Disorder versus the Mermin-Wagner-Hohenberg effect: From classical spin systems to ultracold atomic gases

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We propose a general mechanism of *random-field-induced order* (RFIO), in which long-range order is induced by a random field that breaks the continuous symmetry of the model. We particularly focus on the case of the classical ferromagnetic *XY* model on a two-dimensional lattice, in a uniaxial random field. We prove rigorously that the system has spontaneous magnetization at temperature $T=0$, and we present strong evidence that this is also the case for small $T>0$. We discuss generalizations of this mechanism to various classical and quantum systems. In addition, we propose possible realizations of the RFIO mechanism, using ultracold atoms in an optical lattice. Our results shed new light on controversies in existing literature, and open a way to realize RFIO with ultracold atomic systems.

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I. INTRODUCTION

A. Disordered ultracold quantum gases

Studies of disordered systems constitute a new, rapidly developing, branch of the physics of ultracold gases. In condensed-matter physics (CM), the role of *quenched* (i.e., independent of time) disorder cannot be overestimated: it is present in nearly all CM systems, and leads to numerous phenomena that dramatically change both qualitative and quantitave behaviors of these systems. This leads, for instance, to novel thermodynamical and quantum phases,[1,2] and to strong phenomena, such as Anderson localization.[3–6] In general, disorder can hardly be controlled in CM systems. In contrast, it has been proposed recently, that quenched disorder (or pseudodisorder) can be introduced in a controlled way in ultracold atomic systems, using optical potentials generated by speckle radiations,[7–9] impurity atoms serving as random scatterers,[10] or quasicrystalline lattices.[11] This opens fantastic possibilities to investigate the effect of disorder in controlled systems (for a review in the context of cold gases, see Ref. [12]). Recently, several groups have initiated the experimental study of disorder with Bose-Einstein condensates (BEC),[13–15] and strongly correlated Bose gases.[16] In the center of interest of these works is one of the most fundamental issues of disordered systems that concerns the interplay between Anderson localization and interactions in many-body Fermi or Bose systems at low temperatures. In noninteracting atomic systems, localization is feasible experimentally,[17] but even weak interactions can drastically change the scenario. While weak repulsive interactions tend to delocalize, strong ones in confined geometries lead to Wigner-Mott-type localization.[18] Both experiments and theory indicate that in gaseous systems with large interactions, stronger localization effects occur in the excitations of a BEC,[14,19–21] rather than on the BEC wave function itself. In the limit of weak interactions, a Bose gas enters a *Lifshits glass* phase, in which several BECs in various localized single-atom orbitals from the low-energy tail of the spectrum coexist[22] (for "traces" of the Lifshits glass in the mean-field theory, see Ref. [15]). Finally, note that disorder in Fermi gases, or in Femi-Bose atomic mixtures, should allow one to realize various fermionic disordered phases, such as a Fermi glass, a Mott-Wigner glass, "dirty" superconductors, etc.[12]), or even quantum spin glasses.[23]

B. Large effects by small disorder

One of the most appealing effects of disorder is that even extremely small randomness can have dramatic consequences. The paradigm example in classical physics is the Ising model for which an arbitrarily small random magnetic field destroys magnetization even at temperature $T=0$ in two-dimensions, 2D (Refs. [24] and [25]), but not in $D>2$.[26] This result has been generalized to systems with continuous symmetry in random fields distributed in accordance with this symmetry.[24,25] For instance, the Heisenberg model in a SO(3) symmetrically distributed field does not magnetize up to four dimensions (4D).

In quantum physics, the paradigm example of large effects induced by small disorder is provided by the above-mentioned Anderson localization, which occurs in one dimension and two dimensions in arbitrarily small random potentials.[5] In this paper, we propose an even more intriguing opposite effect, where disorder counterintuitively favors ordering: a general mechanism of *random-field-induced order* (RFIO) by which certain spin models magnetize at a higher temperature in the presence of arbitrarily small disorder than in its absence, provided that the disorder breaks the continuous symmetry of the system.

C. Main results and plan of the paper

As is well known, as a consequence of the Mermin-Wagner-Hohenberg theorem,[27] spin- or field-theoretic

systems with continuous symmetry in dimensions less or equal to 2D cannot exhibit long-range order. The mechanism that we propose here breaks the continuous symmetry, and in this sense acts against the Mermin-Wagner-Hohenberg no-go rule in 2D. In particular, we prove rigorously that the classical *XY* spin model on a 2D lattice in a uniaxial random field magnetizes spontaneously at $T=0$ in the *direction perpendicular* to the magnetic-field axis, and provide strong evidence that this is also the case at small positive temperatures. We discuss generalizations of this mechanism to classical and quantum *XY* and Heisenberg models in 2D and 3D. In 3D, the considered systems do exhibit long-range order at finite $T>0$, but in this case the critical temperature decreases with the "size" of the symmetry group: it is the largest for the Ising model (the discrete group Z_2), higher for the *XY* model [the continuous group U(1)], and the highest for the Heisenberg model [the continuous group SU(2) or SO(3)]. In this case we expect that our mechanism will lead to an increase of the critical temperature for the *XY* and Heisenberg models, and to an increase of the order-parameter value at a fixed temperature for the disordered system in comparison to the nondisordered one. Finally, we propose three possible and experimentally feasible realizations of the RFIO phenomenon using ultracold atoms in optical lattices.

