The non-Abelian T-dual of Klebanov-Witten Background and its Penrose Limits

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Plan of the talk

- Introduction and Motivation
- Non-Abelian T-duality
- pp-waves from duals of $AdS_5 \times S^5$
- pp-waves from duals of $AdS_5 \times T^{1,1}$
- Field Theory Duals
- Conclusion

Itsios, Nastase, Nunez, Sfetsos, Zacarias
(arXiv:1711.09911)
Roychowdhury, Tripathy (To appear)

Introduction and Motivation

- pp-waves $\rightarrow$ exact string background.
- Interacting string states from gauge theory using AdS/CFT
- For orbifolds of $AdS_5 \times S^5$, T-dual description is more natural.
- Non-Abelian T-duality as solution generating technique in sugra.
- Interesting interplay between holography and non-Abelian T-duality
- Generates new dual field theories
- Penrose limits and non-Abelian T-duality
- Completion of the background using holography

Non-Abelian T-duality

- For string vacua with a $U(1)$ isometry, the Abelian T-duality

$$
\begin{aligned}
&g'_{xx} = 1/g_{xx},\, g'_{x\mu} = B_{x\mu}/g_{xx} \\
&g'_{\mu\nu} = g_{\mu\nu} - (g_{x\mu}g_{x\nu} - B_{x\mu}B_{x\nu})/g_{xx} \\
&B'_{x\mu} = g_{x\mu}/g_{xx} \\
&B'_{\mu\nu} = B_{\mu\nu} - (g_{x\mu}B_{x\nu} - g_{x\nu}B_{x\mu})/g_{xx} \\
&\phi' = \phi - (1/2)\log g_{xx}
\end{aligned}
$$

Buscher (1987)

- Symmetry of the string worldsheet action

$$
S = \int (g + B)(\partial X)^2
$$

Non-Abelian T-duality

- Generalizaion to string backgrounds with non-Abelian isometries.
- Standard procedure: gauge a global symmetry of σ-model action, integrate out the gauge field.
- Can map a geometry with non-Abelian isometry to another one with a reduced symmetry.

  de la Ossa, Quevedo (1992)
- Generalized to incorporate RR fields.
- Dualize $SU(2)$ isometry subgroup in $AdS_5 \times S^5$ and $AdS_3 \times S^3 \times T^4$.
- Dual geometry preserves half of the original supersymmetry.

  Sfetsos, Thompson (2010)

Non-Abelian T-duality

- σ-model action for $g \in G$:

$$S = -\int \mathrm{Tr}(g^{-1}\partial_- g g^{-1}\partial_+ g)$$

  Invariant under $G_L \times G_R$ isometry.

- Upon gauging G_L

$$S = -\int \mathrm{Tr}(g^{-1}D_- g g^{-1}D_+ g) + i\mathrm{Tr}(vF)$$

- Integrating out the gauge field, we get the σ-model action for v:

$$S = \int \partial_+ v_i (M^{-1})^{ij} \partial_- v_j$$

- Symmetric and antisymmetric parts of M^{-1} gives the target space metric and B field respectively.

Example

- Consider the near horizon geometry of $D1 - D5$ system

$$ds^2 = ds^2(AdS_3) + ds^2(S^3) + ds^2(T^4)$$

with $\phi = 0$ and three from flux F_3 along AdS_3 and S^3.
- In this case,

$$M_{ij} = \delta_{ij} + \epsilon_{ijk} x_k \text{ , hence } (M^{-1})^{ij} = \frac{1}{1 + r^2}(\delta_{ij} + x_i x_j - \epsilon_{ijk} x_k)$$

- The dual geometry

$$ds^2 = ds^2(AdS_3) + dr^2 + \frac{r^2}{1 + r^2} d\Omega_2^2 + ds^2(T^4)$$

Example

- The dilation, NS-NS and RR fluxes are

$$\begin{aligned}
\phi &= \frac{1}{2}\log(1+r^2) \\
H_3 &= \frac{r^2(3+r^2)}{(1+r^2)^2} dr \wedge \mathrm{Vol}(S^2) \\
F_0 &= 1 \\
F_2 &= \frac{r^3}{1+r^2}\mathrm{Vol}(S^2) \\
F_4 &= -r\, dr \wedge \mathrm{Vol}(AdS_3) + \mathrm{Vol}(T^4)
\end{aligned}$$

- This background solves massive type IIA supergravity

Sfetsos, Thompson (2010)

Penrose limits

- Focus on geometry in the vicinity of a null geodesic
- Consider the geodesic equation

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

- For motion along an isometry direction y this gives $\Gamma^\mu{}_{yy} = 0$ and hence,

$$\partial^\mu g_{yy} = 0$$

- In addition, we need to impose the condition that the geodesic is null

$$ds^2 = 0$$

Dual Backgrounds From $AdS_5 \times S^5$

- Consider the $AdS_5 \times S^5$ solution (with $L = \alpha'^2 g_s N\pi/4$)

$$
\begin{aligned}
ds^2 &= ds^2(AdS_5) + ds^2(S^5) \\
ds^2(AdS_5) &= 4L^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2) \\
ds^2(S^5) &= 4L^2(d\alpha^2 + \sin^2\alpha d\beta^2) \\
&+ L^2\cos^2\alpha(d\theta^2 + d\phi^2 + d\psi^2 + 2\cos\theta d\theta d\psi)
\end{aligned}
$$

