Ultra-high intensity lasers
and
quantum-electrodynamic effects

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Outline

- High-Energy Density Physics and Ultra High Intensity Physics

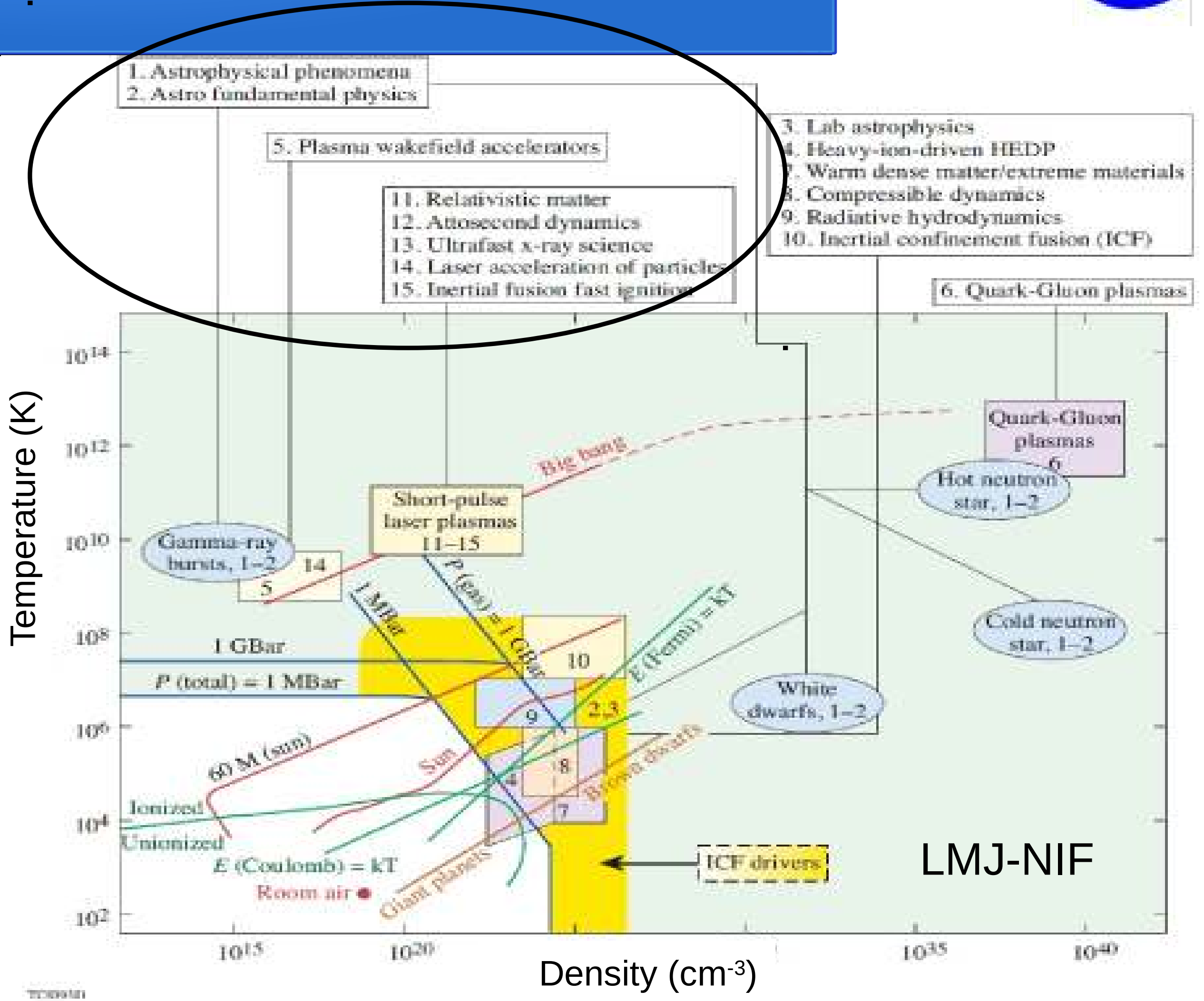
- Quantum Electro-dynamic (QED) processes:
    - Schwinger limit ; Photon-photon collision ; Pair electron-positron creation
    - Astrophysical context: Active Galactic Nuclei...
    - Breit-Wheeler process: Real gamma photon collision in laboratory
    - QED Calculation of differential and total BW cross section
    - Astrophysical application
    - About non-linear and linear BW cross section

Ultra High Intensity (UHI)*

UHI
&
Conditions
Plasmas

Intensity > 10^{19} W/cm^2

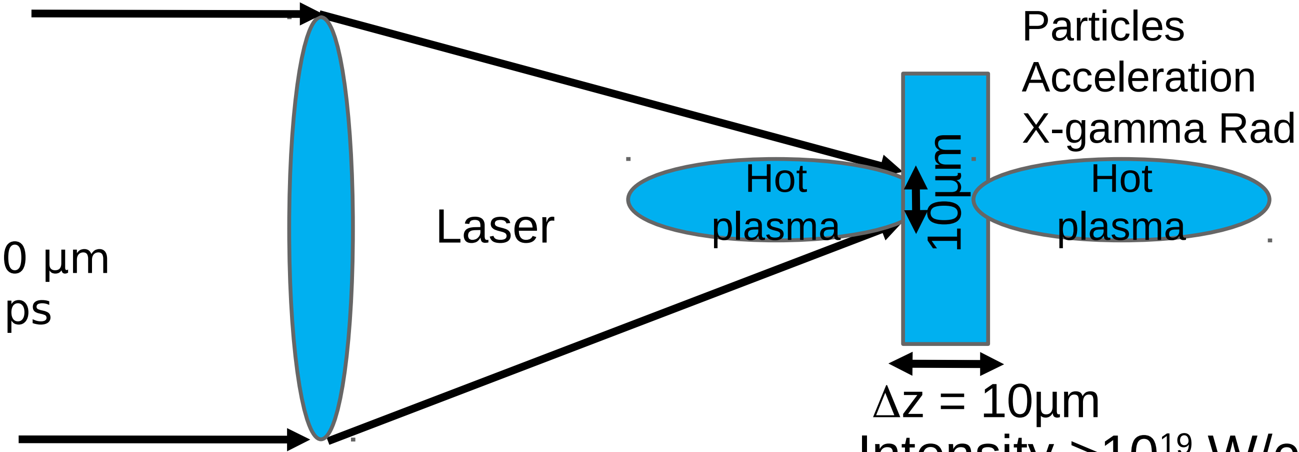
Electronic density: 10^{19} cm^{-3}



*[Ron Davidson et al., National Academies Press (2003); <http://www.nap.edu/>]

Laser-matter interaction accelerates electrons in a relativistic regime (UHI)

Assumed
E_L ~ 1mJ-1 kJ
λ_L= 0.8-1 µm
Focale spot diameter: 10 µm
Pulse duration : 10 fs-1 ps



Particles
Acceleration
X-gamma Radiation

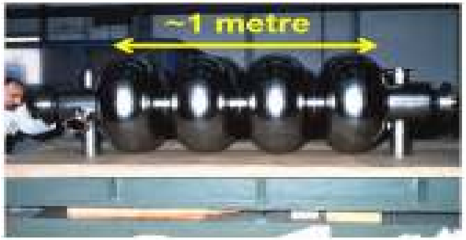
Δz = 10µm
Intensity >10^{19} W/cm^2

accelerating gradient [eV/m] = charge × long. Electricfield (E_Z)

PETAL
Omega-EP
Salle Jaune (LOA)
UHI 100 (Saclay)
Apollon
ELI...

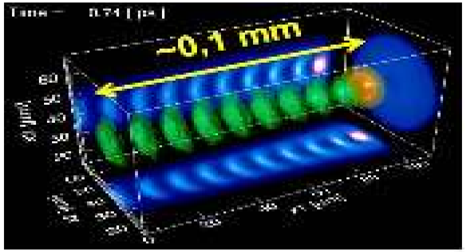
Resonator : RF cavity

E_z<100 MV/m



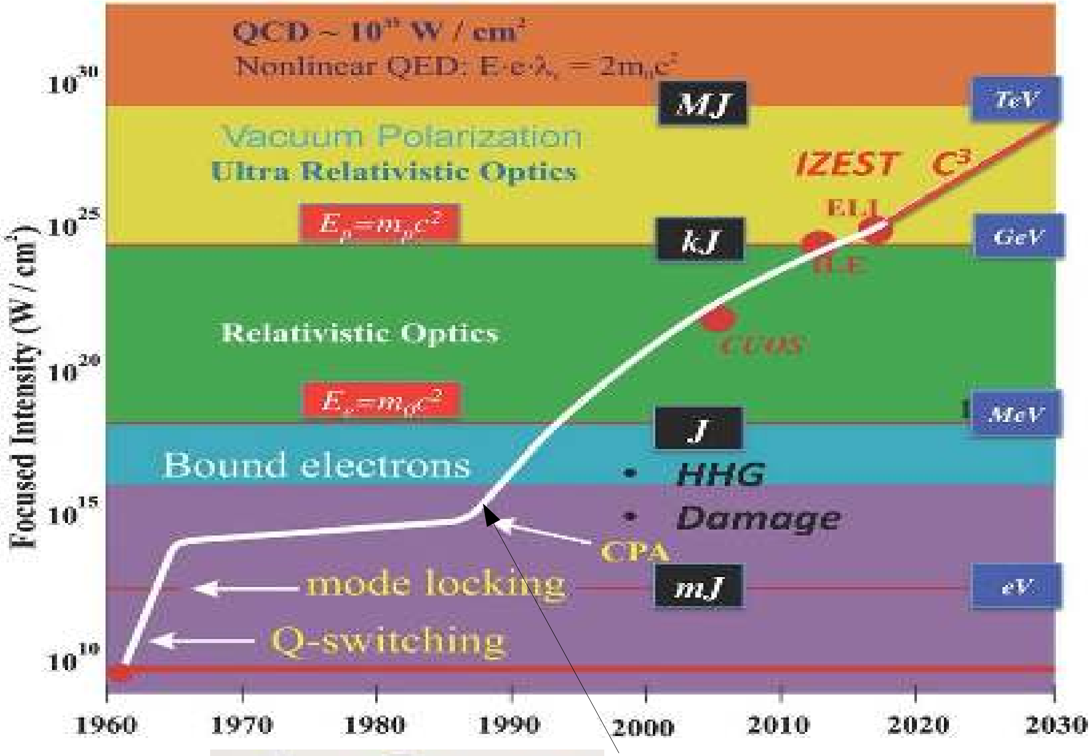
Resonator : plasma

$E_z = m_e c \omega_p / e \approx 100$ GV/m



(for $n_e=10^{19}$ cm^{-3})

History of short pulse laser



Nobel Prize in Physics 2018:
Ashkin P., Mourou G., Strikland D.