The paper is organized as follows. In Sec. II, we present the results concerning the RFIO in the classical *XY* model on a 2D lattice. First, we rigorously prove that the system magnetizes in the direction perpendicular to the direction of the random magnetic field at $T=0$, and then, we present arguments that the magnetization persists in the small $T>0$ case, as well as the results of numerical classical Monte Carlo simulations. Section III is fully devoted to the discussion of the generalizations of the RFIO mechanism to several other classical and quantum spin systems. In Sec. IV, we discuss several experimentally feasible realizations of RFIO in ultracold atomic systems. Finally, we summarize our results in Sec. V.

II. RFIO IN CLASSICAL *XY* MODEL

A. System under study

Consider a classical spin system on the 2D square lattice $\mathbf{Z}^2$, in a random magnetic field **h** (see Fig. 1). The spin variable $\sigma_i=(\cos\theta_i, \sin\theta_i)$, at a site $i \in \mathbf{Z}^2$ is a unit vector in the xy plane. The Hamiltonian (in units of the exchange energy J) is given by

$$H/J = -\sum_{|i-j|=1} \sigma_i \cdot \sigma_j - \epsilon\sum_i \mathbf{h}_i \cdot \sigma_i. \qquad (1)$$

Here the first term is the standard nearest-neighbor interaction of the *XY* model, and the second term represents a small random-field perturbation. The $\mathbf{h}_i$'s are assumed to be independent, identically distributed random, 2D vectors.

For $\epsilon=0$, the model has no spontaneous magnetization m at any positive T. This was first pointed out in Ref. 28, and later developed into a class of results known as the Mermin-Wagner-Hohenberg theorem[27] for various classical, as well as quantum 2D spin systems with continuous symmetry. In
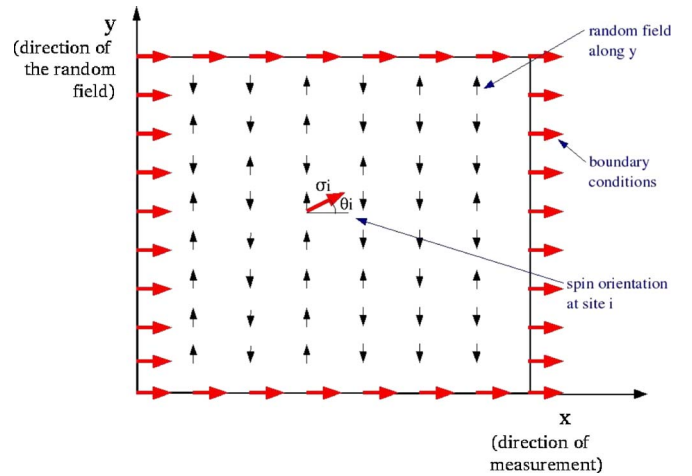

FIG. 1. (Color online) *XY* model on a 2D square lattice in a random magnetic field. The magnetic field is oriented along the y axis, $\mathbf{h}_i = \eta_i \mathbf{e}_y$, where η_i is a real random number. Right boundary conditions are assumed on the outer square, possibly placed at infinity (see text).

higher dimensions the system does magnetize at low temperatures. This follows from the spin-wave analysis,[29] and has been given a rigorous proof in Ref. 30. The impact of a random-field term on the behavior of the model was first addressed in Refs. 24 and 25, where it was argued that if the distribution of the random variables $\mathbf{h}_i$ is invariant under rotations, there is no spontaneous magnetization at any positive T in any dimension $D \leq 4$. A rigorous proof of this statement was given in Ref. 25. Both works use crucially the rotational invariance of the distribution of the random-field variables.

Here we consider the case where $\mathbf{h}_i$ is directed along the y axis: $\mathbf{h}_i = \eta_i \mathbf{e}_y$, where $\mathbf{e}_y$ is the unit vector in the y direction, and η_i is a random real number. Such a random field obviously breaks the continuous symmetry of the interaction and a question arises whether the model still has no spontaneous magnetization in two dimensions. This question has been given contradictory answers in Refs. 31 and 32: while Ref. 31 predicts that a small random field in the y direction does not change the behavior of the model, Ref. 32 argues that it leads to the presence of spontaneous magnetization m in the direction perpendicular to the random-field axis in low (but not arbitrarily low) temperatures. Both works use renormalization-group analysis, with Ref. 32 starting from a version of the Imry-Ma scaling argument to prove that the model magnetizes at zero temperature.

