

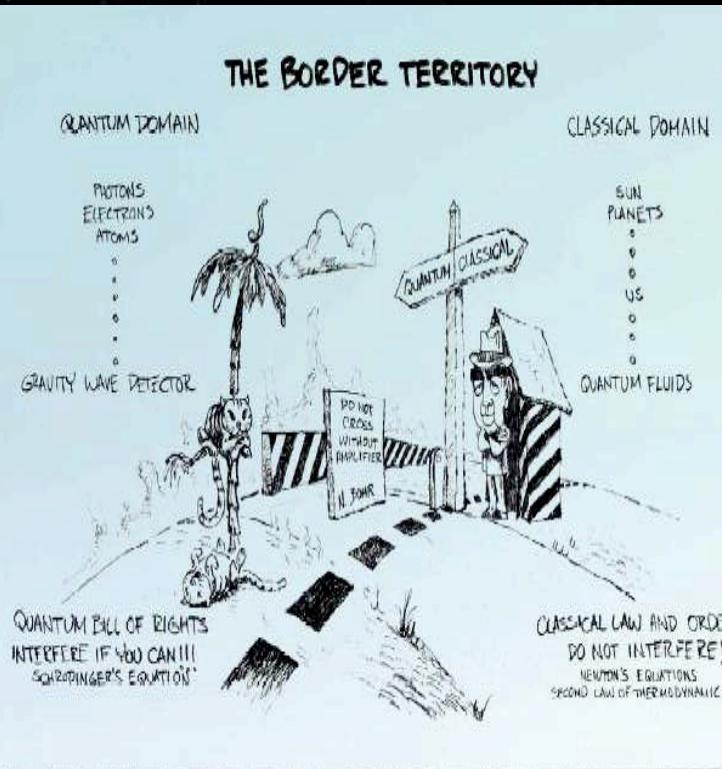


Light-Matter Systems, Quantum Control

Karyn Le Hur



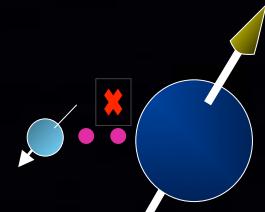
Picture W. Zurek:



**CPHT Ecole
Polytechnique**



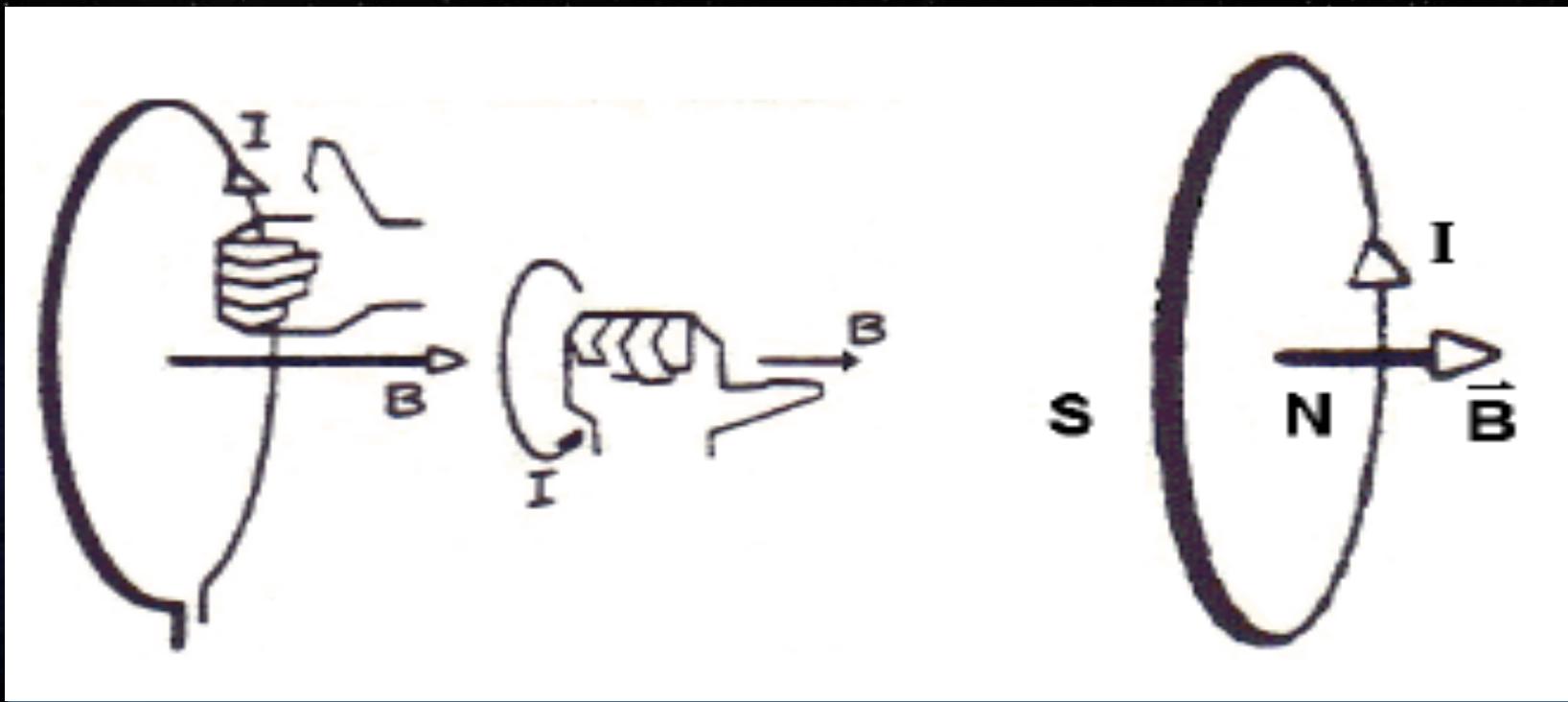
CNRS



ULM January 25th 2016

Angular momentum and Magnetism

10-15 minutes Intro



XIX century
Ampere, Biot, Savart...

XX: Scientists play with "toupies"

Spin?
Neutrino?



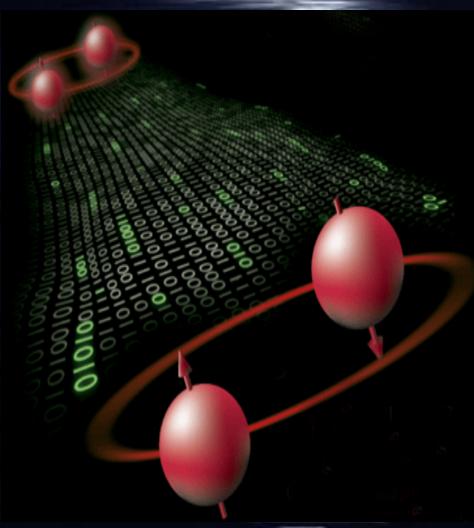
Quantum Weirdness

◆ We move down to the tiny scale of atoms...

Spin

Uhlenbeck-Goudsmit
, 1925

Pauli, 1927
Dirac, 1928



◆ Quantum effects?

particle-wave duality $p=\hbar k$

photo-electric effect $E=\hbar\omega$

quantum superposition,...

Quantum computer

- qubits $|a\rangle, |b\rangle \hat{=} \alpha|0\rangle + \beta|1\rangle, |\alpha|^2 + |\beta|^2 = 1$
- physical implementation:
quantum 2-level-system: $|\uparrow\rangle = |0\rangle, |\downarrow\rangle = |1\rangle$

Schrödinger Cat: Character of the quantum





Quantum Computation: 2-level system as a « Qubit »

The Copenhagen
School (1937):

“ $\hbar/2 = 1$ ”

2 level system

$$S^\pm = S_x \pm iS_y$$

$$S_x = \frac{\hbar}{2} \sigma_x$$

$$S_y = \frac{\hbar}{2} \sigma_y$$

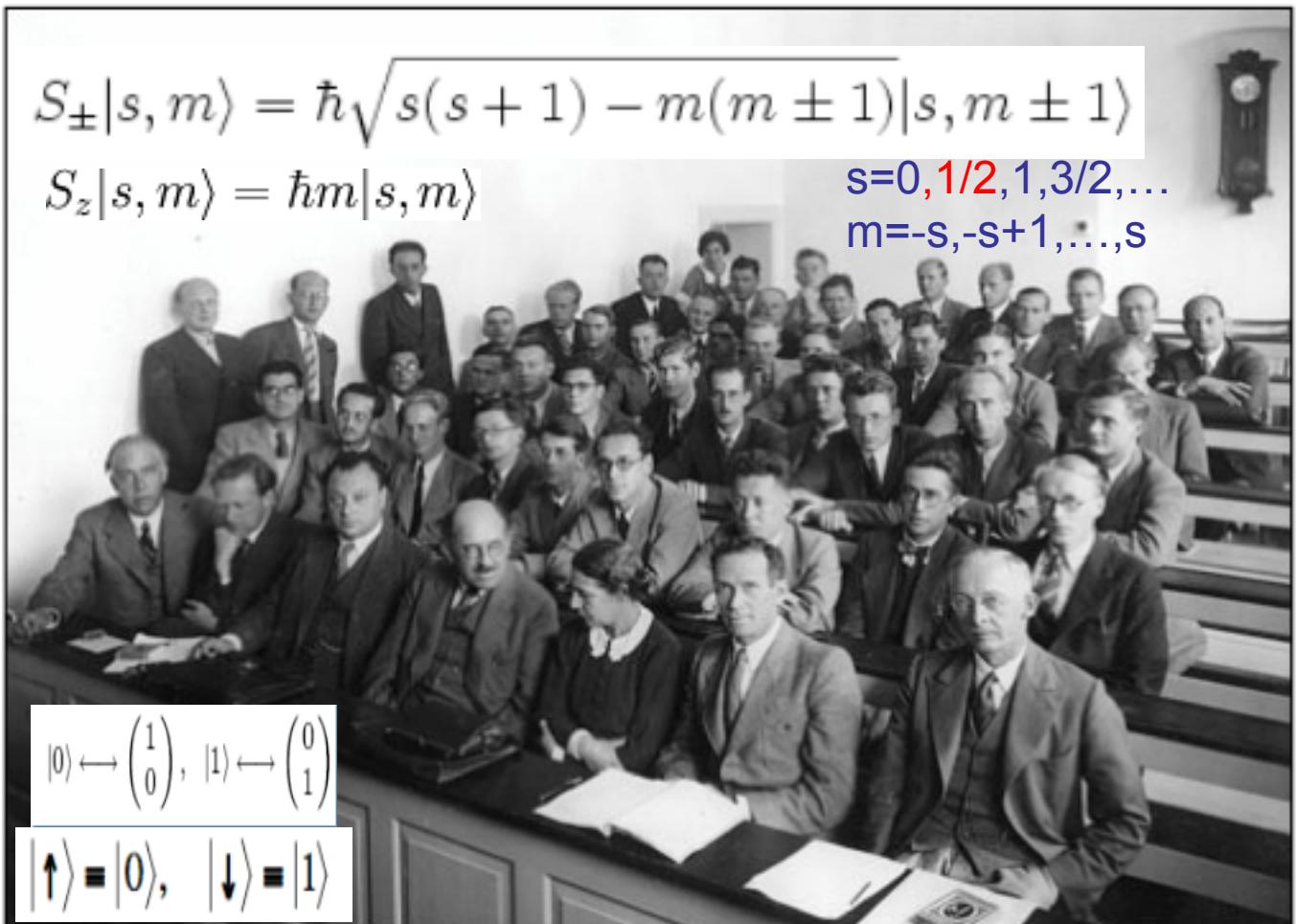
$$S_z = \frac{\hbar}{2} \sigma_z$$

Pauli
matrices

$$S_\pm |s, m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)} |s, m \pm 1\rangle$$

$$S_z |s, m\rangle = \hbar m |s, m\rangle$$

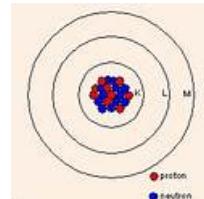
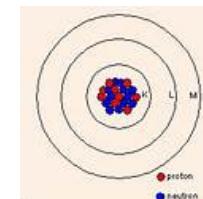
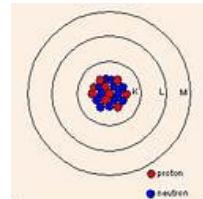
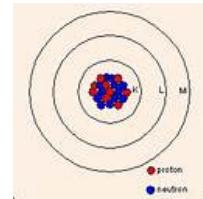
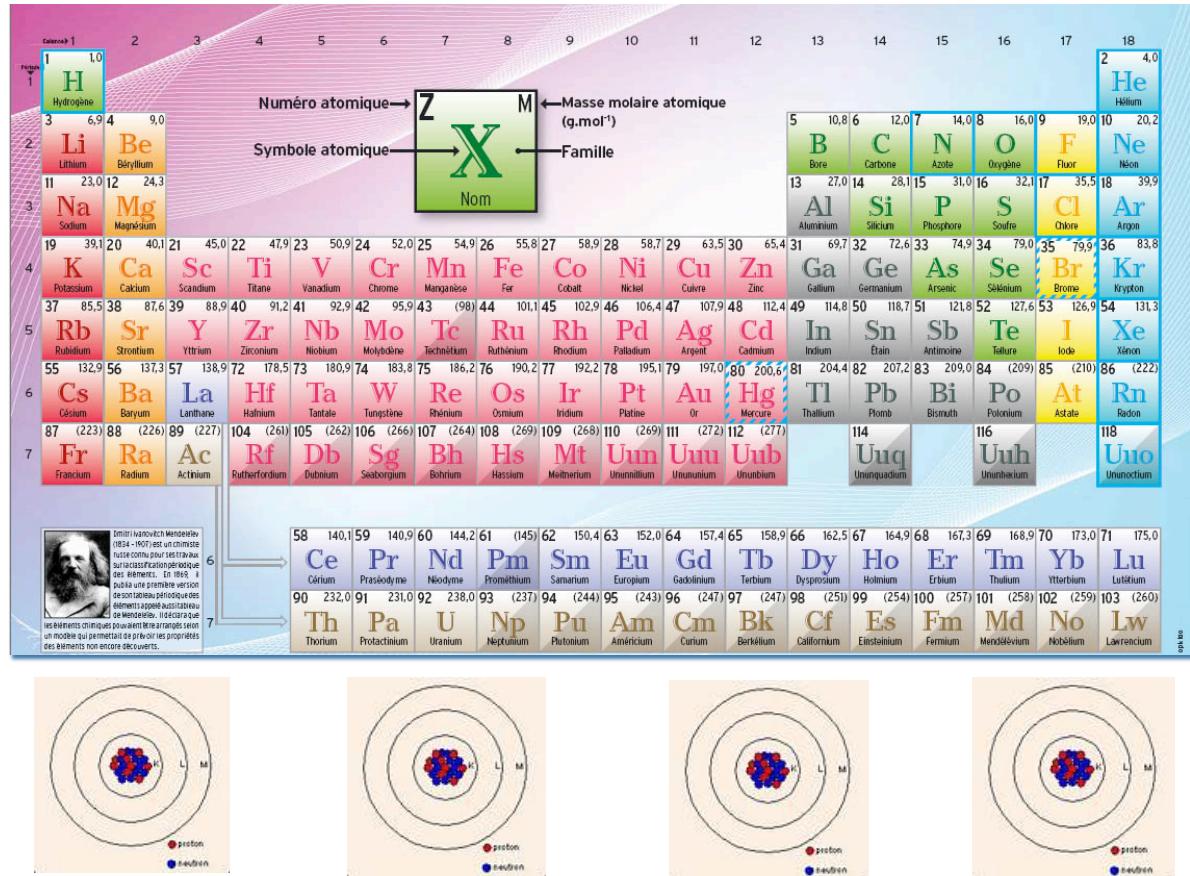
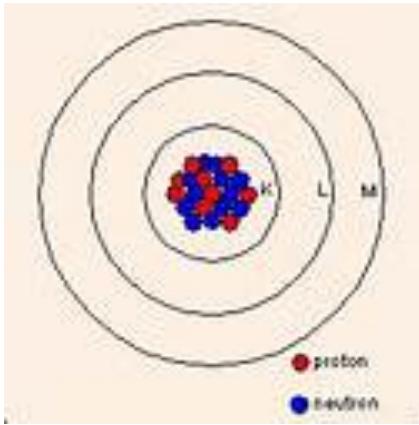
$$\begin{aligned} s &= 0, 1/2, 1, 3/2, \dots \\ m &= -s, -s+1, \dots, s \end{aligned}$$



$$[S_x, S_y] = i\hbar S_z$$

$$|\uparrow\rangle = |s=1/2, S_z=1\rangle \text{ and } |\downarrow\rangle = |s=1/2, S_z=-1\rangle$$

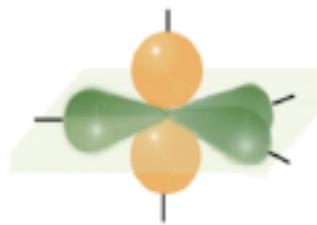
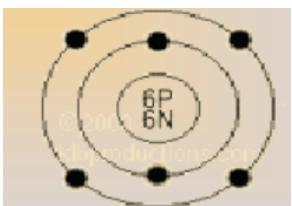
Condensed matter



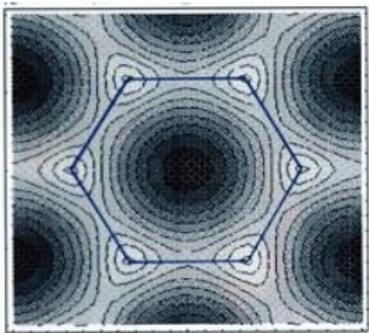
Many-Body electronic properties: important discoveries

- Fermi liquid, Kondo physics
- Superconductivity, High-Tc superconductivity
- Mott, Magnetism, quantum Hall Physics
- Graphene, Topological Insulators, ...

