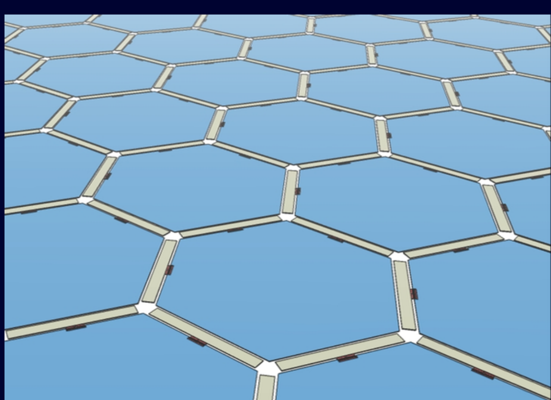
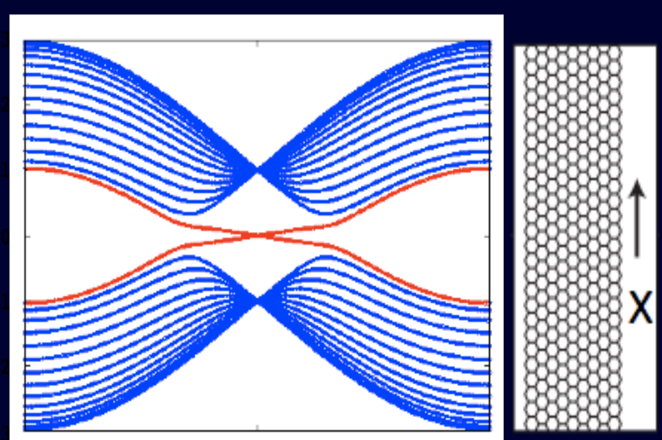
MPIPKS Dresden 2012: From Nano to Macro-scale
June 26th 2012, QUAM 2012

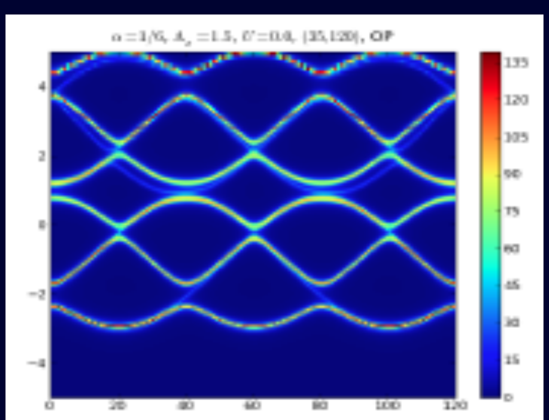
Correlations & Topological Phases

Karyn Le Hur

Ecole Polytechnique, France & CNRS







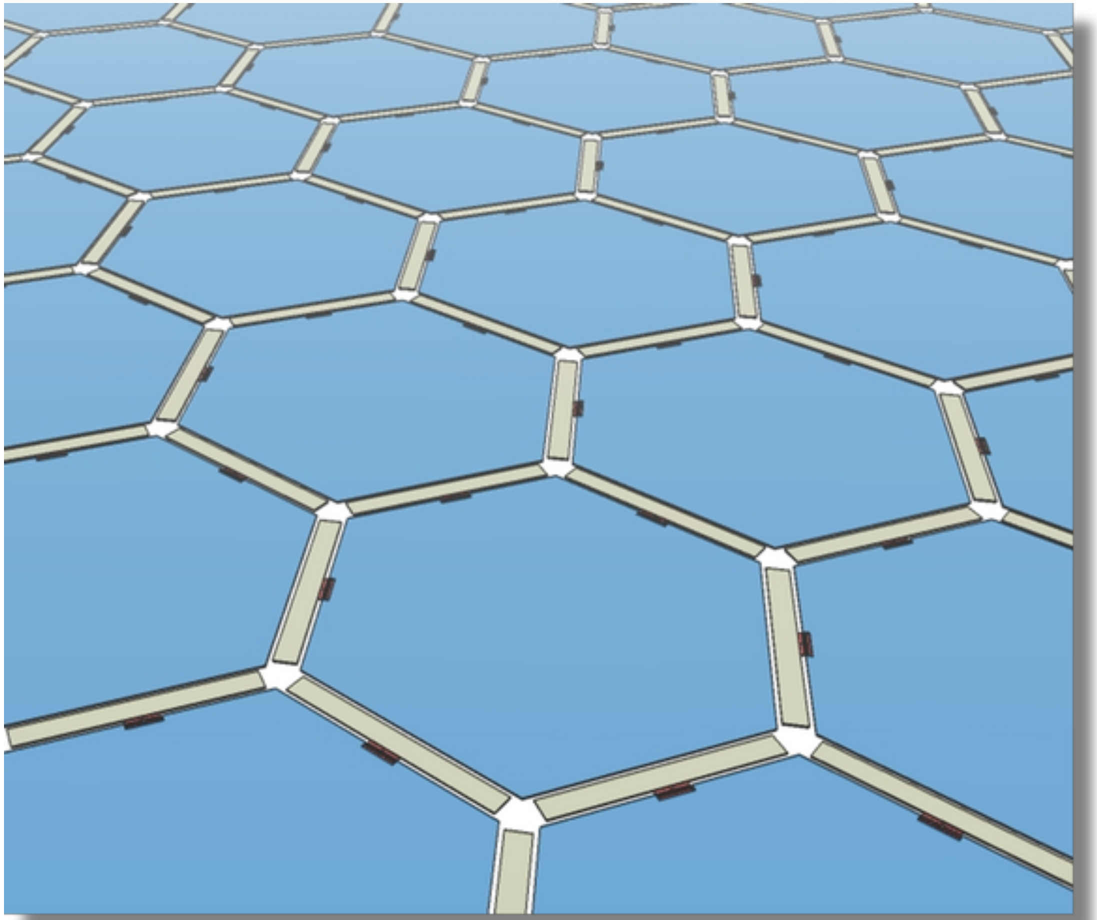
Special Acknowledgment

Bernard Coqblin

Heavy Fermion Systems and Kondo Effect

Coqblin-Schrieffer Model 1969

New phases of matter on the Honeycomb lattice (**example**: the Kitaev model; next talk)



Topological Insulators
Interaction Effects
(Mott physics)

Spin Liquid
d+id Superconductivity
(Laughlin, 1997)

Photons & CQED (A. Houck):
Quantum Hall phases
following Haldane & Raghu
Experiment at MIT, Soljacic
Also M. Hafezi et al. 2011

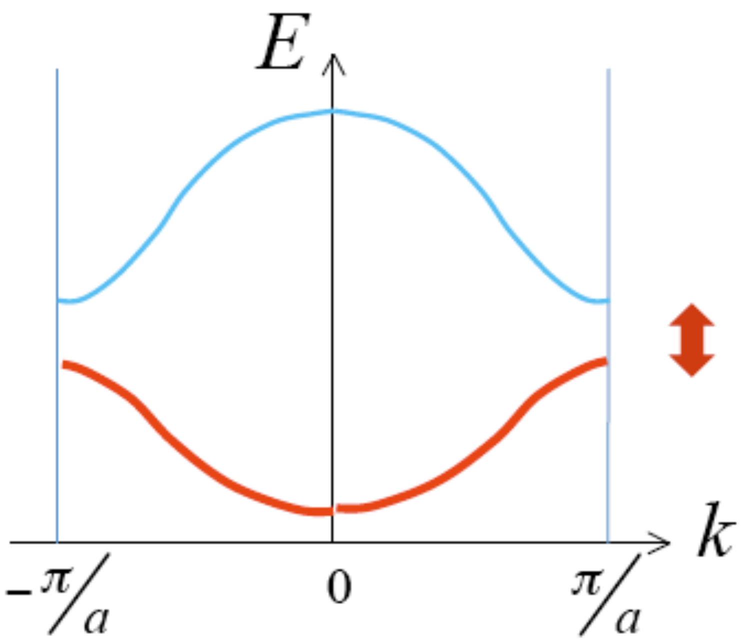
Artificial gauge fields:
Also Relation to **cold atoms**
(I. Spielman; F. Gerbier & J. Dalibard)

Common Language: "Honeycomb Lattice": see colloquium by Hari Manoharan

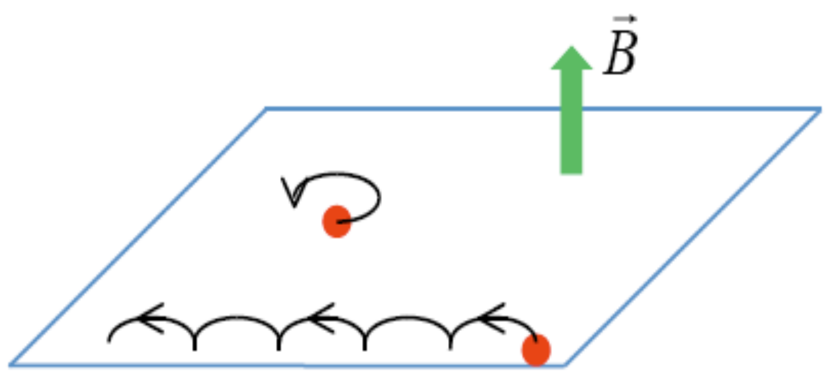
Some Acknowledgments: …

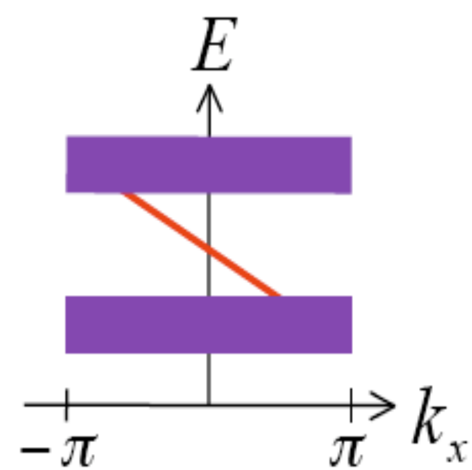
Tianhan Liu (Jussieu, Paris 06 and Ecole Polytechnique)
Benoit Doucot (Jussieu, Paris 06)

Insulators: Basic View

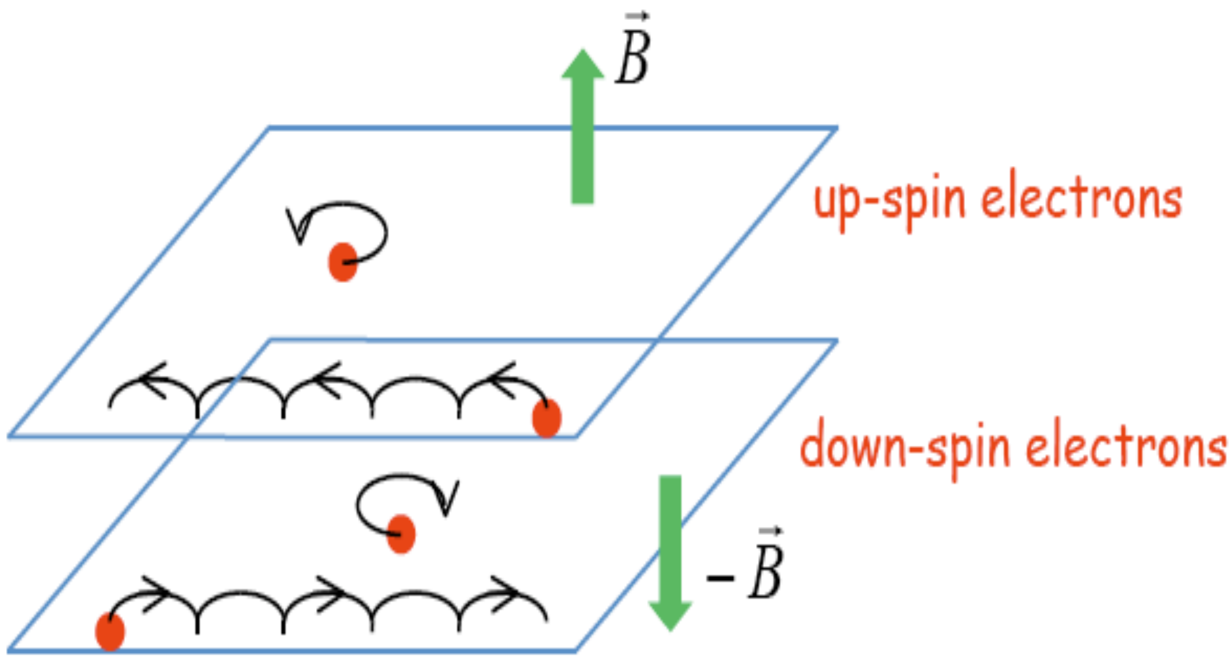


That's all? No





- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda\vec{L}\cdot\vec{\sigma}$
- Gapless helical edge mode (Kramers pair)



No spin rotation symmetry

Microscopic Description: Simple Standard Model

Time reversal invariant of **Haldane model (1988)**: Kane-Mele model

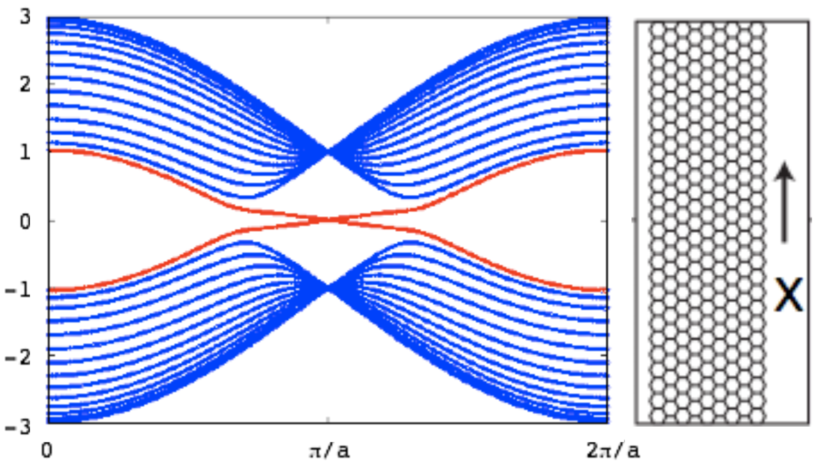
Kane & Mele, PRL 95, 226801 (2005)

see also: **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments**

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^{z}_{\sigma\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'}$$

