Density modulations in an elongated Bose-Einstein condensate released from a disordered potential

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We observe large density modulations in time-of-flight images of elongated Bose-Einstein condensates, initially confined in a harmonic trap and in the presence of weak disorder. The development of these modulations during the time of flight and their dependence with the disorder are investigated. We render an account of this effect using numerical and analytical calculations. We conclude that the observed large density modulations originate from the weak initial density modulations induced by the disorder and not from initial phase fluctuations (thermal or quantum).

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Gaseous Bose-Einstein condensates (BECs) in disordered potentials [1–5] offer controllable systems to study open basic questions on the effects of disorder in quantum media [6]. In this respect, a still debated question relies on the nature of the disorder-induced superfluid-insulator transitions [7] which can originate from strong fluctuations of either the density or the phase. This question can be addressed experimentally with gaseous BECs in optical disorder [1–5] as both density and phase can be measured directly [8]. In addition to its fundamental interest, this study is important for BECs on chips [9] and can also shed some light on the physics of dirty superconductors [10] and granular metals [11].

In the presence of repulsive interactions, Anderson localization is suppressed in a stationary BEC and weak disorder results in small density modulations [12]. However, one may wonder whether weak disorder *may* significantly affect the coherence of a connected BEC and entail large phase fluctuations. It has been suggested [1,13], by analogy with elongated (nondisordered) quasi-BECs, that the observation of large fringes in *time-of-flight* (TOF) images of disordered BECs (see Fig. 1 and [1,14,15]) may signal strong initial phase fluctuations [16–18]. For quasi-BECs, such fringes in TOF images are *indeed* a signature of initial phase fluctuations [17]. However, for disordered BECs, no systematic study of these fringes has been reported so far and their relation to disorder-induced phase fluctuations is still unclear.

In this paper, we report a detailed study of the density modulations in the TOF images of an elongated, nonfragmented three-dimensional (3D) BEC initially placed in a weak 1D disordered potential. Our main experimental result is that the fringes in the TOF images are reproduced from shot to shot when using the *same* realization of the disorder. This excludes disorder-induced phase fluctuations (thermal or quantum) in the trapped BEC as the origin of the fringes observed after TOF for the parameter range of the experiment (relevant also for [1,14,15]). Using analytical and numerical calculations, which do not include initial phase fluctuations, we show that the fringes actually develop during the TOF according to the following scenario. Just after release, the initial weak density modulations (induced by the disorder onto the trapped BEC) imprint a phase with axial modulations and transversal invariance. Then, the resulting axial phase modulations are converted into large axial density modulations.

The experiment is detailed in Refs. [2,5]. We form a cigar-shaped BEC of ^{87}Rb atoms in an Ioffe-Pritchard trap of frequencies $\omega_z/2\pi=6.7$ Hz and $\omega_\perp/2\pi=660$ Hz. The BEC atom number is $N_0\sim 3\times 10^5$, the length $2L_{\mathrm{TF}}\simeq 300$ μm, and the chemical potential $\mu/2\pi\hbar\simeq 4.5$ kHz. We create a 1D speckle (disordered) potential along the z axis. The correlation function is $C(z)=V_{\mathrm{R}}^2\,\mathrm{sinc}^2(z/\sigma_{\mathrm{R}})$ where both amplitude V_{R} and correlation length σ_{R} down to 0.33 μm can be controlled [2,5]. The results presented in this work correspond to $\sigma_{\mathrm{R}}\simeq 1.7$ μm [19]. In the experiment, we wait 300 ms for the BEC to reach equilibrium in the presence of disorder and then switch off abruptly both magnetic and speckle potentials. We then take absorption images of the expanding cloud after a variable time of flight t_{TOF} with typical images shown in Fig. 1.

