Phonon thermal Hall conductivity from scattering with collective fluctuations

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Paris, 2 juin 2022

Collaborators

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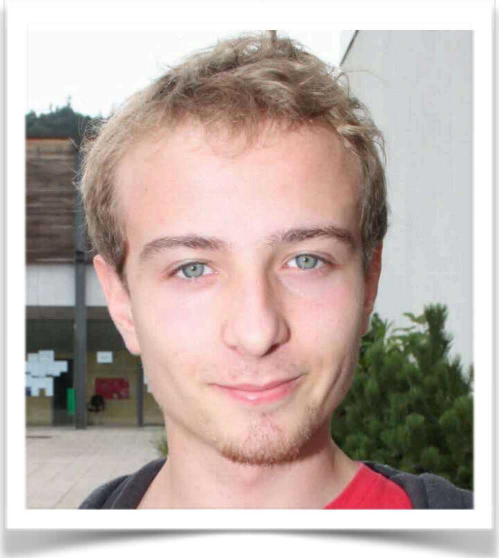

arXiv: 2202.10366

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Leon Balents

KITP

erc European Research Council Established by the European Commission

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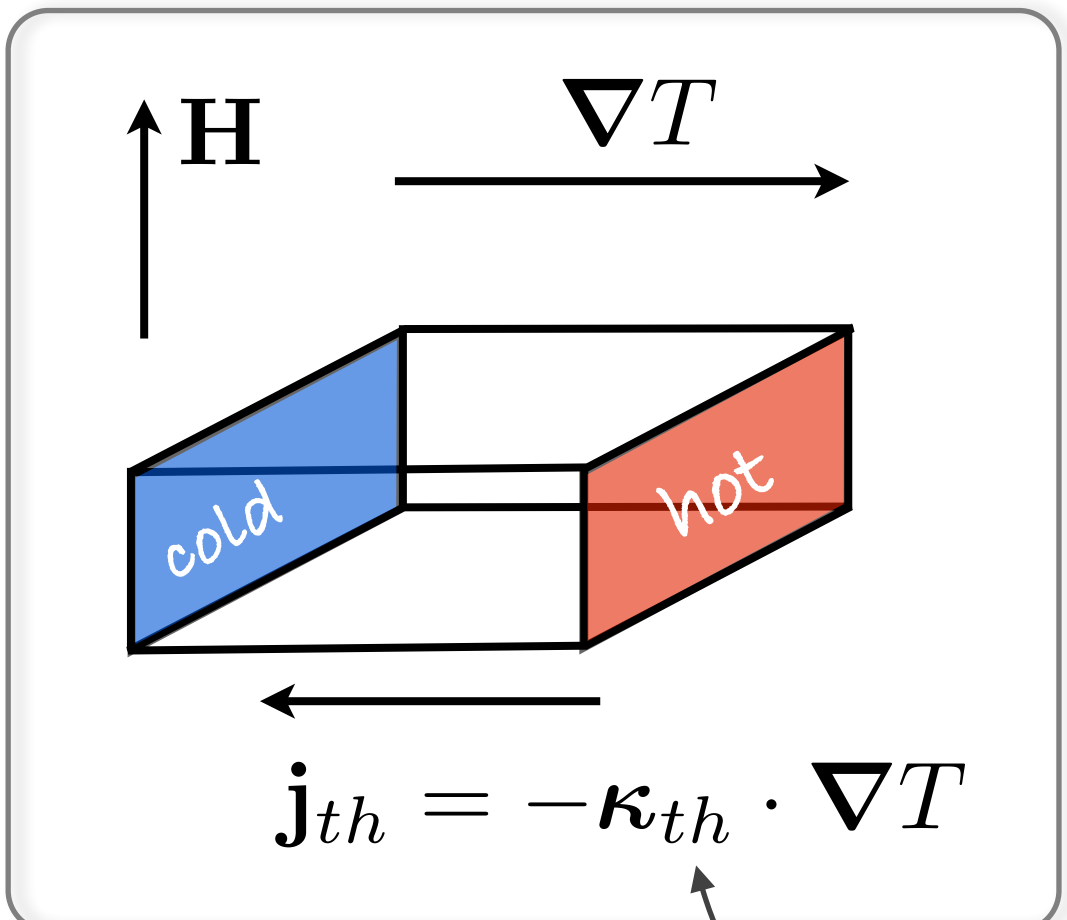
Thermal conductivity

- What is thermal conductivity?



tensor

$\boldsymbol{\kappa}(T, \mathbf{h}, Q, \cdots)$

properties of the system

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa^{xx} & \kappa^{xy} & \kappa^{xz} \\ \kappa^{yx} & \kappa^{yy} & \kappa^{yz} \\ \kappa^{zx} & \kappa^{zy} & \kappa^{zz} \end{pmatrix}$$

κ_L

longitudinal conductivity

dissipative

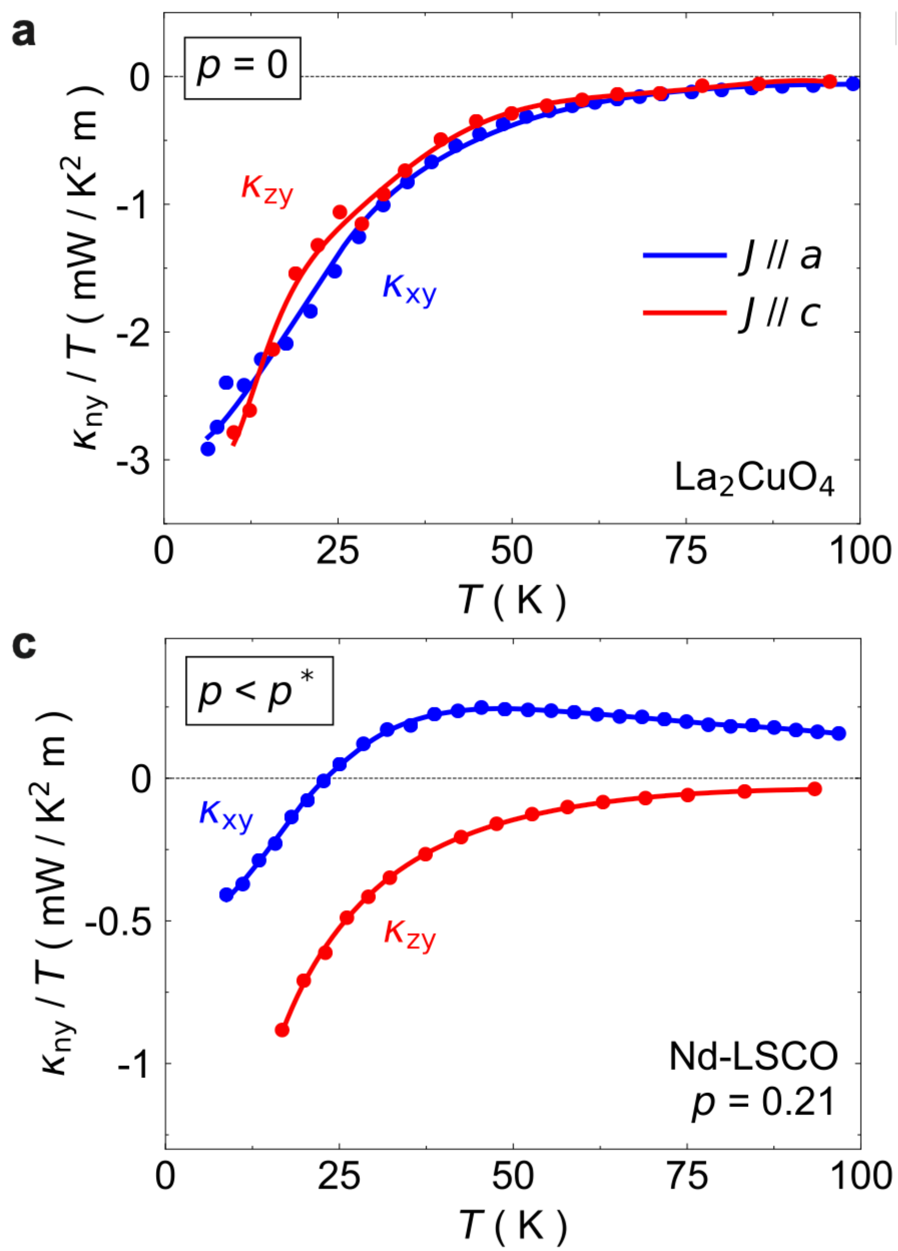
- Thermal *Hall* conductivity: antisymmetric $\underbrace{\boldsymbol{\kappa}_H = (\boldsymbol{\kappa} - \boldsymbol{\kappa}^T)/2}_{\text{isentropic}}$

zero in the presence of time-reversal (Onsager)

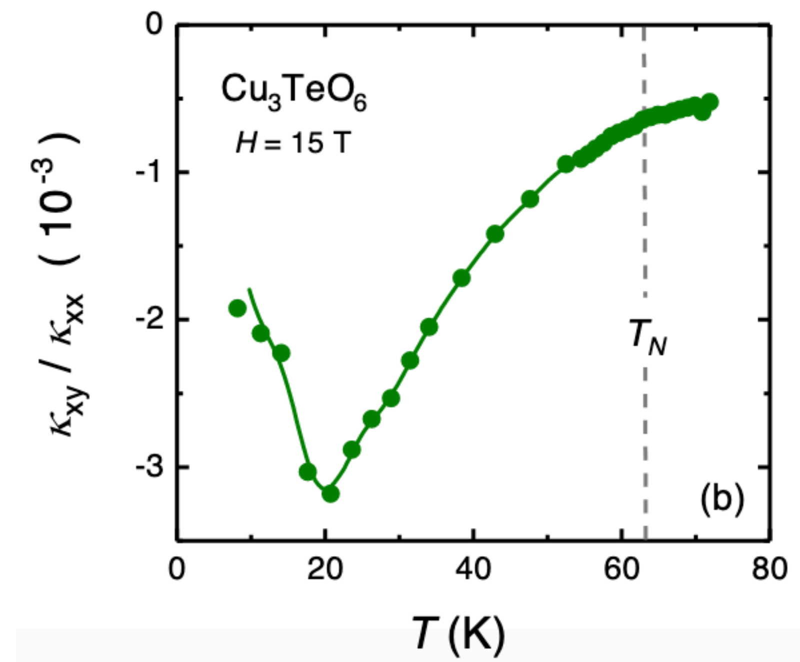
- Thermal Hall *resistivity*: $\boldsymbol{\varrho}_H = \boldsymbol{\kappa}_H^{-1}$

much less dependent on impurity effects than $\boldsymbol{\kappa}_H$

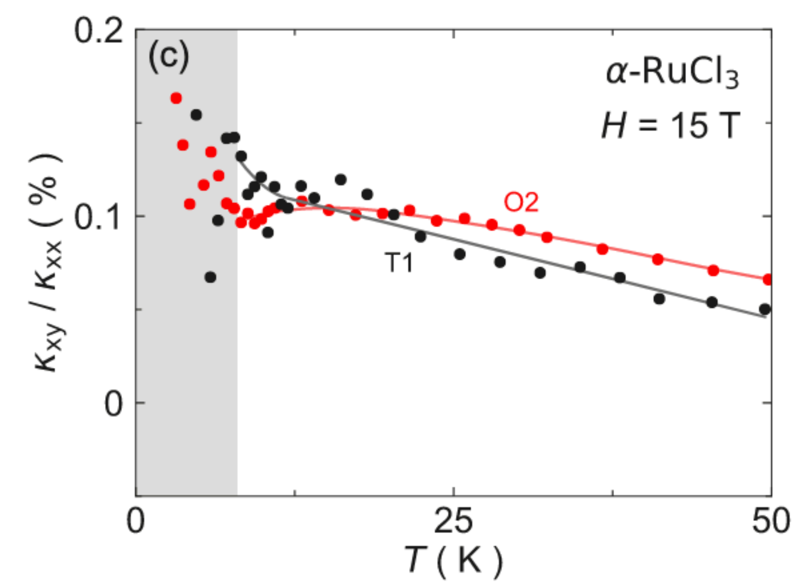
Phonons thermal Hall conductivity?



