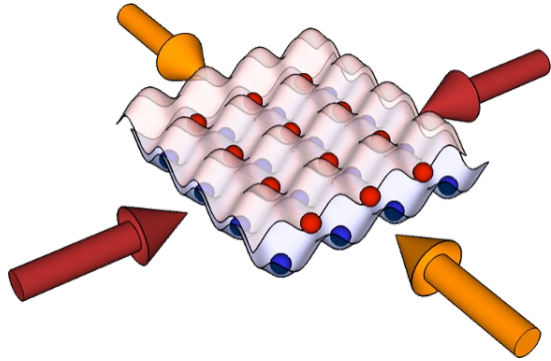


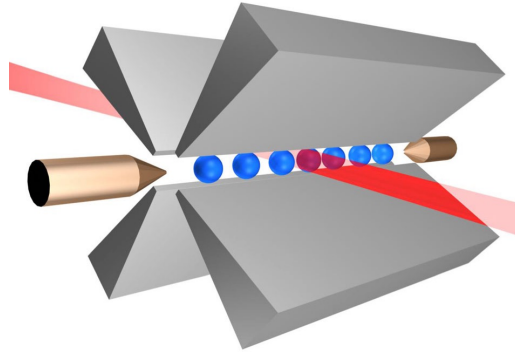
Physics of Quantum Information

Lecture 2: two-level systems and harmonic oscillators

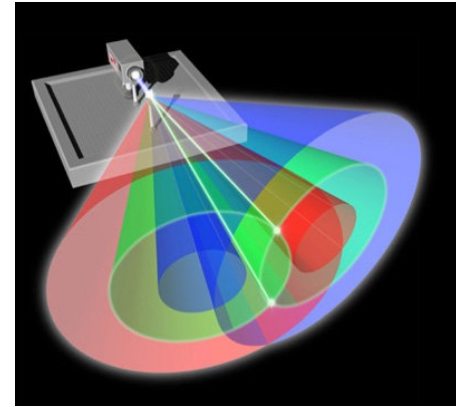
Quantum state engineering with *individual* systems



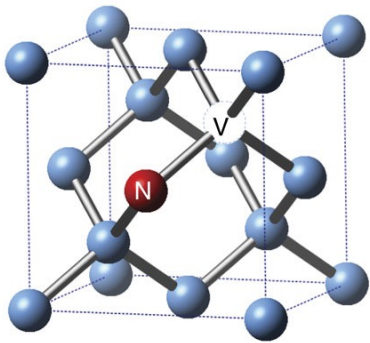
Cold atoms and molecules



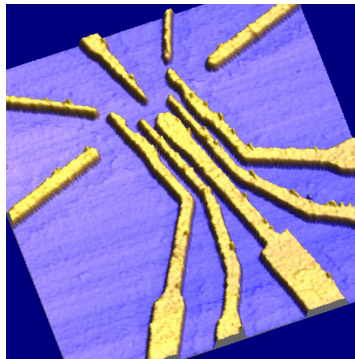
Trapped ions



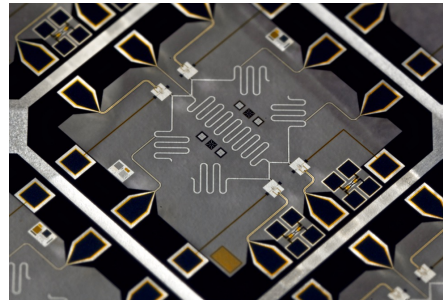
Photons in NL media



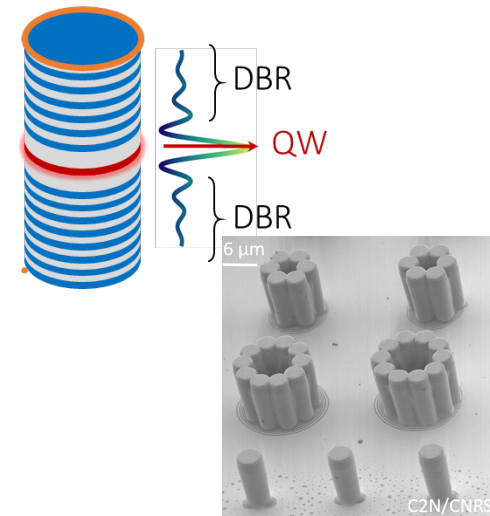
NV centers



Quantum dots

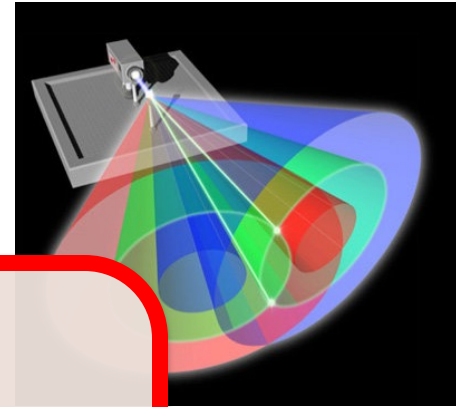
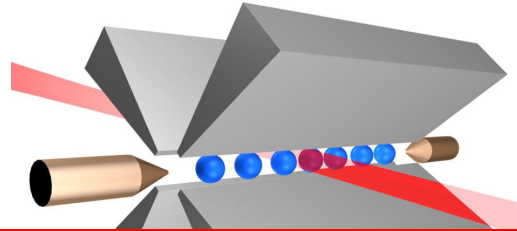
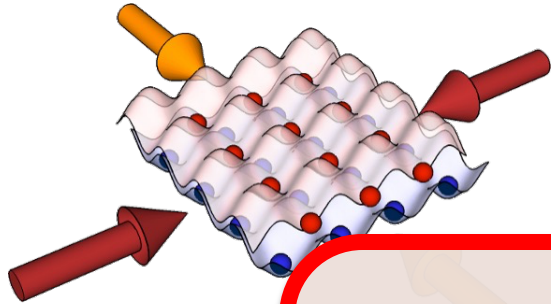


Superconducting qubits



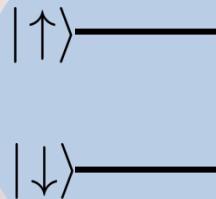
Polaritons in $\frac{1}{2}$ cond.

Quantum state engineering with *individual* systems

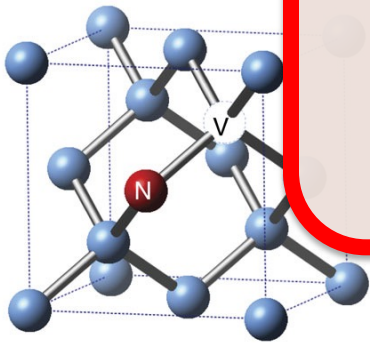


Cold atoms

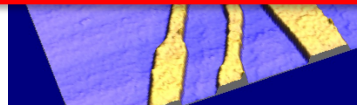
Two-level systems to encode a spin:



Addressable + controlled interactions



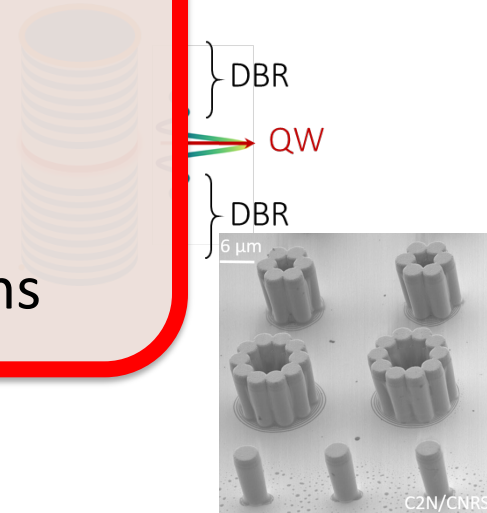
NV centers



Quantum dots

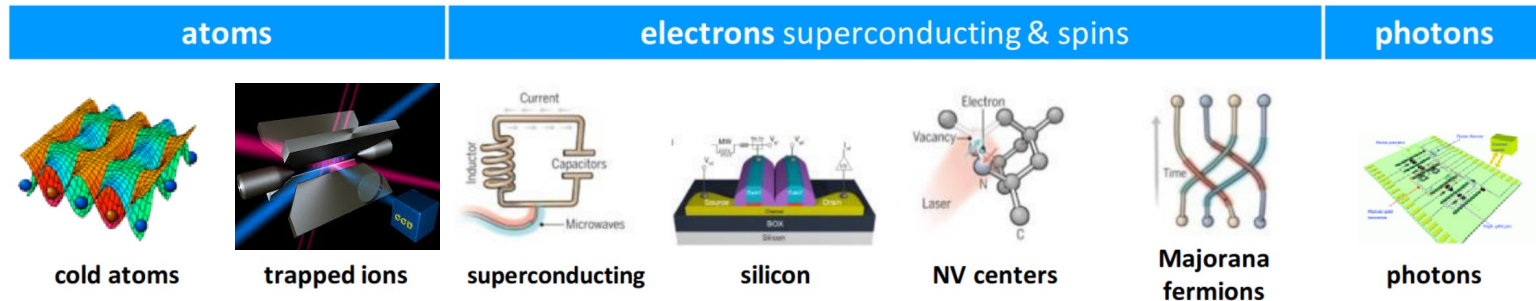


Superconducting qubits



Polaritons in $\frac{1}{2}$ cond.

