

Calcul cinétique du transport électronique non local

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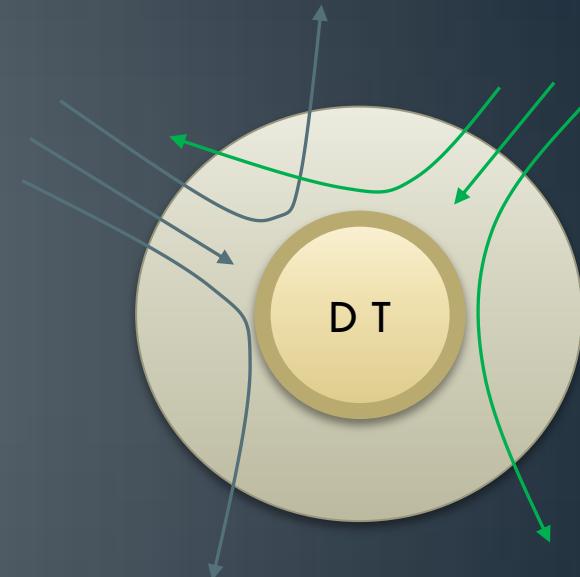
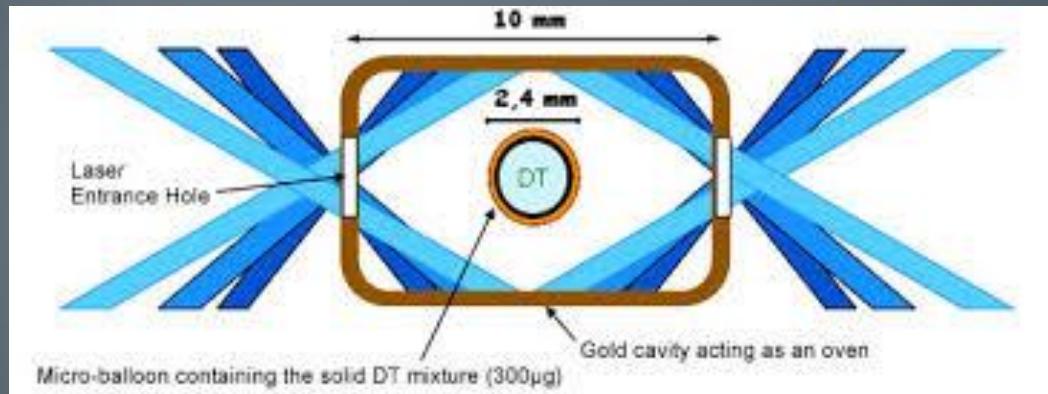
R. Kingham

Tester les modèles de flux délocalisés existants, développer un nouveau modèle de transport délocalisé (si nécessaire)

Plan

- I. Contexte / problématique du transport (électronique) de chaleur
- II. Les modèles numériques
- III. Calculs cinétiques 1D et 2D pour valider/améliorer les modèles simplifiées

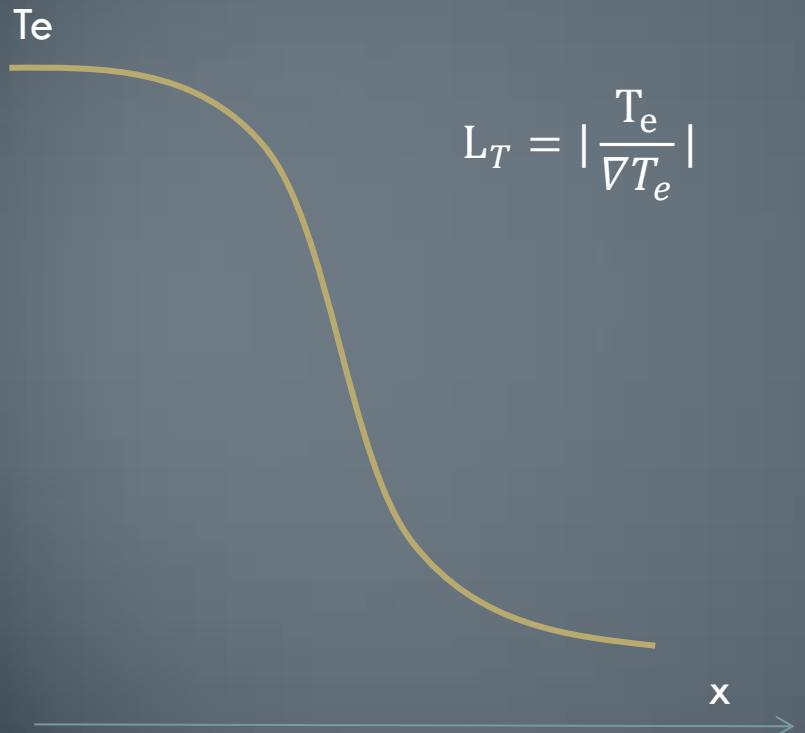
Contexte: FCI (attaque directe et indirecte) implosions magnétisées



Les phénomènes magnéto-hydrodynamiques (advection, détentes, chocs ..) sont dépendants du flux de chaleur

La mesure du flux de chaleur est indirecte & peu d'expériences permettent de valider les modèles existants

Flux de chaleur local



Flux de chaleur local

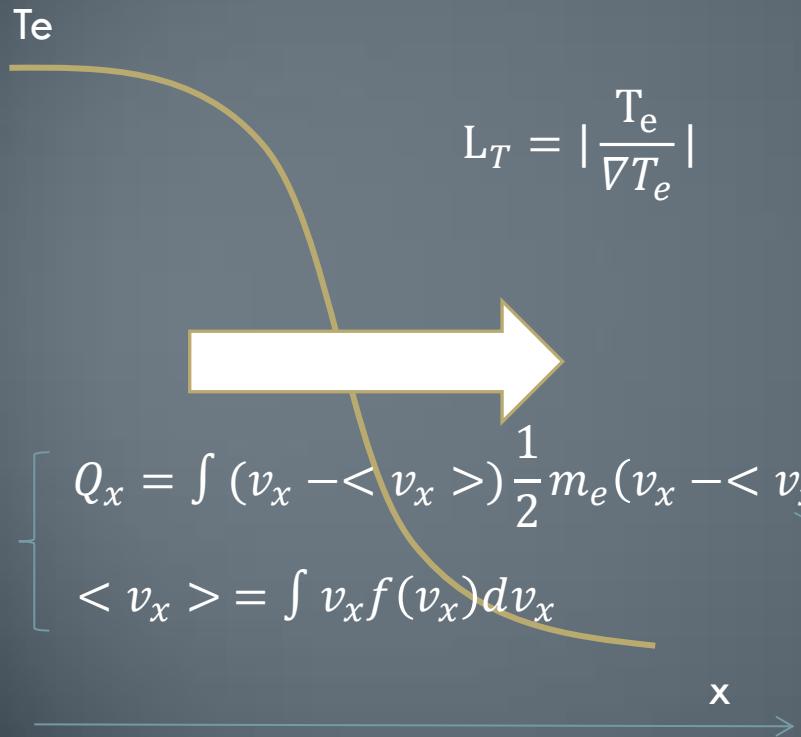
T_e

$$L_T = \left| \frac{T_e}{\nabla T_e} \right|$$

$$\left\{ \begin{array}{l} Q_x = \int (v_x - \langle v_x \rangle) \frac{1}{2} m_e (v_x - \langle v_x \rangle)^2 f(\vec{v}) d\vec{v} \\ \langle v_x \rangle = \int v_x f(v_x) dv_x \end{array} \right.$$

x

Flux de chaleur local



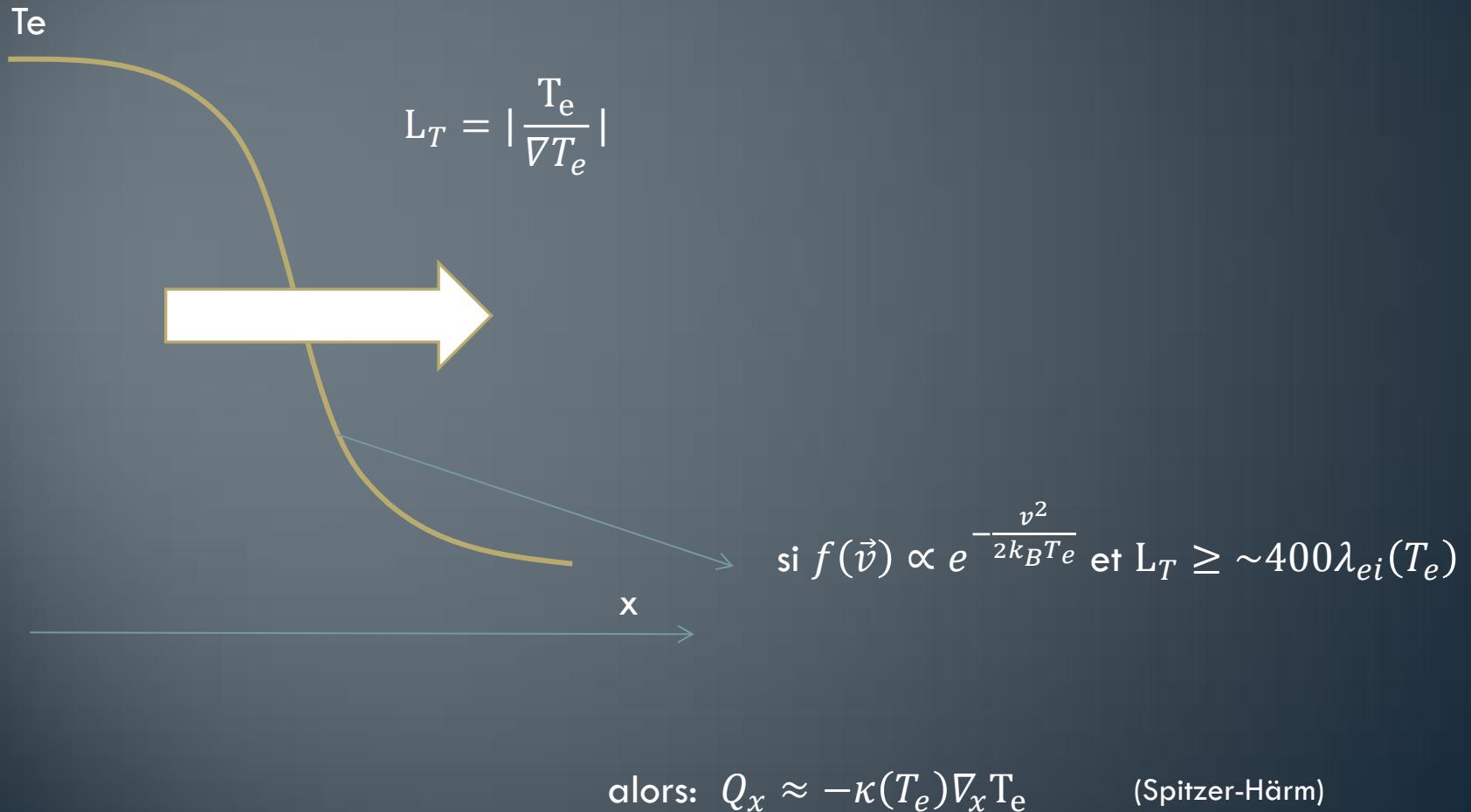
Le (gros) problème:

La théorie linéaire locale (Spitzer-Harm) implique que les électrons contribuant au flux de chaleur ont une vitesse de l'ordre de $3.7\nu_{th}$..