Grissonanche *et al*, 2020



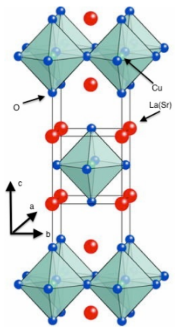
L. Chen *et al*, 2021



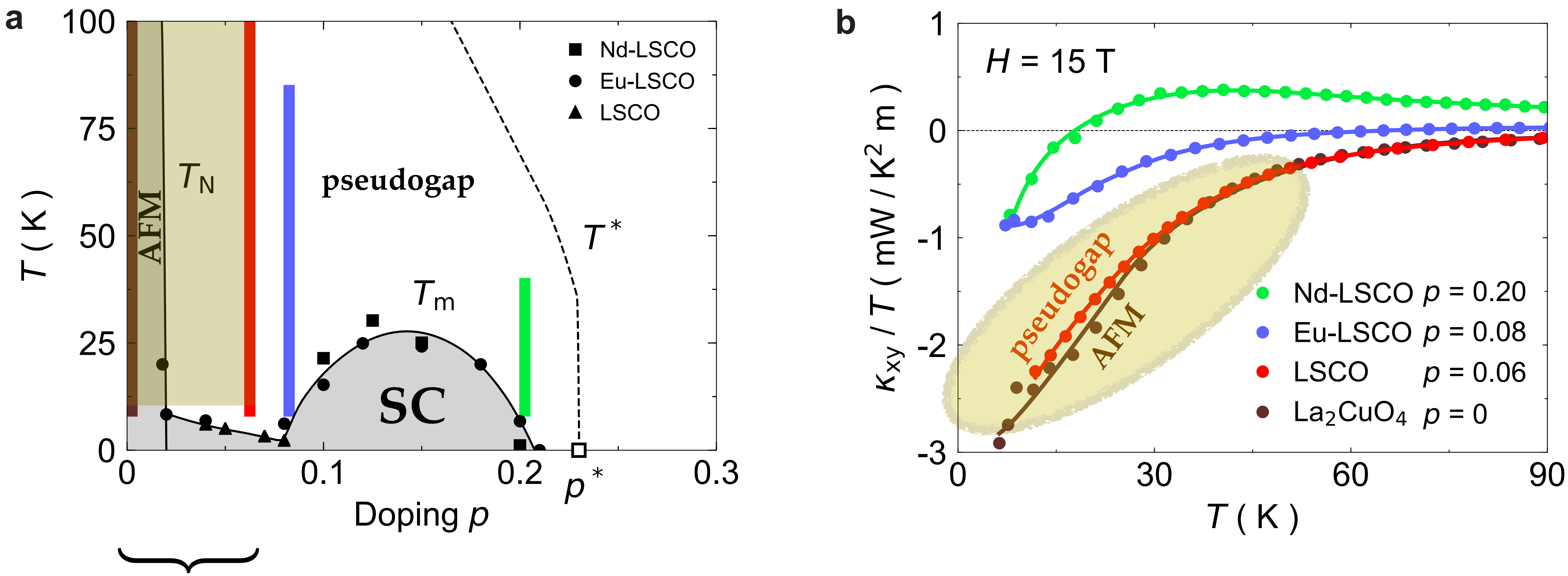
Evidence of a Phonon Hall Effect in the Kitaev Spin Liquid Candidate α-RuCl$_3$

É. Lefrançois,[1] G. Grissonnanche,[1] J. Baglo,[1] P. Lampen-Kelley,[2,3] J. Yan,[2] C. Balz,[4,*] D. Mandrus,[2,3] S. E. Nagler,[4] S. Kim,[5] Young-June Kim,[5] N. Doiron-Leyraud,[1] and L. Taillefer[1,6]

Recent data on cuprates by Taillefer's group



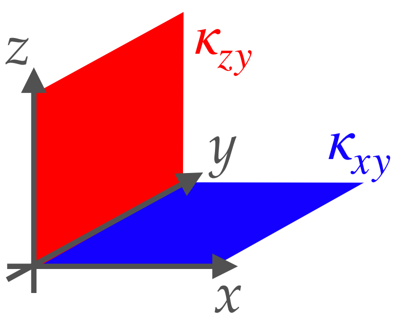
$La_{2-p}Sr_pCuO_4$ Grissonnanche *et al* Nature 2019

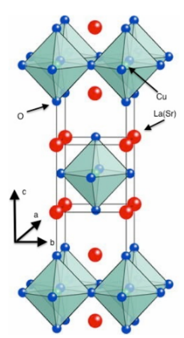


nearly overlapping thermal Hall conductivity curves despite very different phase electronically (insulator v/s bad metal)!

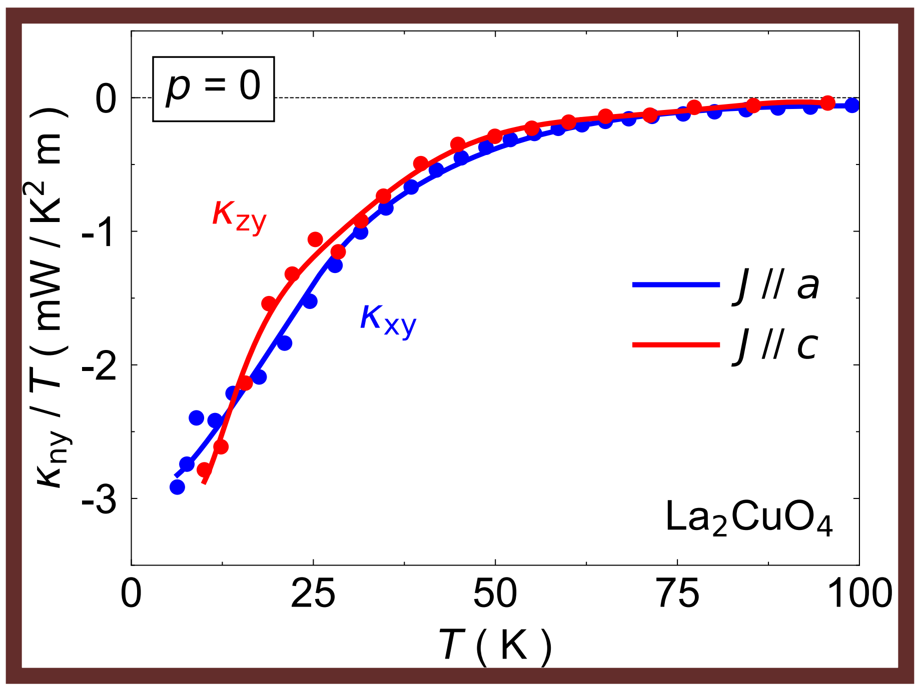
→ phonons?

Out-of-plane propagation (Taillefer's group)

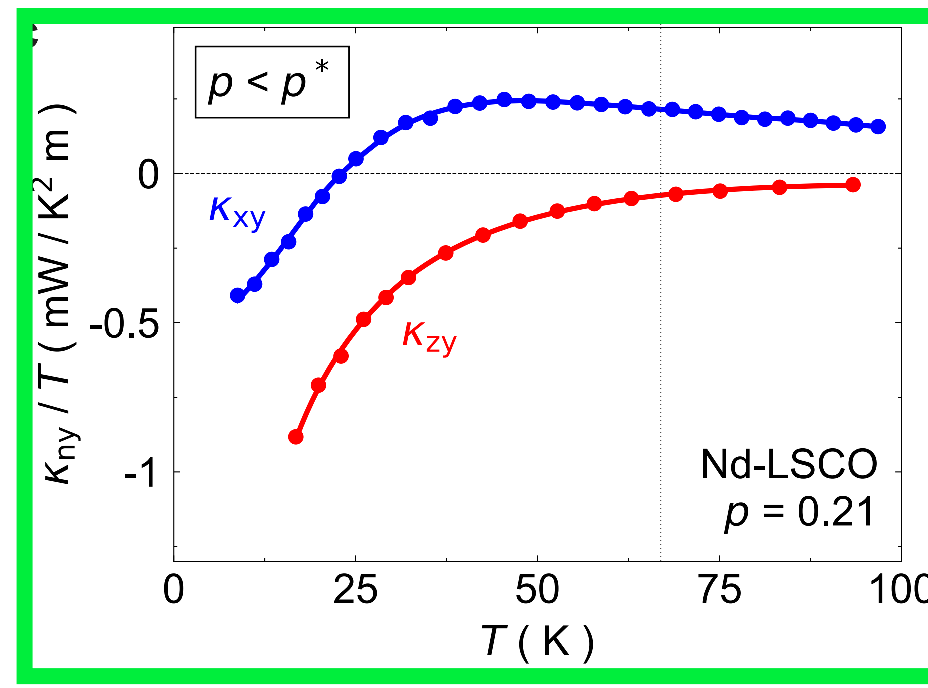




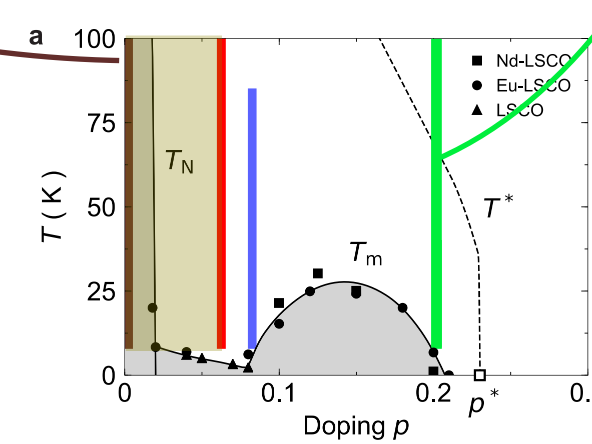
Grissonnanche *et al* 2020



AFM



(bad) metal



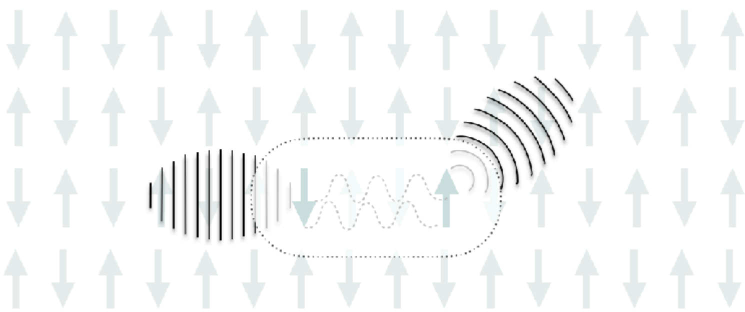
nearly overlapping κ_{xy} and κ_{zy} in AFM:

near-isotropy despite very anisotropic magnetism

phonons?