Band Structure & Magnetism

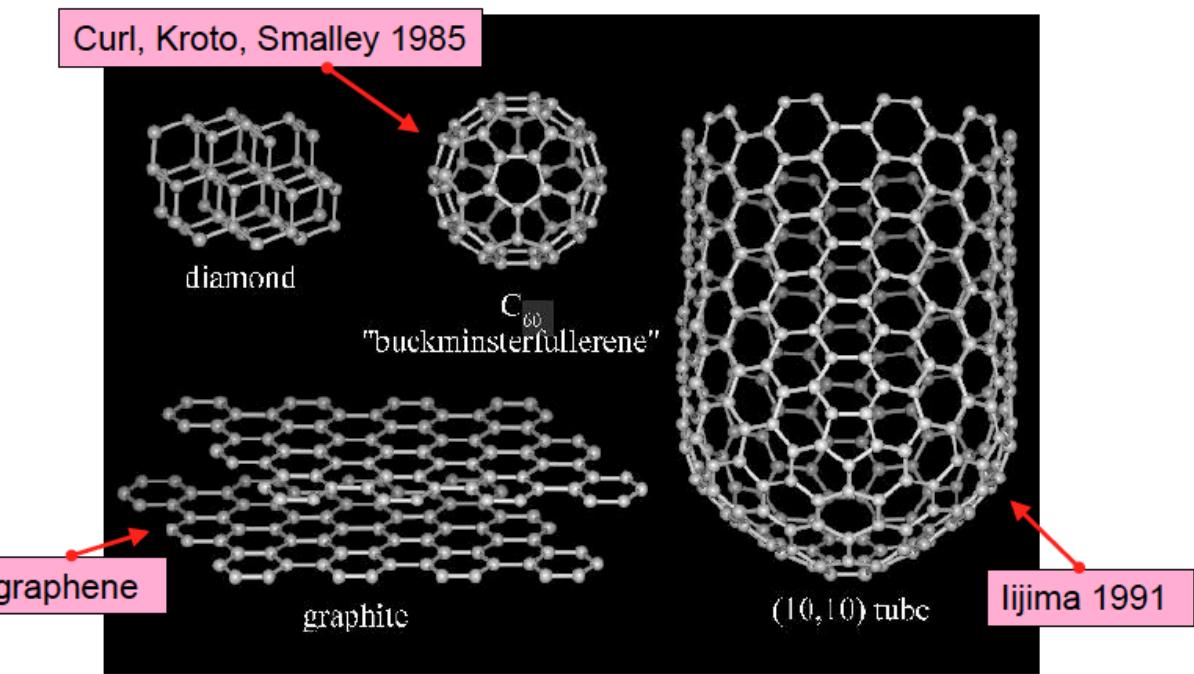


$$t \sim 2.8 \text{ eV}$$
$$a_{\text{C-C}} \sim 1.42 \text{ \AA}$$



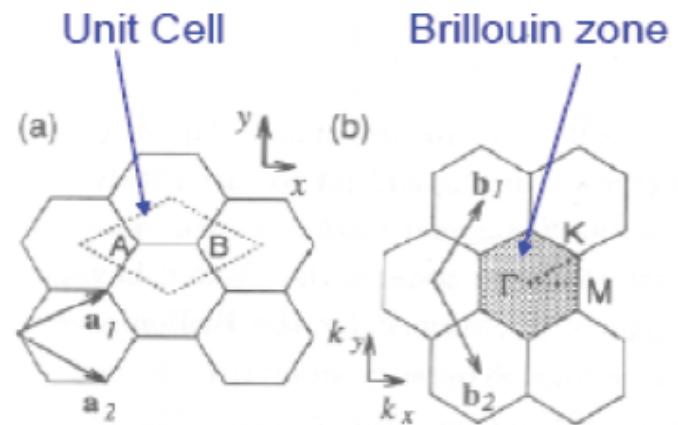
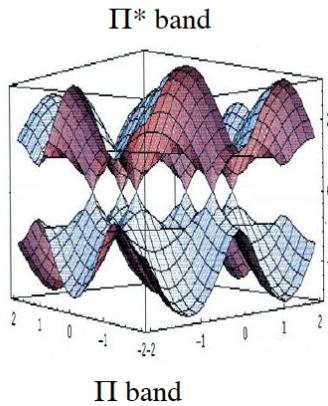
Wallace, 1947

Realized in cold atoms (L. Tarruell)



A. K. Geim & K. S. Novoselov, Nature Materials **6**, 183 (2007)
P. L. McEuen, Physics World **13**, 31 (2000)

$$v_F \sim 3ta/2 \sim 10^6 \text{ m/s} = c/300$$



Magnetism

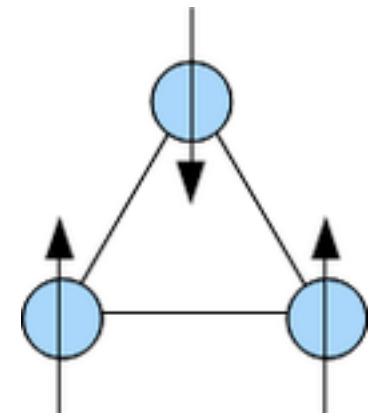
Hubbard model and Superexchange

Sir Nevil Mott: 1 electron per site and it costs
A large energy U to put 2 electrons on the same site

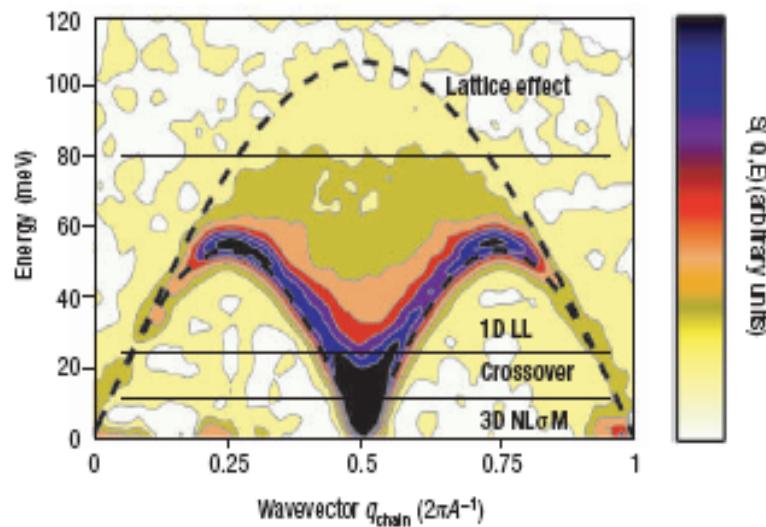
Néel state : up down up down (Goldstone modes)

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow},$$

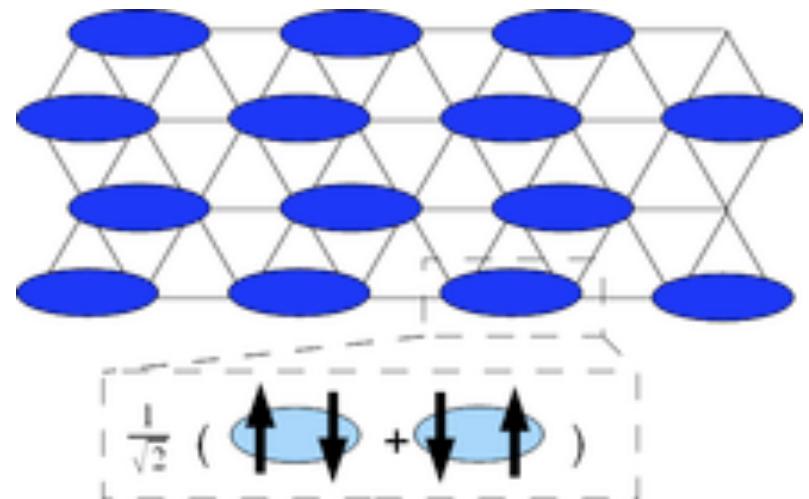
frustration



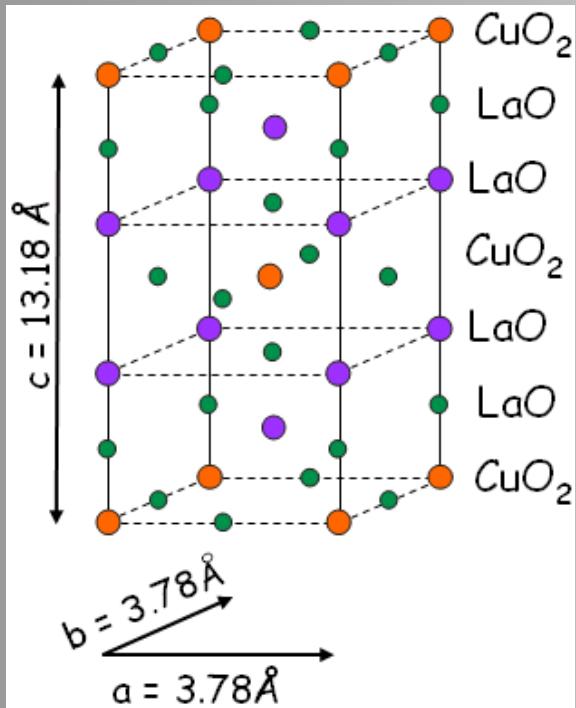
1D exactly solvable: spinons



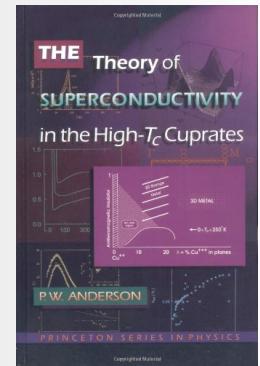
RVB state of Ph. Anderson



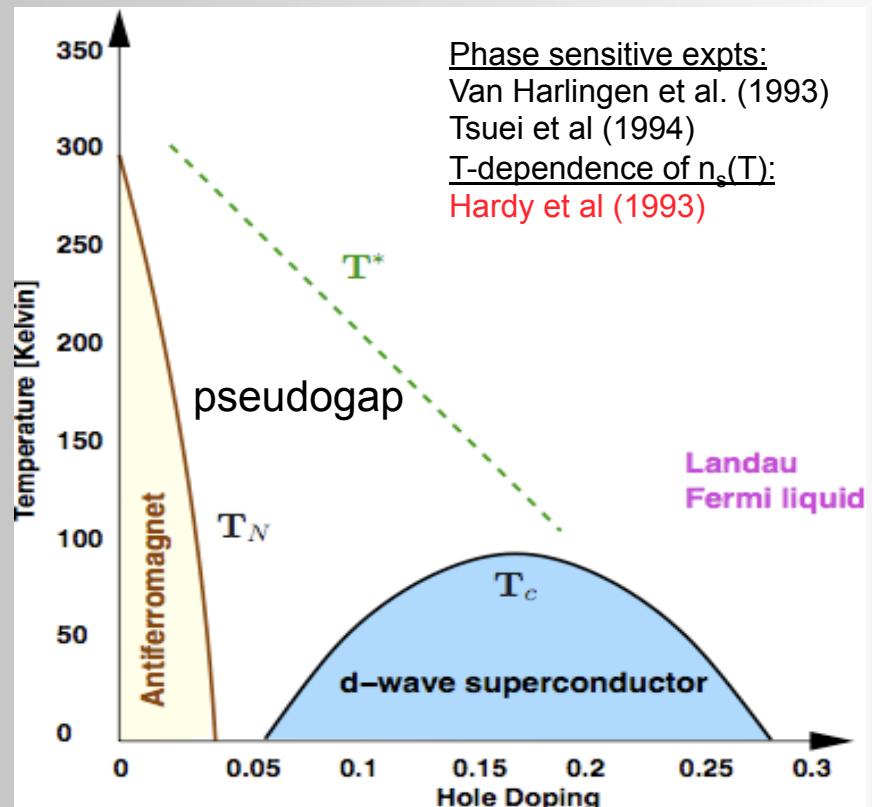
High Temperature Superconductors



1986



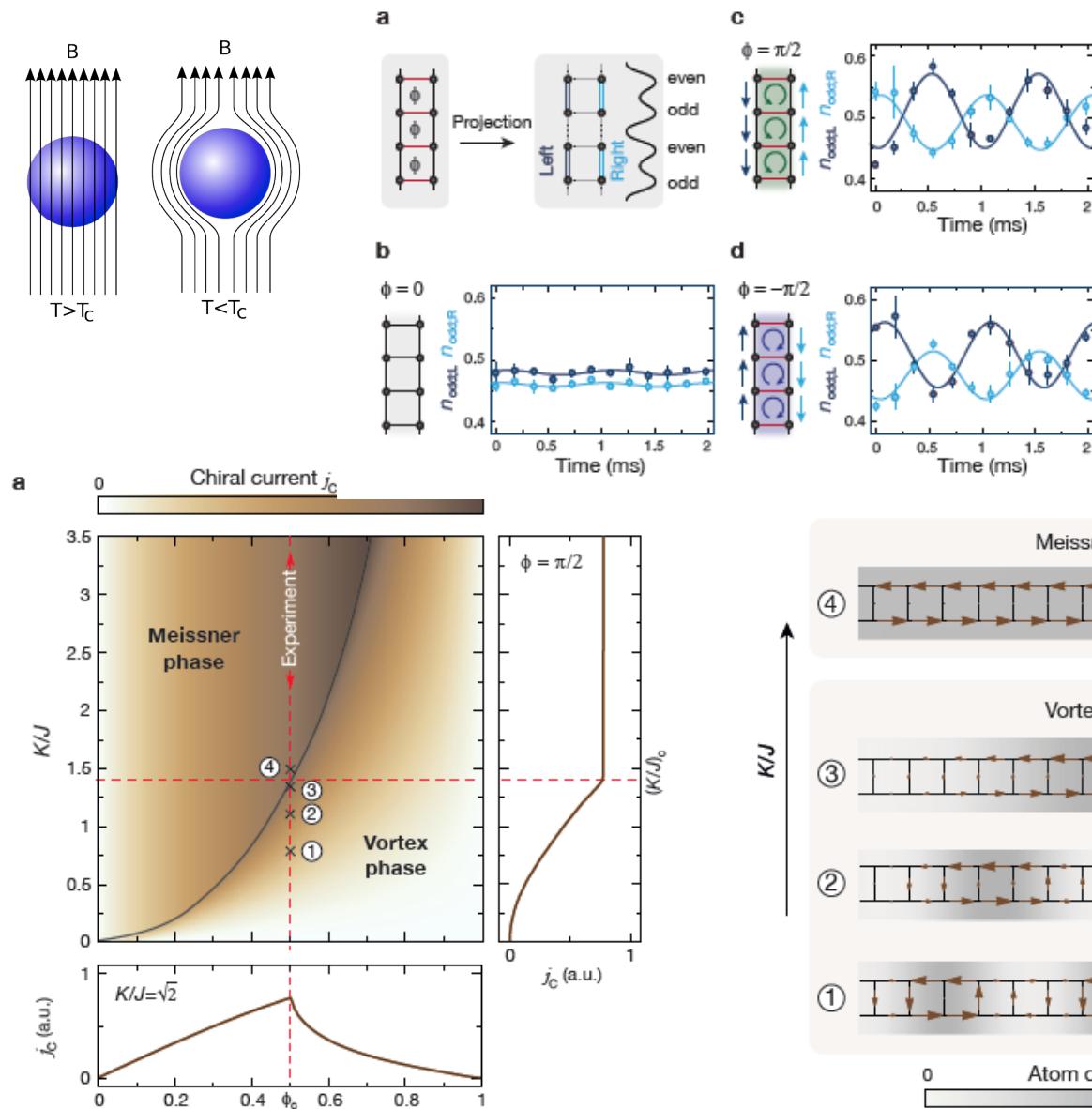
Review Karyn Le Hur & T. M. Rice, 2009



- Coupled CuO₂ layers
- Doping with holes leads to SC
- Nonmonotonic T_c versus doping
- Maximum T_c ~ 150 K
- Electronic SC without phonons?
- Normal phase is not a Fermi liquid at low doping: gap does not follow T_c!

Observation of the Meissner effect with ultracold atoms in bosonic ladders

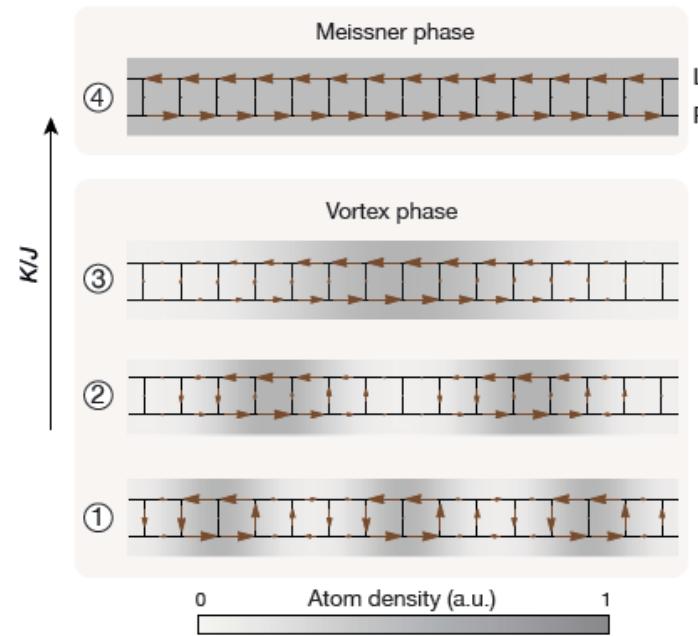
M. Atala^{1,2}, M. Aidelsburger^{1,2}, M. Lohse^{1,2}, J. T. Barreiro^{1,2}, B. Paredes³ & I. Bloch^{1,2}



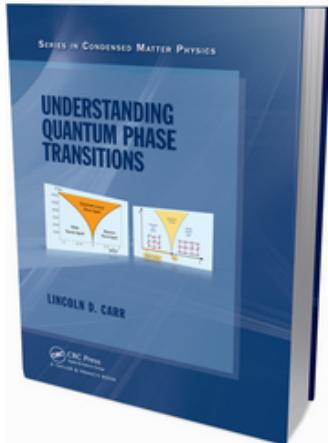
Nature Physics 2014

Theory by
E. Orignac & T. Giamarchi 2001

A. Petrescu & KLH, 2013-2015
DMRG with Guillaume Roux
Muenich: M. Piraud,
F. Heidrich-Meisner, U. Schollwock



General Concepts



Condensed-matter, Light-matter
systems and non-equilibrium dynamics



Quantum information tools for
« sensing »

Quantum systems & topological
protection

Sample two-state systems

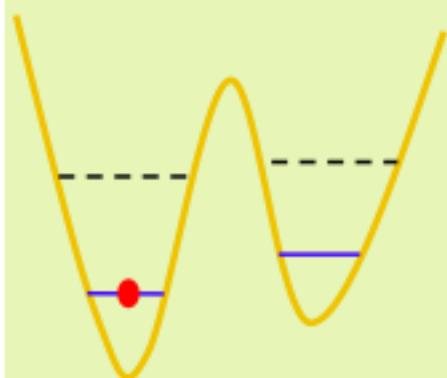
□ Intrinsic two-state

Nuclei spin S=1/2

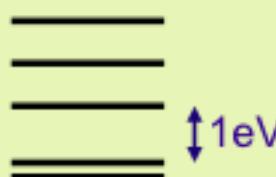
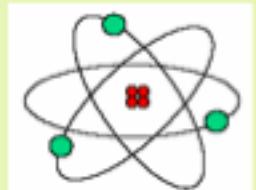
Polarization of photon (electromagnetic cavity)