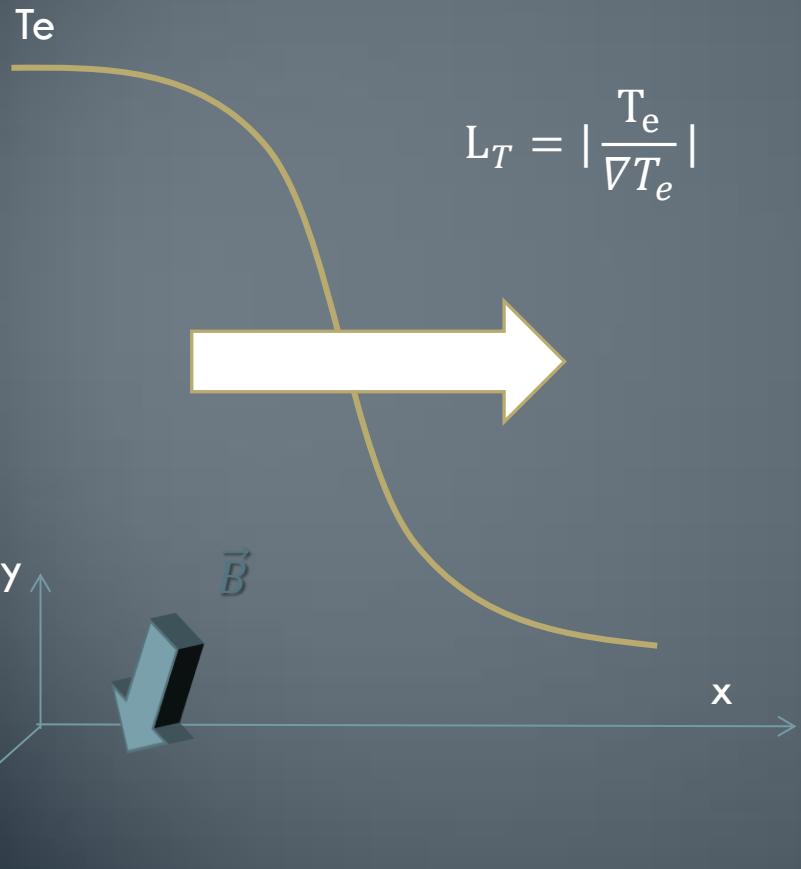
$\lambda_{ei} \propto v^4$ donc

$$L_T = \left| \frac{T_e}{\nabla T_e} \right| \geq \lambda_{ei} (3.7\nu_{th}) \sim 400 \lambda_{ei}(T_e) !!!$$

Flux de chaleur local

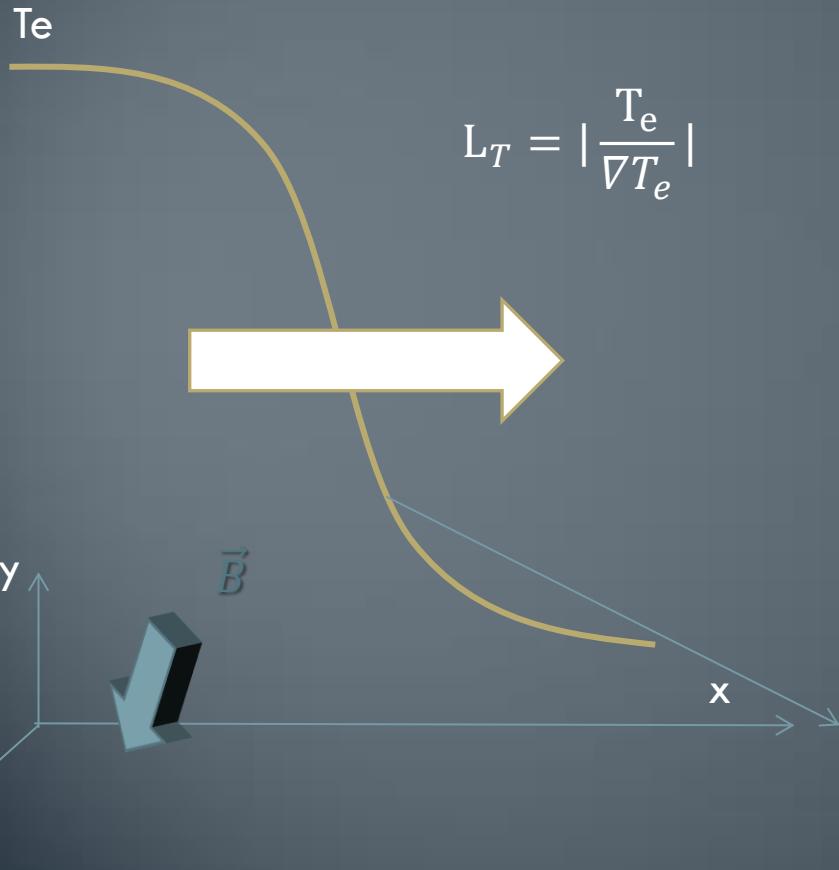


Flux de chaleur local



$$L_T = \left| \frac{\nabla T_e}{T_e} \right|$$

Flux de chaleur local



$$L_T = \left| \frac{\nabla T_e}{T_e} \right|$$

si $f(\vec{v}) \propto e^{-\frac{v^2}{2k_B T_e}}$ et $L_T \geq \sim 400\lambda_{ei}(T_e)$
et $\alpha = \frac{e|B|}{m_e v_{ei}} \ll 1$ Fréquence cyclotron sur fréquence de collision

alors: $Q \approx -\kappa_{\parallel}(T_e) \nabla_{\parallel} T_e - \kappa_{\perp}(T_e, \alpha) \nabla_{\perp} T_e - \kappa_{\wedge}(T_e, \alpha) \nabla_{\wedge} T_e$ (Braginskii, Epperlein-Haines)

Hiérarchie des modèles (classiques) pour estimer un flux de chaleur



Calcul cinétique:

Eq. de Vlasov + collisions
petits angles + Maxwell

Code PIC (Calder, Smilei, ...)

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Simplification des collisions, perturbation/simplification de la fonction de distribution

Calcul cinétique simplifié:

Modèles dits P1, M1... +
Maxwell (simplifié ou non)

Code P1 (Aladin, Impact)

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Pas d'inertie électronique

Cinétique simplifiée et stationnaire:

Modèles SNB, Bychenkov
et al...

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Flux de chaleur local:
Spitzer-Härm, Braginskii,
Epperlein-Haines

Equilibre cinétique partout
(plasma maxwellien)

Code PIC

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Maxwell ..

$$\operatorname{div} \vec{E} = \sum_{\alpha} Z_{\alpha}^* n_{\alpha} - n_e$$

$$\operatorname{rot} \vec{E} + \partial_t \vec{B} = \vec{0}$$

$$\operatorname{div} \vec{B} = 0$$

$$\operatorname{rot} \vec{B} - \partial_t \vec{E} = \sum_{\alpha} \vec{j}_{\alpha} + \vec{j}_e$$

avec: $n_{\alpha} = \iiint f_{\alpha}(\vec{x}, \vec{p}, t) dp^3$ $\vec{j}_{\alpha} = Z_{\alpha}^* \iiint \frac{\vec{p}}{\gamma} f_{\alpha}(\vec{x}, \vec{p}, t) dp^3$ $\gamma = \sqrt{1 + |\vec{p}|^2}$

+ Vlasov ...

$$\partial_t f_{\alpha} + \vec{v} \cdot \nabla_{\vec{x}} f_{\alpha} + \frac{Z_{\alpha}^*}{\tilde{m}_{\alpha}} \left(\vec{E} + \frac{\vec{p}}{\gamma} \wedge \vec{B} \right) \cdot \nabla_{\vec{p}} f_{\alpha} = \sum_{\beta} C_{\alpha\beta} + C_{\alpha\alpha} + C_{\alpha e}$$

$$\partial_t f_e + \vec{v} \cdot \nabla_{\vec{x}} f_e - \left(\vec{E} + \frac{\vec{p}}{\gamma} \wedge \vec{B} + \vec{F}_{rad} \right) \cdot \nabla_{\vec{p}} f_e = C_{ee} + \sum_{\beta} C_{e\beta}$$

Code PIC

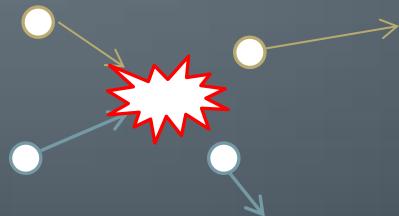
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+ Landau -Beliaev-Budker (collisions élastiques relativistes aux petits angles)

$$C_{\alpha\beta} = \frac{Z_\alpha^2 Z_\beta^2 \log \Lambda_{\alpha\beta}}{8\pi} \sum_\beta \frac{\partial}{\partial \vec{p}} \cdot \iiint U[\vec{p}, \vec{p}'] \times (f_\beta(\vec{p}') \partial_{\vec{p}} f_\alpha - f_\alpha(\vec{p}) \partial_{\vec{p}'} f_\beta) d\vec{p}'^3$$

$$U[\vec{p}, \vec{p}'] = \frac{(\gamma\gamma' - \vec{p} \cdot \vec{p}')^2 ((\gamma\gamma' - \vec{p} \cdot \vec{p}')^2 - 1) Id - \vec{p} \times \vec{p} - \vec{p}' \times \vec{p}' + [\gamma\gamma' - \vec{p} \cdot \vec{p}'][\vec{p} \times \vec{p} + \vec{p}' \times \vec{p}']} {\gamma\gamma' [(\gamma\gamma' - \vec{p} \cdot \vec{p}')^2 - 1]^{3/2}}$$



Modèle P1

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Maxwell ..

$$\operatorname{div} \vec{E} = \sum_{\alpha} Z_{\alpha}^* n_{\alpha} - n_e$$
$$\operatorname{div} \vec{B} = 0$$

$$\operatorname{rot} \vec{E} + \partial_t \vec{B} = \vec{0}$$
$$\operatorname{rot} \vec{B} - \partial_t \vec{E} = \sum_{\alpha} \cancel{\vec{J}_{\alpha}} + \vec{J}_e$$

Code cinétique dit d'approximation P1: *La dimensionnalité est réduite de 3v à 1v*

$$f_e = f_e^0 + \boldsymbol{v} \cdot \mathbf{f}_e^1 \quad f_e^0(\boldsymbol{v}) = \frac{1}{4\pi} \iint_{4\pi} f_e(\boldsymbol{v}) d\Omega \quad f_e^1(\boldsymbol{v}) = \frac{3}{4\pi} \iint_{4\pi} \frac{\boldsymbol{v}}{|\boldsymbol{v}|} f_e(\boldsymbol{v}) d\Omega$$

Cinétique « simplifiée »

$$\partial_t f_e^0 + \frac{\boldsymbol{v}}{3} \nabla_{\boldsymbol{x}} \cdot \mathbf{f}_e^1 - \frac{\mathbf{E}}{3v^2} \cdot \partial_{\boldsymbol{v}} (v^2 \mathbf{f}_e^1) = \frac{Y_{ee}}{v^2} \partial_{\boldsymbol{v}} \left[f_e^0 F(v) + \frac{D(v)}{v} \partial_{\boldsymbol{v}} f_e^0 \right]$$

$$\partial_t f_e^1 + v \nabla_{\boldsymbol{x}} f_e^0 - \mathbf{E} \partial_{\boldsymbol{v}} f_e^0 - \mathbf{B} \wedge \mathbf{f}_e^1 = - \frac{n_i Y_{ei}}{v^3} \mathbf{f}_e^1$$

Modèle SNB [1,2]

Maxwell ..