Physical Qubits



DiVincenzo's criteria (5+2)

1. Well characterized qubits encapsulated in a scalable physical system
2. Ability to initialize the qubits in a trustable state
3. Ability to maintain long coherence times, much longer than the gate operation time
4. Ability to implement a "universal" set of quantum gates
5. Ability to perform useful measurements
6. Ability to interconnect stationary and flying qubits
7. Ability to faithfully transmit flying qubits between specified locations

18

Periodic Table of the Elements

$1s^2 2s^2 \dots (n-1)p^6 ns$

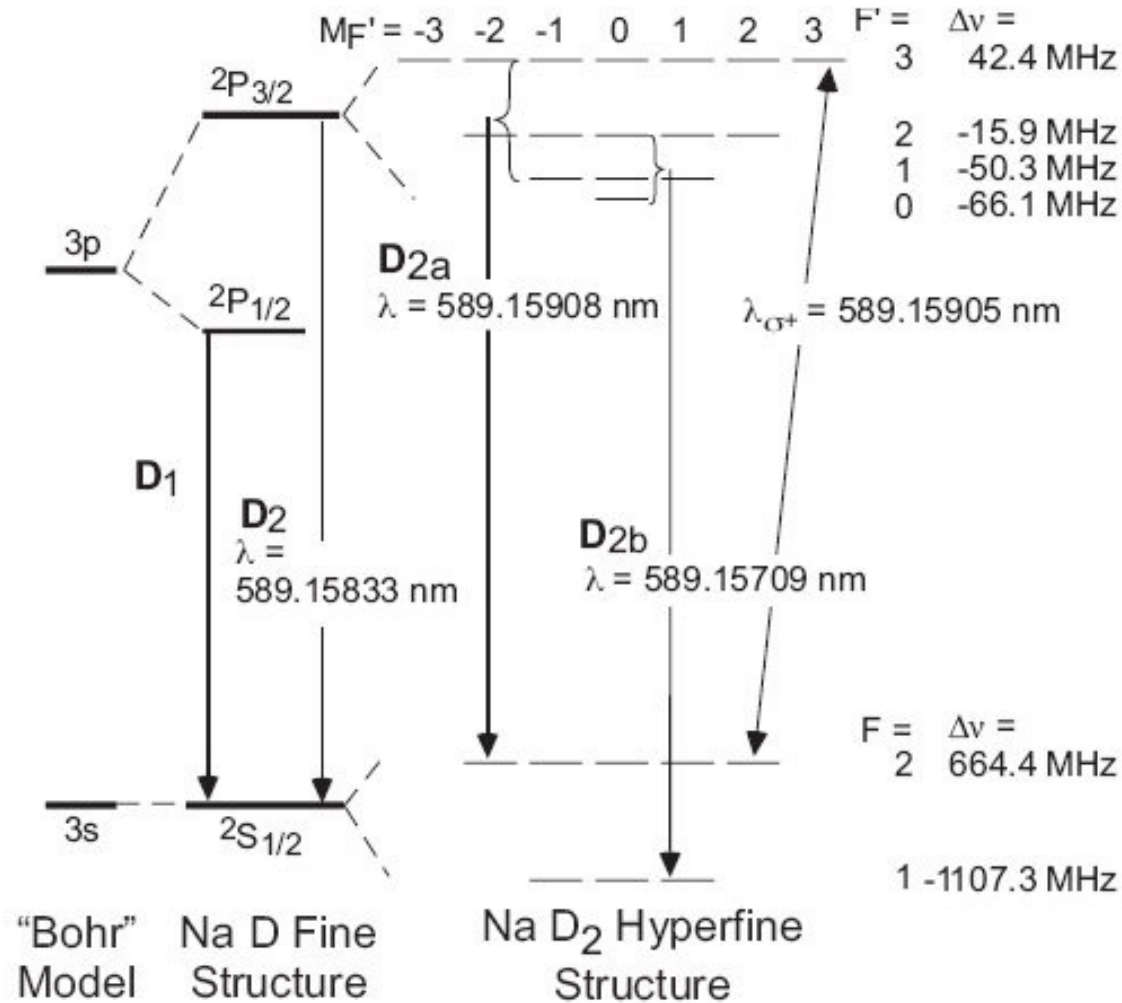
Alkali: 1 external electron

| | | | | | | | | | | | | | | | | | |
|---------------------------------------|--|---------------------------------------|--|--|---|---|--|---|---|--|--|---|--|---|--|---|--|
| 1 H Hydrogen 1.008 | | | | | | | | | | | | | | | | | 2 He Helium 4.003 |
| 3 Li Lithium 6.941 | 4 Be Beryllium 9.012 | | | | | | | | | | | 5 B Boron 10.811 | 6 C Carbon 12.011 | 7 N Nitrogen 14.007 | 8 O Oxygen 15.999 | 9 F Fluorine 18.998 | 10 Ne Neon 20.180 |
| 11 Na Sodium 22.990 | 12 Mg Magnesium 24.305 | | | | | | | | | | | 13 Al Aluminum 26.982 | 14 Si Silicon 28.086 | 15 P Phosphorus 30.974 | 16 S Sulfur 32.066 | 17 Cl Chlorine 35.453 | 18 Ar Argon 39.948 |
| 19 K Potassium 39.098 | 20 Ca Calcium 40.078 | 21 Sc Scandium 44.956 | 22 Ti Titanium 47.867 | 23 V Vanadium 50.942 | 24 Cr Chromium 51.996 | 25 Mn Manganese 54.938 | 26 Fe Iron 55.845 | 27 Co Cobalt 58.933 | 28 Ni Nickel 58.693 | 29 Cu Copper 63.546 | 30 Zn Zinc 65.38 | 31 Ga Gallium 69.723 | 32 Ge Germanium 72.631 | 33 As Arsenic 74.922 | 34 Se Selenium 78.971 | 35 Br Bromine 79.904 | 36 Kr Krypton 84.798 |
| 37 Rb Rubidium 84.468 | 38 Sr Strontium 87.62 | 39 Y Yttrium 88.906 | 40 Zr Zirconium 91.224 | 41 Nb Niobium 92.906 | 42 Mo Molybdenum 95.95 | 43 Tc Technetium 98.907 | 44 Ru Ruthenium 101.07 | 45 Rh Rhodium 102.906 | 46 Pd Palladium 106.42 | 47 Ag Silver 107.868 | 48 Cd Cadmium 112.414 | 49 In Indium 114.818 | 50 Sn Tin 118.711 | 51 Sb Antimony 121.760 | 52 Te Tellurium 127.6 | 53 I Iodine 126.904 | 54 Xe Xenon 131.294 |
| 55 Cs Cesium 132.905 | 56 Ba Barium 137.328 | 57-71 Lanthanides | 72 Hf Hafnium 178.49 | 73 Ta Tantalum 180.948 | 74 W Tungsten 183.84 | 75 Re Rhenium 186.207 | 76 Os Osmium 190.23 | 77 Ir Iridium 192.217 | 78 Pt Platinum 195.085 | 79 Au Gold 196.967 | 80 Hg Mercury 200.592 | 81 Tl Thallium 204.383 | 82 Pb Lead 207.2 | 83 Bi Bismuth 208.980 | 84 Po Polonium [209] | 85 At Astatine [210] | 86 Rn Radon [222] |
| 87 Fr Francium [223] | 88 Ra Radium [226] | 89-103 Actinides | 104 Rf Rutherfordium [261] | 105 Db Dubnium [262] | 106 Sg Seaborgium [266] | 107 Bh Bohrium [264] | 108 Hs Hassium [269] | 109 Mt Meitnerium [268] | 110 Ds Darmstadtium [269] | 111 Rg Roentgenium [272] | 112 Cn Copernicium [277] | 113 Uut Ununtrium [278] | 114 Fl Flerovium [289] | 115 Uup Ununpentium [288] | 116 Lv Livermorium [293] | 117 Uus Ununseptium [294] | 118 Uuo Ununoctium [294] |

| | | | | | | | | | | | | | | |
|---|---------------------------------------|--|---|--|---|---|---|---|---|---|--|--|---|---|
| 57 La Lanthanum 138.905 | 58 Ce Cerium 140.116 | 59 Pr Praseodymium 140.908 | 60 Nd Neodymium 144.243 | 61 Pm Promethium 144.913 | 62 Sm Samarium 150.36 | 63 Eu Europium 151.964 | 64 Gd Gadolinium 157.25 | 65 Tb Terbium 158.925 | 66 Dy Dysprosium 162.500 | 67 Ho Holmium 164.930 | 68 Er Erbium 167.259 | 69 Tm Thulium 168.934 | 70 Yb Ytterbium 173.055 | 71 Lu Lutetium 174.967 |
| 89 Ac Actinium 227.028 | 90 Th Thorium 232.038 | 91 Pa Protactinium 231.036 | 92 U Uranium 238.029 | 93 Np Neptunium 237.048 | 94 Pu Plutonium 244.064 | 95 Am Americium 243.061 | 96 Cm Curium 247.070 | 97 Bk Berkelium 247.070 | 98 Cf Californium 251.080 | 99 Es Einsteinium [254] | 100 Fm Fermium 257.095 | 101 Md Mendelevium 258.1 | 102 No Nobelium 259.101 | 103 Lr Lawrencium [262] |

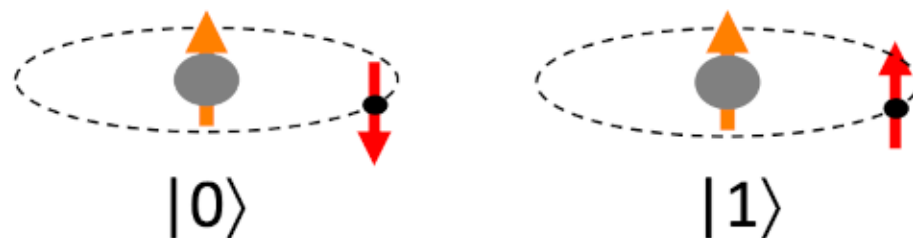
| | | | | | | | | | |
|--------------|----------------|------------------|-------------|-----------|----------|---------|-----------|------------|----------|
| Alkali Metal | Alkaline Earth | Transition Metal | Basic Metal | Semimetal | Nonmetal | Halogen | Noble Gas | Lanthanide | Actinide |
|--------------|----------------|------------------|-------------|-----------|----------|---------|-----------|------------|----------|