Ultra-high-Power

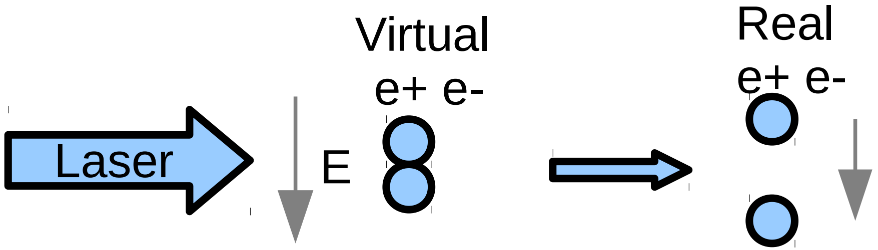
- **Zettawatt = 10^{21} W**
- **Exawatt = 10^{18} W (ELI? Apollon ?…)**
- **Petawatt = 10^{15} W (Total sunlight on earth ~ 100 PW)**
- **Terawatt = 10^{12} W (Total electrical power generated in the world ~ 5 TW)**

https://spie.org/news/4221-exploring-fundamental-physics-at-the-highest-intensity-laser-frontier

Pairs creation e^+e^- and the Schwinger limit

It would occur in a strong electric field in vacuum: QED theory

The electric field separates virtual (e^+, e^-) by a distance of compton length and provided 2 m_ec^2 of energy



$$\lambda_c = \frac{h}{m_e c}$$

$$\text{Energy} = 2eE\lambda_c = 2m_ec^2 \qquad E = m_e^2c^3/eh$$

$$E \simeq E_c \equiv \frac{m_e^2c^3}{e\hbar}$$

Schwinger limit[1] $I_c \simeq 2.3 \times 10^{29}\mathrm{W/cm^2}$

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[1] Schwinger J., Phys. Rev, **82**, 664 (1951)

Photon and Matter

- Matter to Light transformation

$$E = M c^2$$

Energy = Mass (Light velocity)²

c = 300 000 km/s

1 g of matter = 21 000 Tons of TNT (~Hiroshima)

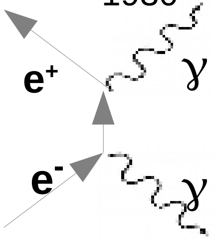
- **Matter-antimatter anihilation**

$$e^+ + e^- \longrightarrow \gamma + \gamma$$

$$2 M_e c^2 = \text{2 gamma Photon}$$

Kinetic energy equivalent to 1 mg, v=570 µm/s (30 hair in 1s)

Theory Dirac Annihilation 1930



Exp. Klemperer 1934

- Light to matter transformation

$$M = E/c^2$$

Mass = Energy / (Light velocity)²

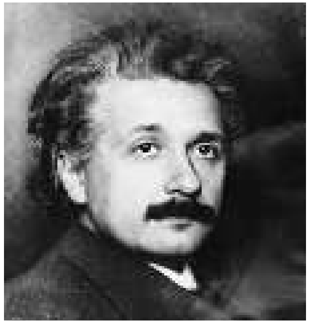
- **What happens when two photon of light collide?**
  - **Visible light: Nothing or almost (see latter)**

$$\gamma + \gamma \longrightarrow \gamma + \gamma$$

  - **Gamma ray light: Light turns into matter and antimatter**

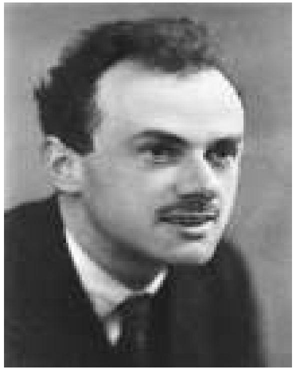
$$\gamma + \gamma \longrightarrow e^+ + e^-$$

Albert Einstein



Crédit : Larousse

Paul A. M. Dirac



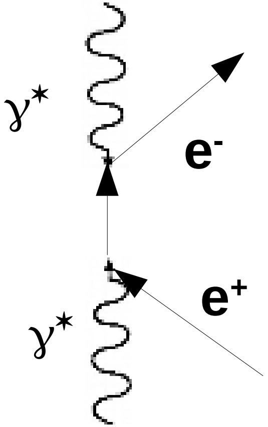
Crédit : Wikipédia

Pairs creation e^+e^- during photons collision

QED processes

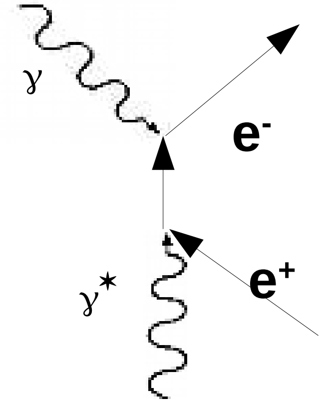
γ^* Virtual photon
γ Real photon

Landau-Lifshitz (1934)



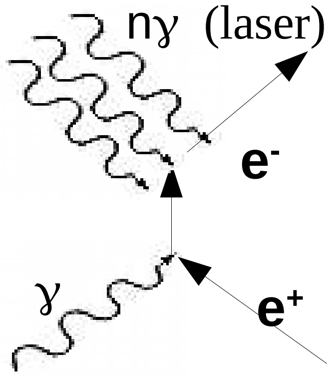
VEPP 2 (Novosibirsk)[1]
ADONE (Frascati)[2] : 1970's

Bethe-Heitler (1934)



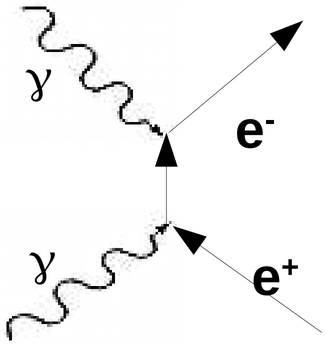
Anderson[3] (1932)

Non-linear
Breit-Wheeler[4] (1962)



SLAC[5] (1997)

Breit-Wheeler[6] (1934)



No observation
until now

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[1] Balakin V. E. et al. Physics Letters 34B 7 (1971)
[2] Baldani Celio R. et al. Physics Letters 86B 2 (1979)
[3] Anderson C. D. Phys. Rev. 43, 491 (1933)
[4] Reiss H. R., J. Math. Phys. 3, 59 (1962)
[5] Burke D. L. et al. PRL 79, 9 (1997)
[6] Breit, G. and Wheeler J. A. Phys. Rev. **46** (1934)

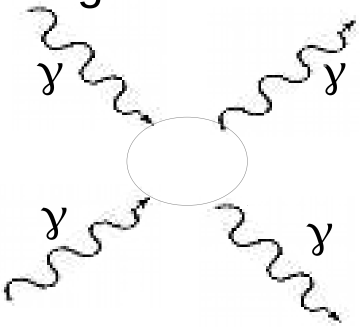
Pure photon-photon collision

Light-light scattering does not occurs in classical electrodynamic (Maxwell equ. are linear)

In QED theory

$$\hbar\omega \leq m_e c^2$$

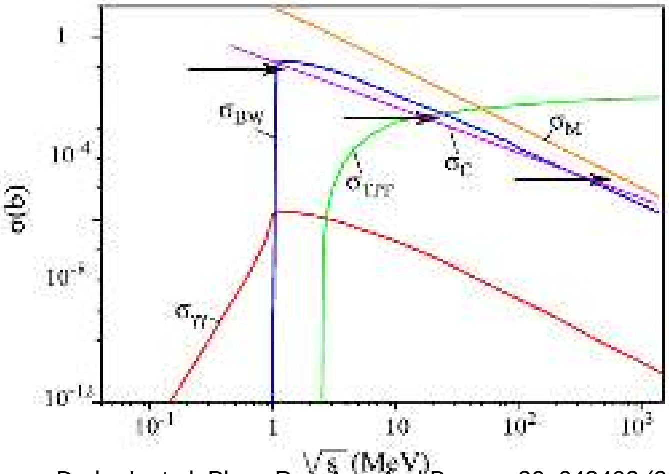
Light diffusion



$$\sigma_{\gamma\gamma} \simeq 3\times10^{-2}\alpha^2 r_e^2 \left(\frac{\hbar\omega}{m_e c^2}\right)^6$$

$$\hbar\omega = 400\,\mathrm{keV}$$

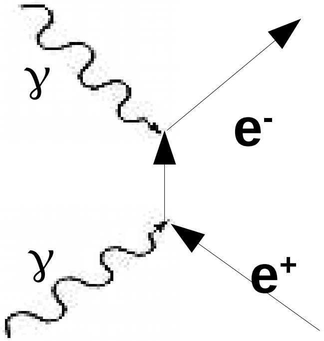
$$\sigma_{\gamma\gamma} \simeq 3.7\times10^{-7} r_e^2$$



Drebo I. et al. Phys. Rev. Acc. And Beams, 20, 043402 (2017)

$$\hbar\omega \geq m_e c^2 = 511\,\mathrm{keV}$$

Breit-Wheeler process



$$\sigma_{BW} \simeq r_e^2$$

$$\boxed{\sigma_{\gamma\gamma} = 3\times10^{-32}\mathrm{cm}^2} \ll \boxed{\sigma_{BW} \simeq 8\times10^{-26}\mathrm{cm}^2}$$

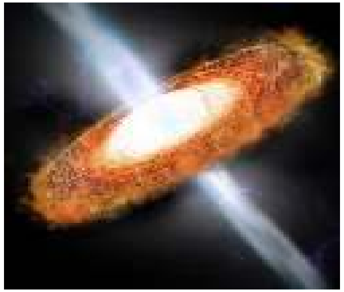
Photon-Photon collision and pair production in astrophysics

Breit-Wheeler process
Collision of two light quanta

$$\gamma + \gamma \longrightarrow e^{+} + e^{-}$$

- Electron pair production in AGN (Active Galaxy nuclei), Blazar, Quasar[1]
- Absorption of high-energy photon in the Universe[2],

cut-off in high energy gamma rays



Artiste composition

- Electron pair production in
  - GRB[3] (Gamma ray burst), Supernovae
  - In pulsar – electron-positron pair plasma
  - Merging neutron start, black hole

Artiste composition

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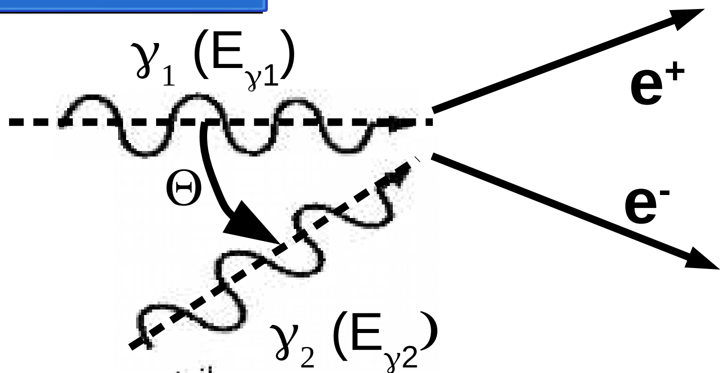
[1] Bonometto, S. and Ress, M. J. MNRAS, **152** 21-25 (1971)
[2] Nikishov A. I., JETP **14** (1962)
[3] Piran, S. Rev. Mod. Phys. **76** (2004)

Why the Breit-Wheeler process is difficult to observe?

$$\gamma_1 + \gamma_2 \longrightarrow e^+ + e^-$$



Breit-Wheeler cross section[1] in Center of Mass frame

$$\sigma_{\gamma\gamma}(s) = \frac{\pi r_e^2}{2}(1-\beta^2)\left[2\beta\left(\beta^2-2\right)+(3-\beta^4)\log\left(\frac{(1+\beta)}{(1-\beta)}\right)\right]$$

$$\beta = \sqrt{1-\frac{1}{s}} \qquad s = \frac{E_{\gamma 1}E_{\gamma 2}}{2m_e^2c^4}(1-\cos\theta)$$

$$\sigma_{\gamma\gamma} \propto r_e^2 = 10^{-26}\mathrm{cm}^2$$

Pair number:

