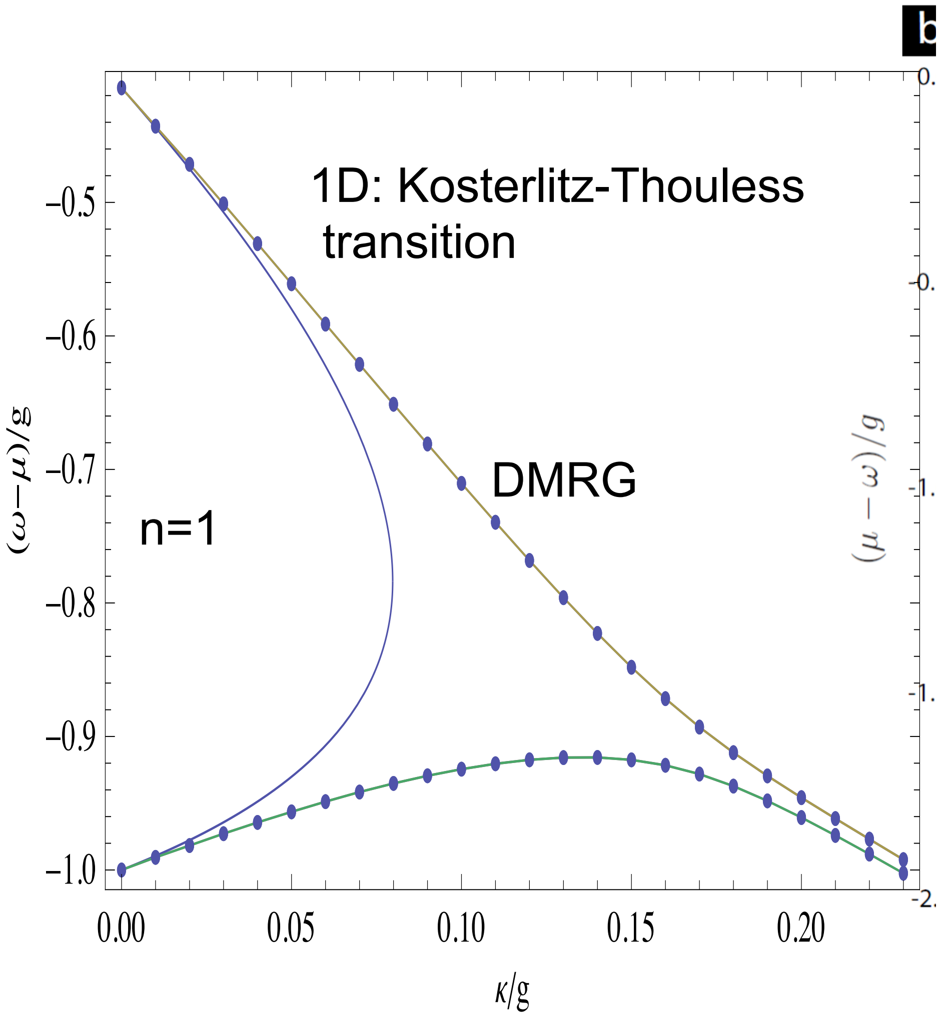
New efforts: QMC, **strong coupling**
T. Flottat, F. Hebert, V. G. Rousseau
G. G. Batrouni, 2016
See also H. Tureci, M. Schiro

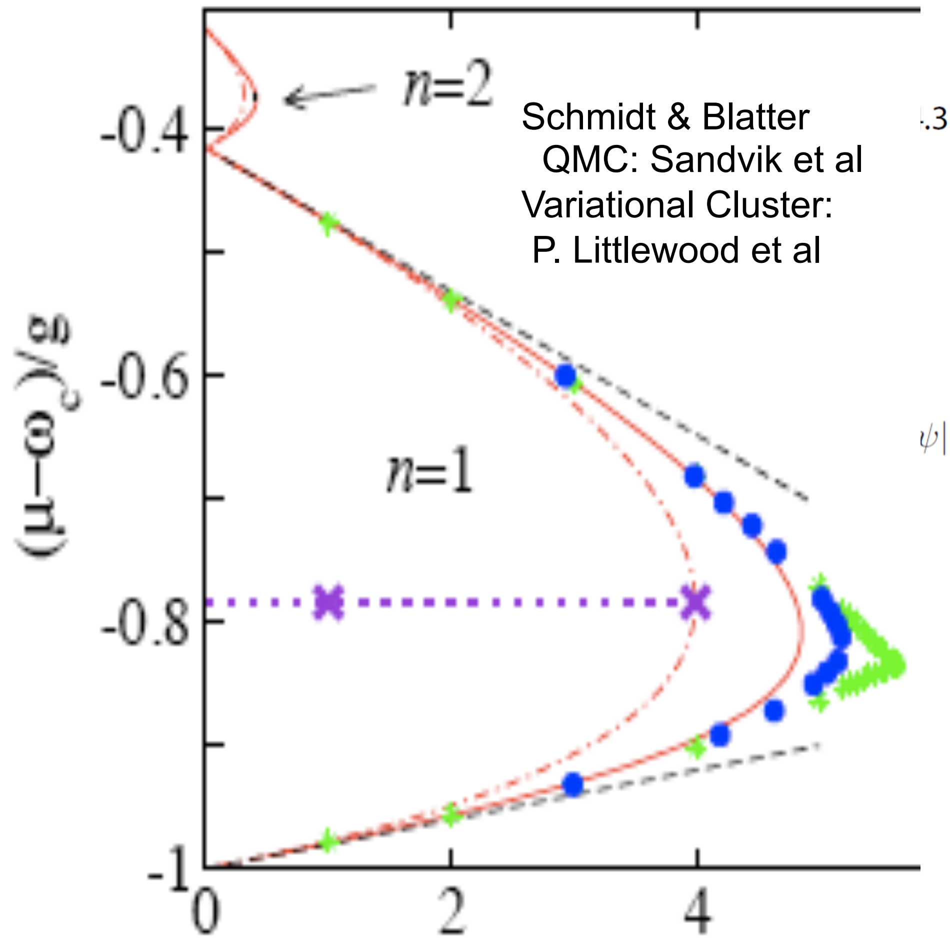
Polariton mapping at the **tip** of a Mott lobe (Schmidt & Blatter, PRL 2009)

1D: Mapping onto XX spin model **between** 2 Mott lobes (J. Koch & KLH, 2009)

MFT results for the JC lattice and Beyond…

H. Francis Song, Yale (unpublished)





Schmidt & Blatter
QMC: Sandvik et al
Variational Cluster:
P. Littlewood et al

Important Issues

Preparation and Measurements

- Insulating state: 1 polariton in each cavity
- Then, detune progressively each cavity
- Homodyne measurement
- Dissipation and cavity loss F. Nissen et al. arXiv:1202.1961
- Relevant

"Disorder" might play a key role: on-site potential, hopping,...

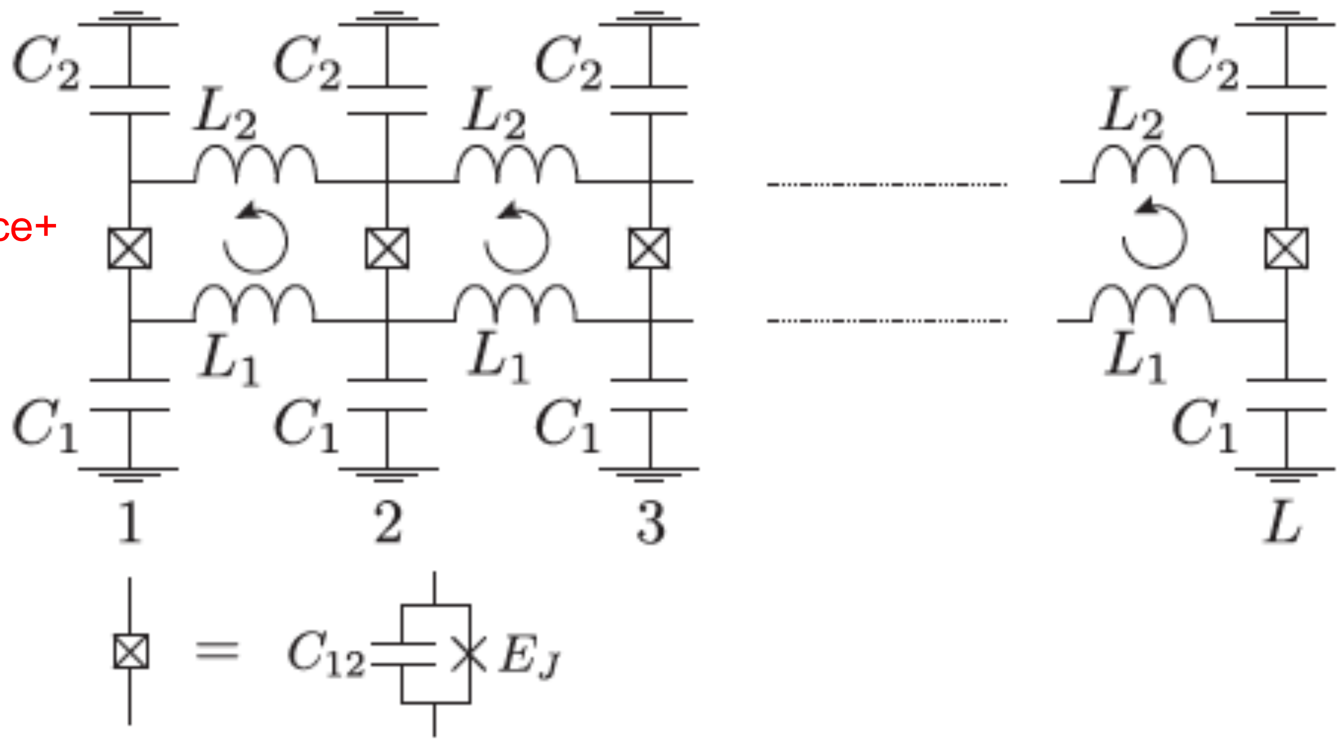
Bose glass phase (M.P.A. Fisher et al., 1989)

See also paper: D. Rossini & R. Fazio, PRL 2007

New efforts on Mott: A. Biella et al. arXiv:1704.089078; J. Lebreuilly et al.

Josephson Junctions & Mott

Capacitance+ Josephson



Mott: commensurate filling (**Cooper pairs in islands**; charging)
quantum Hall state of bosons : example by tuning flux (strong charging terms to achieve a node in Jastrow wave function)

Related to cold atoms: Munich (M. Atala et al. Nature Phys. 2014)

Mott Physics in Boson Systems: Lattice Effects

Bose-Hubbard model of a single lattice boson:

$$H = -t\sum_{<ij>} b_i^\dagger b_j + \sum_i \frac{U}{2} n_i(n_i - 1) - \mu n_i$$

Two-species Bose-Hubbard model:

$$H = -t\sum_{\alpha=1,2}\sum_{<ij>} b_{\alpha i}^\dagger b_{\alpha j} + \sum_{\alpha i} \frac{U}{2} n_{\alpha i}(n_{\alpha i} - 1) - \mu n_{\alpha i}$$

