







Charge orders and Strange metals in Cuprates

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College de France, June 2nd, 2022

Yvan Sidis, J. C. Séamus Davis, Mohammad Hamidian,

Alain Sacuto, Henri Alloul, Nigel Hussey, Dorothée Colson Philippe Bourges, Victor Balédent, Dalila Bounoua, Brigitte Leridon,

Cyril Proust, M-H Julien...



Konstantin Borisovich Efetov (April 29, 1950 – August 11, 2021) was a <u>Russian</u>/German theoretical physicist, recognized leader in the theory of condensed matter, and a teacher of a number of actively working theorists.

Mott transition

Fluctuations



1. The context of strong coupling : doping a Mott insulator

Resonating Valence Bond (RVB) : pairs form and fluctuate





$$\chi_{ij} = \sum_{\sigma} \langle f_{i\sigma}^{\dagger} f_{j\sigma} \rangle,$$
$$\Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{i\uparrow} \rangle.$$



Anderson, Lee, Wen, Nagaosa, Kotliar

 $f_{i\uparrow}^{\dagger}f_{i\uparrow} + f_{i\downarrow}^{\dagger}f_{i\downarrow} + b_i^{\dagger}b_i = 1.$



2. QCP under the SC dome





20 Monthoux, Pines, Lonzarich 07

QCP questionned : an abrupt change at p^* ?



ARPES Bi2212, Chen et al. 2019

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Nernst effect : resolved controversy

Amplitude Fluctuations

Phase fluctuations



Condensate





Recent Exp. developments Charge Order

Presence of competing orders

Charge modulations in strong competition with SC state



Hoffman, 2002

Kapitulnik, 2002



STM measurement of charge density modulation : $Re(\chi_{ij}) = \hat{d}|\chi_{ij}|cos(\boldsymbol{Q}\cdot\boldsymbol{r} + \phi(\boldsymbol{r}))$

d 6 % Occur. 3 B=0T0 -1 -1/2 1/2 0 1 $\phi_y(\mathbf{r})$ (π rad.) -1 $\phi_y(\mathbf{r})$ (π rad.) 20r b 16 8 0ccur. 8 % B = 8.5Ta 0L -1 -1/2 1/2 0 1 -1 +1 $\phi_y(\mathbf{r}) - \phi_0(\mathbf{r})$ (π rad.) $\cos\left(\mathbf{Q}\cdot\mathbf{r}+\phi_y(\mathbf{r})\right)$

B = 0 T random phase distribution :

 $B \neq 0$ *T* centered distribution :

M.H. Hamidian et al., Nat. Phys. **12**, 150 (2015). M.H. Hamidian et al., arXiv:1508.00620 (2015)

Charge order Landscape

YBa₂Cu₃O_{6+x}



Haug, New J. Phys. 2010 T. Wu et al., PRB 2013

Courtesy Y. Sidis

Nematicity

Inversion symmetry

loop currents

anomalous Kerr effect $T_k < T^*$

Xia, PRL 2008

Incipient CDW – $T_m < T^*$

 $Q^* = (\delta, 0)$ and $(0, \delta)$ with $\delta \sim 0.3$

Chang , Nature Phys. 2012 Ghiringhelli, Science 2012

> Stable CDW under magnetic field & Fermi surface reconstruction (NMR, quantum oscillation, ultrasound)

D. LeBoeuf, *Nature* 2007.T. Wu et al., *Nature* 2011.D. LeBoeuf et al., *Nature Physics* 2013.

Emergent symmetry



Sachdev et al (2013) Efetov, Meier, CP (2013)

Pseudo-gap from quantum criticality AFM QCP in d=2

K.B.Efetov, H.Meier, C.P. Nat. Phys. 9, (2013)

Dispersion linearized around 8 hot spots

$$\mathcal{L} = \chi^{\dagger} \left(\partial_{\tau} + \varepsilon (-i\hbar\nabla) + \lambda \vec{\phi}\vec{\sigma} \right) \chi \qquad \langle \phi^{i}_{\omega,\mathbf{k}} \phi^{j}_{-\omega,-\mathbf{k}} \rangle \propto \frac{\delta_{ij}}{(\omega/v_{s})^{2} + (\mathbf{k} - \mathbf{Q})^{2} + a}$$

Composite order parameter

$$c_{\mathbf{p}}^{\mathrm{pp}}\left\langle \left(i\sigma_{2}\right)_{\alpha\beta}\psi_{\alpha,\mathbf{p}}\psi_{\beta,-\mathbf{p}}\right\rangle + c_{\mathbf{p}}^{\mathrm{ph}}\left\langle \delta_{\alpha\beta}\psi_{\alpha,\mathbf{p}}\psi_{\beta,-\mathbf{p}}^{*}\right\rangle,$$

