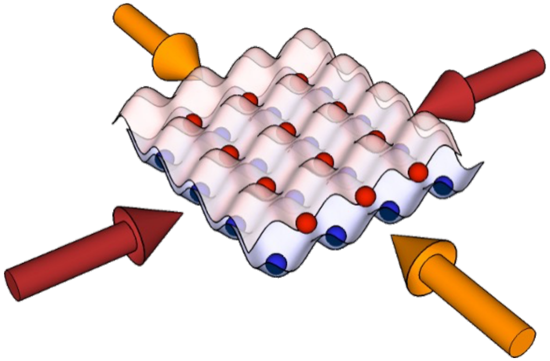
Physics of Quantum Information

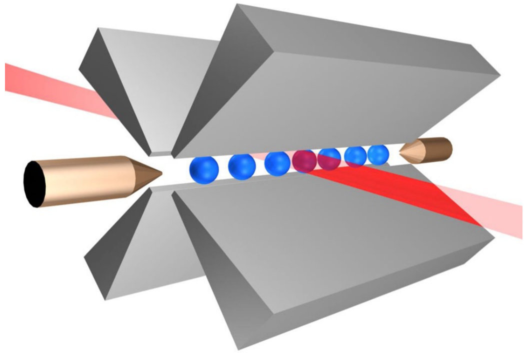
Lecture 2: two-level systems

and harmonic oscillators

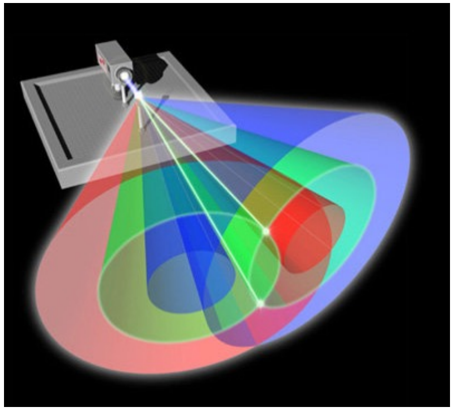
Quantum state engineering with *individual* systems



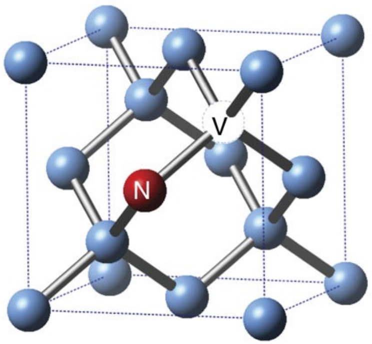
Cold atoms and molecules



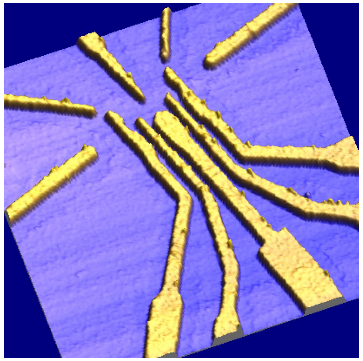
Trapped ions



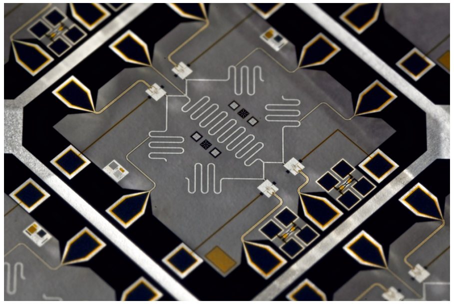
Photons in NL media



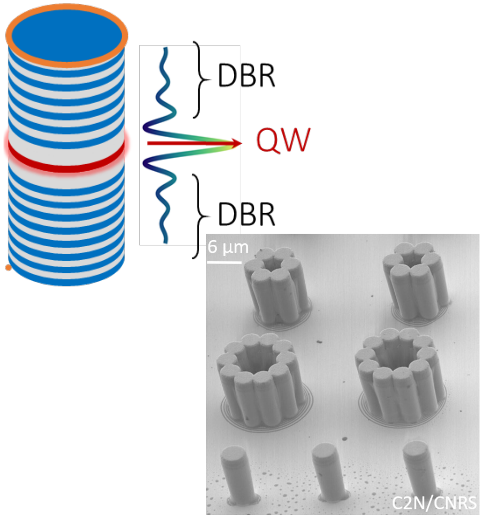
NV centers



Quantum dots

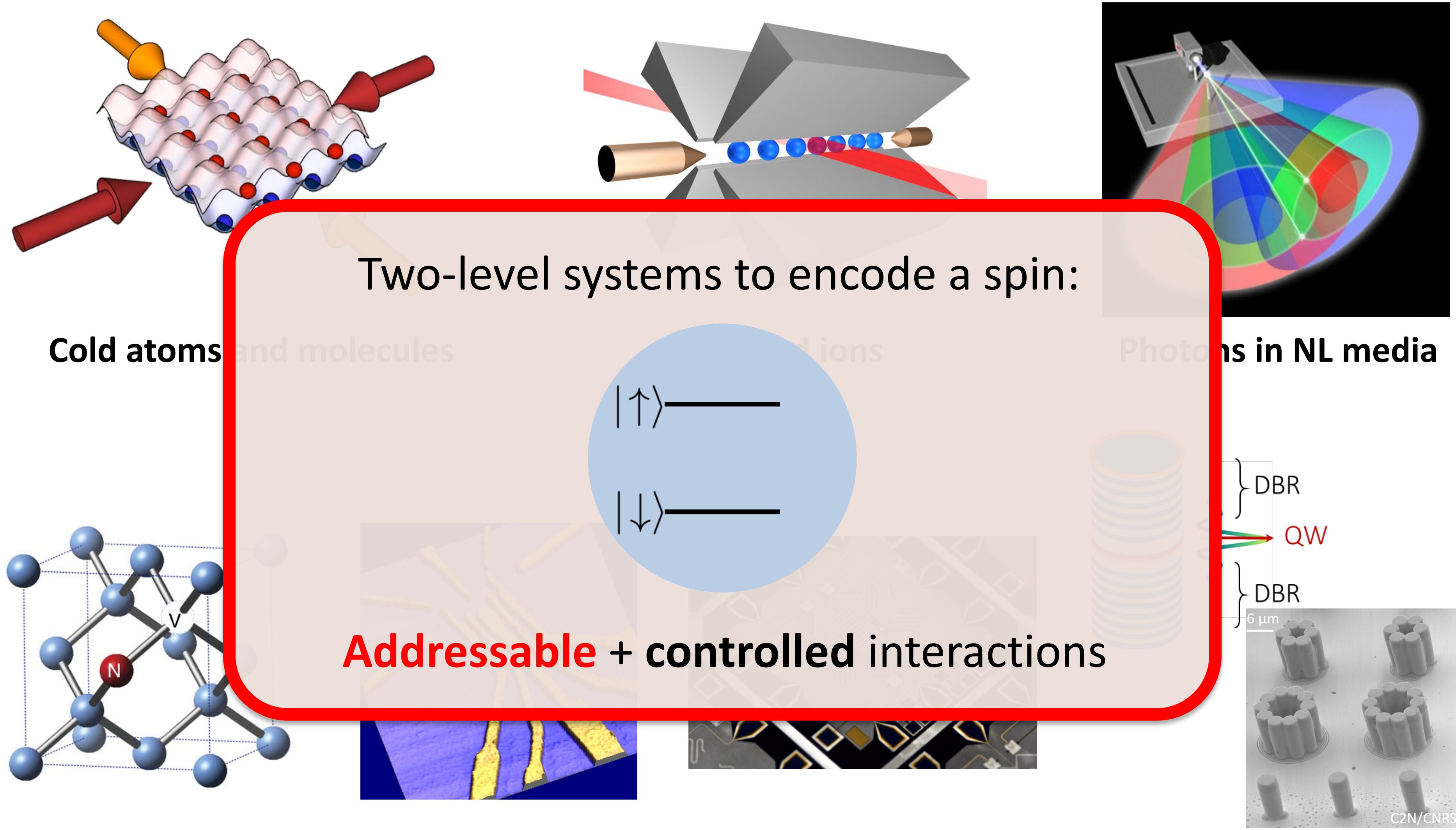


Superconducting qubits



Polaritons in ½ cond.

Quantum state engineering with *individual* systems



Two-level systems to encode a spin:

Addressable + **controlled** interactions

Cold atoms and molecules

ions

Photons in NL media

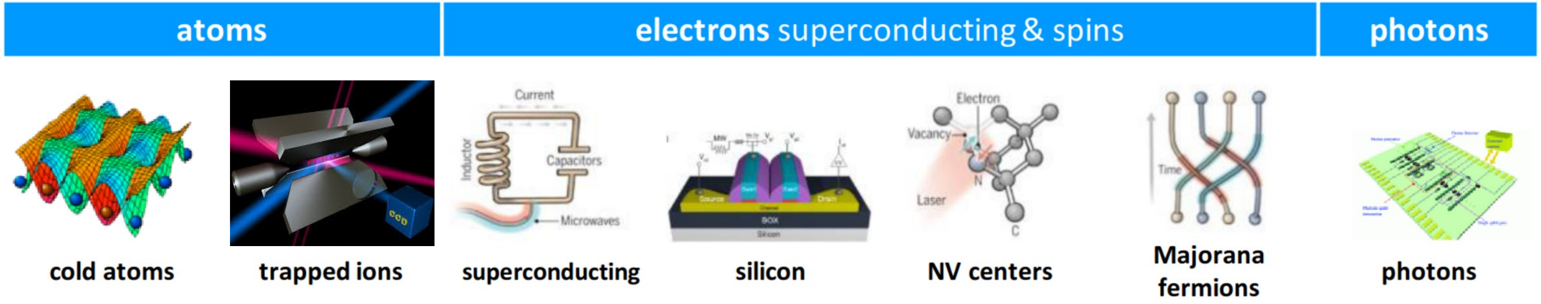
NV centers

Quantum dots

Superconducting qubits

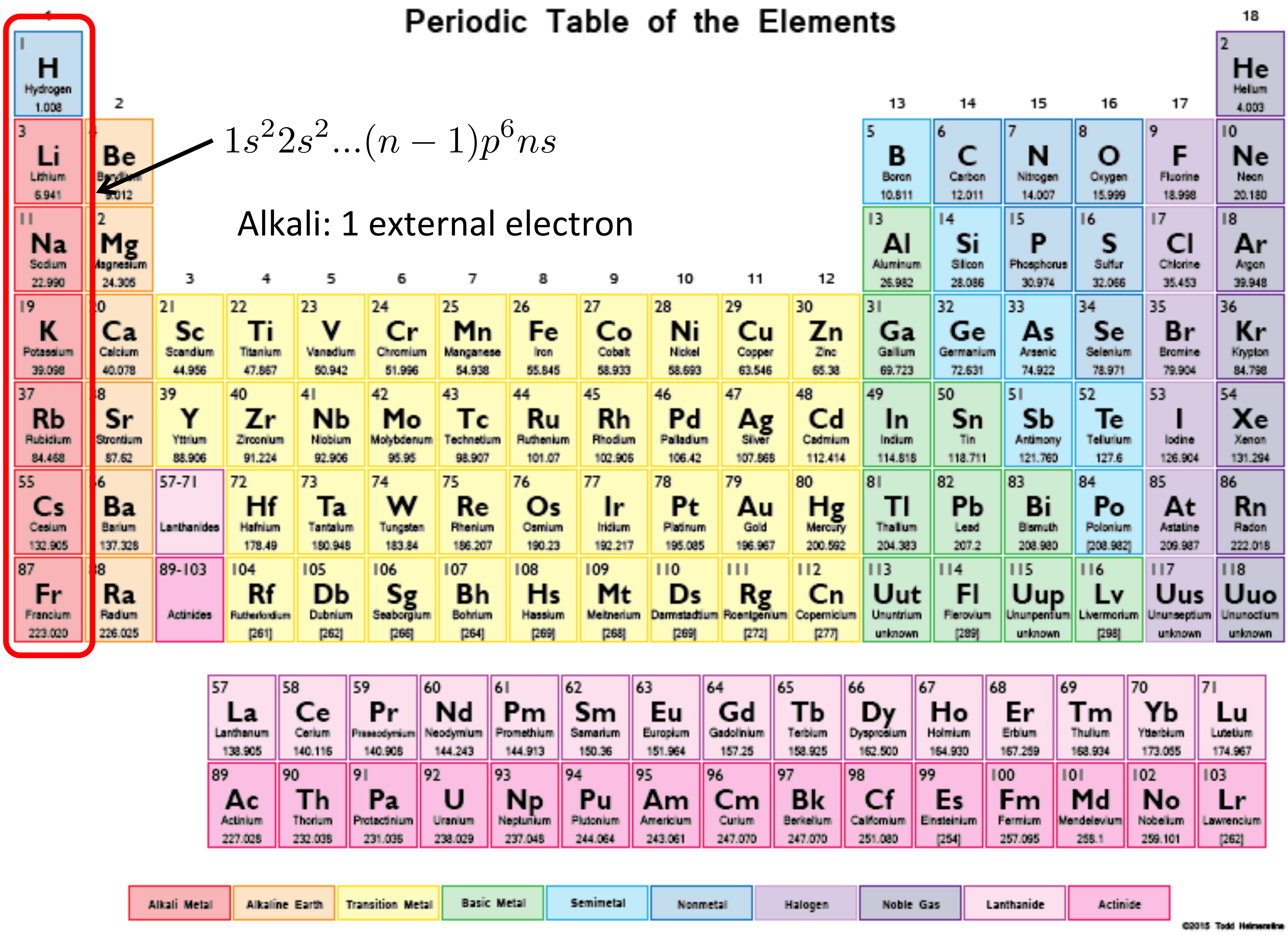
Polaritons in ½ cond.

Physical Qubits



DiVincenzo’s criteria (5+2)

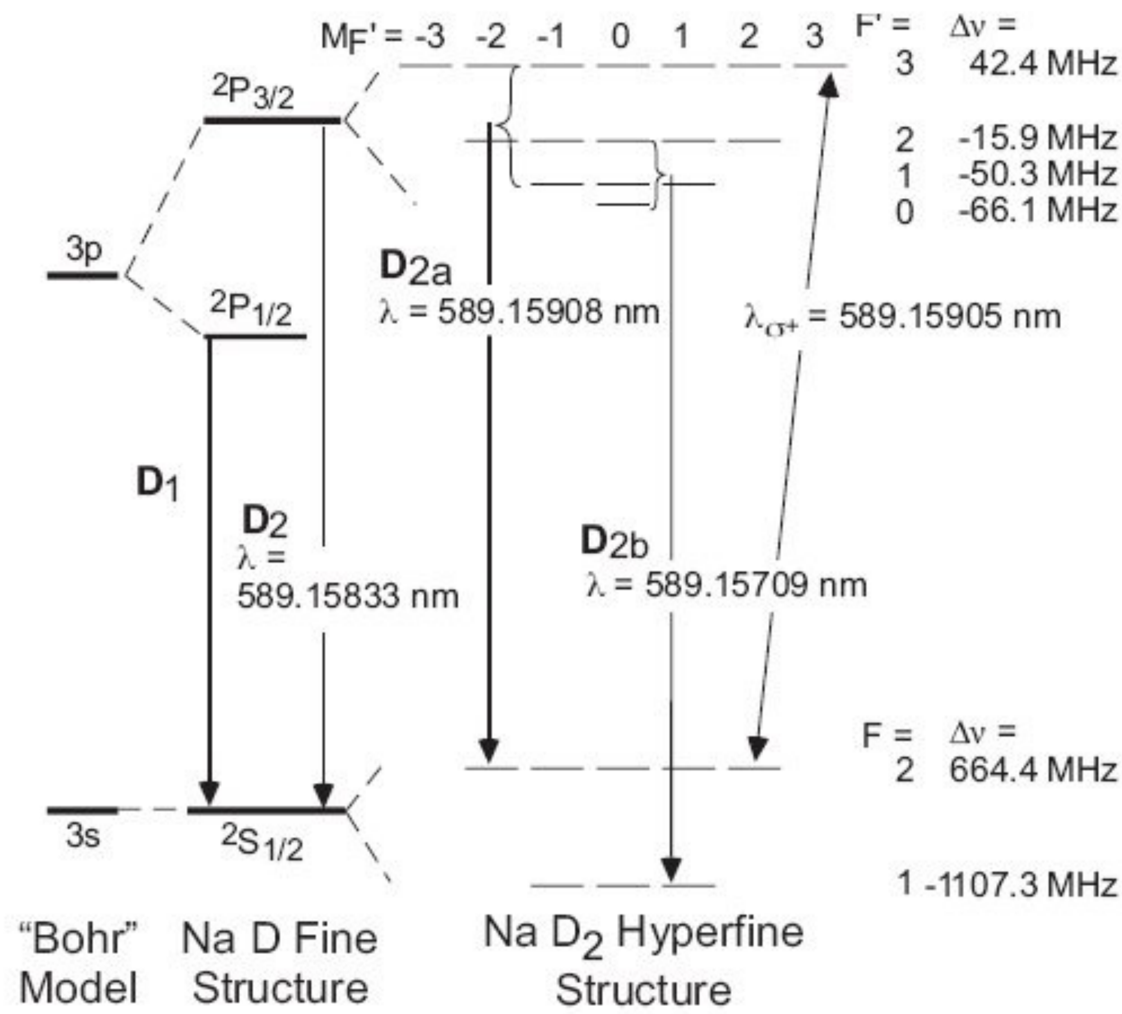
1. Well characterized qubits encapsulated in a scalable physical system
2. Ability to initialize the qubits in a trustable state
3. Ability to maintain long coherence times, much longer than the gate operation time
4. Ability to implement a "universal" set of quantum gates
5. Ability to perform useful measurements
6. Ability to interconnet stationary and flying qubits
7. Ability to faithfully transmit flying qubits between specified locations



$$1s^2 2s^2 \ldots (n-1)p^6 ns$$

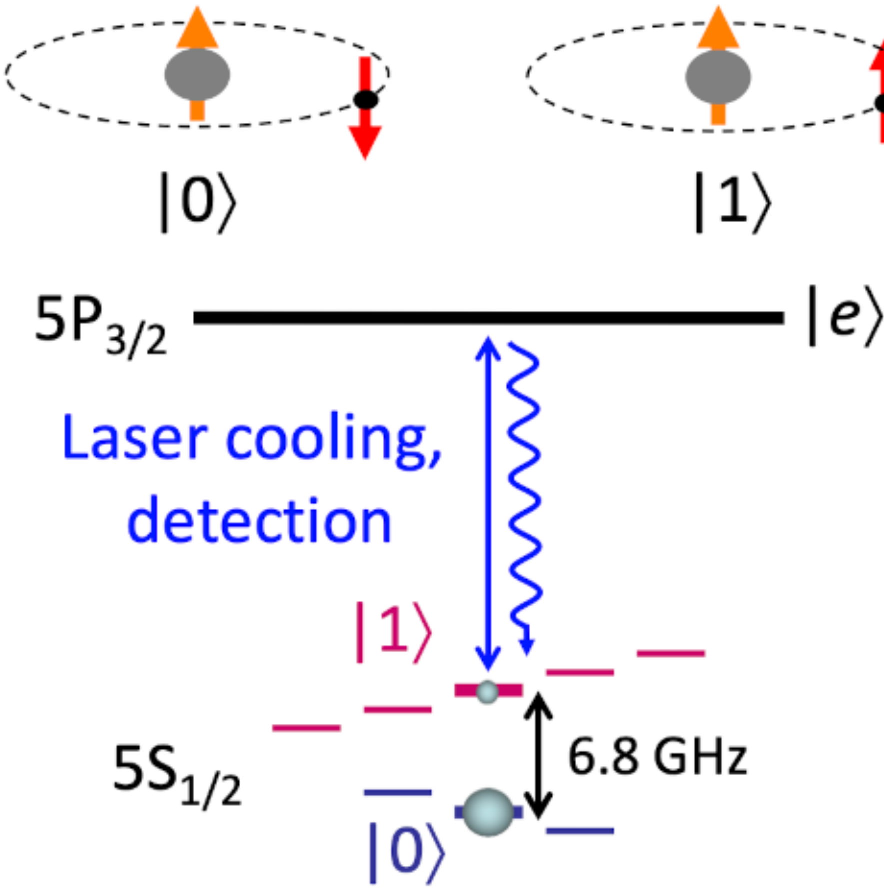
Alkali: 1 external electron

Atomic structure of sodium ($_{11}Na$)