These images show large density modulations along the axis z of the disorder. To measure their amplitude, we first extract a 1D axial density $n_{\mathrm{1D}}(z)$ by integrating the column density over the second transverse direction x. We then define $\eta(z)$ as the normalized deviations of the 1D density from the 1D parabolic Thomas-Fermi (TF) profile $n_{\mathrm{1D}}^0(z)$ which fits best the data (red line in Fig. 1), so that

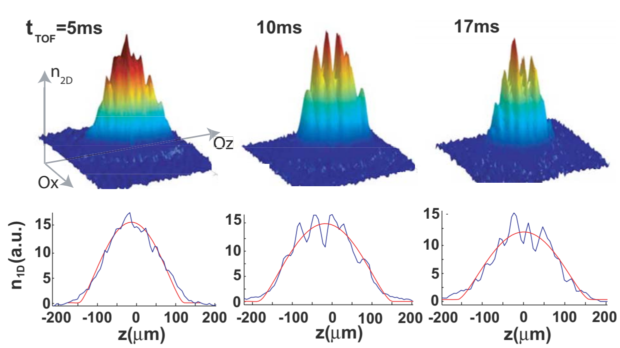

FIG. 1. (Color online) Upper panel: TOF images of an expanding disordered BEC for three different times of flight. The vertical axis represents the column density along the y axis. Lower panel: axial 1D density profiles $n_{\mathrm{1D}}(z)$ (column density integrated along the x axis, blue) and 1D TF parabolic profiles $n_{\mathrm{1D}}^0(z)$ (red). The amplitude of the disorder is $\gamma=V_{\mathrm{R}}/\mu=0.41$.

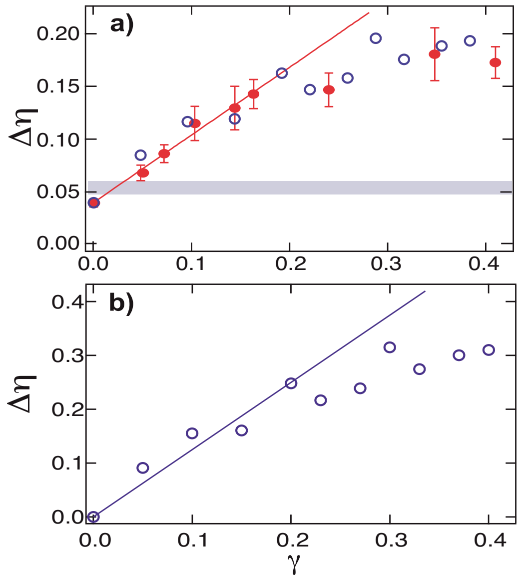

FIG. 2. (Color online) (a) Experimental results (red points) for the density modulations at $\omega_\perp t_{TOF}=62.2$ versus the amplitude of the disorder. The shaded area corresponds to phase fluctuations in our initial elongated BEC, as calculated in Ref. [17] (the error bar reflects the uncertainty on the temperature). (b) Corresponding numerical results, taking into account the finite resolution of our optics, but not the misalignment of the probe beam. The open blue circles in (a) show the same data including an offset accounting for the small initial phase fluctuations and the correction corresponding to the probe angle.

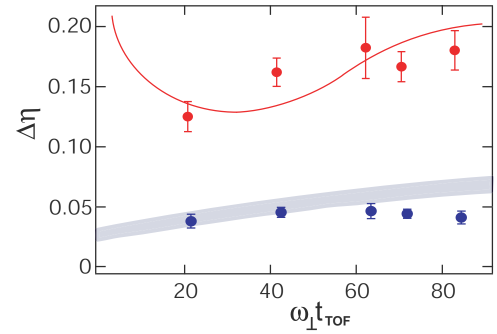

FIG. 3. (Color online) Time evolution of the measured density modulations $\Delta\eta$ during a TOF for $\gamma=0$ (no disorder, blue points) and $\gamma=0.41$ (with disorder, red points). The shaded area corresponds to the calculation of Ref. [17], taking into account the uncertainty on the temperature. The solid red line is the result of numerical calculations for $\gamma=0.4$ (see text).