$$+ \sum_i V_\perp n_{1i} n_{2i} - g\sum_i b_{1i}^\dagger b_{2i} + H.c.$$

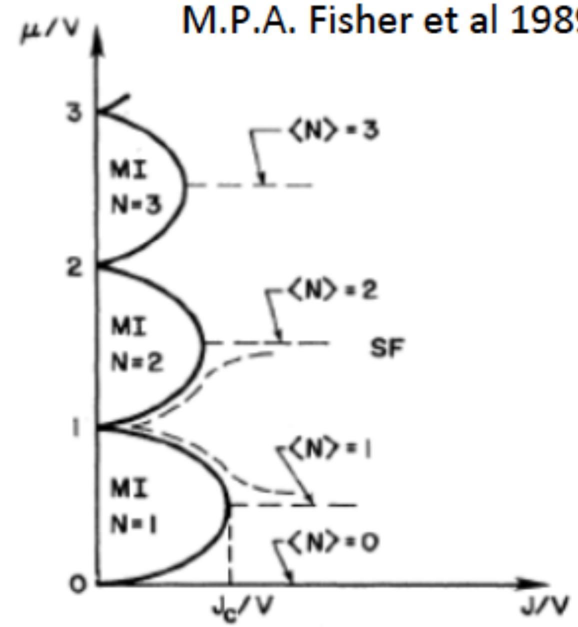
Mott at ρ=1

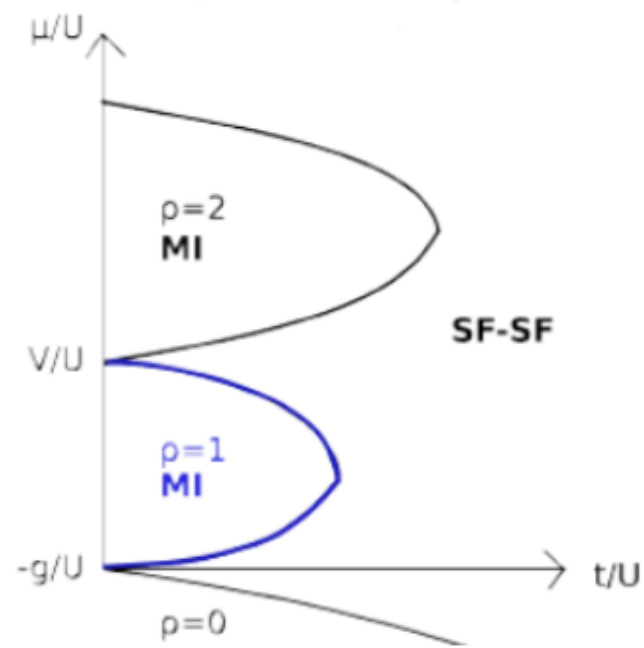
Interchain coherence: Meissner effect

Multicomponent systems: active field in cold atoms

e.g. E. Altman, W. Hofstetter, E. Demler, M. Lukin 2003

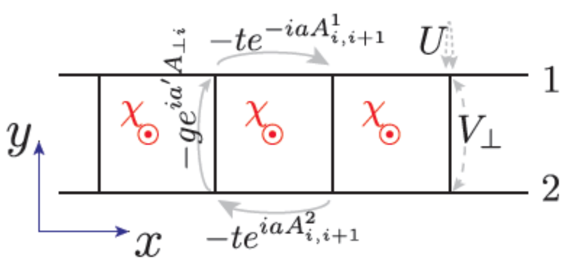
M.P.A. Fisher et al 1989





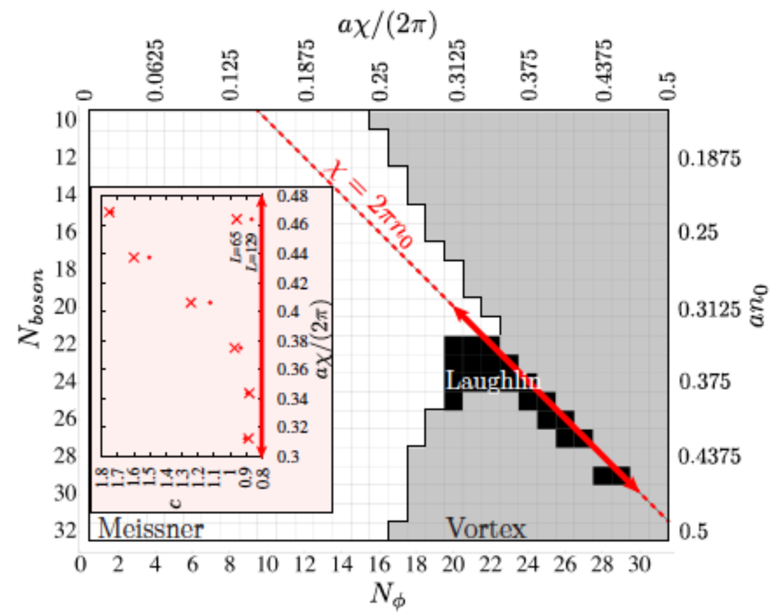
FQHE bosons: 2-leg ladder?

C. L. Kane, Lubensky, Mukhopadyay; Teo & Kane, classification of quantum Hall phases in ladders
Numerical results support bosonic LAUGHLIN PHASE for hard-core bosons with V=0 **finite** systems



A. Petrescu & KLH, PRB 2015 (analytics : V needed for infinite systems)
A. Petrescu, M. Piraud, I. McCulloch, G. Roux, KLH, to appear (see arXiv 2016)
M. Piraud, F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua, U. Schoellwock PRB 2015; See also M. Calvanese et al. PRX 2017 (quantum Hall phases)

DMRG Small densities



Laughlin phase: chiral edge modes with fractional charges
Bipartite fluctuations confirm Laughlin phase 2/5

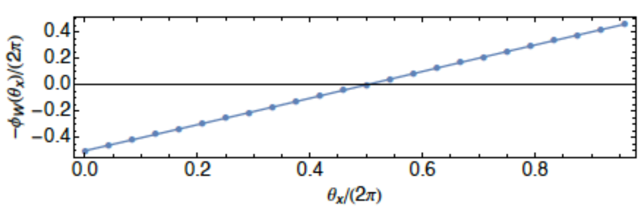
Measurement in quantum wires of fractional charges

H. Steinberg, G. Barak, A. Yacoby, L. N. Pfeiffer, K. W. West
B. Halperin and K. Le Hur, 2008;
see also E. Berg, Y. Oreg, E.-A. Kim, F. von Oppen

K.V. Pham, M. Gabay, P. Lederer, 2000; Safi & Schulz, 1995
Application topological insulators edge modes: Ion Garate & KLH, 2012

$$\sigma_{xy} = \frac{1}{d}\frac{1}{2\pi}\int_0^{2\pi} d\theta_x \frac{\partial}{\partial\theta_x}\phi_W(\theta_x) = \frac{1}{d}\frac{1}{2\pi}\left[\phi_W(\theta_{x,N_x}) - \phi_W(\theta_{x,0})\right].$$

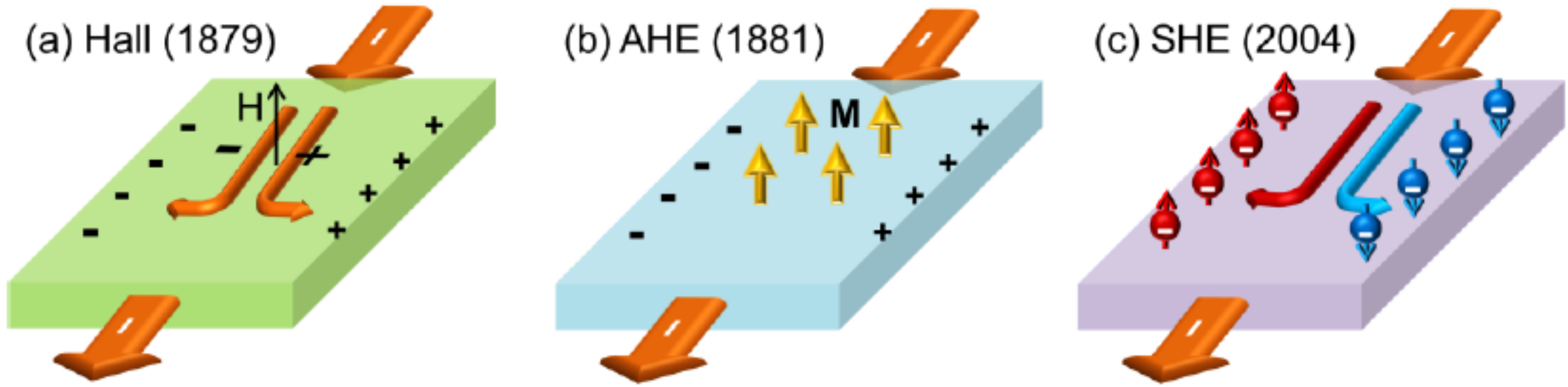
Torus geometry: gap the edges
Thouless Laughlin pump
Experiment in Muenich, Bloch's group
Zak phase (work D. Abanin, E. Demler)
here measures the polarization « 1/2 »



See also F. Grusdt – M. Honing 2014

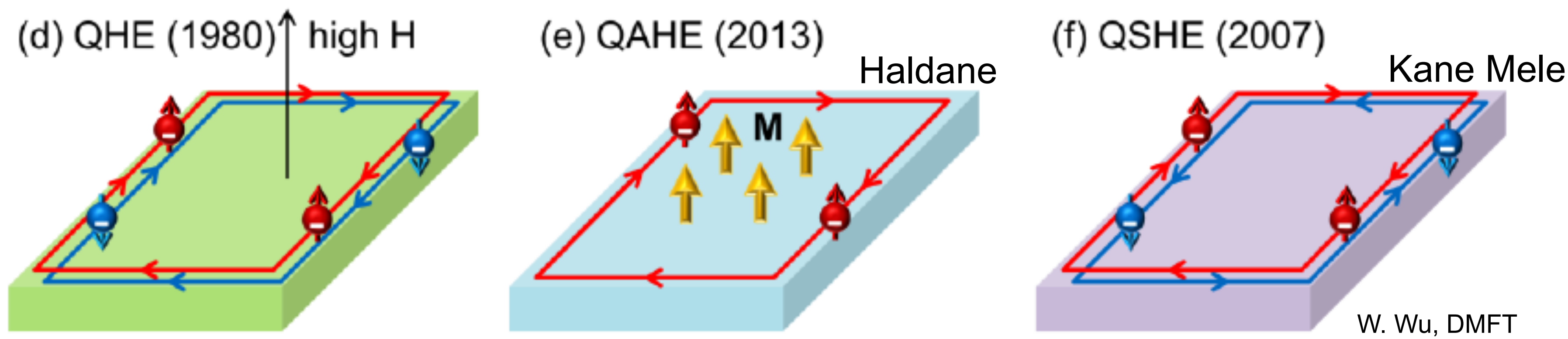
Ground state	Meissner	Vortex	Laughlin
c	1	2	1
N_V	1		>1

Topological states of matter: magnetic fields and spin-orbit coupling



Von Klitzing, Dorda, Pepper;
fractional charges (Grenoble, CEA Saclay, Weizmann)

REALIZED AT WURZBURG IN HGTE (Molenkamp)
3D MERCURY ANALOGUES, PRINCETON (Hasan)



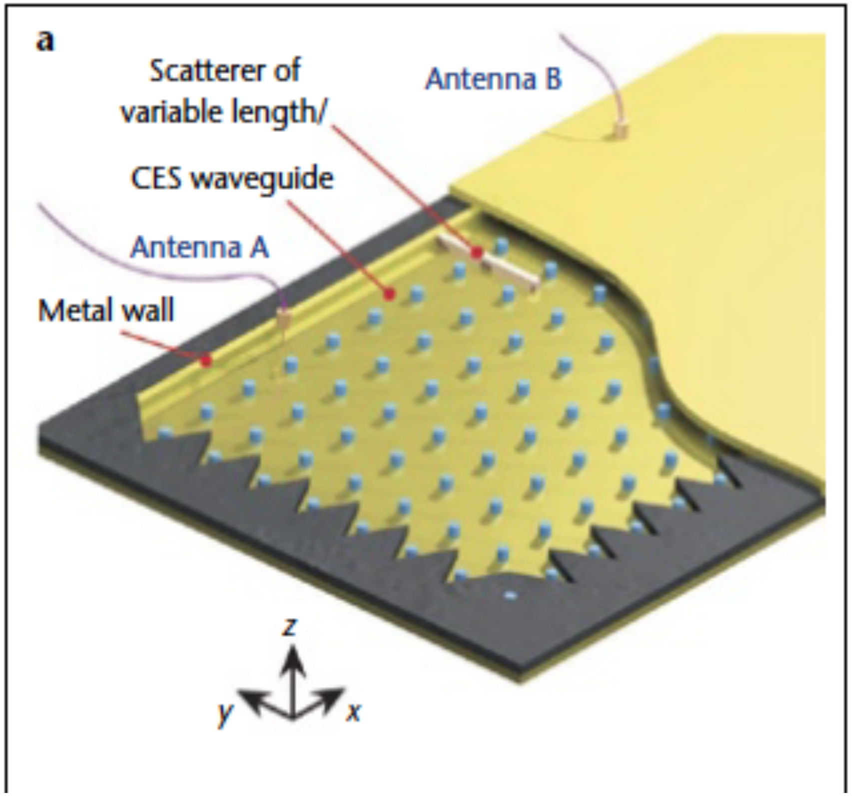
W. Wu, DMFT
China & Yale 2011
College de France
CPHT

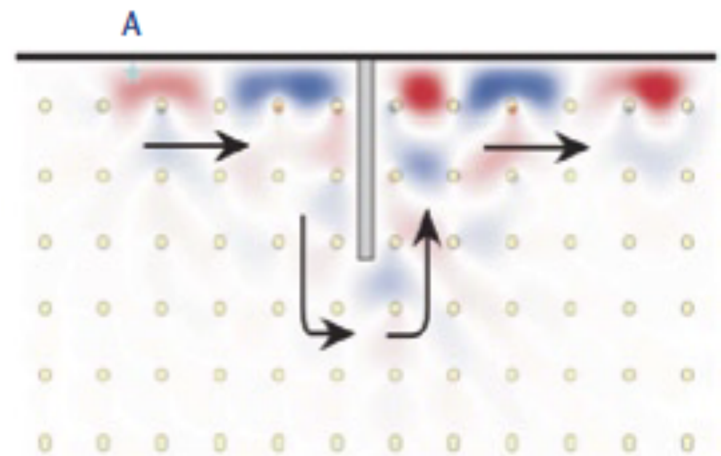
C. Z. Chang and M. Li, Topical Review, arXiv:1510.01754
From material science, to cold atoms and photons

Stable towards interactions: S. Rachel & KLH Kane-Mele-Hubbard model 2010 QSH; D. Pesin & L. Balents, 3D (2010)
C. Varney, K. Sun, M. Rigol, V. Galitski (Maryland) 2010 QAH

One-Way Road in a Photonic Crystal

Chiral edge states channel light waves in one direction, like electrons in the quantum Hall effect





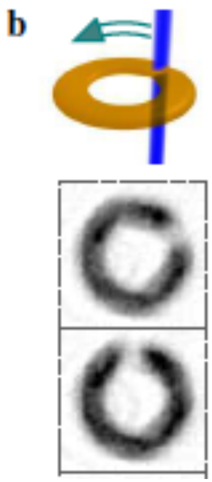
(a) A model of the photonic crystal.
The distance between the ferrite rods is 4 cm.