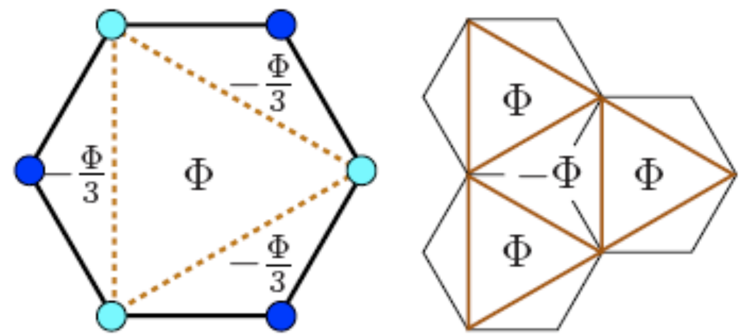
$$\nu_{ij} = \pm 1$$

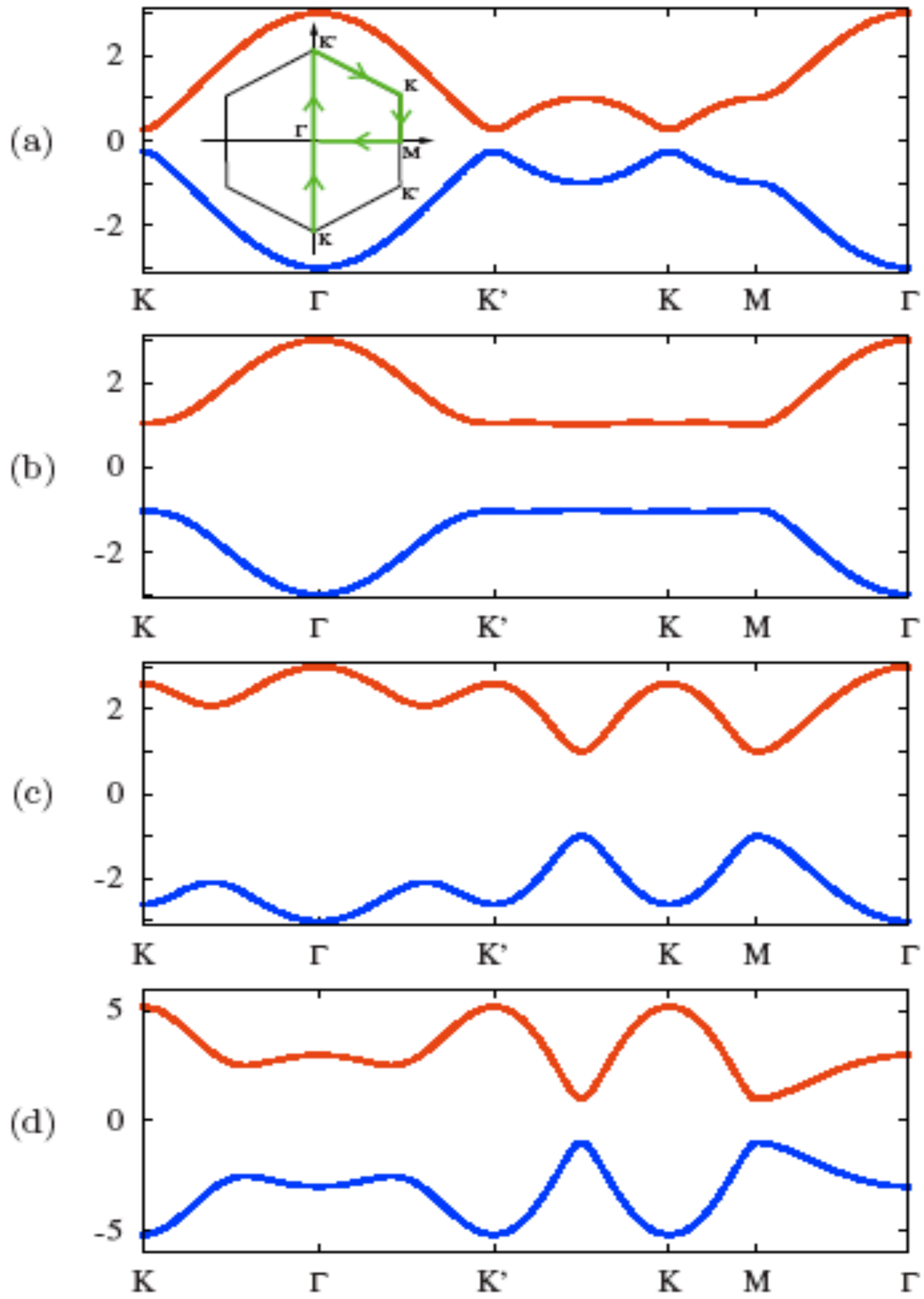
strip geometry:



edge states: Kramer's pair

$$\mathcal{H} \propto \Psi^{\dagger}_{k} \sigma^{z} \tau^{z} \Psi_{k}$$

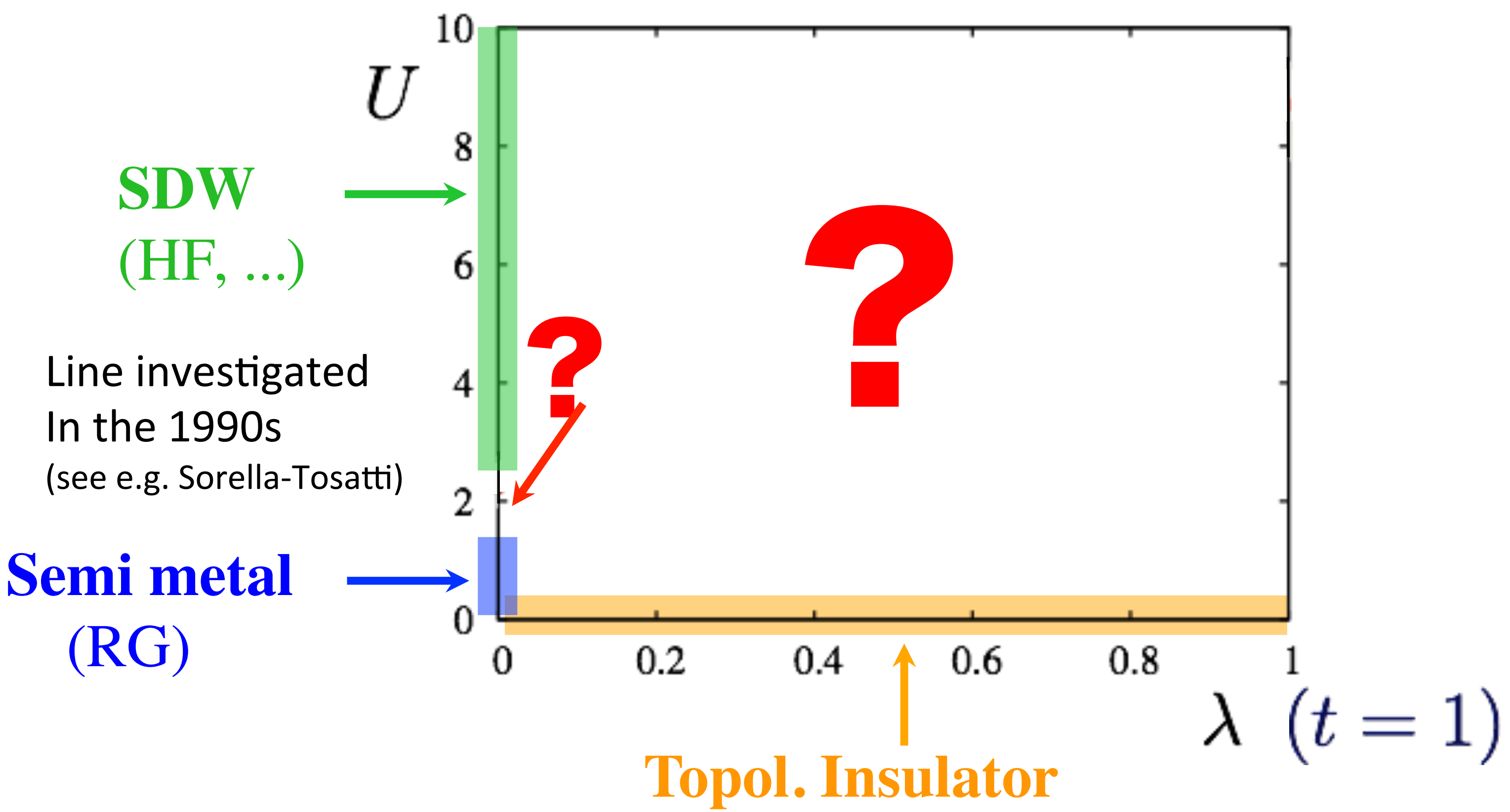




Gap independent of λ for λ~0.2t

Effect of electron-electron interaction?

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^z_{\sigma\sigma'} c^\dagger_{i\sigma} c_{j\sigma'} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



S. Rachel & KLH, PRB **82** 075106 2010

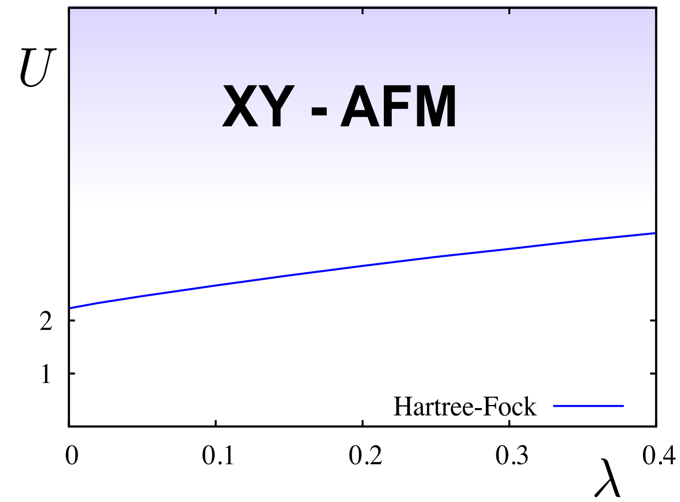
Many other related works: see later for references

Presence of spin orbit coupling: S. Rachel & KLH, PRB 2010

- **Simple Hartree Fock yields transition to magnetically ordered phase (Mott phase)**
- **critical *U* increases with increasing spin orbit coupling**

$$m = (-1)^{\zeta} \langle c_{i\uparrow}^{\dagger} c_{i\uparrow} - c_{i\downarrow}^{\dagger} c_{i\downarrow} \rangle$$

(cf. Sorella & Tosatti EPL 19, 699 (1992))



increase of U_c can be understood frc

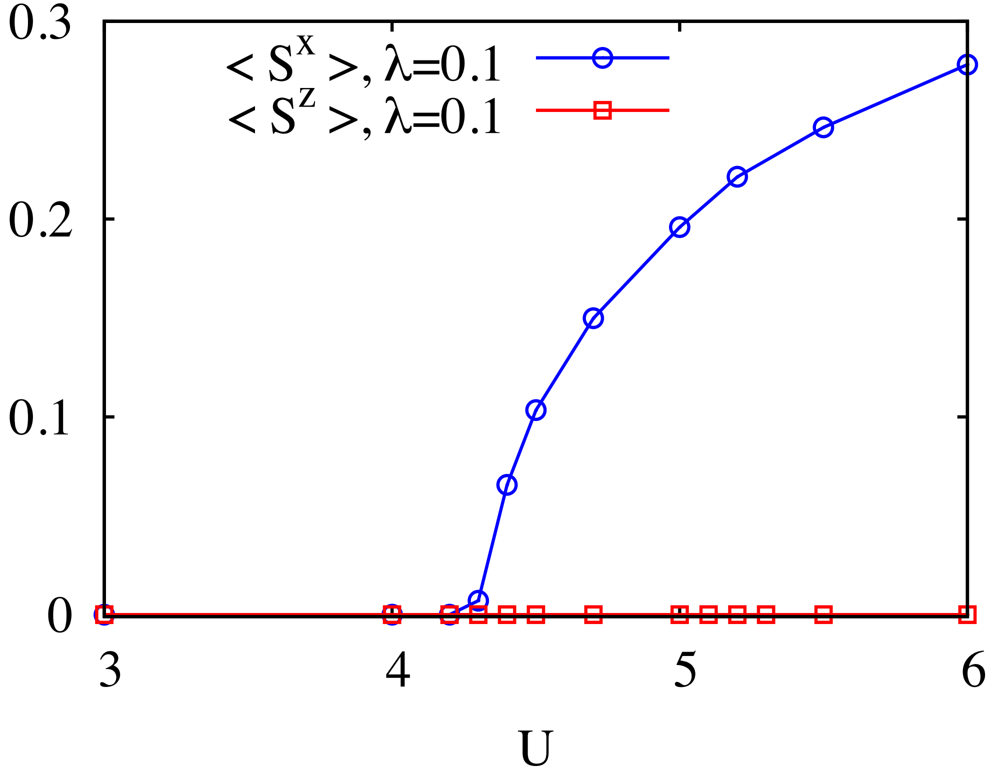
$$\mathcal{H}^{\text{eff}} = \sum_{\langle ij \rangle} \frac{4t^2}{U} \boldsymbol{S}_i \boldsymbol{S}_j + \sum_{\langle\!\langle ij \rangle\!\rangle} \frac{4\lambda^2}{U} \Big(-S_i^x S_j^x - S_i^y S_j^y + S_i^z S_j^z \Big)$$

favors Neel **competes with nearest neighbor term**

$\lambda \neq 0$: **SDW order is turned into XY plane**

Different from (J_1,J_2) model

XY ordering



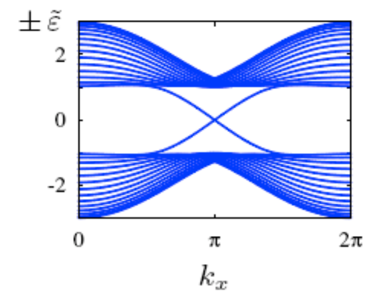
C-DMFT

Real-space version
QMC continous-time
Impurity solver

Some Reviews (not full list):
G. Kotliar et al, RMP 2006
T. Maier et al, RMP 2005