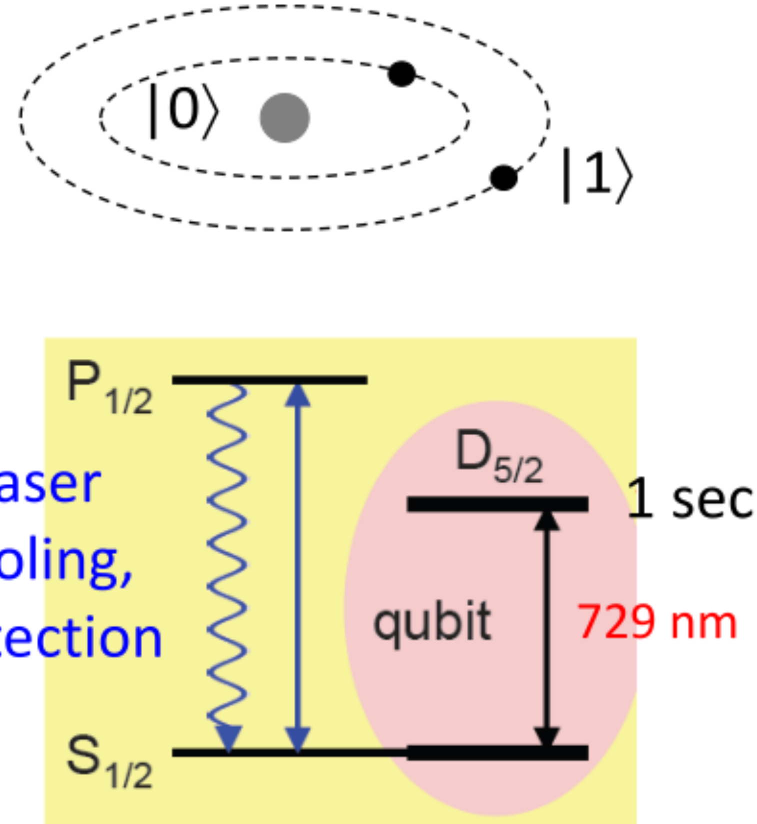
Atomic Qubits

Hyperfine qubit



Rb, Cs...
Be^{+} $^{43}Ca^{+}$ Yb^{+} Cd^{+} ...

Optical qubit



$^{40}Ca^{+}$

See pages 7-8 (detailed calculations in HW1)

Coherent manipulation and Rabi oscillations. As shown above, be the transition dipole electric or magnetic, the Hamiltonian describing the interaction of an atom with an electromagnetic field has the general form

$$\hat{H}(t) = -\frac{\hbar\omega_0}{2}\,\hat{\sigma}_z + \hbar\Omega\,\cos\omega t\,\hat{\sigma}_x = \hbar\begin{pmatrix} -\omega_0/2 & \Omega\,\cos\omega t \\ \Omega\,\cos\omega t & \omega_0/2 \end{pmatrix}_{|0\rangle,|1\rangle}, \tag{12}$$

with $\mp\hbar\omega_0/2$ the energies of the states $|0\rangle$ and $|1\rangle$, respectively.

1. Apply the transformation $|\tilde{\psi}(t)\rangle = \hat{\mathcal{R}}(t)\,|\psi(t)\rangle$ with

$$\mathcal{R}(t) = \exp\left(-i\frac{\omega t}{2}\hat{\sigma}_z\right) = \begin{pmatrix} e^{-i\frac{\omega t}{2}} & 0 \\ 0 & e^{i\frac{\omega t}{2}} \end{pmatrix}_{|0\rangle,|1\rangle}. \tag{13}$$

This transformation amounts to applying a rotation around the Oz axis of angle $\omega t/2$, and hence "move to the frame rotating at frequency ω".

2. The Hamiltonian governing the evolution of the new state $|\tilde{\psi}(t)\rangle$ is found by writing the Schrödinger equation and applying the transformation. It yields the new Schrödinger equation

$$i\hbar\frac{d}{dt}|\tilde{\psi}(t)\rangle = \tilde{H}(t)|\tilde{\psi}(t)\rangle \quad \text{with} \quad \tilde{H} = \mathcal{R}H\mathcal{R}^{-1} + i\hbar\frac{d\mathcal{R}}{dt}\mathcal{R}^{-1}\,. \tag{14}$$

Applying the transformation and using $i\hbar\frac{d\mathcal{R}}{dt} = (\hbar\omega/2)\hat{\sigma}_z\mathcal{R}$, we find

$$\tilde{H} = \frac{\hbar}{2}\begin{pmatrix} \delta & \Omega\,(1+\cancel{e^{-2i\omega t}}) \\ \Omega\,(1+\cancel{e^{+2i\omega t}}) & -\delta \end{pmatrix}_{|0\rangle,|1\rangle}\,, \tag{15}$$

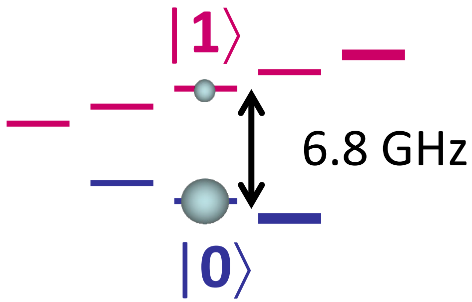
with $\delta = \omega - \omega_0$ the laser detuning wit respect to the atomic transition. Hamiltonian is still time-dependent.

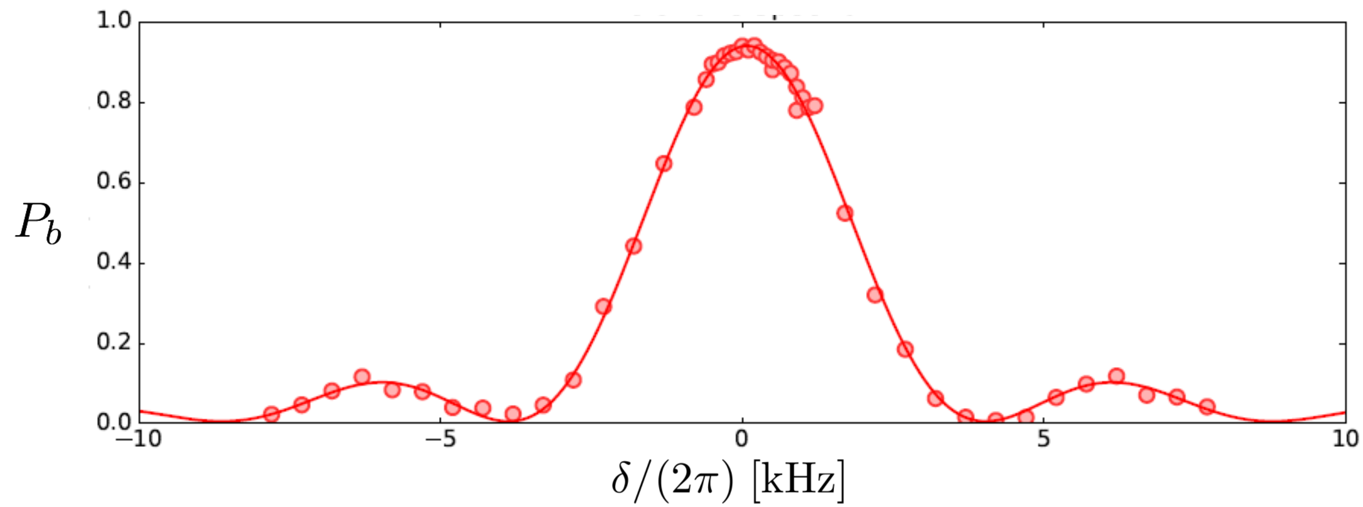
At the end of the procedure, and under the quasi-resonant approximation, we are then left with the effective time-independent Hamiltonian

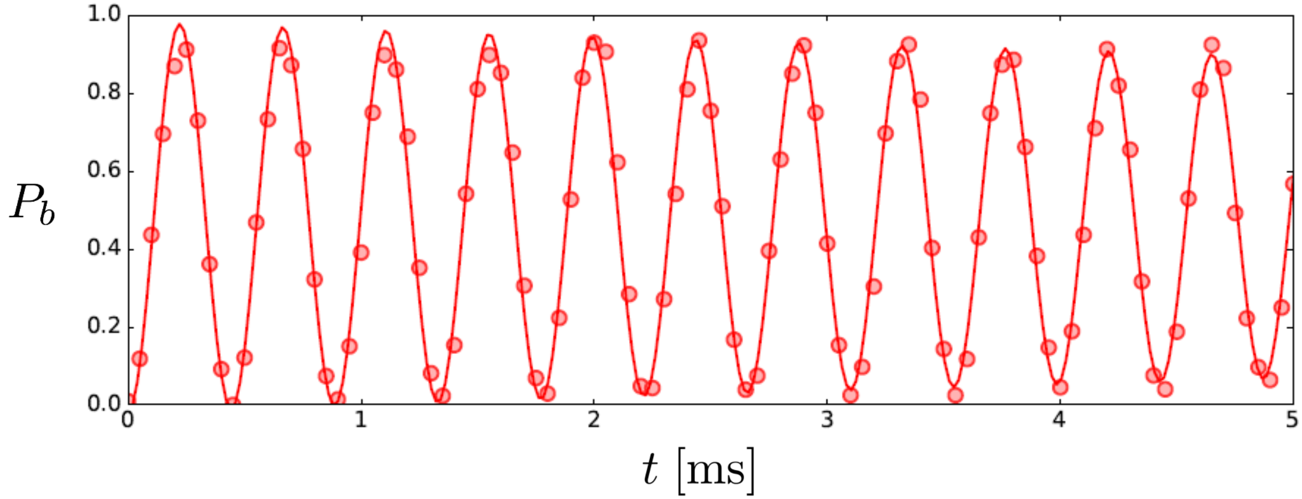
$$\boxed{\tilde{H} = \frac{\hbar}{2}\begin{pmatrix} \delta & \Omega \\ \Omega & -\delta \end{pmatrix}_{|0\rangle,|1\rangle} = \frac{\hbar\delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\hat{\sigma}_x\,.} \tag{20}$$