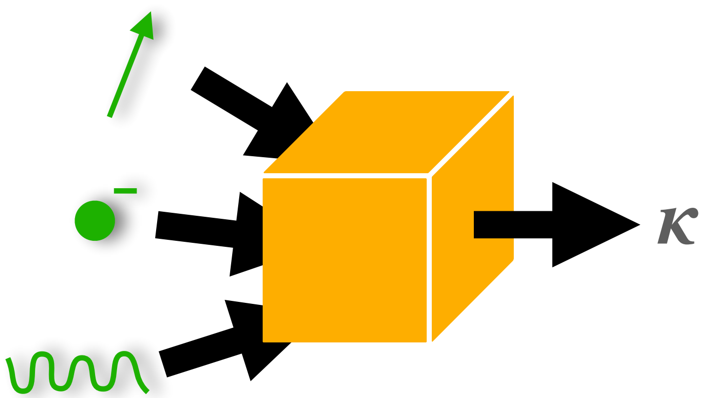
Spoiler alert
our results

Scattering-induced phonon Hall effect probes non-trivial / beyond Gaussian correlations (OTOCs in fact)



Result is obtained for *any* physical degree of freedom

in other words, give us your physical degree of freedom, we will tell you κ



Provide analytical and numerical results for ordered antiferromagnets, fermions (e.g. spinon FS) in terms of microscopic/phenomenological parameters

Scattering

- know it is often *important* for the longitudinal conductivity

- is it *interesting*?
    - as a theoretical question, ask when it provides non-trivial, non-detail-specific information about the system

Hall effect

- intrinsic many-body Hall effect
- Hall effect of *single* particles
  - Berry phase
  - Lorentz force

(left side of Boltzmann's equation)

- scattering with impurities
  - depends on type of impurity
  - skew scattering
  - side jump

Hall effect

- intrinsic many-body Hall effect
- Hall effect of *single* particles
  - Berry phase
  - Lorentz force
- scattering with impurities
  - depends on type of impurity
  - skew scattering
  - side jump

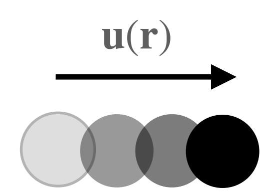
study the thermal Hall effect in the context of phonons (experience no Lorentz force) interacting with other degrees of freedom

Phonons

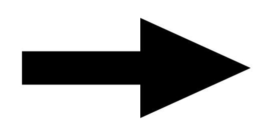
We will **do this very generally** because we can

- fairly challenging to identify all contributions
- not so much understanding so far
- so better to not be too specific

atomic displacement field $\mathbf{u}(\mathbf{r})$



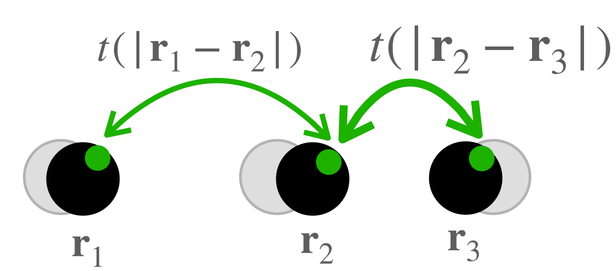
$$\mathscr{E}^{\mu\nu} = \frac{1}{2}\left(\partial_\mu u^\nu + \partial_\nu u^\mu\right)$$



quantization

phonon creation $a^\dagger$ and annihilation a operators

microscopic view of coupling to elasticity:



Phonon coupling

expand interaction Hamiltonian in number of phonon operators:

$$\mathscr{H} = \mathscr{H}_{\text{ph}} \otimes \mathscr{H}_{Q}$$

some operator which couples to a single phonon

some operator which couples to a phonon-conserving 2-phonon operator

...

$$H_{\text{int}} \sim a^{\dagger} Q_{(1)} + a^{\dagger} a \, Q_{(2),1} + a^{\dagger} a^{\dagger} Q_{(2),2} + \text{h.c.} + \cdots$$

e.g. coupling to strain:

$$H_{\text{int}}^{\text{strain}} \sim \mathcal{E}^{\mu\nu} Q^{\mu\nu}$$

e.g. “Raman”

$$\mathbf{u} \times \boldsymbol{\pi}$$

note: the Q’s include coupling strengths λ – $Q \sim \lambda O$

Phonon coupling

expand interaction Hamiltonian in number of phonon operators:

$$\boxed{H_{\text{int}} \sim a^\dagger\, Q_{(1)} + a^\dagger a\, Q_{(2),1} + a^\dagger a^\dagger\, Q_{(2),2} + \text{h.c.} + \cdots}$$

typically the Q represent electronic degrees of freedom
(can also be phonons etc. different from the a's)

e.g.

<u>spins:</u> $Q_{(1)} \sim S^\mu S^\nu$

<u>fermions:</u> $Q \sim c^\dagger c$

<u>gauge field:</u> $Q \sim E$

etc.

Setup

Boltzmann's equation for phonon density N

- phonons are always there
- good quasiparticles (typically weak ph-ph interactions, weak anharmonicity)
- always 3d

<u>Boltzmann:</u>

classical *six* dimensional phase space:

(x, y, z, k_x, k_y, k_z)

$$\boxed{D_t N = \mathscr{C}[\{N\}]}$$

convective derivative

collision terms (rate)

kinetics +

modifications of intrinsic dynamics of individual quasiparticles, e.g. Berry phase effects, etc.

modifications of quasiparticles through scattering
dissipative

Construct collision terms

$$D_t N = \mathscr{C}[\{N\}]$$

convective derivative

collision terms

- Coupling Hamiltonian

$$H' = \sum_{n\boldsymbol{k}} \left(a^\dagger_{n\boldsymbol{k}} Q^\dagger_{n\boldsymbol{k}} + a_{n\boldsymbol{k}} Q_{n\boldsymbol{k}} \right)$$

- Full transition rate

$$\Gamma_{\mathtt{i}\to\mathtt{f}} = \frac{2\pi}{\hbar} |T_{\mathtt{i}\to\mathtt{f}}|^2 \delta(E_{\mathtt{i}} - E_{\mathtt{f}})$$

obtain from Born's approximation (next slide)

$$|\mathrm{g}\rangle = |g_p\rangle \otimes |g_s\rangle$$

phonons Q

- Phonon transition rate

$$\tilde{\Gamma}_{i_p\to f_p} = \sum_{i_s, f_s} \Gamma_{\mathtt{i}\to\mathtt{f}} p_{i_s}$$

$$p_{i_s} = \frac{1}{Z_s} e^{-\beta E_{i_s}}$$

Q subsystem in equilibrium

- Master equation

$$\mathcal{C}_{n\mathbf{k}} = \sum_{i_p, i_f} \tilde{\Gamma}_{i_p\to f_p} (N_{n\mathbf{k}}(f_p) - N_{n\mathbf{k}}(i_p)) p_{i_p}$$

collision terms

In this way we can construct $\mathscr{C}_{n\mathbf{k}}$ for any "Q" subsystem

Transition matrix

In *full many-body space* of phonons + Q (electrons, spins etc):

$$\Gamma_{\mathtt{i}\to\mathtt{f}} = \frac{2\pi}{\hbar}|T_{\mathtt{i}\to\mathtt{f}}|^2\delta(E_{\mathtt{i}} - E_{\mathtt{f}})$$

Born's approximation:

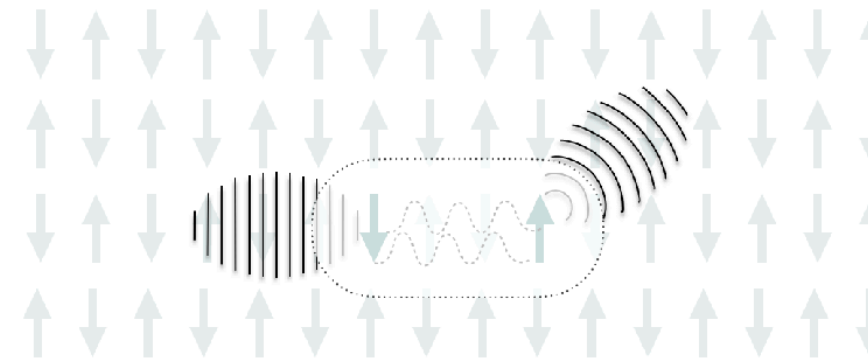
$$\boxed{T_{\mathtt{i}\to\mathtt{f}} = T_{\mathtt{fi}} = \langle\mathtt{f}|H'|\mathtt{i}\rangle + \sum_{\mathtt{n}} \frac{\langle\mathtt{f}|H'|\mathtt{n}\rangle\langle\mathtt{n}|H'|\mathtt{i}\rangle}{E_{\mathtt{i}} - E_{\mathtt{n}} + i\eta} + \cdots}$$

Important point: 1st order term is Hermitian, $H' = (H')^\dagger$
so 1st order T-matrix is effectively time-reversal invariant

- No Hall effect at leading order.

Scattering rates

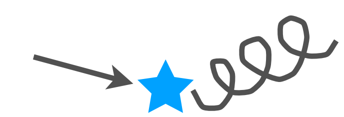
$D_t N = \mathscr{C}[\{N\}]$



<u>diagonal scattering rate:</u> $D \sim \mathscr{C}_{\rm long}[N_{n\mathbf{k}}]$

$$D_{n\mathbf{k}} = -\frac{1}{\hbar^2}\int dt\, e^{-i\omega_{n\mathbf{k}}t}\left\langle [Q_{n\mathbf{k}}(t), Q^\dagger_{n\mathbf{k}}(0)]\right\rangle_\beta + \breve{D}_{n\mathbf{k}}$$

extrinsic scattering,
put in by hand

O(Q²) 

<u>skew scattering rate:</u> $\mathfrak{W} \sim \mathscr{C}_{\rm skew}[N_{n\mathbf{k}}, N_{n'\mathbf{k}'}]$

$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = \frac{2N_{\rm uc}}{\hbar^4}\mathfrak{Re}\int_{t,t_1,t_2} e^{i[\Sigma^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}t+\Delta^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}(t_1+t_2)]}\mathrm{sign}(t_2)\left\langle\left[Q^{-q}_{n\mathbf{k}}(-t-t_2), Q^{-q'}_{n'\mathbf{k}'}(-t+t_2)\right]\left\{Q^{q'}_{n'\mathbf{k}'}(-t_1), Q^{q}_{n\mathbf{k}}(t_1)\right\}\right\rangle$$

commutator *anti-commutator*

O(Q⁴) 

out-of-time-ordered

"Anti-detailed balance" $\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = -\, e^{-\beta(q\omega_{n\mathbf{k}}+q'\omega_{n'\mathbf{k}'})}\, \mathfrak{W}^{\ominus,-q-q'}_{n\mathbf{k}n'\mathbf{k}'}$

Thermal Hall effect

Anti-symmetric part:

skew-scattering rate

$$\kappa_H^{\mu\nu} = \frac{\hbar^2}{k_B T^2} \frac{1}{V} \sum_{n\mathbf{k}n'\mathbf{k}'} J^\mu_{n\mathbf{k}} \frac{e^{\beta\hbar\omega_{n\mathbf{k}}/2}}{2D_{n\mathbf{k}}} \left(\frac{1}{N_{\mathrm{uc}}} \sum_{q=\pm} \frac{\left(e^{\beta\hbar\omega_{n\mathbf{k}}} - e^{q\beta\hbar\omega_{n'\mathbf{k}'}}\right) \mathfrak{W}^{\ominus,+,q}_{n\mathbf{k},n'\mathbf{k}'}}{\sinh(\beta\hbar\omega_{n\mathbf{k}}/2)\sinh(\beta\hbar\omega_{n'\mathbf{k}'}/2)} \right) \frac{e^{\beta\hbar\omega_{n'\mathbf{k}'}/2}}{2D_{n'\mathbf{k}'}} J^\nu_{n'\mathbf{k}'}$$

diagonal rate

$$J^\mu_{n\mathbf{k}} = N^{\mathrm{eq}}_{n\mathbf{k}}\, \omega_{n\mathbf{k}} v^\mu_{n\mathbf{k}}$$