Wei Wu, Stephan Rachel, Wu-Ming Liu and KLH, PRB **85**, 205102 (2012)
(same result can be obtained from "simple" Hartree-Fock theory)
HF shows that the single-particle gap <u>does not</u> close at the transition

Presence of spin orbit coupling



- **Apply (Slave)-Rotor theory of Florens & Georges, PRB 70, 035114 (2004)**
- **See review E. Zhao & A. Paramekanti**
- **Rewrite fermions as rotors (charge degrees of freedom) and spinons**

$$c_{i\sigma} = e^{i\theta_i} f_{i\sigma}$$

- **Introduce constraint**

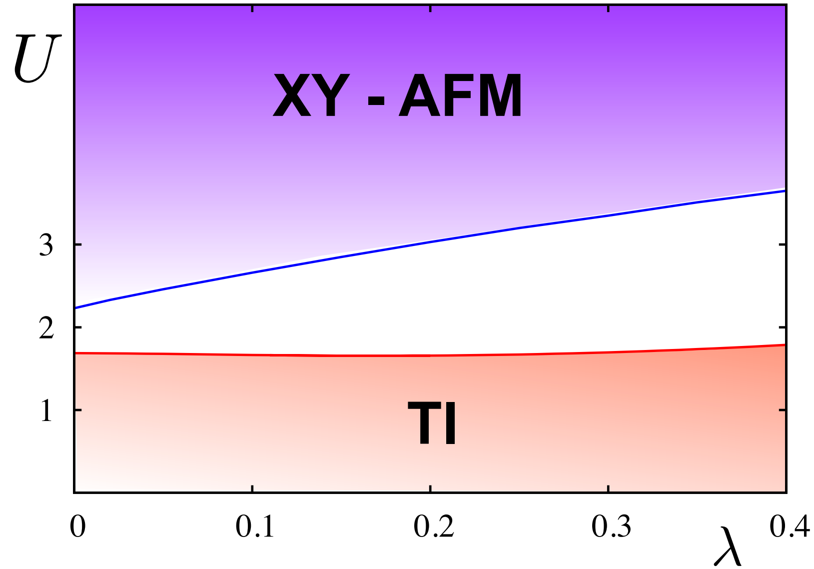
$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + L_i = 1$$

$$\frac{U}{2}\sum_i \left(\sum_\sigma n_{i\sigma} - 1\right)^2 \longrightarrow \frac{U}{2}\sum_i L_i^2$$

- **Hubbard interaction simplifies**

$$L = (i/U)\partial_\tau \theta.$$

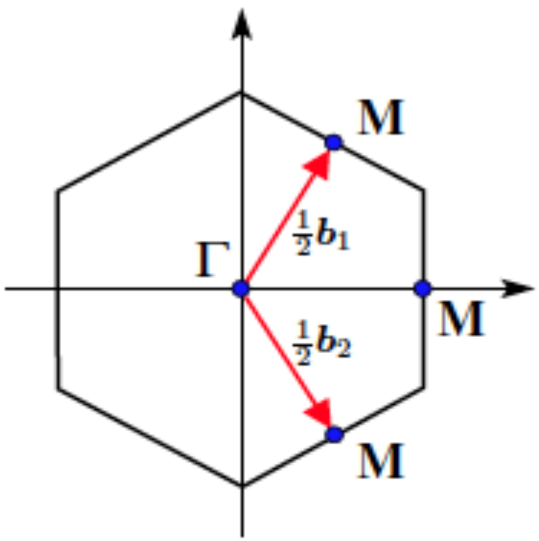
- **Interaction affects rotor only**
- **weak *U*: rotor condense,** $f_\sigma \propto c_\sigma$



S. Rachel & KLH, PRB 2010

[+ other theory arguments]

Z_2 Invariant: Spin Chern number



Time-reversal invariant
Points of the Brillouin zone

Following L. Fu and C. Kane:

$$\mathcal{H}_{\mathbf{k}} = \sum_{a=1}^{5} d_a(\mathrm{k})\Gamma^a$$

$$\mathcal{P} = \tau^x \otimes I = \Gamma^1$$

$$\mathcal{T} = i(I \otimes \sigma^y)K$$

Time-Reversal & Inversion Symmetry

Z_2 invariant given by (here, ν=0 or 1):

$$(-1)^{\nu} = \prod_{i=1}^{4} -\mathrm{sign}[d_1(\Gamma_i)]$$

Single-particle band structure: see also Balents & Moore
Efforts to define top. Invariants for interacting systems:
Qi-S.C. Zhang; V. Gurarie; Y. B. Kim (CDMFT); A. Kitaev; Savrasov (LDA+U)

Edge Theory: Helical Liquid

C. Xu & J. Moore; C. Wu, A. Bernevig & S.-C. Zhang;...

$$H_0 = v_F \int dx \left(\psi^\dagger_{R\uparrow} i\partial_x \psi_{R\uparrow} - \psi^\dagger_{L\downarrow} i\partial_x \psi_{L\downarrow} \right)$$

$\psi^\dagger_{R\uparrow}\psi_{L\downarrow} + \mathrm{h.c.}$ (**elastic**) Backscattering forbidden

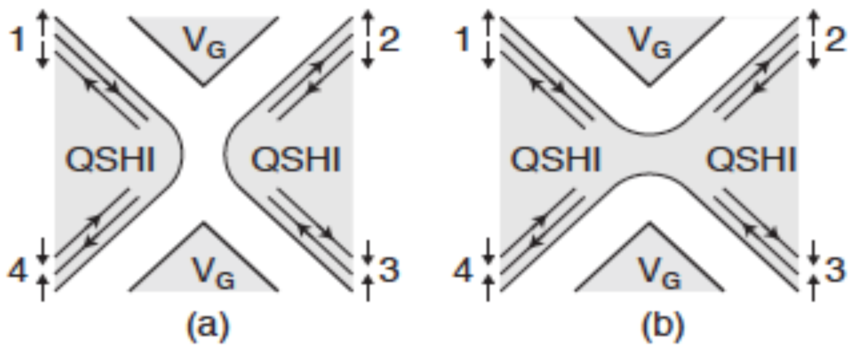
$$H_I = U \int dx \left(\psi^\dagger_{R\uparrow} \psi_{R\uparrow} \psi^\dagger_{L\downarrow} \psi_{L\downarrow} \right)$$

$$H = \int dx \frac{v}{2} \left[\frac{1}{K} \left(\partial_x \phi\right)^2 + K \left(\partial_x \theta\right)^2 \right] - \frac{U m \sin\sqrt{4\pi}\phi}{(\pi a)^2}$$

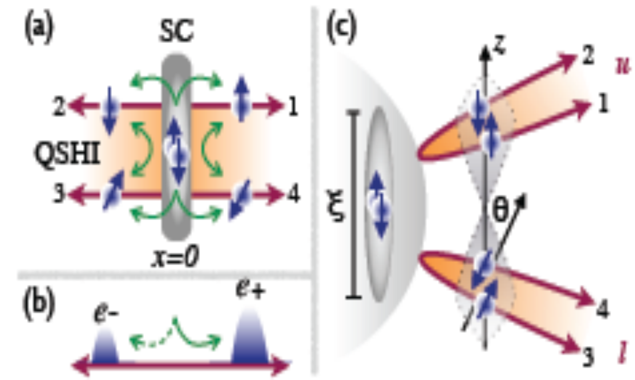
Minimal Model
for TBI/QSH phase

with $m = \langle \psi^\dagger_{R\uparrow}\psi_{L\downarrow} \rangle$

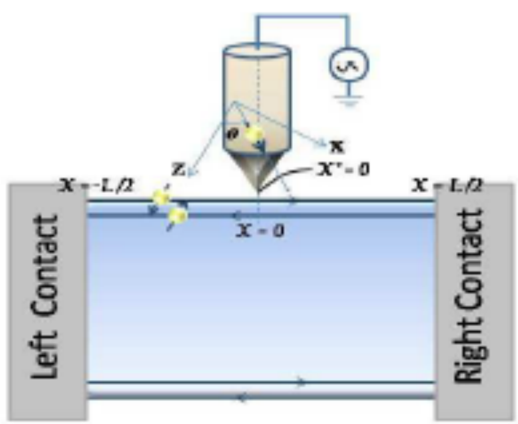
new “Edge Directions” (+Majoranas)



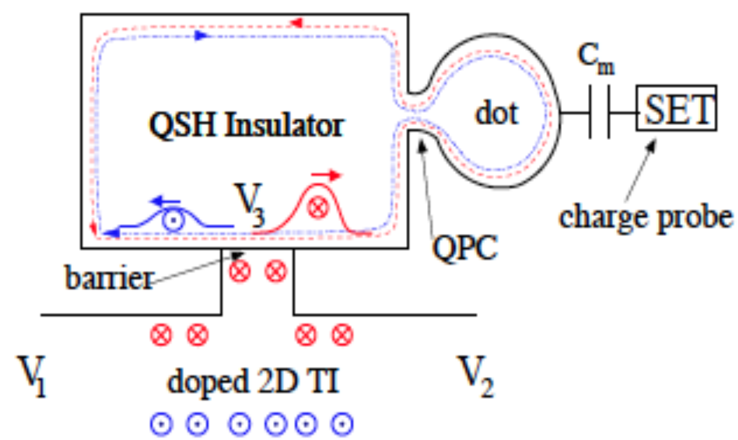
J. Teo & C. Kane, Phys. Rev. B 79, 235321 (2009)
C.-Y. Hou, E.-A. Kim & C. Chamon, PRL 2009



Sato, Tserkovnyak & Loss
Phys. Rev. Lett. **105** (2010) 226401



S. Das and S. Rao,
Phys. Rev. Lett. **106**, 236403 (2011)



I. Garate & KLH, PRB 85, 195465 (2012)
I. Garate (Yale → U. Sherbrooke)

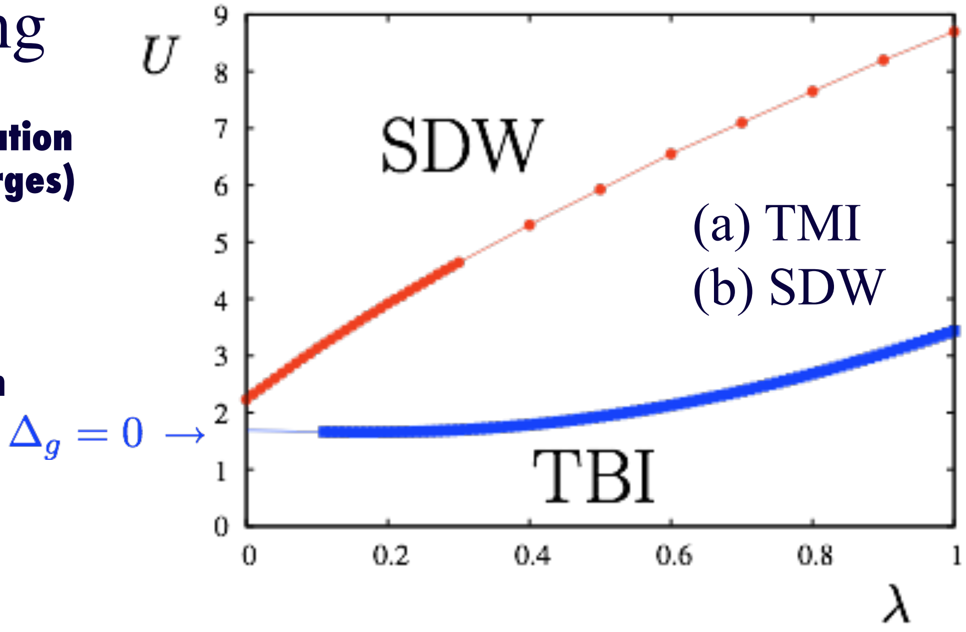
Presence of spin orbit coupling

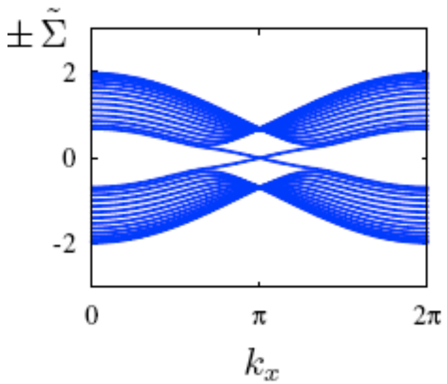
- **More Slave-Rotor: use sigma-model representation**
- **introduce bosonic *X*-fields (S. Florens & A. Georges)**

$$X = e^{i\theta} \qquad |X|^2 = 1$$

- **mean-field decoupling or Hubbard Stratonovich**
- **insulating gap (zero at the transition)**

$$\Delta_g = 2\sqrt{U(\rho + \min \xi_k)}$$





S. Rachel & KLH, PRB 82, 075106 (2010); arXiv:1003.2238, 20 pages

See also S.S. Lee & P. Lee PRL 2005
Young, S. S. Lee, C. Kallin, PRB 2008
Pesin & Balents, Nat. Phys. 2010
Y.-B. Kim & et al. 2010 + many recent works

Mean-Field Solution allows TMI phase:

- Mott gap
- Spin degrees of freedom form a topological Kane-Mele phase

Analogue of S=1 spin Haldane chain (probed through thermal transport?)