Single atom Rabi oscillations

Each point = 200 repetitions



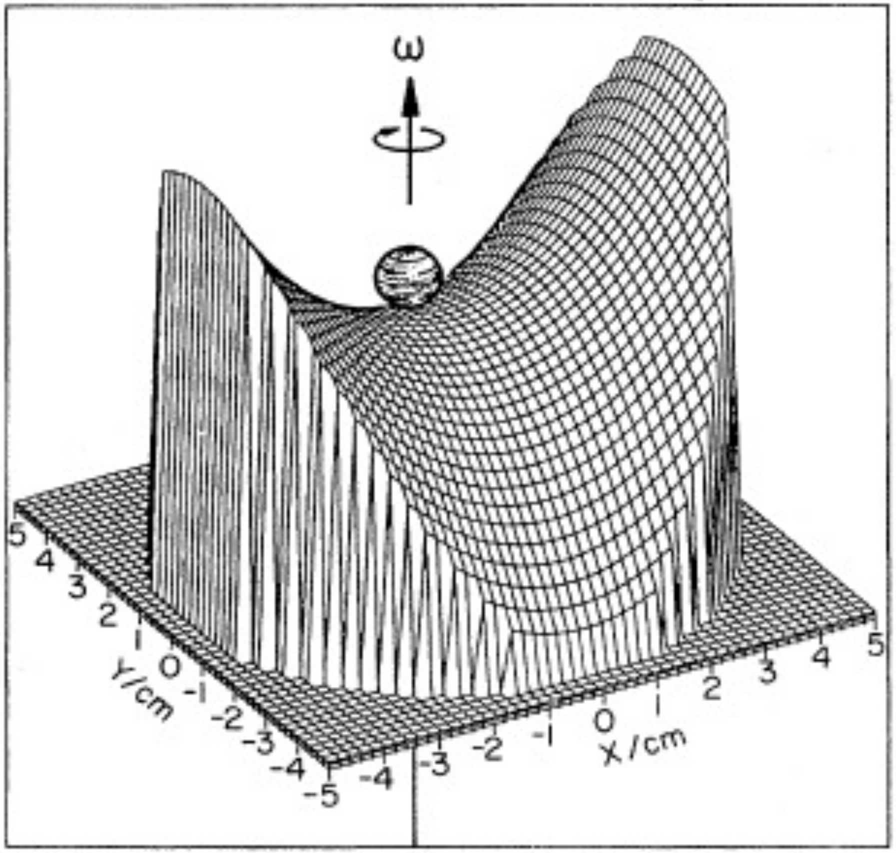




Paul trap for ions

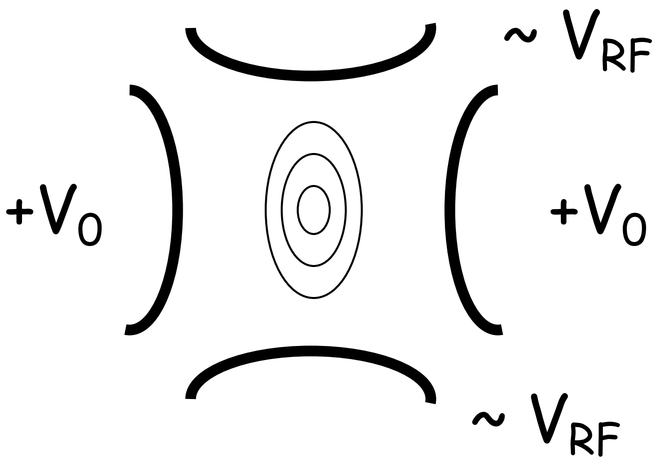
1989 Development of the ion trap technics

W. Paul

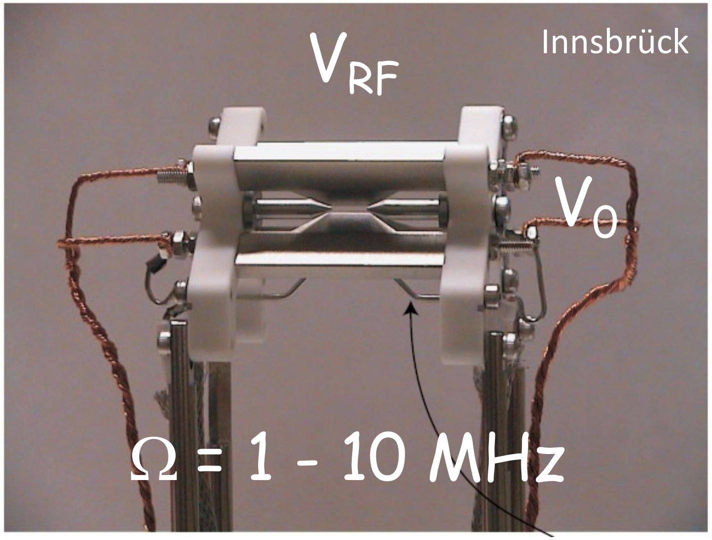


Ion crystals

Paul trap (RF + static)

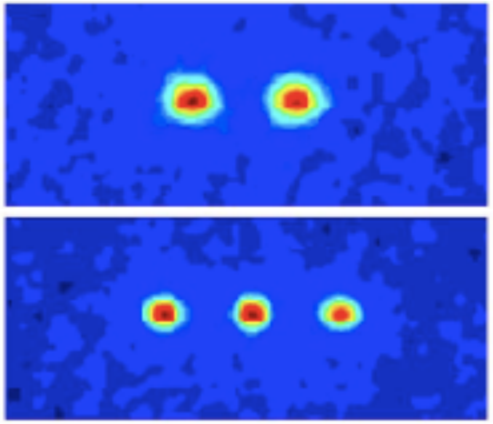


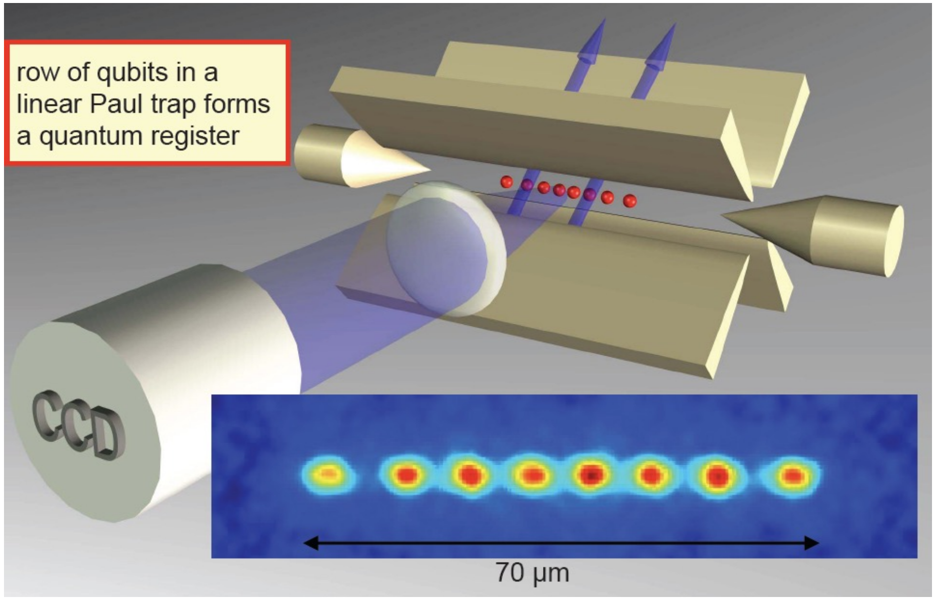
Linear Paul trap



Coulomb cristal

$\Rightarrow$ Ionic crystal



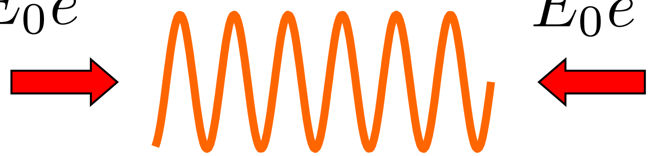


Innsbrück

Ultra-cold atoms in optical lattices

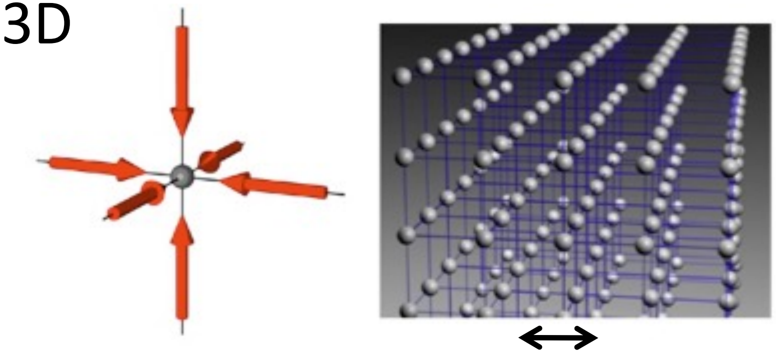
Dipole force: $\mathbf{F} \propto -\nabla I(\mathbf{r})$