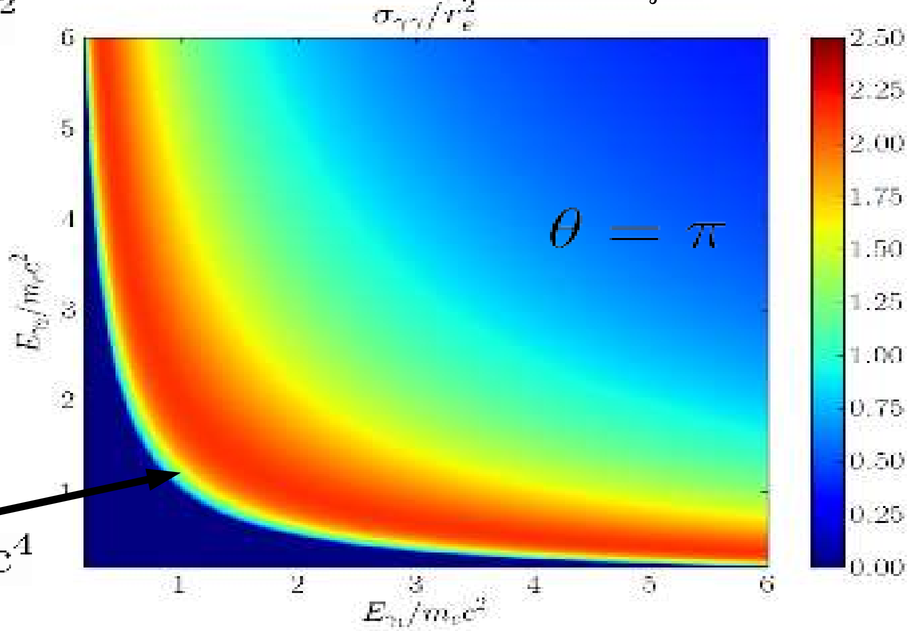
$$\mathrm{N}_p = N_{\gamma_1}N_{\gamma_2}\frac{\sigma_{\gamma\gamma}}{S}$$

$$\mathrm{S{=}10x10}\ \mu\mathrm{m}^2\ ,\mathrm{N}_{\gamma 1} = \mathrm{N}_{\gamma_2}$$

For $\mathrm{N_p} = 10^4$

$$\mathrm{N}_{\gamma_1} = \mathrm{N}_{\gamma_2} = 10^{12}$$

For pair production we need two MeV sources a high number of photons



$$\mathrm{E}_{\gamma_1}\mathrm{E}_{\gamma_2} = \mathrm{m_e^2c^4}$$

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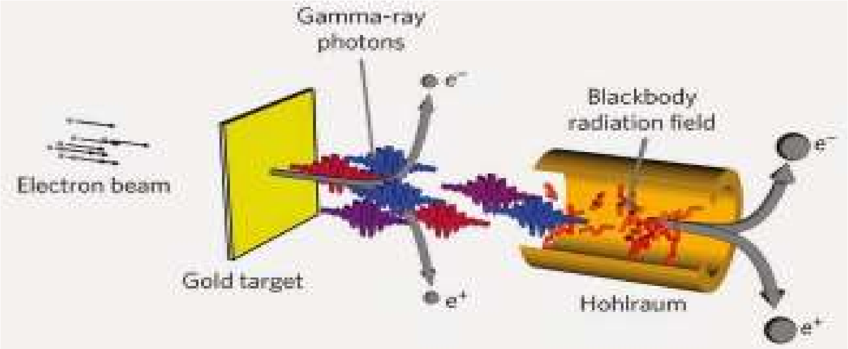
[1] Breit G. and Wheeler J. A. Phys. rev. **46** (1934)

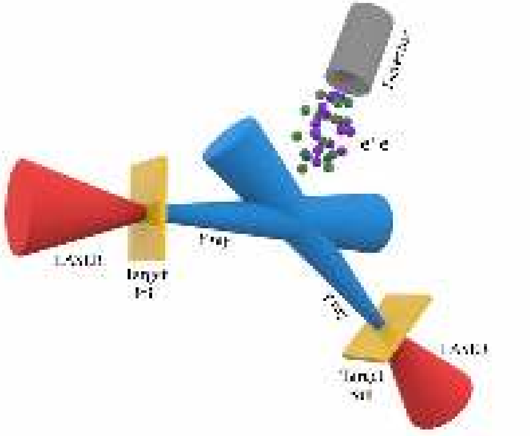
Breit-Wheeler pairs production experimental schemes

GeV – keV photons collision
Brems + X ray Black body



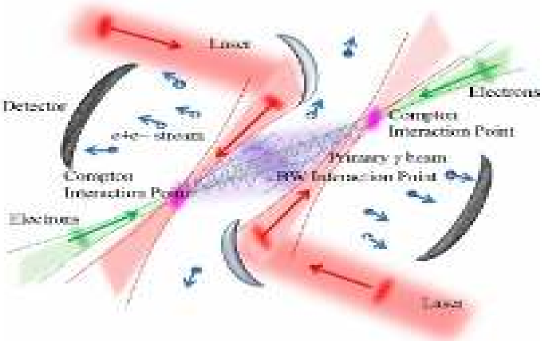
Pike O. et al. Nature Photonics, **8**, 434, (2014)

MeV – MeV photons collision Synch. or Brems beam



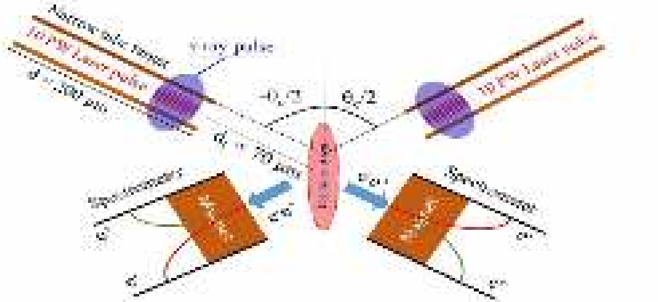
Ribeyre X. et al. Phys. Rev E **93**, 013201 (2016)

MeV Inverse Compton photon beams collision



Dredo I. et al. Phys. Rev. Accel. Beams **20**, 043402 (2017)

10 MeV-10 MeV photons collision from narrow tube target



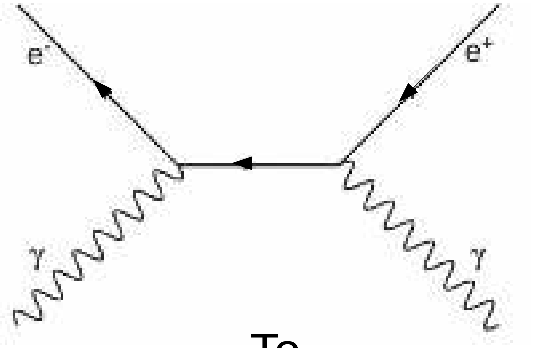
J. Yu et al. PRL **122**,014802 (2019)

Breit-Wheeler process

$$\gamma + \gamma \longrightarrow e^+ + e^-$$

How to pass from :

Feynman Diagram



To

Breit-Wheeler cross section[1] in CM

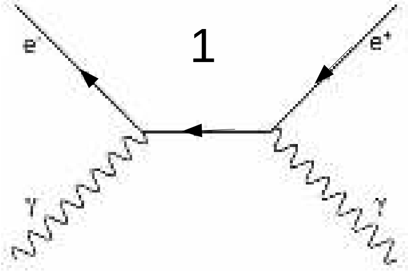
$$\sigma_{\gamma\gamma}(s) = \frac{\pi r_e^2}{2}(1-\beta^2)\left[2\beta\left(\beta^2-2\right)+\left(3-\beta^4\right)\log\left(\frac{1+\beta}{1-\beta}\right)\right]$$

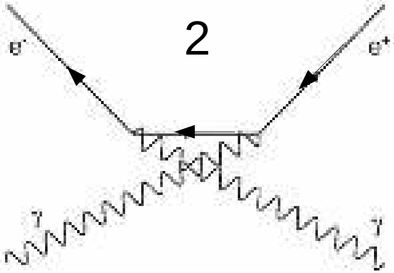
$$\beta = \sqrt{1-\frac{1}{s}} \qquad s = \frac{E_{\gamma_1}E_{\gamma_2}}{2m_e^2c^4}(1-\cos\phi)$$

[1] Breit, G. and Wheeler J. A. PRL **15** (1934)

Calculations Steps

1- Feyman diagrams :


1


2

2- Current of probability

3- Diffusion Matrix: → $i\mathcal{M}_1 \text{ and } \mathrm{i}\mathcal{M}_2$

4- Total diffusion Matrix: → $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$

5- Module of the Diffusion Matrix: for all spin and polarization state →

$$|\overline{\mathcal{M}}|^2 = \frac{1}{4}\sum_{S_e,S'_e}\sum_{\lambda,\lambda'}\mathcal{M}_1\mathcal{M}_1^* + \mathcal{M}_2\mathcal{M}_2^* + 2\mathcal{R}_e(\mathcal{M}_1\mathcal{M}_2)$$

6- Mandelstam variables: Relativistic invariant. →

$$|\overline{\mathcal{M}}|^2 = 2e^4\left[\frac{u-m^2}{(t-m^2)} + \frac{t^2-m^2}{(u-m^2)} - 4m^2\left(\frac{1}{(t-m^2)} + \frac{1}{(u-m^2)}\right) - 4m^4\left(\frac{1}{(t-m^2)} + \frac{1}{(u-m^2)}\right)^2\right]$$

7- Differential cross section center of mass frame →

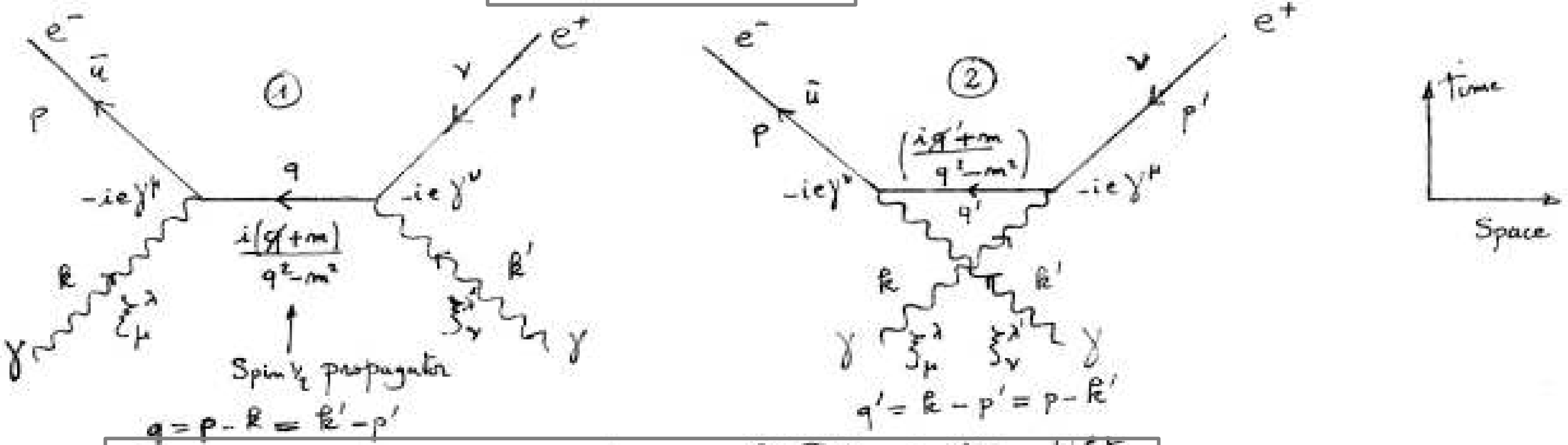
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\overline{\mathcal{M}}^2| = \frac{r_e^2}{4}\beta(1-\beta^2)\left[\frac{1+2\beta^2\sin^4\theta - \beta^4(1+\sin^4\theta)}{(1-\beta^2\cos^2\theta)}\right]$$