Basic idea:

$$A\ \nabla T = -\frac{1}{\tau}\delta n - \frac{1}{\tau_{\mathrm{skew}}}\delta n$$

Fourier's law

$$\delta n = -\tau\ A\ \nabla T - \frac{\tau}{\tau_{\mathrm{skew}}}\delta n$$

$$\approx -\tau\ A\ \nabla T - \frac{\tau^2}{\tau_{\mathrm{skew}}}\ A\ \nabla T$$

Thermal Hall effect

Conductivity versus resistivity:

$$\kappa_H \sim \frac{\tau^2}{\tau_{\mathrm{skew}}}$$

Sensitive to all ordinary scattering mechanisms. Very non-universal.

$$\boldsymbol{\varrho} = \boldsymbol{\kappa}^{-1}$$

matrix inverse

$$\boxed{\varrho_H \sim -\frac{\kappa_H}{\kappa^2} \sim \frac{1}{\tau_{\mathrm{skew}}}}$$

Only sensitive to skew scattering. A better quantity to study.

$$\boxed{\varrho_H \sim \mathfrak{W}^{\ominus,\mathrm{eff}}}$$

Indeed follows from our formulae

Many-body skew scattering

$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = \frac{2N_{\mathrm{uc}}}{\hbar^4}\mathfrak{Re}\int_{t,t_1,t_2} e^{i[\Sigma^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}t+\Delta^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}(t_1+t_2)]}\mathrm{sign}(t_2)\left\langle\left[Q^{-q}_{n\mathbf{k}}(-t-t_2), Q^{-q'}_{n'\mathbf{k}'}(-t+t_2)\right]\left\{Q^{q'}_{n'\mathbf{k}'}(-t_1), Q^{q}_{n\mathbf{k}}(t_1)\right\}\right\rangle$$

What good is it?

- In principle, this can be applied for any Q, could be e.g. quantum critical field etc.
- Can be used to analyze symmetries, *à la* Curie and Onsager
- Any bounds on Hall scattering rate?
- That said, it is very hard to calculate such real-time correlation functions…

Any systems where this might be the dominant contribution, i.e. where ϱ_H probes these OTOC directly?

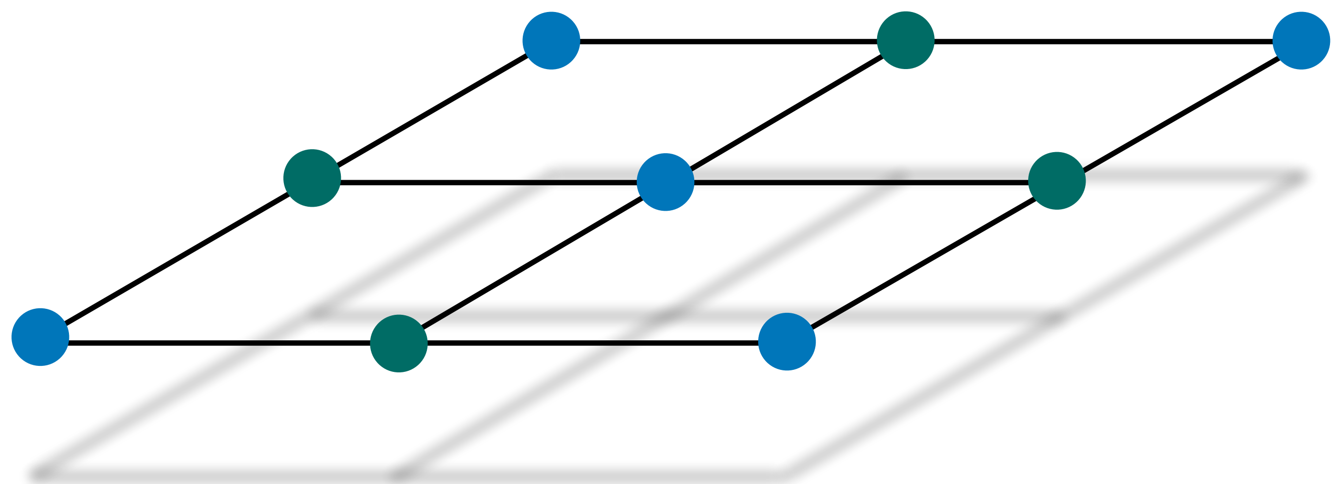
Now calculate these correlation functions for specific systems

Now calculate these correlation functions for specific systems

1st example: magnons

Application to an antiferromagnet

For concreteness, 2d, layered

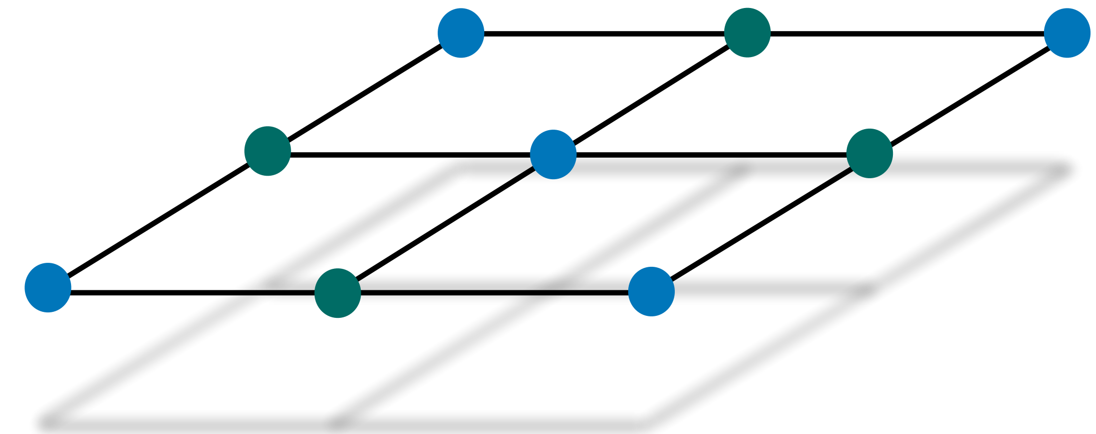


Spin waves

$$H_{\rm NLS} + H_{\rm field} = \sum_{\ell} \sum_{\mathbf{k}} \Omega_{\mathbf{k},\ell} b^{\dagger}_{\mathbf{k},\ell} b_{\mathbf{k},\ell}$$

$b_{\mathbf{k},\ell}, b^{\dagger}_{\mathbf{k},\ell}$ diagonalize magnon hamiltonian

Application to an antiferromagnet



Spin waves

$$H_{\mathrm{NLS}} + H_{\mathrm{field}} = \sum_{\ell} \sum_{\mathbf{k}} \Omega_{\mathbf{k},\ell} b^{\dagger}_{\mathbf{k},\ell} b_{\mathbf{k},\ell}$$

recall interaction hamiltonian:

$$H' = \sum_{n\boldsymbol{k}} \left(a^{\dagger}_{n\boldsymbol{k}} Q^{\dagger}_{n\boldsymbol{k}} + a_{n\boldsymbol{k}} Q_{n\boldsymbol{k}} \right)$$

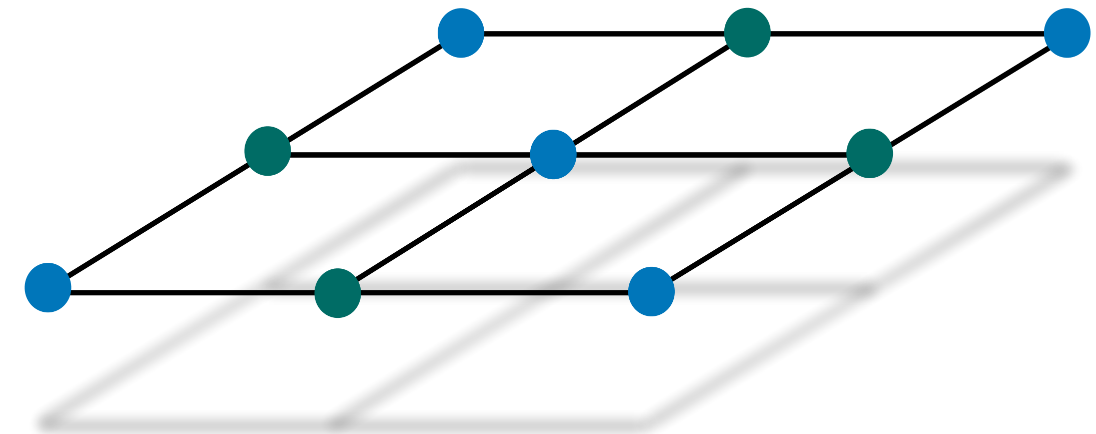
what is Q in this case?

Collective field

$$Q^{q}_{n\mathbf{k}} = \cancel{\sum_{\ell,q_1,z} \mathcal{A}^{n,\ell|q_1 q}_{\mathbf{k}} e^{ik_z z} b^{q_1}_{\ell,\mathbf{k},z}} + \frac{1}{\sqrt{N_{\mathrm{uc}}}} \sum_{\substack{\mathbf{p},\ell,\ell' \\ q_1,q_2,z}} \mathcal{B}^{n,\ell_1,\ell_2|q_1 q_2 q}_{\mathbf{k};\mathbf{p}} \; e^{ik_z z} b^{q_1}_{\ell_1,\mathbf{p}+\frac{q}{2}\mathbf{k},z} b^{q_2}_{\ell_2,-\mathbf{p}+\frac{q}{2}\mathbf{k},z}$$

Negligible phase space

Application to an antiferromagnet



Spin waves

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Negligible phase space

Structure hidden here

General analytical result

- Diagonal scattering rate:

$$D_{n\mathbf{k}} = -\frac{1}{\hbar^2}\int dt\, e^{-i\omega_{n\mathbf{k}}t}\left\langle [Q_{n\mathbf{k}}(t), Q^\dagger_{n\mathbf{k}}(0)]\right\rangle_\beta + \breve{D}_{n\mathbf{k}}$$

- Skew scattering rate:

$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = \frac{2N_{\mathrm{uc}}}{\hbar^4}\mathfrak{Re}\int_{t,t_1,t_2} e^{i[\Sigma^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}t+\Delta^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}(t_1+t_2)]}\mathrm{sign}(t_2)\left\langle\left[Q^{-q}_{n\mathbf{k}}(-t-t_2), Q^{-q'}_{n'\mathbf{k}'}(-t+t_2)\right]\left\{Q^{q'}_{n'\mathbf{k}'}(-t_1), Q^{q}_{n\mathbf{k}}(t_1)\right\}\right\rangle$$

General analytical result

- Diagonal scattering rate:

$$D_{n\mathbf{k}}^{(s)} = \frac{(3-s)\pi}{\hbar^2 N_{\mathrm{uc}}^{2\mathrm{d}}} \sum_{\mathbf{p}} \sum_{\ell,\ell'} \frac{\sinh(\frac{\beta}{2}\hbar\omega_{n\mathbf{k}})}{\sinh(\frac{\beta}{2}\hbar\Omega_{\ell,\mathbf{p}})\sinh(\frac{\beta}{2}\hbar\Omega_{\ell',\mathbf{p}-\mathbf{k}})} \delta(\omega_{n\mathbf{k}} - \Omega_{\ell,\mathbf{p}} - s\Omega_{\ell',\mathbf{p}-\mathbf{k}}) \left| \mathcal{B}^{n,\ell,\ell'|+s-}_{\mathbf{k};-\mathbf{p}+\frac{\mathbf{k}}{2}} \right|^2$$