$E_0 e^{ikx}$ $E_0 e^{-ikx}$



$$I(x) = 2E_0^2(1 + \cos 2kx)$$

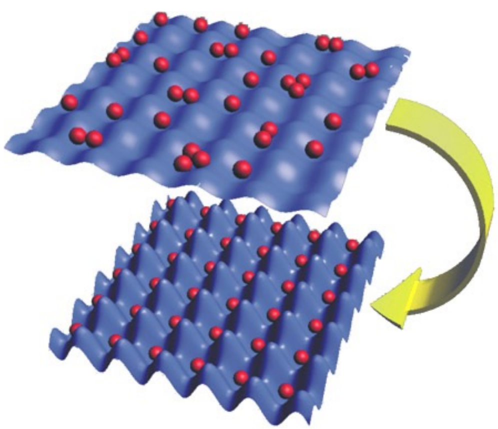
3D



$\lambda/2$ = 0.5 μm

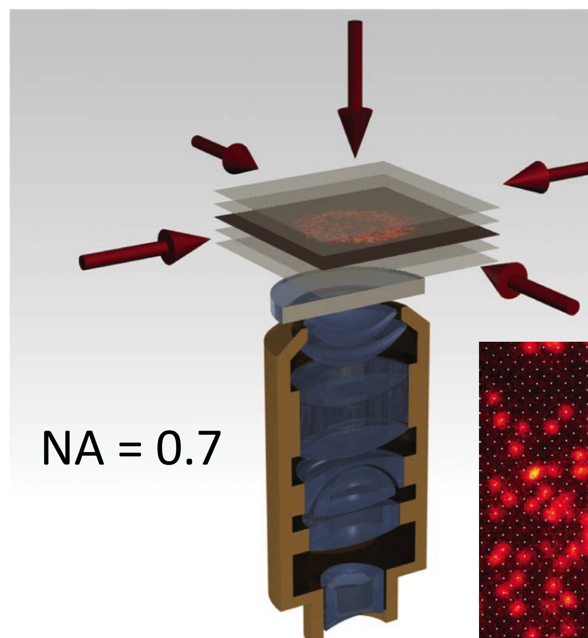
(M. Greiner thesis)

Each site contains 1 atom!



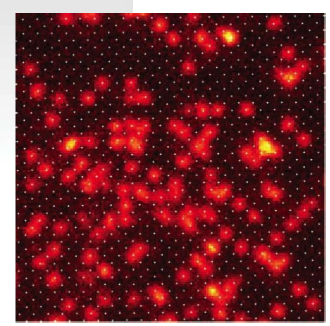
Boson (Rb, Na, ^{7}Li, ^{39}K, ^{4}He*),
Fermion (^{6}Li, ^{40}K),
Magnetic atoms (Cr, Dy…)

Quantum gas microscope



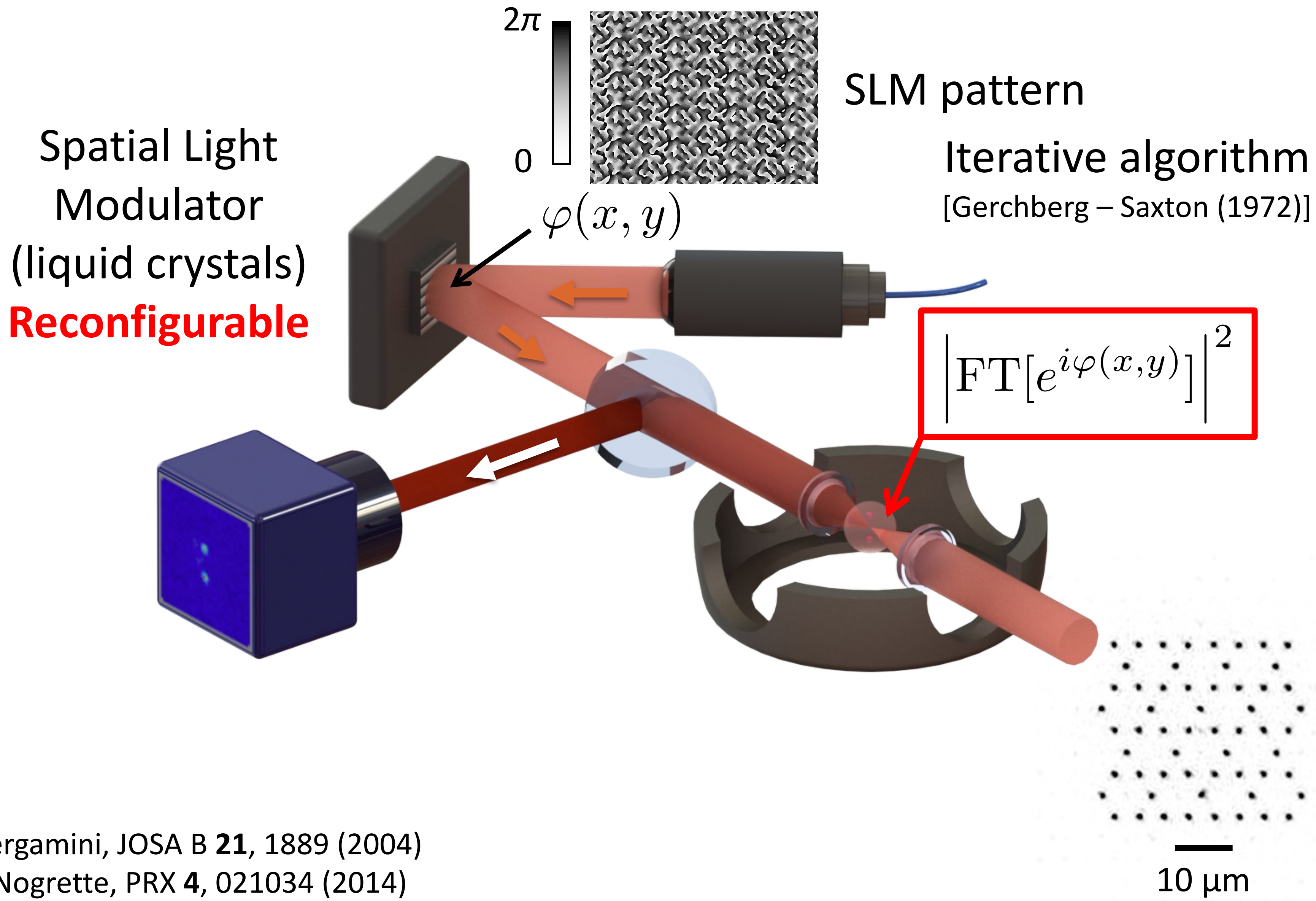
Havard, MPQ

Single-site resolution (< 1 μm)



16 μm

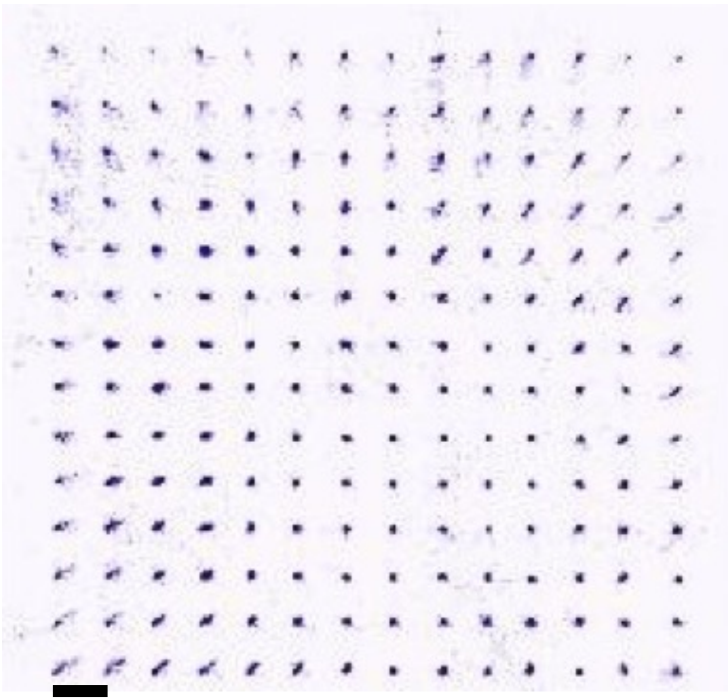
Holographic 2D arrays of tweezers



Bergamini, JOSA B **21**, 1889 (2004)
Nogrette, PRX **4**, 021034 (2014)

Arrays of atoms

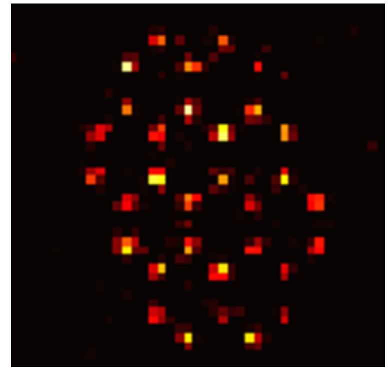
Individual atoms in assembled arrays of tweezers (~200 at.)

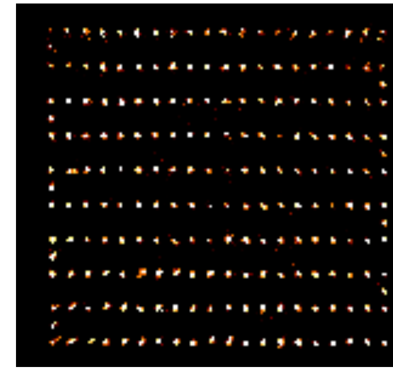


5 μm

Fluorescence: single shot!!