Realizations of AQHE in Photonic crystals: following Haldane & Raghu, PRL 2008 (Dirac points and Faraday effect opens a gap breaking time-reversal symmetry)
Experiment: M. Soljacic et al. Nature **461**, 772 (2009)

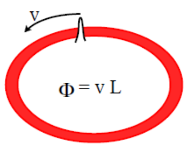
Artificial Gauge Fields & Protection

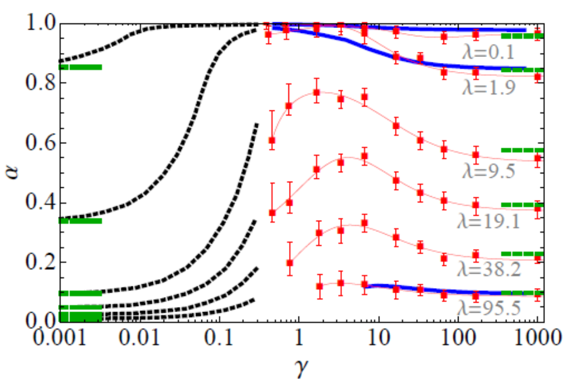
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
I. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...

Atomtronics



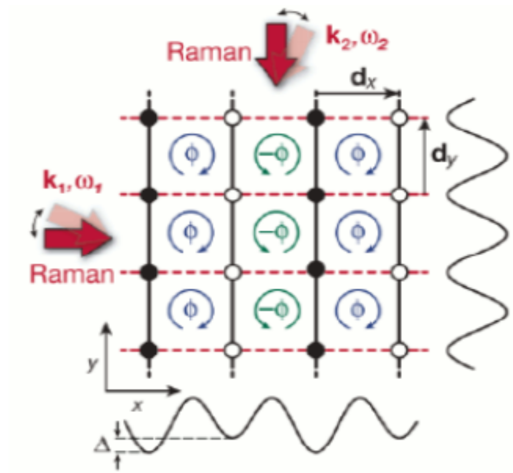
NIST Maryland
William D. Philipps
Gretchen K. Campbell
Nature 2014





Frank Hekking & Anna Minguzzi
(LPMMC Grenoble)
Expérience Villataneuse H. Perrin

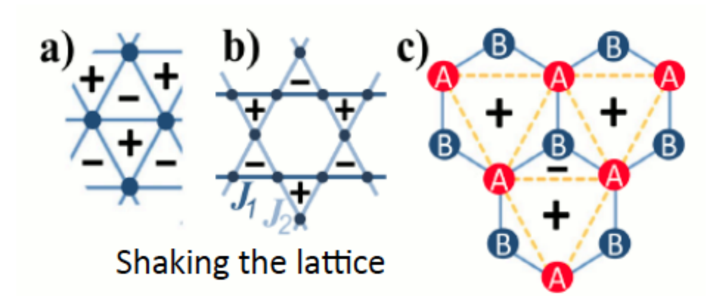
M. Aidelsburger et al. Muenich



Jaksch & Zoller 2003
Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)

Floquet engineering: perturbations periodic in time to engineer topological phases

Hamburg (J. Simonet, C. Weitenberg, K. Sengstock), MIT (W. Ketterle)

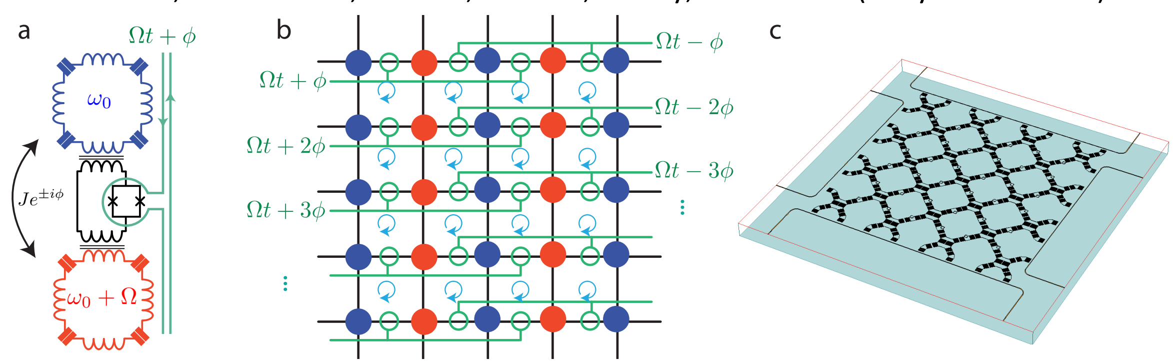


Shaking the lattice

...

Floquet Topological Insulators:
Reviews: J. Cayssol, B. Dora, F. Simon, R. Moessner, arXiv:1211.5623
N. Goldman, J. Dalibard, PRX 2014
P. Delplace, D. Carpentier (Lyon)

K. Fang et al. Nature Photonics 2012
On-going J. Gabelli, J. Esteve, M. Aprili LPS Orsay; progress Santa Barbara, P. Roushan et al 2016
Grenoble: O. Buisson, W. Guichard, N. Roch, C. Naud, L. Levy, V. Bouchiat (early B. Pannetier)



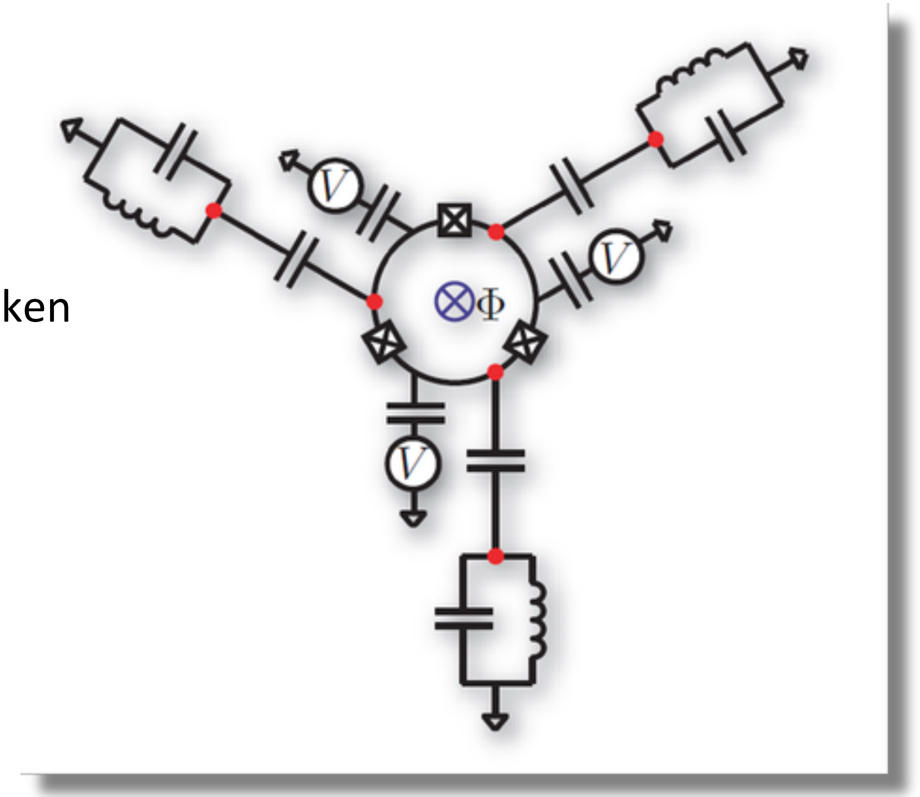
J. Koch, A. Houck, KLH & SM Girvin, PRA 2010

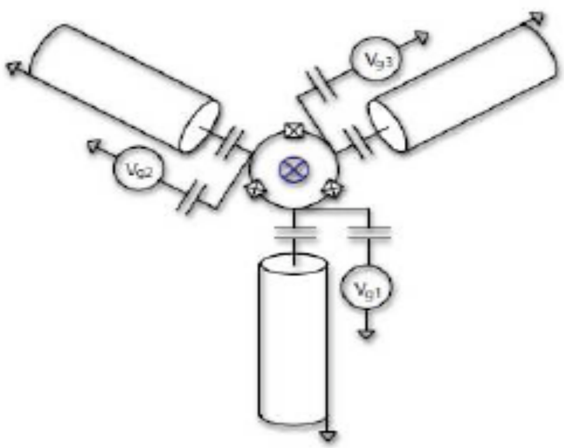
(numerical check at intermediate couplings)

- Josephson ring provides one way to generate complex phase factors
- need magnetic flux to break t-reversal sym. additionally, particle-hole sym. must be broken
- large E_J/E_C: no complex phases, but *tunable coupling strength*!

complex phases for intermediate E_J/E_C

- *random off-set charges can be controlled*

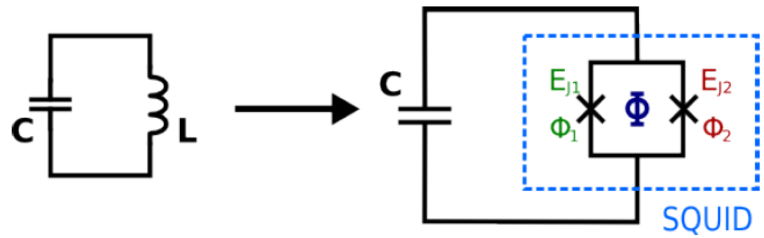




Nano quantum circulator:
A real flux is converted from
Cooper pairs to photons

- **topological phases, Kagome lattice**

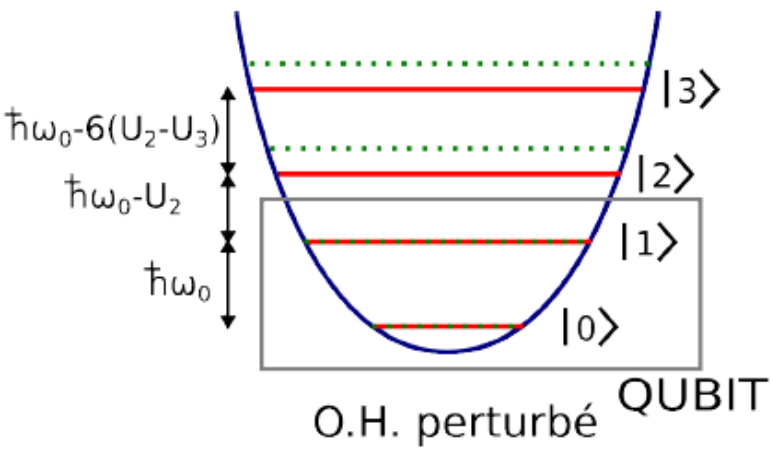
Experiment: Google Santa Barbara

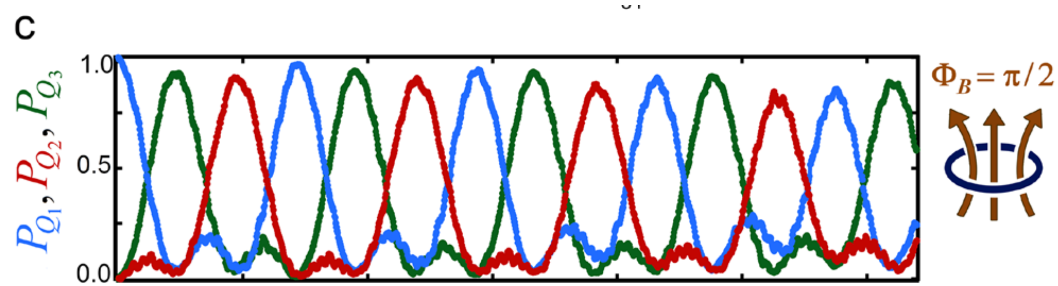


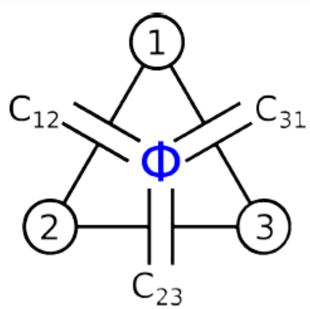
$$\hat{H} = \left[\frac{\hat{Q}^2}{2C} + \frac{\tilde{E}_J}{2}\hat{\varphi}^2\right] + \left[-\frac{\tilde{E}_J}{4!}\hat{\varphi}^4 + \frac{\tilde{E}_J}{6!}\hat{\varphi}^6 + ..\right]$$

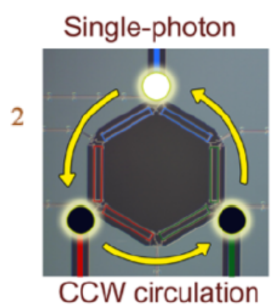
$$= \epsilon_0\left(\hat{N} + \frac{1}{2}\right) - \frac{U_2}{2}\hat{N}(\hat{N}-1) + \frac{U_3}{6}\hat{N}(\hat{N}-1)(\hat{N}-2) + \ldots$$