$$\operatorname{div} \vec{E} = 0$$

$$\operatorname{rot} \vec{E} + \partial_t \vec{B} = \vec{0}$$

$$\operatorname{div} \vec{B} = 0$$

$$\operatorname{rot} \vec{B} = \vec{j}_e$$

$$f_e = f_e^0 + v \cdot f_e^1$$

$$f_e^0 = \text{maxwellienne} + \Delta \mathbf{f}^0$$

$$f_e^1 = f_e^{1,m} + \Delta \mathbf{f}^1$$

Cinétique P1, stationnaire

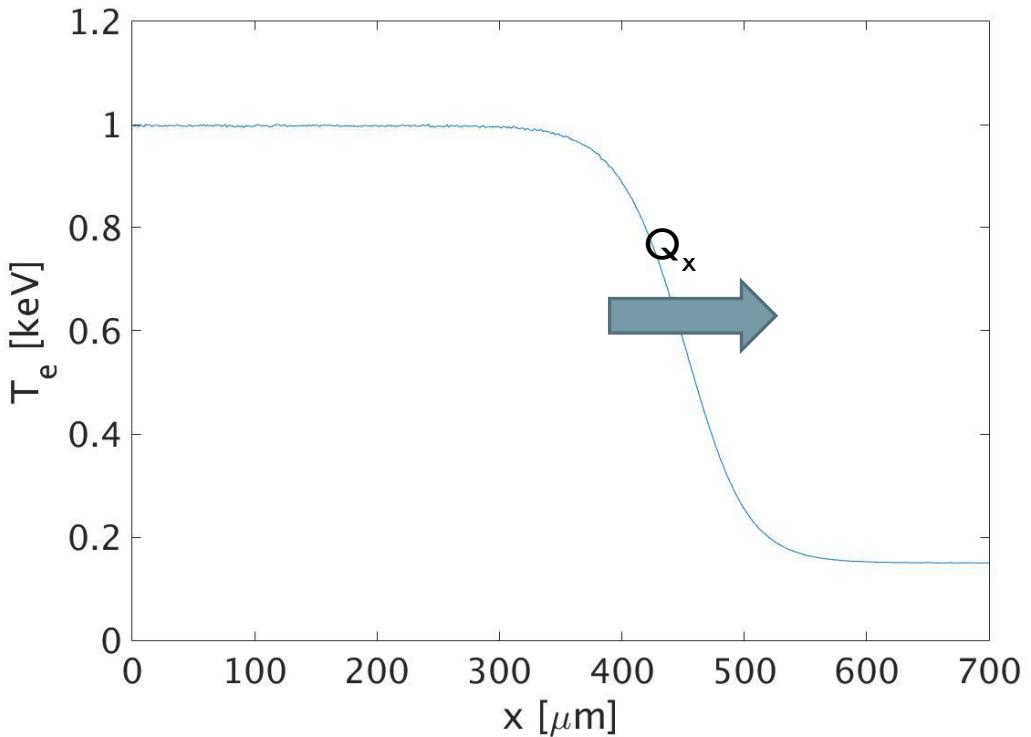
$$\cancel{\partial_t f_e^0} + \frac{v}{3} \nabla_x \cdot f_e^1 - \frac{\mathbf{E}}{3v^2} \cdot \partial_v (v^2 f_e^1) = \frac{Y_{ee}}{v^2} \partial_v \left[f_e^0 F(v) + \frac{D(v)}{v} \partial_v f_e^0 \right]$$

$$\cancel{\partial_t f_e^1} + v \nabla_x f_e^0 - \mathbf{E} \partial_v f_e^0 - \mathbf{B} \wedge f_e^1 = -\frac{n_i Y_{ei}}{v^3} f_e^1$$

[1] Schurtz, Nicolaï & Busquet, Phys. Plasmas 7, 4238 (2000)

[2] Nicolaï, Feugeas & Schurtz, Phys. Plasmas 13, 032711 (2006)

Calcul de flux de chaleur non-local PIC 1D3V



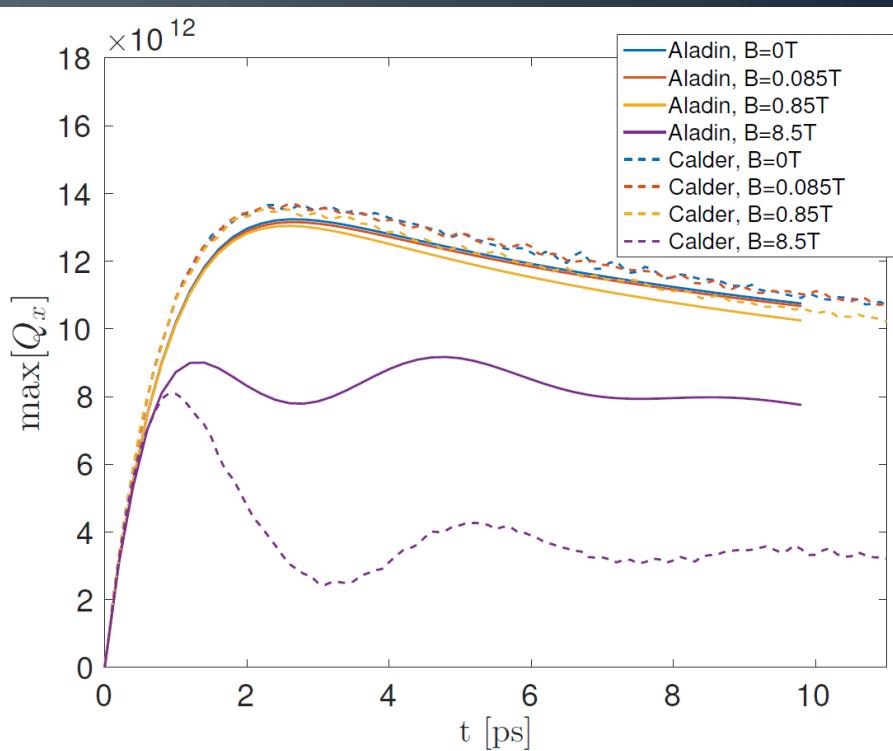
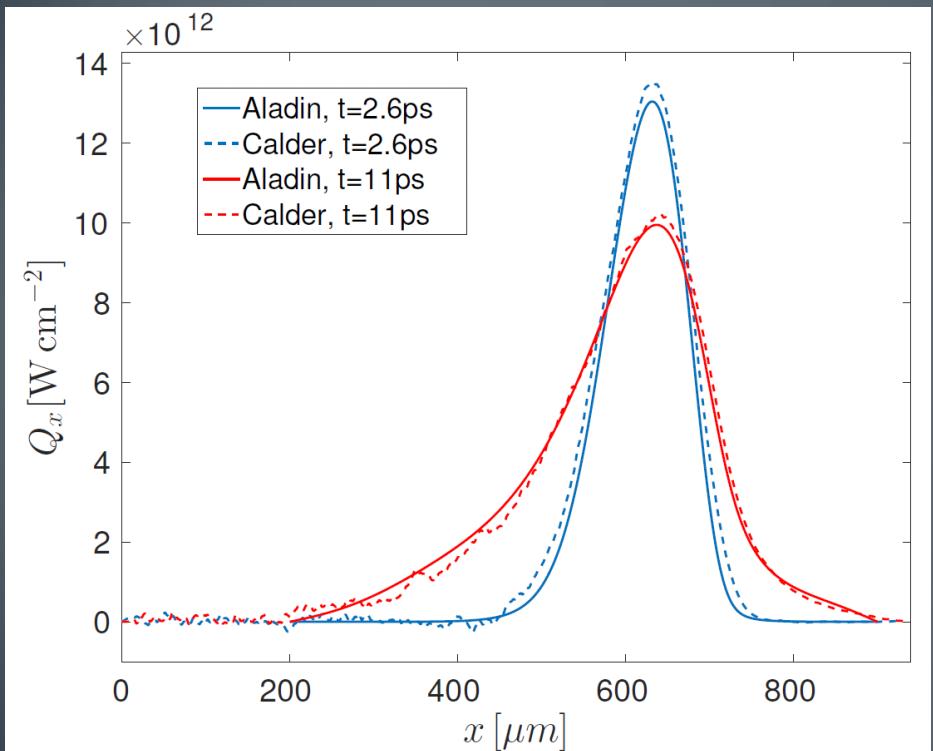
Formation d'un flux de chaleur avec un gradient de température dans un plasma ionisé de densité constante

$$L_T = \left| \frac{T_e}{\nabla T_e} \right| = 2.5\lambda_{ei}(1 \text{ keV}) - 100\lambda_{ei}(175 \text{ eV})$$

$Z=2$, $L=50 \mu\text{m}$

$$\alpha = \frac{e|B|}{m_e v_{ei}} = (0, 0.01, 0.1, 1)$$

$B_z=0.85\text{T}$



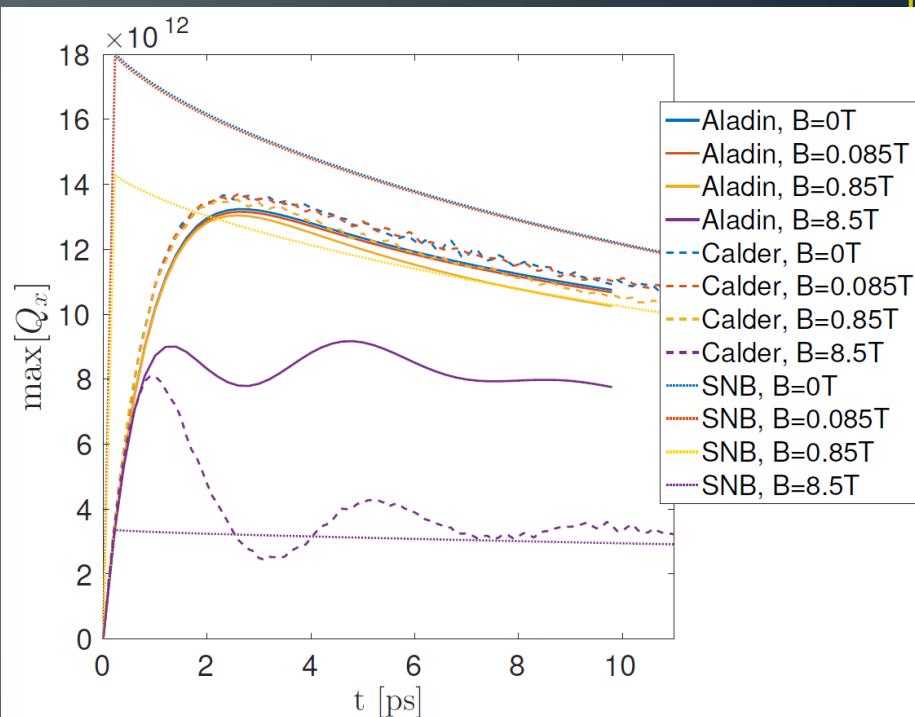
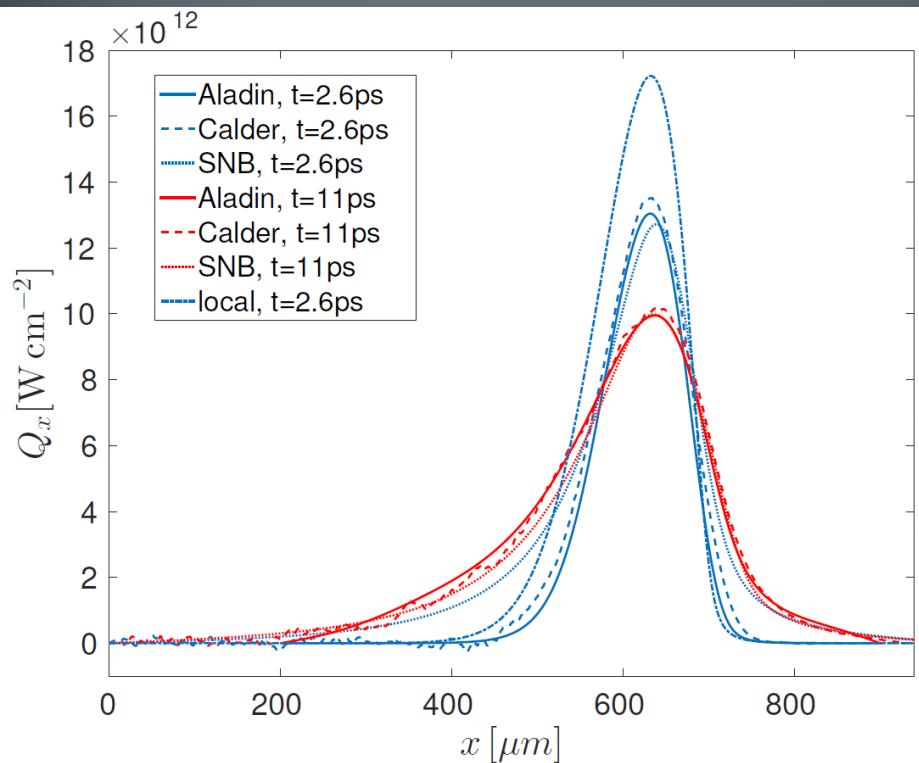
Calder (PIC) ~ 24h sur 1000 processeurs