SU(2) symmetry and fluctuations

$$u = \begin{pmatrix} \Delta_{-} & \Delta_{+} \\ -\Delta_{+}^{*} & \Delta_{-}^{*} \end{pmatrix} \quad \text{with} \quad |\Delta_{+}|^{2} + |\Delta_{-}|^{2} = 1$$

M. Metlitsky and S. Sachdev (2010)



Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa₂Cu₃O_y

Tao Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Mladen Horvatić¹, Claude Berthier¹, W. N. Hardy^{2,3}, Ruixing Liang^{2,3}, D. A. Bonn^{2,3} & Marc-Henri Julien¹





$$f[\psi,\phi] = \alpha_{\psi}|\psi|^{2} + \frac{\beta_{\psi}}{2}|\psi|^{4} + \alpha_{\phi}|\phi|^{2} + \frac{\beta_{\phi}}{2}|\phi|^{4} + \gamma|\psi|^{2}|\phi|^{2},$$

$$\alpha_{\phi} = \alpha'_{\phi} + a_{co}T^2$$

D. Chakraborty et al, Preprint

Non Linear Sigma Model



Topology and local structures

Homotopy classes

$$\Delta_{-,R}^{2} + \Delta_{-,I}^{2} + \Delta_{+,R}^{2} + \Delta_{+,I}^{2} = 1$$
$$\pi_{2}(S_{3}) = 0$$

0(3) non linear σ -model





- 1 / / \ \ \ / \ / - / \

Vortex structure Phase diagram

~ 1 N



 $1 / / \rightarrow$ N $\leftarrow \leftarrow \leftarrow$ (b) Nernst



(c) Pseudogap

Fractionalization of a PDW

The phase diagram





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Fractionalization of a Pair Density Wave

Modulated particle-particle pair :
$$\Delta_{ij}^{PDW} = \left\langle c_{i,\sigma} c_{j,\bar{\sigma}} e^{i \mathbf{Q} \cdot \mathbf{r}_{ij}} \right\rangle$$

PDW fractionalization :

$$\Delta_{ij}^{PDW} = \left[\Delta_{ij}, \chi_{ij}^*\right]$$
$$\Delta_{ij}^* \Delta_{ij} + \chi_{ij}^* \chi_{ij} = 1$$

Uniform particle-particle pair :

Modulated particle-hole pair :

$$\Delta_{ij} = \langle c_{i,\sigma} c_{j,\bar{\sigma}} \rangle \longrightarrow \text{Charge (2)}$$
$$\chi_{ij} = \left\langle c_{i,\sigma}^{\dagger} c_{j,\sigma} e^{i \mathbf{Q} \cdot \mathbf{r}_{ij}} \right\rangle \longrightarrow \text{Translation symmetry}$$

Phase transformation :

$$\begin{cases} \Delta_{ij} \to e^{i\theta} \Delta_{ij} \\ \chi_{ij} \to e^{i\theta} \chi_{ij} \end{cases}$$

Ansatz : $|PG\rangle = \left(\hat{\chi}_{ij} + \hat{\Delta}_{ij}\right)|0\rangle$ + constraint

The phase díagram

the second se

$$S = \frac{1}{2} \int d^2x \sum_{a,b=1}^{2} |\omega_{ab}|^2, \text{ with } \omega_{ab} = z_a \partial_{\mu} z_b - z_b \partial_{\mu} z_a,$$

$$z_1 = \Delta, z_2 = \chi, z_1^* = \Delta^*, z_2^* = \chi^*.$$

$$T = T^*$$

 $\theta \text{ gets to fluctuate}$
We obtain the constraint

$$|\Delta_{ij}|^2 + |\chi_{ij}|^2 = (E^*)^2$$

$$T = T_c$$

 $\phi \text{ gets frozen and we have global phase coherence.}$
Meissner effect.

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STM measurement of charge density modulation : $Re(\chi_{ij}) = \hat{d}|\chi_{ij}|cos(\boldsymbol{Q}\cdot\boldsymbol{r} + \phi(\boldsymbol{r}))$

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The fractionalized PDW supports the symmetry of the Q=0 loop currents as a precursor order parameter

200 K

□ 300 K

300

250

200

150

100

50

0

0.9

_ ≡

1.1

 $l = (|\Delta_Q|^2 - |\Delta_{-Q}|^2)$

Magnetic intensity (a.u.

品中層

H (r.l.u.)

■ 100 K

■ 200 K

🗆 300 K

Ξ

1.1



Sachdev et al (2013) Efetov, Meier, CP (2013)

Strange metals



$$\sigma_{xx} = \frac{ne^2\tau}{m} \qquad \cot\theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{ne^2\tau}{m}$$
$$\sigma_{xy} = \frac{ne^3B\tau^2}{cm} \qquad R_H = \frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{Bm}{ne}$$



Transport in the Strange Metal

Recent controversy

Spectral weight missing in the Strange Metal regime ?