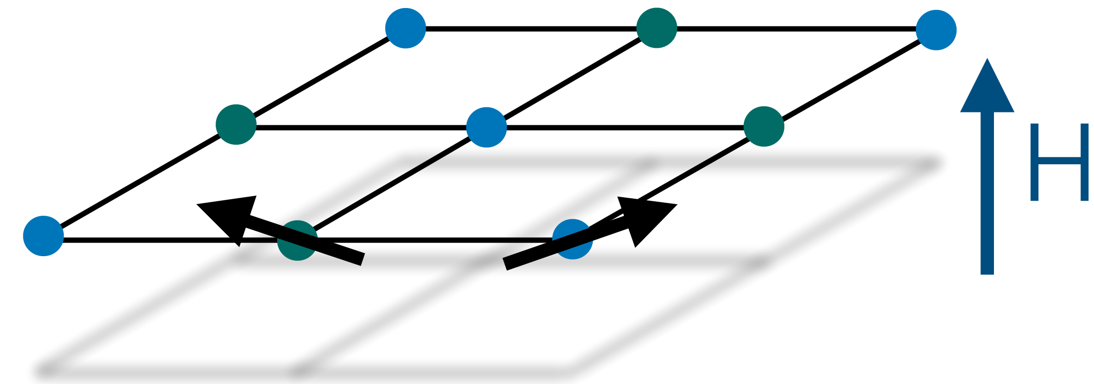
- Skew scattering rate:

$$\begin{aligned}\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k},n'\mathbf{k}'} = \frac{64\pi^2}{\hbar^4} \frac{1}{N_{\mathrm{uc}}^{2\mathrm{d}}} \sum_{\mathbf{p}} \sum_{\{\ell_i,q_i\}} \mathfrak{D}^{nn'|q_1q_2q_3,\ell_1\ell_2\ell_3}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}} \mathfrak{F}^{q_1q_2q_4,\ell_1\ell_2\ell_3}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}} \mathfrak{Im}\Bigg\{ \mathcal{B}^{n\ell_2\ell_3|q_2q_3q}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}+q'\mathbf{k}'} \mathcal{B}^{n'\ell_3\ell_1|-q_3q_1q'}_{\mathbf{k}',\mathbf{p}+\frac{1}{2}q'\mathbf{k}'} \\ \times \mathrm{PP}\Bigg[\frac{\mathcal{B}^{n\ell_1\ell_4|-q_1q_4-q}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}} \mathcal{B}^{n'\ell_4\ell_2|-q_4-q_2-q'}_{\mathbf{k}',\mathbf{p}+q\mathbf{k}+\frac{1}{2}q'\mathbf{k}'}}{\Delta^{qq'}_{n\mathbf{k}n'\mathbf{k}'} + q_1\Omega_{\ell_1,\mathbf{p}} - q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'} - 2q_4\Omega_{\ell_4,\mathbf{p}+q\mathbf{k}}} + \frac{\mathcal{B}^{n'\ell_1\ell_4|-q_1-q_4-q'}_{\mathbf{k}',\mathbf{p}+\frac{1}{2}q'\mathbf{k}'} \mathcal{B}^{n\ell_4\ell_2|q_4-q_2-q}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}+q'\mathbf{k}'}}{\Delta^{qq'}_{n\mathbf{k}n'\mathbf{k}'} - q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'} - 2q_4\Omega_{\ell_4,\mathbf{p}+q'\mathbf{k}'}} \Bigg] \Bigg\}\end{aligned}$$

$$\begin{aligned}\mathfrak{D}^{nn'|q_1q_2q_3,\ell_1\ell_2\ell_3}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}} &= \delta\left(\Sigma^{qq'}_{n\mathbf{k}n'\mathbf{k}'} + q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'}\right)\delta\left(\Delta^{qq'}_{n\mathbf{k}n'\mathbf{k}'} + 2q_3\Omega_{\ell_3,\mathbf{p}+q'\mathbf{k}'} - q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'}\right), \\ \mathfrak{F}^{q_1q_2q_4,\ell_1\ell_2\ell_3}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}} &= q_4\left(2n_{\mathrm{B}}(\Omega_{\ell_3,\mathbf{p}+q'\mathbf{k}'})+1\right)\left(2n_{\mathrm{B}}(\Omega_{\ell_1,\mathbf{p}})+q_1+1\right)\left(2n_{\mathrm{B}}(\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'})+q_2+1\right).\end{aligned}$$

Could be applied to any magnet

Continuum magnons



Hamiltonian:

$$\mathcal{H}_{\mathrm{NLS}} = \frac{\rho}{2}\left(|\underline{\boldsymbol{\nabla}} n_y|^2 + |\underline{\boldsymbol{\nabla}} n_z|^2\right) + \frac{1}{2\chi}(m_y^2 + m_z^2) + \sum_{a,b=y,z} \frac{\Gamma_{ab}}{2} n_a n_b$$

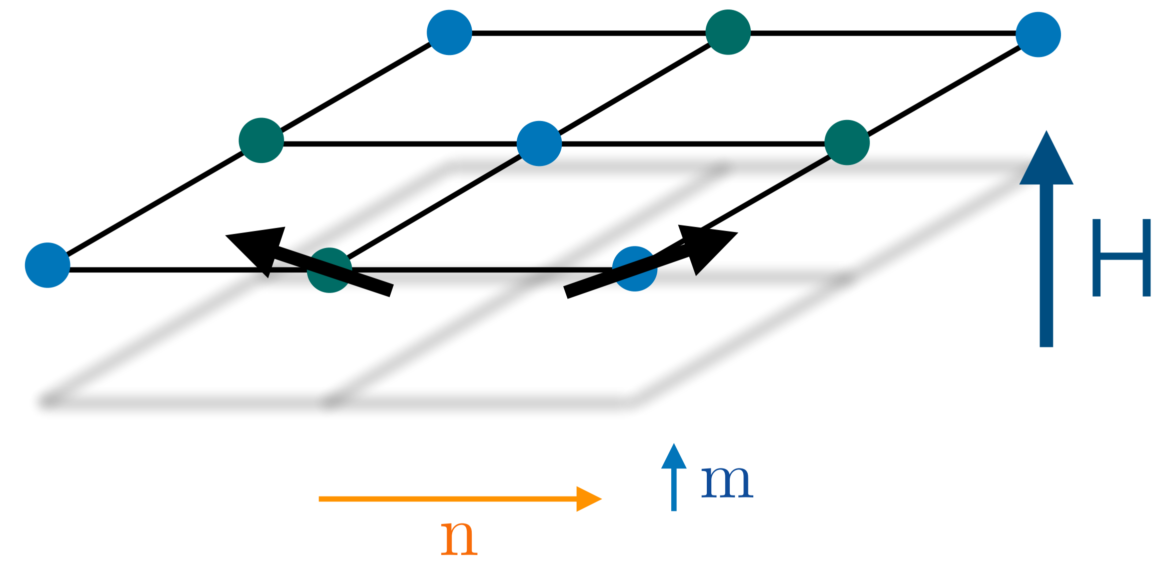
Spin-lattice coupling:

$$\boxed{\mathcal{H}' = \sum_{\substack{\alpha,\beta \\ a,b=x,y,z}} \mathcal{E}_{\mathbf{r}}^{\alpha\beta}\left(\Lambda_{ab}^{(\mathrm{n}),\alpha\beta}\mathrm{n}_a\mathrm{n}_b + \frac{\Lambda_{ab}^{(\mathrm{m}),\alpha\beta}}{n_0^2}\mathrm{m}_a\mathrm{m}_b\right)\Bigg|_{\mathbf{x},z}}$$

$$|\mathbf{n}|^2 + \frac{\mathfrak{a}^4}{\mu_0^2}|\mathbf{m}|^2 = 1, \qquad \mathbf{m}\cdot\mathbf{n} = 0.$$

recall: $H' = \sum_{n\boldsymbol{k}}\left(a_{n\boldsymbol{k}}^{\dagger}Q_{n\boldsymbol{k}}^{\dagger} + a_{n\boldsymbol{k}}Q_{n\boldsymbol{k}}\right)$ $\qquad Q_{n\mathbf{k}} = \frac{1}{\sqrt{N_{\mathrm{uc}}}}\sum_{\substack{\mathbf{p},\ell,\ell' \\ q_1,q_2,z}} \mathcal{B}_{\mathbf{k};\mathbf{p}}^{n,\ell_1,\ell_2|q_1q_2q}\; e^{ik_z z} b_{\ell_1,\mathbf{p}+\frac{q}{2}\mathbf{k},z}^{q_1} b_{\ell_2,-\mathbf{p}+\frac{q}{2}\mathbf{k},z}^{q_2}$

Continuum magnons



Spin-lattice coupling:

$$\mathcal{H}' = \sum_{\substack{\alpha,\beta \\ a,b=x,y,z}} \mathcal{E}_{\mathbf{r}}^{\alpha\beta} \left(\Lambda_{ab}^{(\mathrm{n}),\alpha\beta} \mathrm{n}_a \mathrm{n}_b + \frac{\Lambda_{ab}^{(\mathrm{m}),\alpha\beta}}{n_0^2} \mathrm{m}_a \mathrm{m}_b \right) \Bigg|_{\mathbf{x},z} \qquad |\mathbf{n}|^2 + \frac{\mathfrak{a}^4}{\mu_0^2} |\mathbf{m}|^2 = 1, \qquad \mathbf{m} \cdot \mathbf{n} = 0.$$

Solve NLSM constraints, expand around canted state

$$\boxed{\mathcal{H}' \approx \sum_{\alpha\beta} \mathcal{E}_{\mathbf{r}}^{\alpha\beta} \sum_{a,b=y,z} \sum_{\xi,\xi'=m,n} \lambda_{\xi_a,\xi'_b}^{\alpha\beta} n_0^{-\xi-\xi'} \xi_{a\mathbf{r}} \xi'_{b\mathbf{r}}}$$

n.b. $m\,m$

$n\,n$

$\boxed{m\,n}$ Effective-TRS breaking

Scaling: $\Omega \sim \omega \sim v_{\mathrm{ph}} k \sim k_B T$

- <u>$\mathscr{B}$ coefficients:</u>

recall: $Q_{n\mathbf{k}} = \frac{1}{\sqrt{N_{\mathrm{uc}}}} \sum_{\substack{\mathbf{p},\ell,\ell' \\ q_1,q_2,z}} \mathcal{B}_{\mathbf{k};\mathbf{p}}^{n,\ell_1,\ell_2|q_1 q_2 q} \, e^{ik_z z} b^{q_1}_{\ell_1,\mathbf{p}+\frac{q}{2}\mathbf{k},z} b^{q_2}_{\ell_2,-\mathbf{p}+\frac{q}{2}\mathbf{k},z}$

$$\boxed{\mathcal{B} \sim \left(\frac{k_B T}{M v_{\mathrm{ph}}^2}\right)^{\frac{1}{2}} n_0^{-1} \left(\lambda_{mm} \frac{\chi k_B T}{n_0} + \lambda_{mn} + \lambda_{nn} \frac{n_0}{\chi k_B T}\right) \sim T^{1/2+x}}$$

smallness: ions are heavy.