Aladin (modèle P1) ~ 30 min sur 1 processeur

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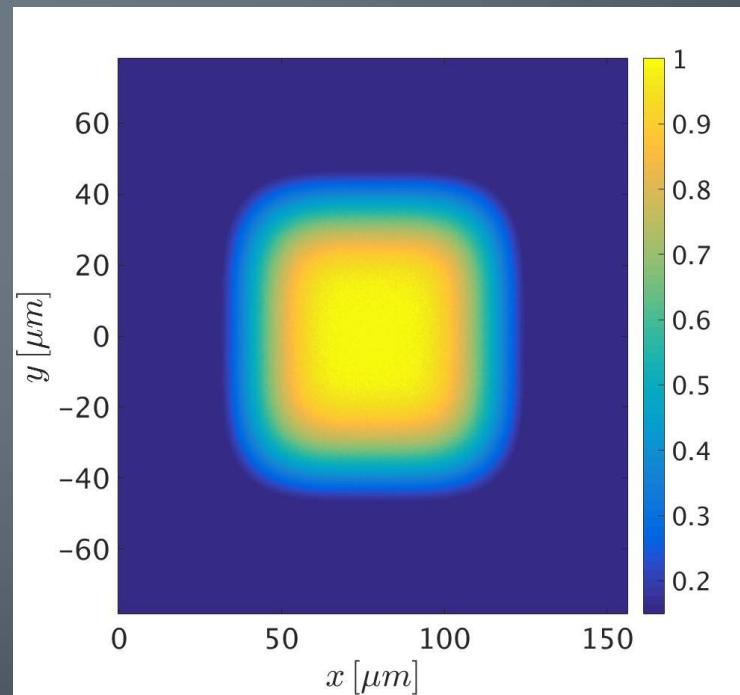
Aladin (modèle P1) $\sim 30 \text{ min}$ sur 1 processeur

SNB dans code hydro \sim minute sur 1 processeur

Cas 2D

$$Te(x, y) = \begin{cases} e^{-\frac{x^4}{40^4}-\frac{y^4}{40^4}} & \text{si } T_e > 0.15 \text{ keV} \\ 0.15 & \text{otherwise} \end{cases}$$

Z=2

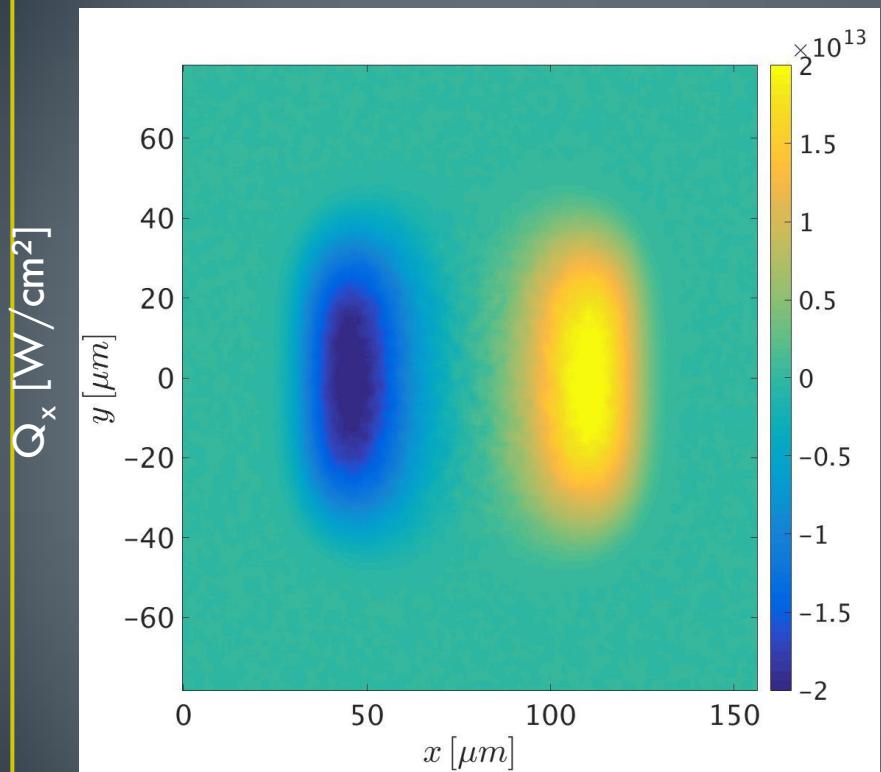


Cas 2D ($B_z=0T$, 0.5ps)

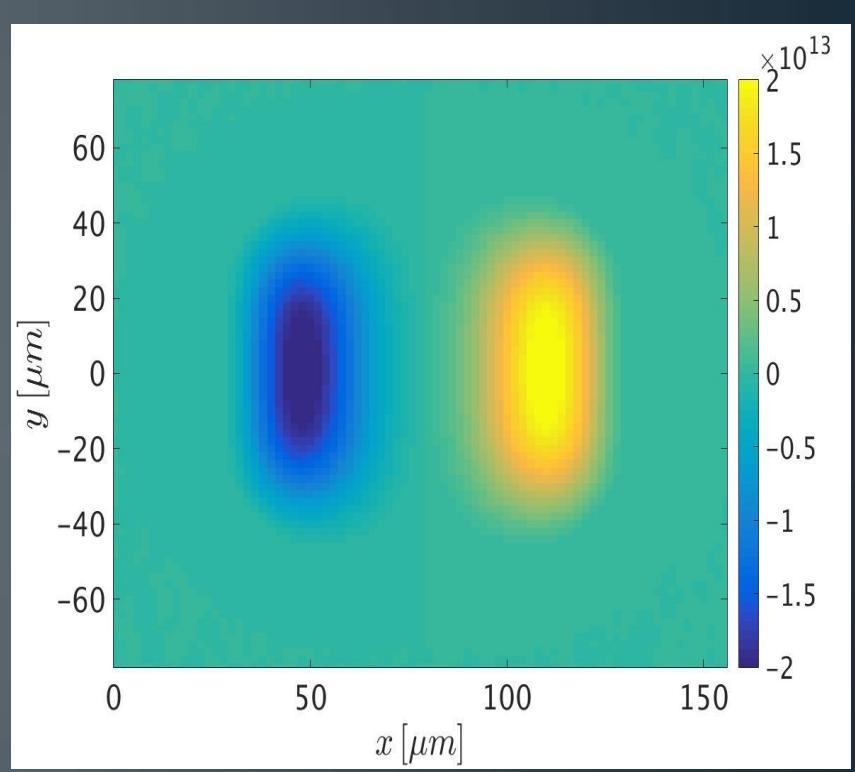
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PIC



Aladin (P1)

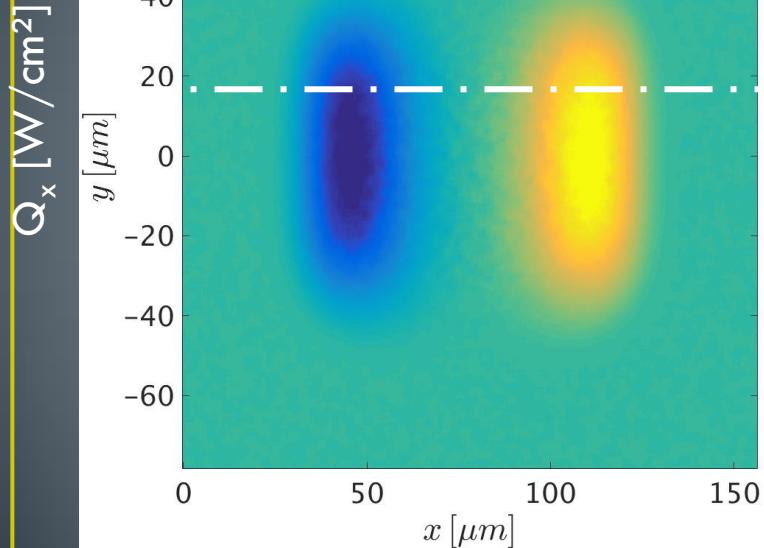


Cas 2D ($B_z=0\text{T}$, 0.5ps)

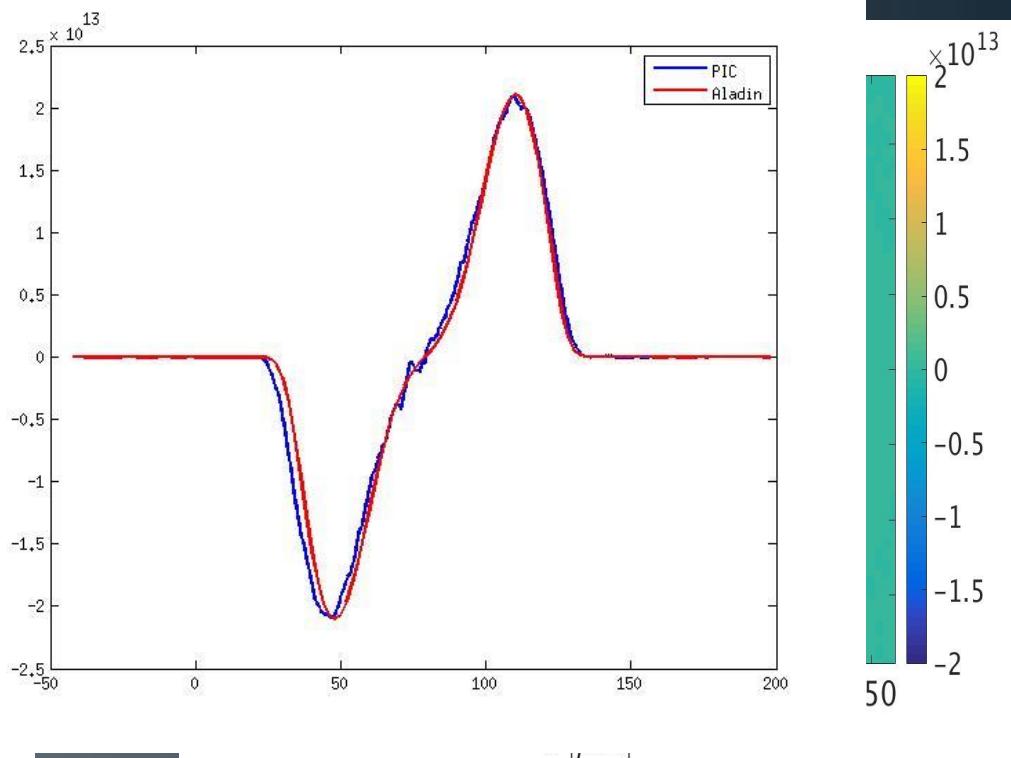
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PIC



Aladin (P1)