The same model was subsequently studied by Feldman,[33] using ideas similar to the argument given in the present paper. As we argue below, however, his argument contains an essential gap, which is filled in the present work. We first present a complete proof that the system indeed magnetizes at $T=0$, and argue that the ground-state magnetization is stable under inclusion of small thermal fluctuations. For this, we use a version of the Peierls contour argument,[34] eliminating first the possibility that Bloch walls or vortex configurations destroy the transition.

B. RFIO at $T=0$

Let us start by a rigorous analysis of the ground state. Consider the system in a square Λ with the "right" boundary conditions, $\sigma_i=(1,0)$, for the sites i on the outer boundary of Λ (see Fig. 1). The energy of any spin configuration decreases if we replace the x components of the spins by their absolute values and leave the y components unchanged. It follows that in the ground-state, x components of all the spins are nonnegative. As the size of the system increases, we expect the x component of the ground-state spins to decrease, since they feel less influence of the boundary conditions and the ground state value of each spin will converge. We thus obtain a well-defined infinite-volume ground state with the "right" boundary conditions at infinity.

We emphasize that the above convergence statement is nontrivial and requires a proof. Physically it is, however, quite natural. A similar statement has been rigorously proven for ground states of the random-field Ising model using Fortuin-Kasteleyn-Ginibre monotonicity techniques.[25,35]

C. Infinite volume limit

A priori this infinite-volume ground state could coincide with the ground state of the random field Ising model, in which all spins have zero x component. The following argument shows that this is *not* the case. Suppose that the spin σ_i at a given site i is aligned along the y axis, i.e., $\cos\theta_i=0$. Since the derivative of the energy function with respect to θ_i vanishes at the minimum, we obtain

$$\sum_{j:|i-j|=1} \sin(\theta_i - \theta_j) = 0. \qquad (2)$$

Since $\cos\theta_i=0$, this implies $\sum_{j:|i-j|=1}\cos\theta_j=0$. Because in the right ground state all spins lie in the (closed) right half-plane $x\geq 0$, all terms in the above expression are non-negative and hence have to vanish. This means that at all the nearest neighbors j of the site i, the ground-state spins are directed along the y axis as well. Repeating this argument, we conclude that the same holds for all spins of the infinite lattice, i.e., the ground state is the (unique) random-field Ising model ground state. This, however, leads to a contradiction, since assuming this, one can construct a field configuration, occurring with a positive probability, which forces the ground-state spins to have nonzero x components. To achieve this we put strong positive ($\eta_i>0$) fields on the boundary of a square and strong negative fields on the boundary of a concentric smaller square. If the fields are very weak in the area between the two boundaries, the spins will form a Bloch wall, rotating gradually from $\theta=\pi/2$ to $\theta=-\pi/2$. Since such a local field configuration occurs with a positive probability, the ground state *cannot have* zero x components everywhere, contrary to our assumption.

We would like to emphasize the logical structure of the above argument, which proceeds indirectly assuming that the ground-state spins (or, equivalently, at least one of them) have zero x components and reach a contradiction. The initial assumption is used in an essential way to argue existence of the Bloch wall interpolating between spins with y components equal to $+1$ and -1. It is this part of the argument that

we think is missing in Ref. 33. Note that this argument applies to strong, as well as to weak random fields, so that the ground state is never, strictly speaking, field-dominated and always exhibits magnetization in the x direction. Moreover, the argument does not depend on the dimension of the system, applying, in particular, in one dimension. We argue below that in dimensions greater than one the effect still holds at small positive temperatures, the critical temperature depending on the strength of the random field (and presumably going to zero as the strength of the field increases).

D. RFIO at low positive T

To study the system at low positive T, we need to ask what are the typical low-energy excitations from the ground state. For $\epsilon=0$, continuous symmetry allows Bloch walls, i.e., configurations in which the spins rotate gradually over a large region, for instance from left to right. The total excitation energy of a Bloch wall in 2D is of order one, and it is the presence of such walls that underlies the absence of continuous symmetry breaking. However, for $\epsilon>0$, a Bloch wall carries additional energy, coming from the change of the direction of the y component of the spin, which is proportional to the area of the wall (which is of the order L^2 for a wall of linear size L in two dimensions), since the ground-state spins are adapted to the field configuration, and hence overturning them will increase the energy per site. Similarly, vortex configurations, which are important low-energy excitations in the nonrandom XY model, are no longer energetically favored in the presence of a uniaxial random field.

We are thus left, as possible excitations, with sharp domain walls, where the x component of the spin changes sign rapidly. To first approximation we consider excited configurations, in which spins take either their ground-state values, or the reflections of these values in the y axis. As in the standard Peierls argument,[34] in the presence of the right boundary conditions, such configurations can be described in terms of contours γ (domain walls), separating spins with positive and negative x components. If m_i is the value of the x component of the spin σ_i in the ground state with the right boundary conditions, the energy of a domain wall is the sum of $m_i m_j$ over the bonds (ij) crossing the boundary of the contour. Since changing the signs of the x components of the spins does not change the magnetic-field contribution to the energy, the Peierls estimate shows that the probability of such a contour is bounded above by $\exp(-2\beta\Sigma_{(ij)}m_i m_j)$, with $\beta=J/k_BT$.