$n_{1D}(z)=n^0_{1D}(z)[1+\eta(z)]$. Finally, we calculate the standard deviation of $\eta(z)$ over a given length L: $\Delta\eta=\sqrt{\frac{1}{L}\int_L dz\ \eta^2(z)}$ [here $\int_L dz\ \eta(z)=0$]. The calculation of $\Delta\eta$ is restricted to 70% of the BEC total length ($L=1.4L_{TF}$) to avoid the edges where thermal atoms are present. Two imperfections reduce the measured $\Delta\eta$ compared to the real value. First, our imaging system has a finite resolution $L_{res}=8.5\ \ \mu$m, larger than the variation scale σ_R of the disorder. This effect is quantified by measuring the modulation transfer function (MTF) of the imaging system. Second, a slight misalignment of the probe beam—which is not exactly perpendicular to axis z—also reduces the contrast of the fringes [17]. This effect is more difficult to quantify as angles smaller than our uncertainty on the probe angle (1°) can drastically reduce the contrast. We find that numerics reproduce our experimental results assuming a misalignment of 0.33°.

We first study the amplitude of the normalized density modulations, $\Delta\eta$, as a function of the amplitude of the disorder $\gamma=V_R/\mu$ at a given time of flight $t_{TOF}=15.3$ ms ($\omega_\perp t_{TOF}=62.2$) with experimental results plotted in Fig. 2(a). In the absence of disorder, we observe nonvanishing density modulations ($\Delta\eta_0\simeq0.037$) larger than the noise in the background of the images ($\Delta\eta_n\simeq0.015$). They are interpreted as small but nonzero phase fluctuations initially present in our elongated BEC [16]. The calculation of their contribution to the density modulations in the TOF images as determined in Ref. [17] agrees with our data [see Figs. 2(a)]. In the presence of disorder, we find that, for small values of γ (typically $\gamma<0.2$), $\Delta\eta$ grows with γ as $\Delta\eta=\Delta\eta_0+0.64(3)\gamma$. For larger values of γ, the disordered BEC is fragmented either in the trap or during the expansion, and $\Delta\eta$ has a maximum value of $\Delta\eta\simeq0.17$ in the experiment [20].

We also perform numerical integrations of the 3D Gross-Pitaevskii equation (GPE) for the expanding disordered BEC and extract $\Delta\eta$ as in the experiments. The numerics do not include any initial phase fluctuations. We find a linear dependence of $\Delta\eta$ versus γ, $\Delta\eta\simeq3.5\gamma$ for the bare numerical results and $\Delta\eta\simeq1.23\gamma$ if we take into account the finite resolution of the imaging system, but not the misalignment of the probe [see Fig. 2(b)]. In fact, we find that the numerics agree with the experiments if, in addition to an offset $\Delta\eta_0$ to mimic the small initial phase fluctuations, we include the systematic correction associated to a probe angle of 0.33° [see Fig. 2(a)].

We now examine the TOF dynamics of the disordered BEC and plot $\Delta\eta$ versus t_{TOF} in Fig. 3. In the presence of disorder, the observed density modulations (red points) are clearly enhanced compared to those in the absence of disorder (blue points). We also observe that the density modulations first develop and then saturate. The dynamics of their development is reproduced well by our numerical calculations (solid red line) if we take into account all imperfections of our imaging system.

After correcting all experimental imperfections, the density modulations in the TOF images turn out to be larger ($\Delta\eta\simeq3.5\gamma$) than the ones of the trapped BEC before TOF ($\Delta\eta=2\gamma$) [21]. One may wonder whether these large density modulations in the TOF images reveal phase fluctuations induced by the disorder in the initial BEC [1,13]. Actually, several arguments lead us to conclude that it is not so. First, our numerics, which reproduce the experimental data, do *not* include initial phase fluctuations. Second, the numerical diagonalization of the Bogolyubov equations indicates that the disorder hardly affect the excitation spectrum of a 1D BEC for the experimental parameters [22]. Last but not least, we have observed identical density modulations in successive experiments performed with the *same* realization of the disorder (see also Refs. [14,15]). Hence, averaging over various images taken with the same realization of the disorder does not wash out the fringes [23] and this excludes initial random fluctuations, quantum or thermal [19].