Atomic structure of sodium ($_{11}\text{Na}$)



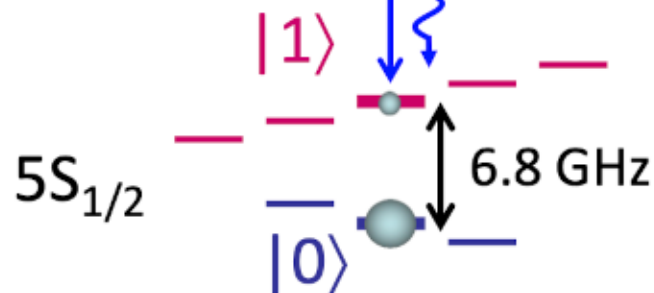
Atomic Qubits

Hyperfine qubit



$5P_{3/2}$ $|e\rangle$

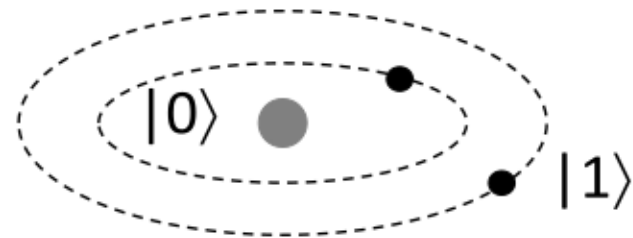
Laser cooling,
detection



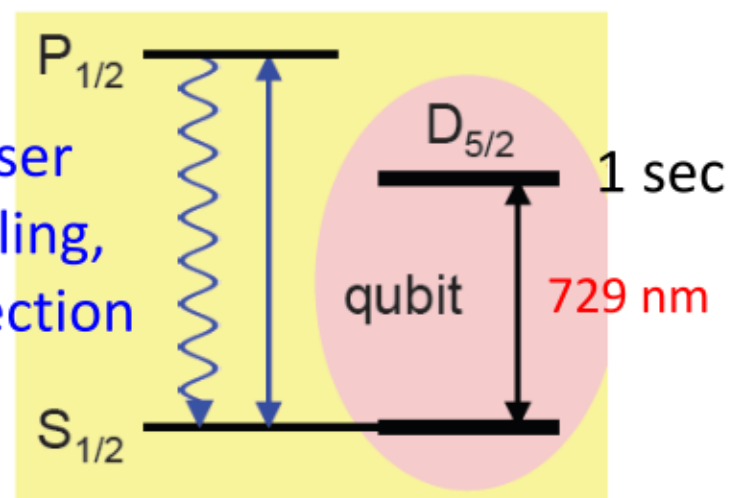
Rb, Cs...

Be⁺ ⁴³Ca⁺ Yb⁺ Cd⁺ ...

Optical qubit



Laser
cooling,
detection



⁴⁰Ca⁺

See pages 7-8 (detailed calculations in HW1)

Coherent manipulation and Rabi oscillations. As shown above, be the transition dipole electric or magnetic, the Hamiltonian describing the interaction of an atom with an electromagnetic field has the general form

$$\hat{H}(t) = -\frac{\hbar\omega_0}{2} \hat{\sigma}_z + \hbar\Omega \cos \omega t \hat{\sigma}_x = \hbar \begin{pmatrix} -\omega_0/2 & \Omega \cos \omega t \\ \Omega \cos \omega t & \omega_0/2 \end{pmatrix}_{|0\rangle,|1\rangle}, \quad (12)$$

with $\mp \hbar\omega_0/2$ the energies of the states $|0\rangle$ and $|1\rangle$, respectively.

1. Apply the transformation $|\tilde{\psi}(t)\rangle = \hat{\mathcal{R}}(t) |\psi(t)\rangle$ with

$$\mathcal{R}(t) = \exp \left(-i \frac{\omega t}{2} \hat{\sigma}_z \right) = \begin{pmatrix} e^{-i \frac{\omega t}{2}} & 0 \\ 0 & e^{i \frac{\omega t}{2}} \end{pmatrix}_{|0\rangle,|1\rangle}. \quad (13)$$

This transformation amounts to applying a rotation around the Oz axis of angle $\omega t/2$, and hence “move to the frame rotating at frequency ω ”.

2. The Hamiltonian governing the evolution of the new state $|\tilde{\psi}(t)\rangle$ is found by writing the Schrödinger equation and applying the transformation. It yields the new Schrödinger equation

$$i\hbar \frac{d}{dt} |\tilde{\psi}(t)\rangle = \tilde{H}(t) |\tilde{\psi}(t)\rangle \quad \text{with} \quad \tilde{H} = \mathcal{R}H\mathcal{R}^{-1} + i\hbar \frac{d\mathcal{R}}{dt} \mathcal{R}^{-1} . \quad (14)$$

Applying the transformation and using $i\hbar \frac{d\mathcal{R}}{dt} = (\hbar\omega/2)\hat{\sigma}_z\mathcal{R}$, we find

$$\tilde{H} = \frac{\hbar}{2} \begin{pmatrix} \delta & \Omega(1 + e^{-2i\omega t}) \\ \Omega(1 + e^{+2i\omega t}) & -\delta \end{pmatrix}_{|0\rangle, |1\rangle} , \quad (15)$$

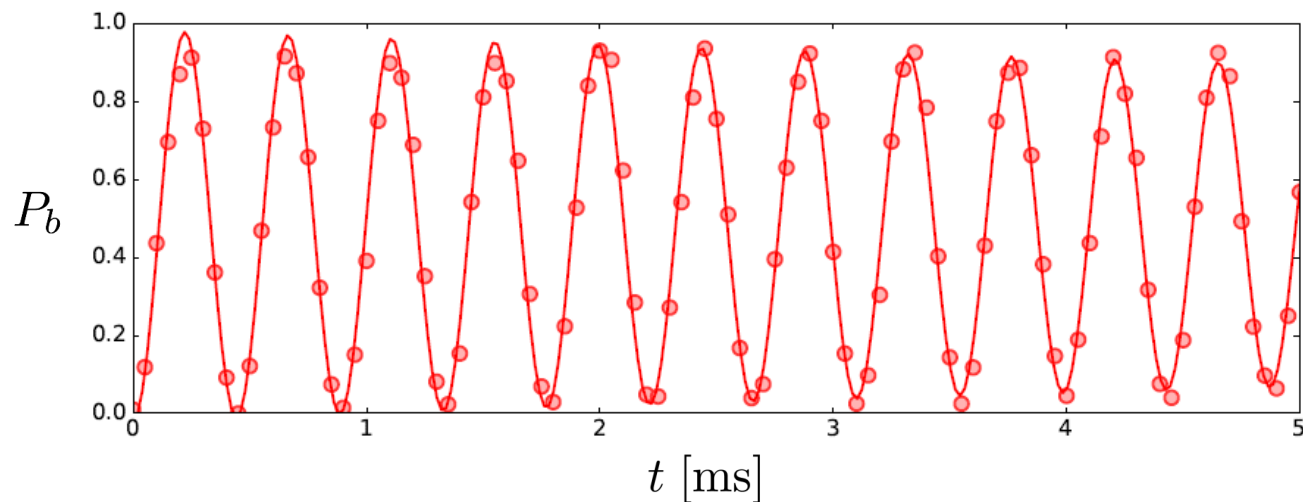
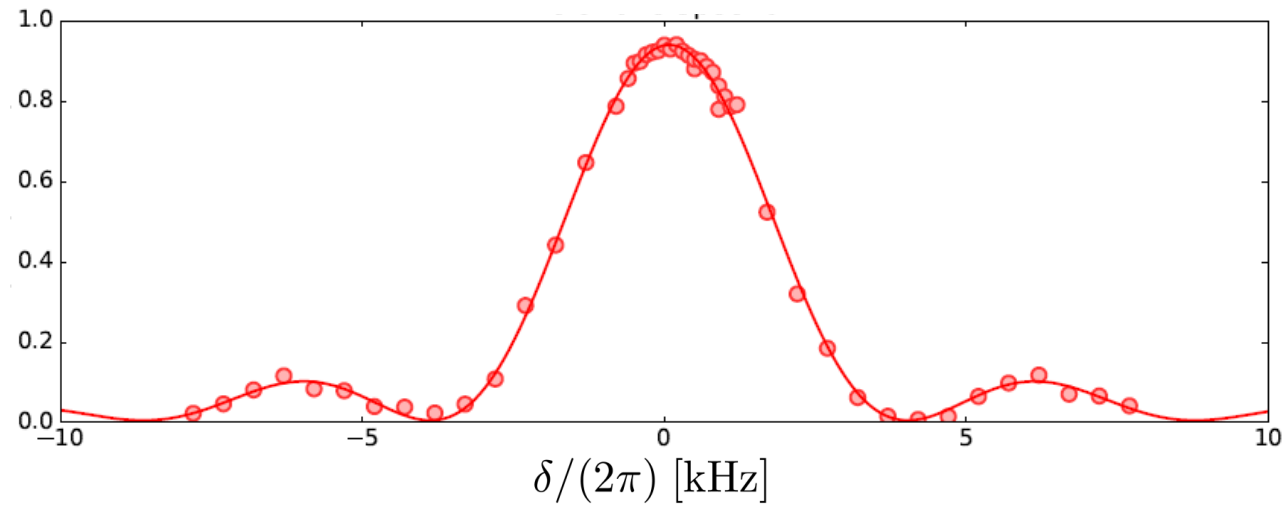
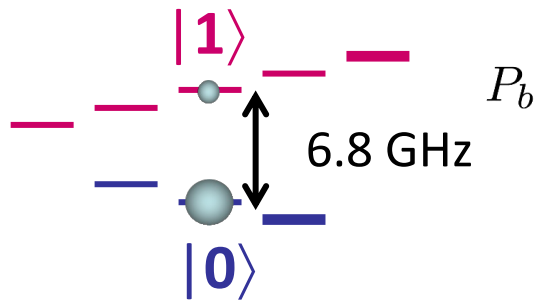
with $\delta = \omega - \omega_0$ the laser detuning with respect to the atomic transition. Hamiltonian is still time-dependent.