We want to show that for a typical realization of the field **h**, (i.e., with probability one), these probabilities are summable, i.e., their sum over all contours containing the origin in their interior is finite. It then follows that at a still lower T, this sum is small, and the Peierls argument proves that the system magnetizes (in fact, a simple additional argument shows that summability of the contour probabilities already implies the existence of spontaneous m). To show that a series of random variables is summable with probability one, it suffices to prove the summability of the series of the expected values. We present two arguments for the last statement to hold.

If the random variables m_i are bounded away from zero, i.e., $m_i > \sqrt{c}$, for some $c > 0$, the moment-generating function of the random variable $\Sigma_{(ij)} m_i m_j$ satisfies

$$\mathbf{E}\left[\exp\left(-\beta \sum_{(ij)} m_i m_j\right)\right] \leq \exp[-c\beta L(\gamma)], \qquad (3)$$

with $L(\gamma)$ denoting the length of the contour γ. The sum of the probabilities of the contours enclosing the origin is thus bounded by $\Sigma_\gamma \exp[-c\beta L(\gamma)]$. The standard Peierls-Griffiths bound proves the desired summability.

The above argument does not apply if the distribution of the ground state m contains zero in its support. For unbounded distribution of the random field this may very well be the case, and then another argument is needed. If we assume that the terms in the sum $\Sigma_{(ij)} m_i m_j$ are independent and identically distributed, then $\mathbf{E}[\exp(-2\beta\Sigma_{(ij)} m_i m_j)]$ $= \mathbf{E}[\exp(-2\beta m_i m_j)]^{L(\gamma)} = \exp\{L(\gamma)\log \mathbf{E}[\exp(-2\beta m_i m_j)]\}$ and we just need to observe that $\mathbf{E}[\exp(-2\beta m_i m_j)] \to 0$ as $\beta \to \infty$ (since the expression under the expectation sign goes pointwise to zero and lies between 0 and 1) to conclude that $\mathbf{E}[\exp(-2\beta\Sigma_{(ij)} m_i m_j)]$ behaves as $\exp[-g(\beta)L(\gamma)]$ for a positive function $g(\beta)$ with $g(\beta) \to \infty$ as $\beta \to \infty$. While $m_i m_j$ are not, strictly speaking, independent, it is natural to assume that their dependence is weak, i.e., their correlation decays fast with the distance of the corresponding bonds (ij). The behavior of the moment-generating function of their sum is then qualitatively the same, with a renormalized rate function $g(\beta)$, still diverging as $\beta \to \infty$. As before, this is enough to carry out the Peierls-Griffiths estimate, which implies spontaneous magnetization in the x direction. We remark that our assumption about the fast decay of correlations implies that the sums of $m_i m_j$ over subsets of $\mathbf{Z}^2$ satisfy a large deviation principle analogous to that for sums of independent random variables and the above argument can be restated using this fact.

E. Numerical Monte Carlo simulations

Based on the above discussion it is expected that the RFIO effect predicted here will lead to the appearance of magnetization m in the x direction of order 1 at low temperatures in systems much larger than the correlation length of typical excitations. For small systems, however, the effect may be obscured by finite size effects, which, due to long-range power-law decay of correlations, are particularly strong in the XY model in 2D. In particular, the 2D XY model shows finite magnetization (m) in small systems,[36] so that RFIO is expected to result in an increase of the magnetization.

We have performed numerical Monte Carlo simulations[37] for the 2D XY classical model [Hamiltonian (1), with $\epsilon = 1$]. We generate a random magnetic field $\mathbf{h}_i = \eta_i \mathbf{e}_y$ in the y direction. The η_i's are independent random real numbers, uniformly distributed in $[-\sqrt{3}\Delta h^y, \sqrt{3}\Delta h^y]$. Note that Δh^y is thus the standard deviation of the random field $\mathbf{h}_i$. Boundary conditions on the outer square correspond to $\sigma_i = (1,0)$ (see Fig. 1). The calculations were performed in 2D lattices with up to 200×200 lattice sites for various temperatures. The results are presented in Fig. 2.
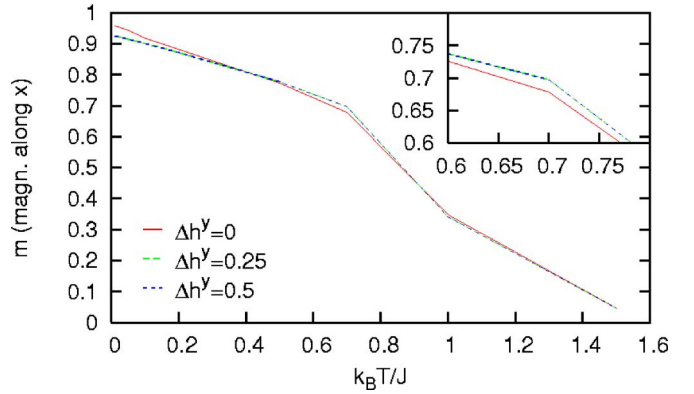

FIG. 2. (Color online) Results of the Monte Carlo simulation for the classical 2D XY model in a 200×200 lattice. The inset is a magnetification of the main figure close to $T = 0.7 J/k_B$.