Our answer : two types of carriers, fermions and charge-2 bosons with finite momentum





H/T scaling in the SM phase

 $\rho(H,T) - \rho(0,0) = \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2}$





• Incoherent $\sigma_{xy}=0$

• Planckían límít

Ayres et al., preprint 2020, courtesy N.Hussey

Presence of « another species » in this regime : MEELS experiment



Mitrano et al., 2018

Jamming transition

Paír tunelíng ín LSCO : noíse measurement



Zhou et al., 2019

Our proposal : Charged bosons in the Strange Metal phase

$$\mathcal{D}^{-1}(\mathbf{q}, i\omega_n) = \gamma |\omega_n| + \mathbf{q}^2 + \mu(T)$$

$$\sigma_{xx} \left(i\omega \to \omega + i\delta \right) = \frac{\sigma_0^b \tau}{\left(1 - i\frac{\gamma\omega}{2\mu} \right)},$$



$$\sigma_{xx} = \frac{ne^2}{m}\tau_{xx}$$

$$\cot \theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{c}{eB} \frac{\tau_x}{\tau_{xy}^2}$$

$$\sigma_{xy} = \frac{ne^3B}{cm}\tau_{xy}^2$$



Averaging the Hall conductivity around the Fermi surface with hot and cold spots

> Kokalj, McKenzie and Hussey, 2012



 $\tau_{xx}^{-1} \sim T$ $\tau_{xy}^{-1} \sim T^{1.5}$

Predictions ARPES

Why the system would want to do this ?

$$\begin{split} H &= \sum_{i,j,\sigma} c_{i,\sigma}^{\dagger} t_{ij} c_{j,\sigma} + J \sum_{\langle i,j \rangle} \mathbf{S}_{i} \cdot \mathbf{S}_{j} \\ &+ V \sum_{\langle i,j \rangle} n_{i} n_{j} \end{split}$$

b)

a)

$$\chi_{ij} = \frac{1}{2} < c_{i\sigma}^{\dagger} c_{j\sigma} > \qquad \Delta_{ij}^{*} = < c_{i}^{\dagger} \uparrow c_{j}^{\dagger} \downarrow >$$

 $= 0.017 \ eV,$

$$\Psi_{ij} = (\hat{\Delta}_{ij}, \hat{\chi}_{ij})^t \qquad |\Psi_{ij}| = E^*$$

Condensation energy

$$E_{PG} = \frac{1}{2\tilde{J}} |\Psi_{k=k_F}|^2 = 0.$$

$$E_{SC} = \frac{1}{2L} |\Delta_{k=k_F}|^2 = 0.$$

$$E_{SC} = \frac{1}{2J_{-}} |\Delta_{k=k_F}|^2 = 0.014 \ eV,$$
$$E_{CDW} = \frac{1}{2J_{+}} |\chi_{k=k_F}|^2 = 0.011 \ eV.$$

Mean-field decoupling

$$\Delta_k = \sum_{\sigma} \sigma c_{k,\sigma} c_{-k,\bar{\sigma}}$$
$$\chi_k^Q = \sum_{\sigma} c_{k,\sigma}^{\dagger} c_{k+Q,\sigma}$$

Energy scales :

 $\begin{aligned} \Delta_k &\sim 3J - V \\ \chi_k^Q &\sim 3J + V \end{aligned}$

Opening a gap in the Fermi surface



M. Grandadam et al. PRB (2020)

ARPES, Bi2201

Comparison with CDMFT': « hidden fermion»



M. Grandadam et al. 2021

Raman scattering



Solving gap equations

$$\Delta_{k,\omega} = -\frac{1}{\beta} \sum_{q,\Omega} \frac{J_{-}(q,\Omega) \,\Delta_{k+q}}{\left(\omega + \Omega\right)^2 - \xi_{k+q}^2 - \Delta_{k+q}^2},$$

$$\chi_{k,\omega} = -\frac{1}{\beta} \sum_{q,\Omega} \frac{J_+(q,\Omega) \,\chi_{k+q}}{(\omega + \Omega - \xi_{k+q}) \,(\omega + \Omega - \xi_{k+Q+q}) - \chi_{k+q}^2},$$





Same order of magnitude for Δ and χ

M. Grandadam et al. PRB (2019)

p

Phonon Softening

Anomalous softening of phonons

Blackburn, 2013



S. Sarkar et al. (2020)



Conclusions

• Charge orders are a key players in cuprate physics: natural competitor of superconductivity