$\epsilon_0 = \sqrt{4\tilde{E}_J E_C}$ et U_2, U_3 sont des paramètres **positifs**.
$U_2, U_3 \propto E_C$ et en régime $E_C << \tilde{E}_J$, $\boxed{U_2, U_3 << \epsilon_0}$

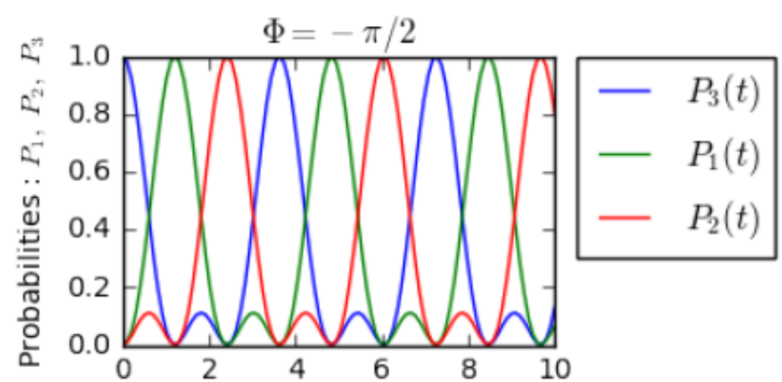






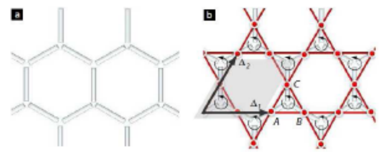


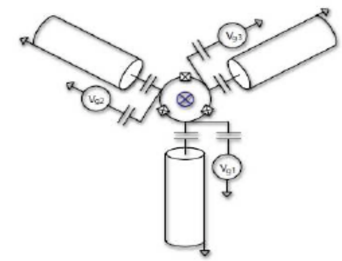
P. Roushan et al. Nat. Phys 2017



F. Appas 3rd year student Ecole Polytechnique 2016, internship (together with T. Goren, K. Plekhanov)

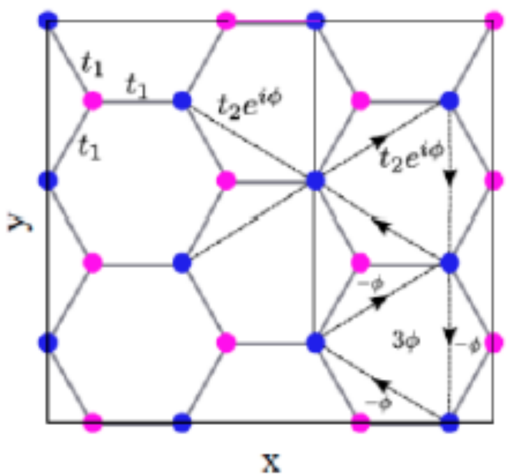
Quantum Anomalous Hall Effect

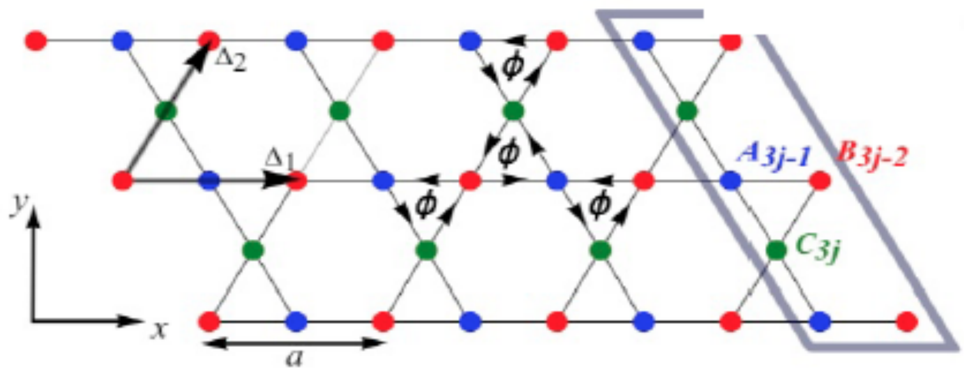




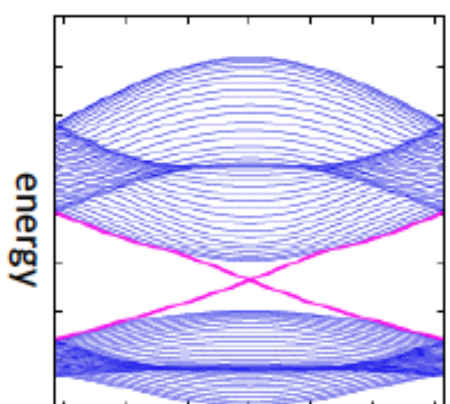
F. D. M. Haldane 1988

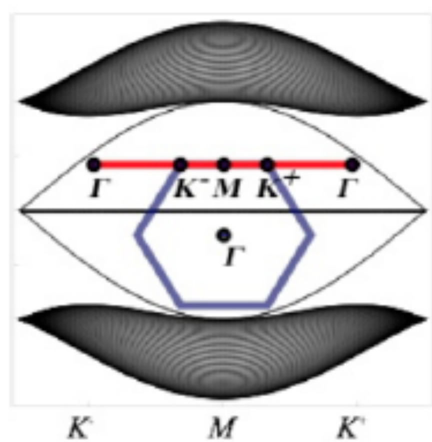
Kagome version:
A. Petrescu, A. A. Houck and KLH, 2012
J. Koch, A. Houck, KLH, S. Girvin 2010

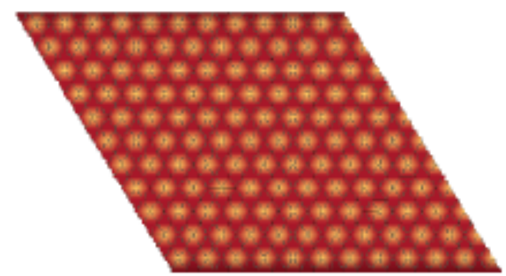




Flat bands observed in polaritons (**LPN Marcoussis: Jacqueline Bloch, A. Amo**)







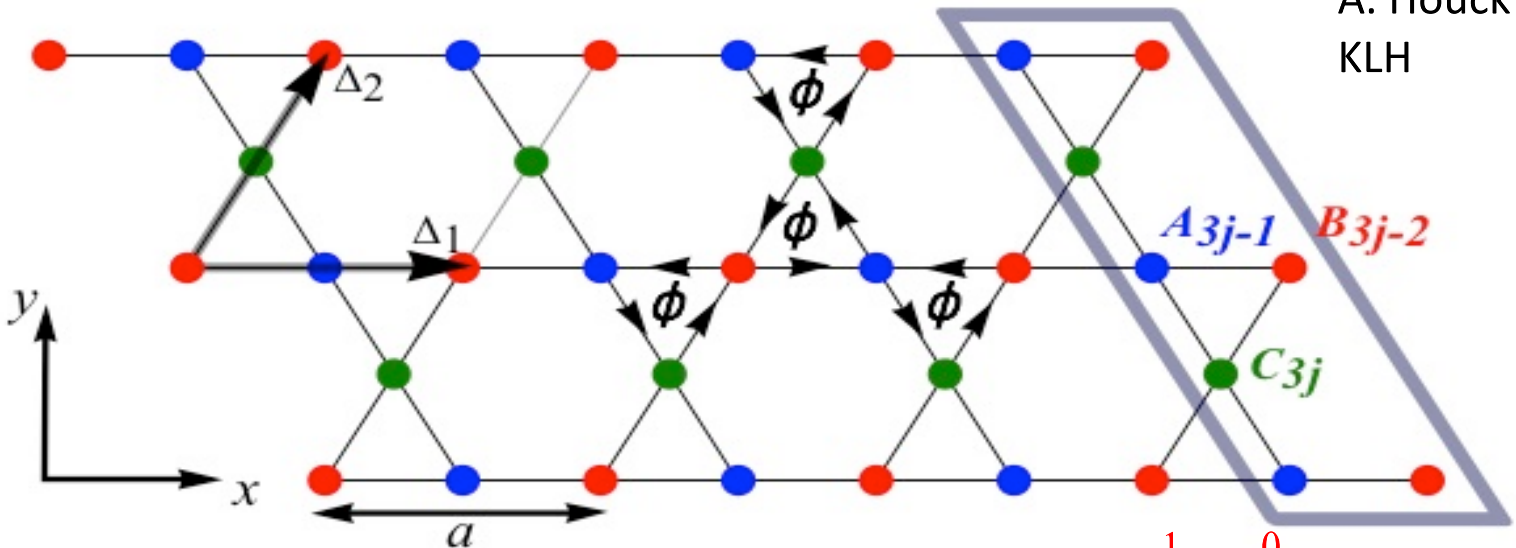
Localization in
Hexagon rings

Graphene
+gap

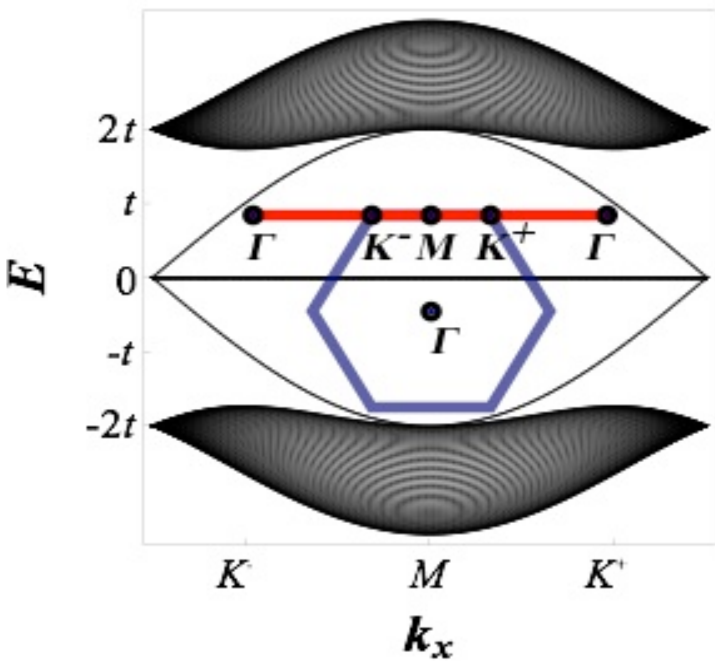
Figure from KLH, Henriet, Petrescu, Roux, Schiro Académie of Sciences 2016

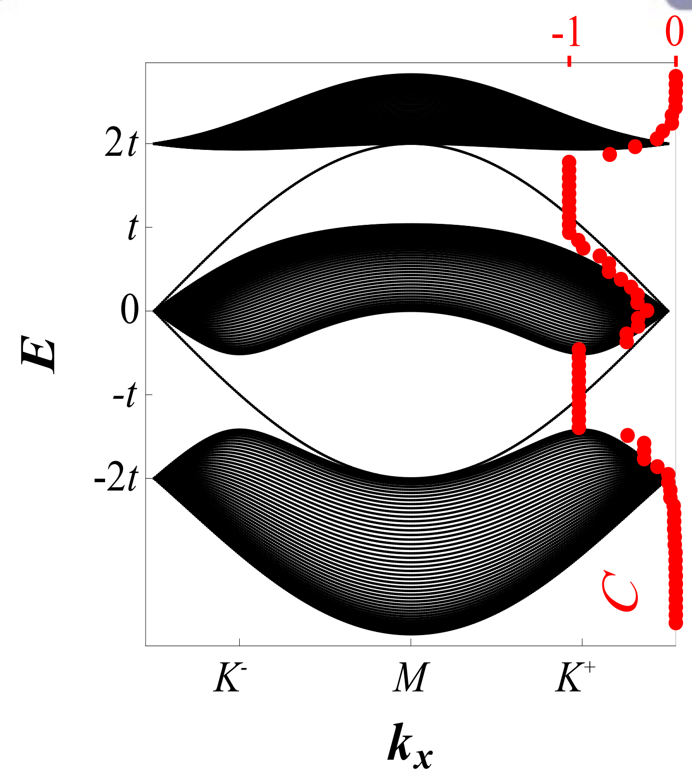
Topological Phases?