□ Truncated two-state

Particle in double well

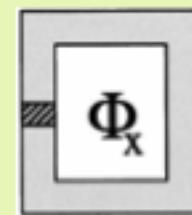


Atom



Condensed matter systems

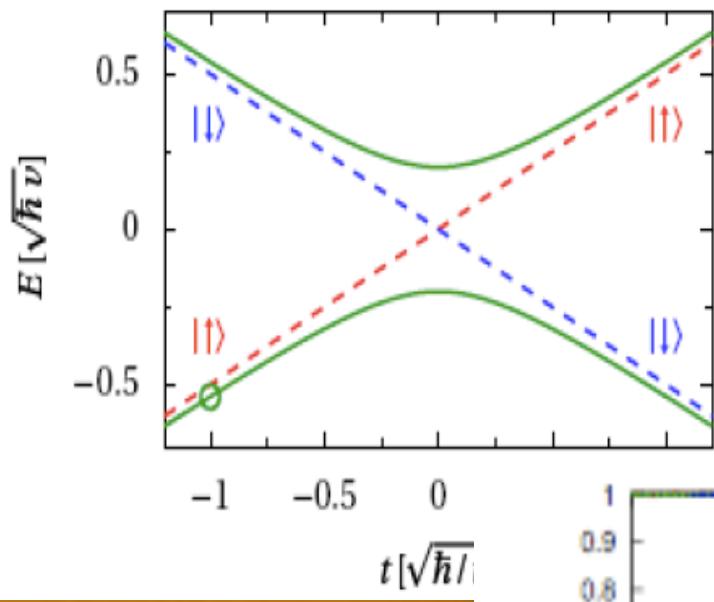
▪ rf SQUID



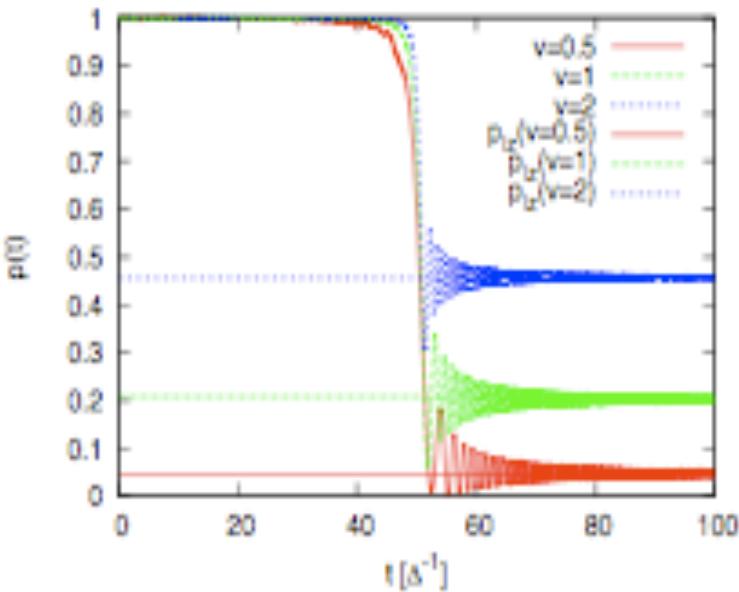
▪ Quantum dot



Main question: OPEN limit (A. Rivas and S. F. Huelga, review 2011)



Avoided crossing

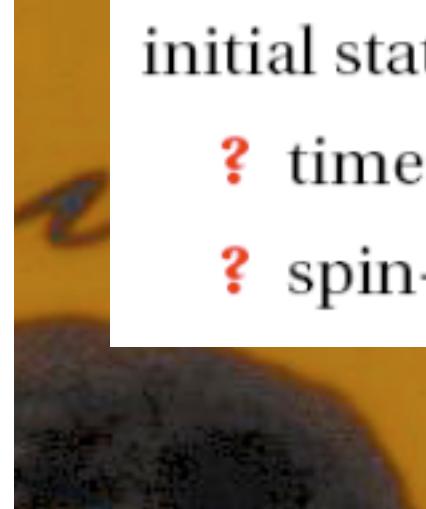


atomic/molecular collisions

Chemical reaction dynamics

NanoSystems & Nanomagnets, BECs...

The Majorana formula and the Landau–Zener–Stückelberg treatment of the avoided crossing problem

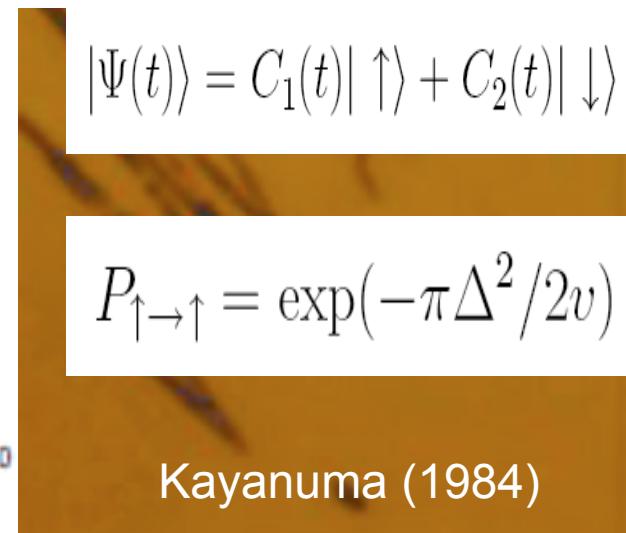


initial state: $|\psi(t = -\infty)\rangle = |\uparrow\rangle$

? time evolution

? spin-flip probability $P_{\uparrow \rightarrow \downarrow}$

$$H = \frac{vt}{2}\sigma^z + \frac{\Delta}{2}\sigma^x$$



$$P_{\uparrow \rightarrow \downarrow} = \exp(-\pi\Delta^2/2v)$$

Kayanuma (1984)

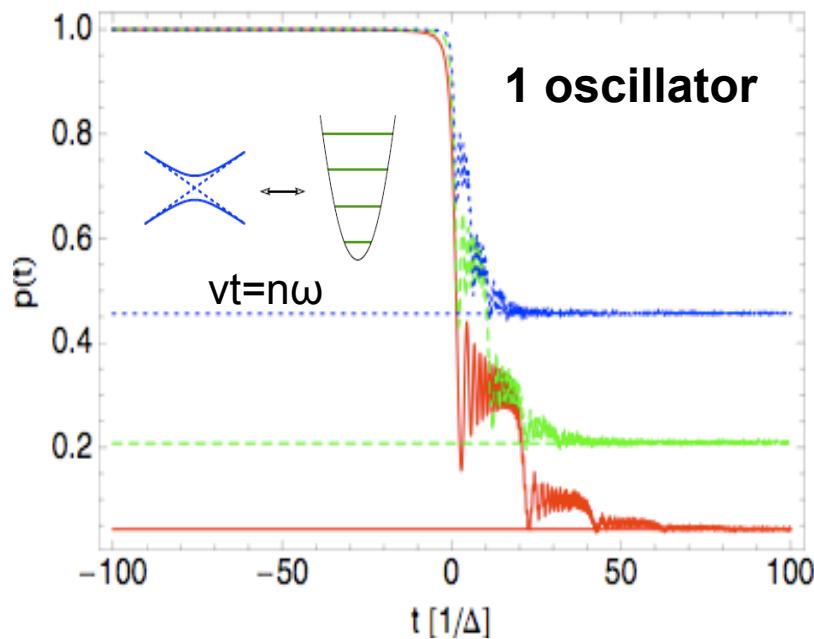
L.D. Landau, 1932
C. Zener, 1932,
E. Majorana, 1932
E.C.G. Stückelberg, 1932

Effect of an environment: many-body dynamics ...

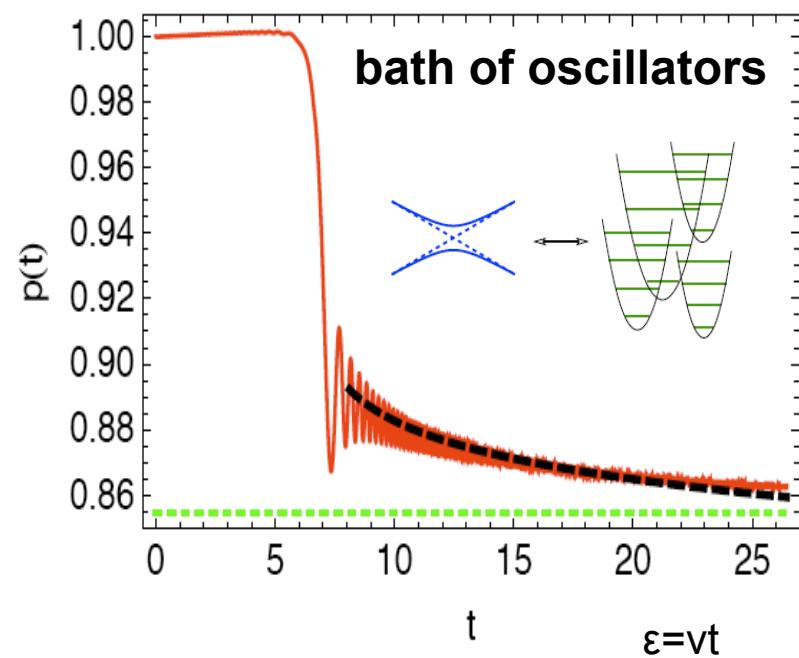
In the case of a longitudinal coupling with the environment, the long-time probability remains unchanged (1 spin)

Theorem (Hanggi, Kayanuma et al, 2008)

Survey: Kayanuma, 1987



Stochastic approach



P. Orth, A. Imambekov, K. Le Hur, 2010, review 2013
New developments in arrays: interactions can change $p(t)$...:
Loic Henriet & KLH 2015 (PRB, accepted)

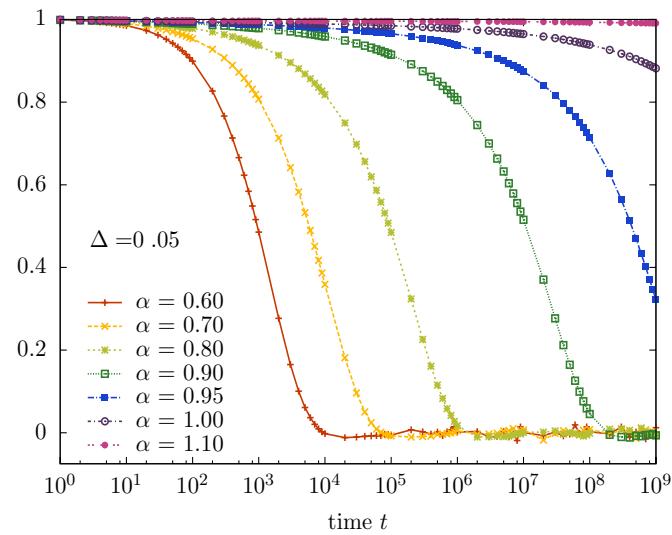
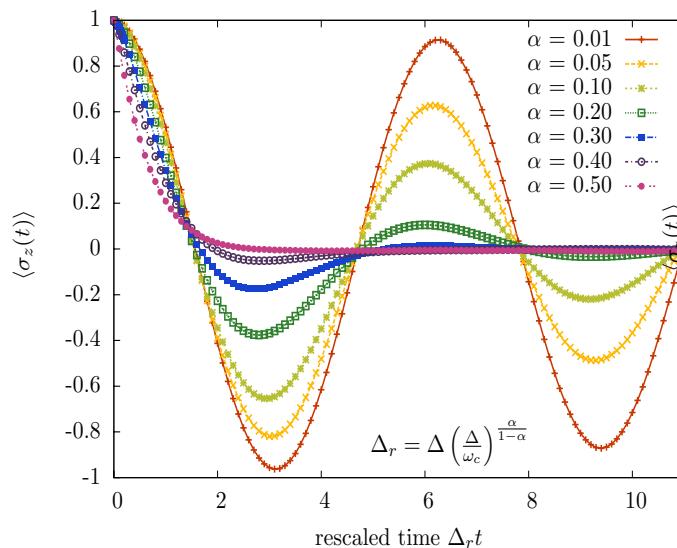
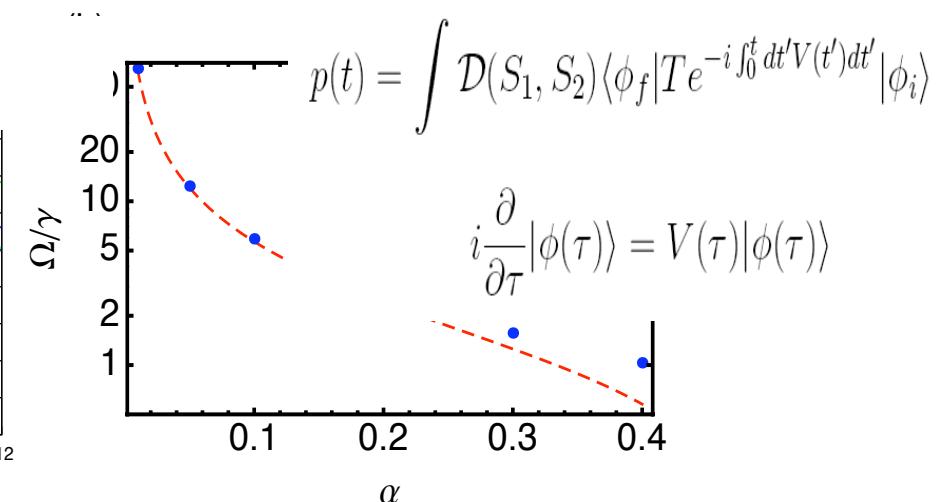
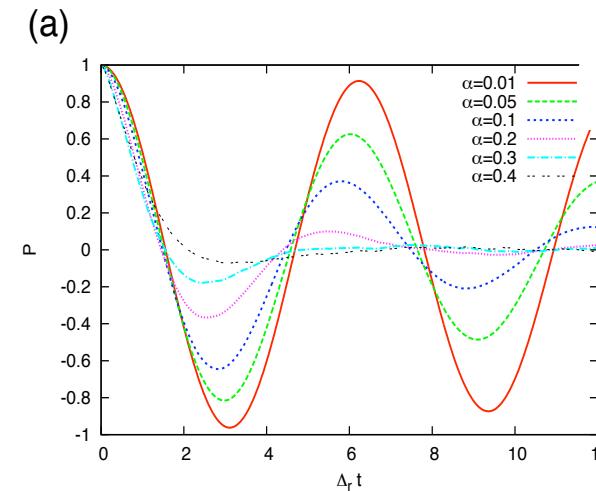
Example of non-Markovian Bath (ohmic)

Develop analytical Approaches & numerics

P. Orth, A. Imambekov, K. Le Hur, stochastic approach

P. Orth, D. Roosen, W. Hofstetter, KLH: time-dependent Wilson NRG

Stochastic Approach



Time-dependent NRG

Lot of works: this for example A. Leggett al. RMP 1987; book of U. Weiss
Dissipative quantum systems

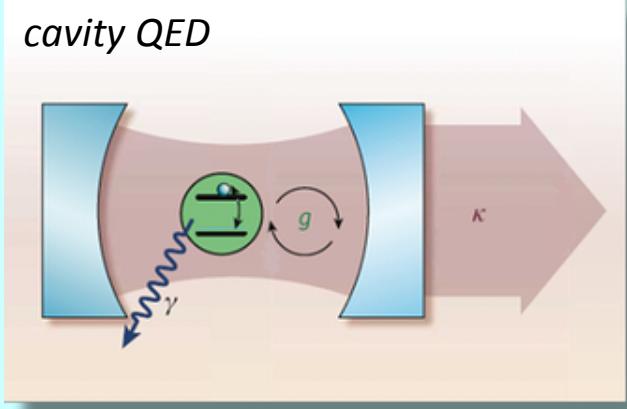
Cavity & Circuit QED

Prix Nobel S. Haroche

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity

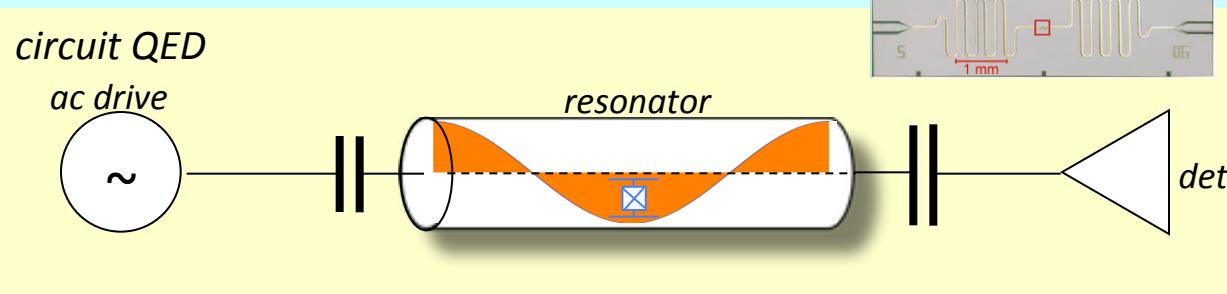
$$\begin{aligned}2g &= \text{vacuum Rabi frequency} \\ \gamma &= \text{atomic relaxation rate} \\ \kappa &= \text{photon escape rate}\end{aligned}$$



Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\omega_a\sigma_z + \omega_r a^\dagger a + g(\sigma_- a^\dagger + \sigma_+ a) + (H_{\text{drive}} + H_{\text{baths}})$$

- same concept works for superconducting qubits!



Experiments in
SPEC CEA Saclay

LPS Orsay

Santa Barbara...