At very small temperature, the system magnetizes in the absence of disorder (m approaches 1 when T tends to 0) due to the finite size of the lattice.[36] In this regime, a random field in the y direction tends to induce a small local magnetization, parallel to $\mathbf{h}_i$, so that the magnetization in the x direction, m, is slightly reduced. At higher temperatures ($T \simeq 0.7 J/k_B$ in Fig. 2), the magnetization is significantly smaller than one in the absence of disorder. This is due to non-negligible spin-wave excitations. In the presence of small disorder, these excitations are suppressed due to the RFIO effect discussed in this paper. We indeed find that, at $T = 0.7 J/k_B$, m increases by 1.6% in presence of the uniaxial disordered magnetic field. At larger temperatures, excitations, such as Bloch walls or vortices are important and no increase of the magnetization is found when applying a small random field in the y direction.

III. RFIO IN OTHER SYSTEMS

The RFIO effect predicted above may be generalized to other spin models, in particular, those that have finite correlation length. Here we list the most spectacular generalizations:

A. Heisenberg ferromagnet (HF) in random fields of various symmetries

Here the interaction has the same form as in the XY case, but spins take values on a unit sphere. As for the XY Hamiltonian, if the random-field distribution has the same symmetry as the interaction part, i.e., if it is symmetric under rotations in three dimensions, the model has no spontaneous magnetization up to four dimensions.[24,25] If the random field is uniaxial, e.g., oriented along the z axis, the system still has a continuous symmetry (rotations in the xy plane), and thus cannot have spontaneous magnetization in this plane. It cannot magnetize in the z direction either, by the results of Ref. 25. Curiously enough, a field distribution with an intermediate symmetry may lead to symmetry breaking. Namely, arguments fully analogous to the previous ones imply that if the random field takes values in the yz plane with a distribution invariant under rotations, the system will magnetize in

the x direction. We are thus faced with the possibility that a planar-field distribution breaks the symmetry, while this is broken neither by a field with a spherically symmetric distribution nor by a uniaxial one.

B. Three-dimensional *XY* and Heisenberg models in a random field of various symmetries

We have argued that the 2D *XY* model with a small uniaxial random field orders at low T. Since in the absence of the random field spontaneous magnetization occurs only at $T=0$, this can be equivalently stated by saying that a small uniaxial random field raises the critical temperature T_c of the system. By analogy, one can expect that the (nonzero) T_c of the *XY* model in 3D becomes higher and comparable to that of the 3D Ising model, in the presence of a small uniaxial field. A simple mean-field estimate suggests that T_c might increase by a factor of 2. The analogous estimates for the Heisenberg model in 3D suggest an increase of T_c by a factor $3/2$ (or 3) in a small uniaxial (or planar rotationally symmetric) field, respectively. These conjectures are the subject of a forthcoming project.

C. Antiferromagnetic systems

By flipping every second spin, the classical ferromagnetic models are equivalent to antiferromagnetic ones (on bipartite lattices). This equivalence persists in the presence of a random field with a distribution symmetric with respect to the origin. Thus the above discussion of the impact of random fields on continuous symmetry breaking in classical ferromagnetic models translates case by case to the antiferromagnetic case.

D. Quantum systems

All of the effects predicted above should, in principle, have quantum analogs. Quantum fluctuations might, however, destroy the long-range order, so each of the discussed models should be carefully reconsidered in the quantum case. Some models, such as the quantum spin $S=1/2$ Heisenberg model, for instance, have been widely studied in literature.[38] The Mermin-Wagner theorem[27] implies that the model has no spontaneous magnetization at positive temperatures in 2D. For $D>2$ spin-wave analysis[39–41] shows the existence of spontaneous magnetization (though a rigorous mathematical proof of this fact is still lacking). In general, one does not expect major differences between the behaviors of the two models at $T \neq 0$. It thus seems plausible that the presence of a random field in the quantum case is going to have effects similar to those in the classical Heisenberg model. Similarly, one can consider the quantum Heisenberg antiferromagnet (HAF) and expect phenomena analogous to the classical case, despite the fact that unlike their classical counterparts, the quantum HF and HAF systems are no longer equivalent. We expect to observe spontaneous staggered magnetization in a random uniaxial *XY* model, or random planar field HF. A possibility that a random field in the z direction can enhance the antiferromagnetic order in the xy plane has been pointed out in Ref. 42.
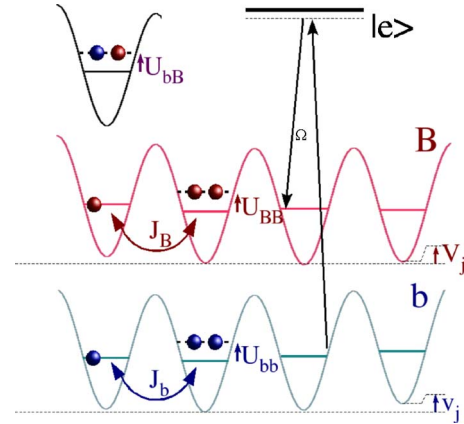