A. Petrescu,
A. Houck &
KLH



Φ = π/6





Karplus-Luttinger, 1954

D. Haldane, 2004

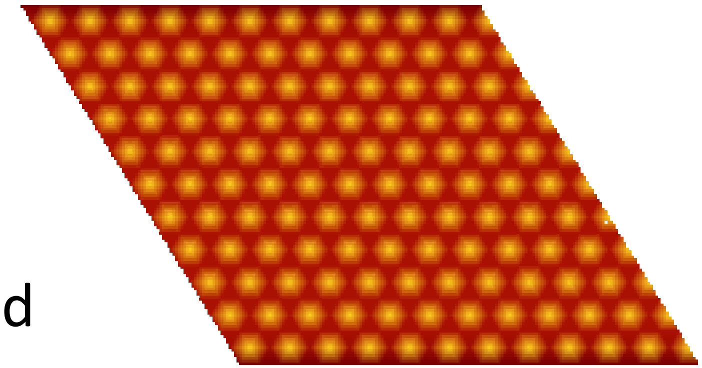
See also
D. Bergman
& G. Refael, 2010
J. Meyer

Φ = π/4

Paper with A. Petrescu
and A. Houck 2012

LDOS

$\Phi = \pi/6$

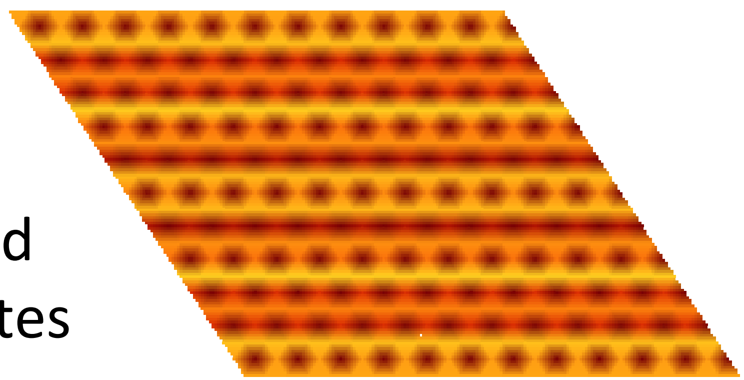


Flat
Band

Edge
States
QHE

$\Phi = \pi/6$
disorder

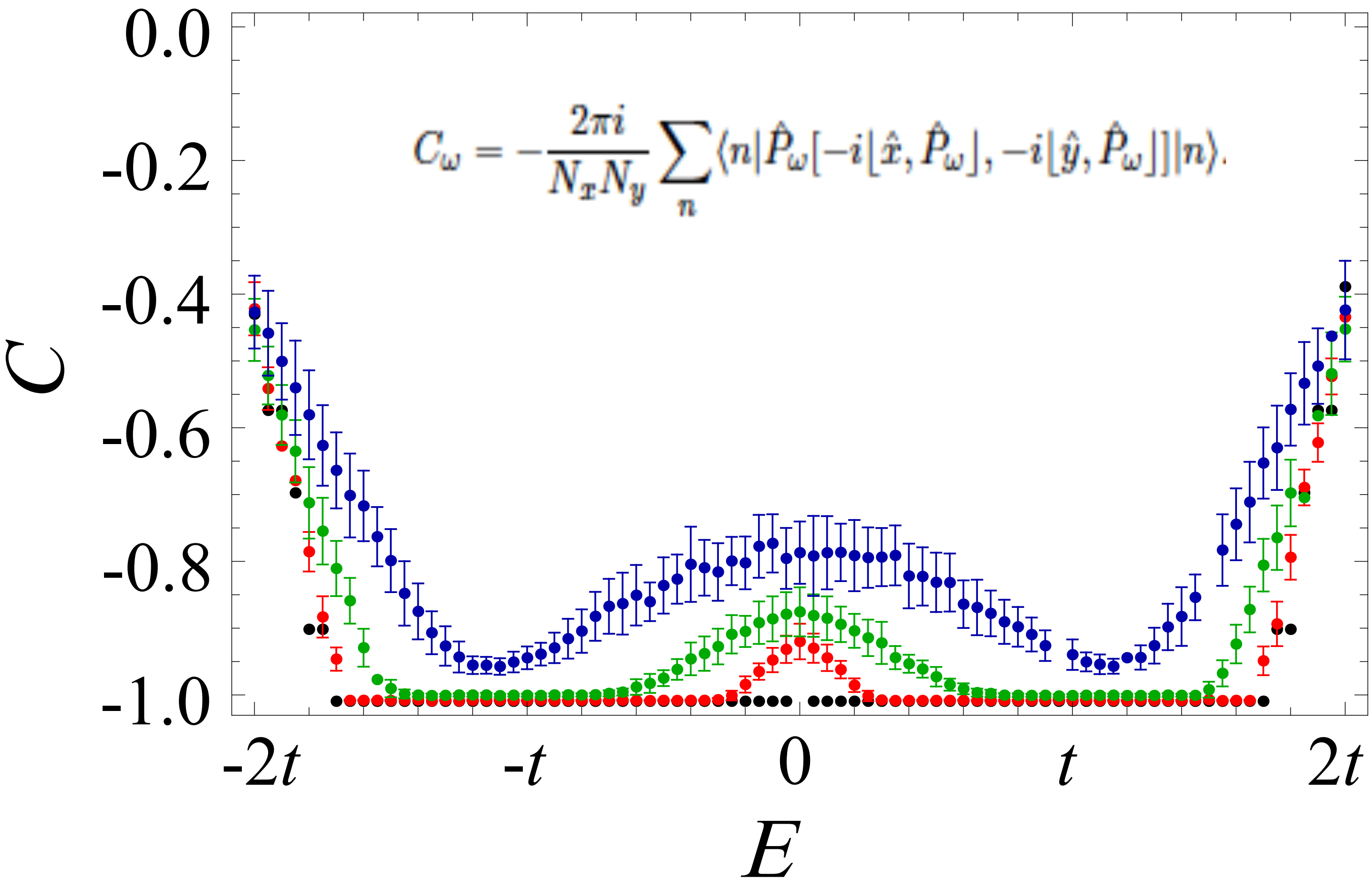
$\Phi = \pi/4$



AHE
Extended
Bulk states

on-going work with Ariane Soret (PhD), Eric Akkermans
On Casimir effect with disorder

Disordered case at Φ=π/6



Real Space computation of Chern number following J. Bellissard; E. Prodan (non-commutative geometry)

Kagome lattice: why interesting…

Flat band (search for ferromagnetism)

A. Mielke; H. Tasaki; E. Lieb

Exotic Topological Phases: fractional quantum Hall state

E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011

N. Regnault and A. Bernevig, PRB 2012,…

Spin liquid search, classical degeneracies

Experimentally relevant: 2D Materials (Orsay; Princeton;…)

Cold atoms: Berkeley; see D. Stamper-Kurn group, 2011

L. Balents, Nature 464, 199 (2010)

S. Yang, D. Huse and S. White, Science (2011)

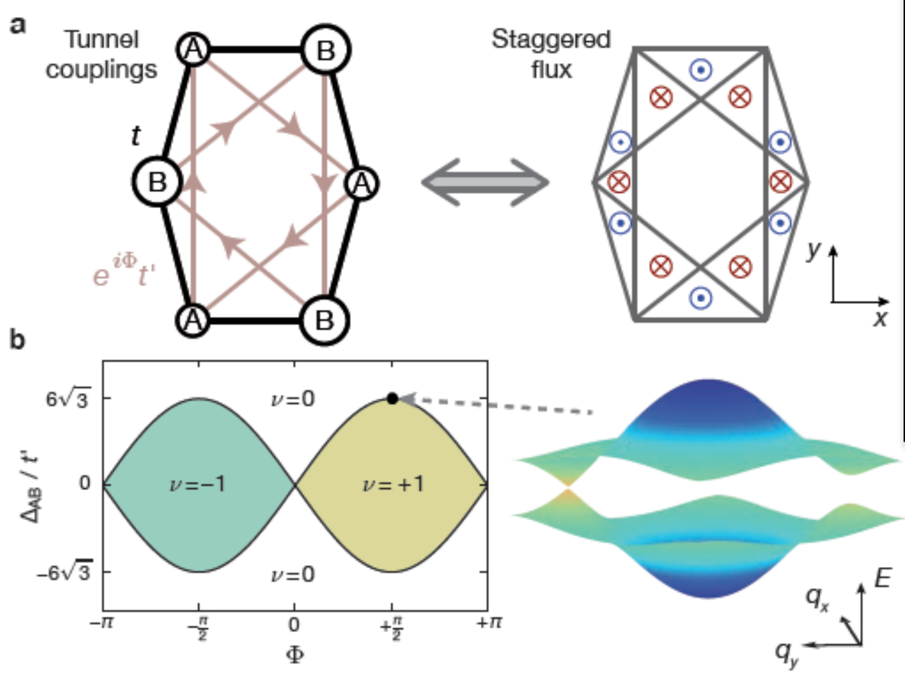
Work by Claire Lhuillier and co-authors,…

Other Experimental observations

- Ultra-cold atoms – see for example Esslinger's experiment (ETH)
- Ultra-cold atoms: importance of Floquet-type point of view

Rubidium atom

Jotzu et al.



<u>Modulation of optical lattice</u>

$$\mathbf{r}_{\rm lat} = -A\Big(\cos(\omega t)\mathbf{e}_x + \cos(\omega t - \varphi)\mathbf{e}_y\Big),$$

$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{r}}_{\rm lat}(t)$$

$$\hat{H}_{\rm lat}(t) = \sum_{\langle ij\rangle} t_{ij}\hat{c}_i^\dagger \hat{c}_j + \sum_i \left(\boldsymbol{F}(t)\cdot\boldsymbol{r}_i\right)\hat{c}_i^\dagger \hat{c}_i$$

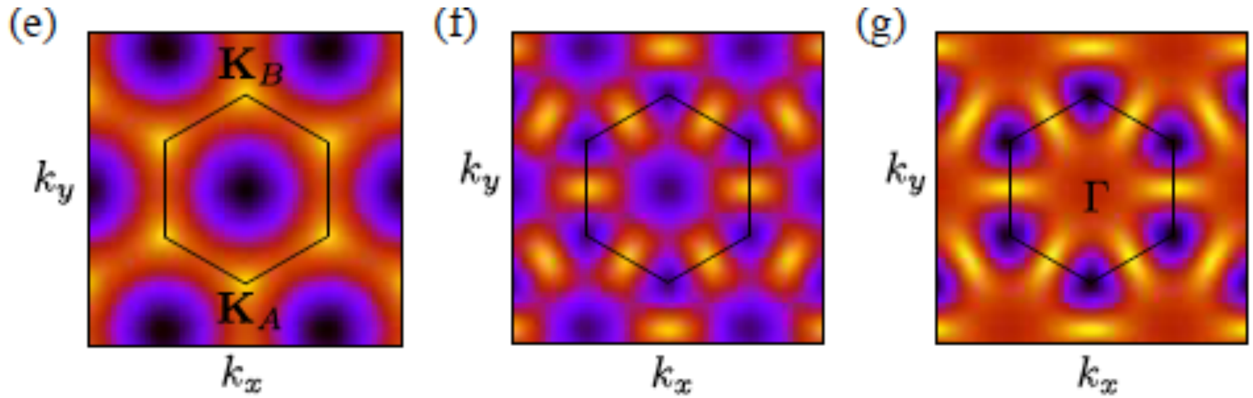
$$\hat{U}(T,t_0) = \mathcal{T} e^{-i\int_{t_0}^{t_0+T}\hat{H}(t)dt} = e^{-iT\hat{H}_{\rm eff}(t_0)}$$

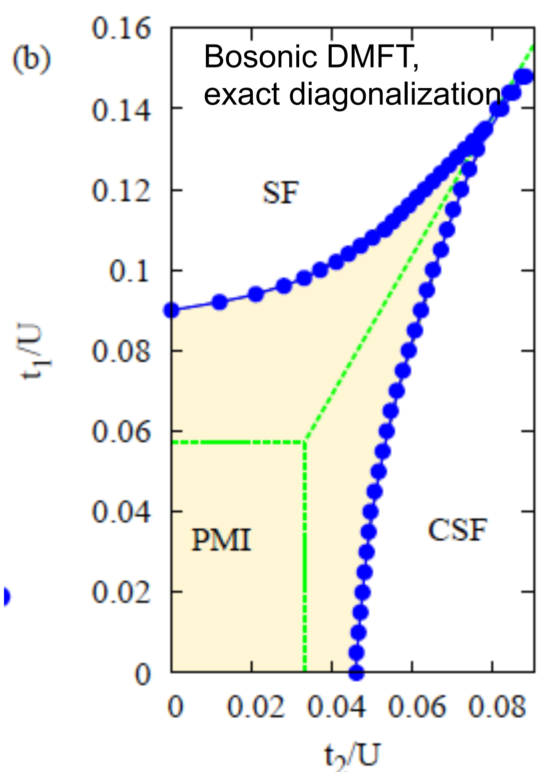
K. Plekhanov, G. Roux, KLH 2016

T : Hamiltonian periodic in time

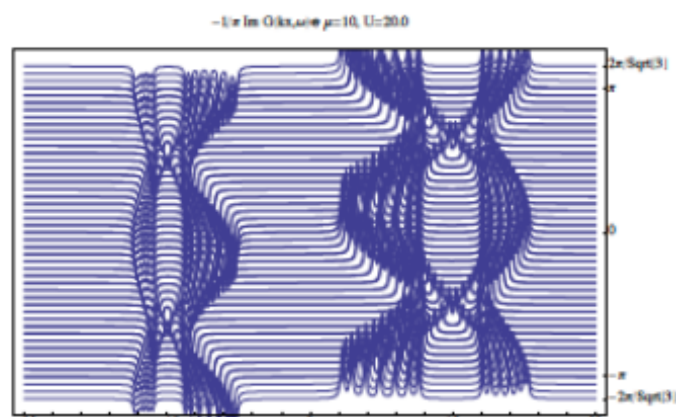
Exotic bosonic phases: Haldane model

I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB)
K. Plekhanov, G. Roux, KLH recent paper PRB 2017