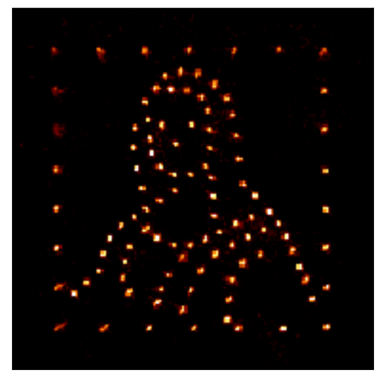
~3-10 μm

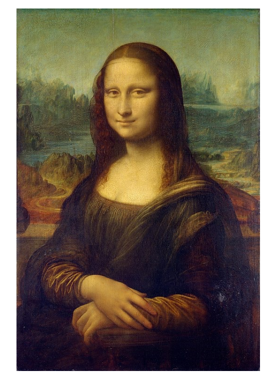




~100 μm

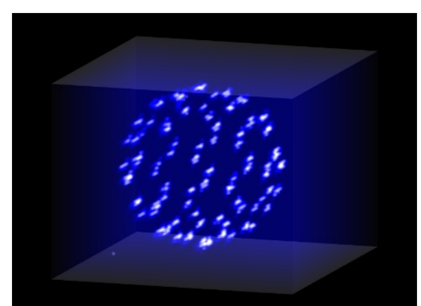
Schymik, PRA 2021

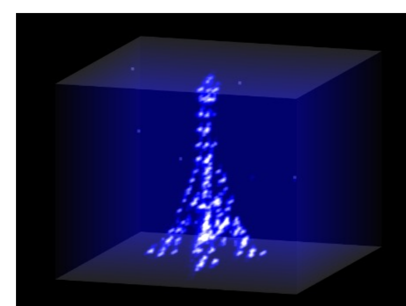




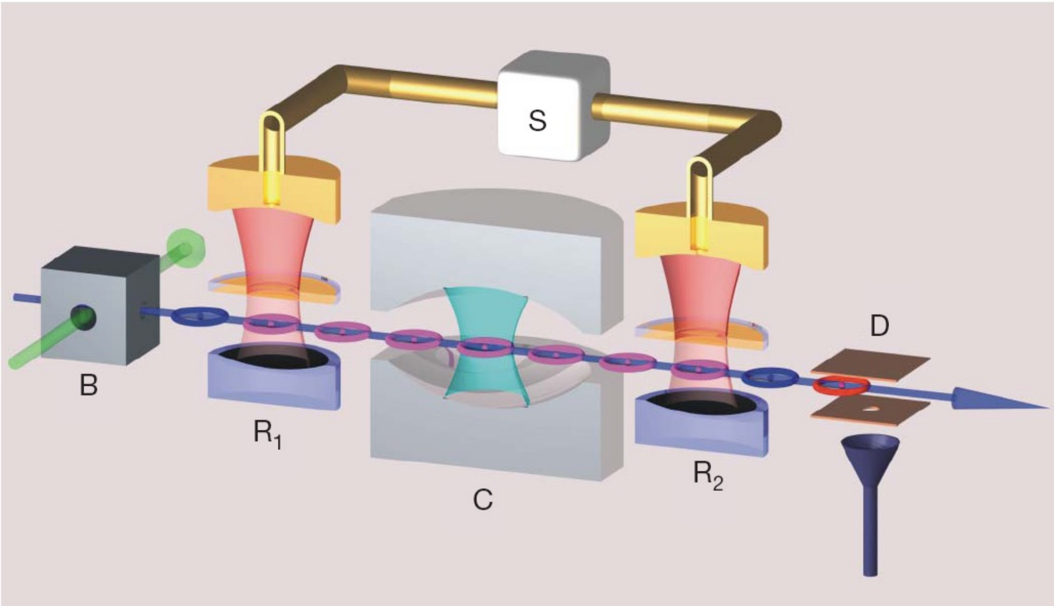
L. da Vinci

(Averaged)



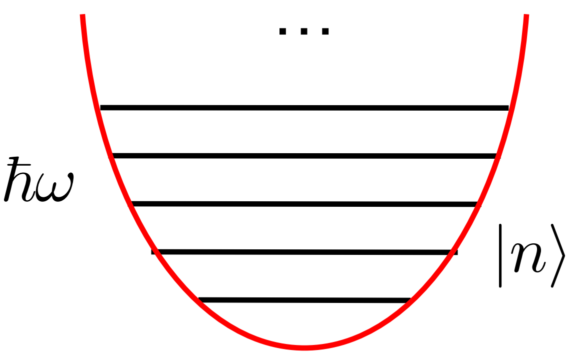


Photons in a cavity

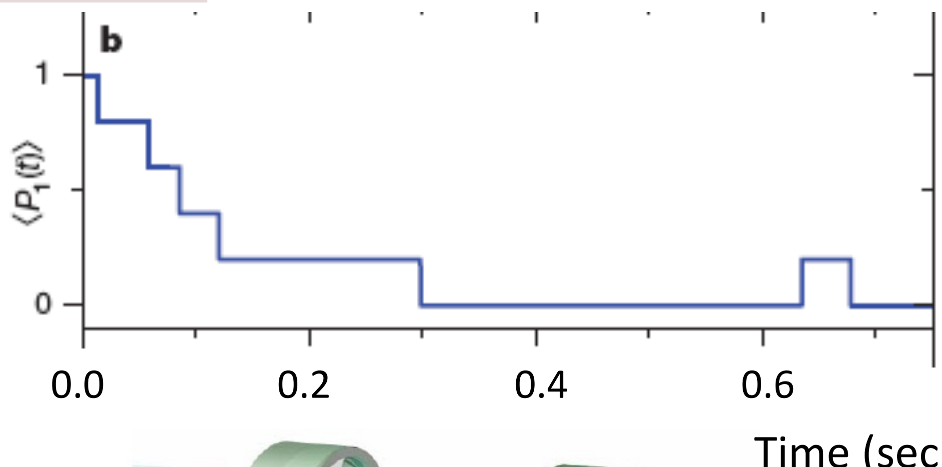


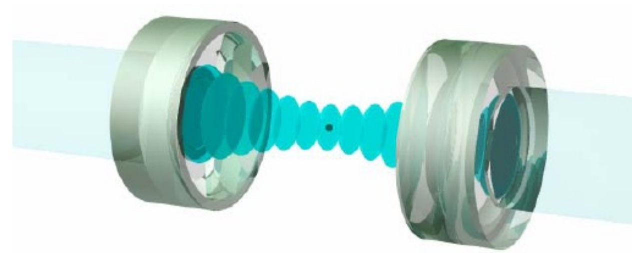
S. Haroche
2012

$$\hat{E} = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\,(\hat{a} + a^{+})$$



Cavity field, ω





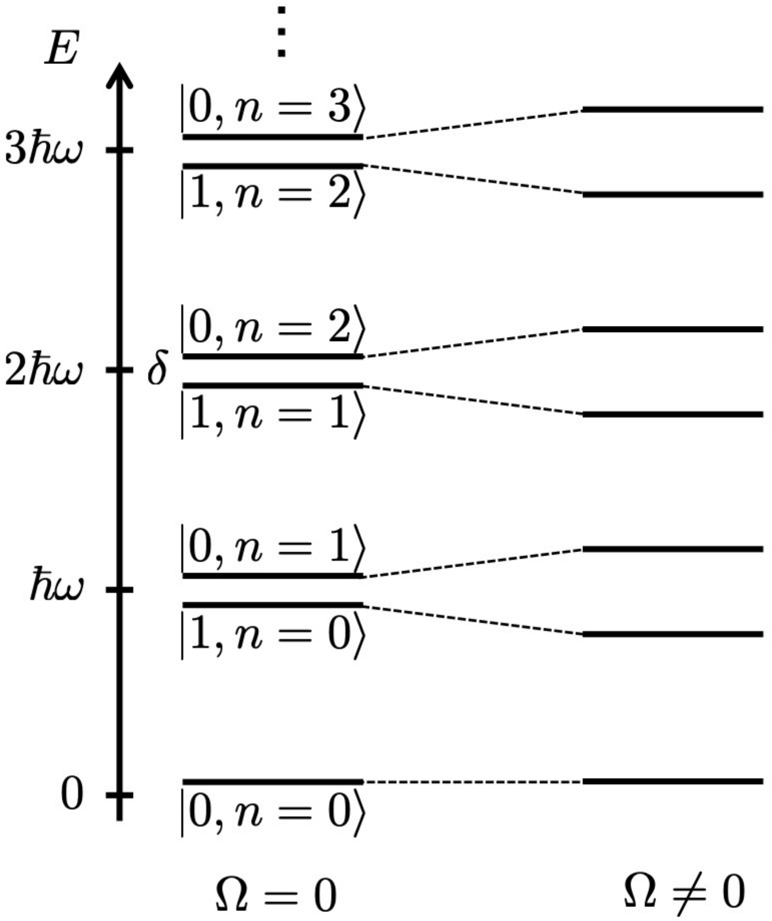
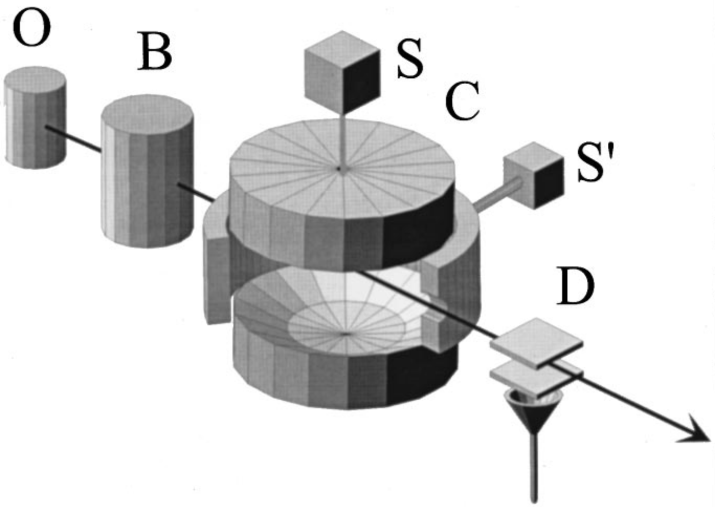
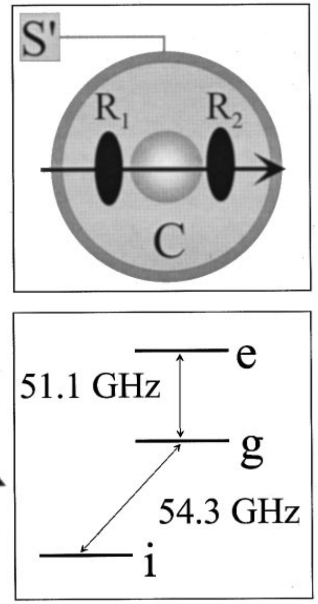


Figure 3: Spectrum of the Jaynes-Cummings Hamiltonian for $\delta > 0$.