FIG. 3. (Color online) Atomic-level scheme of a two-component Bose mixture in a random optical lattice used to design spin models in random magnetic fields (see text).

IV. TOWARDS THE EXPERIMENTAL REALIZATION OF RFIO IN ULTRACOLD ATOMIC SYSTEMS

Further understanding of the phenomena described in this paper will benefit from experimental realizations and investigations of the above-mentioned models. Below, we discuss the possibilities to design quantum simulators for these quantum spin systems using ultracold atoms in optical lattices (OL).

A. Two-component lattice Bose gas

Consider a two-component Bose gas confined in an OL with on-site inhomogeneities. The two components correspond here to two internal states of the same atom. The low-T physics is captured by the Bose-Bose Hubbard model (BBH) (Ref. 43) (for a review of ultracold lattice gases see Ref. 44):

$$H_{\text{BBH}} = \sum_j \left[\frac{U_b}{2} \mathbf{n}_j(\mathbf{n}_j - 1) + \frac{U_B}{2} \mathbf{N}_j(\mathbf{N}_j - 1) + U_{bB}\mathbf{n}_j\mathbf{N}_j \right]$$
$$+ \sum_j (v_j\mathbf{n}_j + V_j\mathbf{N}_j) - \sum_{\langle j,l \rangle} [(J_b\mathbf{b}_j^{\dagger}\mathbf{b}_l + J_B\mathbf{B}_j^{\dagger}\mathbf{B}_l) + \text{H.c.}]$$
$$- \sum_j \left(\frac{\Omega_j}{2} \mathbf{b}_j^{\dagger}\mathbf{B}_j + \text{H.c.} \right), \tag{4}$$

where $\mathbf{b}_j$ and $\mathbf{B}_j$ are the annihilation operators for both types of bosons in the lattice site j, $\mathbf{n}_j = \mathbf{b}_j^{\dagger}\mathbf{b}_j$ and $\mathbf{N}_j = \mathbf{B}_j^{\dagger}\mathbf{B}_j$ are the corresponding number operators, and $\langle j, l \rangle$ denote a pair of adjacent sites in the OL. In Hamiltonian (4), (i) the first term describes on-site interactions, including interaction between bosons of different types, with energies U_b, U_B, and U_{bB}; (ii) the second accounts for on-site energies; (iii) the third describes quantum tunneling between adjacent sites, and (iv) the fourth transforms one boson type into the other with a probability amplitude $|\Omega|/\hbar$. The last term can be implemented with an optical two-photon Raman process if the two Bosonic "species" correspond to two internal states of the same atom (see also Fig. 3). Possibly, both on-site energies v_j, V_j, and the Raman complex amplitude Ω_j can be made site-dependent using speckle laser light.[13–15]

Consider the limit of strong repulsive interactions $(0 < J_b, J_B, |\Omega_j| \ll U_b, U_B, U_{Bb})$ and a total filling factor of 1 (i.e., the total number of particles equals the number of lattice sites). Proceeding as in the case of Fermi-Bose mixtures, recently analyzed by two of the authors in Ref. 23, we derive an effective Hamiltonian, H_{eff}, for the Bose-Bose mixture. In brief, we restrict the Hilbert space to a subspace $\mathcal{E}_0$ generated by $\{\Pi_j | n_j, N_j\rangle\}$ with $n_j + N_j = 1$ at each lattice site, and we incorporate the tunneling terms via second-order perturbation theory as in Ref. 23 We then end up with

$$H_{\text{eff}} = -\sum_{\langle j,l\rangle} (J_{j,l} \mathcal{B}_j^\dagger \mathcal{B}_l + \text{H.c.}) + \sum_{\langle j,l\rangle} K_{j,l} \mathcal{N}_j \mathcal{N}_l + \sum_j \mathcal{V}_j \mathcal{N}_j$$
$$- \sum_j \left(\frac{\Omega_j}{2} \mathcal{B}_j + \text{H.c.}\right), \tag{5}$$