Strong coupling cluster expansion in Mott



Similar models on <u>square</u> lattice:
L. K. Lim, C. M. Smith and A. Hemmerich,
Phys. Rev. Lett. 100, 130402 (2008) and PRA 2010

FFLO analogue in Heisenberg-Kitaev doped models
Tianhan Liu, Cécile Repellin,
Benoît Douçot, Nicolas Regnault
Karyn Le Hur, 2016

Non-trivial chiral
Edge excitations
In Mott phase

In progress,
Bosonic-Kane-Mele-Hubbard
K. Plekhanov, I. Vasic, A. Petrescu
R. Nirwan, G. Roux, W. Hofstetter,
KLH (chiral spin state)

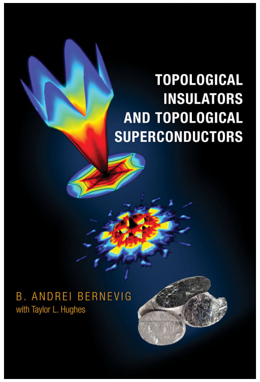
Spin-orbit coupling

Kane & Mele, PRL 95, 226801 (2005); Fu-Kane

see also: **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)**

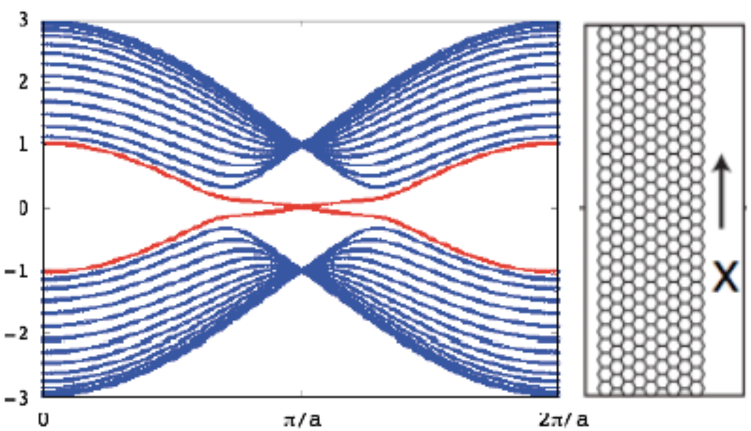
Also realizations in photon systems for example: M. Hafezi, S. Mittal, J. Fan, A. Migdall, J. Taylor (2013)
Mikael C. Rechtsman, Julia M. Zeuner, Yonatan Plotnik, Yaakov Lumer, Stefan Nolte, Mordechai Segev, Alexander Szameit (2013)

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^{z}_{\sigma\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'} \qquad \nu_{ij} = \pm 1$$



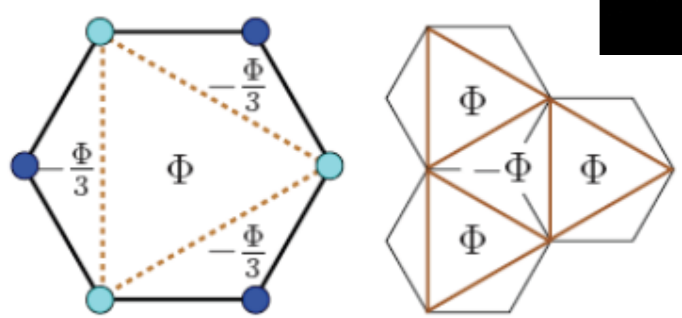
strip geometry:

QSH



edge states: Kramers's pair

$$\mathcal{H} \propto \Psi^{\dagger}_{k} \sigma^{z} \tau^{z} \Psi_{k}$$



Joel Moore, perspective Nature 2010

Half-filling

D. Carpentier, P. Delplace, K. Gavitski, M. Fruchart, N. Regnault
Gilles Montambaux, Jean-Noel Fuchs, Mark Goerbig, F. Piechon

Stable towards (moderate) interactions
S. Rachel and K. Le Hur, 2010; Wei Wu numerics

Also 3D analogues: Bismuth … Weyl fermions
QCD and flavor models

XXI, Detect the Majorana in topological SCs: L. Kouwenhoven Delft, 2012

See F. Wilczek, Majorana returns, Nat. Physics 2009

They appear accidently in spin chains: via Jordan-Wigner transformation (1928)
Generalization of Dirac algebra for harmonic oscillators 1925 (group theory)
high energy physics (neutrino…)

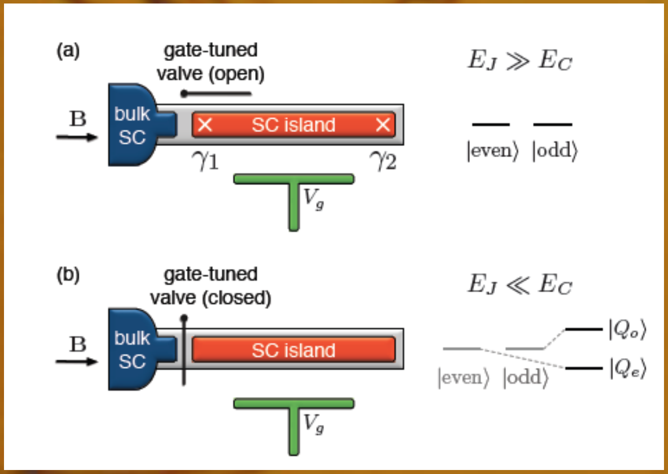
Particle and its own antiparticle

γ

Proposals:
Alexei Kitaev
Nick Read
Leonid Levitov
Hans Mooij
Liang Fu
Charles Kane
Carlo Beenakker
Matthew Fisher
Bert Halperin
Pascal Simon…

Challenge taking into account that the man who discovered the Majorana disappeared 1938

Progress in nano-engineering to reveal the Majoranas (see Bieri Cooper, Egger, Altand, C. Mora, E. Eriksson, J. Meyer, M. Houzet…)

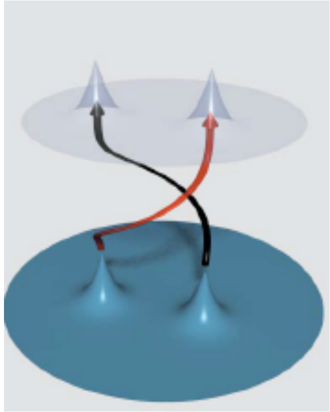


T. Kontos, A. Cottet (ENS)

D. Aaasen et al. arXiv 2015
Charles Marcus group 2016
Also Ali Yazdani, Princeton

Note: recent work on 2 coupled topological SC chains
Loic Herviou, Christophe Mora, KLH 2016

The Majorana fermion states must be occupied in pairs, since the entire physical system can only occupy real fermion states.
So only combinations of Majorana fermions can be occupied



This occupied state is inherently delocalized – it has weight in two spatially separated vortex cores.

N. Read & D. Green
N. Read & G. Moore
D. Ivanov, Volovik

$$\hat{c}^{\dagger}|\Psi_0\rangle = (\hat{\gamma}_1 + i\hat{\gamma}_2)\,|\Psi_0\rangle$$

Exchange of 1 and 2 $\gamma_1 \to \gamma_2$
$\gamma_2 \to -\gamma_1$

$$(\hat{\gamma}_2 + i\hat{\gamma}_1)\,|\Psi_0\rangle = i\,(\hat{\gamma}_1 - i\hat{\gamma}_2)\,|\Psi_0\rangle = i\hat{c}|\Psi_0\rangle$$

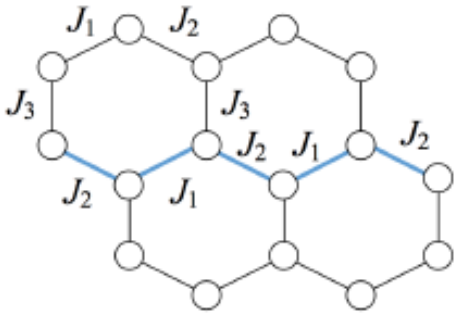
Different final state! – Non-Abelian statistics.

Application qubits : quantum computing

Sankar Das Sarma, Michael Freedman, Chetan Nayak arXiv:1501.02813

New spin chains in circuit QED (J1, J2) ···

Kitaev model 2006
Magnetic analogues, solvable
Spin liquids and BCS superconductors

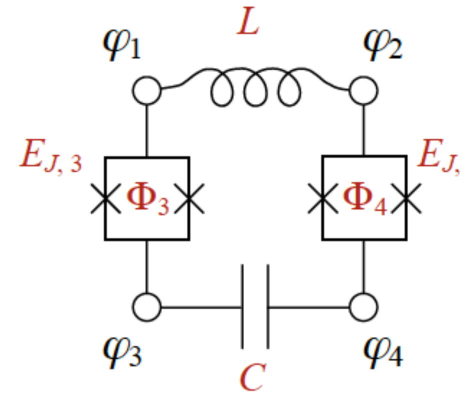


Recent efforts M. Hermanns, S. Trebst
J. Vidal, S. Dusuel,…
T. Liu, B. Douçot, C. Repellin, N. Regnault, KLH

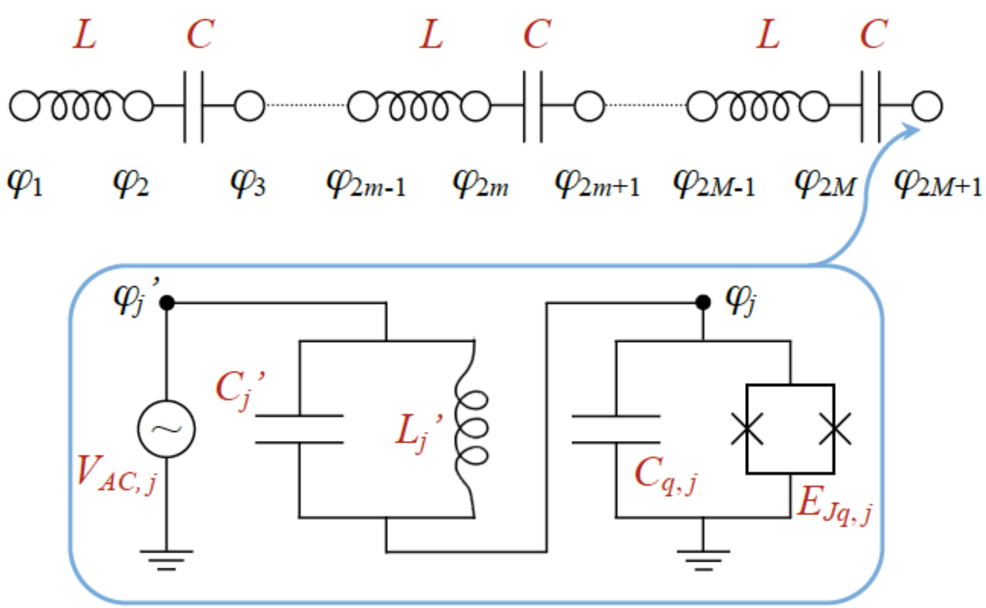
Simulation of New Devices with SC devices and Transmons

Anderson RVB states and Majoranas, p-wave SC
KLH, Ariane Soret, Fan Yang (24 pages, for theory and mapping) : arXiv:1703.07322
Possible braiding and applications quantum computing, « loop » devices in link with Sachdev-Ye-Kitaev models
Precise Device engineering in progress, Fan Yang master project M2 below

Su-Schrieffer-Heeger and Rice-Mele model with LC chains
T. Goren, K. Plekhanov, F. Appas, G. Roux, KLH − **in progress**
Probe of topology, Bloch bands, and transport with photons



Coupling
4 Majoranas



A 4-site toric code
has been realized

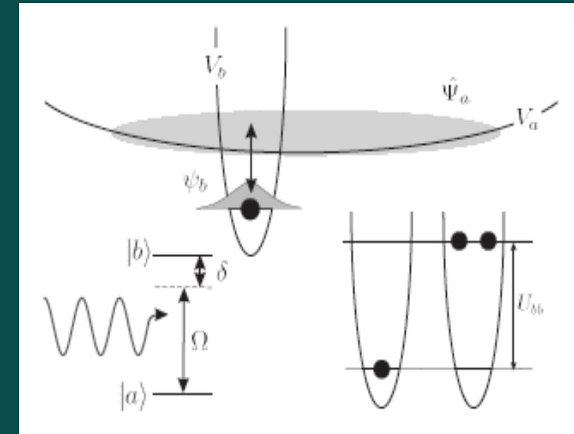
Y. P. Zhong et al
PRL 117, 110501 (2016)

Exemple Realization of a Kitaev spin chain (emergent Majorana chain)

Loic Herviou, C. Mora and KLH (collaboration with P. Roushan, C. Neill − google Santa Barbara
on generalized quenches and bi-partite fluctuations **in XY and Ising quantum spin chains**)