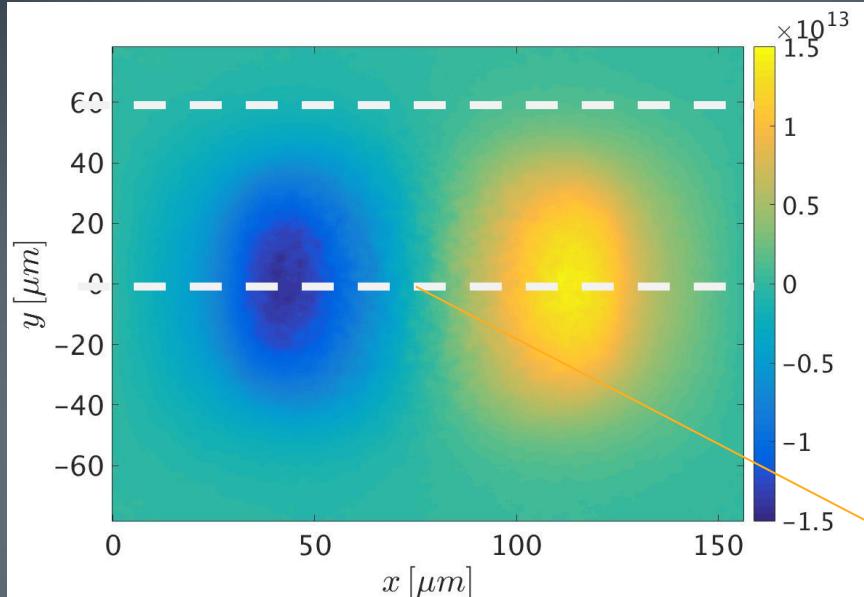
Cas 2D ($B_z=0T$, 2ps)

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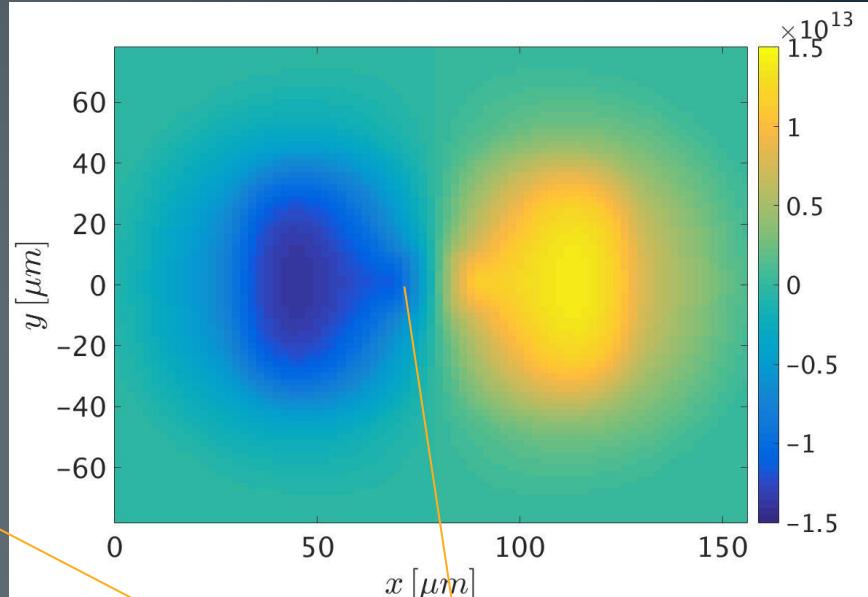


PIC

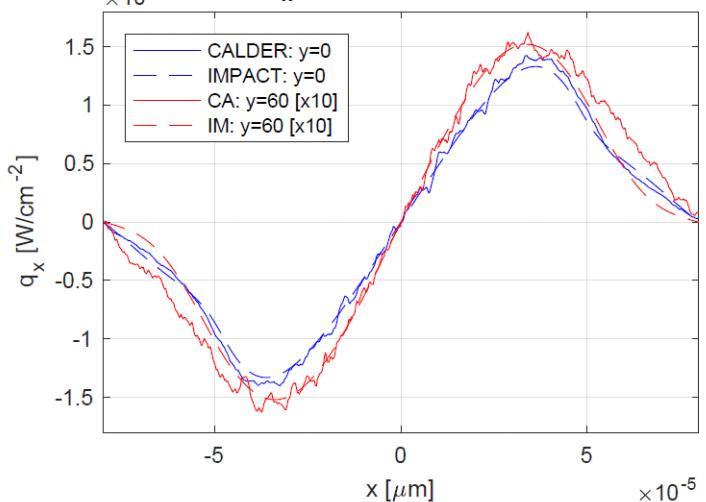
Q_x [W/cm²]



Aladin (P1)



$q_x(x)$ [W/cm⁻²] t=2ps



Impact (P1)

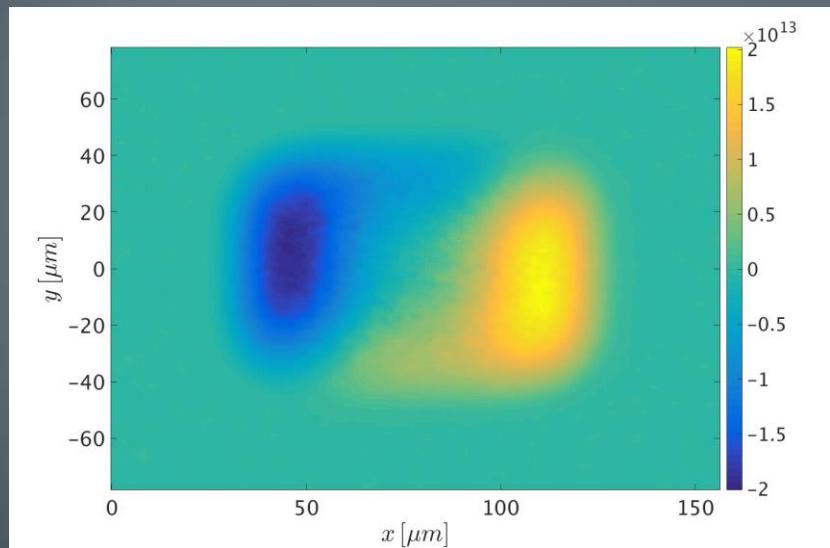
Léger désaccord au temps long avec Aladin (P1) mais bon accord avec impact (P1)

Cas 2D ($B_z=8.5\text{T}$, 0.5ps)

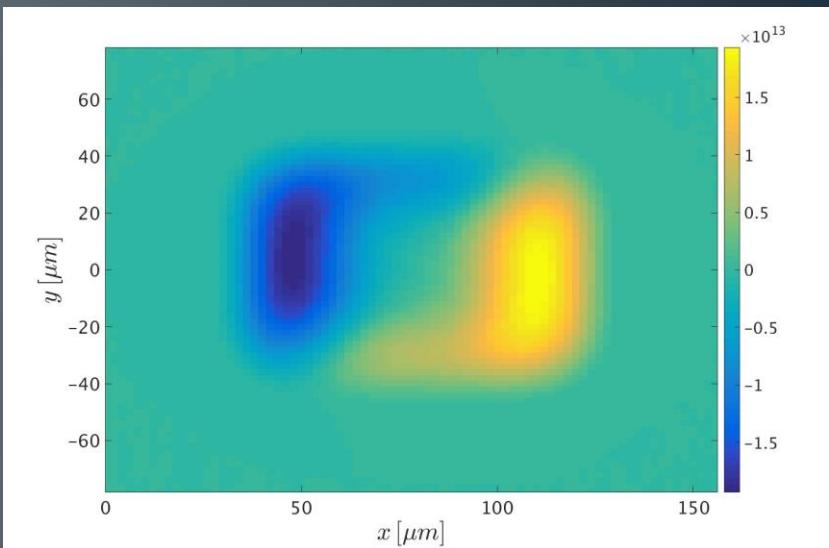
$$\alpha = \frac{e|B|}{m_e v_{ei}} \sim 1$$

PIC

$Q_x [\text{W/cm}^2]$



Aladin (P1)



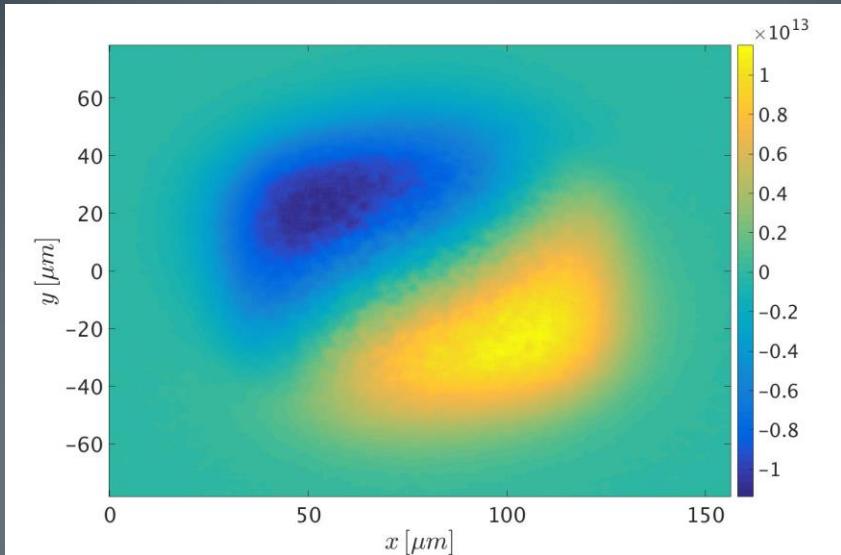
Cas 2D ($B_z=8.5\text{T}$, 2ps)

DE LA RECHERCHE À L'INDUSTRIE

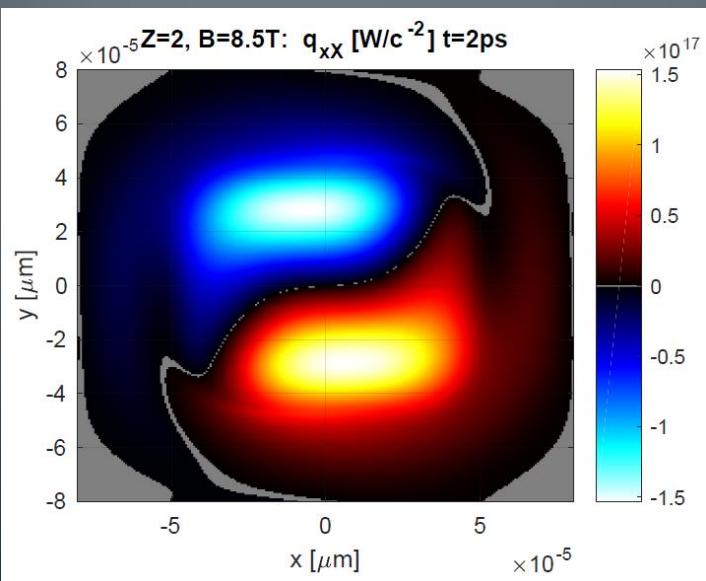
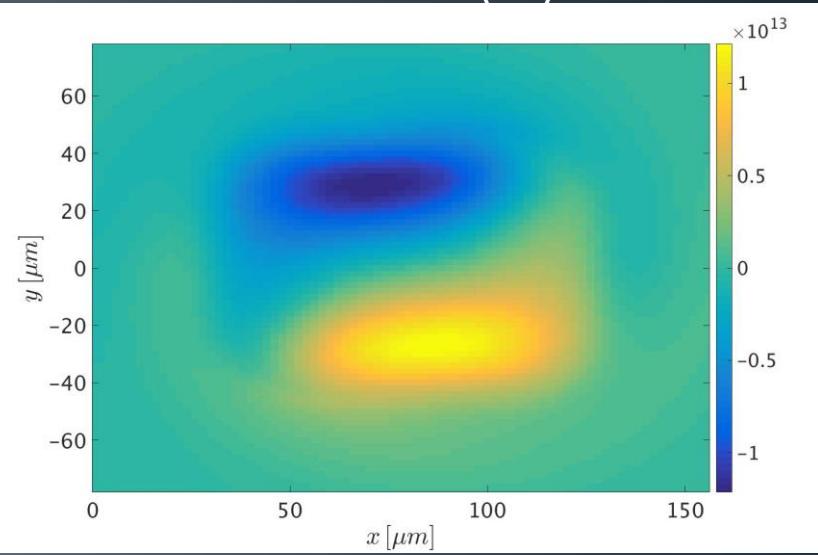