Jaynes-Cummings Ladder

in the base $|n, +_z\rangle$ and $|n + 1, -_z\rangle$

$$H = \begin{pmatrix} n\omega_0 + \frac{\Delta}{2} & \frac{g}{2}\sqrt{n+1} \\ \frac{g}{2}\sqrt{n+1} & (n+1)\omega_0 - \frac{\Delta}{2} \end{pmatrix}$$

We have the following eigenvalues and eigenstates ($N > 1$):

$$E_{N+} = N\omega_0 - \frac{\delta}{2} + \frac{1}{2}\sqrt{\delta^2 + Ng^2}$$

$$E_{N-} = N\omega_0 - \frac{\delta}{2} - \frac{1}{2}\sqrt{\delta^2 + Ng^2}$$

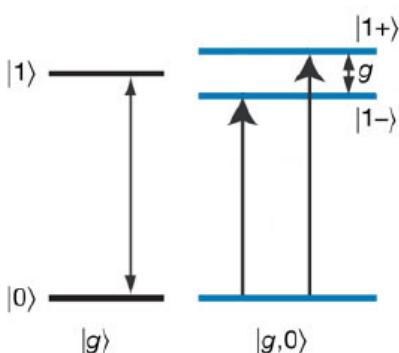
$$|N+\rangle = \alpha_n |N-1, +_z\rangle + \beta_n |N, -_z\rangle$$

$$|N-\rangle = -\beta_n |N-1, +_z\rangle + \alpha_n |N, -_z\rangle$$



Eigenstates are **Polaritons**

$$N = a^\dagger a + \frac{1}{2}(\sigma^z + 1)$$



$$\beta_N = \cos(1/2 \tan^{-1} \frac{g\sqrt{N}}{\delta}) \text{ and } \alpha_N = \sin(1/2 \tan^{-1} \frac{g\sqrt{N}}{\delta})$$

$\delta = \omega_0 - \Delta$ is the detuning

Photon Blockade: Photons go one by one nonlinearities

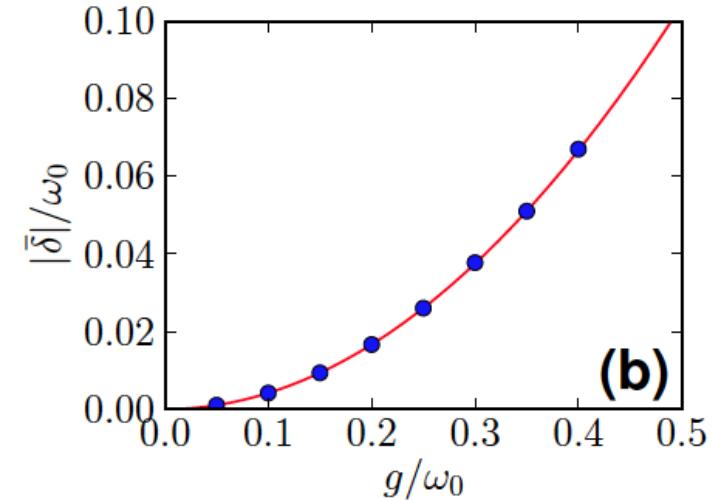
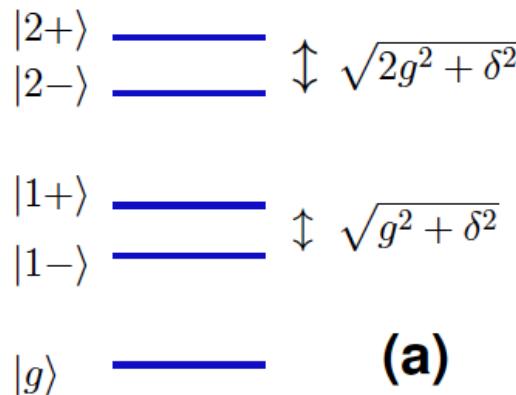
$$\rho_S(t_0) = |+_z\rangle\langle+_z|$$

Rabi oscillations between 1- & 1+ states

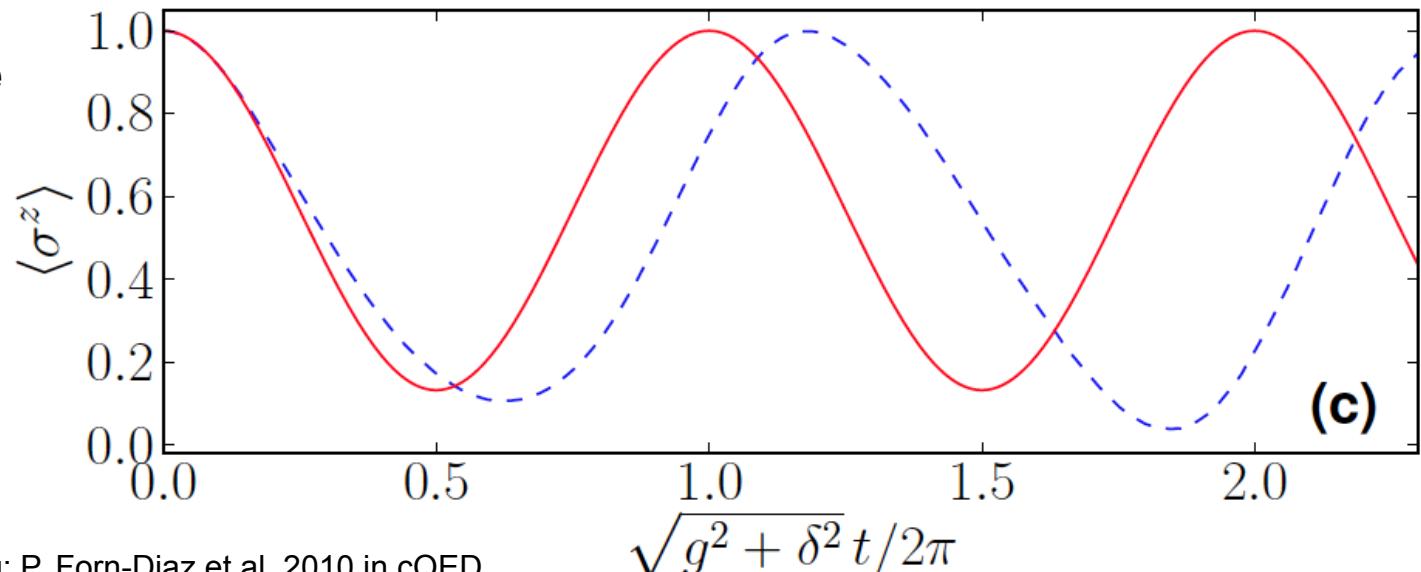
$$\langle \sigma^z(t) \rangle = 1 - 2 \sin^2(\sqrt{g^2 + \delta^2} \frac{t}{2}) [1 - \cos^2(\tan^{-1} \frac{g}{\delta})]$$

Bloch-Siegert shift

C. Cohen-Tanoudji
1968, Cargèse lectures
S. Haroche 1971



**Rabi model in the strong coupling:
New features**

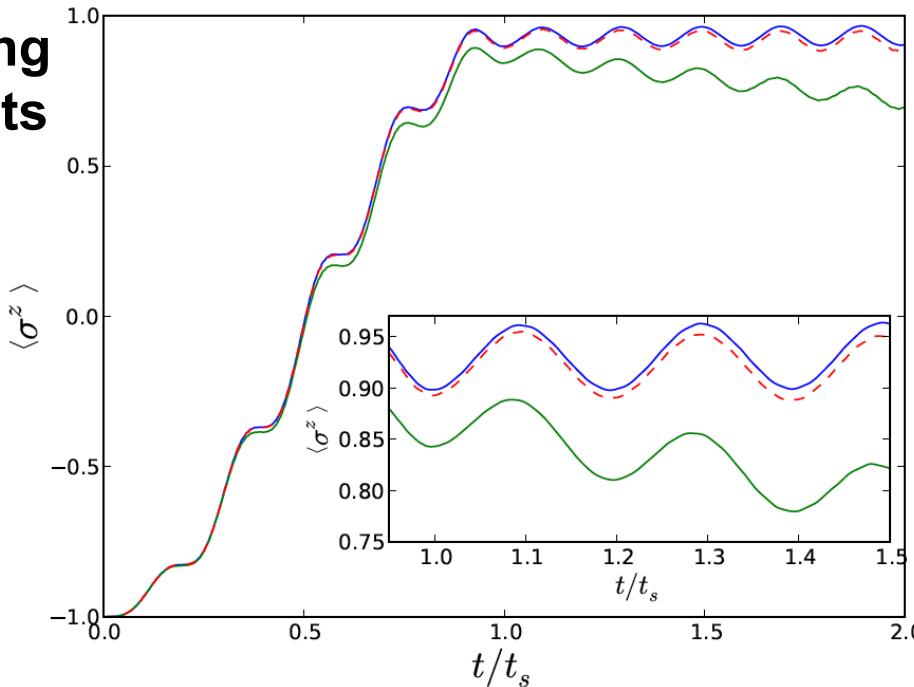
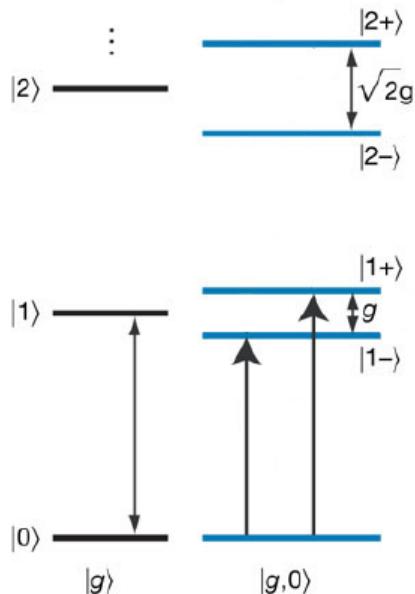


Example of strong AC driving Effects

Π rotation to achieve 1 polariton

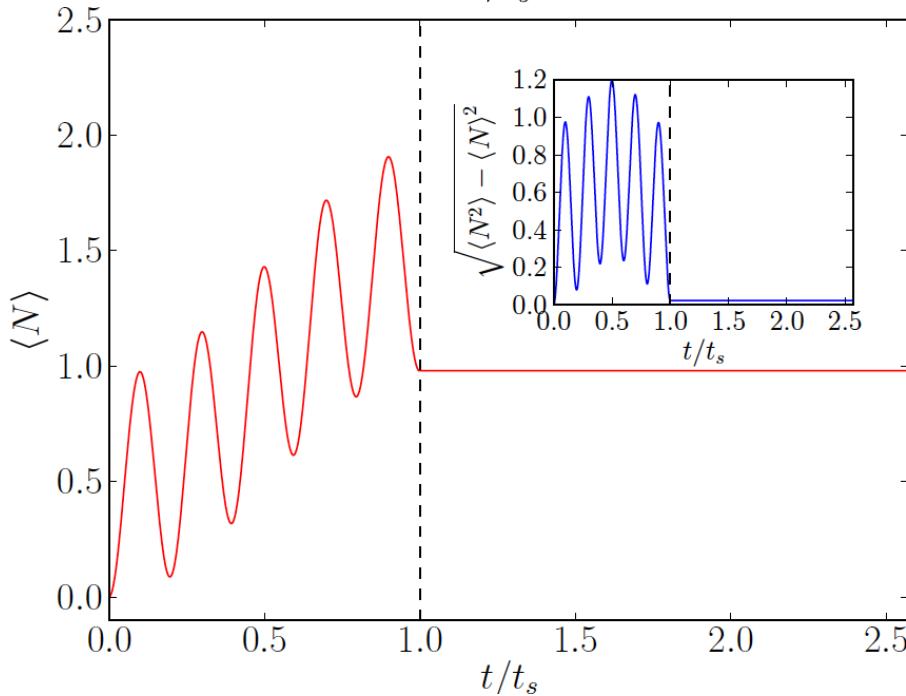
$$N = a^\dagger a + \frac{1}{2} (\sigma^z + 1)$$

Small g limit:
Jaynes-Cummings



L. Henriet
Z. Ristivojevic
P. P. Orth, KLH, 2014
(arXiv:1401.4558)
Rabi model

Dissipation effects



Π rotation also useful to measure Berry phase

Experiments
Zuerich, 2007 (Ramsey)
Boulder, 2014
Santa Barbara 2014

COLD-ATOMIC Quantum IMPURITIES

A. Recati et al. PRL **94**, 040404 (2005)

Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)

Single Atom: Ph. Grangier et al. Science **309**, 454 (2005)

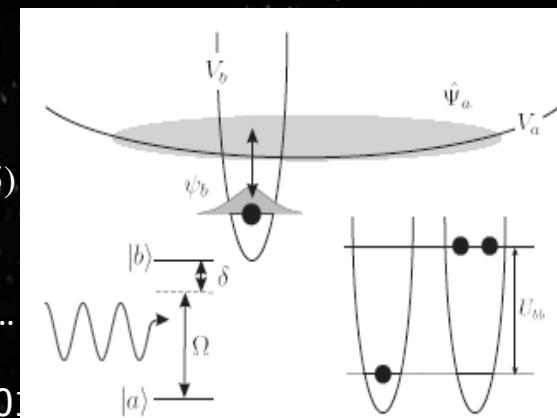
A. Fuhrmanek, Y. R. P. Sortais, P. Grangier, A. Browaeys
Phys. Rev. A **82**, 023623 (2010).

D. Porras, F. Marquardt, J. von Delft, J. I. Cirac (2007),...

M. Knap et al. Phys. Rev. X **2**, 041020 (2012)

M. Knap, D. A. Abanin, E. Demler, PRL **111**, 265302 (2013)

J. Bauer, C. Salomon, E. Demler PRL **111**, 215304 (2013)



RC circuits

M. Büttiker, H. Thomas, and A. Pretre, Phys. Lett. A **180**, 364 - 369,(1993)

J. Gabelli et al., Science **313**, 499 (2006); G. Feve et al. 2007 (LPA ENS)

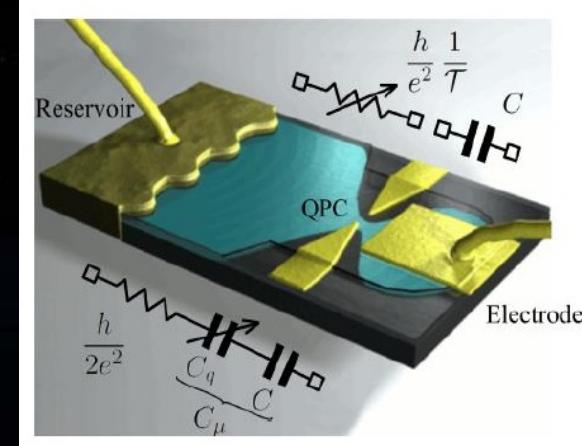
J. Gabelli et al. Rep. Progress 2012

C. Mora and K. Le Hur, Nature Phys. **6**, 697 (2010)

Y. Hamamoto, et al. Phys. Rev. B **81**, (2010) 153305

Y. Etzioni, B. Horovitz, P. Le Doussal, PRL **106**, 166803 (2011)

M. Filippone, KLH, C. Mora; P. Dutt, T. Schmidt, C. Mora, KLH, 2013



Hybrid Photon-Nano Systems, Impurities with Photons

K. Le Hur, Phys. Rev. B **85**, 140506(R) (2012)

A. Leclair, F. Lesage, S. Lukyanov and H. Saleur (1997)

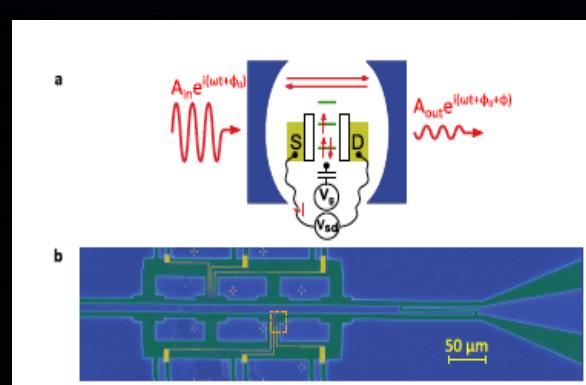
M. Goldstein, M. H. Devoret, M. Houzet and L. I. Glazman, 2012

Grenoble: S. Florens, H. Baranger, N. Roch and collaborators

M. Hofheinz et al. arXiv:1102.0131

M. Delbecq et al. PRL **107**, 256804 (2011)

M. Schiro & KLH, arXiv 1310.8070, PRB 2014

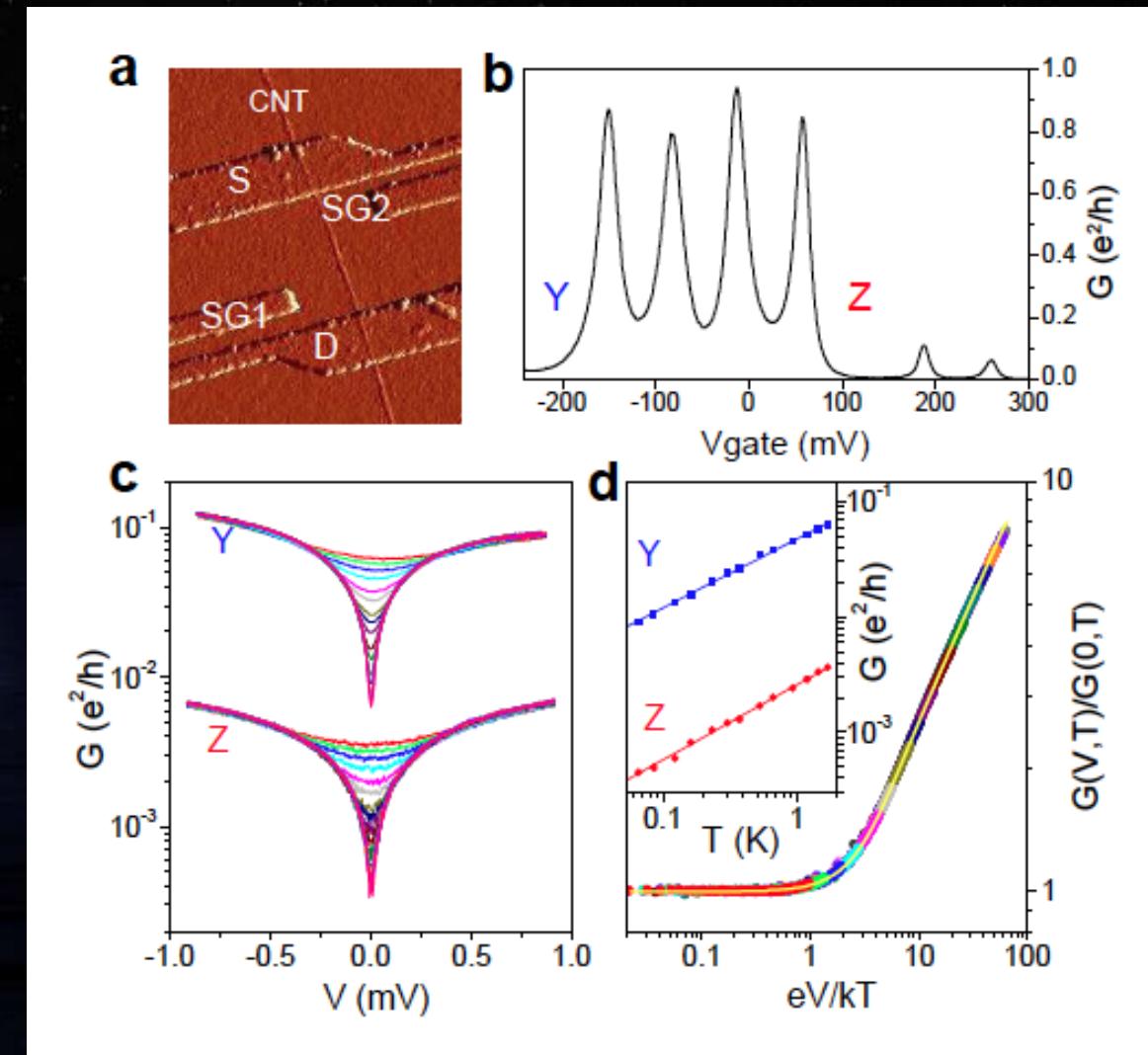


Collaboration with
C.-H. Chung, P. Woelfle,
M. Vojta, G. Finkelstein
PRL 2009, PRB 2013

H. T. Mebrahtu, I. V. Borzenets, H. Zheng, Y. V. Bomze, A. I. Smirnov, S. Florens, H. U. Baranger, G. Finkelstein
Nature Physics, 9 732 (2013)

similar experiments
at LPN Marcoussis
F. Pierre group

Theory: I. Safi &
M. Albert



Feedback on the circuit Quantum Electrodynamics?

$$\mathcal{H} = \sum_{kl} \omega_k b_{kl}^\dagger b_{kl} + (a + a^\dagger) \sum_{kl} g_k (b_{kl}^\dagger + b_{kl}) + \mathcal{H}_{sys}$$

Anderson-Holstein model

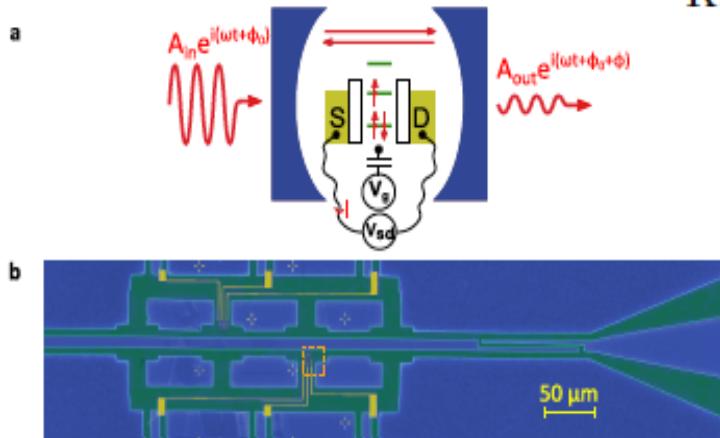
$$t(\omega) \equiv \frac{\langle V_R^{out}(\omega) \rangle}{\langle V_L^{in}(\omega) \rangle} = i J(\omega) \chi_{xx}^R(\omega)$$

$$\lambda x n + \omega_0 a^\dagger a$$

Input-Output Theory:

A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).