8- Total BW cross section: →

$$\sigma = 2\pi\int_{-1}^{1}\frac{d\sigma}{d\Omega}d\cos\theta$$

$$\sigma_{BW}(s) = \frac{\pi r_e^2}{2}(1-\beta^2)\left[2\beta\left(\beta^2-2\right)+(3-\beta^4)\log\left(\frac{1+\beta}{1-\beta}\right)\right]$$

$\boxed{\gamma + \gamma \rightarrow e^- + e^+}$



Rem : There is two Feynmann diagrams because of Photon indiscernability.

e^-, e^+ : Lepton : Spin $1/2$ a $-1/2$
γ : Boson : Spin $1, -1$; 0 not allowed in QED (the photon mass is zero)

p, p' : Energy-momentum quadrivector, electron - positron
k, k' : Energy-momentum quadrivector of photons

$\bar{u}, v$: Dirac Spinor : the Probability function $\Psi(\underline{x})$ is defined by $\Psi(\bar{x}) = u(\bar{p}, s_e)\, e^{-ip\underline{x}}$; $\underline{x}$ quadrivector position
$\xi^\lambda_\mu\ \xi^{\lambda'}_\nu$: Wave functions of Photon : λ, λ' Spin state : polarisation.

γ^μ : Dirac Matrix and Feynmann Slash is defined by : $\not{a} = \gamma^\mu a_\mu$

The Dirac equation is

$$\boxed{\left(i\gamma^\mu \partial_\mu - m\right)\Psi(\underline{x}) = 0}$$

$$\boxed{\text{Diagram ①}}$$

Current of probability

$$\begin{cases} j_\mu(\gamma, e^-) = -e\,\overbrace{\bar{u}(\vec{p}_e, s_e)}^{\text{final}}\,(-ie\gamma^\mu)\,\overbrace{\xi_\mu^\lambda}^{\text{initial}}\,e^{i(p-k)x} \\ j^\mu(\gamma, e^+) = -e\,v(\vec{p}_e', s_e)\,(-ie\gamma^\nu)\,\xi_\nu^{\lambda'}\,e^{i(p'-k')x} \end{cases}$$

Diffusion Matrix :

$$-i\mathcal{M}_1 = e^2\left[\bar{u}(\vec{p}_e\, s_e)\,(-ie\gamma^\mu)\,\xi_\mu^\lambda\right]\underbrace{\left[i\,\frac{(\not{q}+m)}{q^2-m^2}\right]}_{\text{Fermion propagator (Spin 1/2)}}\left[v(\vec{p}_e', s_e')\,(-ie\gamma^\nu)\,\xi_\nu^{\lambda'}\right]$$

$$\boxed{-i\mathcal{M}_1 = e^2\,\bar{u}(\vec{p}_e, s_e)\left[\xi_\mu^\lambda\,(-ie\gamma^\mu)\left(\frac{i(\not{q}+m)}{q^2+m}\right)(-ie\gamma^\nu)\,\xi_\nu^{\lambda'}\right]v(p_e', s_e')}$$

$$\boxed{\text{Diagram ②}}$$

$$\boxed{-i\mathcal{M}_2 = e^2\,\bar{u}(\vec{p}_e, s_e)\left[\xi_\nu^{\lambda'}\,(-ie\gamma^\nu)\left(\frac{i(\not{q}'+m)}{q^2-m^2}\right)(-ie\gamma^\mu)\,\xi_\mu^{\lambda'}\right]v(p_e', s_e')}$$

Mandelstam Variables s, t, u Lorentz invariant

$$\begin{cases} s = (k+k')^2 = (p+p')^2 \\ t = (p-k)^2 = (p'-k')^2 \\ u = (p-k')^2 = (p'-k)^2 \end{cases}$$

3 variables not independent

because $q^2=(p-k)^2=t$ and $q'^2=(k-p')=u$

$$\boxed{\mathcal{M}_1 = \frac{e^2}{(t-m^2)}\,\bar{u}(\vec{p},s_e)\,\xi^{\lambda}_{\mu}\,\gamma^{\mu}\,(\not{q}+m)\cdot\gamma^{\nu}\,\xi^{\lambda'}_{\nu}\,v(\vec{p}',s'_e)}$$

Then for $\mathcal{M}_2$:

$$\boxed{\mathcal{M}_2 = \frac{e^2}{(u-m^2)}\,\bar{u}(\vec{p},s_e)\,\xi^{\lambda'}_{\nu}\,\gamma^{\nu}\,(\not{q}'+m)\,\gamma^{\mu}\,\xi^{\lambda}_{\mu}\,v(\vec{p}',s'_e)}$$

The Total Diffusion Matrix : $\boxed{\mathcal{M}=\mathcal{M}_1+\mathcal{M}_2}$

$$\overline{|\mathcal{M}|}^2 = \frac{1}{4}\sum_{s_e,s'_e}\sum_{\lambda,\lambda'}|\mathcal{M}|^2$$

$$\boxed{\begin{array}{l}
\overline{|\mathcal{M}|}^2 = \frac{1}{4}\sum_{s_e,s'_e}\sum_{\lambda,\lambda'}\left(\mathcal{M}_1\mathcal{M}_1^* + \mathcal{M}_2\mathcal{M}_2^* + 2\,\mathrm{Re}(\mathcal{M}_1\mathcal{M}_2^*\right) \\
\\
\qquad \mathcal{M}_1 = \frac{e^2}{t-m^2}\,\bar{u}(\vec{p},s_e)\,\not{\varepsilon}^{\lambda}\,(\not{q}+m)\,\not{\varepsilon}^{\lambda'}\,v(\vec{p}',s'_e) \\
\\
\qquad \mathcal{M}_2 = \frac{e^2}{u-m^2}\,\bar{u}(\vec{p},s_e)\,\not{\varepsilon}^{\lambda'}\,(\not{q}'+m)\,\not{\varepsilon}^{\lambda}\,v(\vec{p}',s'_e)
\end{array}}$$

Where

$\not{\varepsilon}^{\lambda} = \xi^{\lambda}_{\mu}\gamma^{\mu}$

$\not{\varepsilon}^{\lambda'} = \xi^{\lambda'}_{\nu}\gamma^{\nu}$

④

$\boxed{\mathcal{M}_1 \mathcal{M}_1^* \text{ term}}$

$$\mathcal{M}_1 \mathcal{M}_1^* = \frac{e^4}{(t-m^2)^2} \left(\bar{u}(\vec{p}, s_e)\, \not{\varepsilon}^{\lambda} (\not{q}+m)\, \not{\varepsilon}^{\lambda'} v(\vec{p}', s_e') \right) \left(\bar{u}(\vec{p}, s_e)\, \not{\varepsilon}^{\lambda} (\not{q}+m)\, \not{\varepsilon}^{\lambda'} v(\vec{p}', s_e') \right)^{+}$$

Hermite conjugation

$(\ )^* = (\ )^+$

$$\mathcal{M}_1 \mathcal{M}_1^* = \frac{e^4}{(t-m^2)} \bar{u}(\vec{p}, s_e)\, \not{\varepsilon}^{\lambda} (\not{q}+m)\, \not{\varepsilon}^{\lambda'} v(\vec{p}', s_e') \quad v^{+}(\vec{p}', s_e') (\not{\varepsilon}^{\lambda'})^{+} (\not{q}+m)^{+} (\not{\varepsilon}^{\lambda})^{+} (\bar{u}(\vec{p}', s_e'))^{+}$$

Properties : $\gamma^{\mu +} = \gamma^0 \gamma^\mu \gamma^0 \rightsquigarrow (\gamma^0)^+ = \gamma^0 \qquad \gamma^0\gamma^\mu = -\gamma^\mu\gamma^0 \qquad (\gamma^0)^2 = \mathbb{1}$

Clifford Algebra

Then $\boxed{(\not{\varepsilon}^{\lambda})^{+} = -(\not{\varepsilon}^{\lambda})^{*} \quad \text{and} \quad (\not{\varepsilon}^{\lambda'})^{+} = -(\not{\varepsilon}^{\lambda'})^{*}}$

$\rightarrow (\bar{u}(\vec{p}', s_e))^{+} = (u^{+}(\vec{p}, s_e)\, \gamma^0)^{+}$ Dirac adjoint

$\rightarrow \boxed{(\bar{u}(\vec{p}, s_e))^{+} = \gamma^0 u(\vec{p}, s_e)}$

$\rightarrow \boxed{(\not{q}+m)^{+} = -\not{q} + m}$

$$\boxed{\mathcal{M}_1 \mathcal{M}_1^* = \frac{e^4}{(t-m^2)} \bar{u}(\vec{p}, s_e)\, \not{\varepsilon}^{\lambda} (\not{q}+m)\, \not{\varepsilon}^{\lambda'} v(\vec{p}', s_e') \cdot \bar{v}(\vec{p}', s_e')\, \gamma_0\, \not{\varepsilon}^{\lambda' *} (\not{q}+m)\, \not{\varepsilon}^{\lambda *} \gamma^0 u(\vec{p}, s_e)}$$

$$\mathcal{M}_{11} = \frac{1}{4} \sum_{s_e, s_e'} \sum_{\lambda, \lambda'} \mathcal{M}_1 \mathcal{M}_1^*$$

We can show that :

$$\boxed{\mathcal{M}_{11} = \frac{e^4}{4(t-m^2)^2} \underset{\nwarrow \text{Trace}}{\mathrm{Tr}} \left[(\not{p}' - m) \gamma^\nu (\not{q} + m) \gamma^\mu (\not{p} + m) \gamma_\mu (\not{q} + m) \gamma_\nu \right]}$$

Then for $\mathcal{M}_{22} = \frac{1}{4} \sum_{s_e s_e'} \cdot \sum_{\lambda, \lambda'} \mathcal{M}_2 \mathcal{M}_2^*$

$$\boxed{\mathcal{M}_{22} = \frac{e^4}{4(u-m^2)^2} \mathrm{Tr} \left[(\not{p}' - m) \gamma^\mu (\not{q}' + m) \gamma^\nu (\not{p} + m) \gamma_\nu (\not{q}' + m) \gamma_\mu \right]}$$

Then for $\mathcal{M}_{12} = \frac{1}{4} \sum_{s_e s_e'} \sum_{\lambda \lambda'} \mathcal{M}_1 \mathcal{M}_2^*$

$$\boxed{\mathcal{M}_{12} = \frac{e^4}{4(t-m^2)(u-m^2)} \mathrm{Tr} \left[(\not{p}' - m) \cdot \gamma^\nu (\not{q} + m) \gamma^\mu (\not{p} + m) \cdot \gamma_\nu (\not{q}' + m) \gamma_\mu \right]}$$

γ Matrix properties : $\gamma^\mu \gamma_\mu = 4$; $\gamma^\mu \not a \gamma_\mu = -2 \not a$; $\gamma^\mu \not a \not b \gamma_\mu = 4ab$; $\gamma^\mu \not a \not b \not c \gamma_\mu = -2 \not c \not b \not a$

Then : $\boxed{\gamma^\mu (\not p + m) = -2(\not p - 2m)}$

because circular index permutation not change the Trace :

$$\mathcal{M}_{11} = \frac{e^4}{(t-m^2)^2} \mathrm{Tr}\left[(\not p' + 2m)(\not q + m)(\not p - 2m)(\not q + m) \right]$$

$$\mathcal{M}_{22} = \frac{e^4}{(u-m^2)^2} \mathrm{Tr}\left[(\not p' + 2m)(\not q' + m)(p - 2m)(\not q' + m) \right]$$

For $\mathcal{M}_{12}$ we found :

$$\boxed{\mathcal{M}_{12} = \frac{e^4}{4(t-m^2)(u-m^2)} \mathrm{Tr}\left[-8qq' \not p \not p' - 4m^2 \not p \not q + 4m^2 (\not p + \not q)(\not p' - \not q') + 4m^2 \not q' \not p' + 8m^4 \right]}$$

After Trace calculations

and used the Mandelstam variables s, t, u.