At the end of the procedure, and under the quasi-resonant approximation, we are then left with the effective time-independent Hamiltonian

$$\boxed{\tilde{H} = \frac{\hbar}{2} \begin{pmatrix} \delta & \Omega \\ \Omega & -\delta \end{pmatrix}_{|0\rangle, |1\rangle} = \frac{\hbar\delta}{2} \hat{\sigma}_z + \frac{\hbar\Omega}{2} \hat{\sigma}_x .} \quad (20)$$

Single atom Rabi oscillations

Each point = 200 repetitions

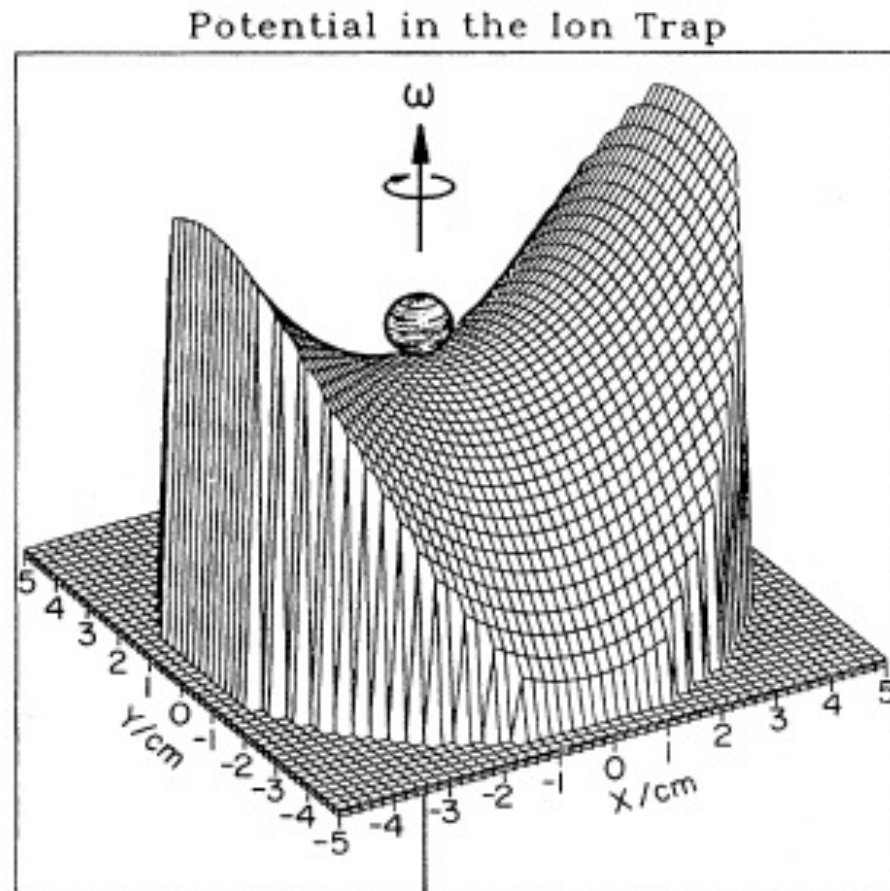


Paul trap for ions



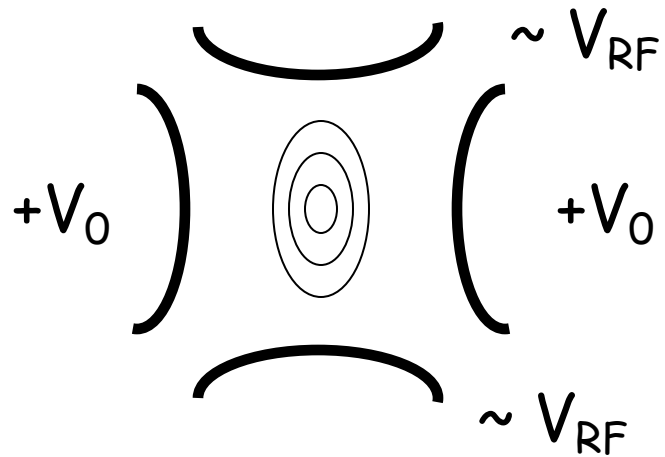
1989 Development of
the ion trap technics

W. Paul

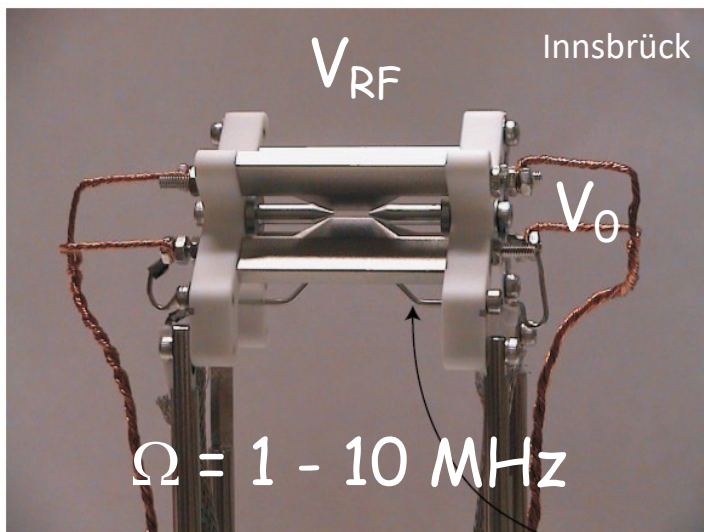


Ion crystals

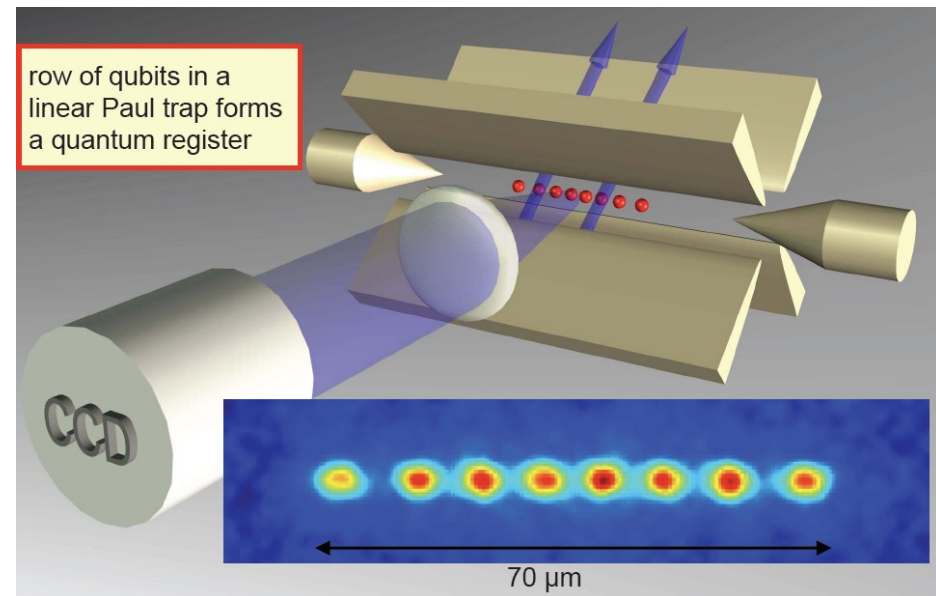
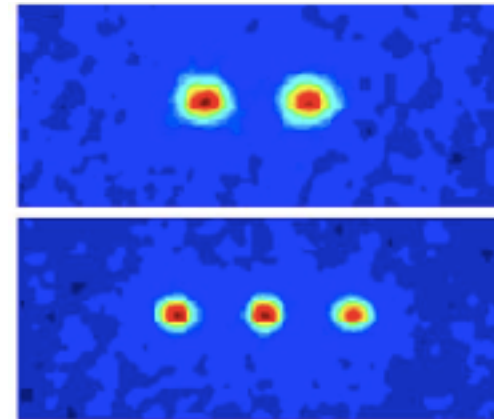
Paul trap (RF + static)



Linear Paul trap



Coulomb crystal
 \Rightarrow Ionic crystal



Innsbrück

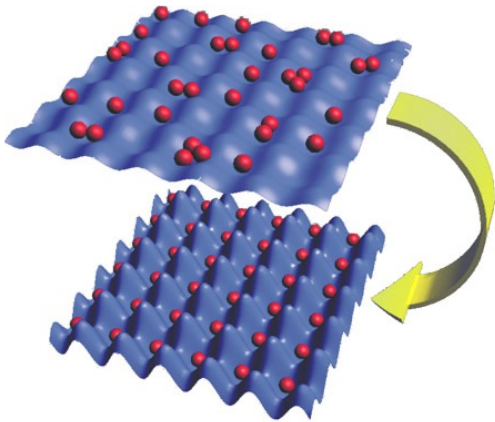
Ultra-cold atoms in optical lattices

Dipole force: $\mathbf{F} \propto -\nabla I(\mathbf{r})$



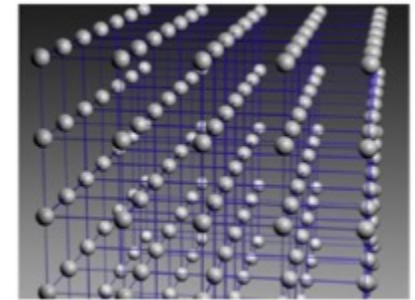
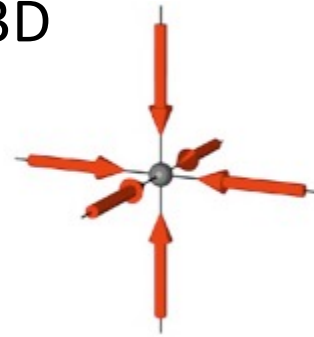
$$I(x) = 2E_0^2(1 + \cos 2kx)$$

Each site contains 1 atom!