K. Le Hur, Phys. Rev. B **85**, 140506 (2012).



$$\chi_{xx}^R(t) = -i\theta(t) \langle [x(t), x(0)] \rangle_{H_{sys}}$$

$$\tan \varphi(\omega) \equiv \frac{\text{Im } t(\omega)}{\text{Re } t(\omega)} = \frac{\text{Re } \chi_{xx}^R(\omega)}{\text{Im } \chi_{xx}^R(\omega)}.$$

The retarded photon Green's function can be written in Fourier space in terms of the photon self-energy $\Pi^R(\omega)$ as

$$\chi_{xx}^R(\omega) = \frac{\omega_0}{\omega^2 - \omega_0^2 - \omega_0 \Pi^R(\omega)} \quad (15)$$

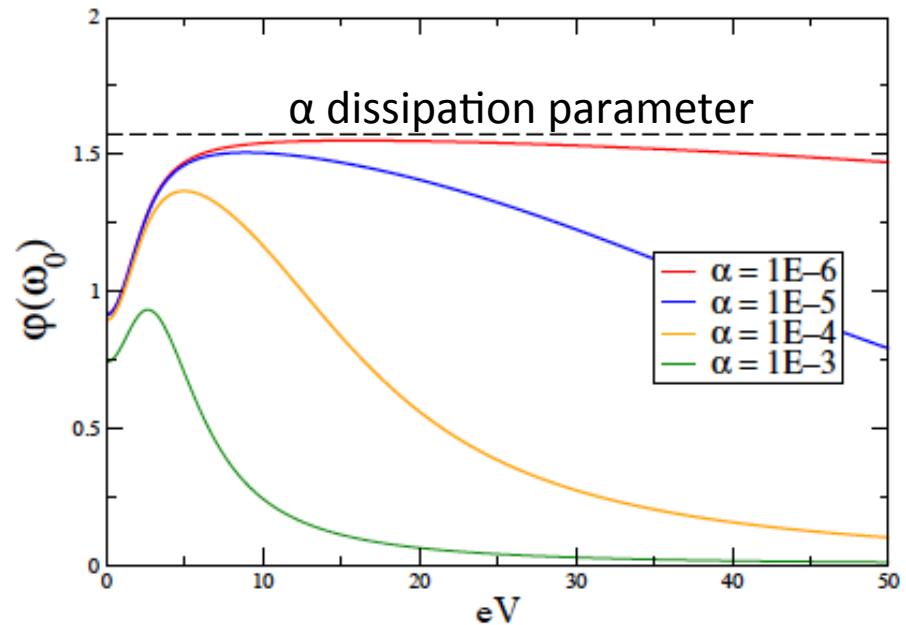
where $\Pi^R(\omega)$ includes both the effects of frequency renormalization and the damping due to the environment.

$$\Pi^R(t, t') = \Lambda^R(t, t') \equiv \lambda^2 \chi_{el}(t - t') \quad (17)$$

with $\chi_{el}(t - t') = -i\theta(t - t') \langle [n(t), n(t')] \rangle_{el}$ the electronic charge susceptibility. For an Anderson Impurity Model which exhibits a Fermi-Liquid type of ground state this must satisfy the Korringa-Shiba relation⁵⁷ which implies

$$\text{Im}\chi_{el}(\omega) = \pi\omega \left[(\text{Re}\chi_{el\uparrow}(0))^2 + (\text{Re}\chi_{el\downarrow}(0))^2 \right]. \quad (18)$$

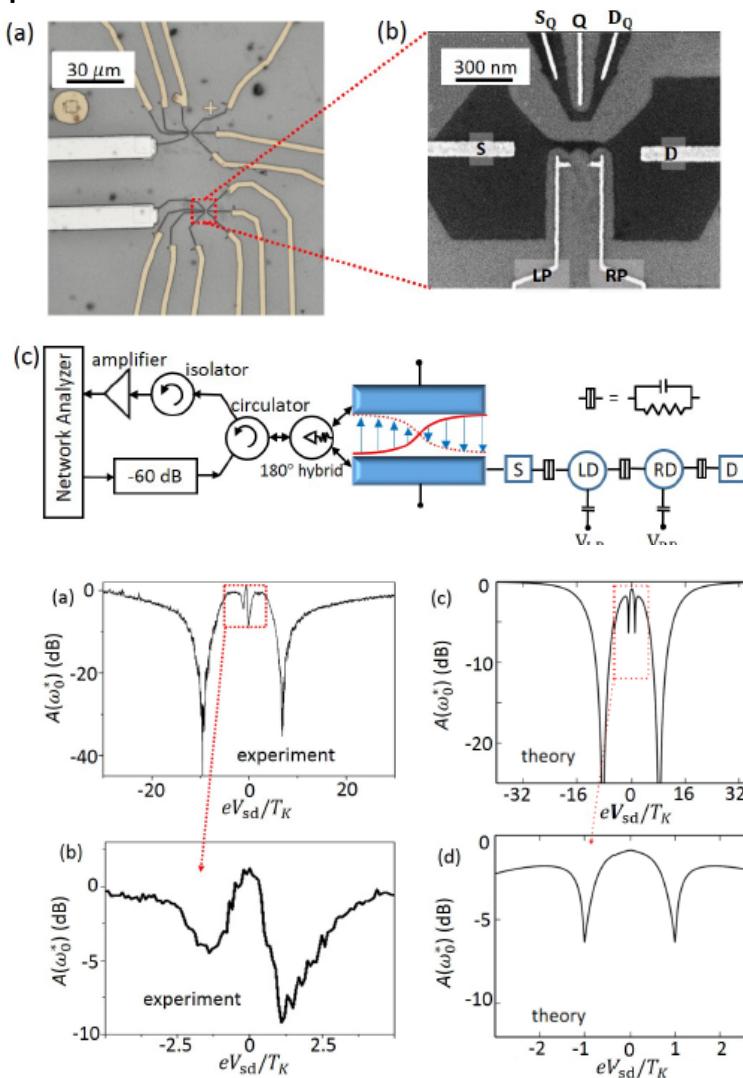
At low-frequency: importance in RC circuits
Anderson model, M. Fillipone, KLH, C. Mora
 Phys. Rev. Lett. **107**, 176601 (2011)



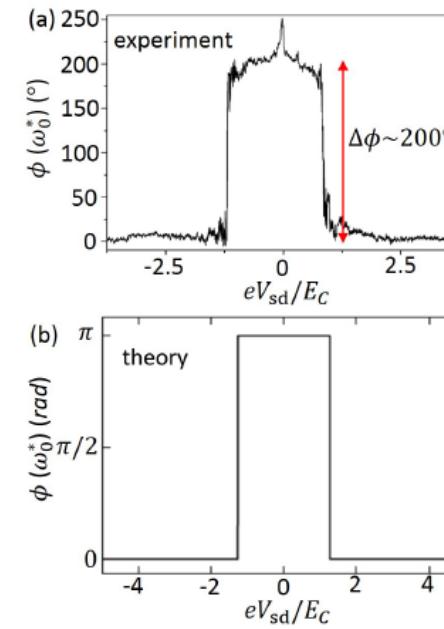
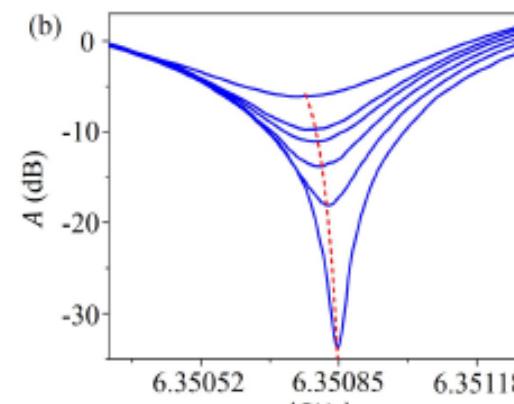
Marco Schiro & KLH,
 Phys. Rev. B **89**, 195127 (2014)

Example of new Hybrid Systems

graphene



T_K is a new energy scale: the Kondo energy scale



2015

Submitted
To PRX

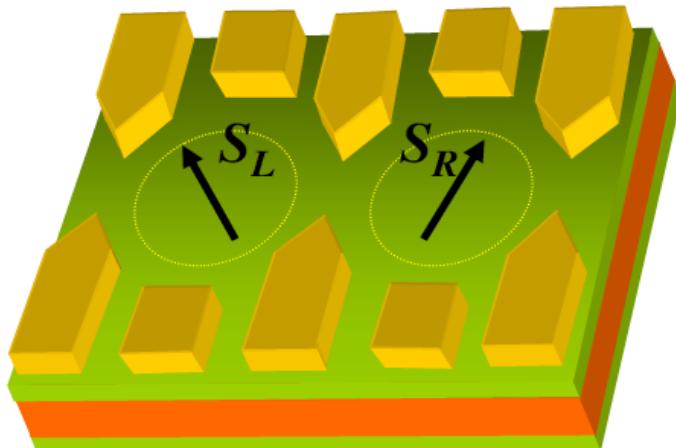
Phase of
 Π in the
reflection

Guang-Wei Deng^{†, 1, 2} Loïc Henriet^{†, 3} Da Wei,^{1, 2} Shu-Xiao Li,^{1, 2} Hai-Ou Li,^{1, 2} Gang Cao,^{1, 2} Ming Xiao,^{1, 2} Guang-Can Guo,^{1, 2} Marco Schiró,⁴ Karyn Le Hur,³ and Guo-Ping Guo^{1, 2, *}

What are the questions and challenges

Either find a way to minimize decoherence at sufficiently relevant temperatures:
certainly one needs to understand the quantum matter in a better way
Error correction codes

Or Find another type of qubits: topological aspects, **Majorana Fermions**



Spin Qubits
Loss & DiVincenzo

Qubit candidates (?)

- NMR
- quantum dots
- Cold atoms
- Superconducting circuits
- Ion traps
- Polar molecules
- Semiconductor impurities,...

2 spins in a cold-atom bath:

P. Orth, D. Roosen, W. Hofstetter, KLH
2010 (TD-NRG and analytics)
Loic Henriet & KLH, 2015 stochastic

Majorana fermion is its own antiparticle:
Advantage, delocalized object occurring in exotic condensed matter systems
topological protection? IMPORTANT point

XXI, Detect the Majorana in topological SCs: L. Kouwenhoven Delft, 2012

See F. Wilczek, Majorana returns, Nat. Physics 2009

They appeared first
In spin chains: via Jordan-Wigner
transformation (1928)

Challenge taking
into account that the
Man who discovered
the Majorana
disappeared 1938

Proposals:

Alexei Kitaev

Nick Read

Leonid Levitov

Hans Mooij

Liang Fu

Charles Kane

...

Note: recent work on 2 coupled topological SC chains
Loic Herviou, Christophe Mora, KLH 2016

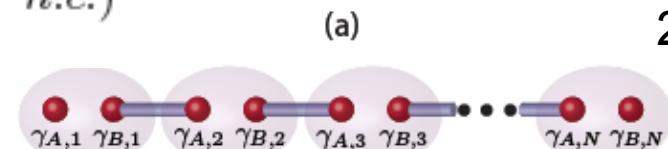


Recent Implementation: quantum wires (Delft 2012, for example)

Review: J. Alicea et al. 2010

$$H = -\mu \sum_{x=1}^N c_x^\dagger c_x - \sum_{x=1}^{N-1} (tc_x^\dagger c_{x+1} + |\Delta| e^{i\phi} c_x c_{x+1} + h.c.)$$

Kitaev chain
2001



Difficulty to find p-wave SCs in nature?

Equivalent quantum Ising spin chain

Take advantage of spin-orbit coupling

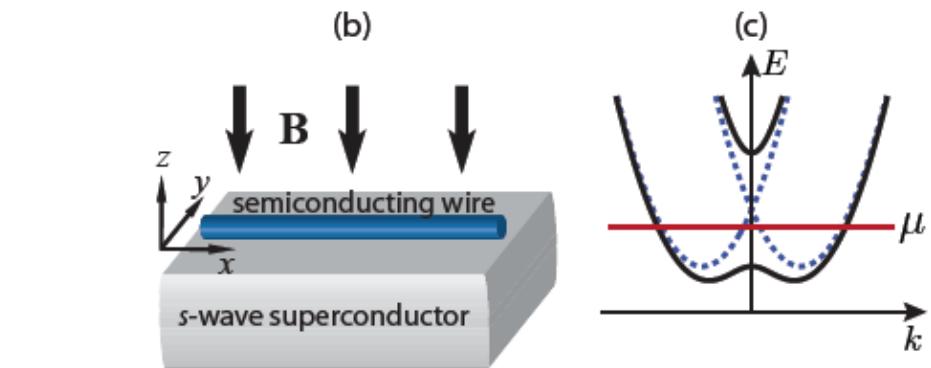
$$c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x}),$$

$$H = -it \sum_{x=1}^{N-1} \gamma_{B,x} \gamma_{A,x+1}.$$

Theory Proposals (2010):

G. Refael, Y. Oreg, F. von Oppen

R. Lutchin, J. Sau and S. das Sarma

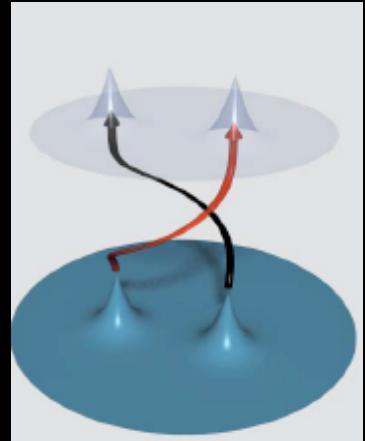


$$\begin{aligned} \mathcal{H} = & \int dx \left[\psi_x^\dagger \left(-\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\hbar u \hat{\mathbf{e}} \cdot \boldsymbol{\sigma} \partial_x \right. \right. \\ & \left. \left. - \frac{g\mu_B B_z}{2} \sigma^z \right) \psi_x + (|\Delta| e^{i\varphi} \psi_{\downarrow x} \psi_{\uparrow x} + h.c.) \right]. \end{aligned}$$

Dresselhaus or Rashba

Braiding and Strange Gates

The Majorana fermion states must be occupied in pairs, since the entire physical system can only occupy real fermion states.
So only combinations of Majorana fermions can be occupied



This occupied state is inherently delocalized – it has weight in two spatially separated vortex cores.

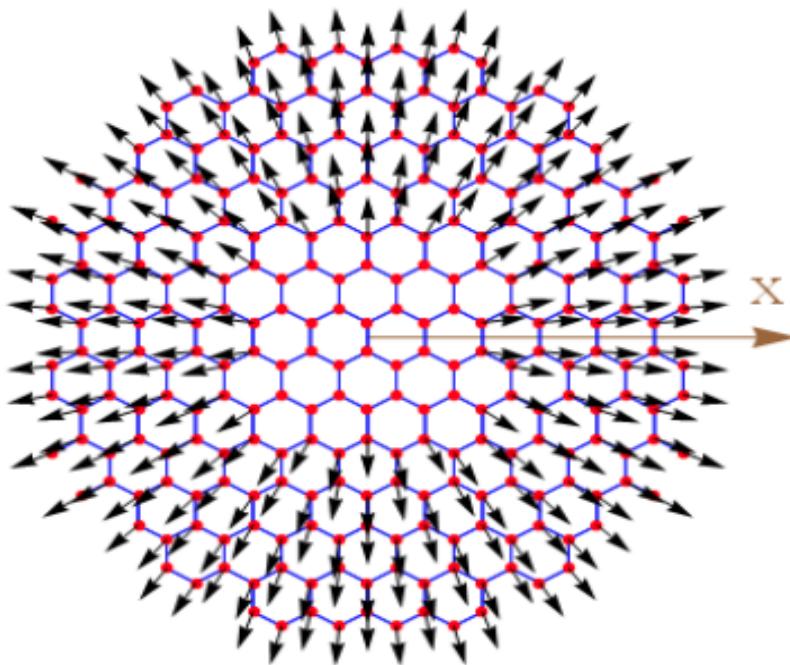
$$\hat{c}^\dagger |\Psi_0\rangle = (\hat{\gamma}_1 + i\hat{\gamma}_2) |\Psi_0\rangle$$

Exchange of 1 and 2 $\gamma_1 \rightarrow \gamma_2$
 $\gamma_2 \rightarrow -\gamma_1$

$$(\hat{\gamma}_2 + i\hat{\gamma}_1) |\Psi_0\rangle = i(\hat{\gamma}_1 - i\hat{\gamma}_2) |\Psi_0\rangle = i\hat{c}|\Psi_0\rangle$$

Different final state! – Non-Abelian statistics.