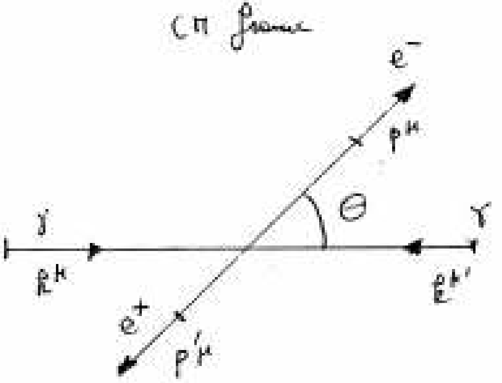
$$\boxed{\begin{aligned}
\mathcal{M}_{11} &= \frac{2e^4}{(t-m^2)^2}\left[(s-2m^2)(m^2-t)-(m^2+t)^2\right]\\
\mathcal{M}_{22} &= \frac{2e^4}{(u-m^2)^2}\left[(s-2m^2)(m^2-u)-(m^2+u)^2\right]\\
\mathcal{M}_{12} &= \frac{2e^4m^2}{(t-m^2)(u-m^2)}\left[s-4m^2\right]
\end{aligned}}$$

$$|\overline{\mathcal{M}}|^2 = \mathcal{M}_{11}+\mathcal{M}_{22}+2\mathrm{Re}(\mathcal{M}_{12})$$

$$\boxed{|\overline{\mathcal{M}}|^2 = 2e^4\left[\frac{u-m^2}{(t-m^2)}+\frac{t-m^2}{(u-m^2)}-4m^2\left(\frac{1}{(t-m^2)}+\frac{1}{(u-m^2)}\right)-4m^4\left(\frac{1}{(t-m^2)}+\frac{1}{(u-m^2)}\right)^2\right]}$$

Differential cross section
in centre of mass frame.

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\,\frac{p_f}{p_i}\,|\mathcal{M}|^2}$$



$$\begin{cases} p^\mu(E,\vec{p})\;;\; p'^\mu=(E,-\vec{p}) \qquad E=\sqrt{p^2+m^2} \\ k^\mu=(E_\gamma,\vec{k})\;;\; k'^\mu=(E_\gamma,-\vec{k}) \qquad E_\gamma=\|\vec{k}\| \end{cases}$$

Energy conservation $\boxed{E_\gamma = E = \|\vec{k}\|}$

Then

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\,\frac{\|\vec{p}\|}{E_\gamma}\,|\mathcal{M}|^2}$$

$$\boxed{\begin{aligned} s &= (k+k')^2 = 4E_\gamma^2 \\ t &= (p-k)^2 = m^2 - 2(E_\gamma^2 - \|\vec{p}\|\,E_\gamma\cos\theta) \\ u &= (p-k')^2 = m^2 - 2(E_\gamma^2 + \|\vec{p}\|\,E_\gamma\cos\theta) \end{aligned}}$$

unit system here $\hbar = c = 1 = \varepsilon_0 \Rightarrow \boxed{\alpha = \frac{e^2}{4\pi}}$

$$\boxed{\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E_\gamma^3}\|\vec{p}\|\Bigg\{ &\frac{E_\gamma^2+E_\gamma\|\vec{p}\|\cos\theta}{E_\gamma^2-E_\gamma\|\vec{p}\|\cos\theta} + \frac{E_\gamma^2-E_\gamma\|\vec{p}\|\cos\theta}{E_\gamma^2+E_\gamma\|\vec{p}\|\cos\theta} + 2m^2\left[\frac{1}{E_\gamma^2-E_\gamma\|\vec{p}\|\cos\theta} + \frac{1}{E_\gamma^2+E_\gamma\|\vec{p}\|\cos\theta}\right] \\ &- m^4\left[\frac{1}{E_\gamma^2-E_\gamma\|\vec{p}\|\cos\theta} + \frac{1}{E_\gamma^2+E_\gamma\|\vec{p}\|\cos\theta}\right]^2\Bigg\}\end{aligned}}$$

⑧

Final expression of the differential cross section

because
$\|\vec{p}\|^2 = E_\gamma^2 - m^2$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2\beta}{8m^2}(1-\beta^2)\left\{\left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)+\left(\frac{1-\beta\cos\theta}{1+\beta\cos\theta}\right)+2(1-\beta^2)\left(\frac{1}{1-\beta\cos\theta}+\frac{1}{1+\beta\cos\theta}\right) - (1-\beta^2)^2\left(\frac{1}{1-\beta\cos\theta}+\frac{1}{1+\beta\cos\theta}\right)^2\right\}}$$

where: $\beta = \frac{\|\vec{p}\|}{E_\gamma}$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{r_e^2}{4}\beta(1-\beta^2)\left(\frac{1+2\beta^2\sin^2\theta-\beta^4(1+\sin^4\theta)}{(1-\beta^2\cos^2\theta)}\right)}$$

where $r_e = \frac{\alpha}{m_e}$

Total differential cross section

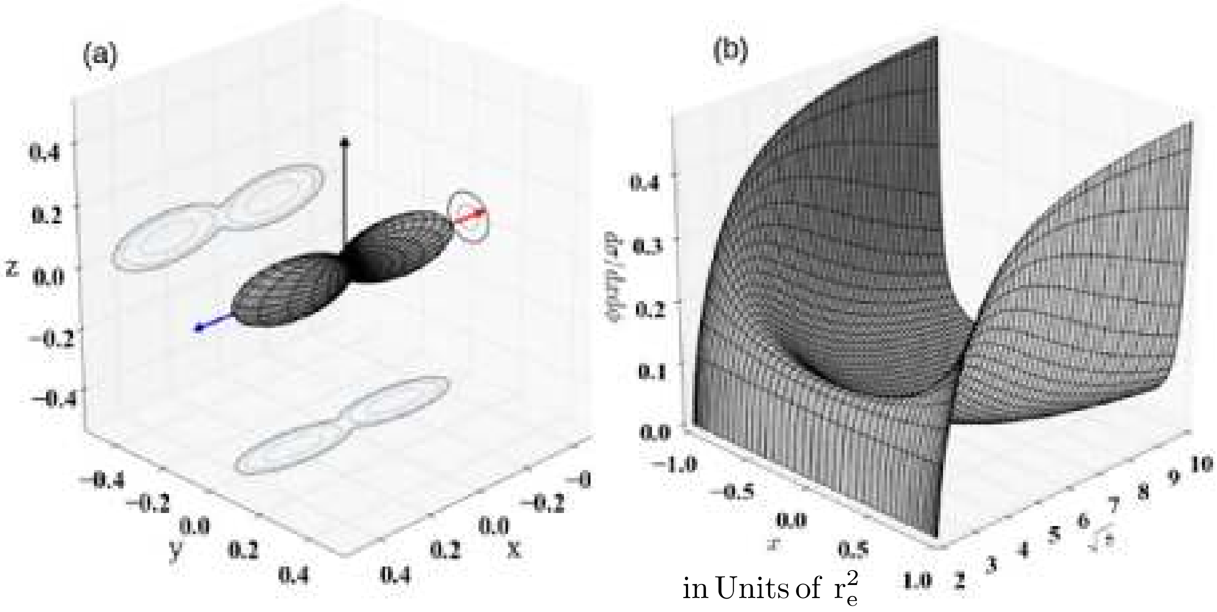
$$\sigma_{cm} = 2\pi\int_{-1}^{1}\left.\frac{d\sigma}{d\Omega}\right|_{cm} d\cos\theta$$

$$\boxed{\sigma_{cm} = \frac{\pi r_e^2}{2}(1-\beta^2)\left[-2\beta(2-\beta^2)+(3-\beta^4)\ln\left(\frac{1+\beta}{1-\beta}\right)\right]}$$

Thanks to the help of J. C. Caillon (CENBG) «Physique des particules et interactions fondamentales » Lecture

Breit Wheeler differential cross section

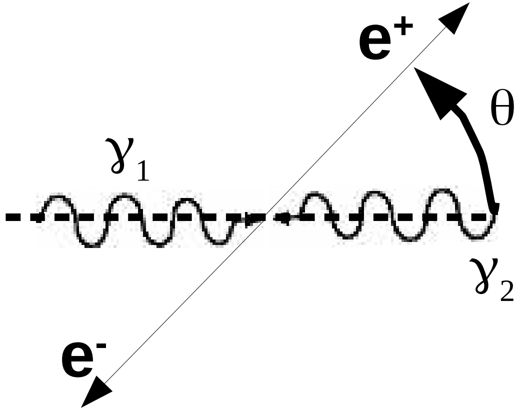
$$\frac{d\sigma_{BW}}{d\Omega} = \frac{r_e^2}{4}\beta(1-\beta^2)\left[\frac{1+2\beta^2\sin^4\theta-\beta^4(1+\sin^4\theta)}{(1-\beta^2\cos^2\theta)}\right]$$