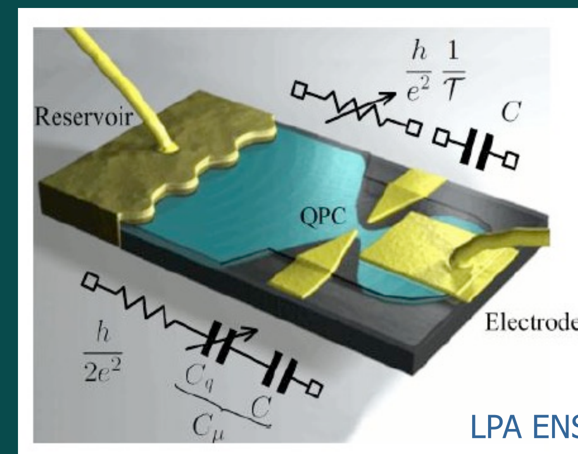
COLD-ATOMIC Quantum IMPURITIES

A. Recati et al. PRL **94**, 040404 (2005)
Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)
Single Atom: Ph. Grangier et al. Science **30**9, 454 (2005)
A. Fuhrmanek, Y. R. P. Sortais, P. Grangier, A. Browaeys
Phys. Rev. A 82, 023623 (2010).
D. Porras, F. Marquardt, J. von Delft, J. I. Cirac (2007),...
M. Knap et al. Phys. Rev. X 2, 041020 (2012)
M. Knap, D. A. Abanin, E. Demler, PRL 111, 265302 (2013)
J. Bauer, C. Salomon, E. Demler PRL 111, 215304 (2013)



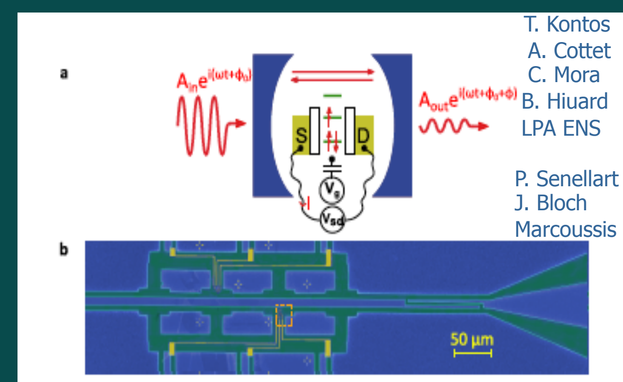
RC circuits

M. Buettiker, H. Thomas, and A. Pretre, Phys. Lett. A 180, 364 - 369,(1993)
J. Gabelli et al., Science **313**, 499 (2006); G. Feve et al. 2007 (LPA ENS)
J. Gabelli et al. Rep. Progress 2012
C. Mora and K. Le Hur, Nature Phys. 6, 697 (2010)
Y. Hamamoto, et al. Phys. Rev. B **81**, (2010) 153305
Y. Etzioni, B. Horovitz, P. Le Doussal, PRL **106**, 166803 (2011)
M. Filippone, KLH, C. Mora; P. Dutt, T. Schmidt, C. Mora, KLH, 2013,…

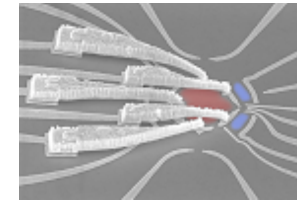


Hybrid Photon-Nano Systems, Impurities with Photons

K. Le Hur, Phys. Rev. B 85, 140506(R) (2012)
A. Leclair, F. Lesage, S. Lukyanov and H. Saleur (1997)
M. Goldstein, M. H. Devoret, M. Houzet and L. I. Glazman, 2012
Grenoble: S. Florens, H. Baranger, N. Roch and collaborators
M. Hofheinz et al. arXiv:1102.0131
M. Delbecq et al. PRL 107, 256804 (2011)
M. Schiro & KLH, arXiv 1310.8070, PRB 2014 …

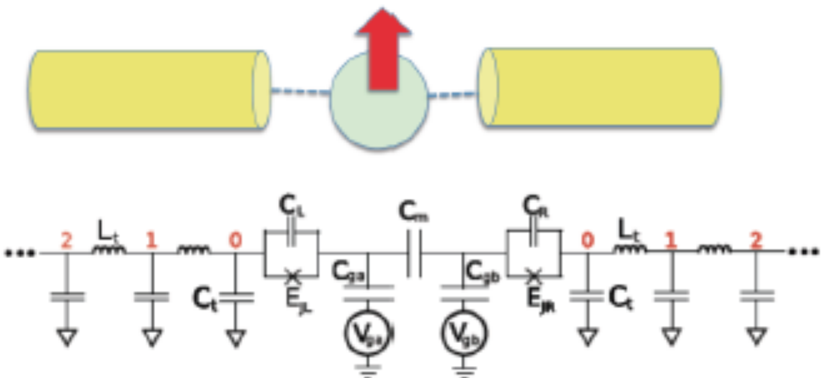


One way motion of light with Kondo « correlations »

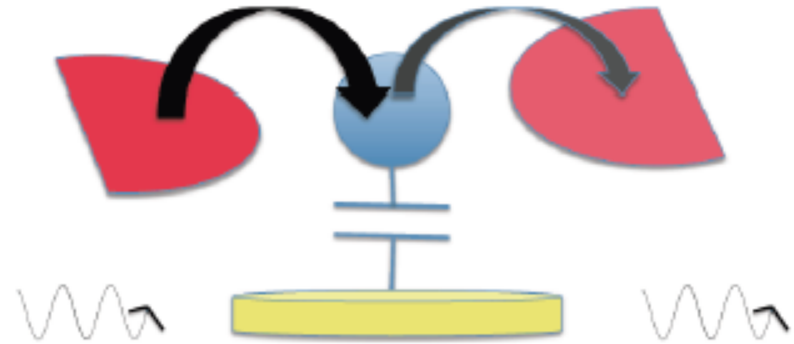


D. Goldhaber-Gordon

Kondo physics and Heavy fermions
Book by Alex Cyril Hewson, Cambridge University Press
Ph. Nozières 1974, Nobel Prize Kenneth Wilson

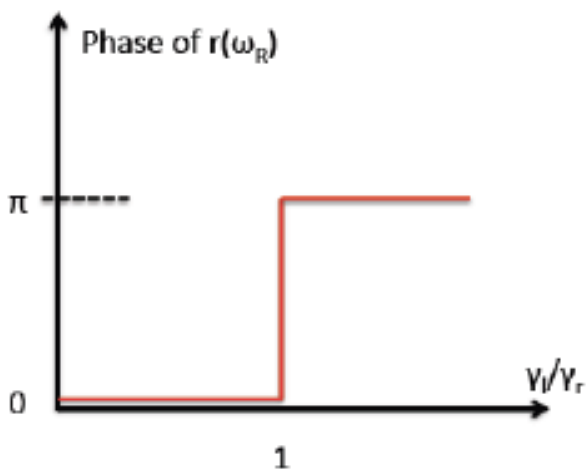


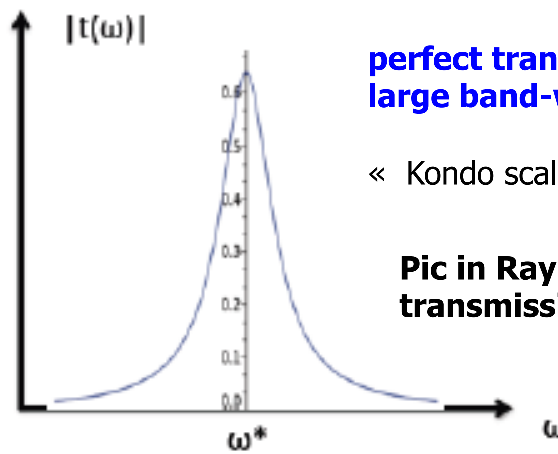
K. Le Hur 2012
M. Goldstein, M. Devoret, M. Houzet, L. Glazman
2013



M. Schiro & KLH, 2014

analogy Friedel sum rule for electrons in DC transport
L. Kouwenhoven & L. Glazman, physics world 2001





perfect transmission
large band-width: application

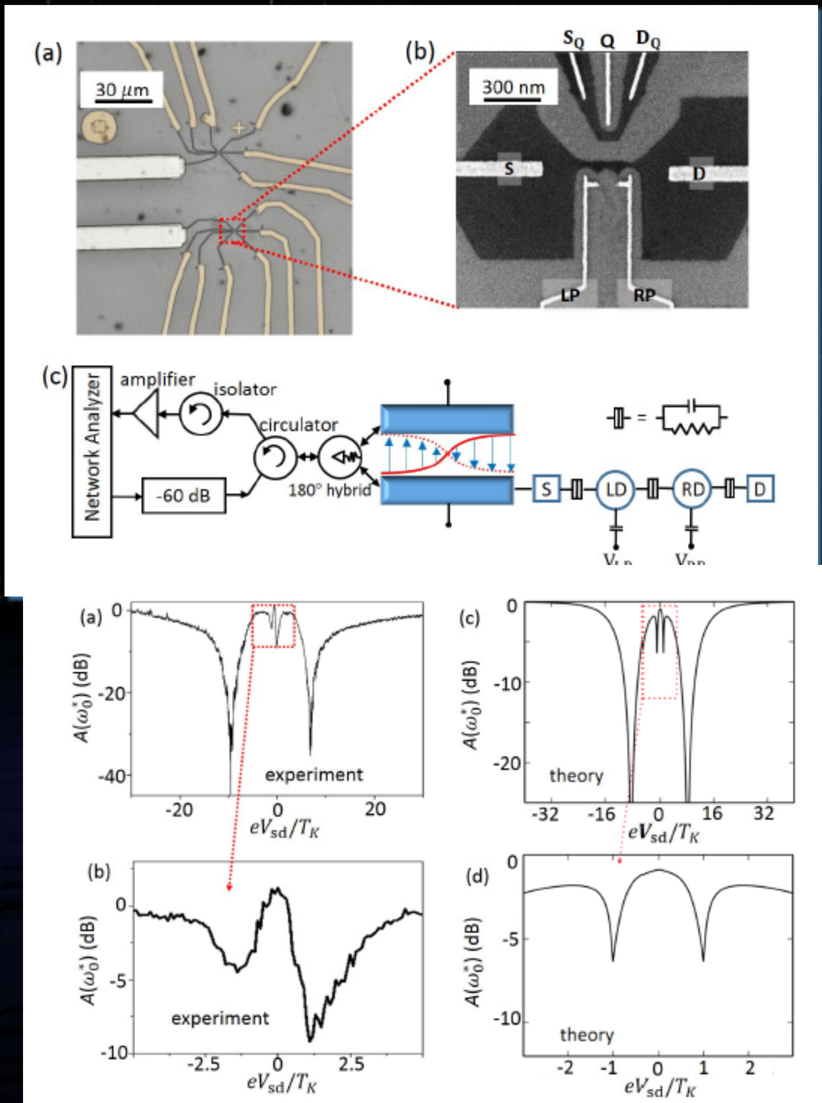
« Kondo scale »

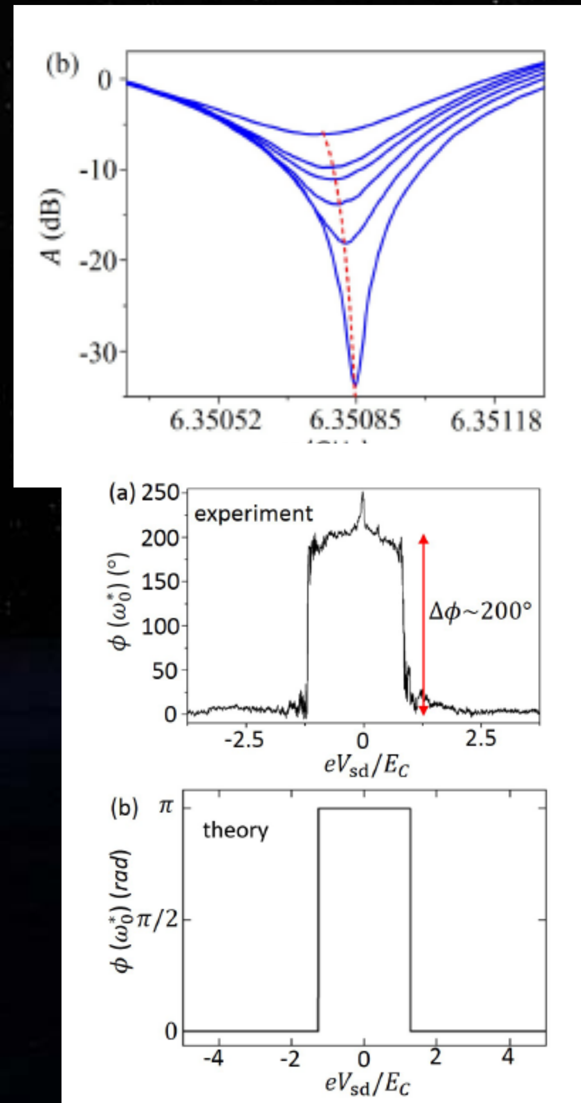
Pic in Rayleigh transmission

Experiment N. Roch Grenoble
Also Garching, Waterloo, Maryland …

KLH, L. Henriet, A. Petrescu, K. Plekhanov, G. Roux, M. Schiro Académie of Sciences 2016
Detail of Friedel phase in the new review: 2017

Explore Hybrid Kondo System in graphene





arXiv: 2015

Phase of Π observed

T=30mK

2 cooling procedures

T_K is a new energy scale: the Kondo energy scale (amplitude & DC transport favor SU(4) Kondo physics)