2D: Direct Transition from TBI to XY

$$\mathcal{L}_{MF} = m \sum_{a=\pm} \left(f^{\dagger}_{\uparrow a} \tau^z f_{\uparrow a} - f^{\dagger}_{\downarrow a} \tau^z f_{\downarrow a} \right)$$

Monopole insertion = “spin flip” operator
Localized +2π flux of the gauge field implies that a single extra spin-up spinon will be Induced along with the gauge flux, while one spin-down spinon will be depleted

Fermions are gapped:

$$\mathcal{L}_{Maxwell} = (1/2e^2) \sum_{\mu} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

Monopoles only cost a finite action: monopole propagator is long-ranged

Here, this implies magnetic order in the XY plane: <S^+> is finite

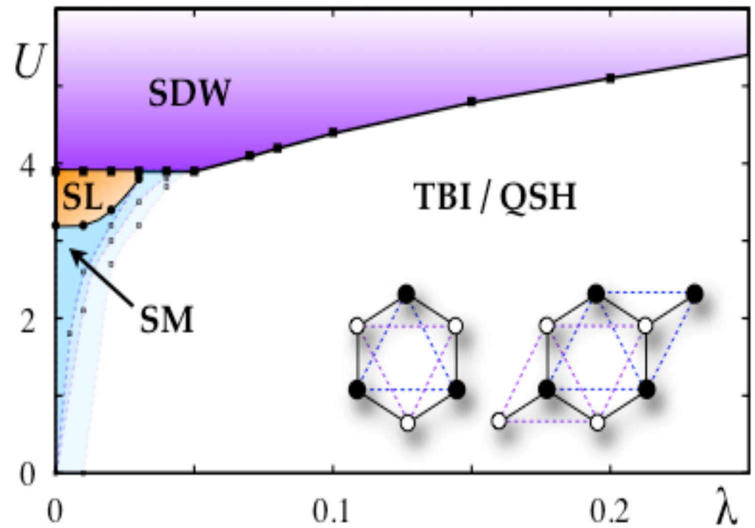
Polyakov’s gauge field argument: see also S. S. Lee & P. Lee; Y. Ran et al; M. Hermele…

Phase Diagram

Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

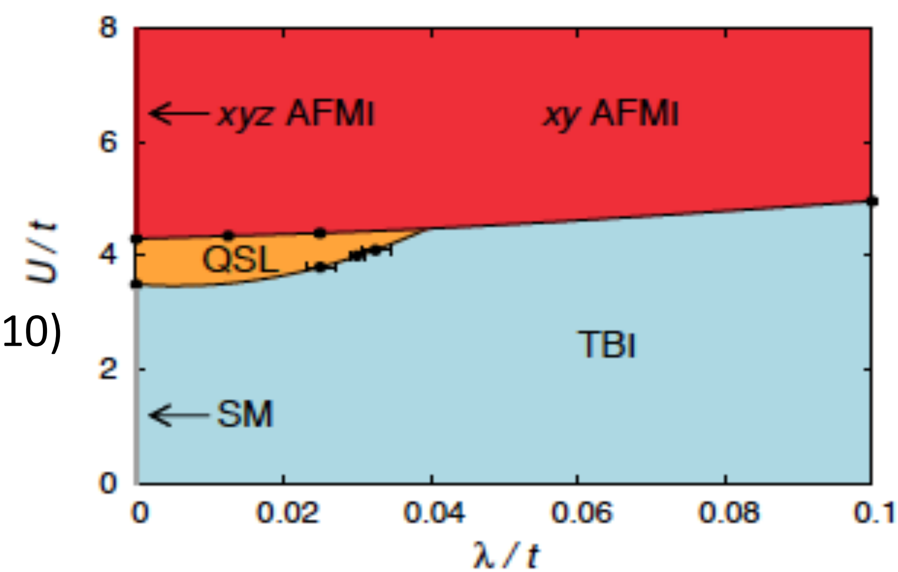
Real-space version
QMC continous-time
Impurity solver



3D XY
S. Rachel & KLH, 2010
Griset & C. Xu, 2011
D.-H. Lee, 2011

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)



M. Hohenadler et al.
arXiv:1111.3949

Phys. Rev. Lett. **106**,
100403 (2011)

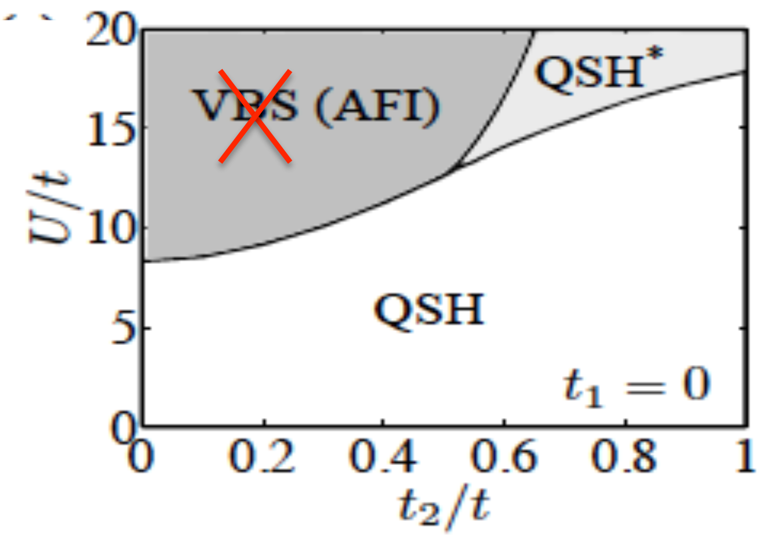
Connection to reality?

(Takagi; Gegenwart; S. Jullian;…)

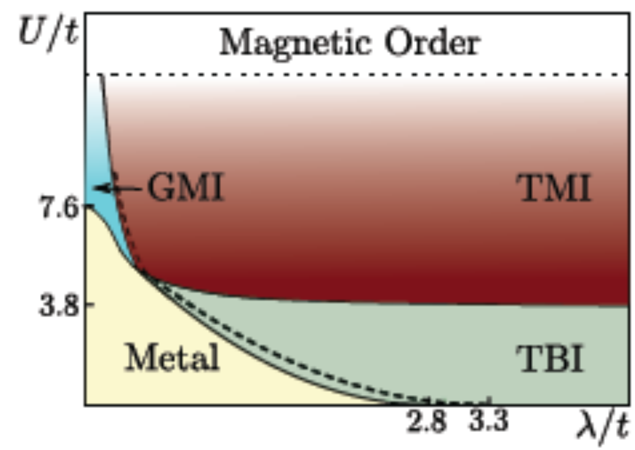
- Na_2IrO_3: honeycomb layers (experiment: e.g. Gegenwart)

  Spin-orbit coupling slightly different:

  Shitade et al. PRL **102** 256402 (2009)



Ruegg & G. Fiete, PRL 2012



D. Pesin & L. Balents, Nature Phys. 2010

Relevance for 3D pyrochlore Iridates (?)

Y. B. Kim and co-authors

Also quest for Weyl fermions

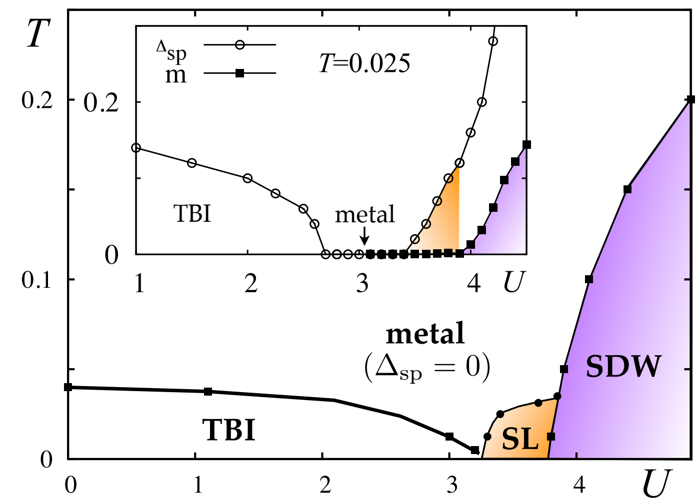
- GaAs heterostr.: A. Singha et al. Science 2011
- Cold atoms: L. Tarruel et al. (Esslinger) 2012

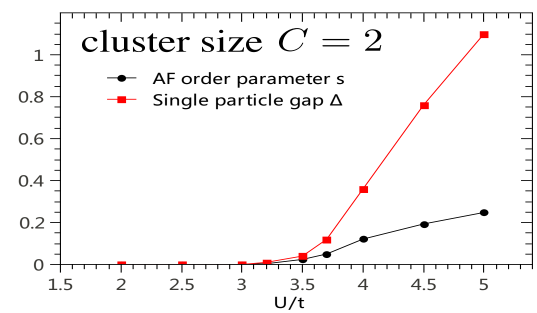
“Spin Liquid” (?)