Superconducting Graphene



Topological defect In quantum matter

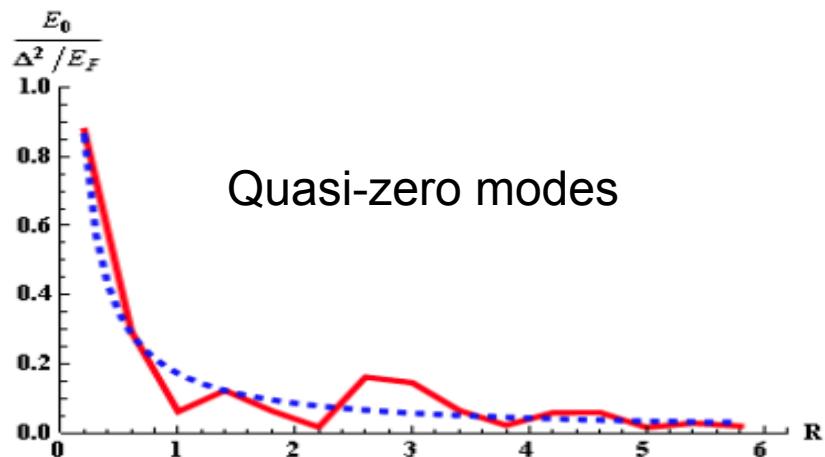
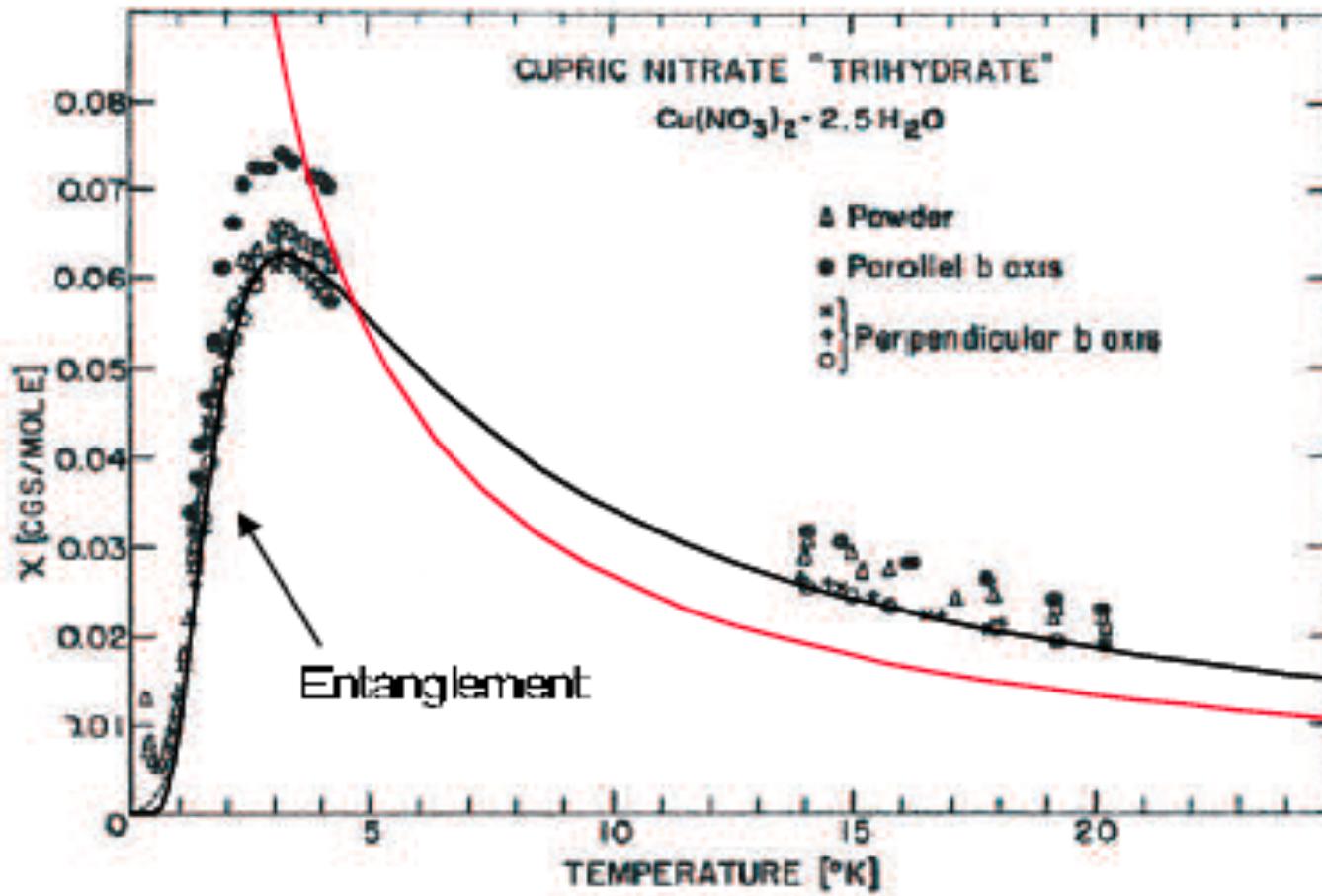


FIG. 9: Ratio of lowest energy scale to de Gennes energy scale, with varying vortex core size. The vortex core sizes range from 0.2 to 6.0 in increments of 0.4, and the lattice patch is circular with a radius of 6.0. The nearest neighbor distance is $1/\sqrt{3}$. The raw data is denoted by the continuous (red) curve, and the fit is denoted by a dashed (blue) curve.

D. Bergman & KLH
2010

p+ip SC better
N. Read & D. Green

Quantum Entanglement in Bulk Properties of Solids:
Quantum spin chain different from Ising classical chain? How?

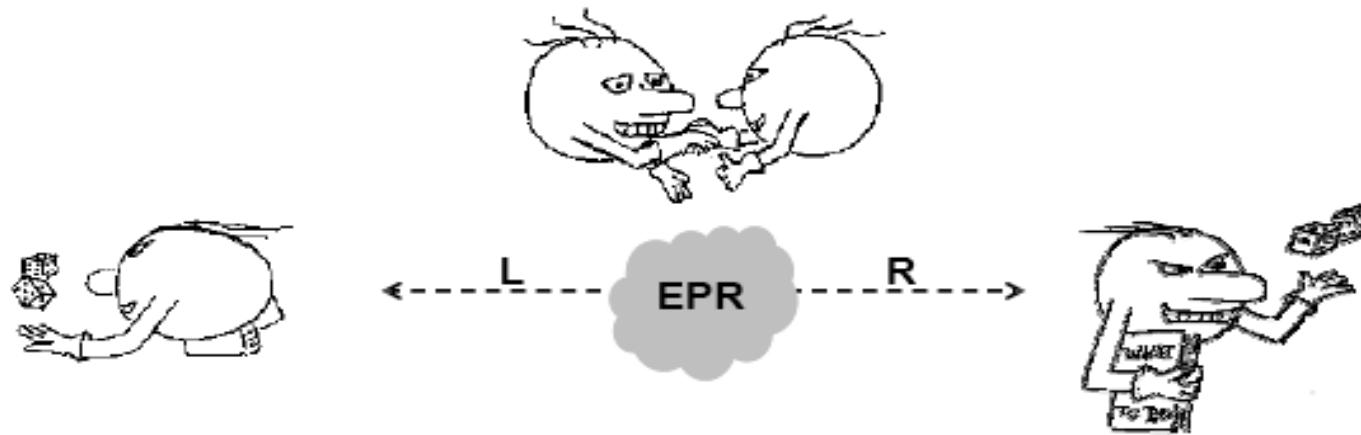


Brukner,
Vedral,
Zeilinger, 2004

$$\chi \geq \frac{g^2 \mu_B^2 N}{kT} \frac{1}{6}$$

implies separable state (?)

What is Entanglement? Spooky action at Distance (Einstein)



Simple example: 2 Qbits forming a singlet pair

$$|\Psi_S\rangle = \frac{1}{\sqrt{2}} (| \uparrow_A \rangle | \downarrow_B \rangle - | \downarrow_A \rangle | \uparrow_B \rangle)$$

Wave function is NOT factorizable into individual wave functions...
Quantum states of 2 (or more) particles are linked together

Detection (for photons) lies on violation of Bell's inequalities
(see for example experiment by A. Aspect, P. Grangier, G. Roger 1981)
also J. Dalibard

Entanglement Entropy

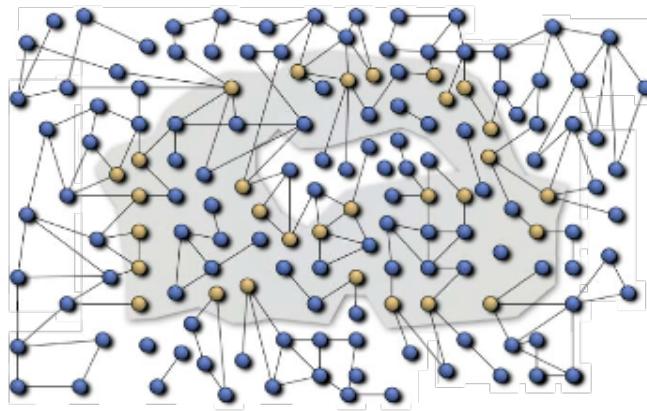
Review: J. Eisert, M. Cramer, M. Plenio RMP 2010

quantum mechanics:
entropy > 0 without an objective lack of information

non-degenerate
pure ground state

$$\rho_0 = |\psi\rangle\langle\psi|$$

$$\Rightarrow S(\rho_0) = 0$$



von Neumann entropy
 $S(\rho) = -\text{tr}(\rho \log_2 \rho)$

shaded region A
remainder B

$$\rho_A = \text{tr}_B(\rho)$$

$$\Rightarrow S(\rho_A) \neq 0$$

Entanglement
Entropy

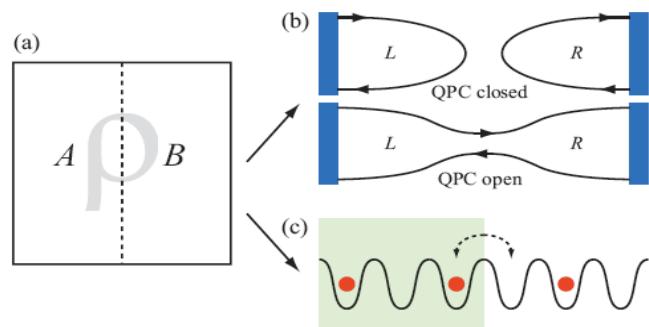
Is this Entropy Important?

Perhaps: Beckenstein-Hawking black hole entropy scales as the area and not volume; entanglement origin ? (Bombelli et al 1986) non-gravitational view of black holes...

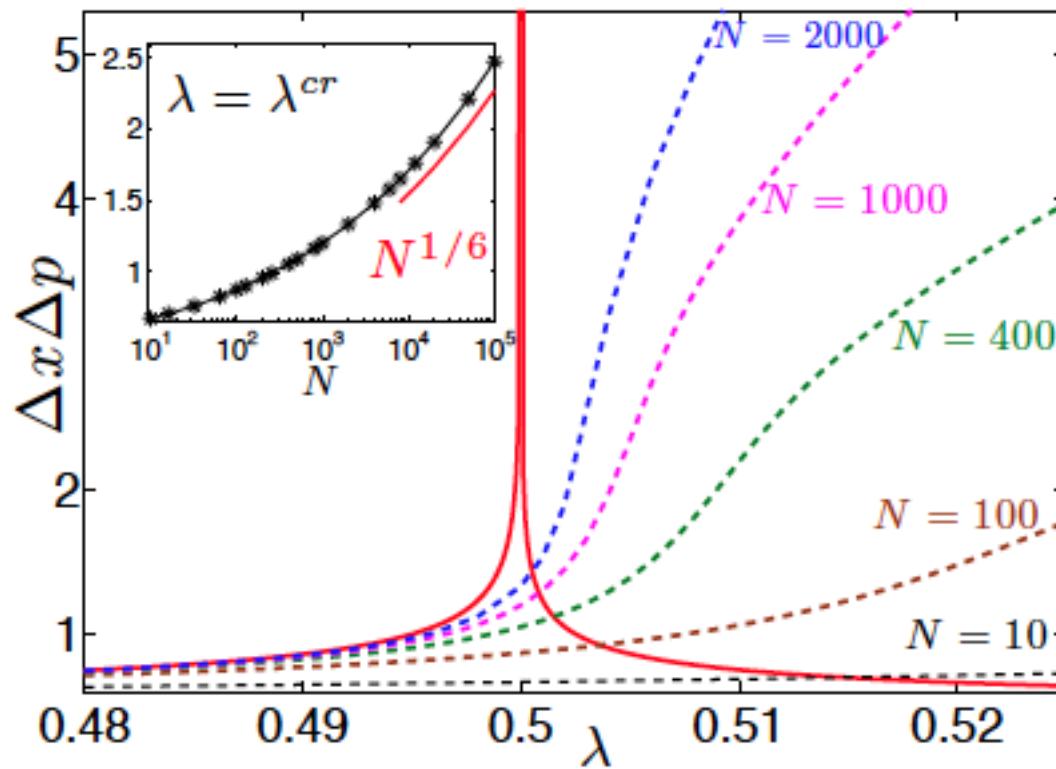
But, other possible interpretations (holographic principle)

Our Understanding: Take free electrons (quasiparticles)

Mixing entropy at zero temperatures



Entanglement between light and matter



Example
Dicke model
(changing the
Number of atoms)

**Heisenberg
principle**

$$S \sim \log_2 \{\Delta x \Delta p\} \quad \text{if} \quad \Delta x \Delta p \gg 1$$

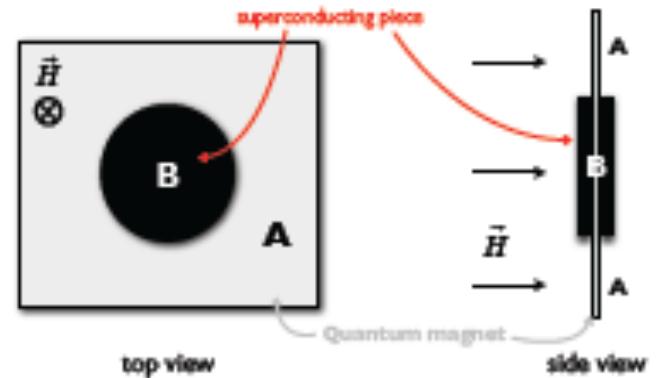
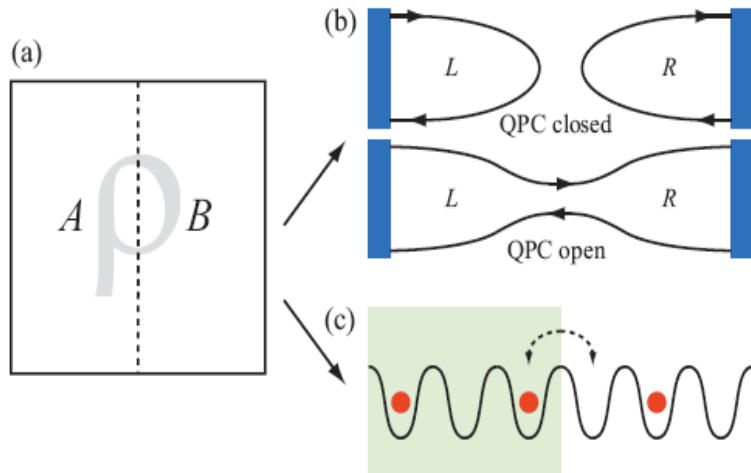
Pierre Nataf, Mehmet Dogan and Karyn Le Hur, arXiv: 1204.3065
See also Refael, Silva, Klich

Different spin-boson model Angela Kopp & KLH, 2007; **review** KLH Annals of Physics 2008

Kondo model: B. Alkurtass, A. Bayat, I. Affleck, S. Bose, H. Johannesson, P. Sodano, E. Sorensen, KLH, 2015

Note that quantifying entanglement in many-body systems is not unique:
 Here, we have chosen cases where naturally we can divide the system
 into two pieces; **still this gives new ways to think about “fluctuations”**

Interacting cases: see paper by J. Cardy (PRL 2011); P. Calabrese etc...