Antiferromagnet: order-parameter (n) has strongest correlations

- <u>Diagonal scattering rate:</u>

$$D^{(s)}_{n\mathbf{k}} = \frac{(3-s)\pi}{\hbar^2 N_{\mathrm{uc}}^{2\mathrm{d}}} \sum_{\mathbf{p}} \sum_{\ell,\ell'} \frac{\sinh(\frac{\beta}{2}\hbar\omega_{n\mathbf{k}})}{\sinh(\frac{\beta}{2}\hbar\Omega_{\ell,\mathbf{p}}) \sinh(\frac{\beta}{2}\hbar\Omega_{\ell',\mathbf{p}-\mathbf{k}})} \quad \delta(\omega_{n\mathbf{k}} - \Omega_{\ell,\mathbf{p}} - s\Omega_{\ell',\mathbf{p}-\mathbf{k}}) \left|\mathcal{B}^{n,\ell,\ell'|+s-}_{\mathbf{k};-\mathbf{p}+\frac{\mathbf{k}}{2}}\right|^2$$

$$\frac{1}{\tau} \sim T^{d-1}|\mathcal{B}|^2 \sim T^{d+2x}$$

$\sim T^{d+2}, T^{d}, T^{d-2}$

$$\longrightarrow \quad \boxed{\kappa_L \sim T^{3-d-2x}}$$

d: dimensionality of Q correlations

Scaling: $\Omega \sim \omega \sim v_{\rm ph} k \sim k_B T$

- <u>$\mathscr{B}$ coefficients:</u>

recall: $Q_{n\mathbf{k}} = \frac{1}{\sqrt{N_{\rm uc}}} \sum_{\substack{\mathbf{p},\ell,\ell' \\ q_1,q_2,z}} \mathcal{B}^{n,\ell_1,\ell_2|q_1 q_2 q}_{\mathbf{k};\mathbf{p}} \, e^{ik_z z} b^{q_1}_{\ell_1,\mathbf{p}+\frac{q}{2}\mathbf{k},z} b^{q_2}_{\ell_2,-\mathbf{p}+\frac{q}{2}\mathbf{k},z}$

$$\boxed{\mathcal{B} \sim \left(\frac{k_B T}{M v_{\rm ph}^2}\right)^{\frac{1}{2}} n_0^{-1} \left(\lambda_{mm} \frac{\chi k_B T}{n_0} + \lambda_{mn} + \lambda_{nn} \frac{n_0}{\chi k_B T}\right) \sim T^{1/2+x}}$$

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$$\boxed{\frac{1}{\tau} \sim T^{d-1} |\mathcal{B}|^2 \sim T^{d+2x} \\ \sim T^{d+2},\ T^{d},\ T^{d-2}}$$

Spin–phonon interactions in a Heisenberg antiferromagnet: II. The phonon spectrum and spin–lattice relaxation rate

d=3

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Received 11 March 1974

$$\frac{1}{\tau_{\rm SL}} \simeq \frac{b_1 S^2 (r^2-1)}{D^{10}} \left(\frac{5T_{\rm D}^3}{12\pi^4} + \frac{\pi^2 D^3}{24V}\right) Q_0^2 T^5$$

Scaling: Hall

From the formula:

$$\mathfrak{W}^{\ominus} \sim T^{d-3}\mathcal{B}^4$$

Effective-TRS breaking: one factor of m-n coupling:

$$\boxed{\mathfrak{W}^{\ominus} \sim T^{d-1}\lambda_{mn}\left(\lambda_{mm}T + \lambda_{nn}T^{-1}\right)^3 \sim T^{d-1+3x}}$$

This gives Hall resistivity:

$$\boxed{\varrho_H \sim \mathfrak{W}^{\ominus,\mathrm{eff}} \sim T^{d-1+3x}}$$

Check: numerical calculation

Many parameters: loosely inspired by Copper Deuteroformate Tetradeuterate (CFTD)

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Spin Dynamics of the 2D Spin $\frac{1}{2}$ Quantum Antiferromagnet Copper Deuteroformate Tetradeuterate (CFTD)

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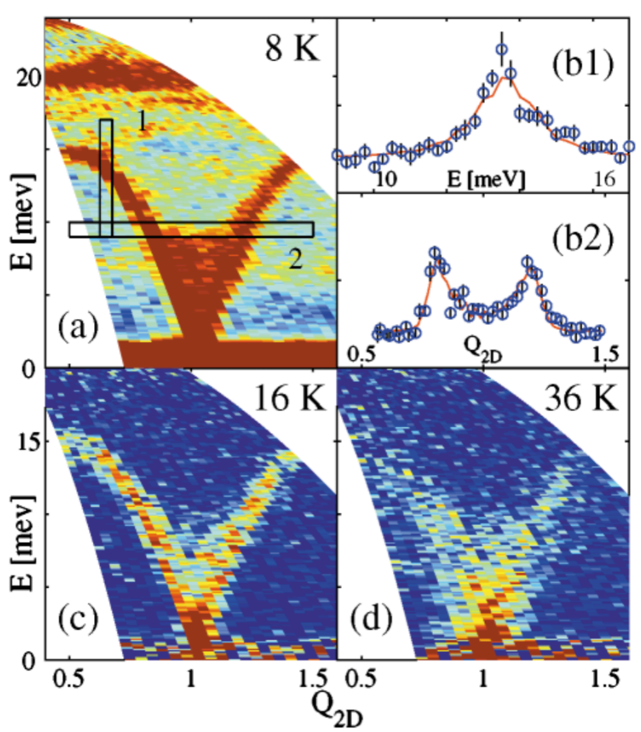


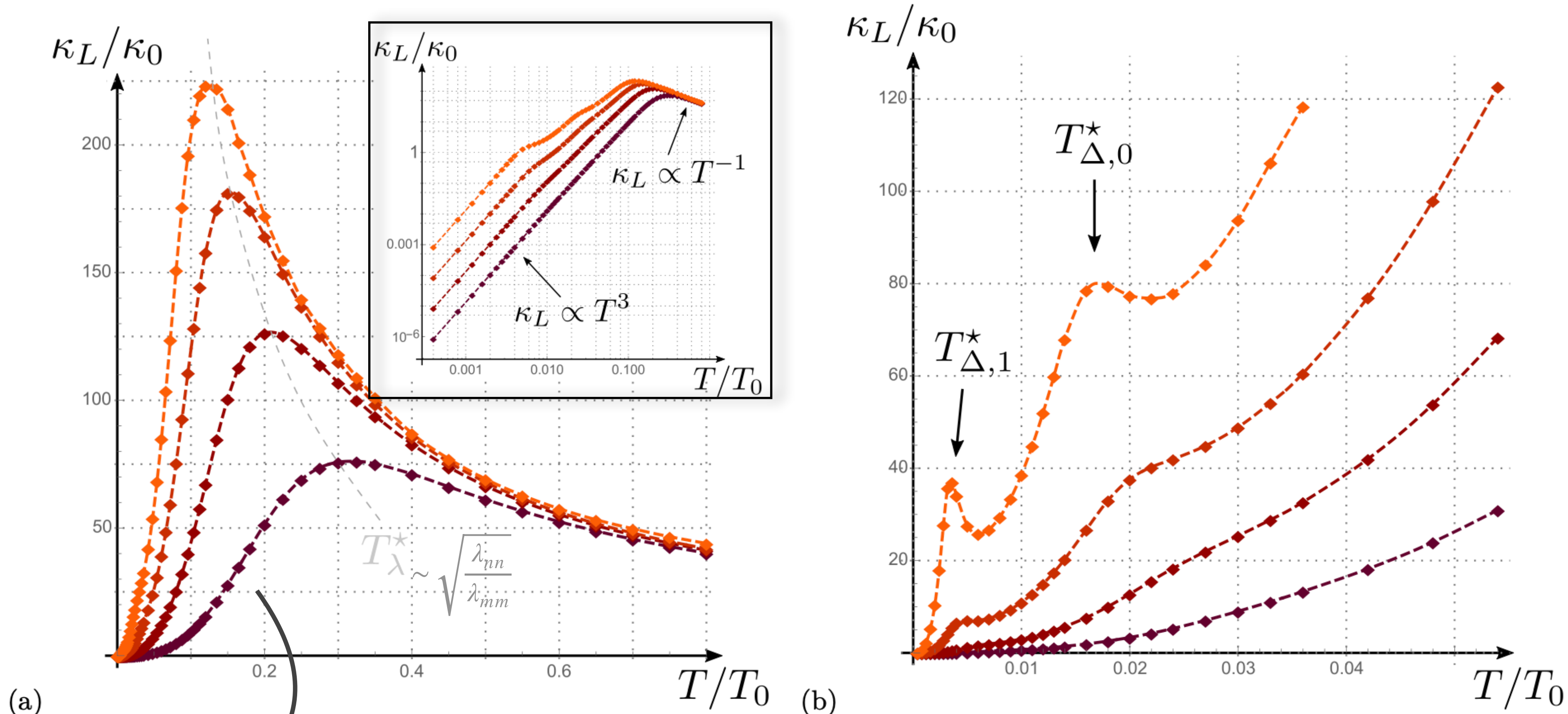
Good match of magnon and phonon phase space

$\frac{v_{\rm m}}{v_{\rm ph}}$	$\chi\epsilon_0\mathfrak{a}^2$	n_0	$\frac{M_{\rm uc}v_{\rm ph}\mathfrak{a}}{\hbar}$	m_0^x	m_0^y	m_0^z	$\frac{\Delta_0}{\epsilon_0}$	$\frac{\Delta_1}{\epsilon_0}$
2.5	0.19	1/2	$8 \cdot 10^3$	0	0.0 0.05	0.05 0.0	0.2	0.04

ξ	$\Lambda_1^{(\xi)}$	$\Lambda_2^{(\xi)}$	$\Lambda_3^{(\xi)}$	$\Lambda_4^{(\xi)}$	$\Lambda_5^{(\xi)}$	$\Lambda_6^{(\xi)}$	$\Lambda_7^{(\xi)}$
n = 0	12.0	10.0	14.0	10.0	12.0	0.6	0.8
m = 1	−10.0	−12.0	−14.0	−12.0	−10.0	−0.8	−0.6

TABLE I: Numerical values of the fixed dimensionless parameters used in all numerical evaluations. The upper and lower entries for m_0^y and m_0^z correspond to the two cases for calculating ϱ_H^{xy} and ϱ_H^{xz}, respectively. The couplings $\Lambda_i^{(\xi)}$ are given in units of $\epsilon_0/\mathfrak{a}$.