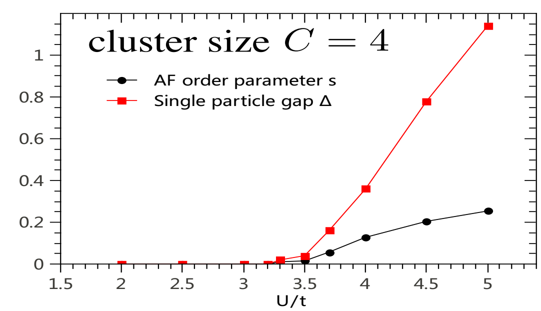
Numerical Facts: CDMFT

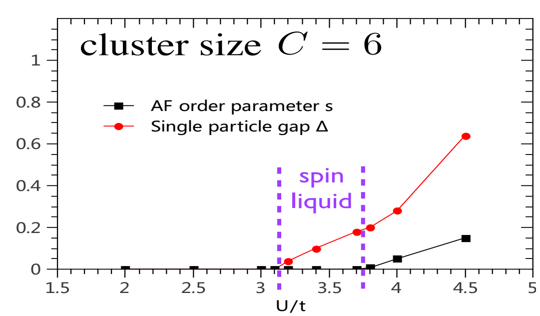
Single-particle gap

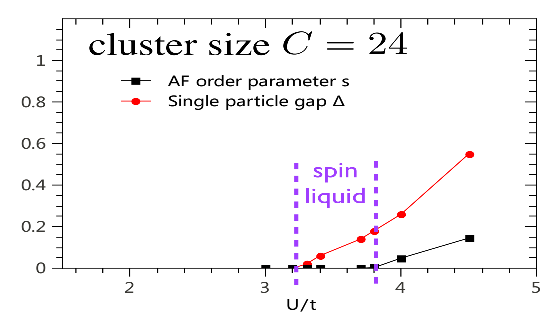
No long-range order







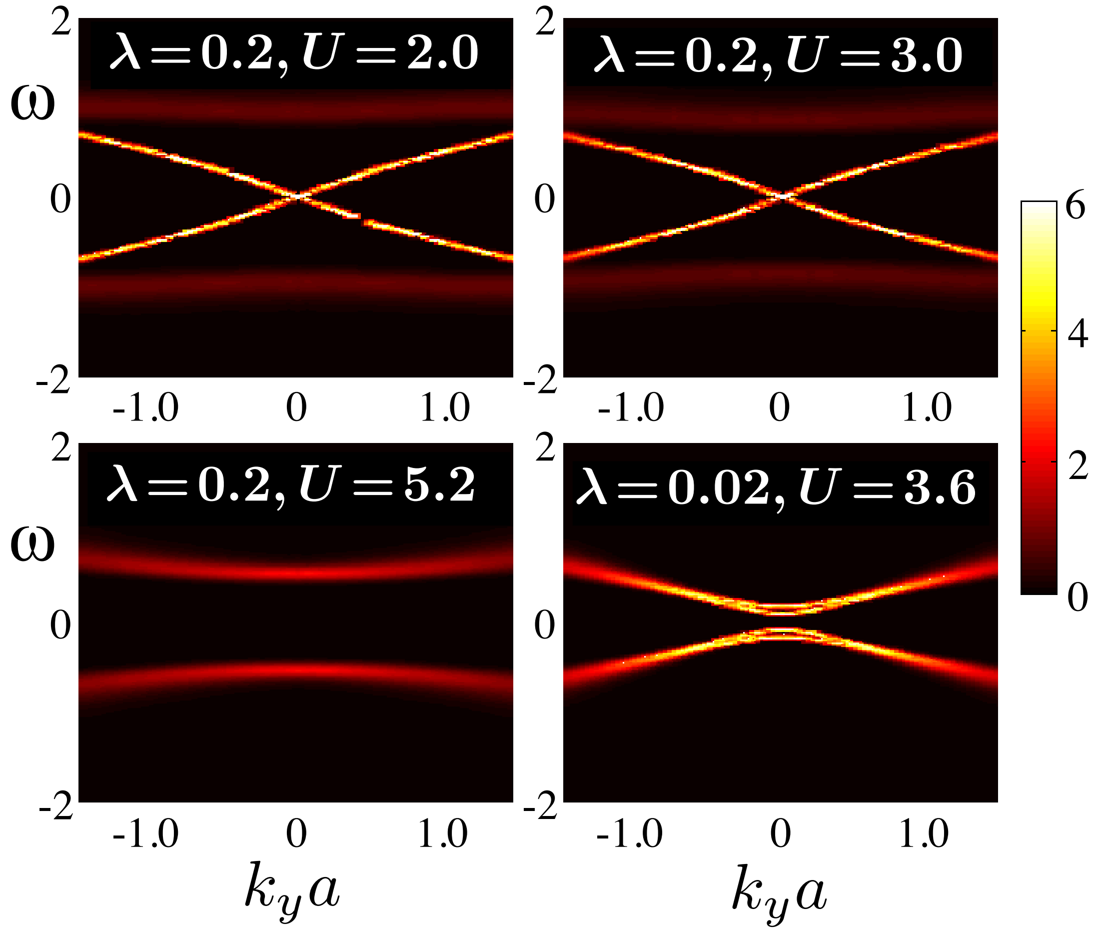




Wei Wu

CDMFT

(No) Edge States in $A_s(k,\omega)$



CDMFT

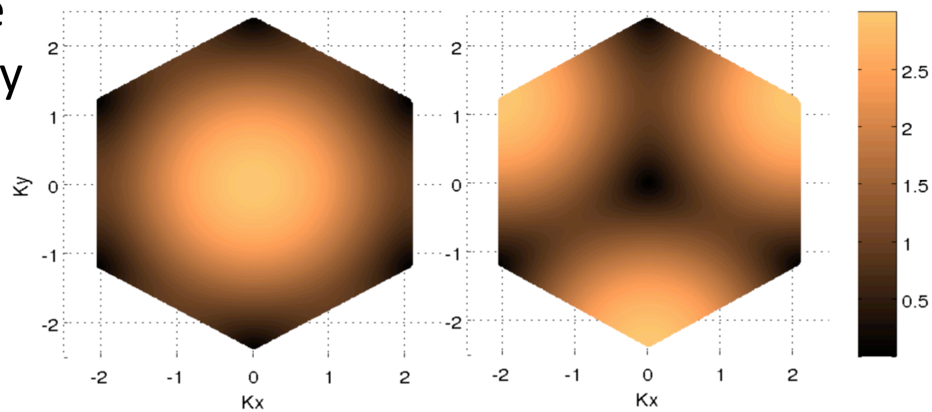
Analytical Development

Possibility of gapped spin liquid

J_1-J_2 model; Tran & Kim, PRB 2011 & Clark, Abanin & Sondhi PRL 2010

**D+iD spin gap :
plausible scenario?**

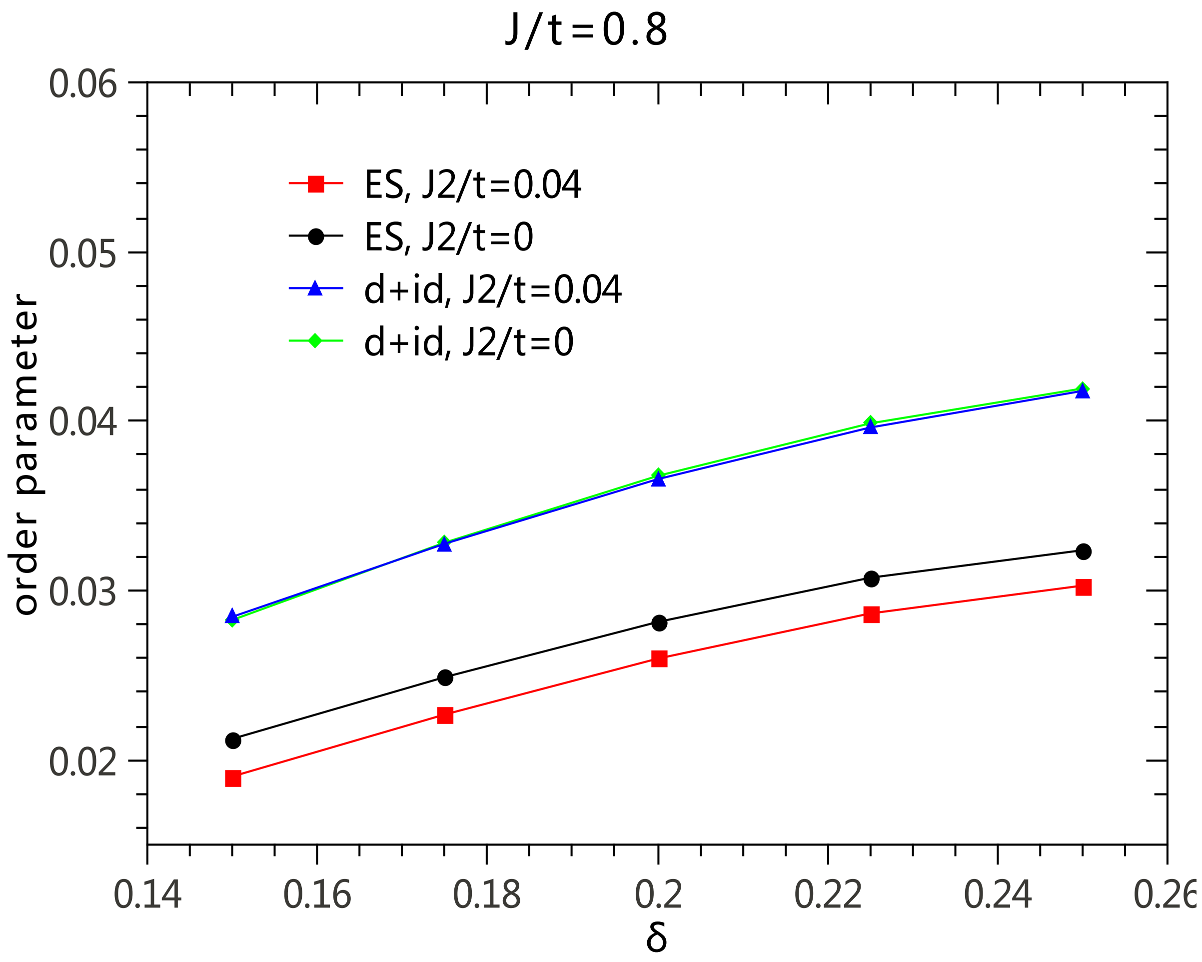
Difficulty to solve this model exactly



$$\Delta_{\mathbf{k}}^{d+id} = \cos(\tfrac{\pi}{3})\Delta_{x^2-y^2} + i\sin(\tfrac{\pi}{3})\Delta_{xy}.$$

Other variants of spin liquid: C. Xu & S. Sachdev; Vaezi et al; …

d+id Pairing



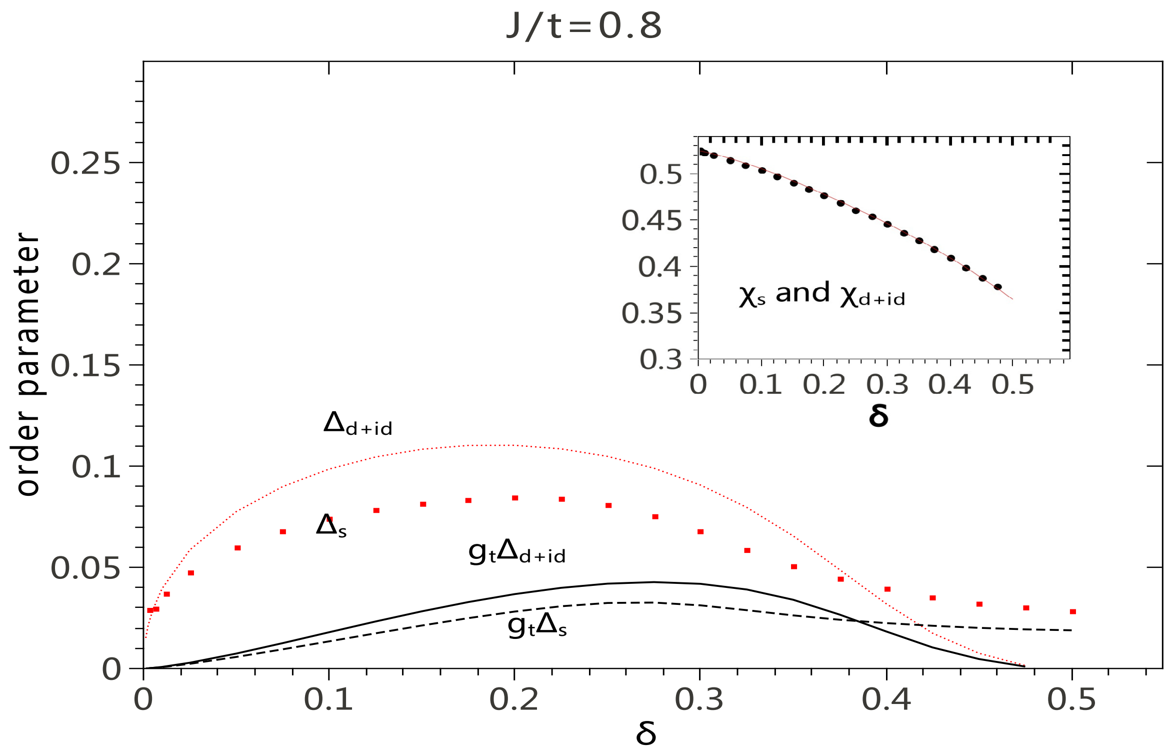
In progress

Other possible SC Pairings: B. Rosenow's talk

RMFT, Method review: KLH & T. Maurice Rice, Annals of Physics **324**, 1452 (2009)
Superconductivity close to the Mott state: high-Tc SCs and cold atoms

Unconventional SC

RMFT



O. Vafek, review

This shows that d+id SC might arise below half-filling

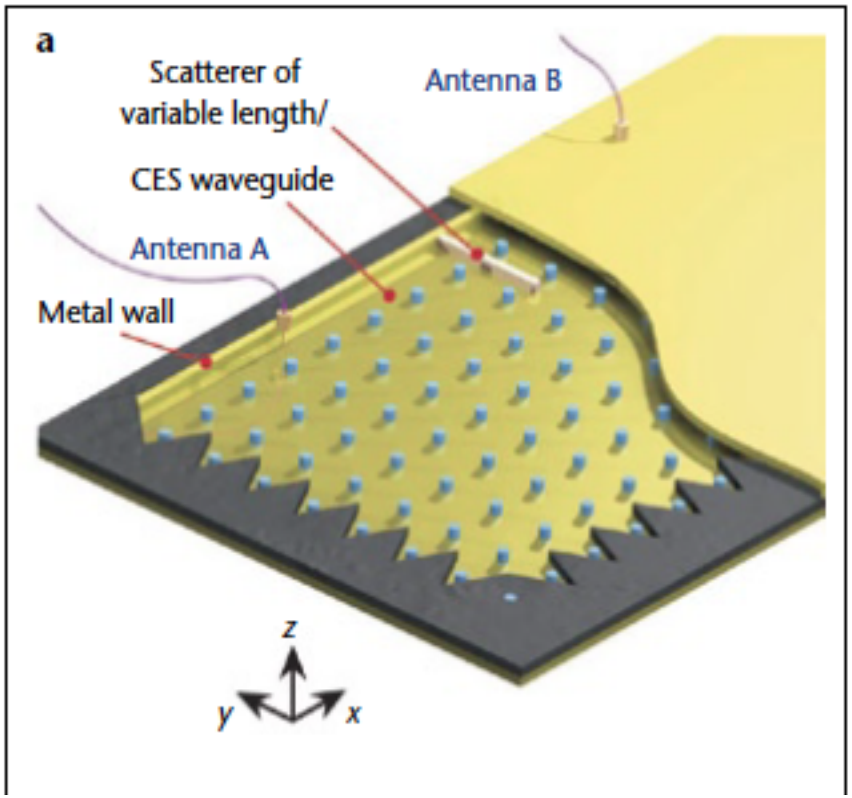
A. Black-Schaffer & Doniach; Baskaran; L. Balents & X. Wen et al. (numerics/progress)…

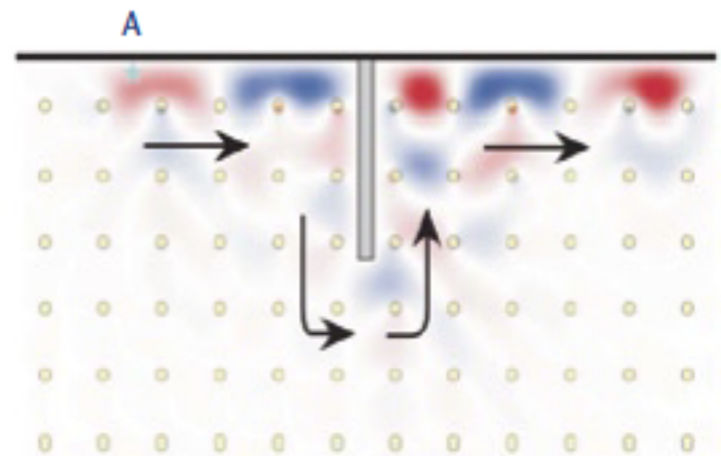
Heavily doped graphene: R. Nankdishore, L. Levitov & A. Chubukov, Nature Phys. 2012;…

fRG: C. Honerkamp; R. Thomale …

One-Way Road in a Photonic Crystal

Chiral edge states channel light waves in one direction, like electrons in the quantum Hall effect





(a) A model of the photonic crystal. The distance between the ferrite rods is 4 cm.