PIC

$Q_x [\text{W/cm}^2]$



Aladin (P1)



Impact (P1)

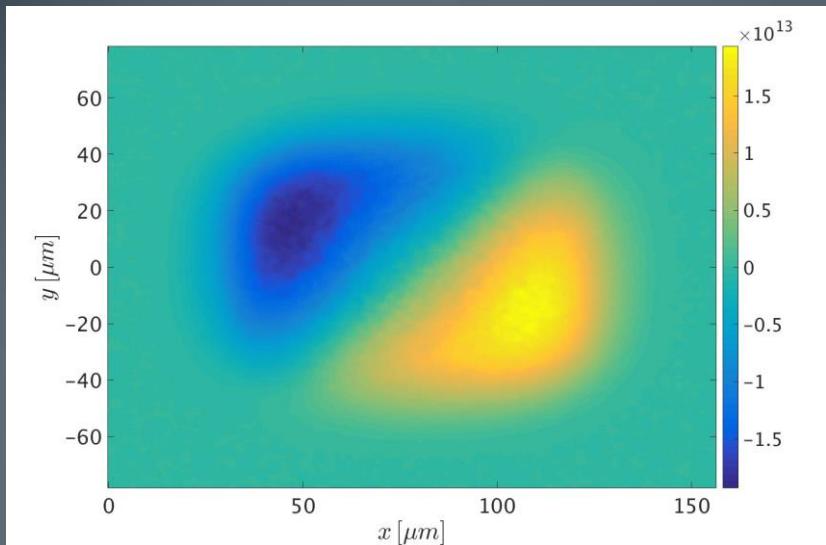
Conclusion/Perspectives

- Code PIC efficace (mais coûteux!) pour tester les modèles de flux de chaleur
- Excellent accord entre les modèles simplifiés P1 et le calcul cinétique complet en 1D
- Tester les flux de chaleur en 2D avec champ magnétique (en cours)
- Jusqu'où peut-on pousser le modèle SNB (utilisé dans les codes hydrodynamiques)
- Extraire les effets Nernst, Righi-Leduc, Peltier, Ettingshausen avec le code PIC (?)

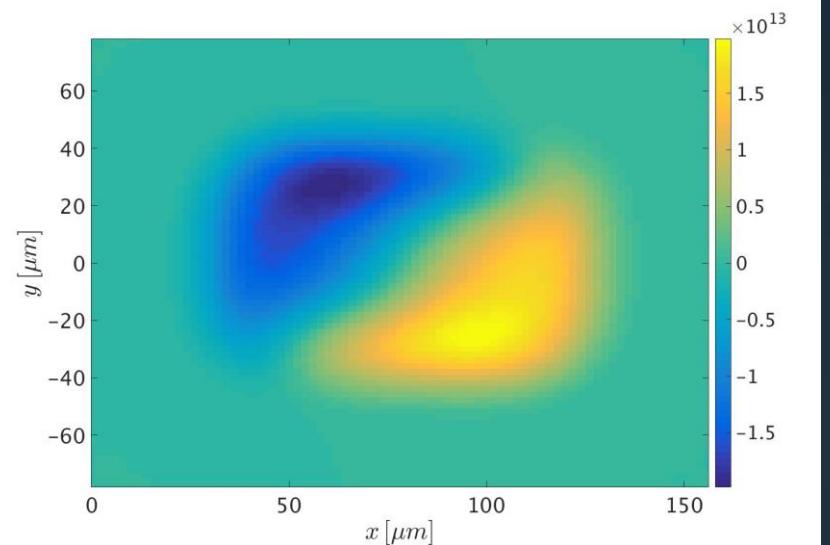
Cas 2D ($B_z = 8.5\text{T}$, 1ps)

PIC

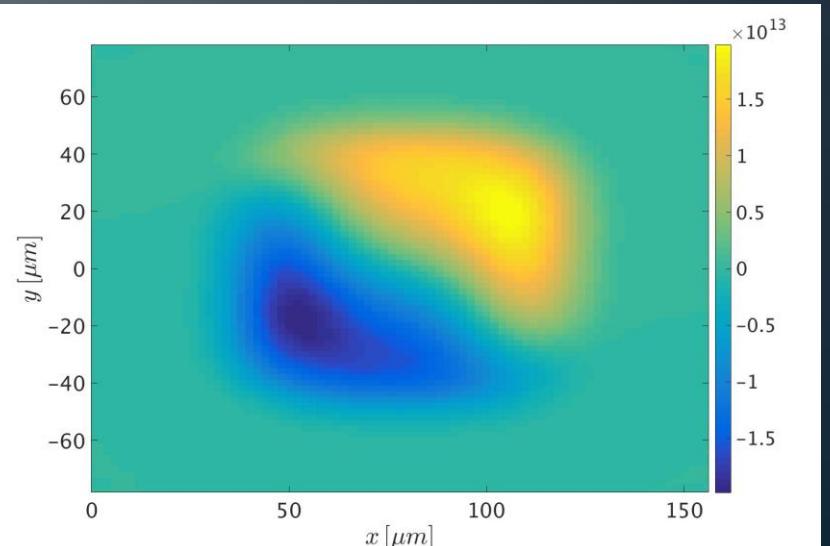
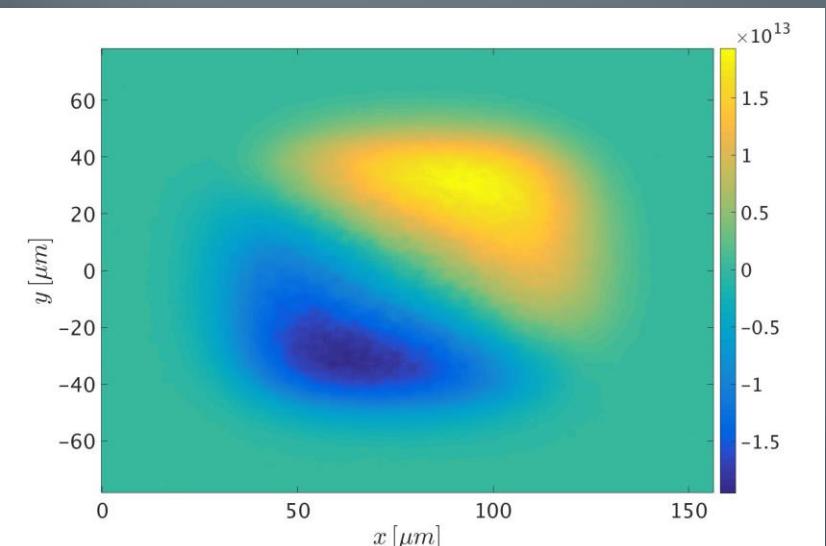
$Q_x [\text{W/cm}^2]$



Aladin (P1)

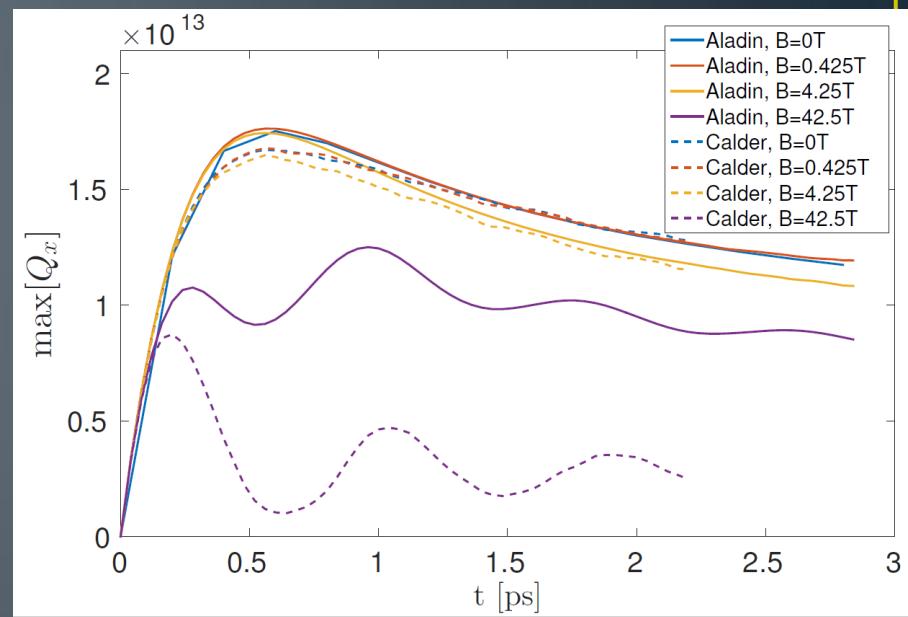
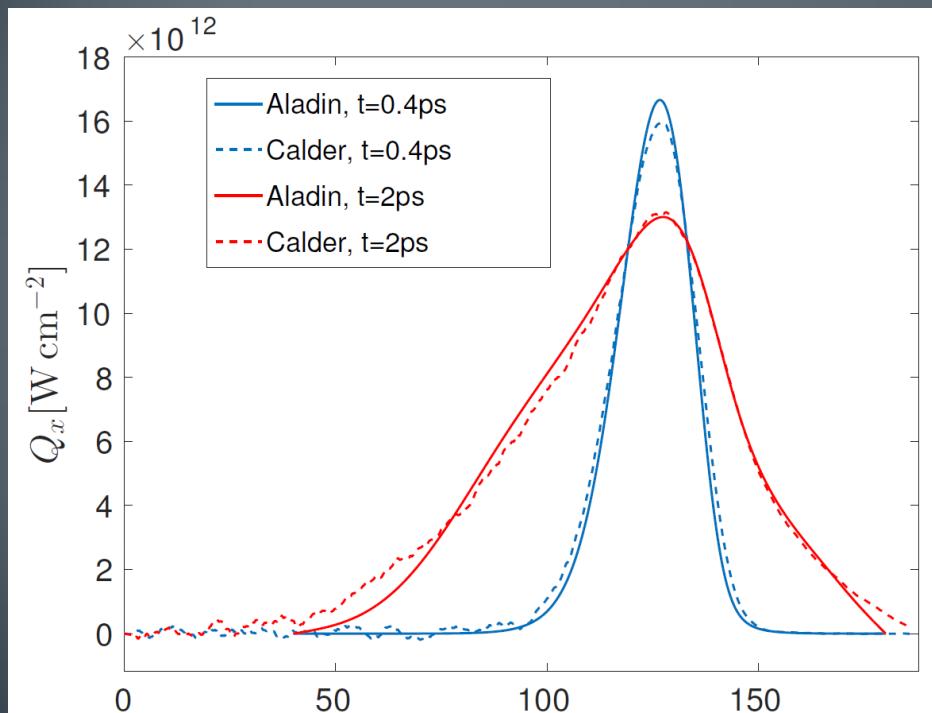