Diagonal conductivity



$D_{tot} = D_{n\mathbf{k}} + \breve{D}$

for different extrinsic scattering strengths $\breve{D}$

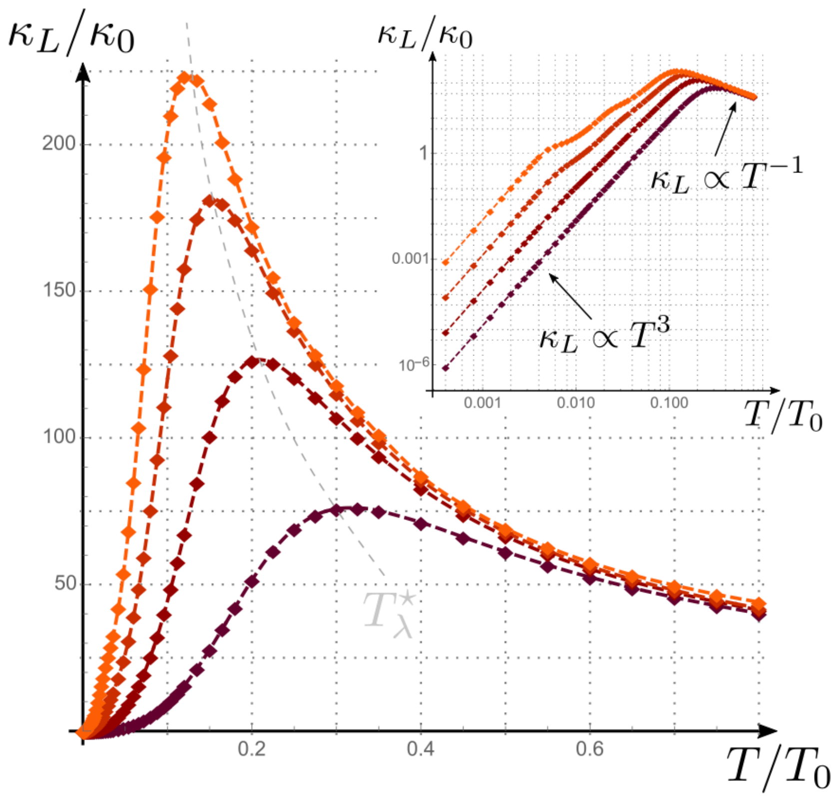
zoom near T=0

$$\boxed{\kappa_L \sim T^{3-d-2x}} \qquad \boxed{\kappa_L \sim T^{-1}}$$

$T < T^\star_\lambda$ $\qquad$ $T > T^\star_\lambda$

One can see Heisenberg regimes ($\lambda_{mm} \gg \lambda_{nn}$ — T^{-1}), anisotropic regime, extrinsic regime ($\breve{D}$ — T^3)

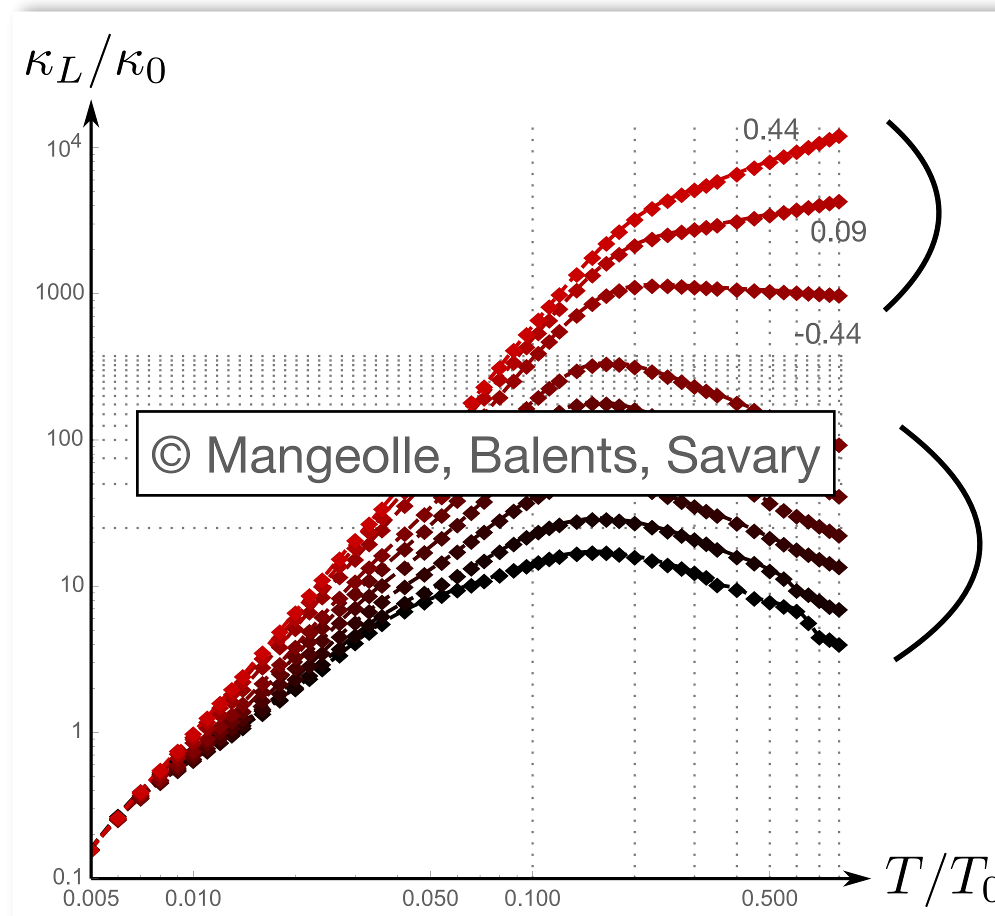
Scaling of κ_L for $T > T^\star_\lambda$ is actually very subtle



$v_m/v_{ph} = 2.5$

$$\kappa_L \underset{T > T^\star_\lambda}{\sim} T^{-1}$$

$v_m/v_{ph} = 4.125$ $v_m/v_{ph} = 3.75$ $v_m/v_{ph} = 3.325$



$\kappa_L \sim T^{3-8\left(\frac{v_m}{v_{ph}}-1\right)^{-1}}$

© Mangeolle, Balents, Savary

$\kappa_L \sim T^{-1}$

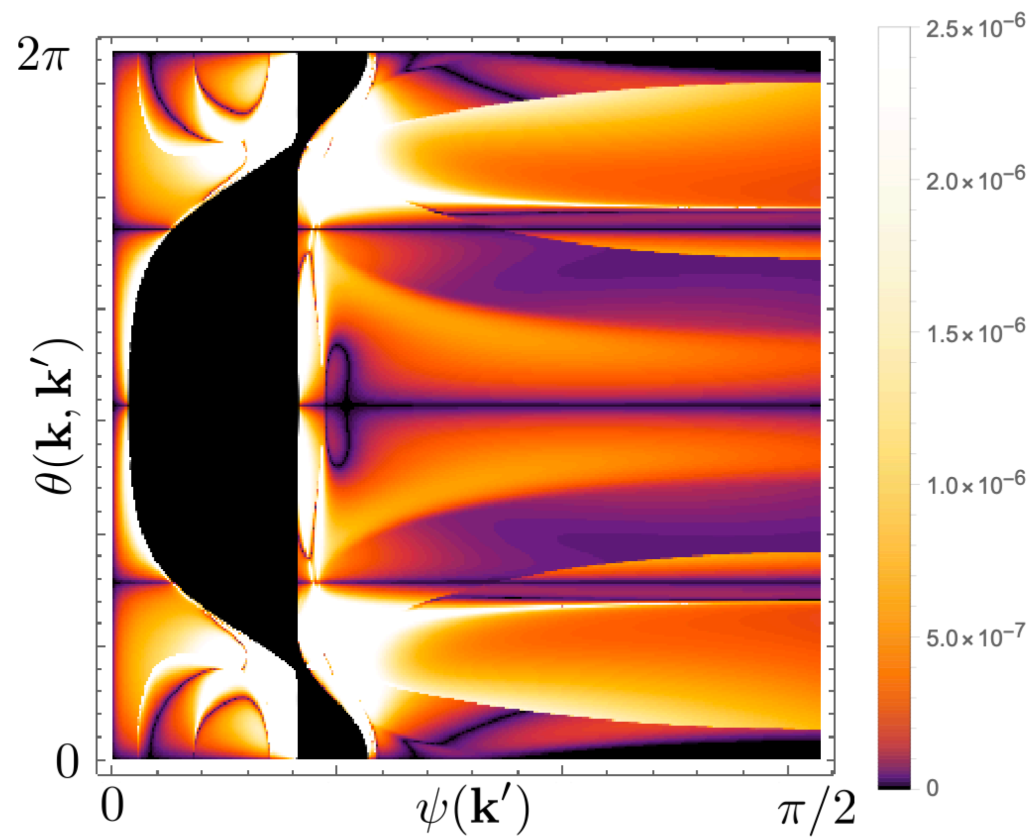
v_m/v_{ph}**-dependent power law for** $v_m/v_{ph} > 3$ **!!**

{1.5, 1.75, 2., 2.25, 2.5, 2.75, 3.325, 3.75, 4.125}

Skew scattering

Cut (fix 2 out of 6 variables) through the skew scattering rate:

$\mathfrak{W}^{\ominus}$

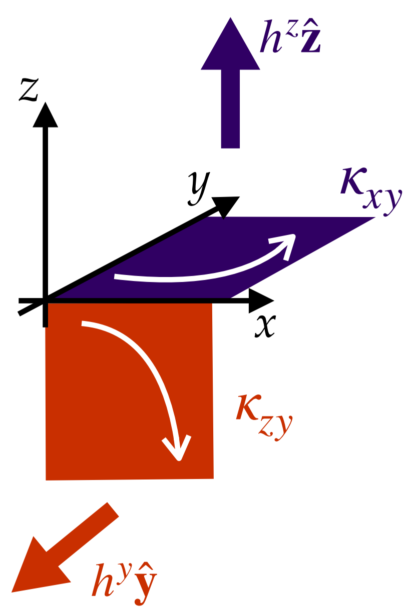


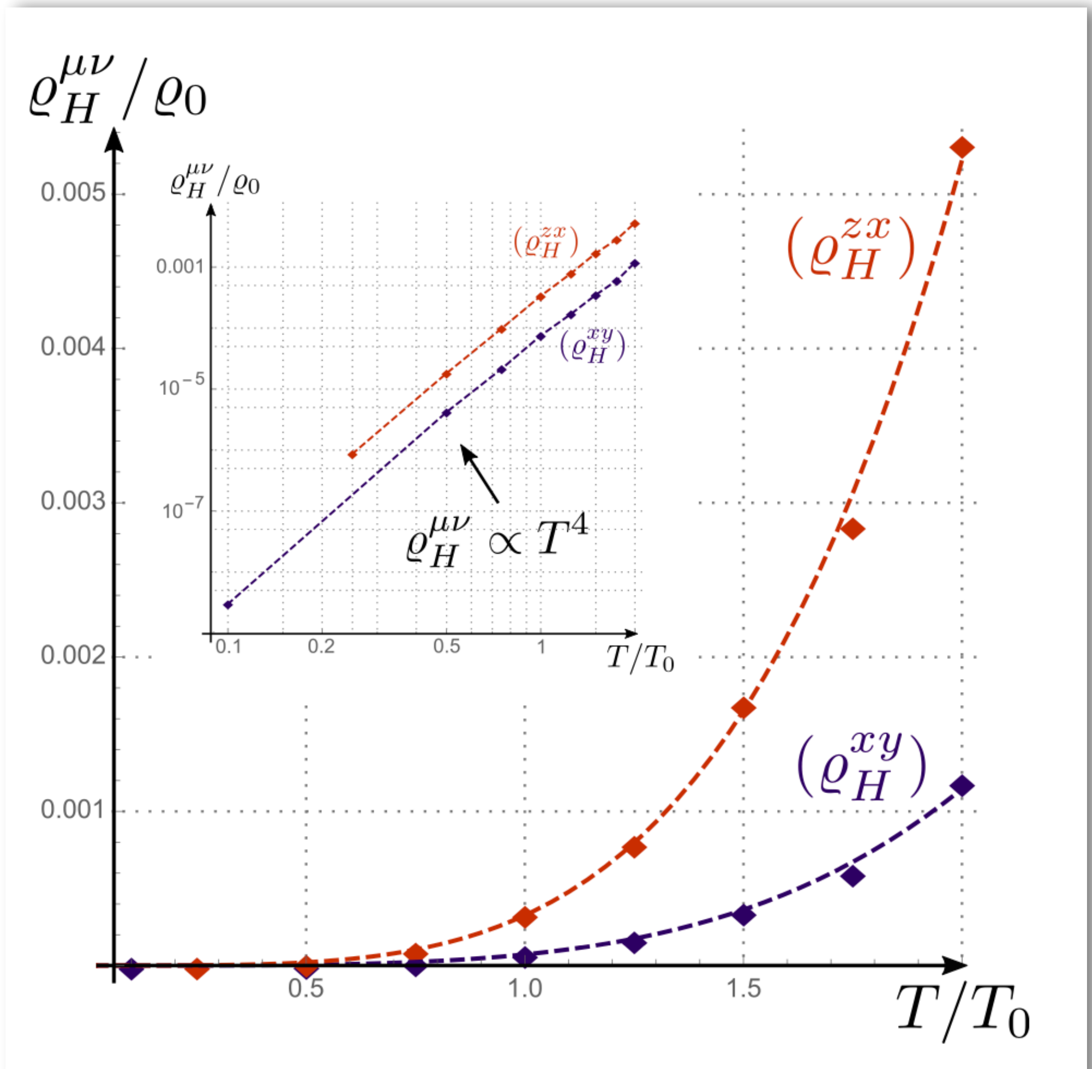
A very complex object, lots of phase space features