$E_{\gamma_1} = E_{\gamma_2} = 4\,\mathrm{MeV}$ $\theta_\mathrm{p} = 40°$

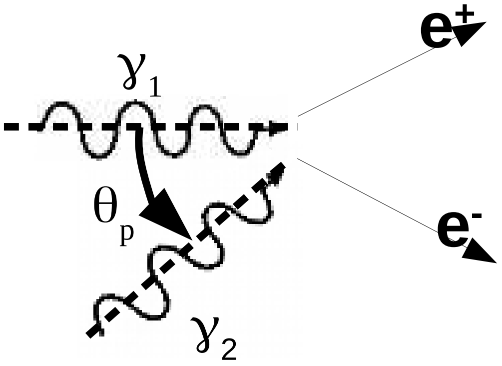
in Units of $\mathrm{r_e^2}$

$x = \cos\theta$



$$\beta - \sqrt{1-\frac{1}{s}}$$

$$\sqrt{s} = E_{cm} = \sqrt{2E_{\gamma_1}E_{\gamma_2}(1-\cos\theta_p)}$$

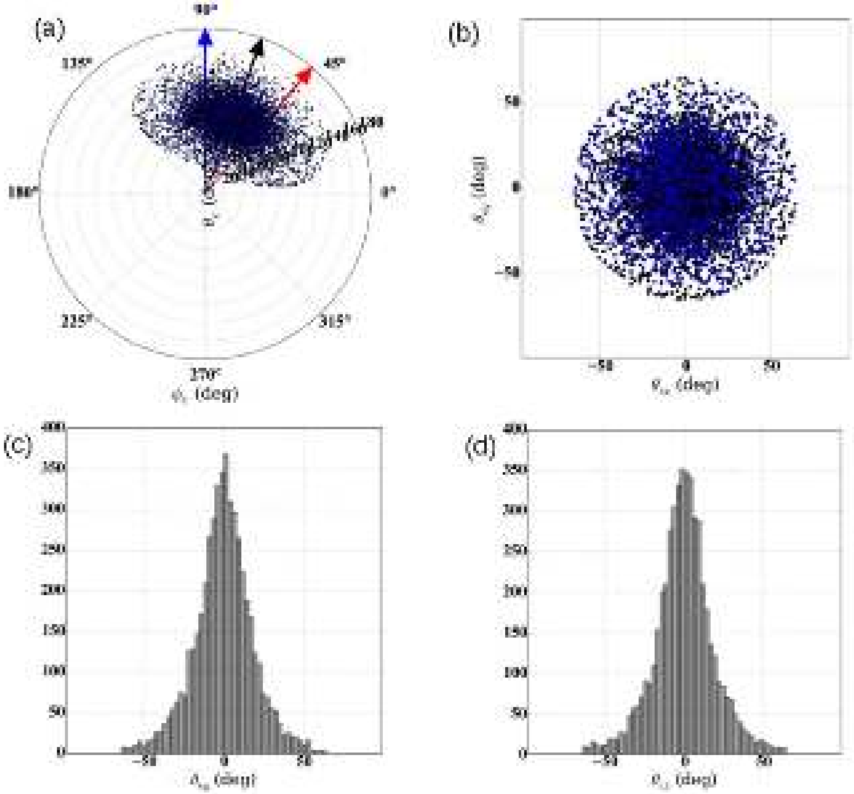


[1] Ribeyre X. et al. PPCF 59 014024 (2017)

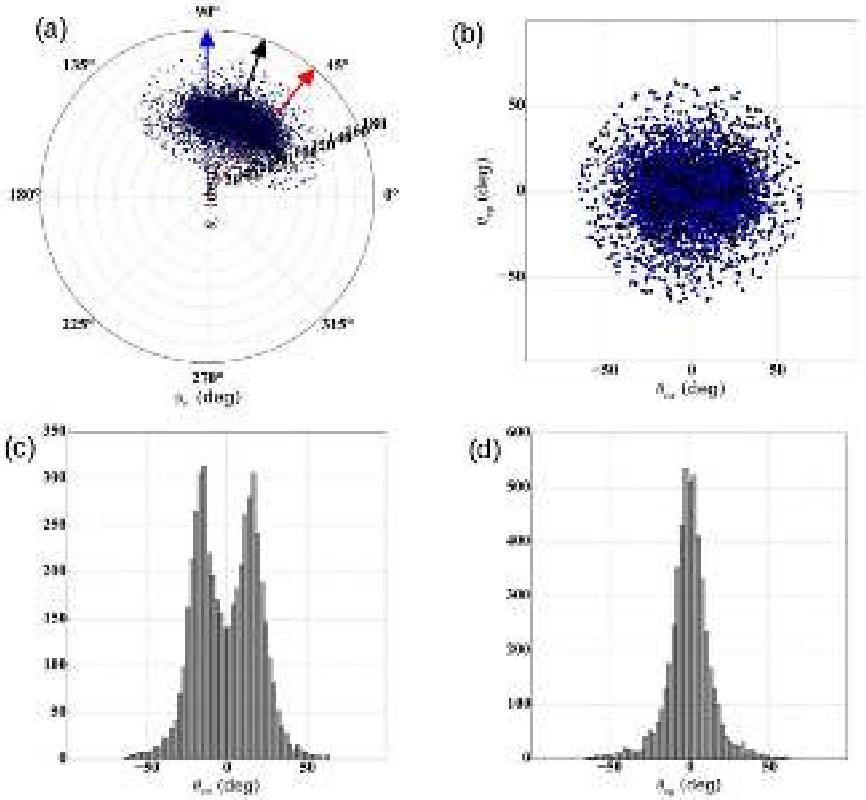
[2] Ribeyre X. et al. PPCF 60 104001 (2018)

Effect of Breit Wheeler differential cross section On pair beaming

In lab. Frame $E_{\gamma_1} = E_{\gamma_2} = 4\,\mathrm{MeV}\ \theta_\mathrm{p} = 40°$



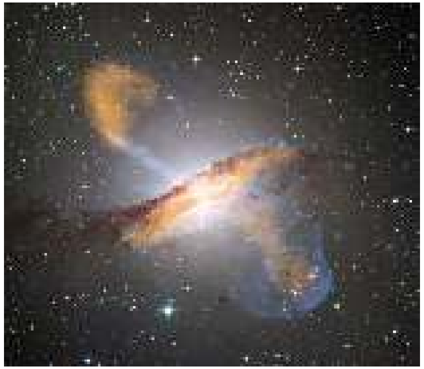
Without BW diff cross section



With BW diff cross section

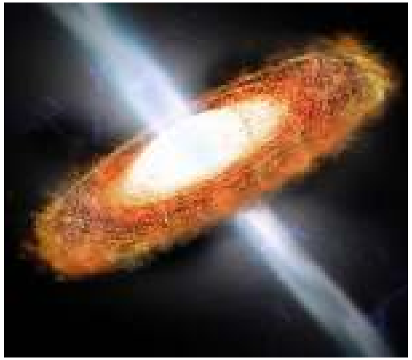
Example of matter-antimatter production in univers

The Centaurus A Galaxy and its jet from the supermassive black hole



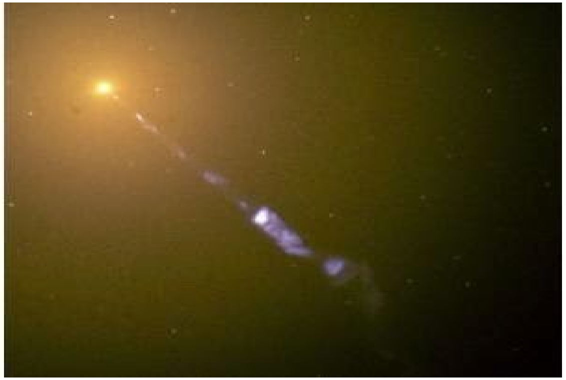
Crédit : NASA-ESO

Active Galactic Nuclei

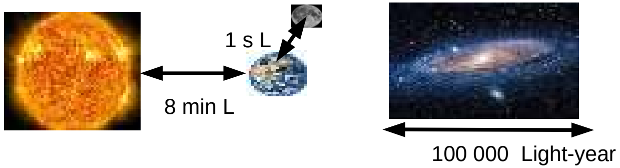


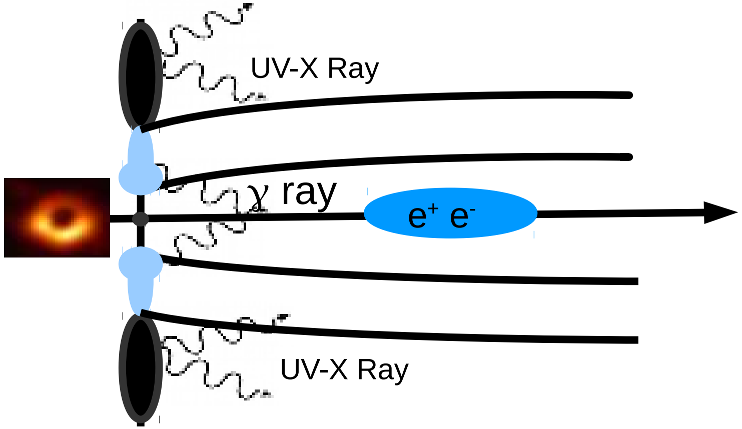
Artiste view

The active Galaxy M87 and its particle jet
Jet size 5000 AL

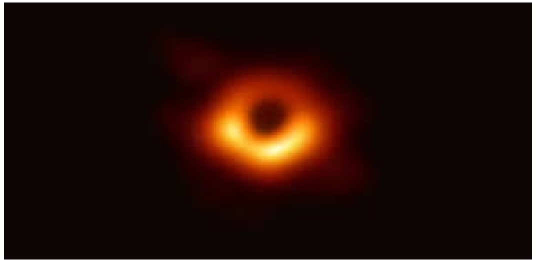


Crédit : HST/NASA/ESA





The central black hole of the giant galaxy M87.
April 2019



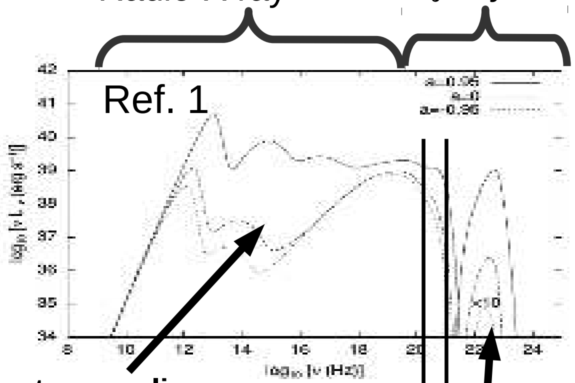
Crédit : EHT Collaboration

BW Pairs beaming and Active Galactic Nuclei (AGN) and ultra-relativistic pair beam

Active Galactic Nuclei (AGN)

Radio-X ray

γ ray

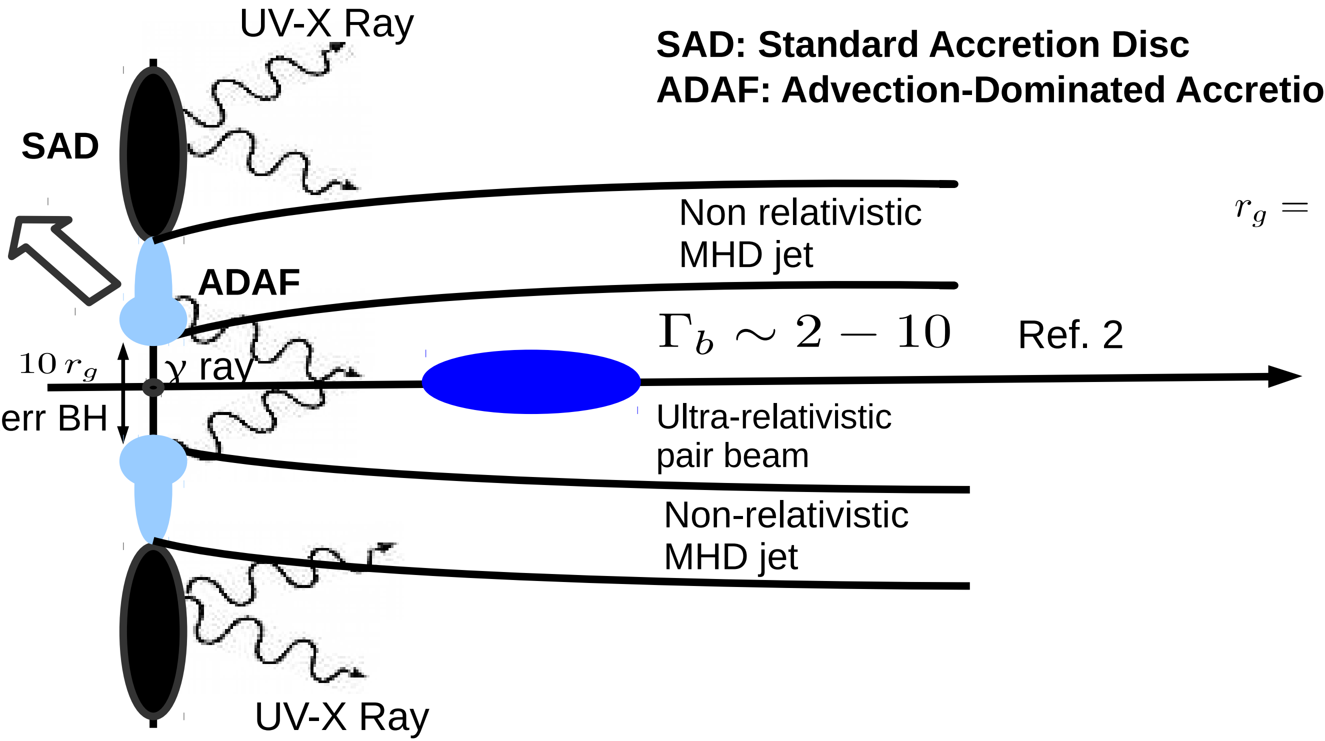


Electron cooling processes:
Synchrotron,
Bremsstrahlung,
Inverse Compton Radiation