Realizations of AQH effect in Photonic crystals: following Haldane & Raghu (lattice of rods and Faraday effect opens a gap breaking time-reversal symmetry)

Experiment: M. Soljacic et al. Nature **461**, 772 (2009)

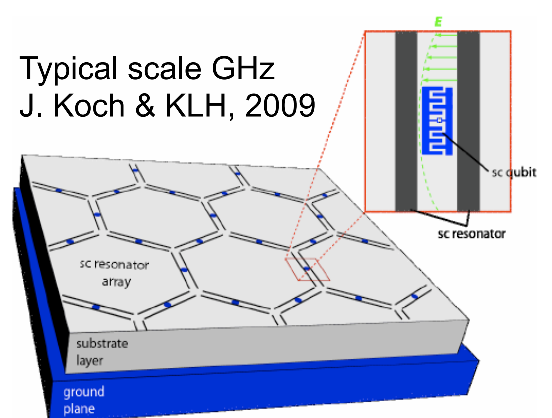
n.b: QSH phase, M. Hafezi et al Nature Phys. 2011; A. Khanikaev, arXiv:1204.5700

Systems of interacting photons: Theory surveys

► M. Hartmann et al., Laser & Photonics Review 2, 527 (2008)
A. Tomadin & R. Fazio, J. Opt. Soc. Am B 27, A130 (2010)
I. Carusotto and C. Ciuti, arXiv:1205.6500

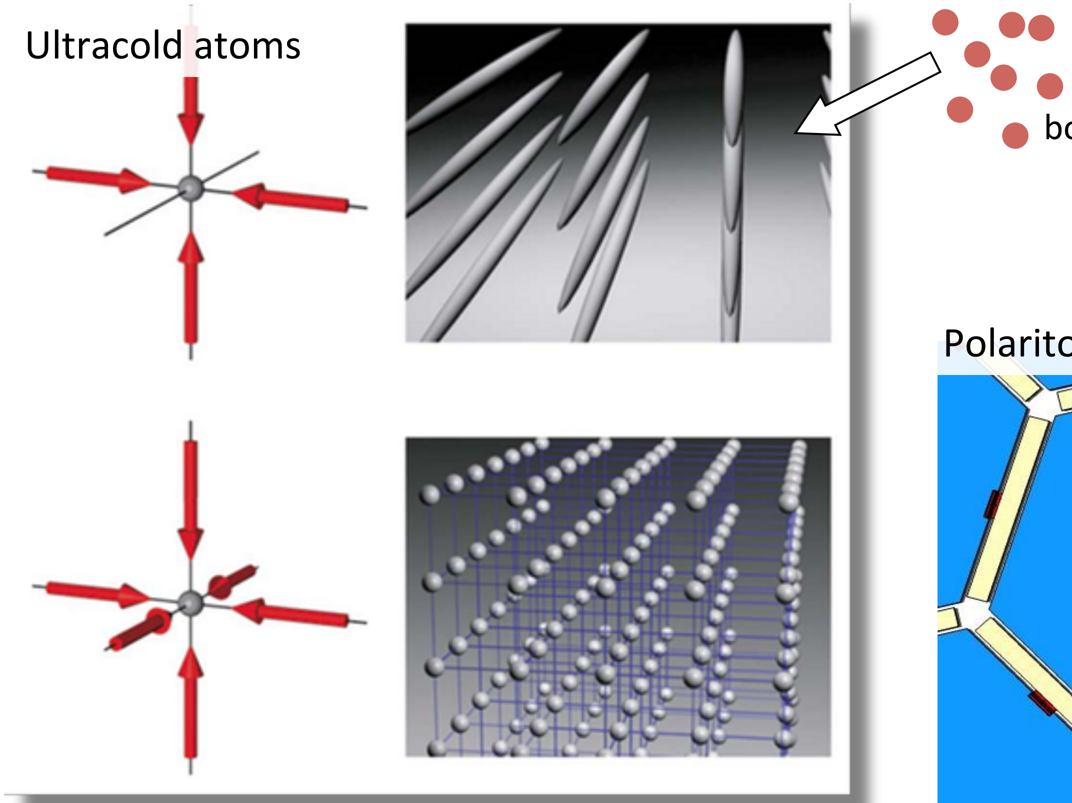
proposed realizations

* photonic band gap cavities
* arrays of silicon micro-cavities
- fibre based cavities
- Progress circuit QED: A. Houck; H. Tureci; J. Koch
Nature Physics 2012

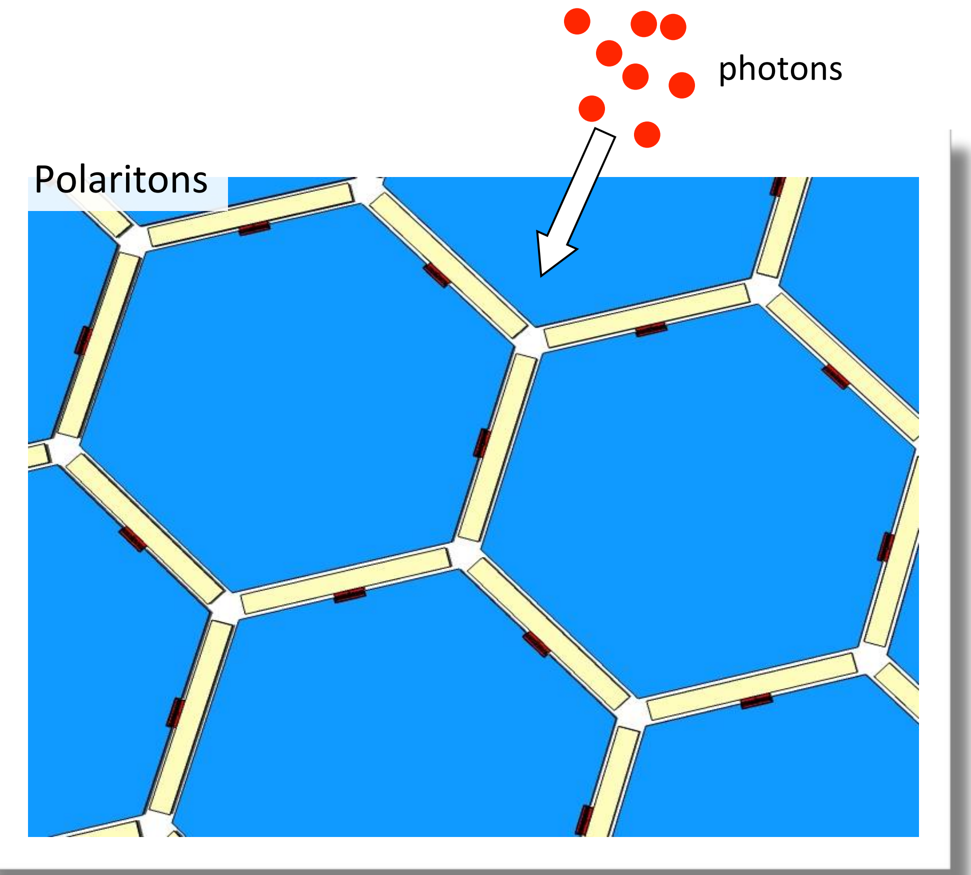


+ tunability
+ access to single lattice site
- must be treated as open system
+ interesting: transitions between
different steady states

Interacting bosons on a lattice

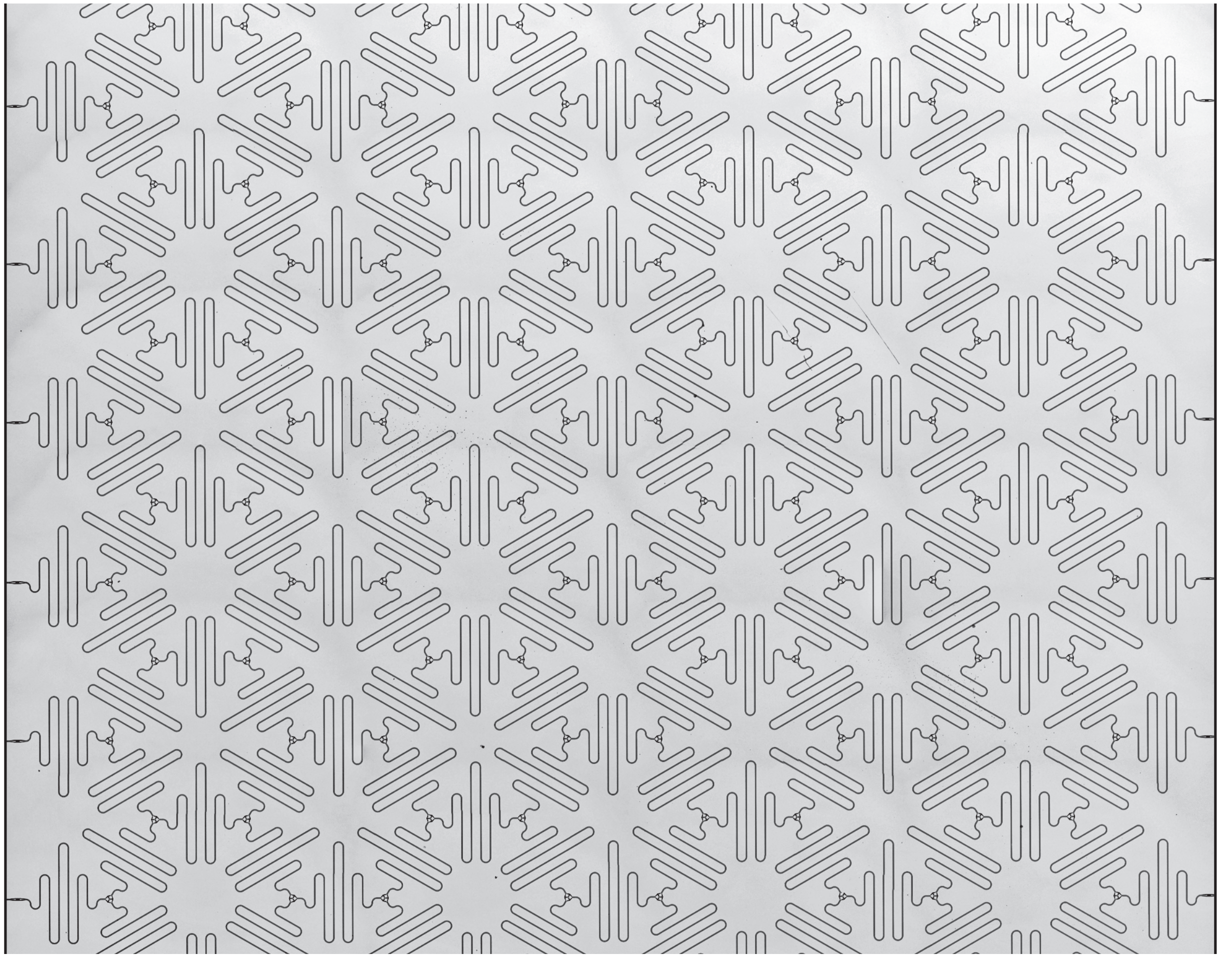


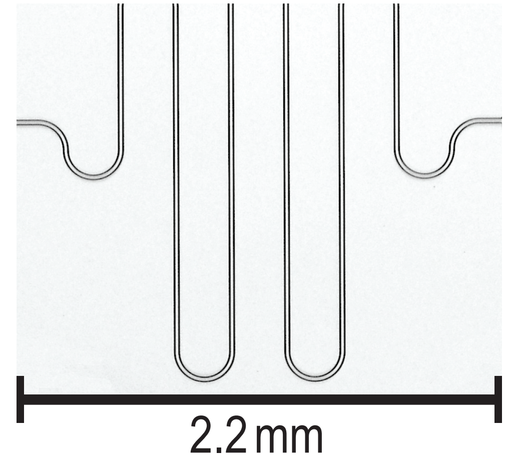
D. Jaksch et al., Phys. Rev. Lett. **81**, 3108 (1998)
M. Greiner et al., Nature **415**, 39 (2002)
I. Bloch et al., Rev. Mod. Phys. **80**, 885 (2008)

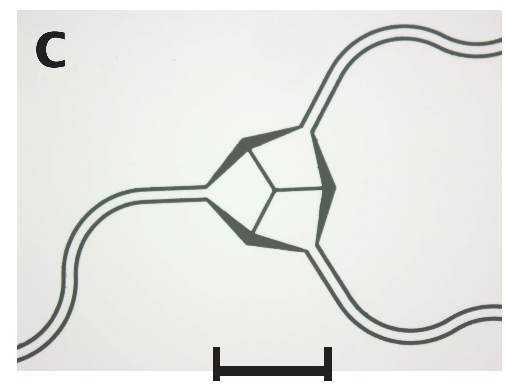


Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and Karyn Le Hur, PRA **80**, 023811 (2009)…

On-going experiment
A. Houck at princeton







Experimental progress: arXiv:1203.5363

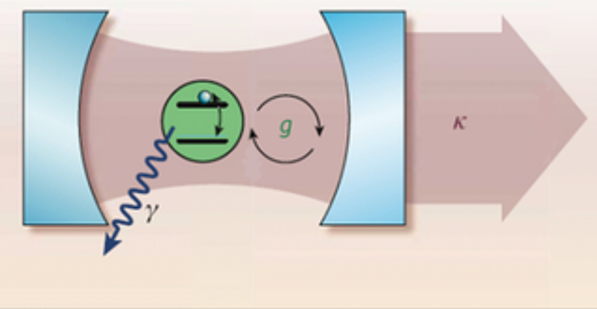
- Review: A. Houck; H. Tureci; J. Koch Nature Physics 2012

Cavity & Circuit QED: 1 cavity a lot of activity…

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity

cavity QED



2g = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

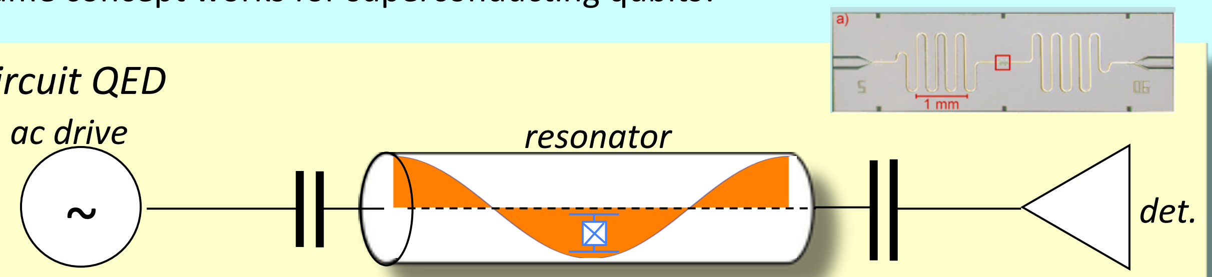
Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\omega_a \sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\mathrm{drive}} + H_{\mathrm{baths}}\right)$$

See also A. Blais et al

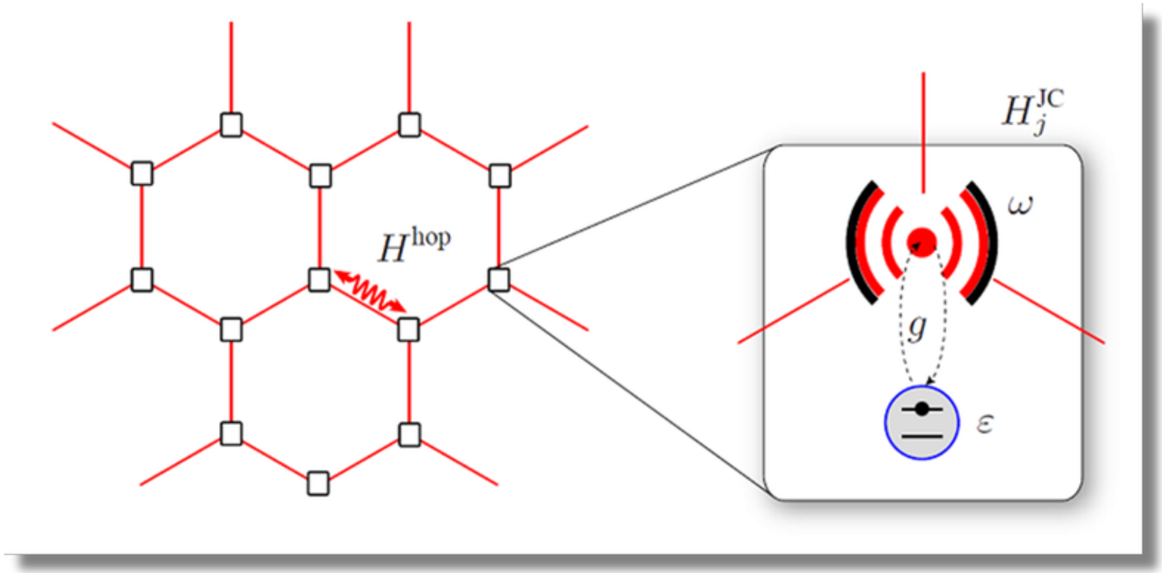
- same concept works for superconducting qubits!