where $\mathcal{B}_j = \mathbf{b}_j^\dagger \mathbf{B}_j \mathcal{P}$, $\mathcal{P}$ is the projector onto $\mathcal{E}_0$ and $\mathcal{N}_j = \mathcal{B}_j^\dagger \mathcal{B}_j$. Hamiltonian H_{eff} contains (i) a hopping term $J_{j,l}$, (ii) an interaction term between neighbor sites $K_{j,l}$, (iii) inhomogeneities $\mathcal{V}_j$, and (iv) a creation/annihilation term. Note that the total number of composites is not conserved except for a vanishing Ω. The coupling parameters in Hamiltonian (5) are (Ref. 45):

$$J_{j,l} = \frac{J_b J_B}{U_{bB}} \left[\frac{1}{1 - \left(\frac{\delta_{j,l}}{U_{bB}}\right)^2} + \frac{1}{1 - \left(\frac{\Delta_{j,l}}{U_{bB}}\right)^2} \right], \tag{6}$$

$$K_{j,l} = -\frac{4J_b^2/U_b}{1 - \left(\frac{\delta_{j,l}}{U_b}\right)^2} + \frac{2J_b^2/U_{bB}}{1 - \left(\frac{\delta_{j,l}}{U_{bB}}\right)^2}$$
$$- \frac{4J_B^2/U_B}{1 - \left(\frac{\Delta_{j,l}}{U_B}\right)^2} + \frac{2J_B^2/U_{bB}}{1 - \left(\frac{\Delta_{j,l}}{U_{bB}}\right)^2}, \tag{7}$$

$$\mathcal{V}_j = V_j - v_j + \sum_{\langle j,l\rangle} \left[\frac{4J_b^2/U_b}{1 - \left(\frac{\delta_{j,l}}{U_b}\right)^2} - \frac{J_b^2/U_{bB}}{1 - \frac{\delta_{j,l}}{U_{bB}}} \right.$$
$$\left. - \frac{J_B^2/U_{bB}}{1 + \frac{\Delta_{j,l}}{U_{bB}}} + \frac{4J_B^2/U_B}{1 - \left(\frac{\Delta_{j,l}}{U_B}\right)^2} \right], \tag{8}$$

where $\delta_{j,l} = v_j - v_l$ and $\Delta_{j,l} = V_j - V_l$. Hamiltonian H_{eff} describes the dynamics of *composite particles* whose annihilation operator at site j is $\mathcal{B}_j = \mathbf{b}_j^\dagger \mathbf{B}_j \mathcal{P}$. In contrast to the case of Fermi-Bose mixtures discussed in Ref. 23, where the composites are fermions, in the present case of Bose-Bose mixtures, they are *composite Schwinger bosons* made of one B boson and one b hole.

Since the commutation relations of $\mathcal{B}_j$ and $\mathcal{B}_j^\dagger$ are those of Schwinger bosons,[46] we can directly turn to the spin representation[46] by defining $\mathbf{S}_j^x + i\mathbf{S}_j^y = \mathcal{B}_j$ and $\mathbf{S}_j^z = 1/2 - \mathcal{N}_j$, where $\mathcal{N}_j = \mathcal{B}_j^\dagger \mathcal{B}_j$. It is important to note that since Raman processes can convert b bosons into B bosons (and conversely), $\Sigma_j \langle \mathcal{N}_j \rangle$ *is not fixed* by the total number of bosons of each species, i.e., the z component of m, $\Sigma_j \langle \mathbf{S}_j^z \rangle$ is not con-

strained. For small inhomogeneities $(\delta_{j,l} = v_j - v_l, \Delta_{j,l} = V_j - V_l \ll U_b, U_b, U_{bB})$, Hamiltonian H_{eff} is then equivalent to the anisotropic Heisenberg XXZ model[46] in a random field

$$H_{\text{eff}} = -J_\perp \sum_{\langle j,l\rangle} (\mathbf{S}_j^x \mathbf{S}_l^x + \mathbf{S}_j^y \mathbf{S}_l^y) - J_z \sum_{\langle j,l\rangle} \mathbf{S}_j^z \mathbf{S}_l^z$$
$$- \sum_j (h_j^x \mathbf{S}_j^x + h_j^y \mathbf{S}_j^y + h_j^z \mathbf{S}_j^z), \tag{9}$$

where

$$J_\perp = \frac{4J_b J_B}{U_{bB}}, \tag{10}$$

$$J_z = 2\left(\frac{2J_b^2}{U_b} + \frac{2J_B^2}{U_B} - \frac{J_b^2 + J_B^2}{U_{bB}}\right), \tag{11}$$

$$h_j^x = \Omega_j^R, \quad h_j^y = -\Omega_j^I, \quad h_j^z = \mathcal{V}_j - \zeta J_z/2, \tag{12}$$

with ζ the lattice coordination number, $\mathcal{V}_j = V_j - v_j + \zeta[4J_b^2/U_b + 4J_B^2/U_B - (J_b^2 + J_B^2)/U_{bB}]$ and $\Omega_j = \Omega_j^R + i\Omega_j^I$. In atomic systems, all these (possibly site-dependent) terms can be controlled almost at will.[23,44,47] In particular, by employing various possible control tools one can reach the HF $(J_\perp = J_z)$ and XY $(J_z = 0)$ cases.