Boson (Rb, Na, ^7Li , ^{39}K , $^4\text{He}^*$),
Fermion (^6Li , ^{40}K),
Magnetic atoms (Cr, Dy...)

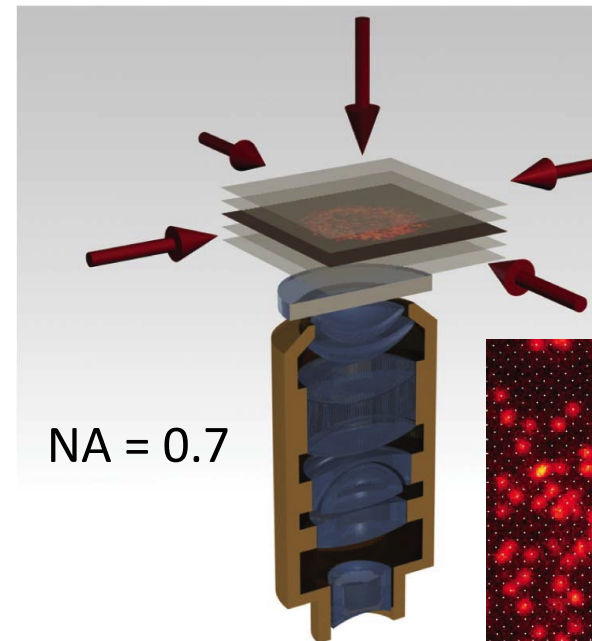
3D



(M. Greiner thesis)

$$\lambda/2 = 0.5 \mu\text{m}$$

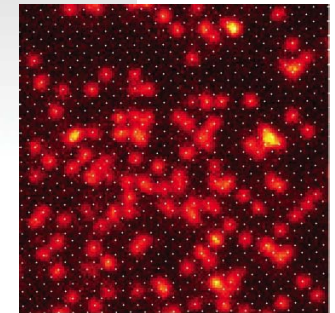
Quantum gas microscope



Single-site
resolution
($< 1 \mu\text{m}$)

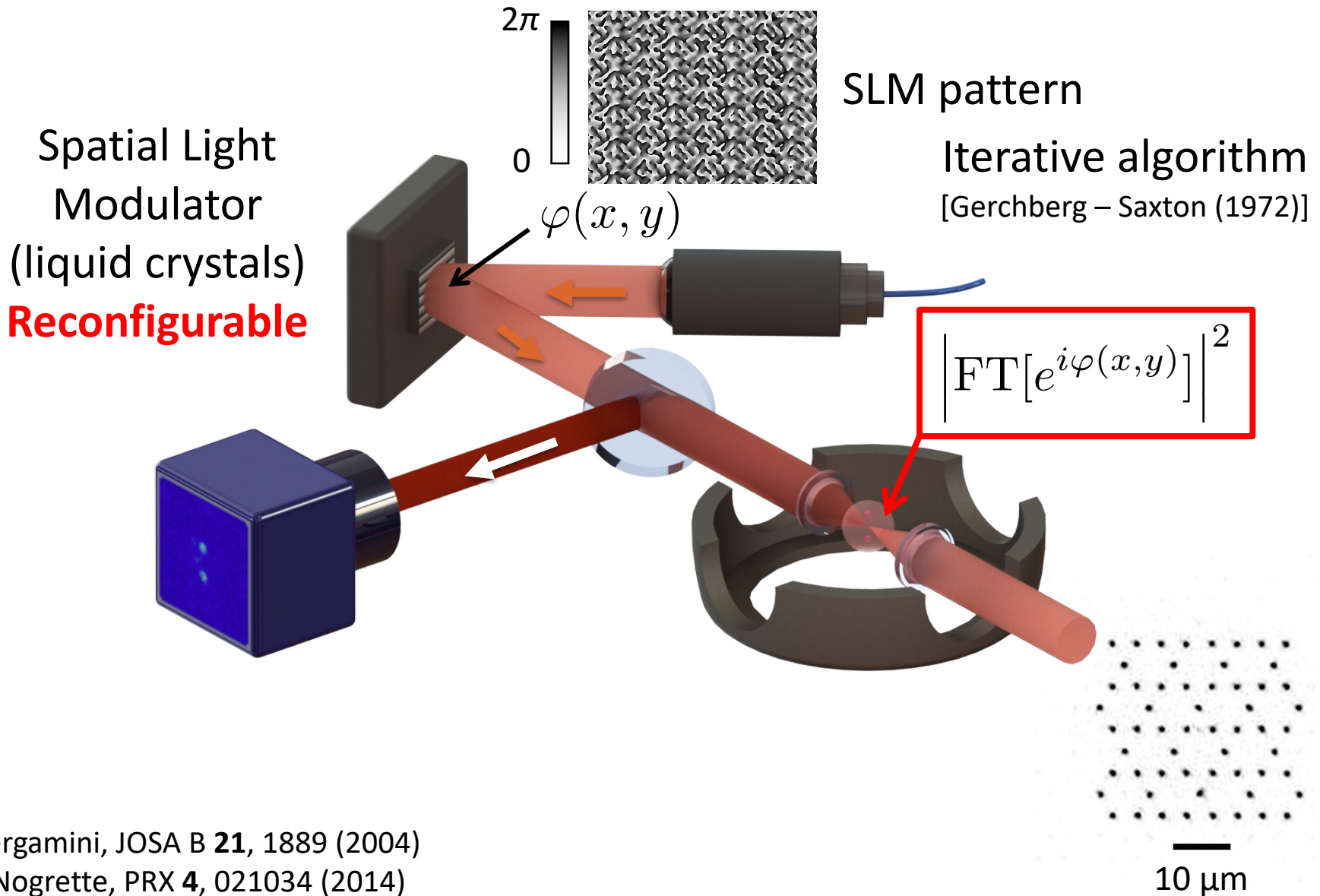
$\text{NA} = 0.7$

Havard, MPQ



$16 \mu\text{m}$

Holographic 2D arrays of tweezers

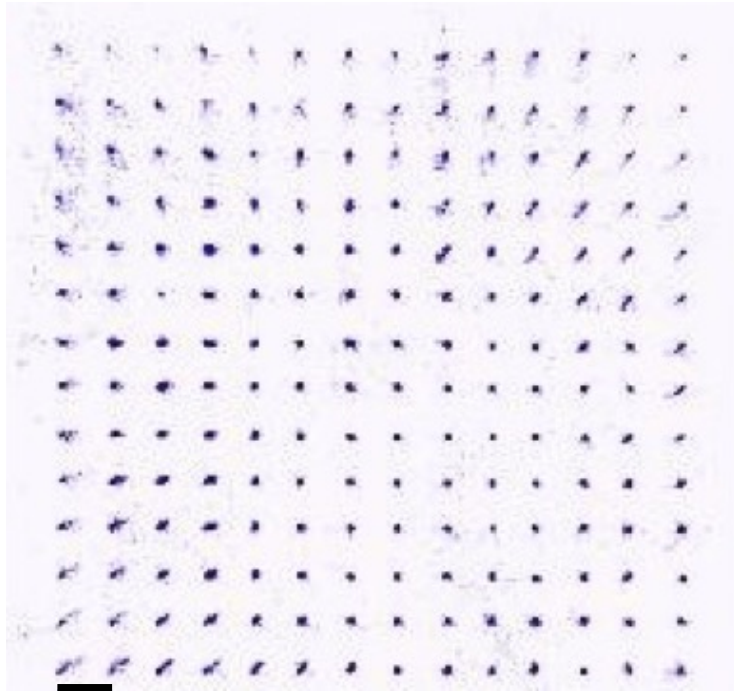


Bergamini, JOSA B **21**, 1889 (2004)

Nogrette, PRX **4**, 021034 (2014)

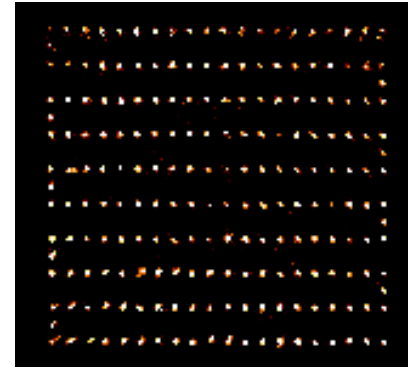
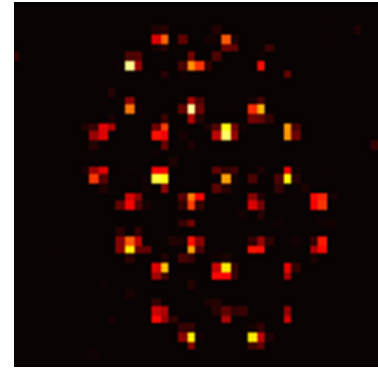
Arrays of atoms

Individual atoms in assembled
arrays of tweezers (~ 200 at.)