We now develop an analytical model for the evolution of the BEC density profile during the TOF, which shows explicitly how a weak disorder leads to large density modulations after a long-enough TOF, *without initial phase fluctuations*. Although the probability of fragmentation is small for weak-enough disorder, it may happen that the BEC is fragmented into a small number of fragments. However, for the considered $t_{\mathrm{TOF}} \lesssim 1/\omega_z$, the fragments will only weakly overlap as the axial expansion is small. Therefore, we neglect fragmentation in our model.

In the absence of disorder, the TOF expansion of a BEC initially trapped in a harmonic potential in the Thomas-Fermi regime is self-similar [24], so that

$$\psi(\vec{r},t) = \Big[\prod_j b_j(t)\Big]^{-1/2} \phi(\{x_j/b_j(t)\},t) e^{i\theta_0(\vec{r},t)}, \qquad (1)$$

with $j=1,\ldots,3$ the spatial directions, $\theta_0(\vec{r},t)=(m/2\hbar)\times\Sigma_j(\dot{b}_j/b_j)x_j^2$ the dynamical phase, and ϕ the (time-independent) wave function of the BEC in the trap. The scaling factors $b_j(t)$ are governed by the equations $\ddot{b}_j=\omega_j^2/(b_j\Pi_k b_k)$ with the initial conditions $b_j(0)=1$ and $\dot{b}_j(0)=0$ [24]. In the presence of disorder, we use the scaling (1) and we write the (now time-dependent) wave function $\phi(\rho,z,t)=\sqrt{\tilde{n}(\rho,z,t)}e^{i\tilde{\theta}(\rho,z,t)}$ where $\rho=\sqrt{x^2+y^2}$ is the radial coordinate. In the absence of phase fluctuations, $\phi(\rho,z,t=0)$ is real (up to a homogeneous phase) as it is the ground state of the trapped, disordered BEC. The TOF dynamics is then governed by two coupled equations for the density $\tilde{n}$ and the phase $\tilde{\theta}$ which are equivalent to the complete time-dependent GPE.

Let us introduce now a couple of approximations. First, in elongated 3D BECs, the expansion for $\omega_z t \lesssim 1$ is mainly radial and $b_z(t)\simeq 1$. Second, we assume not too large perturbation of the density, so that $\tilde{n}=\tilde{n}_0+\delta\tilde{n}$ with $\tilde{n}_0$, the density in the absence of disorder, and $\delta\tilde{n}\ll\tilde{n}_0$. Using the local density approximation, we can neglect all the spatial derivatives of $\tilde{n}_0$. We also neglect the radial derivatives of $\delta\tilde{n}$ since the 1D disorder induces short-range spatial inhomogeneities mainly along the z axis. We are thus left with the equations

$$\partial_t \delta\tilde{n} = -(\hbar/m)\tilde{n}_0 \partial_z^2 \tilde{\theta}, \qquad (2)$$

$$-\hbar\partial_t\tilde{\theta} = g\,\delta\tilde{n}/b_\perp^2 + (\hbar^2/2m)[|\partial_z\tilde{\theta}|^2 - \partial_z^2\delta\tilde{n}/2\tilde{n}_0]. \qquad (3)$$

In a first stage, the initial small inhomogeneities of the density induced by the 1D disordered potential before TOF, $\delta\tilde{n}\simeq -V(z)/g$ [12], hardly evolve since $\partial_t\delta\tilde{n}(t=0)=0$. They are, however, crucial as they act as an inhomogeneous potential which induces the development of a phase modulation $\tilde{\theta}(z,t)$ at the beginning of the TOF [25]. From Eqs. (2) and (3), we find

$$\tilde{\theta}(z,t) \simeq \arctan(\omega_\perp t)[V(z)/\hbar\omega_\perp], \qquad (4)$$