$Q_y [\text{W/cm}^2]$



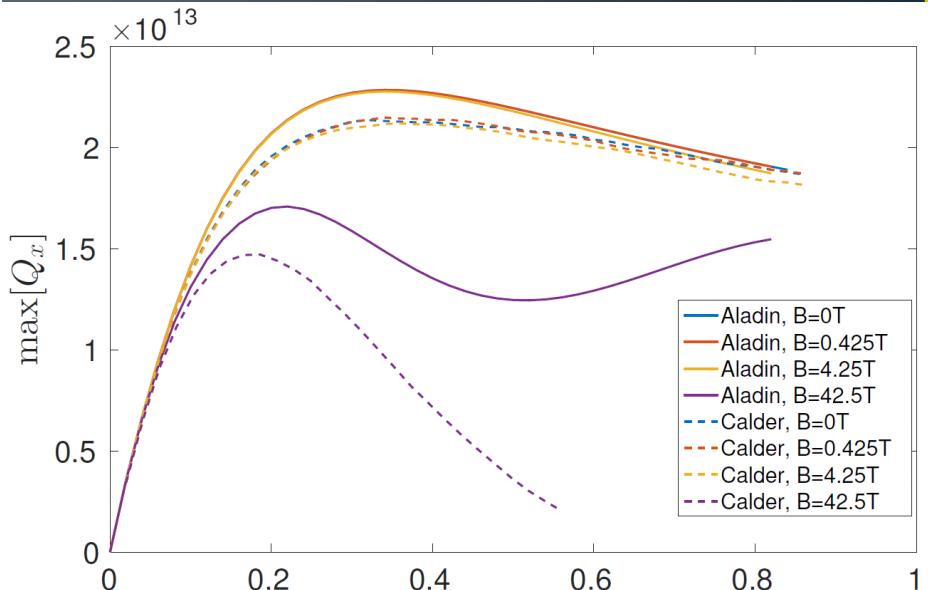
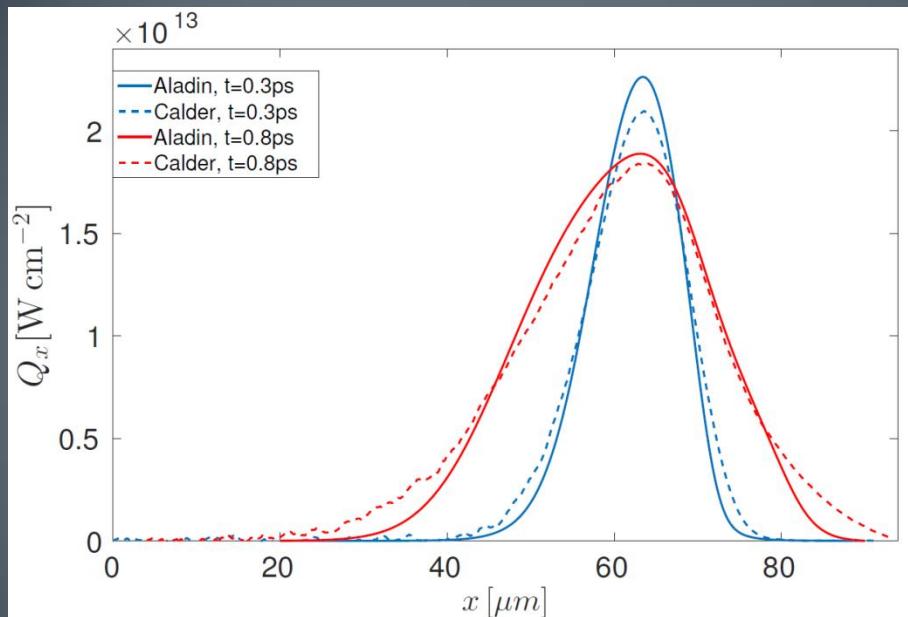
$Z=10$, $L=10 \mu\text{m}$

$B_z=0\text{T}$

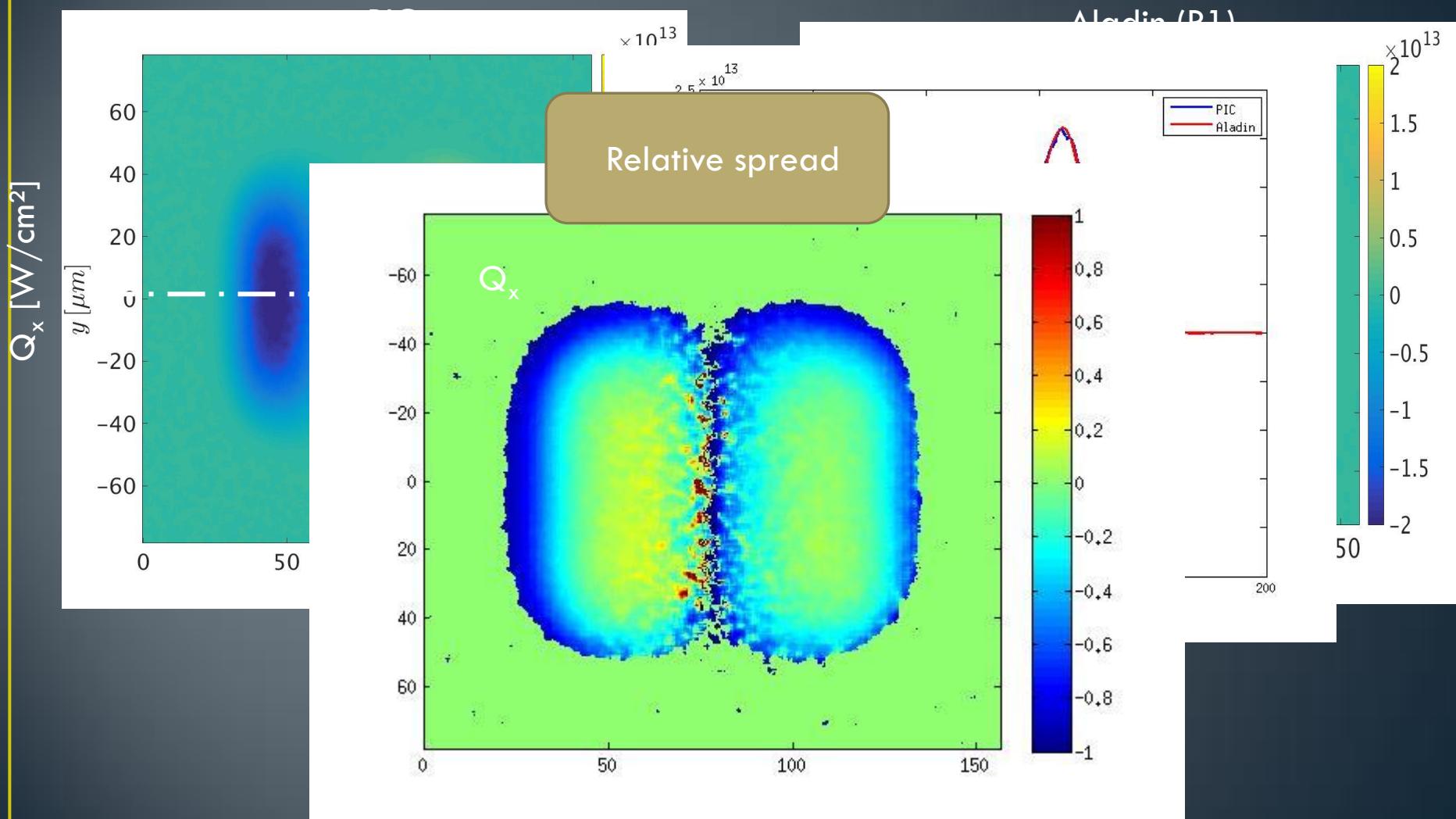


$Z=10$, $L=5 \mu\text{m}$

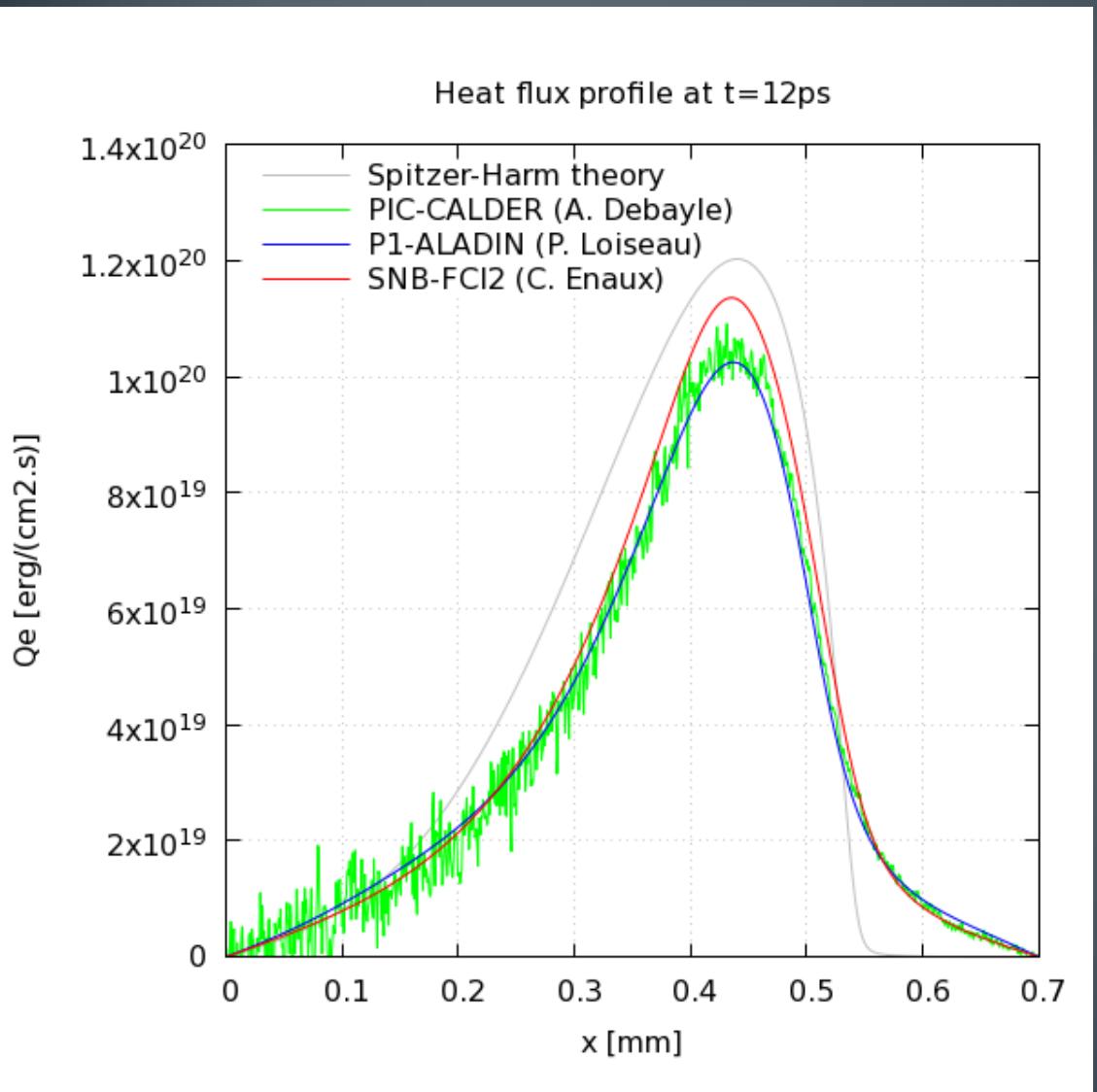
$Bz=4.25\text{T}$



Cas 2D ($B_z=0T$, 0.5ps)