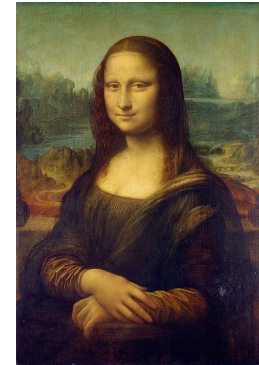
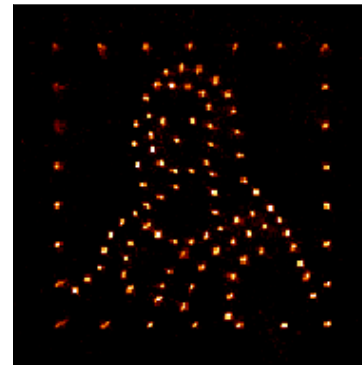
5 μm

$\sim 3\text{-}10\ \mu\text{m}$



$\sim 100\ \mu\text{m}$

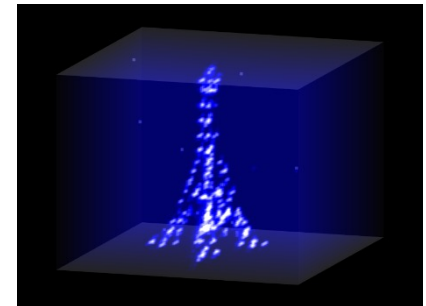
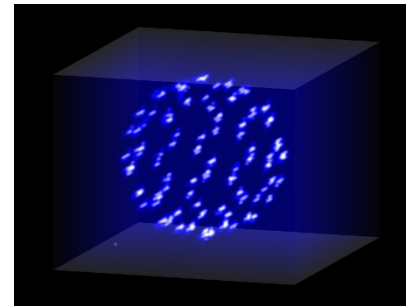
Schymik,
PRA 2021



L. da Vinci

Fluorescence: single shot!!

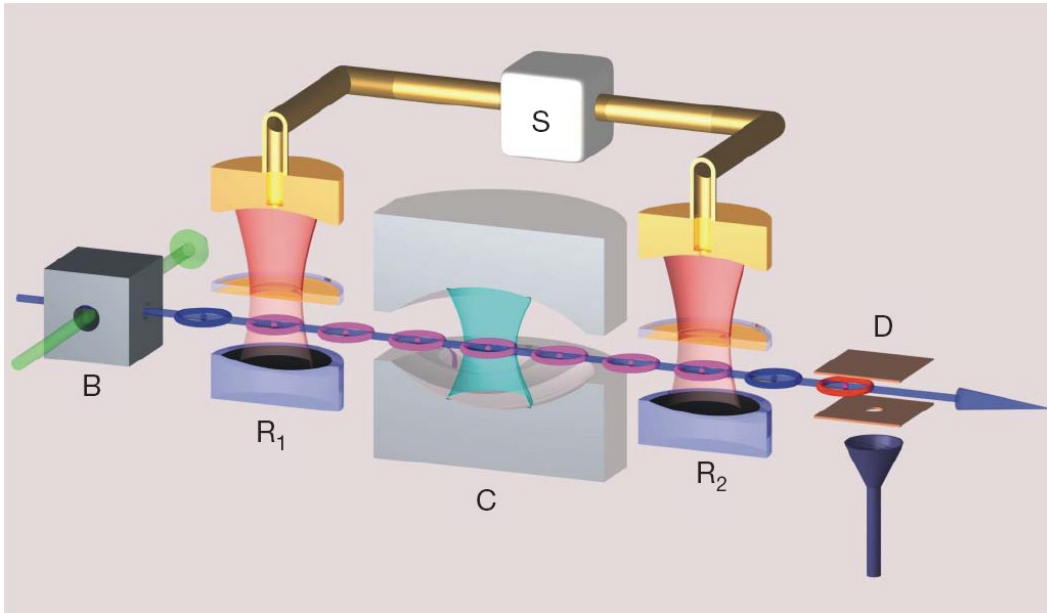
(Averaged)



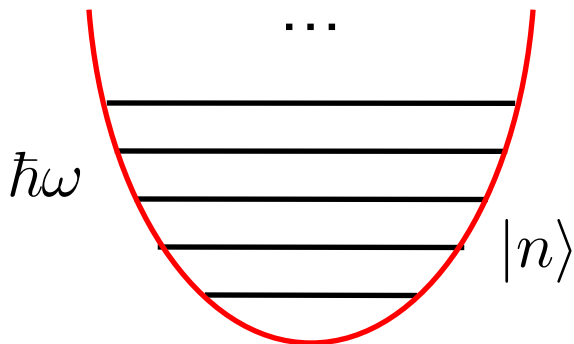
Photons in a cavity



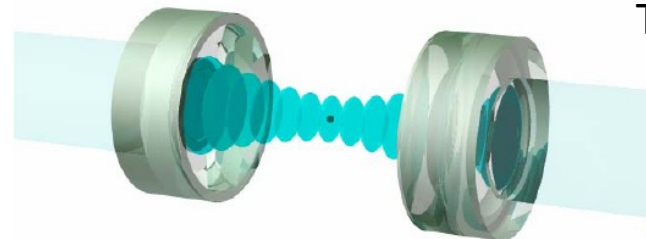
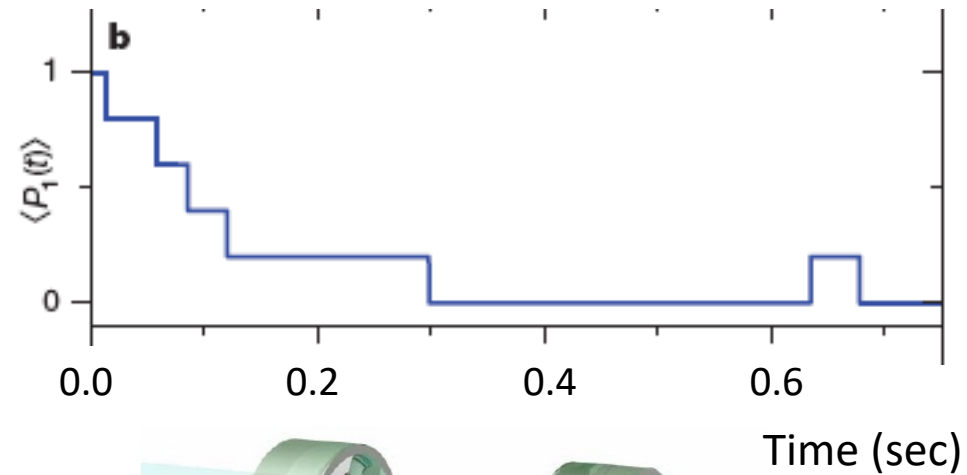
S. Haroche
2012



$$\hat{E} = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} (\hat{a} + a^+)$$



Cavity field, ω



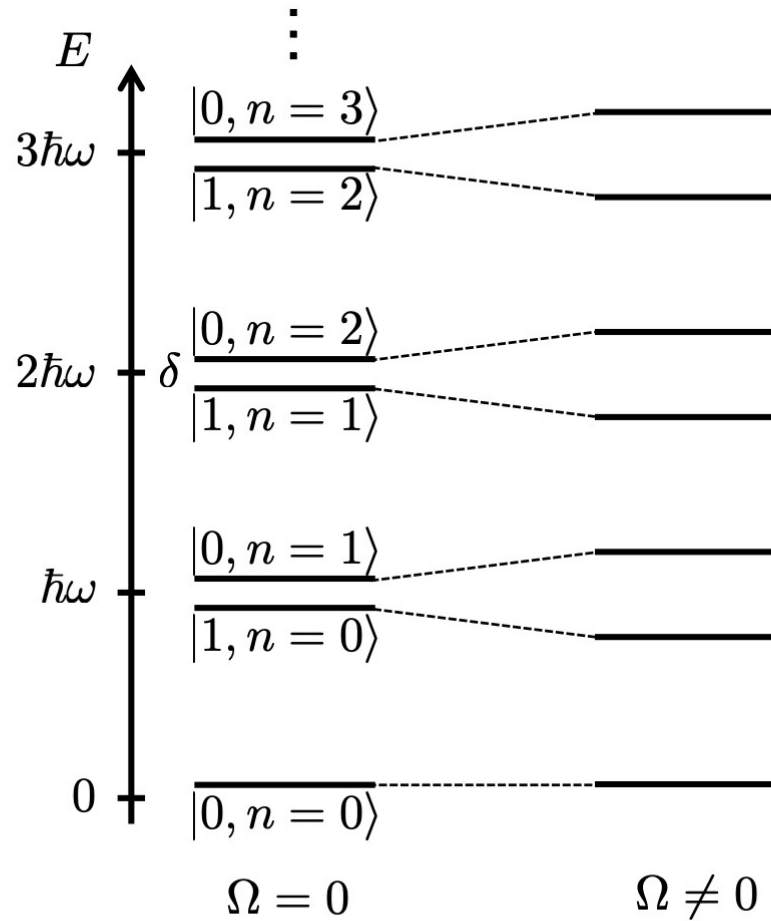
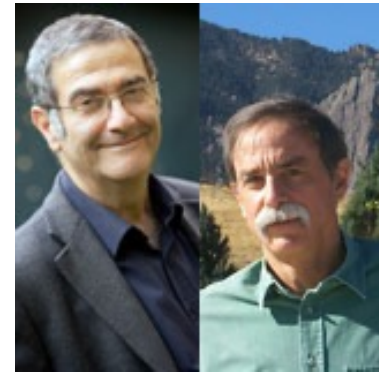
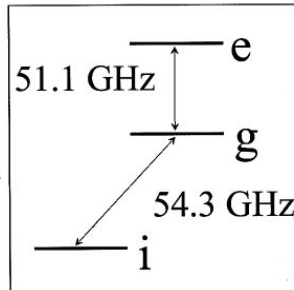
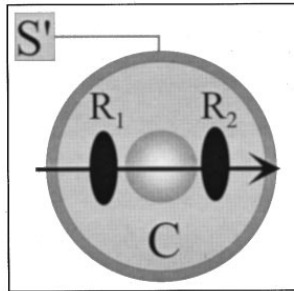
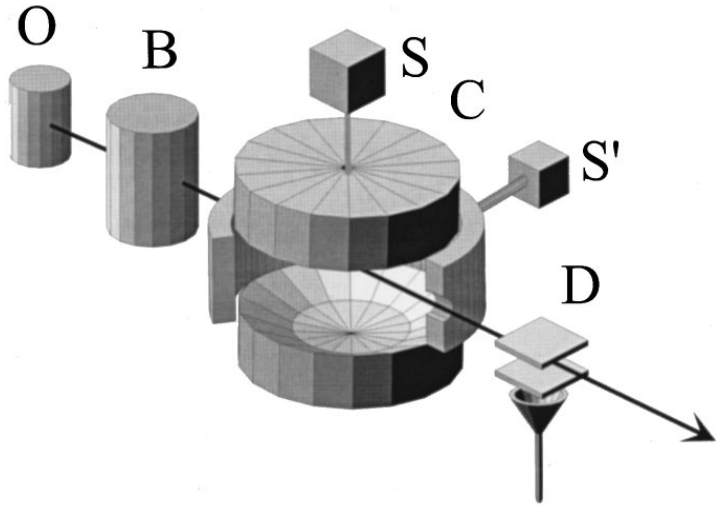