$$\delta\tilde{n} \simeq -V(z)/g - [\tilde{n}_0\partial_z^2 V(z)/m\omega_\perp^2]F(\omega_\perp t), \qquad (5)$$

where $F(\tau)=\int_0^\tau d\tau' \arctan(\tau')=\tau\arctan(\tau)-\ln\sqrt{1+\tau^2}$. From Eq. (5), we then find

$$\Delta\eta(t) \simeq 2\left(\frac{V_{\mathrm{R}}}{\mu}\right)\left[1-\frac{2}{3}\left(\frac{\mu}{\hbar\omega_\perp}\right)^2\left(\frac{\xi}{\sigma_{\mathrm{R}}}\right)^2 F(\omega_\perp t)\right]. \qquad (6)$$

Hence, $\Delta\eta$ first slightly decreases. It is easily understood as an interacting BEC initially at rest will tend to fill its holes when released from the disordered potential. The solution [Eqs. (4) and (5)] is valid as long as the contribution of the last two terms in Eq. (3) remains small [i.e., for $\omega_\perp t\ll(\sigma_{\mathrm{R}}/\xi)^2$ and $\omega_\perp t\ll(\sigma_{\mathrm{R}}/\xi)(\hbar\omega_\perp/\sqrt{V_{\mathrm{R}}\mu})$]. In addition, it requires that the density modulations not vary much [i.e., the second term on the right-hand side (rhs) of Eq. (5) is small compared to the first one]. For the experimental parameters, the last condition is the most restrictive. It defines a typical time $t_0=(1/\omega_\perp)F^{-1}[(\sigma_{\mathrm{R}}/\xi)^2(\hbar\omega_\perp/\mu)^2/2]$ during which the radial expansion imprints a phase modulation due to the initial inhomogeneities of the BEC density created by the disorder before the TOF. In particular, if $t_0\gg 1/\omega_\perp$, the phase modulations freeze at $\tilde{\theta}(z)\simeq(\pi/2)[V(z)/\hbar\omega_\perp]$.

In a second stage, the phase modulations are converted into density modulations similarly as thermal phase fluctuations do during the TOF of an elongated quasi-BEC [17]. For $t\gtrsim t_1$ where t_1 is a typical time much longer than $1/\omega_\perp$ (see below), the scaling parameter $b_\perp(t)$ becomes large so that the first term on the rhs of Eq. (3) can now be neglected. Assuming small phase gradients, we are left with the equation $\partial_t^2\delta\tilde{n}_k+\hbar^2k^4\delta\tilde{n}_k/4m^2=0$ where $\delta\tilde{n}_k(\rho,t)$ is the 1D Fourier transform of $\delta\tilde{n}$ along z and whose solution reads $\delta\tilde{n}_k(t)=\delta\tilde{n}_k(t_1)\cos[(\hbar k^2/2m)(t-t_1)]+[2m\delta\dot{\tilde{n}}_k(t_1)/\hbar k^2]\sin[(\hbar k^2/2m)\times(t-t_1)]$. If $t_0\gg 1/\omega_\perp$, we can take $1/\omega_\perp\ll t_1\leq t_0$ and the exact value of t_1 does not matter much (we use $t_1=t_0$). If $t_0\lesssim 1/\omega_\perp$, the determination of t_1 is not straightforward, but can be found through fitting procedures, for instance. Then, according to Eq. (5), $\delta\tilde{n}_k(t_1)\simeq -V(z)/g$ is mainly determined by the initial density modulations of the trapped BEC while $\delta\dot{\tilde{n}}_k(t_1)\simeq -[\tilde{n}_0\partial_z^2V(z)/m\omega_\perp]\arctan(\omega_\perp t_1)$ results from the phase modulations created in the first stage of the TOF. For $\hbar\omega_\perp\ll\mu$ as in the experiment, the cosine term can be neglected and we find

$$\Delta\eta(t) \simeq \sqrt{8}(V_{\mathrm{R}}/\hbar\omega_\perp)\arctan(\omega_\perp t_1)I(\sigma_{\mathrm{R}},t-t_1), \qquad (7)$$

where $I(\sigma_{\mathrm{R}},t)=\sqrt{\int_0^1 d\kappa(1-\kappa)\sin^2[(2\hbar t/m\sigma_{\mathrm{R}}^2)\kappa^2]}$ for a speckle potential.