$$1 - 5\,\mathrm{MeV}$$

$$p + p \rightarrow p + p + \pi_0$$

$$\pi_0 \rightarrow \gamma + \gamma$$



UV-X Ray

SAD

ADAF

$10\, r_g$

γ ray

Kerr BH

UV-X Ray

Non relativistic
MHD jet

$\Gamma_b \sim 2 - 10$

Ref. 2

Ultra-relativistic
pair beam

Non-relativistic
MHD jet

SAD: Standard Accretion Disc
ADAF: Advection-Dominated Accretion Flow

$$r_g = \frac{2GM}{c^2}$$

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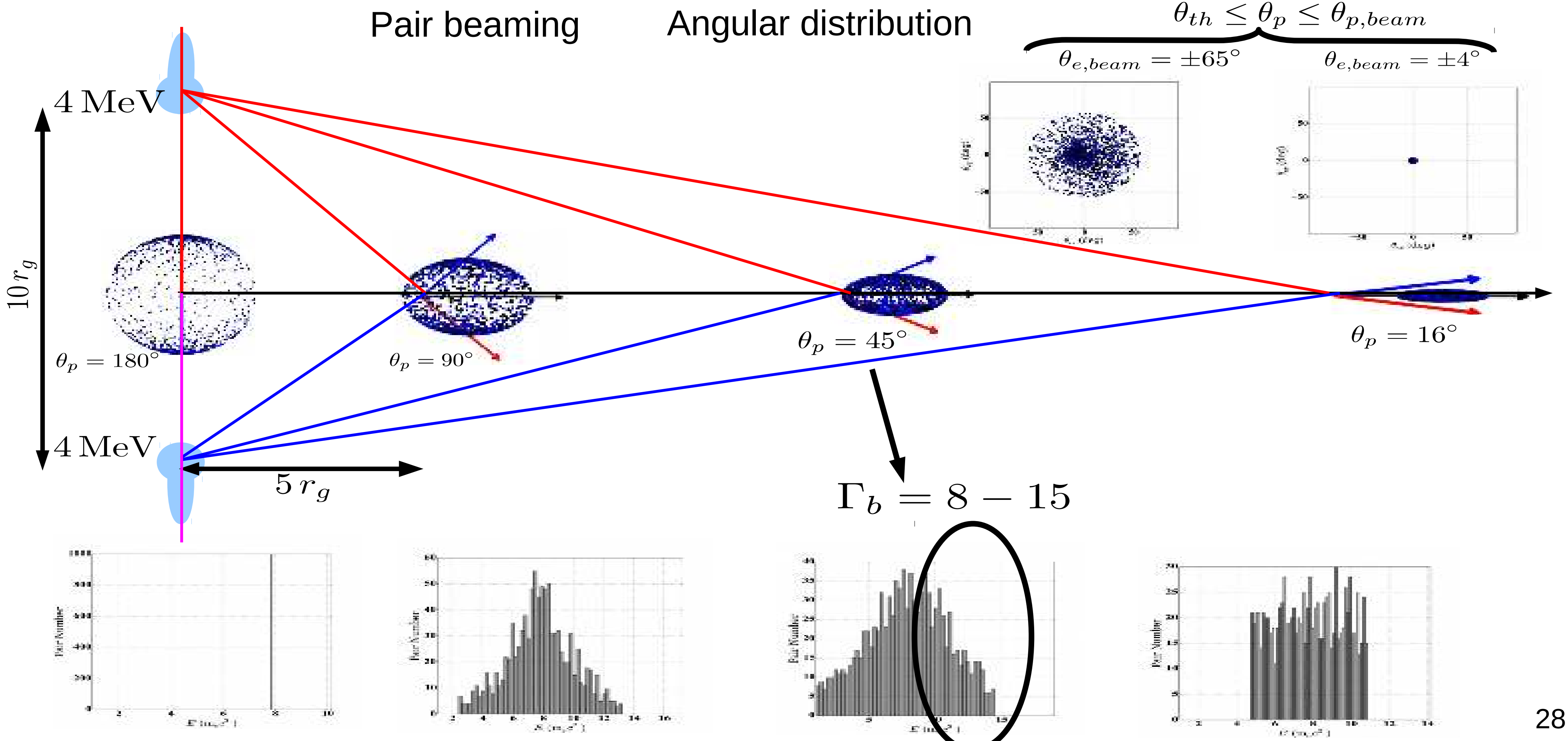
[1] Oka K & Manmoto T. MNRAS **340**,543 (2003)
[2] Vuillaume T. et al. A&A **581** A18 (2015)

Pairs beaming from BW process
4 MeV-4MeV



Pair beaming

Angular distribution

$\underbrace{\theta_{e,beam} = \pm 65^\circ \quad \theta_{e,beam} = \pm 4^\circ}_{\theta_{th} \leq \theta_p \leq \theta_{p,beam}}$