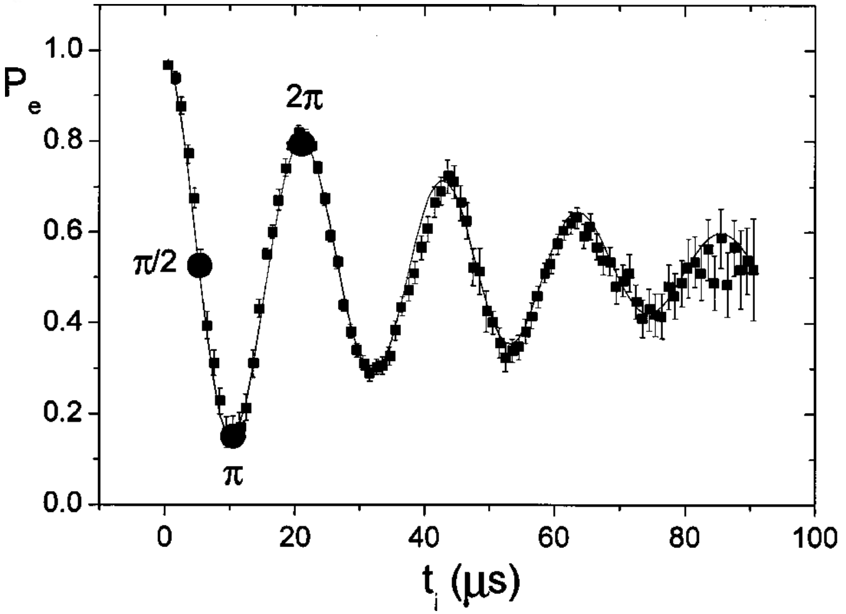
“Single-photon” Rabi oscillations





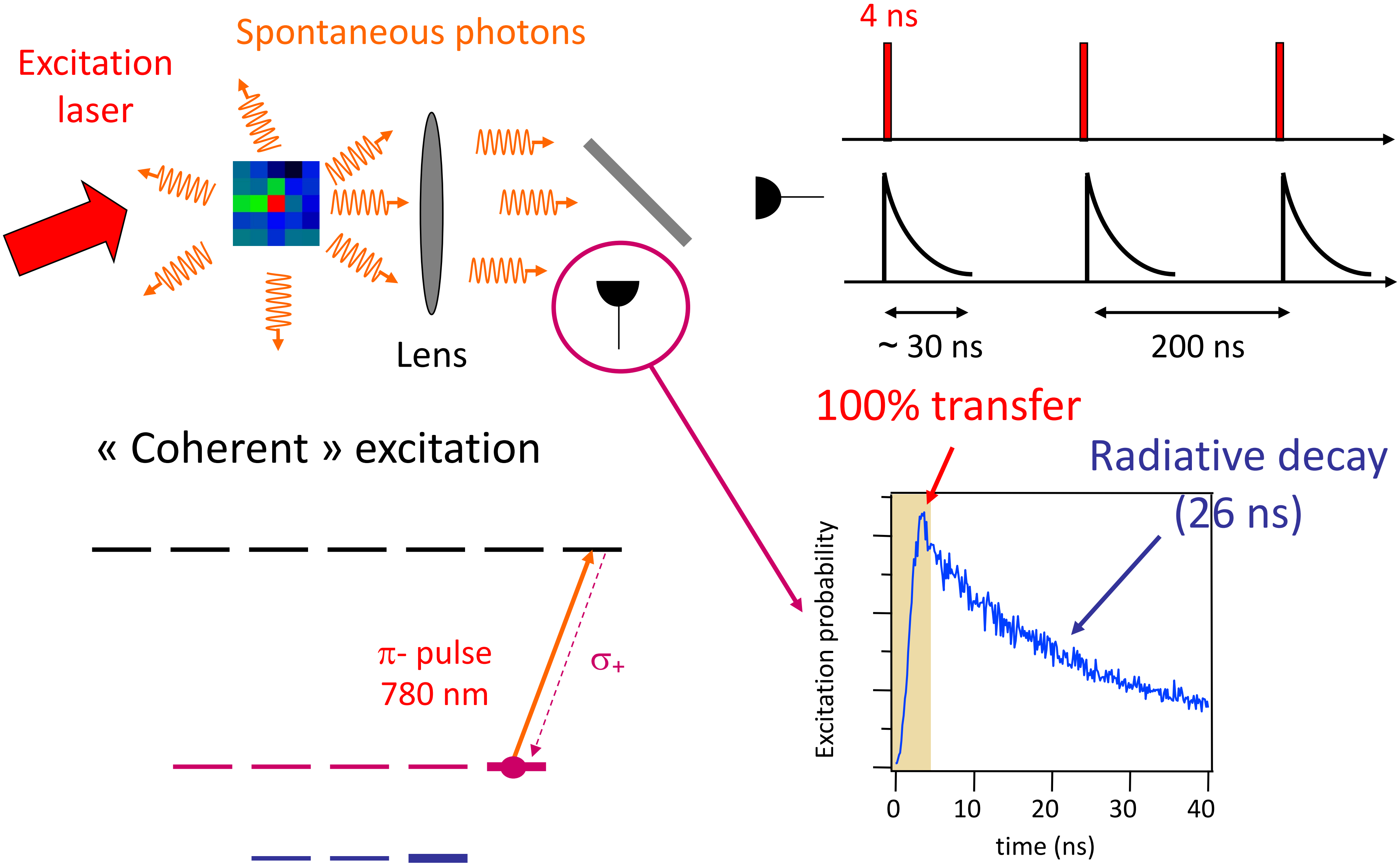
Nobel Prize 2012

S. Haroche, D.J. Wineland



M. Brune, PRL **76**, 1800 (1996)

Triggered emission of single photons by an atom

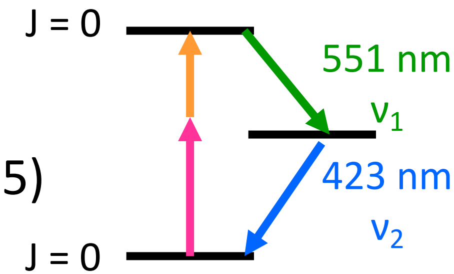


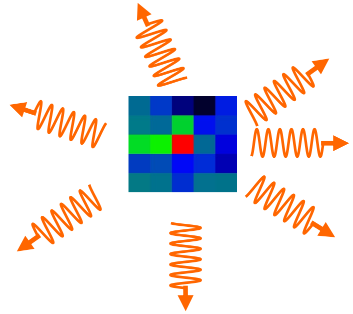
Darquié *et al*, Science **309**, 454 (2005)

« Microscopic » single photon sources

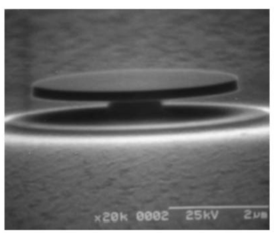
Based on photon emission by a **single** microscopic dipole

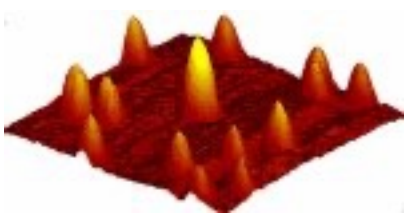
Grangier, Aspect (1985)



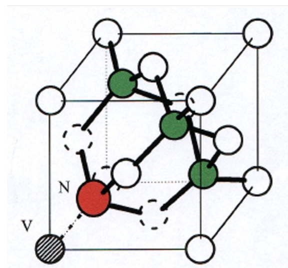


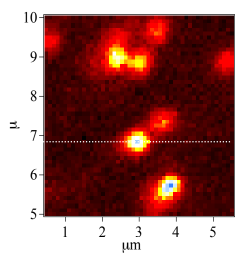
Single trapped atom (2005)



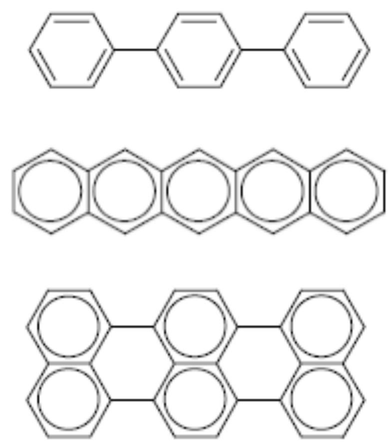


Quantum dots (+ cavity)(2001)

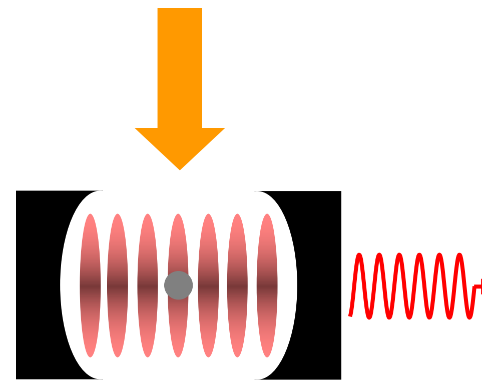




NV center (2000)



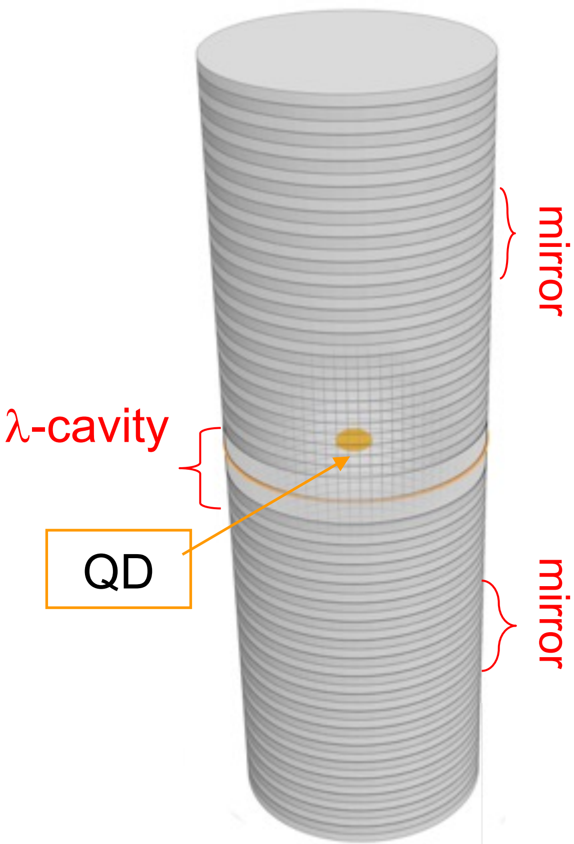
Molecules in solid (1999-2000)