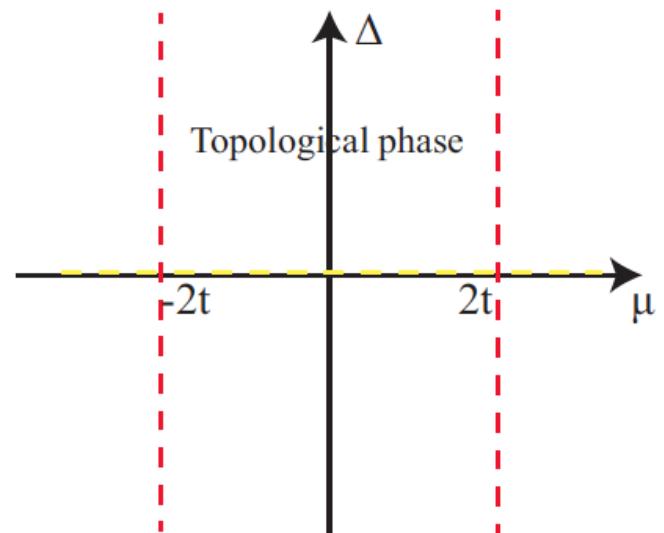
Also, experimental progress in cold atom systems to measure bi-partite fluctuations

- Measure of local spin susceptibility, T. Esslinger ETH Zuerich 2012
- Parity number correlation functions (Harvard, Muenich)
- Correlation functions in SC qubit systems (John Martinis Group/Google/Santa Barbara)

NEW Fluctuations and Kitaev chain

L. Herviou, C. Mora and KLH, 2016

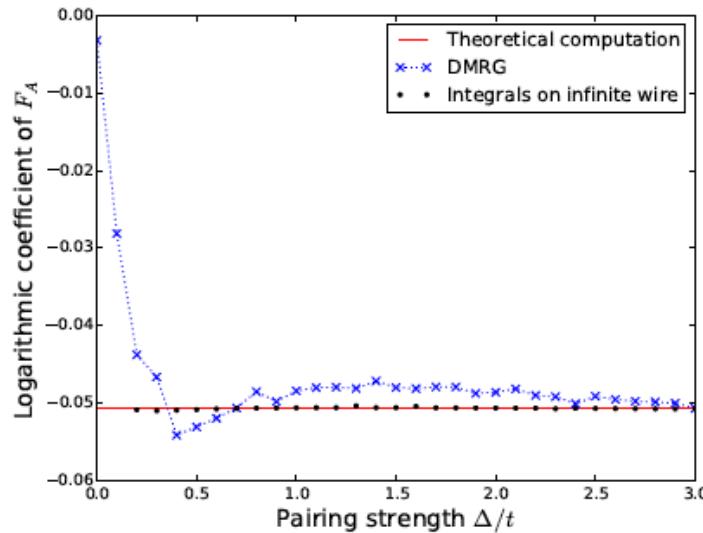
$$H_K\{c\} = -\mu \sum_{j=1}^L c_j^\dagger c_j + \sum_{j=1}^{L-1} -t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) \\ + \Delta(c_j^\dagger c_{j+1}^\dagger + c_{j+1} c_j).$$



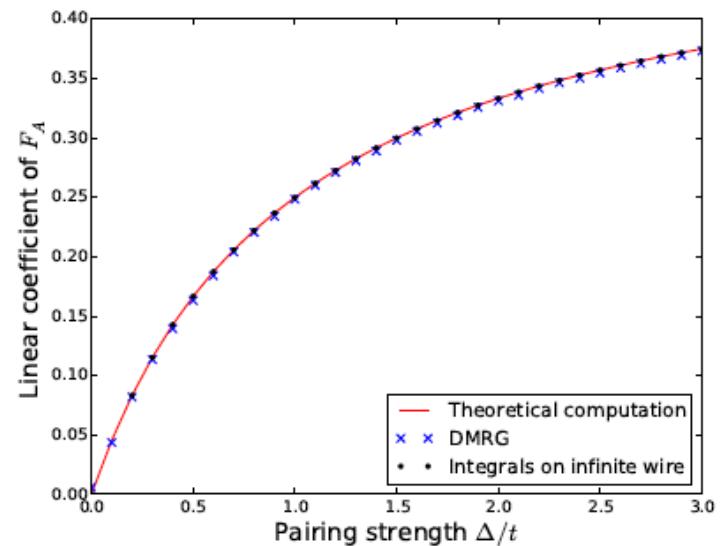
At criticality (linear term due to NO charge conservation)

$$F_A(l) = \frac{l}{4} - \frac{1}{2\pi^2} \log(l) - \frac{\gamma_{\text{euler}} + 2\log(2)}{2\pi^2} + \mathcal{O}(1).$$

$F = C_2$

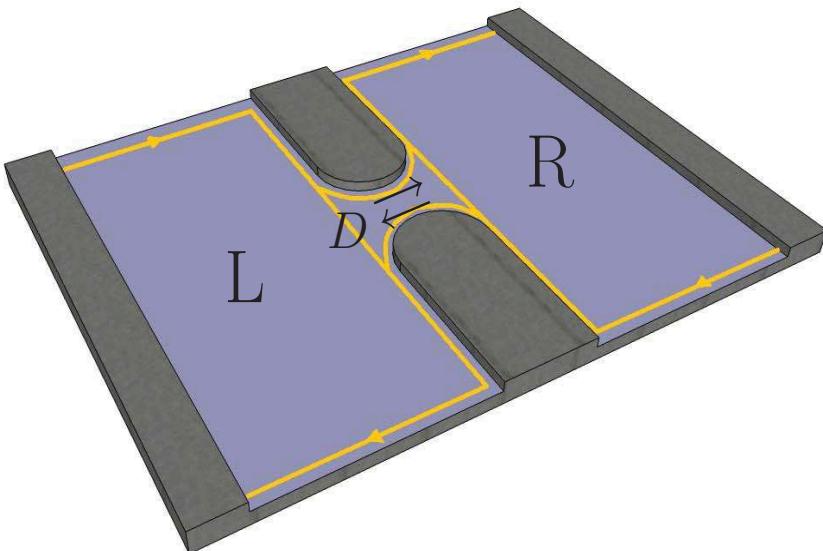


(b) Logarithmic coefficient of the charge fluctuations.



(a) Linear coefficient of the charge fluctuations.

Role of Nanotechnology & Math



Bipartite Fluctuations as a Probe of Many-Body Entanglement

H. Francis Song, Stephan Rachel, Christian Flindt, Israel Klich, Nicolas Laflorencie, Karyn Le Hur (review)

Phys. Rev. B **85**, 035409 (2012) [27 pages]
(Editors' Suggestion) + long Suppl. Material

Entanglement entropy of free fermions

$$\mathcal{S} = \lim_{K \rightarrow \infty} \sum_{n=1}^{K+1} \alpha_n(K) C_n,$$

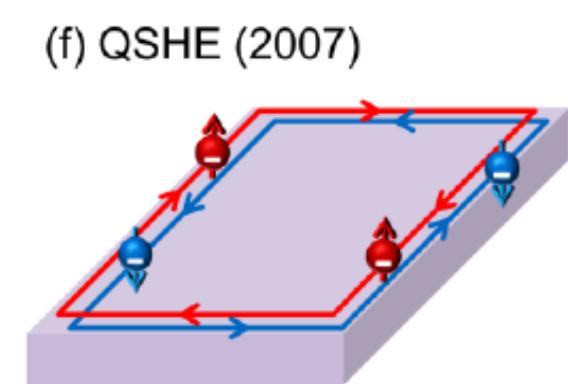
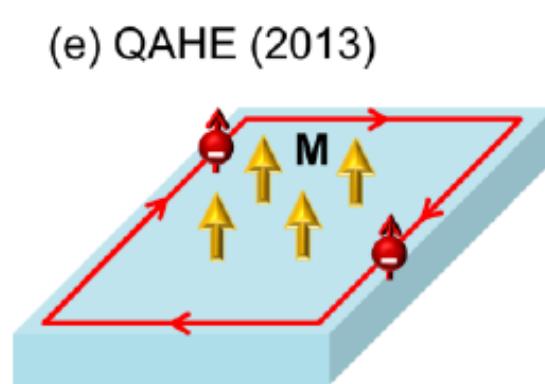
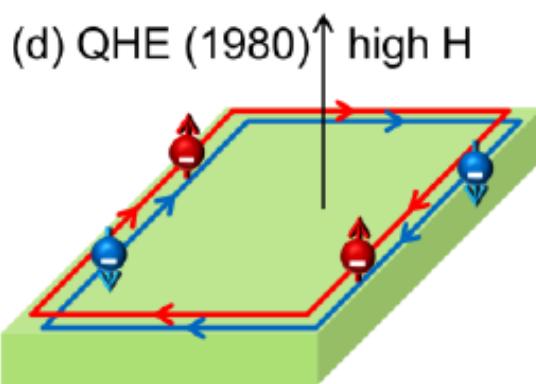
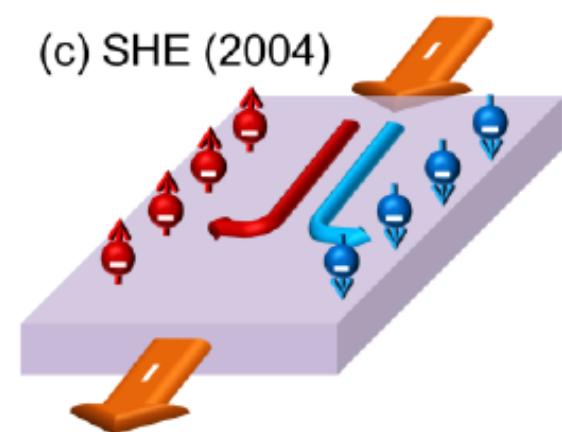
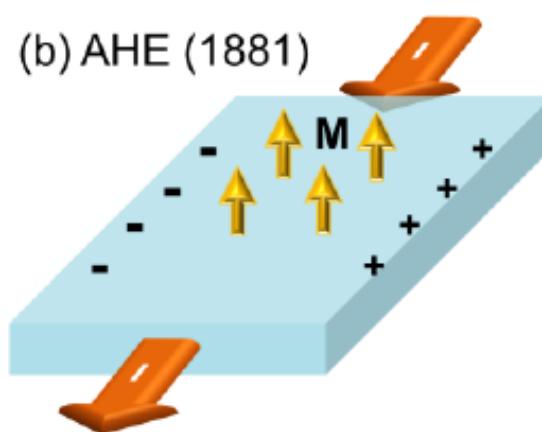
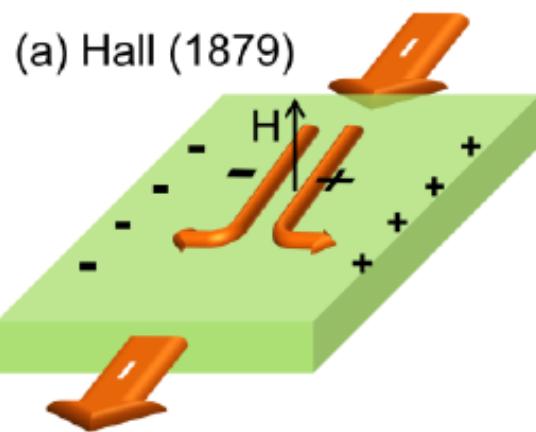
where

$$\alpha_n(K) = \begin{cases} 2 \sum_{k=n-1}^K \frac{S_1(k, n-1)}{k! k} & \text{for } n \text{ even,} \\ 0 & \text{for } n \text{ odd.} \end{cases}$$

Here $S_1(n, m)$ are **unsigned Stirling numbers of the first kind**.

Practically, K is the number of available cumulants and should be taken to be even.

More on Topological states of matter



Cold Atoms:

Jaksch & Zoller 2003

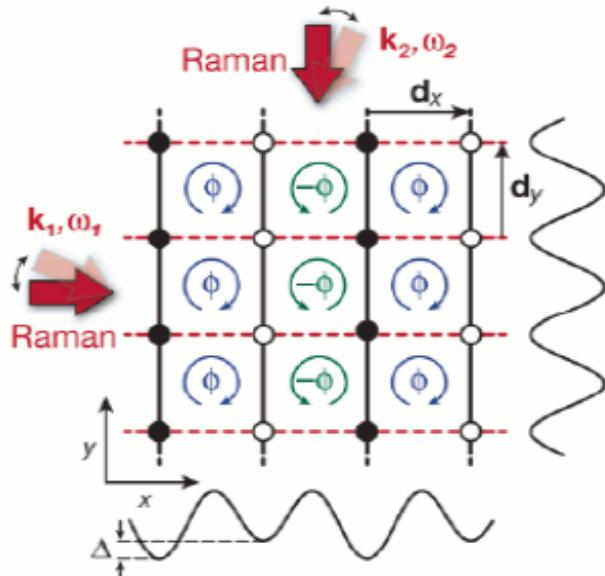
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
J. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- **Ways to implement magnetic fields & gauge fields**

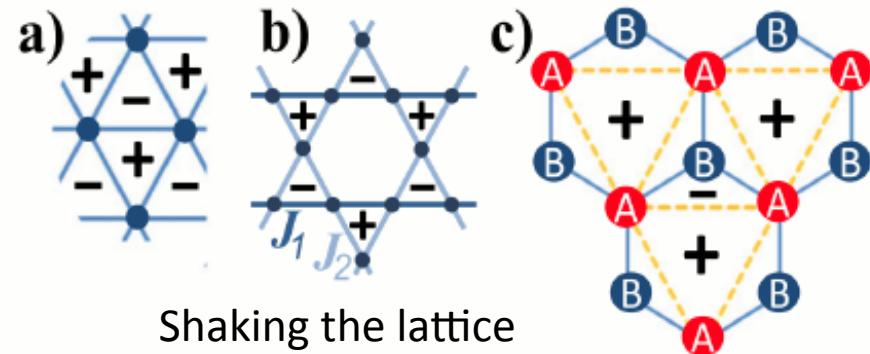
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)

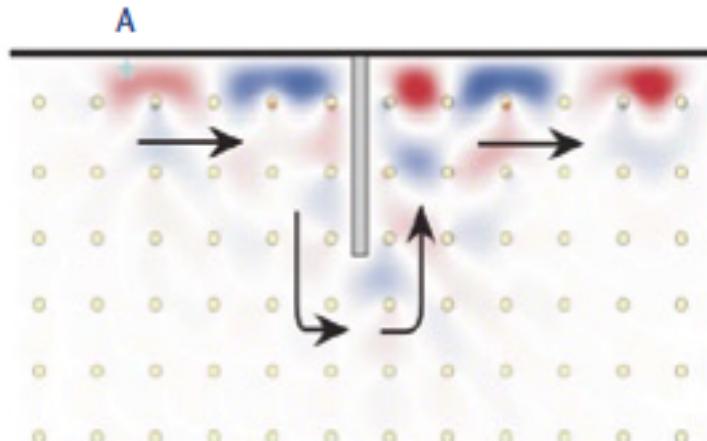
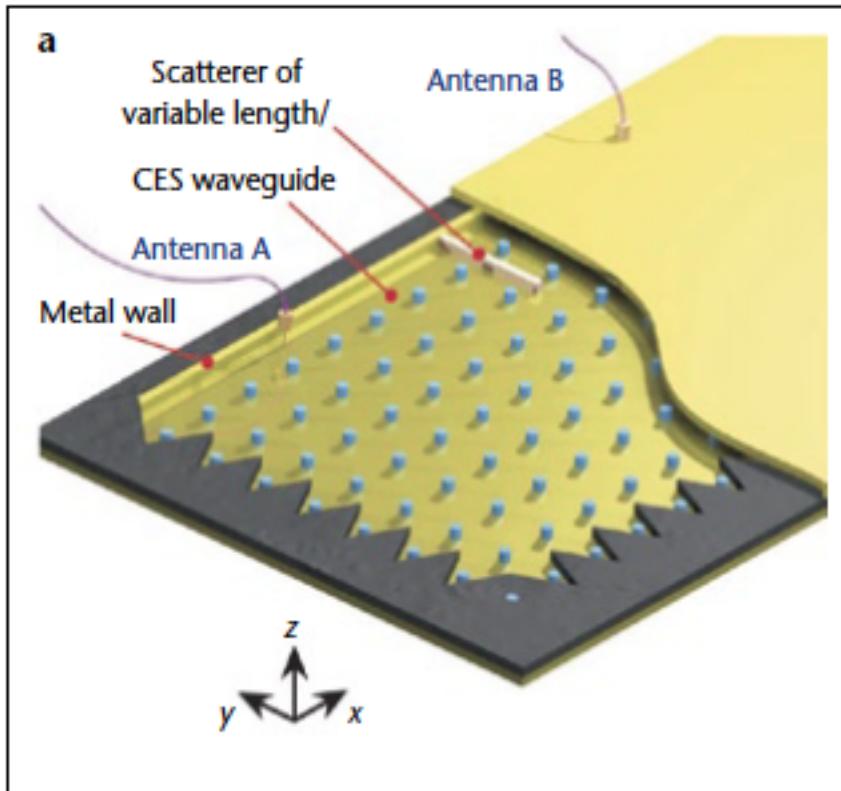


Floquet Topological Insulators:

Reviews: J. Cayssol, B. Dora, F. Simon,
R. Moessner, arXiv:1211.5623
N. Goldman, J. Dalibard, PRX 2014

One-Way Road in a Photonic Crystal

Chiral edge states channel light waves in one direction, like electrons in the quantum Hall effect



(a) photonic crystal.
The distance between
the ferrite rods is 4 cm.

Realizations of AQHE in Photonic crystals: following Haldane & Raghu, PRL 2008
(Dirac points and Faraday effect opens a gap breaking time-reversal symmetry)

Experiment: M. Soljacic et al. Nature **461**, 772 (2009) + Review 2014

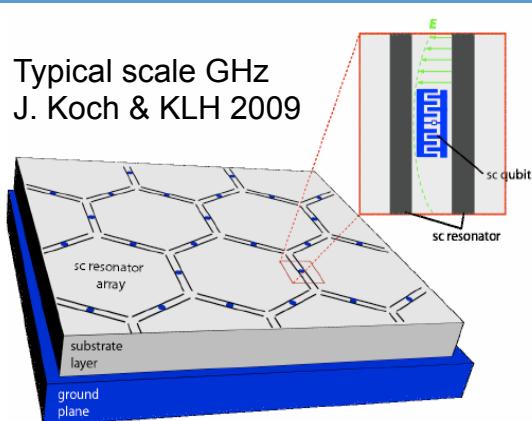
Review: KLH; Loic Henriet; A. Petrescu; K. Plekhanov; G. Roux; M. Schiro 2015 (see arXiv)
(invited review CRAS, I. Carusotto, J. Bloch and A. Amo)

Systems of interacting photons: Theory surveys

- M. Hartmann et al., Laser & Photonics Review 2, 527 (2008)
A. Tomadin & R. Fazio, J. Opt. Soc. Am B 27, A130 (2010)
J. Larson ; I. Carusotto and C. Ciuti, RMP 2012

realizations: superfluidity of polaritons **Stanford at Grenoble-EPFL, LKB ENS, LPN Marcoussis, Pittsburgh**

- * photonic band gap cavities
- * arrays of silicon micro-cavities
- * fibre based cavities
- * cQED Array current realization (A. Houck; H. Tureci; J. Koch 2012 & S. Schmidt, J. Koch 2012)



some pros and cons

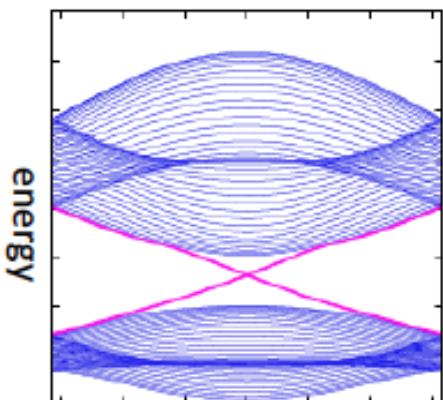
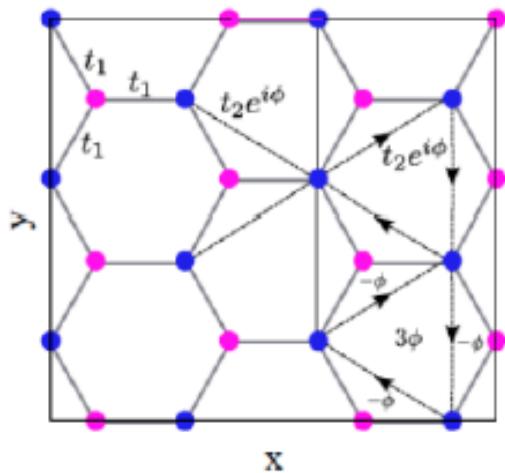
- + tunability
- + access to single lattice site
- must be treated as open system
- + interesting: transitions between different steady states