- The self-dual five form field

$$F_5 = \frac{2}{g_s L}(1 + *)\mathrm{Vol}(AdS_5)$$

- Abelian T-duality along ψ gives (ϕ, H_3, F_4 and)

$$ds^2 = 4L^2\Big(ds^2(AdS_5) + d\Omega_2^2(\alpha,\beta)\Big) + \frac{L^2 d\psi^2}{\cos^2\alpha} + L^2\cos^2\alpha d\Omega_2^2(\chi,\xi)$$

Dual Backgrounds From $AdS_5 \times S^5$

- The isometry directions are ξ, β and ψ.
- For motion along the ξ isometry, there are two geodesics

$$\{\alpha = 0, \chi = \pi/2\} \text{ and } \{\alpha = \pi, \chi = \pi/2\}$$

- Make the following expansion around the first geodesic

$$r = \frac{\bar{r}}{2L}\ , \quad \alpha = \frac{x}{2L}\ , \quad \psi = \frac{y}{L}\ , \quad \chi = \frac{\pi}{2} + \frac{z}{L}\ ,$$
$$t = x^+\ , \quad \xi = 2\,x^+ + \frac{x^-}{L^2}\ ,$$

Itsios et. al. (2017)

Dual Backgrounds From $AdS_5 \times S^5$

- The resulting pp wave metric,

$$\begin{aligned} ds^2 &= 4\,dx^+dx^- + d\bar{r}^2 + \bar{r}^2 d\Omega_3^2 + dx^2 + x^2 d\beta^2 + dz^2 + dy^2 \\ &- \left(\bar{r}^2 + x^2 + 4\,z^2\right)(dx^+)^2\ , \end{aligned}$$

- Other background fields

$$B_2 = 2\,y\,dz \wedge dx^+\ , \qquad e^{-2\Phi} = \frac{1}{\tilde{g}_s^2}\ ,$$

$$F_4 = \frac{4\,x}{\tilde{g}_s} dx \wedge d\beta \wedge dz \wedge dx^+ .$$

- Same solution for the second geodesic.

Itsios et. al. (2017)

Dual Backgrounds From $AdS_5 \times S^5$

- No pp-wave along β as well as ψ directions.
- For motion on (ψ, ξ) plane, the geodesic at $\{\alpha = 0, \chi = \pi/2\}$ admit pp-wave.
- After T-dualizing along an $SU(2)$ direction, the geometry becomes

$$
\begin{aligned}
ds^2 &= 4L^2 ds^2(\mathrm{AdS}_5) + 4L^2 d\Omega_2^2(\alpha, \beta) \\
&+ \frac{\alpha'^2 d\tilde{\rho}^2}{L^2 \cos^2 \alpha} + \frac{\alpha'^2 L^2 \tilde{\rho}^2 \cos^2 \alpha}{\alpha'^2 \tilde{\rho}^2 + L^4 \cos^4 \alpha} d\Omega_2^2(\chi, \xi)
\end{aligned}
$$

- Nonzero ϕ, H_3, F_2 and F_4.
- Isometry along β, ξ does not give pp-wave geometry.
- pp-wave for combined motion in ρ, ξ.

Itsios et. al. (2017)

Dual Backgrounds From $AdS_5 \times T^{1,1}$

- For coincident $D3$ at conical singularity, the gravity dual is $AdS_5 \times T^{1,1}$, constant dilation & self-dual F_5.
- The $T^{1,1}$ metric is

$$
\begin{aligned}
ds^2_{T^{1,1}} &= \frac{1}{6}\, d\Omega_2^2(\theta_1, \phi_1) + \frac{1}{6}\, d\Omega_2^2(\theta_2, \phi_2) \\
&+ \frac{1}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 .
\end{aligned}
$$

- Isometry along ϕ_1, ϕ_2, ψ directions
- Abelian T-duality along ψ gives

$$
d\hat{s}^2 = L^2 ds^2(\mathrm{AdS}_5) + \frac{L^2}{6}\left[d\Omega_2^2(\theta_1, \phi_1) + d\Omega_2^2(\theta_2, \phi_2) + 54 d\psi^2\right] .
$$

plus dilaton, H_3 and F_4.