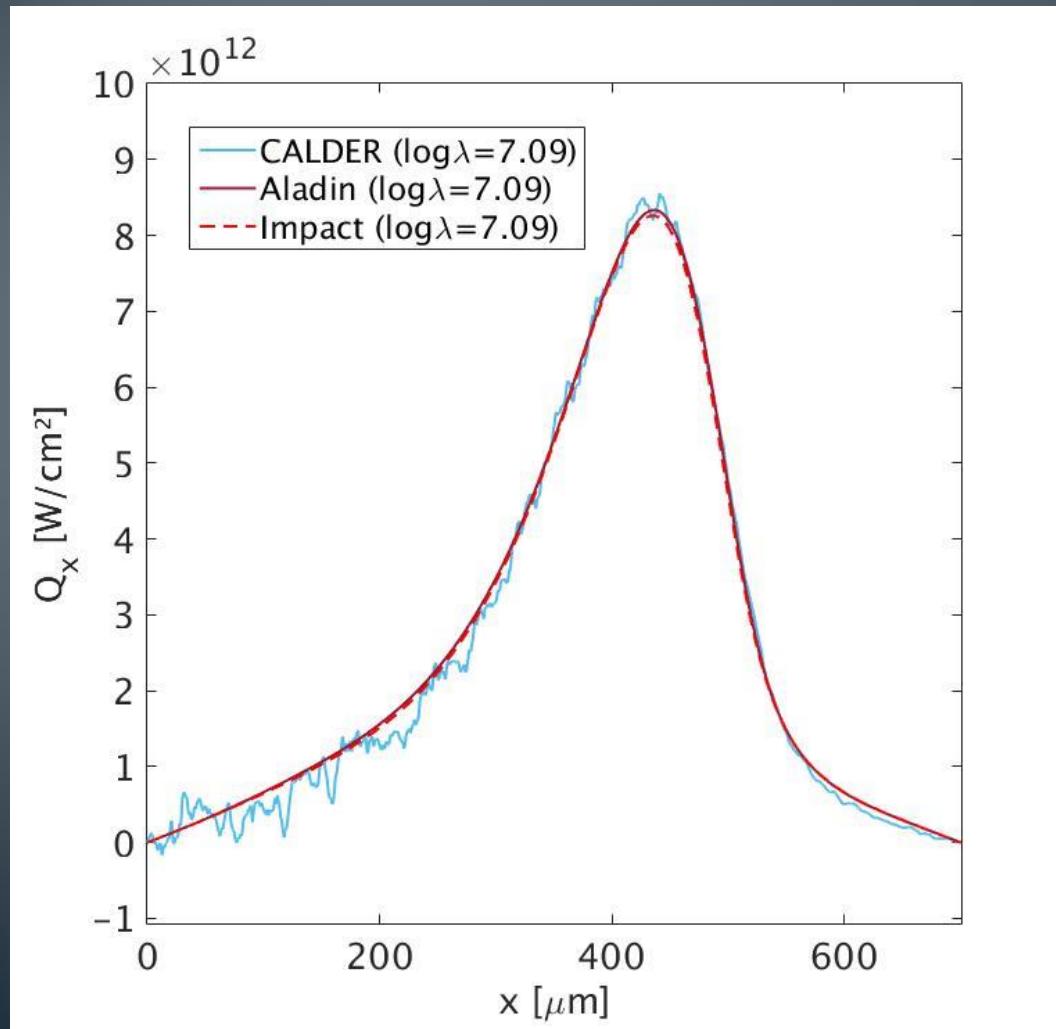
Comparaison Z=2



$$L_T = \left| \frac{T_e}{\nabla T_e} \right| = 2.5\lambda_{ei}(1 \text{ keV}) - 100\lambda_{ei}(175 \text{ eV})$$

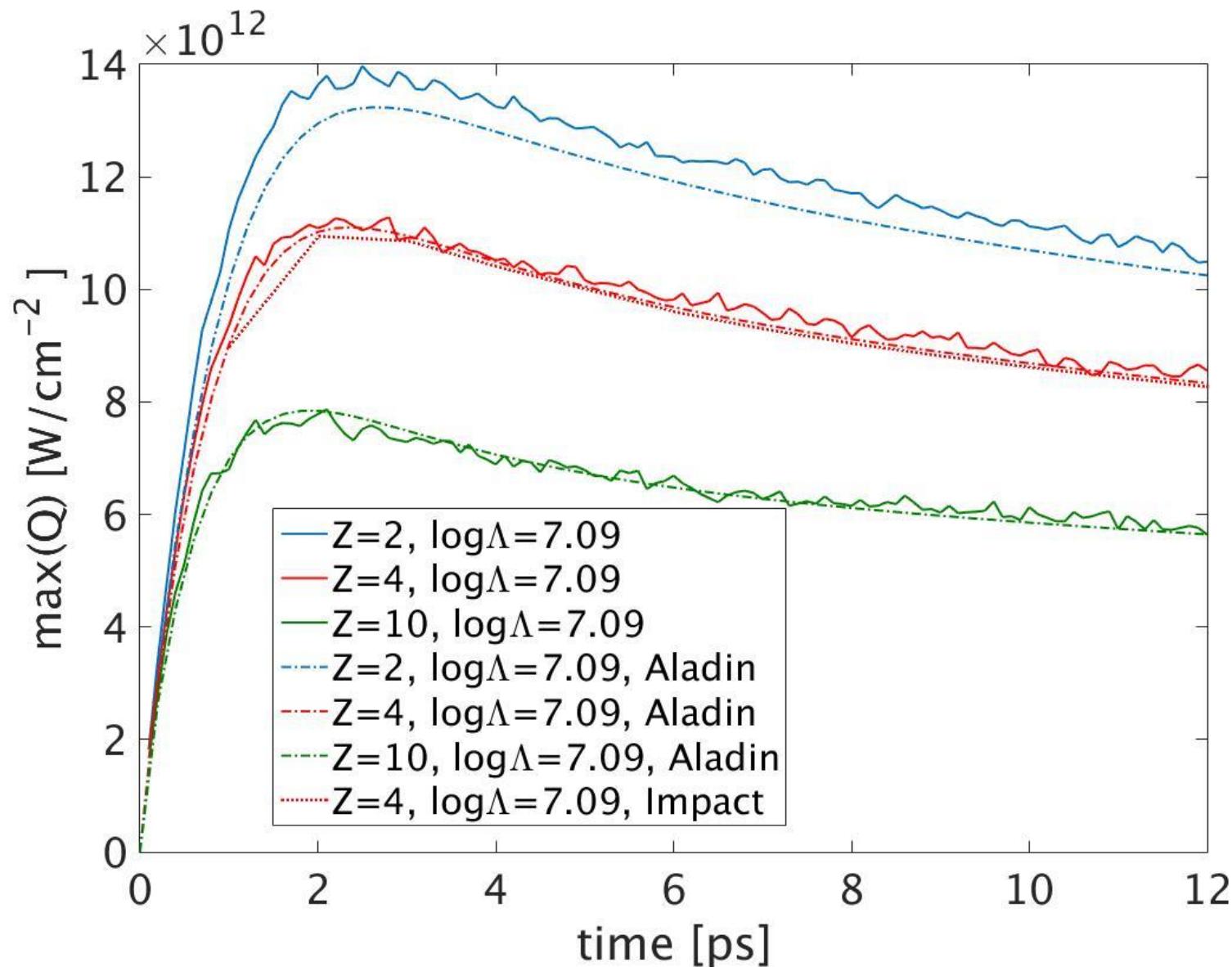
Comparaison Z=4

Q flux @ 12 ps



Impact=code cinétique
simplifié anglais (modèle P1)

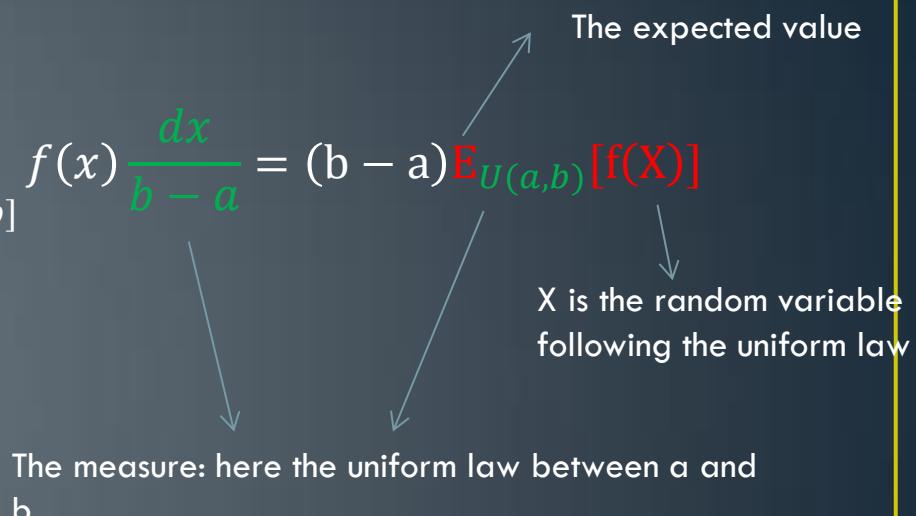
Comparaison du maximum du flux en temps (Z=2,4,10)



Appendix D

Some examples of the MC spirit: the integral..

$$A = \int_{[a,b]} f(x)dx = (b-a) \int_{[a,b]} f(x) \frac{dx}{b-a} = (b-a) E_{U(a,b)}[f(X)]$$



To estimate A, just calculate:

$$A \approx (b-a) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

with the expectation x_i selected according the random law $U(a,b)$

Appendix D

Some examples of the MC spirit: the Vlasov equation and the PIC code

$$\left\{ \begin{array}{l} \partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \cdot \nabla_{\mathbf{p}} f = 0 \\ f(t=0, \mathbf{x}, \mathbf{p}) = g(\mathbf{x}, \mathbf{p}) \end{array} \right.$$

Following the characteristics:

$$\left\{ \begin{array}{l} \frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \\ \frac{d\mathbf{x}}{dt} = \mathbf{v} \end{array} \right. \quad \text{we have: } \frac{df(t, \mathbf{x}(t), \mathbf{p}(t))}{dt} = 0$$

Thus...

$$f(t, \mathbf{x}(t), \mathbf{p}(t)) = f(0, \mathbf{x}(0), \mathbf{p}(0)) = g(\mathbf{x}, \mathbf{p})$$

To introduce the analogy with the ray tracing: note that the Jacobian $J(t) = 1$ here because in virtue of the Liouville theorem:

$$\frac{1}{J} \frac{dJ}{dt} = \operatorname{div}_{\mathbf{p}, \mathbf{x}} \left[\frac{q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B})}{\mathbf{v}} \right] = 0$$

Appendix D

In the MC spirit, we want to introduce an expected value. To do so, we must accept to approximate the distribution function on a small phase space volume.

Let's introduce the indicator function (in 1D1V for simplicity) :

$$\phi_{i,j}(x, p) = \begin{cases} \frac{1}{\Delta p \Delta x} & \text{if } x_i \leq x < x_{i+1} \text{ and } p_j \leq p < p_{j+1} \\ 0 & \text{else} \end{cases}$$

One must verify:
 $\int_{R^2} \phi_{i,j} dx dp = 1$

and: $\alpha = \int_{R^6} g(x, p) dx dp$ (which by the way « corresponds » to the number of particles)

Then the distribution function in the vicinity of $\phi_{i,j}$, noted $F_{i,j}$ is:

$$F_{i,j} = \int_{R^6} \phi_{i,j}(x_t, p_t) f(t, x_t, p_t) dx_t dp_t = \int_{R^6} \phi_{i,j}(x_t, p_t) g(x_0, p_0) dx_t dp_t$$

where x_t and p_t stand for $x(t)$ $p(t)$ and x_0 p_0 for $x(0)$ and $p(0)$

Appendix D

$$F_{i,j} = \int_{R^6} \phi_{i,j}(x_t, p_t) g(x_0, p_0) dx_t dp_t$$

$$F_{i,j} = \int_{R^6} \phi_{i,j}(x_t, p_t) g(x_0, p_0) J(t) dx_0 dp_0 \quad \text{where}$$

$$\begin{cases} J(t) = \det |M| = 1! \\ M_{ij} = \frac{\partial x_t p_t}{\partial x_0 p_0} \end{cases}$$

Because the phase space is conserved ($J(t)=1$) we thus have:

$$F_{i,j} = \alpha \int_{R^6} \phi_{i,j}(x_t, p_t) \frac{g(x_0, p_0)}{\alpha} dx_0 dp_0 = \alpha E_{g(x,p)} [\phi_{i,j}(X_t, P_t)]$$

X_t and P_t are the random variables, following the deterministic characteristics with the initial condition x_0, p_0 following the initial law given in green

The measure: here the normalized initial distribution function

$$F_{i,j} \approx \frac{\alpha}{N} \sum_{k=1}^N \phi_{i,j}[x_k(t), p_k(t)]$$

The mean value of the distribution function at the cell $\phi_{i,j}$ is given by the statistical mean of the “macro-particles” sited inside.