- Fractionalizing a PDW or a more complex boson
- Entangling particle-hole and particle-particle pairs at T*
- Explains recent Raman, phonon softening
- ARPES : back-bending, poles in self-energy (cf. DMFT studies)
- Can a charge-2 boson explain the mystery of strange metal and Hall resistivity ?
- Exp. predictions with mesoscopic noise, Josephson effects
- Numerical check in strong coupling approaches



Díscussions of the data and a few Refs

Maxence Grandadam, Catherine Pépin arXiv:2012.11226

Anurag Banerjee, Maxence Grandadam, Hermann Freire, Catherine Pépin arXiv:2009.09877

Saheli Sarkar, Maxence Grandadam, Catherine Pépin arXiv:2009.02975

Maxence Grandadam, Debmalya Chakraborty, Xavier Montiel, Catherine Pépin arXiv:2002.12622

D. Chakraborty, M. Grandadam, M. H. Hamidian, J. C. S. Davis, Y. Sidis, C. Pépin arXiv:1906.01633

Saheli Sarkar, Debmalya Chakraborty, Catherine Pépin arXiv:1906.08280

C. Pépin, D. Chakraborty, M. Grandadam, S. Sarkar arXiv:1906.10146

Quantum Criticality Or Cross-Over ?

QCP questioned : an abrupt change at p^* ?

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ARPES Bi2212, Chen et al. 2019

O(3) Non Linear Sigma Model



The context of strong coupling : doping a Mott insulator



$$H = P \left[-\sum_{\langle ij \rangle,\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{i\sigma} + J \sum_{\langle ij \rangle} \left(\mathbf{S}_{i} \cdot \mathbf{S}_{j} - \frac{1}{4} n_{i} n_{j} \right) \right] P$$

Anderson, Lee, Nagaosa, Rice etc...

P: projection on no double occupancy

The extend of the Cooper pairs phase fluctuations regime Nernst effect (Ong, Behnia), transport (Rullier-Albenque, Sebastian), Squid spectroscopy (Lesueur)...

The presence of a partner to SC pairing inhibits the visibility of phase fluctuations in transport and Nernst effect (Orgard, 2017)



Hsu et al, (2017)



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SU(2) symmetry and fluctuations

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⁸⁹Y NMR Evidence for a Fermi-Liquid Behavior in YBa₂Cu₃O_{6+x}

H. Alloul, T. Ohno,^(a) and P. Mendels

Physique des Solides, Université de Paris-Sud, 91405 Orsay, France (Received 15 May 1989)

We report NMR shift ΔK and T_1 data of ⁸⁹Y taken from 77 to 300 K in YBa₂Cu₃O_{6+x} for 0.35 < x < 1, from the insulating to the metallic state. A Korringa law and therefore a Fermi-liquid picture is found to apply for the spin part K_s of ΔK . The spin contribution $\chi_s(x,T)$ to χ_m is singled out, as the T variation of ΔK scales linearly with the macroscopic susceptibility χ_m . This implies that Cu(3d) and O(2p) holes do not have independent degrees of freedom. Their hybridization, which has a σ character, hardly varies with doping. These results put severe constraints on theoretical models of high- T_c cuprates.

PACS numbers: 74.70.Vy, 75.20.En, 76.60.Cq, 76.60.Es





FIG. 1. The shift ΔK of the ⁸⁹Y line, referenced to YCl₃ plotted vs T, from 77 to 300 K. The lines are guides to the eye.



Fractionalization in the PG phase ?





Is fractionalization compatible with the observation of Bogoliubov QP in the anti-nodal region ?

Coherence of the electrons ?

Amplitude Fluctuations

Phase fluctuations



Condensate





The concept of SU(2) symmetry

C.N.Yang & S-C. Zhang (1989)

Pseudo-Spins

$$\eta^{+} = \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{Q}\downarrow}$$
$$\eta_{z} = \sum_{\mathbf{k}} \left(c^{\dagger}_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} + c^{\dagger}_{\mathbf{k}+\mathbf{Q}\downarrow} c_{\mathbf{k}+\mathbf{Q}\downarrow} - 1 \right)$$

l=1 representation

$$\Delta_{1} = -\frac{1}{\sqrt{2}} \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow},$$
$$\Delta_{0} = \frac{1}{2} \sum_{\mathbf{k},\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{Q}\sigma},$$
$$\Delta_{-1} = -\Delta_{1}^{\dagger},$$

$$\begin{bmatrix} \eta^{\pm}, \Delta_m \end{bmatrix} = \sqrt{l (l+1) - m (m \pm 1)} \Delta_{m \pm 1},$$
$$[\eta_z, \Delta_m] = m \Delta_m.$$



0(3) non linear σ -model



Topological structure: Skyrmions in the pseudo spin space