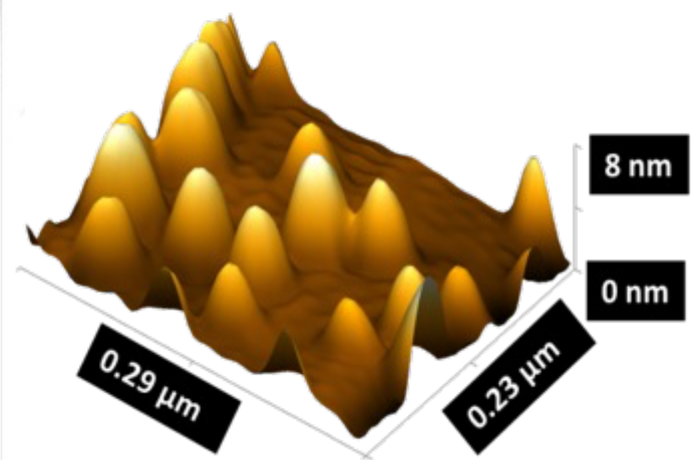


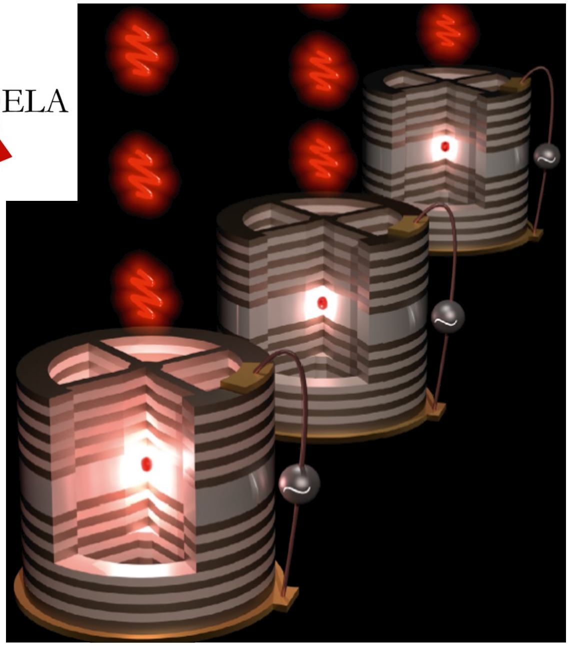
CQED with ions, Atoms (2002-2004)

Sources triggered by the excitation = « deterministic »

Commercial single-photon sources

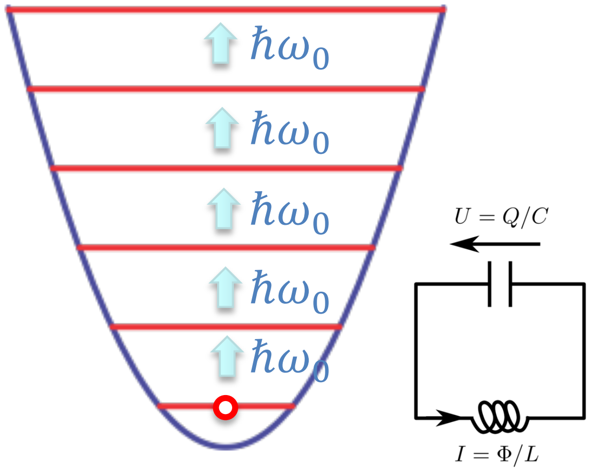




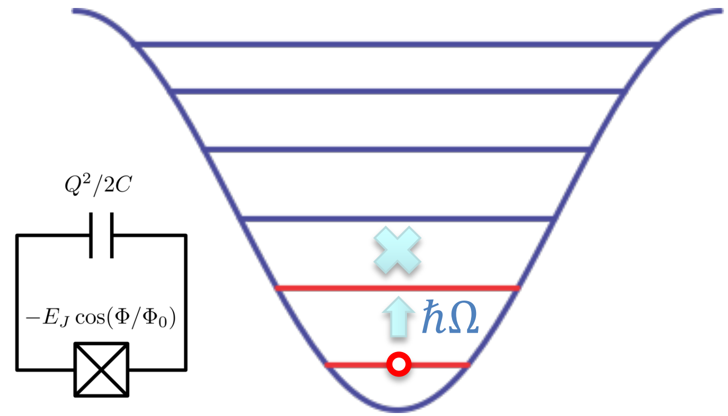


Flux: 80×10^6 s^{-1}

Superconducting circuit and qubits

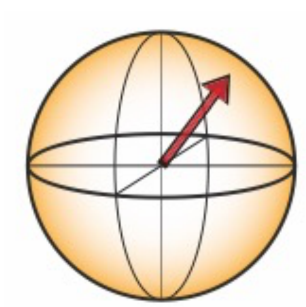


Oscillateur harmonique



Oscillateur anharmonique

Système à 2 niveaux
(spin)

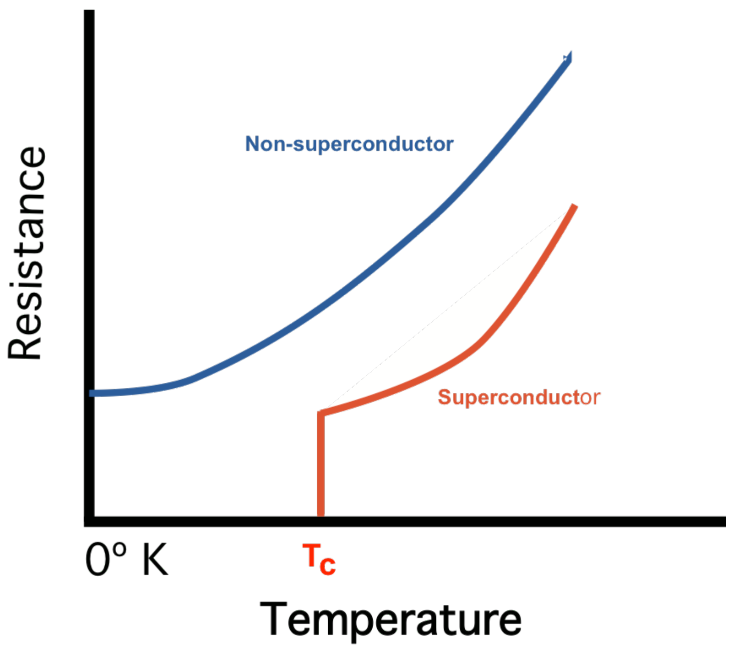


$$\hat{H} = \frac{\hbar\Omega}{2}(|1\rangle\langle 1| - |0\rangle\langle 0|) = -\frac{\hbar\Omega}{2}\hat{\sigma}_z$$

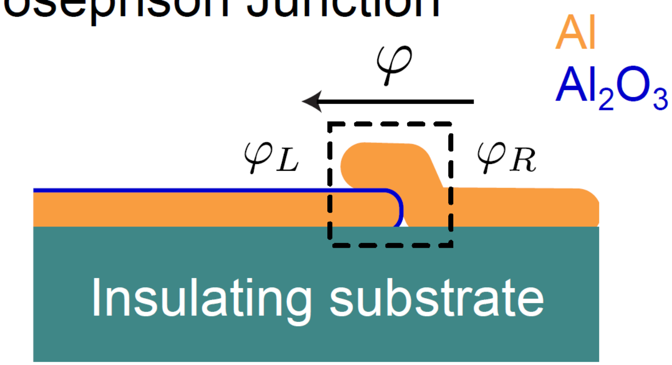
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

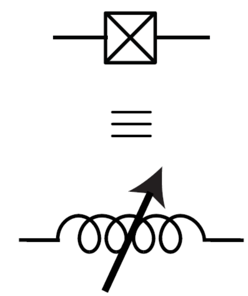
Superconducting circuit and qubits

Superconductivity



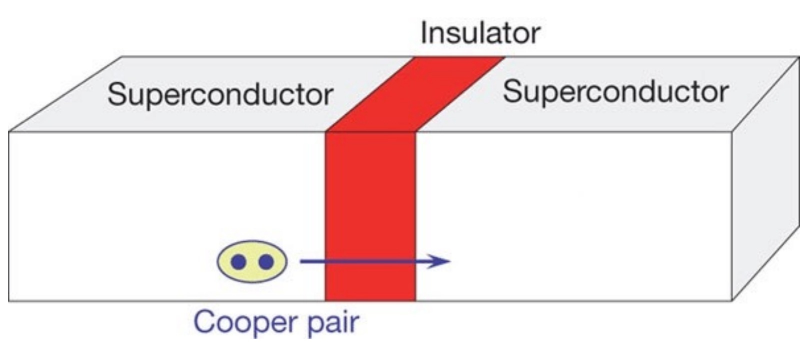
Josephson Junction

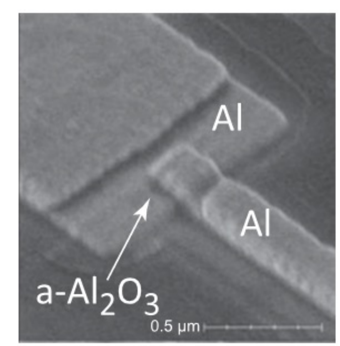




$$I = I_c \sin\varphi \quad \longrightarrow \quad I = L_J(\varphi)\varphi$$

$$V = \varphi_0 \times \dot{\varphi}$$





Superconducting circuit and qubits

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