Numerical integrations of the complete 3D GPE confirm the expected behavior of $\Delta\eta$ during the TOF at short [Eq. (6)] and long [Eq. (7)] times as shown in Fig. 4. This validates our scenario in a quantitative manner.

Three remarks are in order. First, we find that, due to the development of phase modulations in the first stage of the TOF, the density modulations in the expanded BEC ($\Delta\eta\propto V_{\mathrm{R}}/\hbar\omega_\perp$) can be larger than those in the trapped BEC ($\Delta\eta\propto V_{\mathrm{R}}/\mu$) if $\mu>\hbar\omega_\perp$. Second, the density pattern is completely determined by the realization of the disorder. Third, Eq. (7) shows that the density modulations saturate at $\Delta\eta\simeq\sqrt{2}(V_{\mathrm{R}}/\hbar\omega_\perp)\arctan(\omega_\perp t_1)$ for very long times t [26]. These properties are in qualitative agreement with the experimental observations.

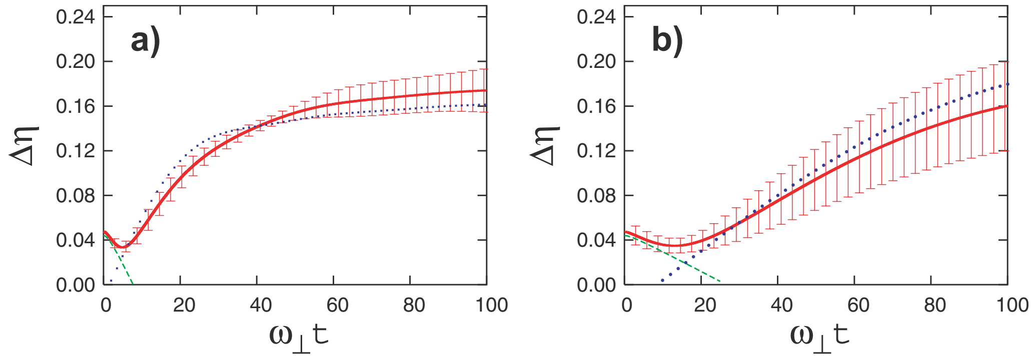

FIG. 4. (Color online) Dynamics of density modulations as obtained numerically (solid red line with error bars) and comparisons with Eq. (6) (green dashed line) and Eq. (7) (blue dotted line). (a) The parameters are the same as in the experiment (in particular, $\sigma_R = 1.7\ \mu$m) with $V_R = 0.02\mu$. Here $\omega_\perp t_0 \simeq 3$ and we have used $t_1 = 0.5 t_0$. (b) Same as (a), but with $\sigma_R = 3.4\ \mu$m. Here, $\omega_\perp t_0 \simeq 8.5 \gg 1$, so that we have used $t_1 = t_0$ (see text).

In conclusion, we have shown that the large fringes observed in this work and Refs. [1,14,15] in TOF images of disordered BECs *do not* rely on initial disorder-induced phase fluctuations. They actually result from the TOF process of a BEC with small initial density modulations, but without phase fluctuations. A phase modulation determined by the weak initial modulations of the BEC density first develops and is later converted into large density modulations. Our analytical calculations based on this scenario agree with numerical calculations and experimental observations. Nevertheless, our results do not exclude that, in different regimes, disorder might enhance phase fluctuations. This question is crucial in connection to the nature of superfluid-insulator transitions in the presence of disorder. Dilute BECs can help answering it as their phase coherence can be probed with accuracy [17,18]. Revealing this possible effect via TOF images requires taking into account the phase modulation which develops during the first stage of the TOF as demonstrated here.

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