Interacting photons:

M. Lukin, E. Demler et al:
Fermionizing light

Quantum Anomalous Hall Effect

F. D. M. Haldane 1988

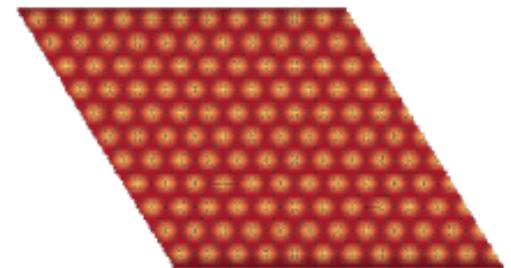
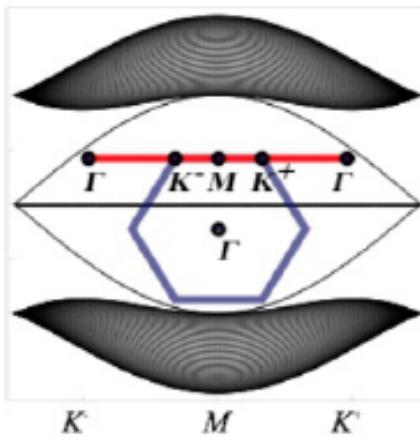
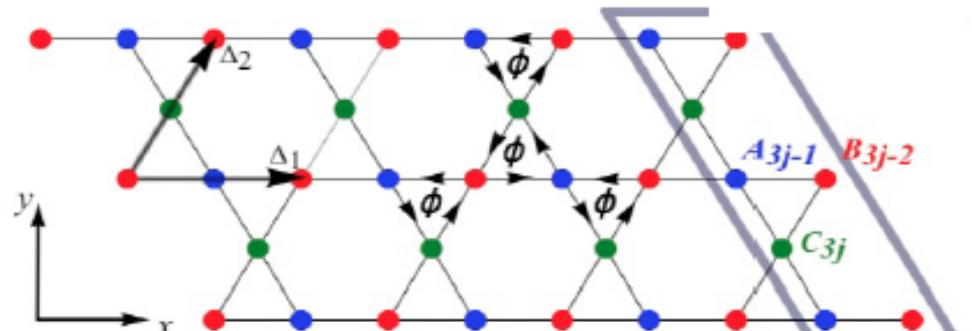


Graphene
+gap

Kagome version:

A. Petrescu, A. A. Houck and KLH, 2012

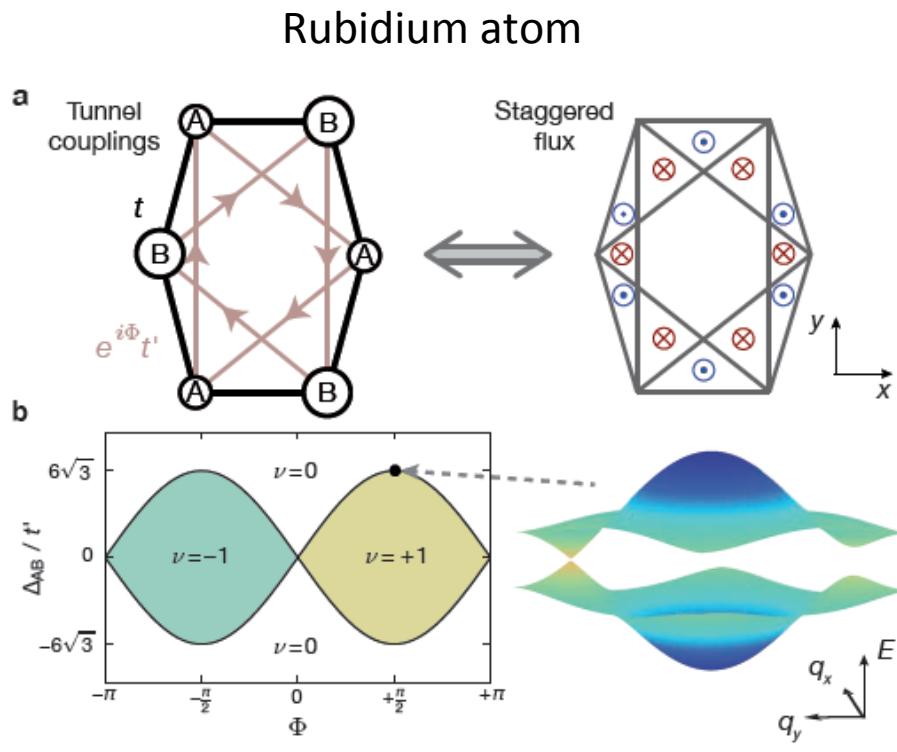
See also J. Koch, A. Houck, KLH, S. Girvin 2010



Experiment in real materials: C.-Z. Chang et al. 2013

Other Experimental observations

- Ultra-cold atoms – see for example Esslinger's experiment (ETH)
- Ultra-cold atoms: importance of Floquet-type point of view



Modulation of optical lattice

$$\mathbf{r}_{\text{lat}} = -A \left(\cos(\omega t) \mathbf{e}_x + \cos(\omega t - \varphi) \mathbf{e}_y \right),$$

$$\mathbf{F}(t) = -m \ddot{\mathbf{r}}_{\text{lat}}(t)$$

$$\hat{H}_{\text{lat}}(t) = \sum_{\langle ij \rangle} t_{ij} \hat{c}_i^\dagger \hat{c}_j + \sum_i (\mathbf{F}(t) \cdot \mathbf{r}_i) \hat{c}_i^\dagger \hat{c}_i$$

$$\hat{U}(T, t_0) = \mathcal{T} e^{-i \int_{t_0}^{t_0+T} \hat{H}(t) dt} = e^{-iT\hat{H}_{\text{eff}}(t_0)}$$

Chiral Bosonic Phases on the Haldane Honeycomb Lattice

I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB 2015)

$$\mathcal{H} = \mathcal{H}_H + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i,$$

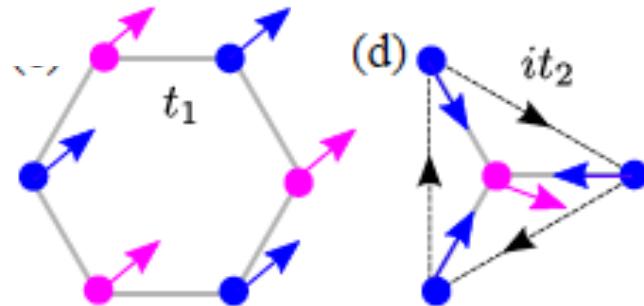
Phase-angle variables $b_i^\dagger = \sqrt{n} e^{i\theta_i}$

chiral SF:

nonuniform phase,
plaquette currents

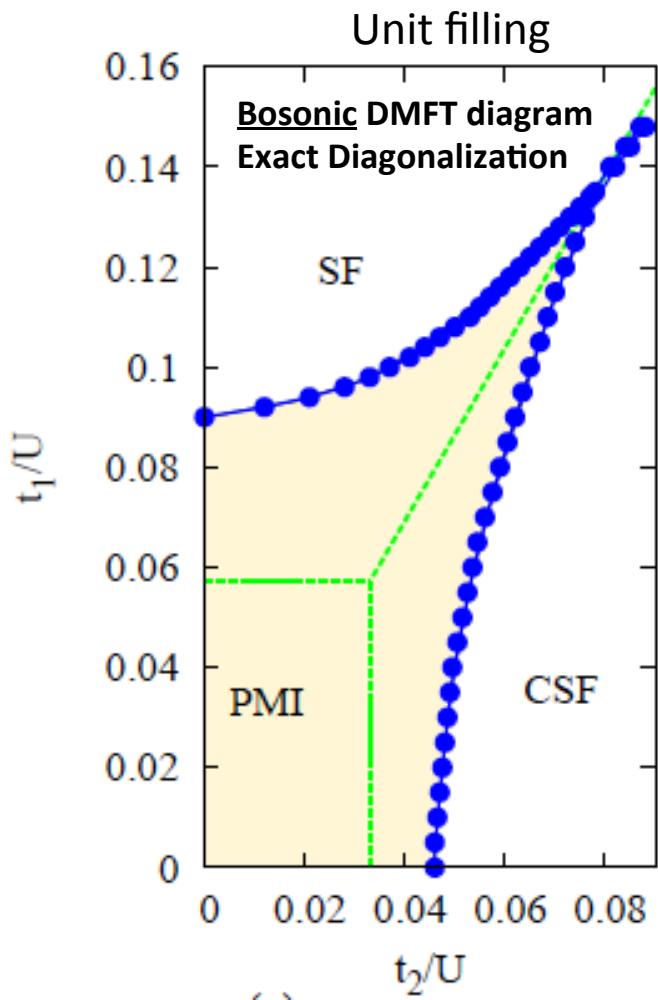
SF:

uniform phase,
“Meissner current”



Similar models on square lattice:

L. K. Lim, C. M. Smith and A. Hemmerich,
Phys. Rev. Lett. 100, 130402 (2008) and PRA 2010



Topological Insulators

Kane & Mele, PRL 95, 226801 (2005); Fu-Kane

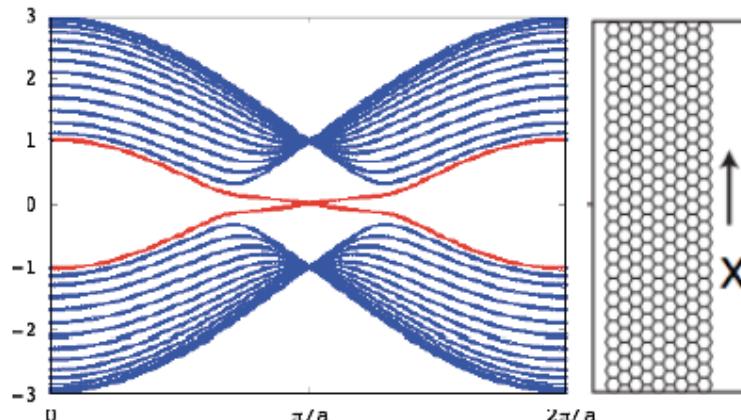
see also: Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)

Also realizations in photon systems for example: [M. Hafezi, S. Mittal, J. Fan, A. Migdall, J. Taylor](#) (2013)

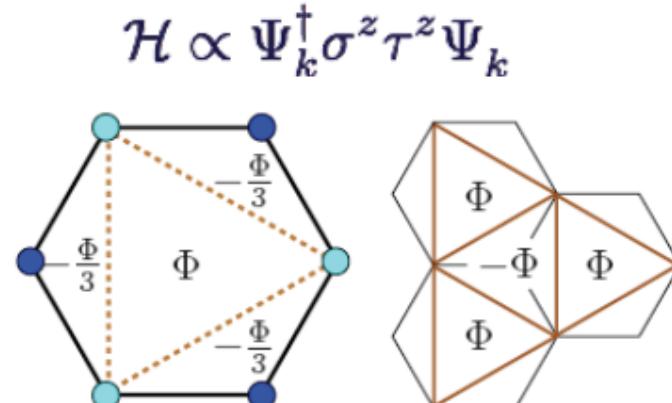
[Mikael C. Rechtsman](#), [Julia M. Zeuner](#), [Yonatan Plotnik](#), [Yaakov Lumer](#), [Stefan Nolte](#), [Mordechai Segev](#), [Alexander Szameit](#) (2013)

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma_{\sigma\sigma'}^z c_{i\sigma}^\dagger c_{j\sigma'}$$

strip geometry:



edge states: Kramers's pair



Quantum Spin Hall Effect

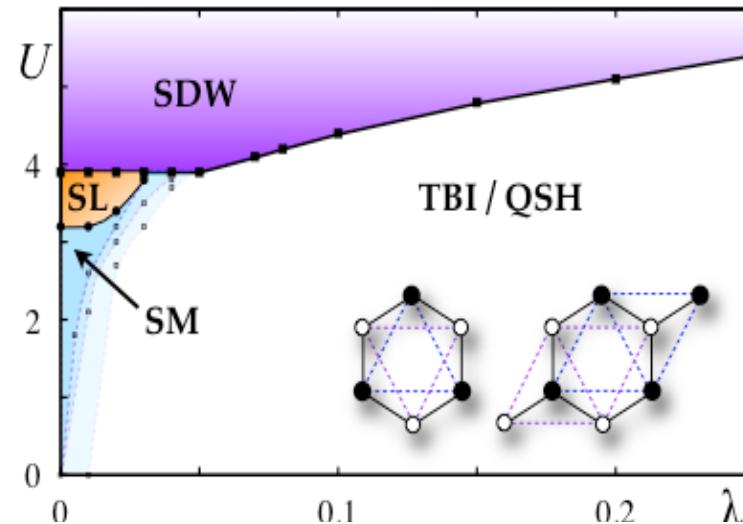
Interaction effects: S. Rachel & KLH, 2010
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Wei Wu,
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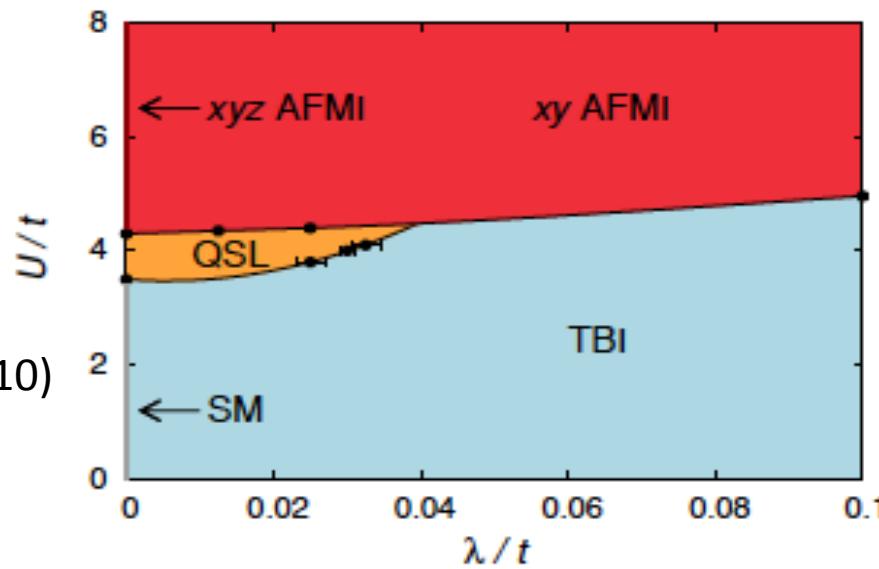
CDMFT

A. Georges, G. Kotliar
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Z.Y. Meng et al.
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Analytics:

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Reviews: Hohenadler & Assaad, 2013
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Connection to reality?

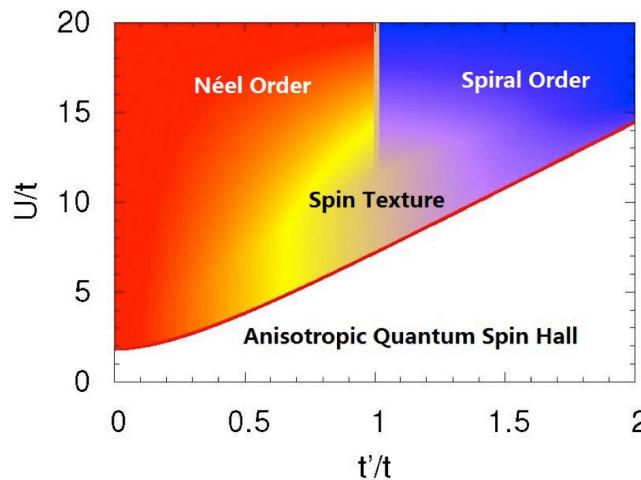
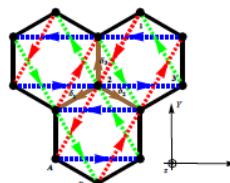
- Na_2IrO_3 : anisotropic spin-orbit coupling (thin films: arXiv:1303:5245, M. Jenderka et al)

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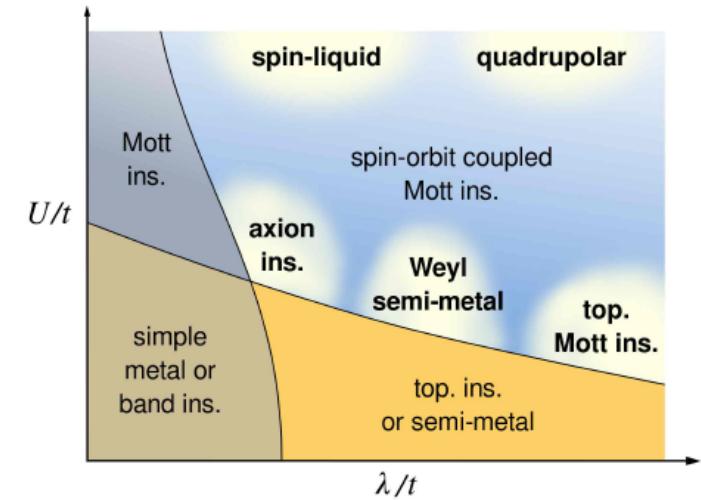
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**α Lithium Iridates
and Spiral order**
R. Coldea



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Summary & Thanks on quantum

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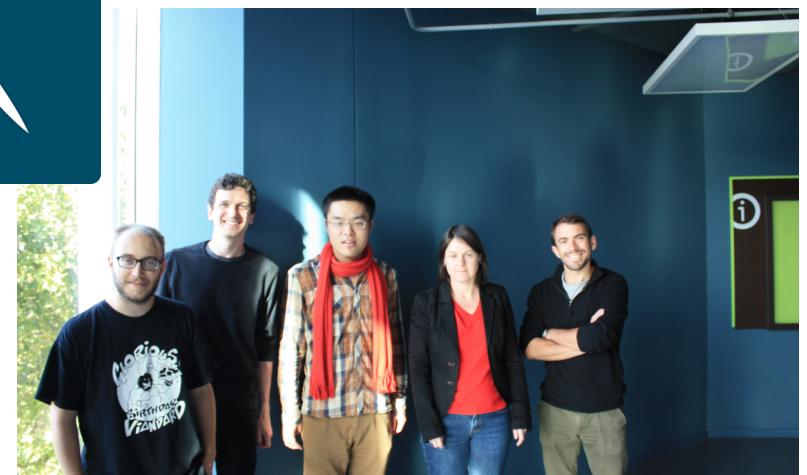
Palaiseau 2015



More Informations at:

<https://www.cpht.polytechnique.fr/cphtlehur/Karyn.LeHur.html>

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