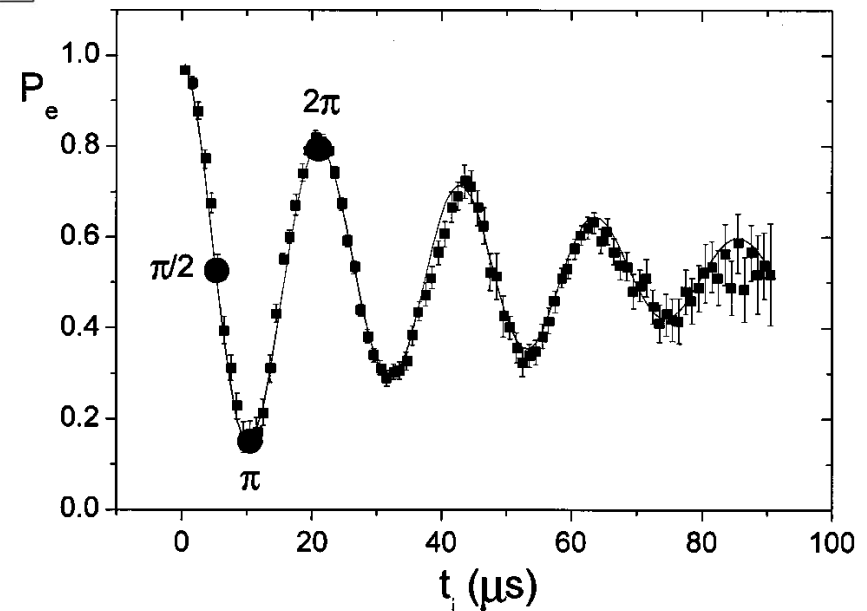
Figure 3: Spectrum of the Jaynes-Cummings Hamiltonian for $\delta > 0$.

“Single-photon” Rabi oscillations

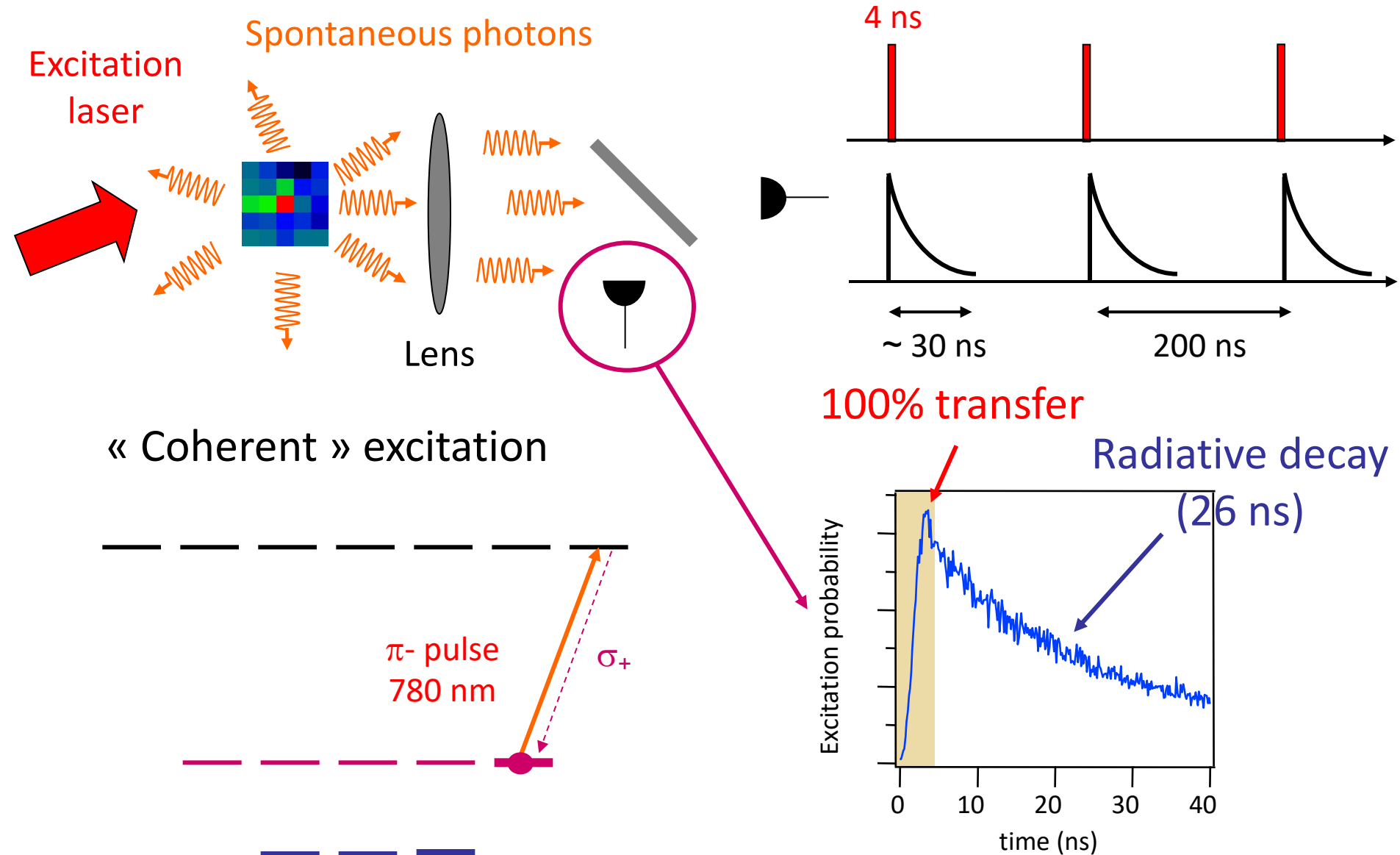


Nobel Prize 2012
S. Haroche, D.J. Wineland

M. Brune, PRL **76**, 1800 (1996)



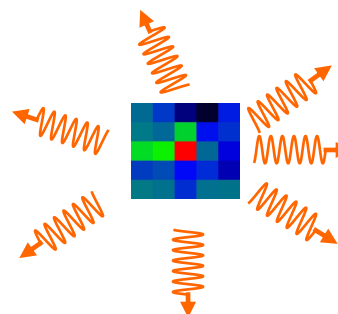
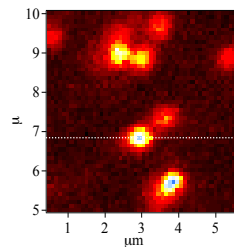
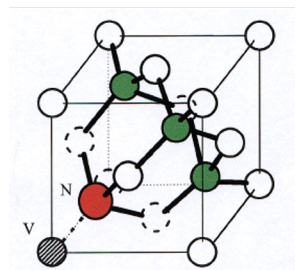
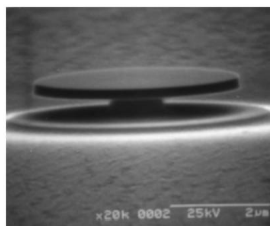
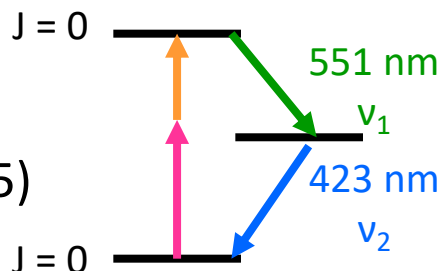
Triggered emission of single photons by an atom



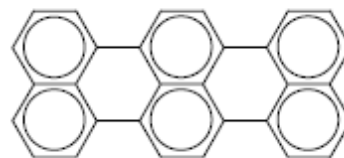
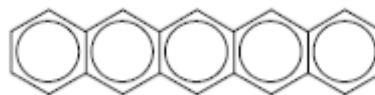
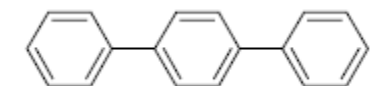
« Microscopic » single photon sources

Based on photon emission by a **single** microscopic dipole

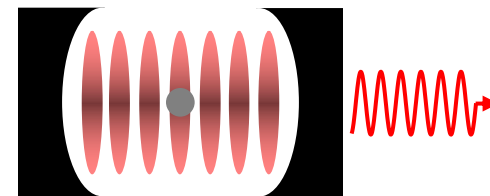
Grangier,
Aspect (1985)



Single trapped atom (2005)



Molecules in solid
(1999-2000)