circuit QED



Reviews: J.M. Raimond et al., Rev. Mod. Phys. **73**, 565 (2001); R. J. Schoelkopf, S.M. Girvin, Nature **451**, 664 (2008)

The Jaynes-Cummings "Lattice" Model



Jaynes-Cummings model: 1963
(famous model in quantum optics)

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)

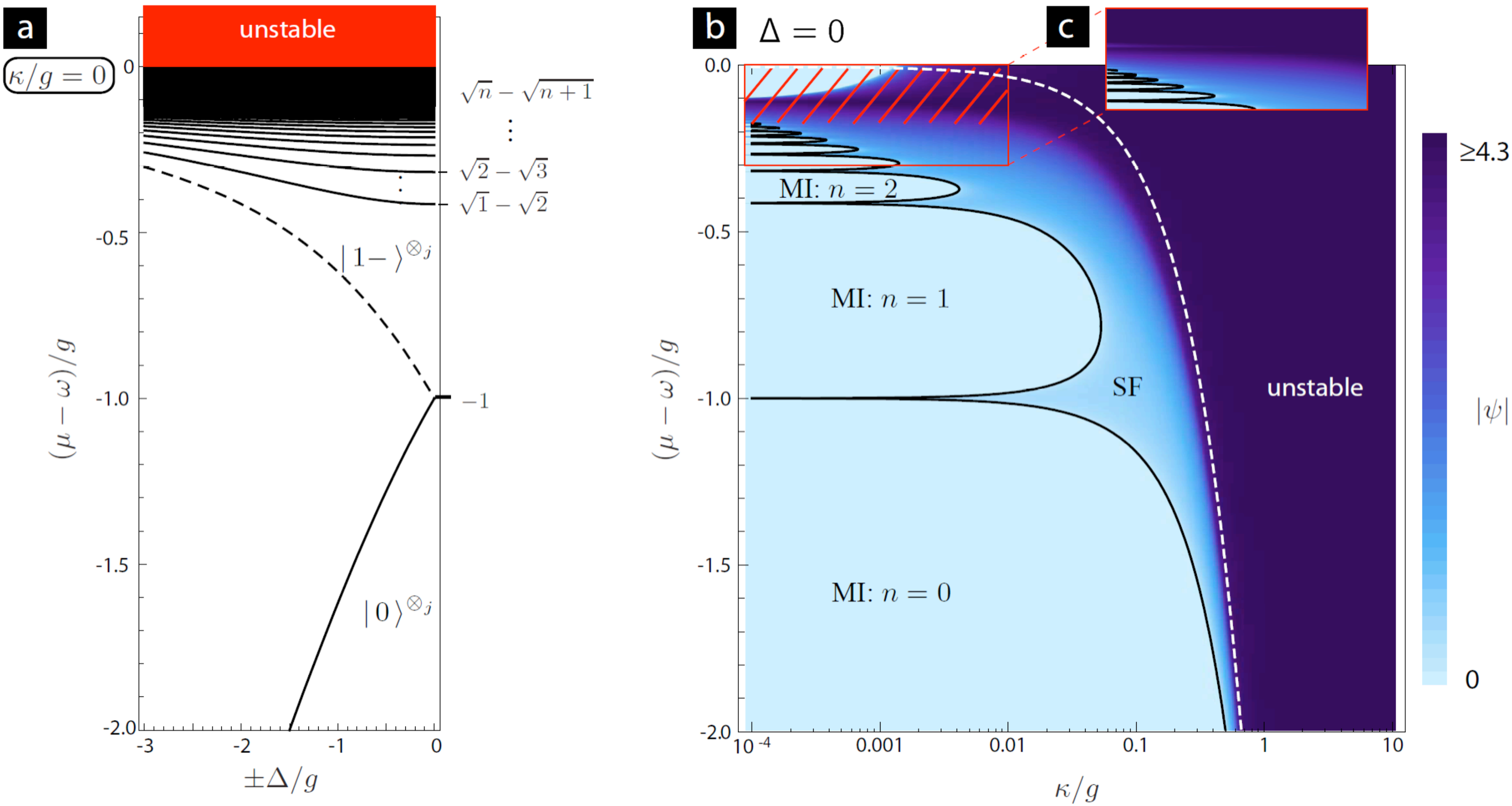
Other groups: H. Tureci, R. Fazio, G. Blatter, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik,...

Jaynes-Cummings lattice model $H = \sum_j H_j^{\text{JC}} + H^{\text{hop}} - \mu N$ "chemical potential"

- *Jaynes-Cummings:* $H_j^{\text{JC}} = \omega a_j^\dagger a_j + \varepsilon \sigma_j^+ \sigma_j^- + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j)$
- *nearest-neighbor photon hopping:* $H^{\text{hop}} = -\kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i)$
- *polariton number:* $N = \sum_j (a_j^\dagger a_j + \sigma_j^+ \sigma_j^-)$

MFT results for the JC lattice

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)

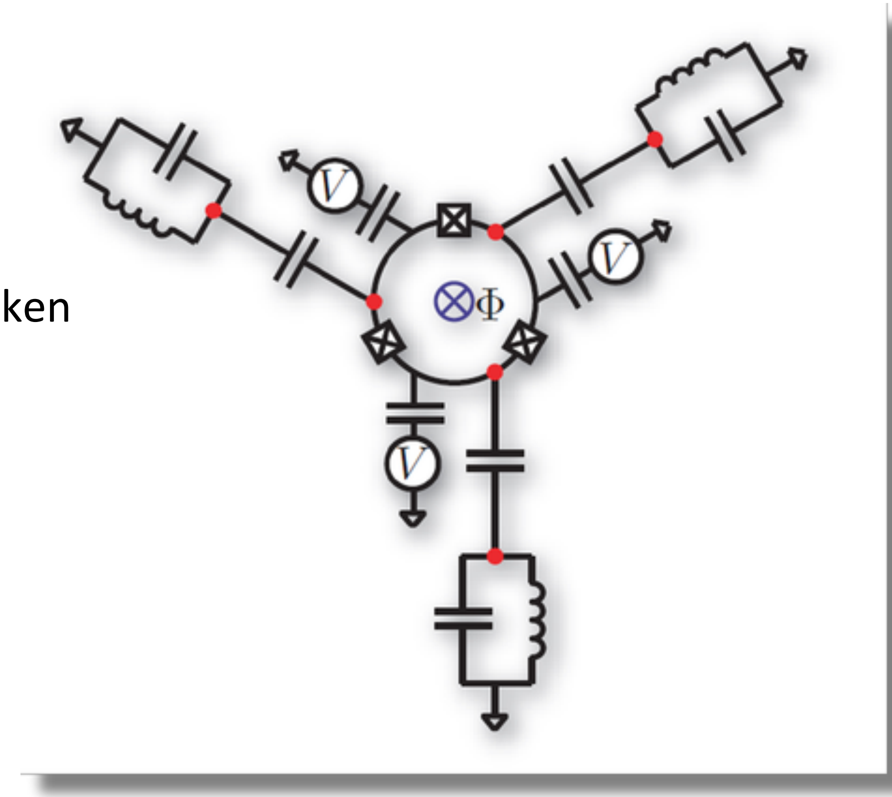


Jens Koch and KLH, PRA **80**, 023811 (2009)

J. Koch, A. Houck, KLH & SM Girvin, PRA 2010

(numerical check at intermediate couplings)

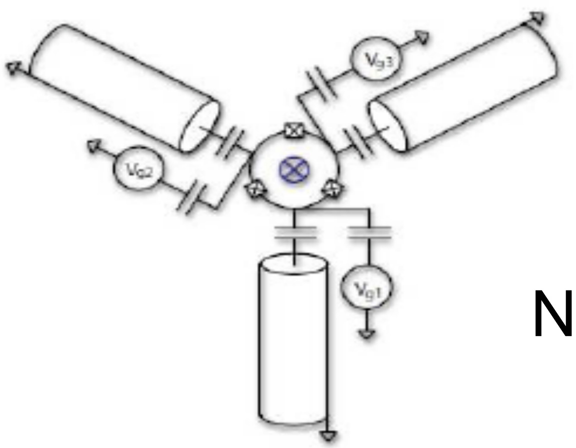
- Josephson ring provides one way to generate complex phase factors



- need magnetic flux to break t-reversal sym. additionally, particle-hole sym. must be broken
- large E_J/E_C: no complex phases, but *tunable coupling strength*!

complex phases for intermediate E_J/E_C

- *random off-set charges can be controlled*



Nano quantum circulator

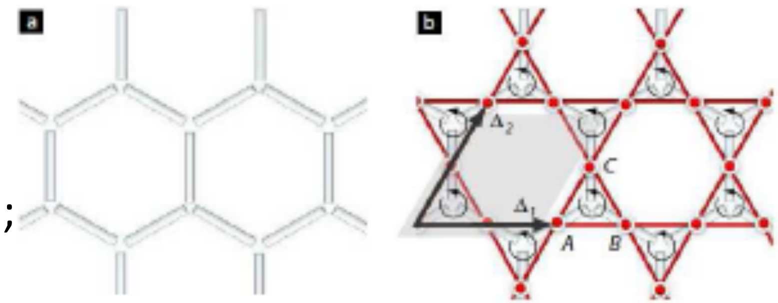
- **topological phases, Kagome lattice**

Breaking T-reversal symmetry

SC Waveguides
with circulators

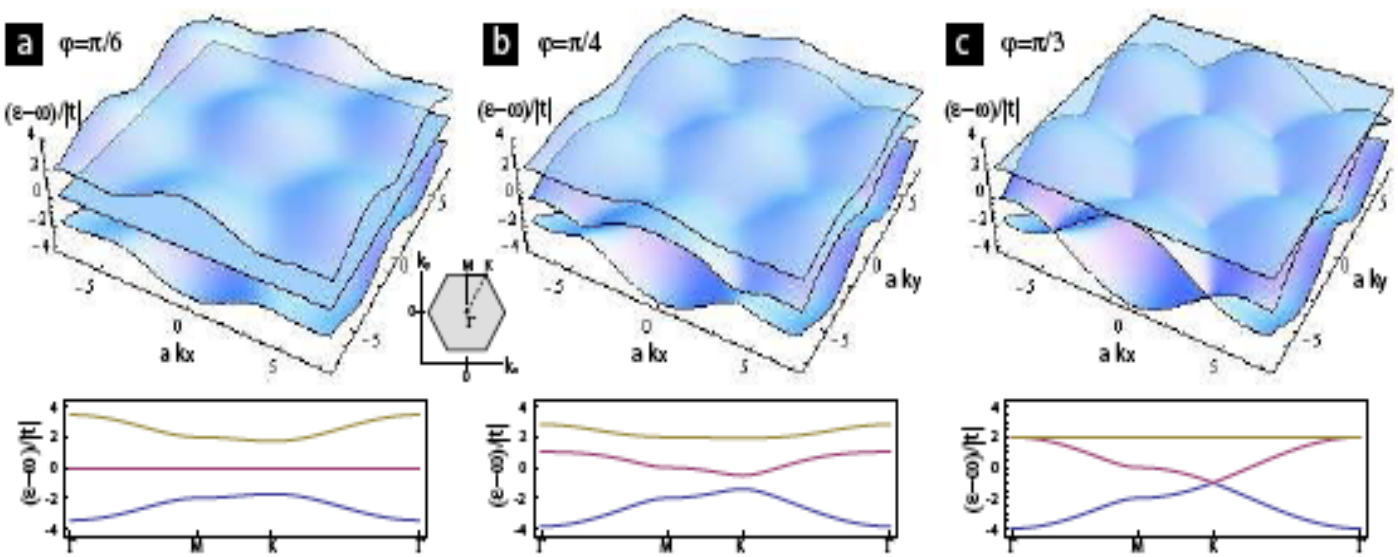
Analogy to cold
Atoms: I. Spielman;
F. Gerbier & J. Dalibard;
N. Trivedi