Guang-Wei Deng†,[1,2] Loïc Henriet†,[3] Da Wei,[1,2] Shu-Xiao Li,[1,2] Hai-Ou Li,[1,2] Gang Cao,[1,2] Ming Xiao,[1,2] Guang-Can Guo,[1,2] Marco Schiró,[4] Karyn Le Hur,[3] and Guo-Ping Guo[1,2,*]

Transmission line

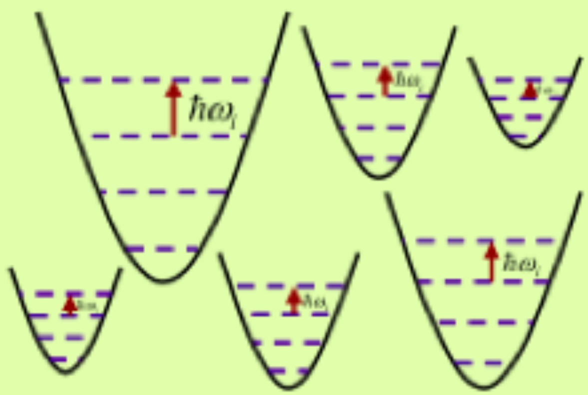
Kosterlitz-Thouless transition and topology

- Model the environment by quantum harmonic oscillators

$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987)
U. Weiss book, quantum dissipative systems, 1999

$$H_B = \sum_i \left(\frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2 x_i^2}{2} \right)$$



Bosonic bath

$$\frac{1}{2}\left\langle \sum_i \lambda_i x_i(t) \cdot \sum_i \lambda_i x_i(0) \right\rangle_\omega = \hbar J(\omega) \coth(\omega / 2k_B T)$$

Ohmic dissipation

$$J(\omega) = \alpha \pi \hbar \omega / 2$$

Dissipation strength

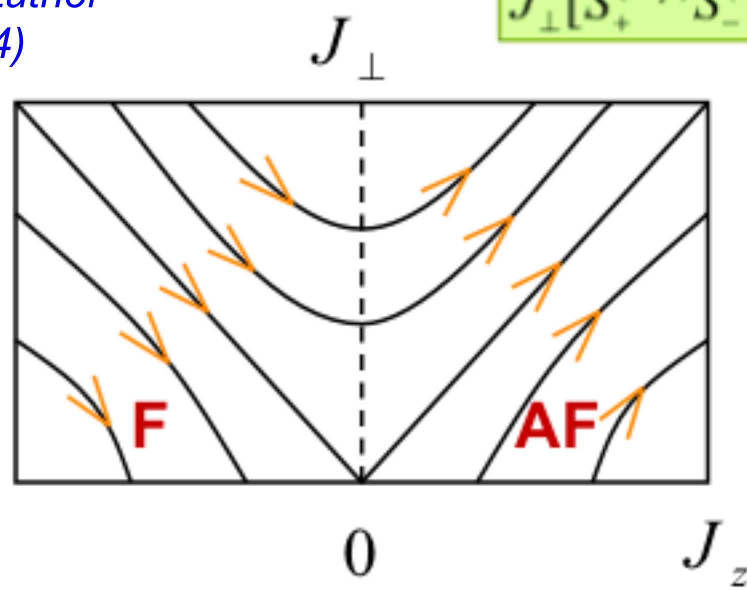
Analogy to another quantum impurity Kondo problem

V.J. Emery & A. Luther
PRB 9, 215 (1974)

$$J_\perp [S_+^{(imp)} S_-^{(e)} + h.c.] + J_z S_z^{(imp)} S_z^{(e)}$$

$$\mathbf{S}^e\,(\mathbf{x=0}) = \sum_{\substack{\mathbf{k,k'}\\ \alpha,\beta}} \mathbf{c}_{\mathbf{k}\alpha}^{\dagger}\, \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}\, \mathbf{c}_{\mathbf{k'}\beta}$$

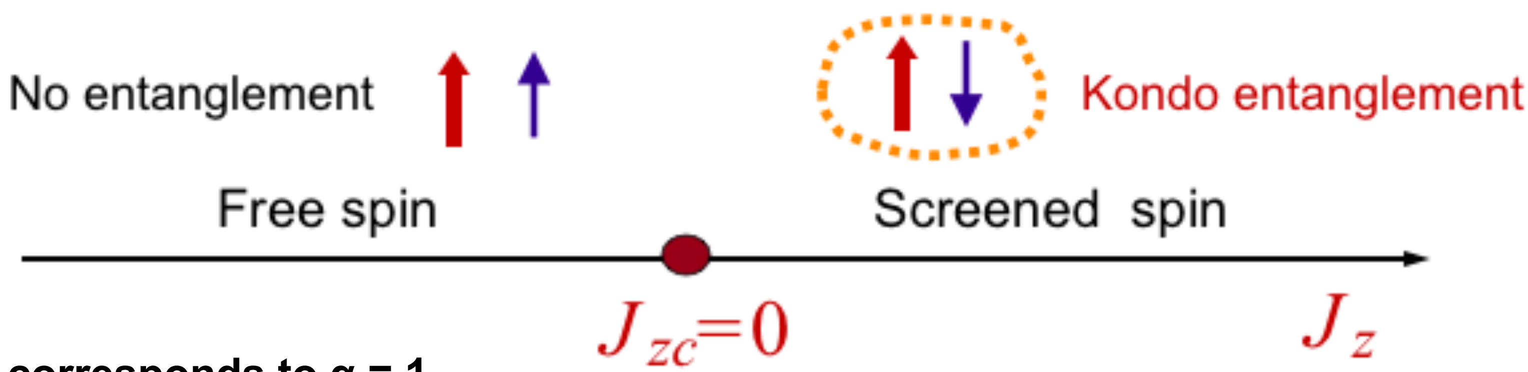
Perturbative calculations



Bethe Ansatz:
See N. Andrei; A. Tsvelik; P. Wiegmann
CFT: Affleck-Ludwig

See also K. Wilson, NRG
Ph. Nozières, **Fermi liquid**, 1974
Coqblin-Schrieffer

Small $J_\perp$



Jzc = 0 corresponds to α = 1

Berezinskii-Kosterlitz-Thouless:

2D XY models: Superconductors, ^{4}He, Cold atomic bosons

$$H = -J \sum_{\langle i;j \rangle} \cos(\varphi_i - \varphi_j)$$

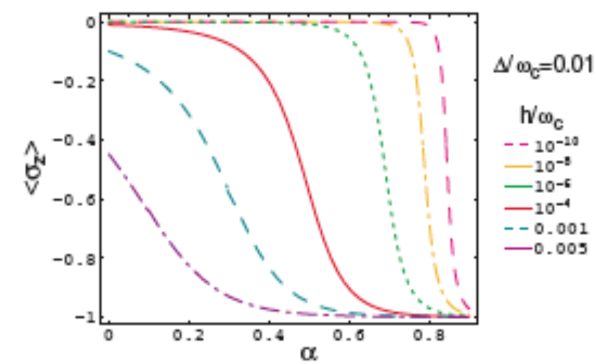
SC order parameter = $|\Psi| \exp(i\varphi)$

$S_x + iS_y = \exp(i\varphi)$

KT transition: High Temperature disordered phase (free vortices)

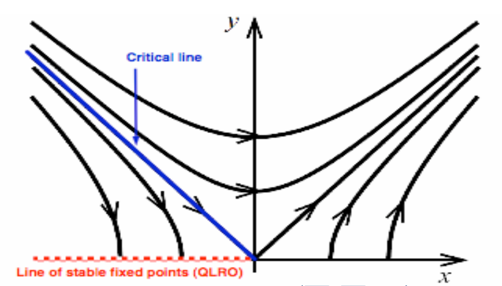
Low-Temperature quasi-long range order

KLH 2008



Universal Jump of Superfluid density at T_{KT}

(vortex fugacity)



Simple summary

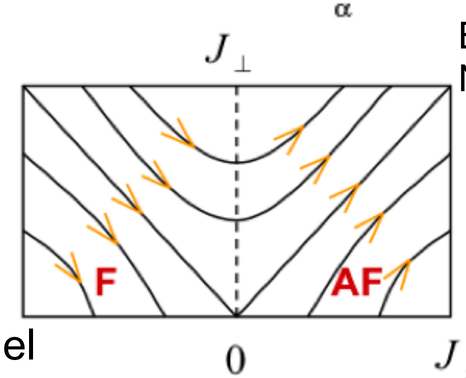
$(T-T_{KT})$

P. W. Anderson 1969

Related RG of Kondo model

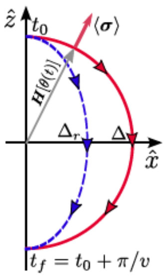
Bethe Ansatz

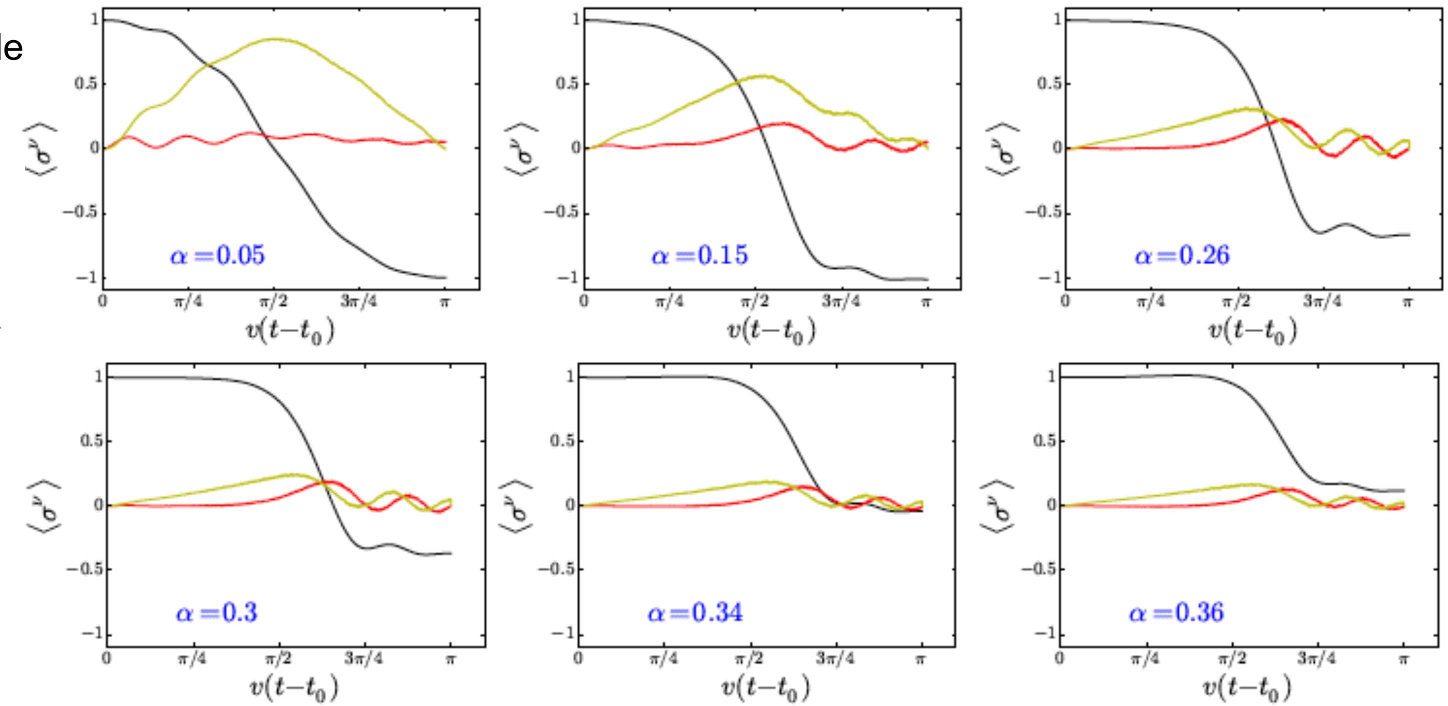
NRG



Driven effects: stochastic approach
L. Henriet, A. Sclocchi, P. Orth and KLH, 2017

Polar angle = vt





[12] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys **59**, 1 (1987).
[13] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B **1**, 4464 (1970).

[35] P. P. Orth, A. O. Imambekov, and K. Le Hur, Phys. Rev. A **82**, 032118 (2010).
[36] P. P. Orth, A. O. Imambekov, and K. Le Hur, Phys. Rev. B **87**, 014305 (2013).
[37] L. Henriet, Z. Ristivojevic, P. P. Orth, and K. Le Hur, Phys. Rev. A **90**, 023820 (2014).
[38] L. Henriet and K. Le Hur, Phys. Rev. B **93**, 064411 (2016).
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Summary

Many-Body physics in cQED and Josephson systems.

Why?

Condensed-matter physics simulators

(complement efforts in materials, cold atoms)

What's new: novel limits (driven, dissipation, AC)

Ramsey protocol in tunnel junctions (2 pulses):

T. Goren, KLH and E. Akkermans 2016

Novel applications: topology

either by topology, or Kondo

Little device : google