Dual Backgrounds From $AdS_5 \times T^{1,1}$

- Isometries along ϕ_1, ϕ_1, ψ
- No constraint for ψ.
- For ϕ_1 isometry, geodesic at $\theta_1 = \pi/2$.
- Same pp-wave solution as in $AdS_5 \times S^5$.
- For motion along ϕ_1, ψ directions, geodesic at $\theta_1 = \pi/2, \theta_2 = 0$.
- Expand around this geodesic

$$r = \frac{\bar{r}}{L},\ \theta_1 = \frac{\pi}{2} + \frac{z}{L},\ \theta_2 = \frac{x}{L}.$$

Dual Backgrounds From $AdS_5 \times T^{1,1}$

- This expansion leads to pp-wave solution

$$\begin{aligned} ds^2_{pp} &= 2dudv + d\bar{r}^2 + \bar{r}^2 d\Omega_3^2 \\ &+ dz^2 + dx^2 + x^2 d\beta^2 + dw^2 - (\bar{r}^2 + 36J^2 z^2) du^2. \end{aligned}$$

- Dilaton $\exp(-2\hat{\phi}) = \lambda^2/\tilde{g}_s^2$, NS-NS three-form flux

$$\hat{H}_3 = 2\, dz \wedge dw \wedge du + 2x\sqrt{1-6J^2}\, dx \wedge d\beta \wedge du.$$

- The RR fields at Penrose limit are,

$$\hat{F}_2 = 0,\ \hat{F}_4 = \frac{4\sqrt{6}}{3\tilde{g}_s} Jx\, du \wedge dz \wedge dx \wedge d\beta.$$

Dual Backgrounds From $AdS_5 \times T^{1,1}$

- Applying non-Abelian T-duality on an $SU(2)$ isometry, the metric

  $$d\hat{s}^2 = L^2 ds^2_{AdS_5} + L^2 d\hat{s}^2_{T^{1,1}},$$

  where,

  $$\begin{aligned} d\hat{s}^2_{T^{1,1}} &= \lambda_1^2 d\Omega_2^2(\theta_1,\phi_1) + \frac{\lambda_2^2\lambda^2}{\Delta} x_1^2 (d\psi + \cos\theta_1 d\phi_1)^2 \\ &+ \frac{1}{\Delta}((x_1^2+\lambda^2\lambda_2^2)dx_1^2 + (x_2^2+\lambda_2^4)dx_2^2 + 2x_1x_2dx_1dx_2), \end{aligned}$$

  with

  $$\lambda_1^2 = \lambda_2^2 = 1/6, \lambda^2 = 1/9, \text{ and } \Delta = \lambda_2^2 x_1^2 + \lambda^2(x_2^2+\lambda_2^4),$$

- For motion along ϕ_1, the geodesic is at $x_1 = 0,\ x_2 = 0$ and $\theta_1 = \frac{\pi}{2}$.

Dual Backgrounds From $AdS_5 \times T^{1,1}$

- The resulting pp-wave solution is

$$\begin{aligned} ds^2 &= 2dudv + d\bar{r}^2 + \bar{r}^2 d\Omega_3^2 \\ &+ dz^2 + dy_1^2 + y_1^2 d\psi^2 + dy_2^2 - 6(\bar{r}^2 + 6z^2)du^2. \end{aligned}$$

- The dilaton

$$e^{-2\hat{\Phi}} = \frac{8}{\tilde{g}_s^2}\lambda^2\lambda_2^4,$$

- Three form flux

$$\hat{H}_3 = 2\sqrt{6}\, du \wedge dz \wedge dy_2.$$

- The RR fields

$$\hat{F}_2 = \frac{8}{3\sqrt{3}\tilde{g}_s}\, du \wedge dz, \;\; \hat{F}_4 = 0.$$

QFT Duals

- Abelian T-dual of $AdS_5 \times S^5/Z_k \Longleftrightarrow$ Circular Quiver with k-nodes, $SU(N)^k$ gauge group.
- $N = 2$ Vector multiplets at each node
- $N = 2$ bifundamental hypermultiplets between adjecent nodes
- NATD: Infinitely long quiver with increasing gauge group

$$SU(N) \times SU(2N) \times SU(3N) \times \cdots$$

- Quiver terminates upon completion.

Lozano, Nunez (2016)

QFT Duals

- Adjoint complex scalar X_1 from node i vector multiplet.
- $(\bar{i}, i+1)$-bifundamental complex scalar V_i
- $(i, \overline{i+1})$-bifundamental W_i.
- $U(1)_R : X_i \to e^{i\alpha} X_i, d^2\theta \to e^{-i\alpha} d^2\theta$
- $V_i \to e^{i\alpha} V_i, W_i \to e^{-i\alpha} W_i$
- $SU(2)_R$ which rotates V_i and $\overline{W}_i$.
- Superpotential

$$W = \sum_{i=1}^{k} \int d^2\theta \, \mathrm{Tr}_{i+1} \left[V_i X_i W_i \right] \, .$$

Conclusion

- QFT duals for Abelian T-dual of $AdS_5 \times T^{1,1}$ are intersecting branes in type IIA.
- For non-Abelian T-duals, we have $N = 1$ linear quivers from M5-branes.
- Construct QFT duals for these pp-wave backgrounds.
- Study other backgrounds (with AdS_3 factors).