H. Manoharan: molecular graphene



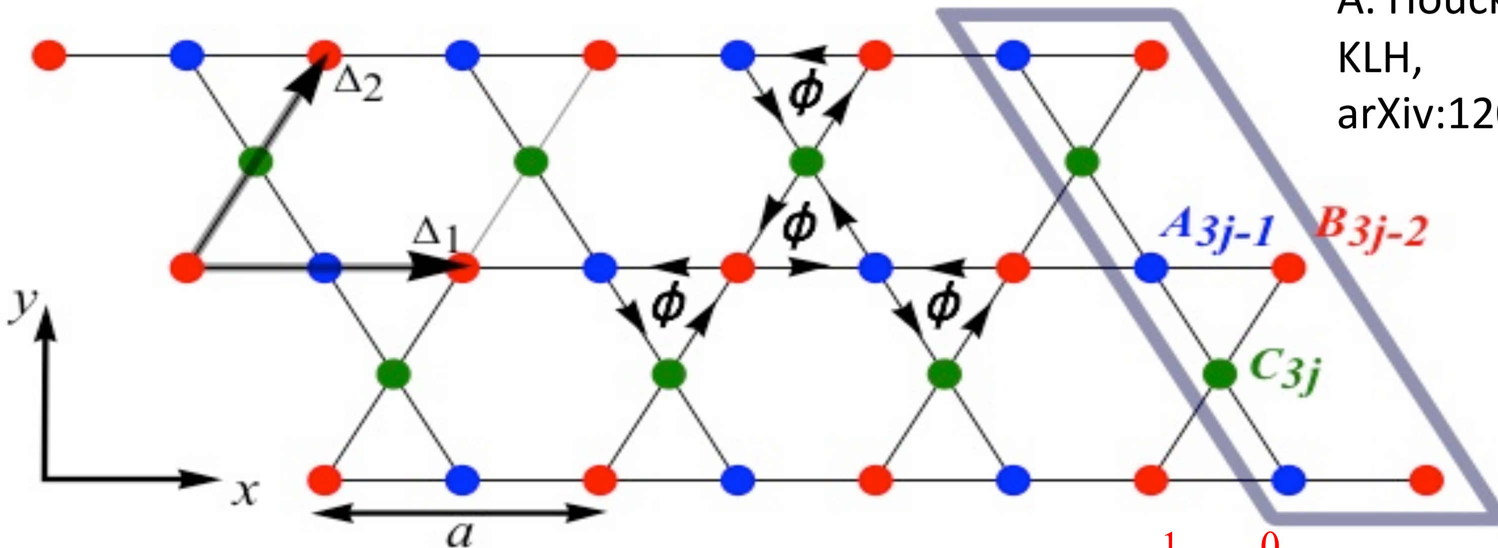
J. Koch, A. Houck, KLH
and S. Girvin
PRA **82**, 043811 (2010)

A. Greentree & A. Martin,
Physics 3, **85** (2010)

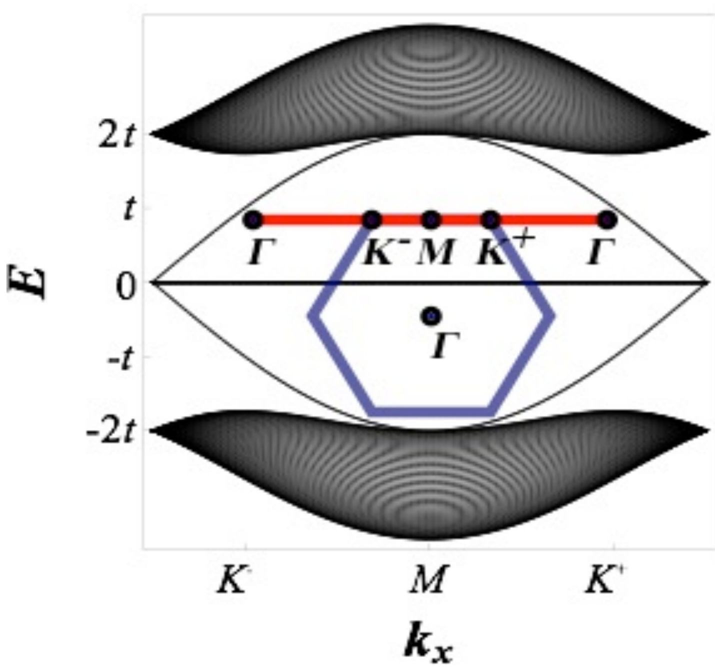


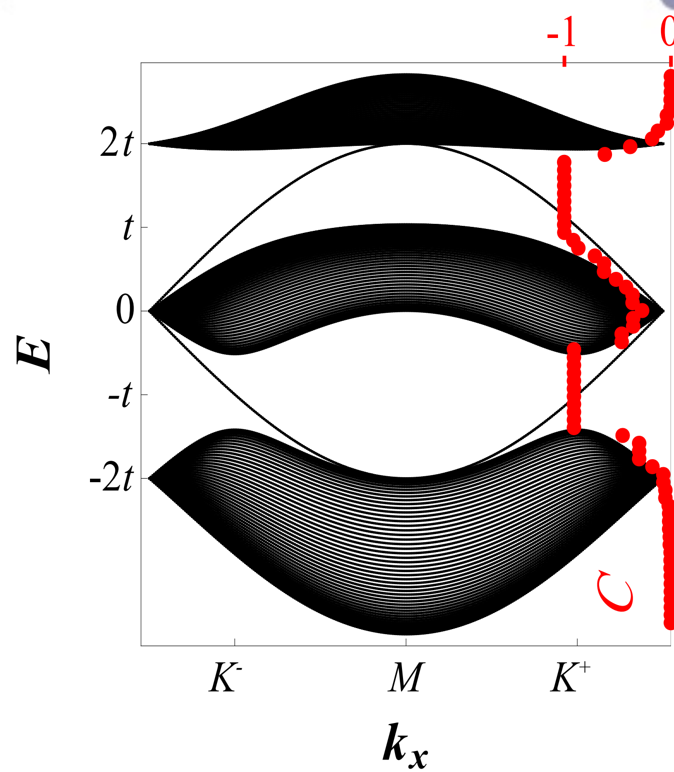
Topological Phases?

A. Petrescu,
A. Houck &
KLH,
arXiv:1206.1539



Φ = π/6





Karplus-Luttinger, 1954

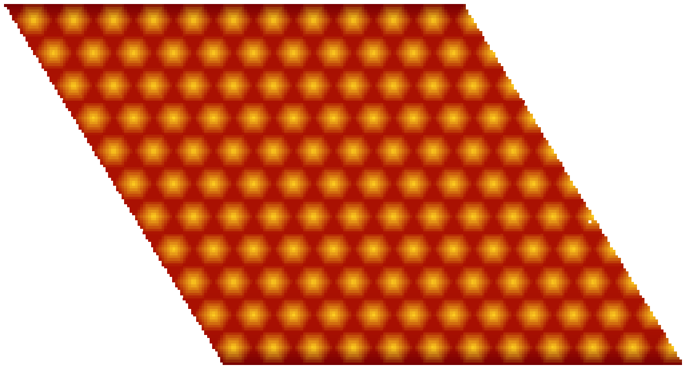
D. Haldane, 2004

See also
D. Bergman
& G. Refael, 2010

Φ = π/4

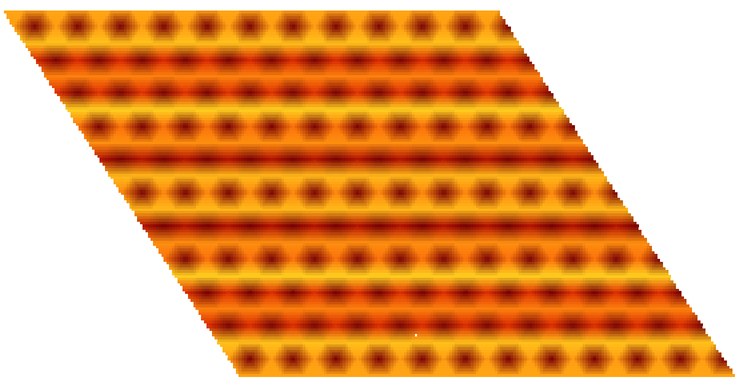
LDOS

Φ = π/6

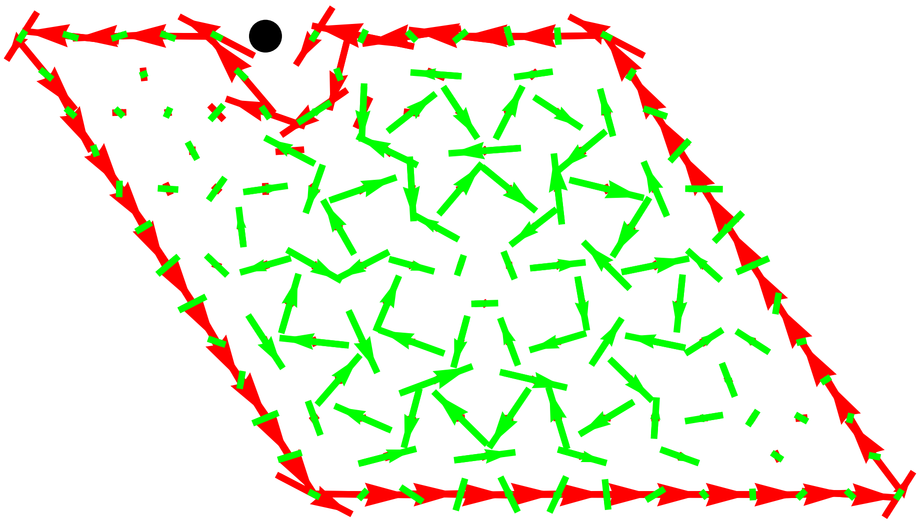


Φ = π/6
disorder

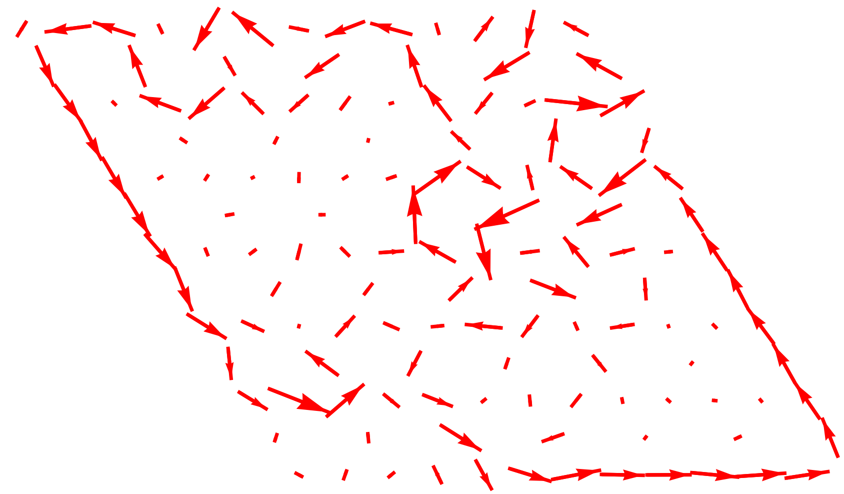
Φ = π/4



Quantum versus Anomalous Hall Effect of Light…



Red: situation at $\Phi = \pi/6$
Green: situation at $\Phi = \pi/4$



situation at $\Phi = \pi/6$
disordered case

Chern number **non-quantized** for AHE and measurable…

Synthetic B-field: Loops in k space and interference experiment

See also related idea by D. Price and N. Cooper, PRA 2012

Kagome lattice: why interesting…

Flat band (search for ferromagnetism)

A. Mielke; H. Tasaki; E. Lieb

Exotic Topological Phases:

H. M. Guo & M. Franz, PRB 2009

E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011

N. Regnault and A. Bernevig, PRB 2012,…

Spin liquid search, classical degeneracies

Experimentally relevant: 2D Materials (Orsay; Princeton;…)

Cold atoms: Berkeley; see D. Stamper-Kurn group, 2011

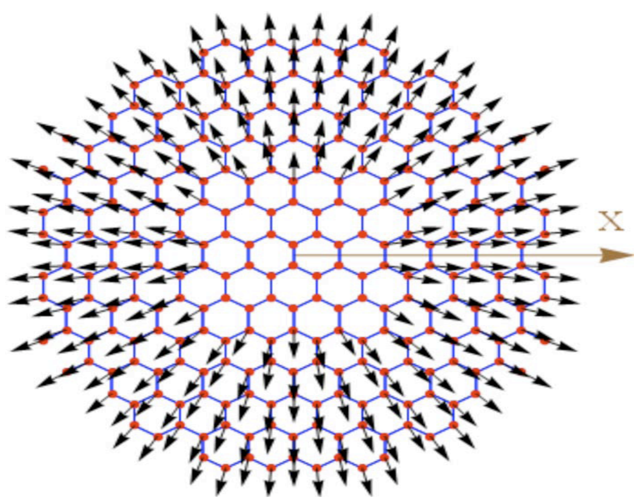
L. Balents, Nature 464, 199 (2010)

S. Yang, D. Huse and S. White, Science (2011)

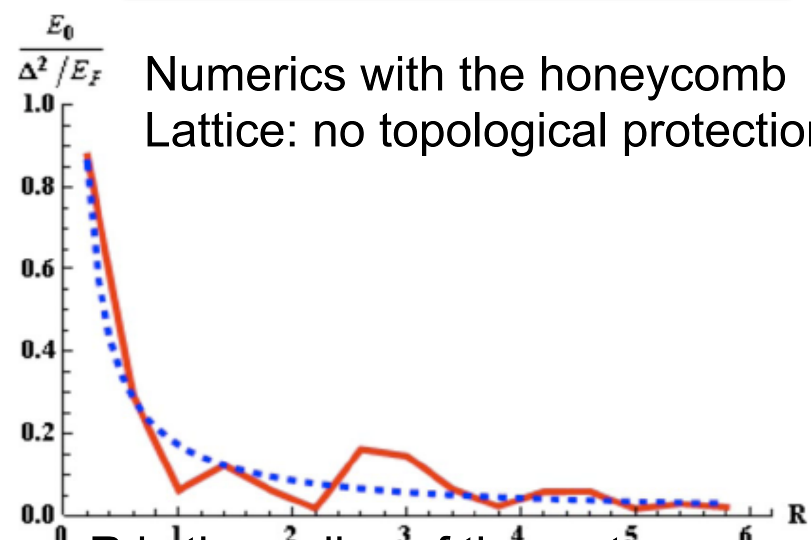
Summary: New Mysteries on the Honeycomb Lattice

- Mott & spin-orbit coupling; spin liquid & d+id SC
- Experimental effort: Iridates and Artificial Lattices
- Majoranas (J. Meyer; next talk)

4 Majorana fermions
s-wave superconductor:
attractive on-site interactions
D. Bergman & KLH, PRB 2009
P. Ghaemi & F. Wilczek



¼ of SC graphene
(topological insulators coupled to
s-wave SCs: Fu-Kane or 1D wires)



Numerics with the honeycomb Lattice: no topological protection

R is the radius of the vortex