B. Bose lattice gas embedded in a BEC

The quantum ferromagnetic XY model in a random field may be alternatively obtained using the same BBH model, but with strong state dependence of the optical dipole forces. One can imagine a situation in which one component (say b) is in the strong interaction limit, so that only one b atom at a site is possible, whereas the other (B) component is in the Bose condensed state and provides only a coherent BEC "background" for the b atoms. Mathematically speaking, this situation is described by Eq. (4), in which n_i's can be equal to 0 or 1 only, whereas B_i's can be replaced by a classical complex field (condensate wave function). In this limit the spin $S = 1/2$ states can be associated with the presence or absence of a b atom in a given site. In this way, setting $v_j = 0$ and $\Omega_j^I = 0$, one obtains the quantum version of the XY model (1) with $J = J_b$ and a uniaxial random field in the x direction with the strength determined by Ω_j^R.

C. Two-component Fermi lattice gas

Finally, the $S = 1/2$ antiferromagnetic Heisenberg model may be realized with a fermi-fermi mixture at half filling for each component. This implementation might be of special importance for future experiments with lithium atoms. As recently calculated,[48] the critical temperature for the Néel state in a 3D cubic lattice is of the order of 30 nK. It is well known that in a 3D cubic lattice the critical temperatures for the antiferromagnetic Heisenberg, the XY and the Ising models are $T_{c,XY} \simeq 1.5 T_{c,Heis}$, and $T_{c,Ising} \simeq 2T_{c,Heis}$. The estimates of these critical temperatures can be, for instance, obtained applying the Curie-Weiss mean-field method to the classical models. Suppose that we put the Heisenberg antiferromagnet

in a uniaxial (respectively, planar) random field, created using the same methods as discussed above, i.e., we break the SU(2) symmetry and put the system into the universality class of *XY* (respectively, Ising) models. Mean-field estimates suggest then that we should expect the increase of the critical temperature by factor 1.5 (respectively, 2), that is, up to $\simeq 45$ (respectively, 90) nK. Even if these estimates are too optimistic, and the effect is two or three times smaller, one should stress, that even an increase by, say 10 nK, is of great experimental relevance and could be decisive for the achievment of an antiferromagnetic state.

We would like to stress that similar proposals, as the three discussed above, have been formulated before,[49] but none of them treat simultaneously essential aspects for the present schemes: (i) disordered fields, but not bonds; (ii) arbitrary directions of the fields; (iii) possibility of exploring Ising, *XY*, or Heisenberg symmetries; (iv) realizing the coherent source of atoms; and (v) avoiding constraints on the magnetization along the *z* axis.

It is also worth commenting on what are the most important experimental challenges that have to be addressed in order to achieve RFIO. Evidently, for the proposals involving the strong interaction limit of two-component Bose or Fermi systems, the main issue is the temperature, which has to be of the order of tens of nanokelvins. Such temperatures are starting to be achievable nowadays (for a careful discussion in the context of Fermi-Bose mixtures see Ref. 50), and there exist several proposals for supplementary cooling of lattice gases, using laser (photons) or couplings to ultracold BEC (phonon cooling) that can help [for reviews see Refs. 44 and 47].

V. SUMMARY

In this paper, we have proposed a general mechanism of *random-field-induced order* (RFIO), occurring in systems with continuous symmetry, placed in a random field that breaks, or reduces this symmetry. We have presented rigorous results for the case of the 2D classical ferromagnetic *XY* model in a random uniaxial field, and proved that the system has spontaneous magnetization at temperature $T=0$. We have presented also a rather strong evidence that this is also the case for small $T>0$. Several generalizations of this mechanism to various classical and quantum systems were discussed. We have presented also detailed proposals to realize RFIO in experiments using two-component Bose lattice gases, one-component Bose lattice gases embedded in BECs, or two-component Fermi lattice gases. Our results shed light on controversies in existing literature, and open the way to realize RFIO with ultracold atoms in an optical lattice.

It is worth mentioning two further realizations of RFIO studied by us recently. RFIO occurs in a two-component trapped Bose gas at $T=0$, when the gas is condensed and the two components are coupled by Raman transition of random strength, but fixed phase. Although such a system belongs to the universality class of the (trapped, i.e., located in an inhomogenous field) *XY* model, it exhibits the RFIO effect in a much stronger manner than the *XY* model discussed in the present paper. We have found this observation important enough to devote a separate detailed paper to it.[51] Similarly, we have studied numerically RFIO in 1D for quantum *XY* and Heisenberg chains.[52] In such systems, even at $T=0$, magnetization vanishes, but amazingly enough the RFIO effect seems to work at the level of the magnetic susceptibilities. Adding a random field confined to a certain axis (respectively, plane), *increases* siginificantly the magnetic suceptibility in the perpedicular directions (respectively, direction).

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