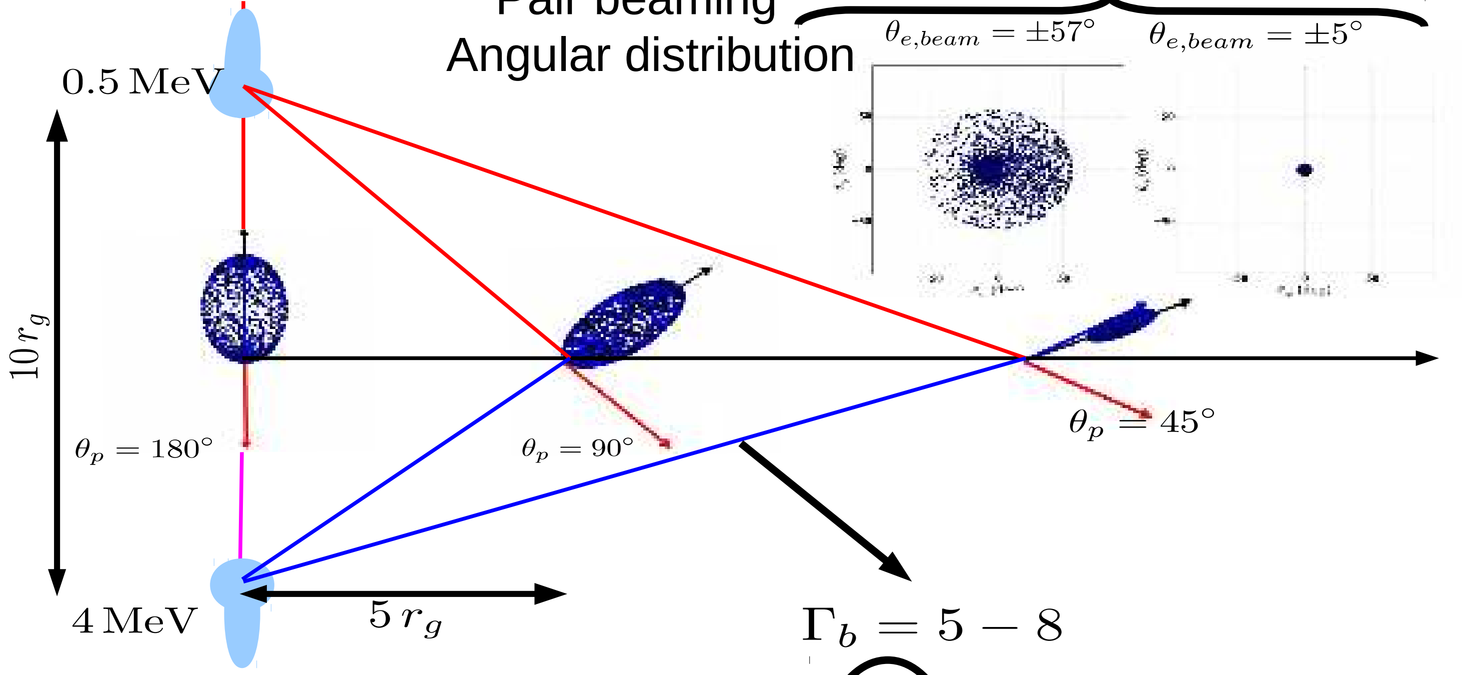
$\Gamma_b = 8 - 15$

Pairs beaming from BW process
4 Mev- 0.5 MeV

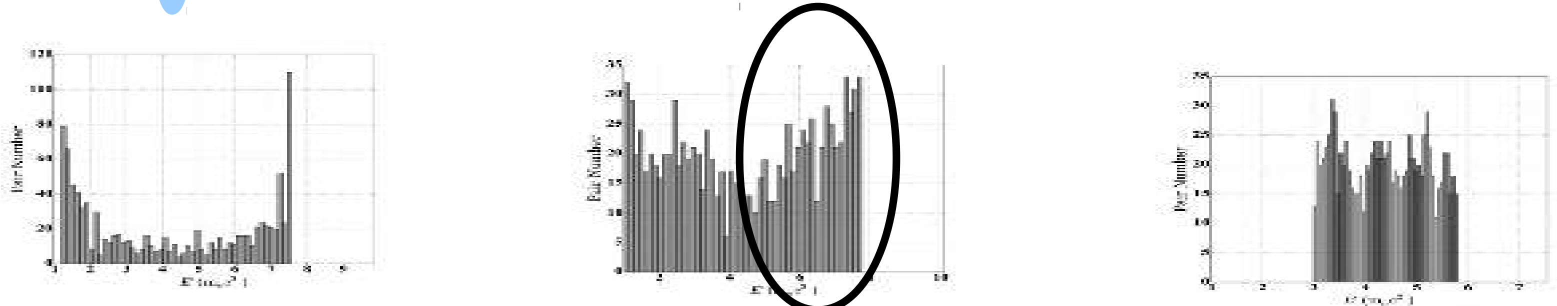
Pair beaming
Angular distribution

$$\underbrace{\theta_{th} \le \theta_p \le \theta_{p,beam}}$$

$\theta_{e,beam} = \pm 57°$ $\theta_{e,beam} = \pm 5°$



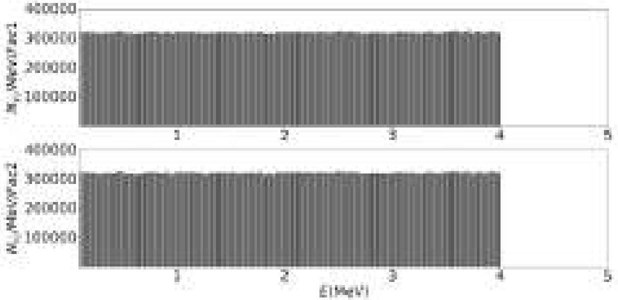
$\Gamma_b = 5 - 8$

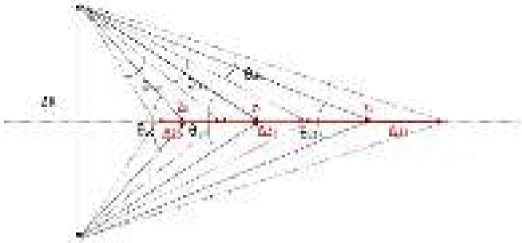


More accurate model for pair distribution and pair beaming

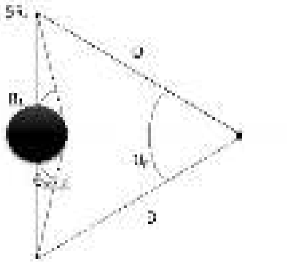
Spectral and angular effects

Gamma photons spectra



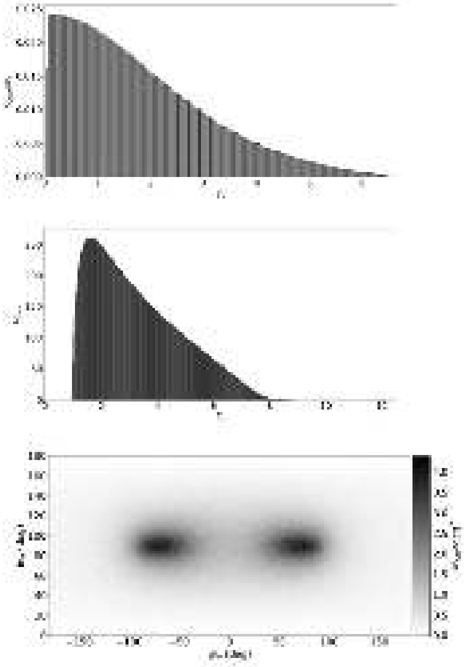


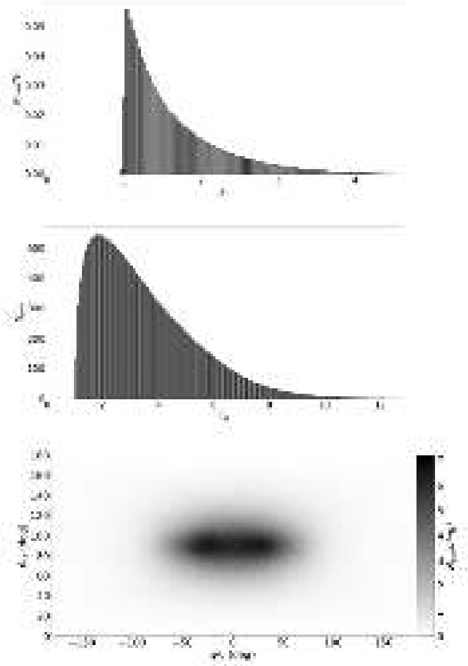
Without black hole



With black hole

$$R_s = \frac{2GM}{c^2}$$





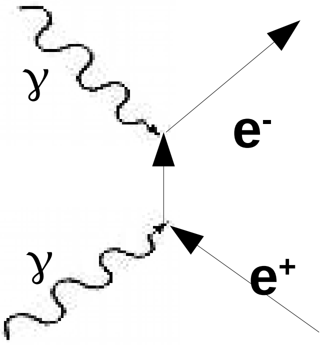
$\theta_\gamma \in [16 : \theta_{pseuil}]$
Angles de collisions

$z_* = z/R_s$ Altitude
normalisée

Γ_π Facteur de Lorentz
des paires

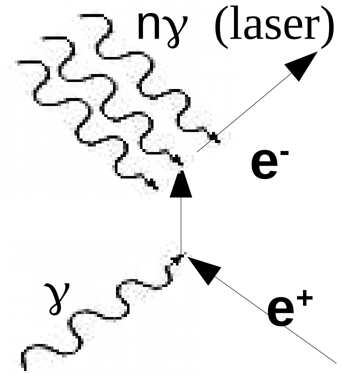
About non-linear Breit-Wheeler and linear Breit-wheeler processes

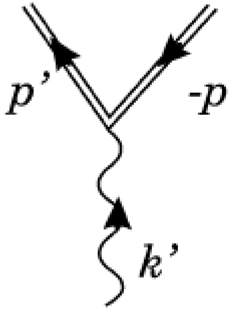
Breit-Wheeler[1] (1934)



-Collision of two gamma photons

Non-linear Breit-Wheeler[2] (1962)





- « Collision of one gamma photon and several low energy (laser) photon »

- Desintegration of one gamma photon in electromagnetic field (n laser photon)

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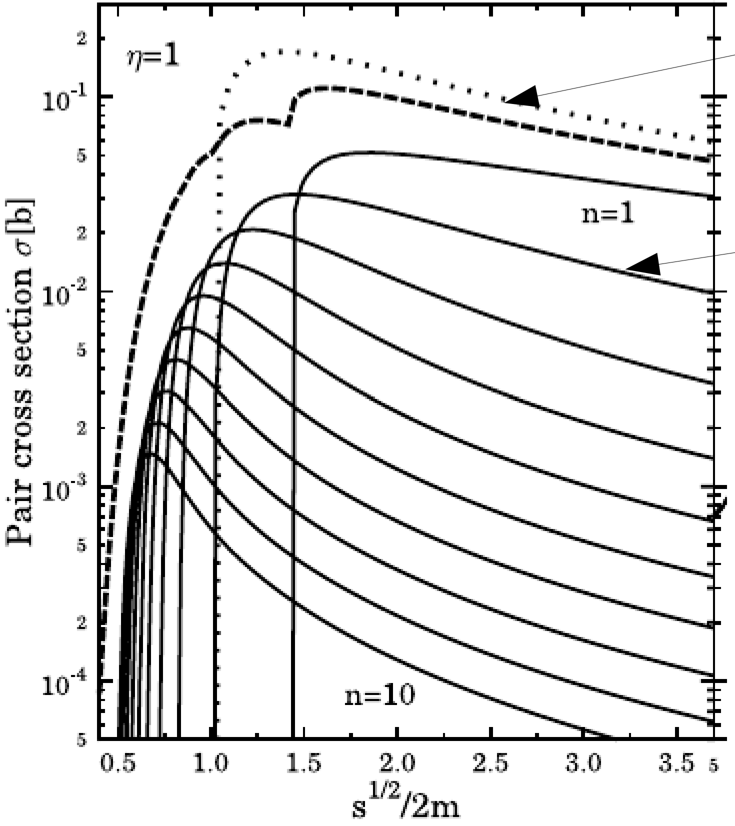
[1] Breit, G. and Wheeler J. A. Phys. Rev. **46** (1934)

[2] Reiss H. R., J. Math. Phys. 3, 59 (1962)

Total cross section comparison



Linear BW

Non linear BW for different n

Ref : Greiner W. and Reinhardt J., Quantum Electrodynamics, Springer (2009)

About non-linear Breit-Wheeler and linear Breit-wheeler processes (1)

For BW process the total cross section is :

$$\gamma + \gamma \rightarrow e^+ + e^- \qquad \bar{\sigma}_{\text{pair}} = \frac{\pi}{2}\frac{\alpha^2}{m_0^2}(1-v^2)\left[(3-v^4)\ln\frac{1+v}{1-v} - 2v(2-v^2)\right] .$$

Same as before

For non-linear BW the total cross section writes :

$$\gamma + n\omega \rightarrow e^+ + e^- \qquad \sigma = \frac{2\pi\alpha^2}{s}\frac{1}{\eta^2}\sum_{n>n_0}^{\infty}\int_1^{u_n} \mathrm{d}u \, \frac{1}{u\sqrt{u(u-1)}}$$

$$\times\left[2J_n^2(z) + \eta^2\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)(2u-1)\right] .$$

Where n is
the number of photons
and

$$\eta = \frac{e\sqrt{|\langle A_\mu A^\mu\rangle|}}{mc^2} = \frac{ea}{mc^2}$$

$$= \frac{e|\boldsymbol{E}|}{\omega mc} = a_0$$

$$s = 4n\omega\omega' \qquad u_n = \frac{ns}{4m_0^2} \qquad z = \sqrt{-Q^2} = \frac{8m^2}{s}\eta\sqrt{1+\eta^2}\sqrt{u(u_n-u)} .$$

About Non-linear Breit-Wheeler and Linear Breit-wheeler processes (2)

If n=1 and $a_0 << 1$

$$u_1 = \frac{s}{4m_0^2}$$

BW non-linear

$$\sigma \simeq \frac{2\pi\alpha^2}{s}2\left[\ln\frac{1+\sqrt{1-1/u_1}}{1-\sqrt{1-1/u_1}}\left(1+\frac{1}{u_1}-\frac{1}{2u_1^2}\right)-\left(1+\frac{1}{u_1}\right)\sqrt{1-\frac{1}{u_1}}\right]$$

$$v = \sqrt{1-\frac{1}{u_1}}$$

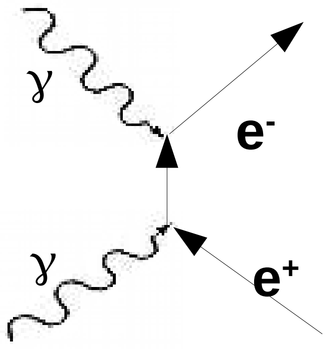
BW linear

$$\bar{\sigma}_{\mathrm{pair}} = \frac{\pi}{2}\frac{\alpha^2}{m_0^2}(1-v^2)\left[(3-v^4)\ln\frac{1+v}{1-v}-2v(2-v^2)\right] .$$

Equivalent total cross section between BW-linear and BW non-linear for n= 1 and low a_0

About Non-linear Breit-Wheeler and Linear Breit-wheeler processe (4)

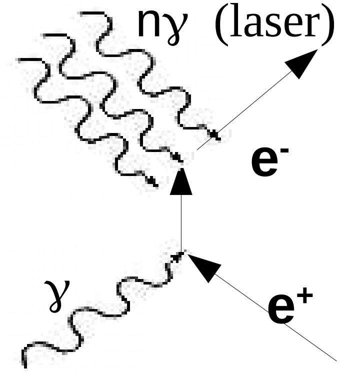
Breit-Wheeler[1] (1934)



If n=1 and a_0 << 1

$$a_0 \propto \sqrt{I}\lambda$$

Non-linear Breit-Wheeler[2] (1962)



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[1] Breit, G. and Wheeler J. A. Phys. Rev. **46** (1934)

[2] Reiss H. R., J. Math. Phys. 3, 59 (1962)

Thank you for your Attention !