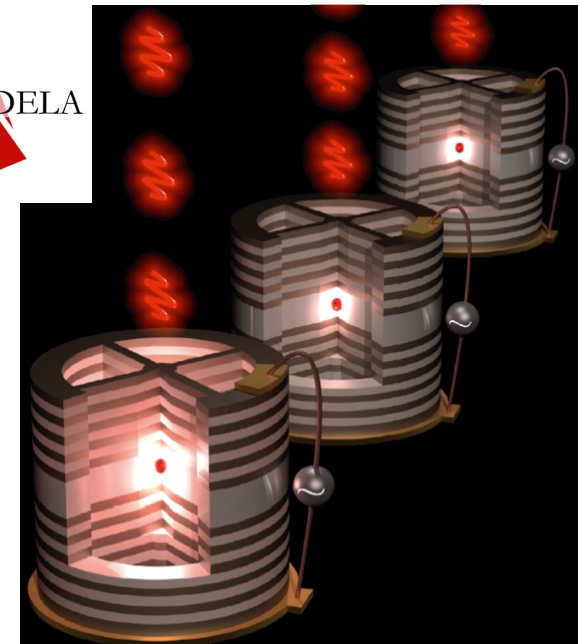
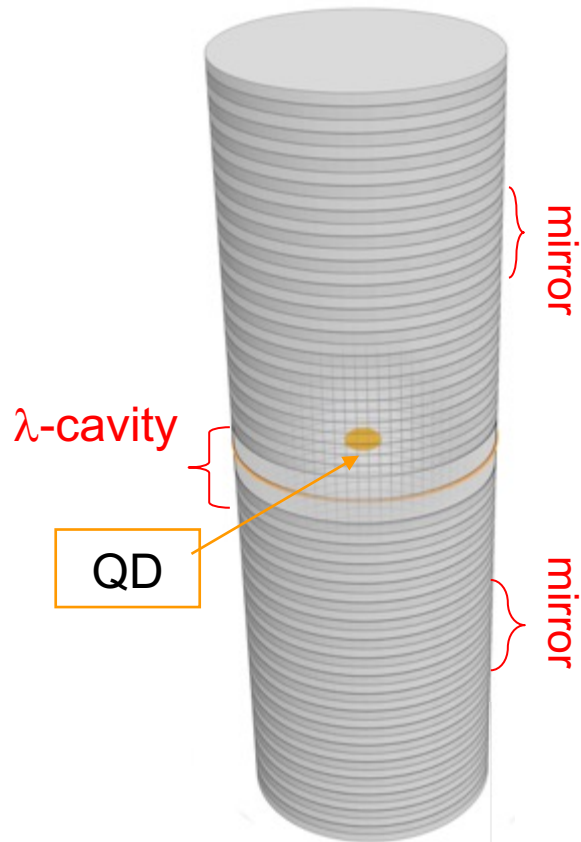
CQED with ions,
Atoms (2002-2004)



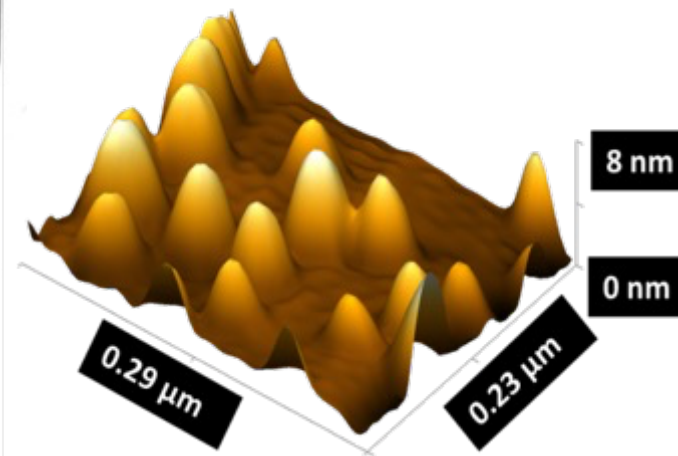
Quantum dots
(+ cavity)(2001)

Sources **triggered** by the excitation = « deterministic »

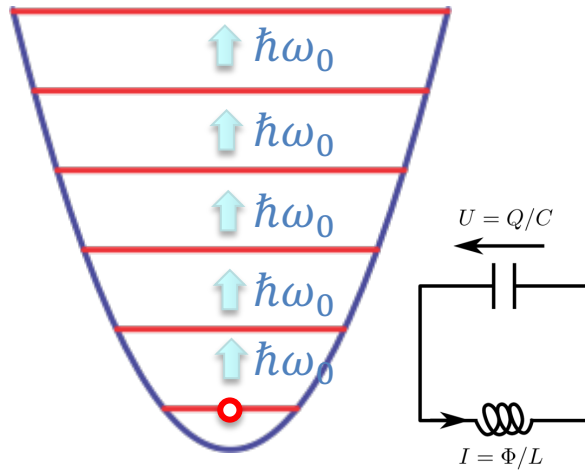
Commercial single-photon sources



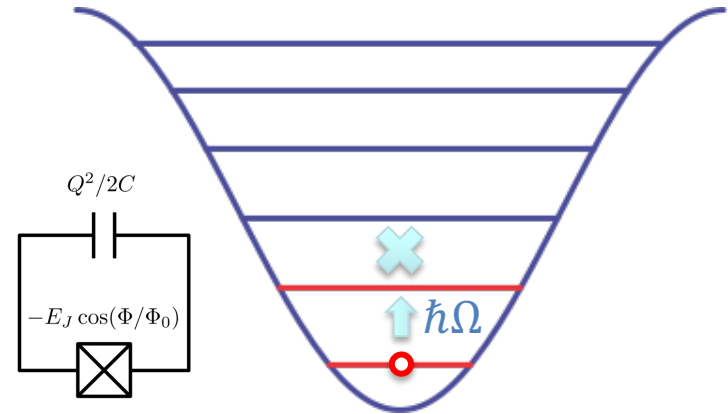
Flux: $80 \times 10^6 \text{ s}^{-1}$



Superconducting circuit and qubits

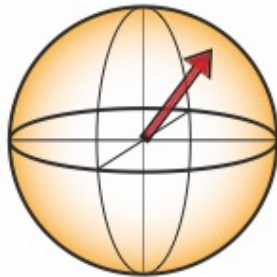


Oscillateur harmonique



Oscillateur anharmonique

Système à 2 niveaux
(spin)

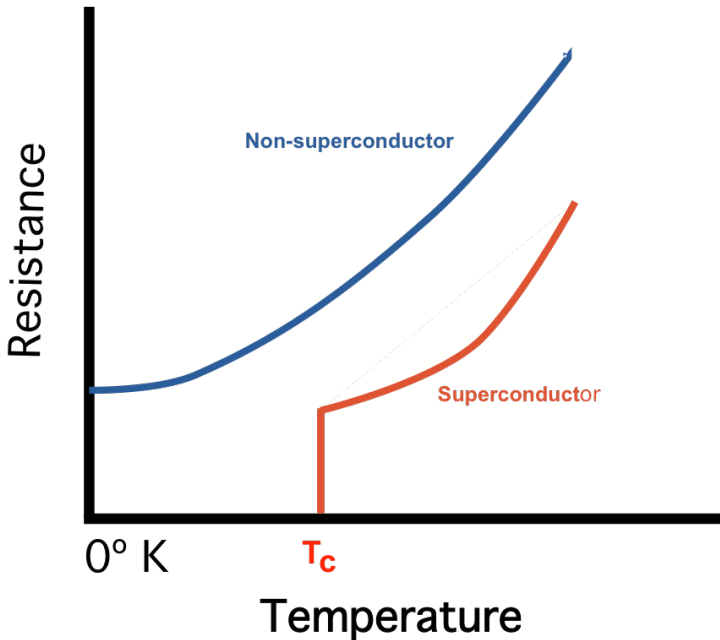


$$\hat{H} = \frac{\hbar\Omega}{2} (|1\rangle\langle 1| - |0\rangle\langle 0|) = -\frac{\hbar\Omega}{2} \hat{\sigma}_z$$

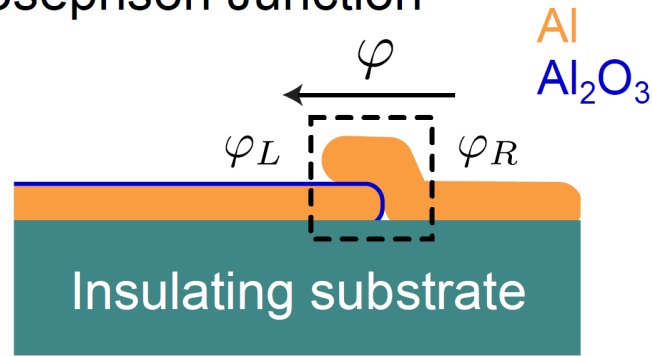
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Superconducting circuit and qubits

Superconductivity



Josephson Junction

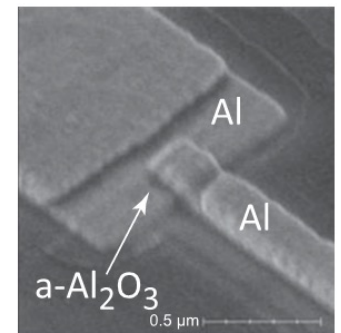
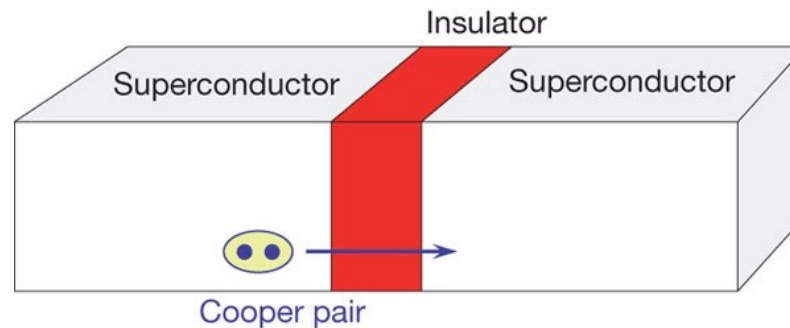
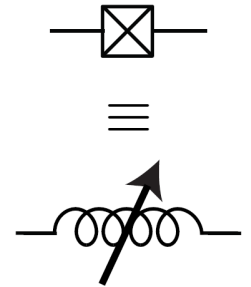


$$I = I_c \sin \varphi$$

$$V = \varphi_0 \times \dot{\varphi}$$



$$I = L_J(\varphi)\varphi$$



Superconducting circuit and qubits

IBM



Google