Thermal Hall resistivity

$$\kappa_0 = k_B v_{\text{ph}}/\mathfrak{a}^2$$

$$\varrho_0 = \kappa_0^{-1}$$





Observe T^4 behavior
(Heisenberg regime)

Larger effect with current perpendicular to plane, even though we took the magnetism *strictly* 2d (magnons do not propagate in z direction)

$$\varrho_0^{\text{CFTD}} \approx 5.88 \text{ K} \cdot \text{m} \cdot \text{W}^{-1}$$

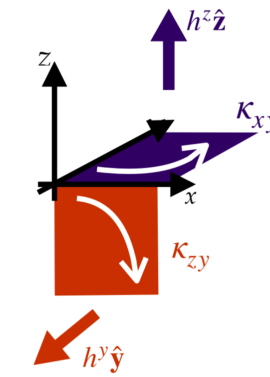
$$\varrho_0^{\text{LCO}} \approx 2.6 \text{ K} \cdot \text{m} \cdot \text{W}^{-1}$$

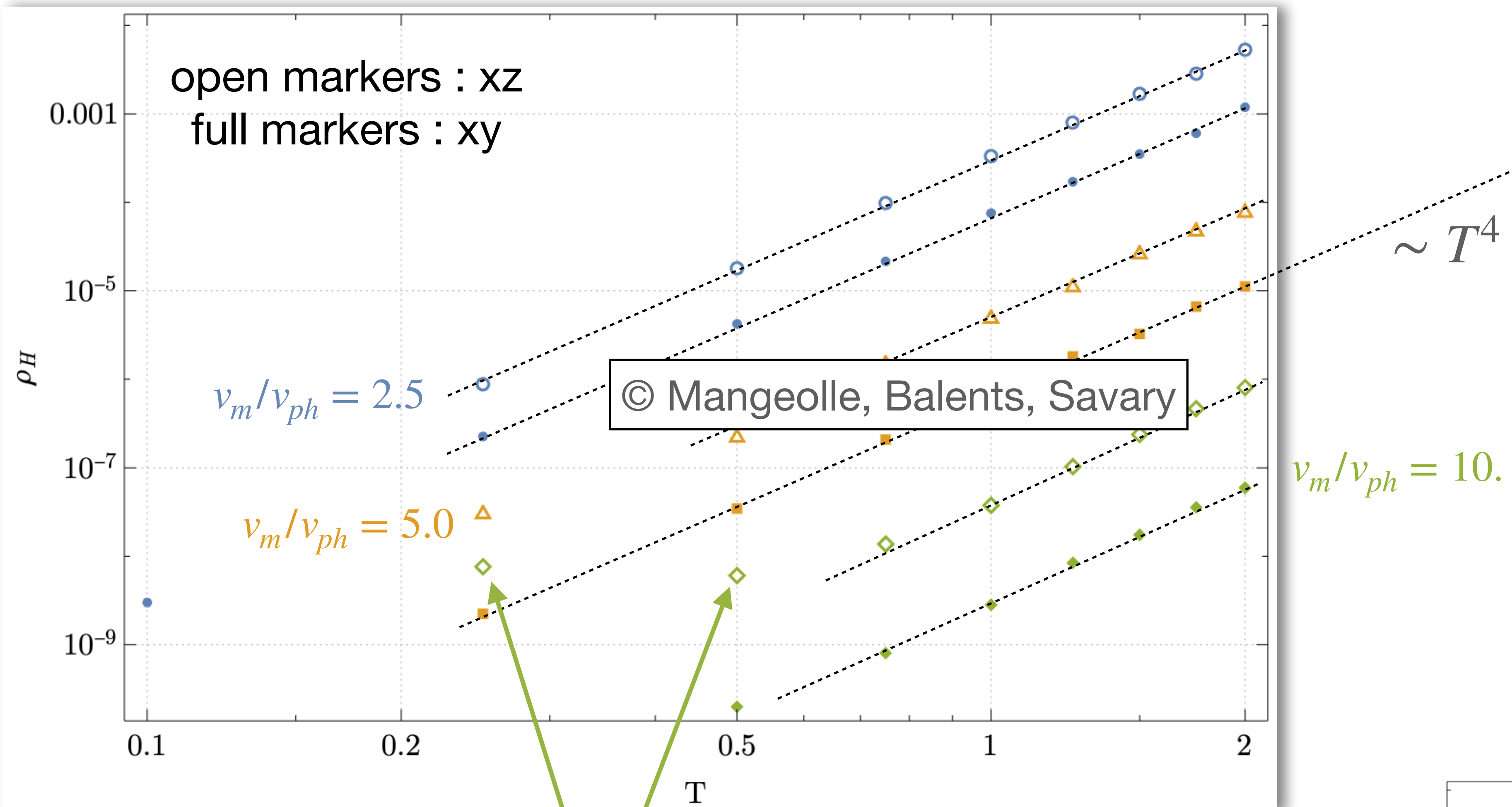
$$\kappa_{xx}^{\text{LCO}} \approx 10 \text{ W} \cdot \text{K}^{-1} \cdot \text{m}^{-1}$$

$$\kappa_{xy}^{\text{LCO}} \approx 40 \text{ mW} \cdot \text{K}^{-1} \cdot \text{m}^{-1}$$

$$(\varrho_H/\varrho_0)^{\text{LCO}} \approx 1.5 \times 10^{-4}$$

Hall resistivity ϱ_H as a function of v_m/v_{ph}





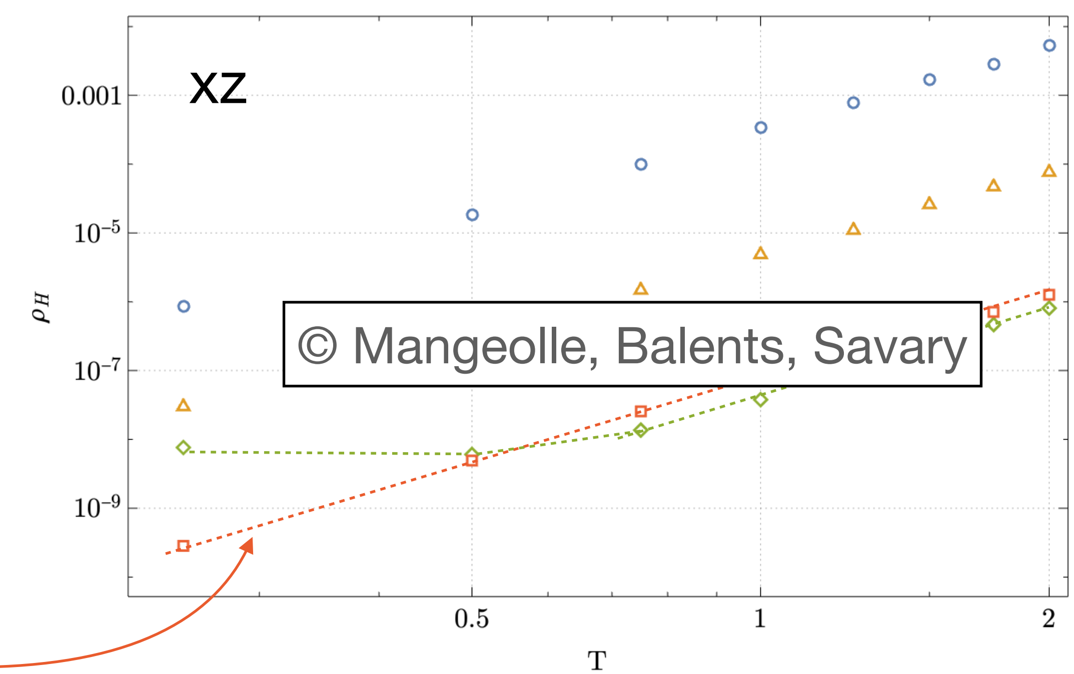
onset of mn and nn terms

results:

$\varrho_H \sim T^4$

$\varrho_H^{xy} < \varrho_H^{xz}$

$\varrho_H \searrow$ when $\dfrac{v_m}{v_{ph}} \nearrow$



~ same parameters but $\lambda_{nn} = 0$

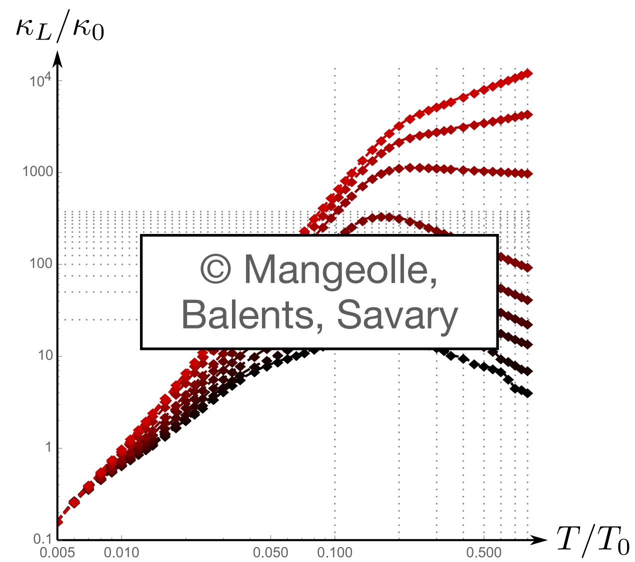
Now calculate these correlation functions for specific systems

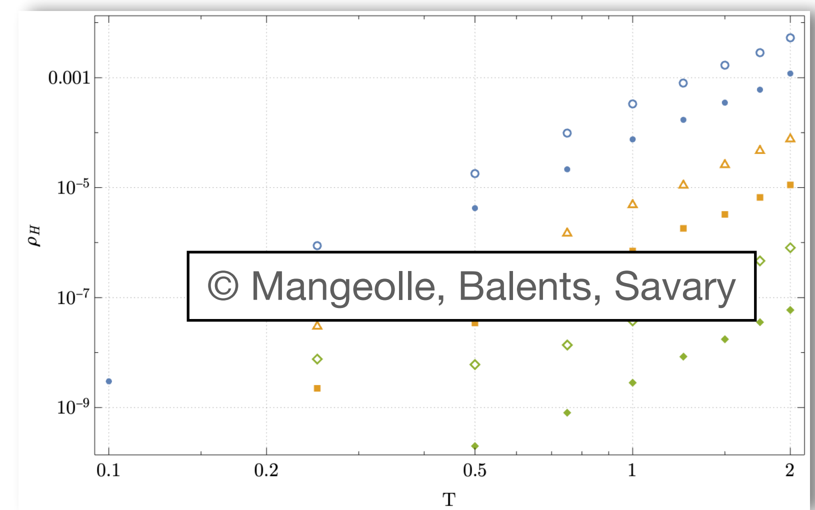
other examples: fermions —
electrons, spinons…

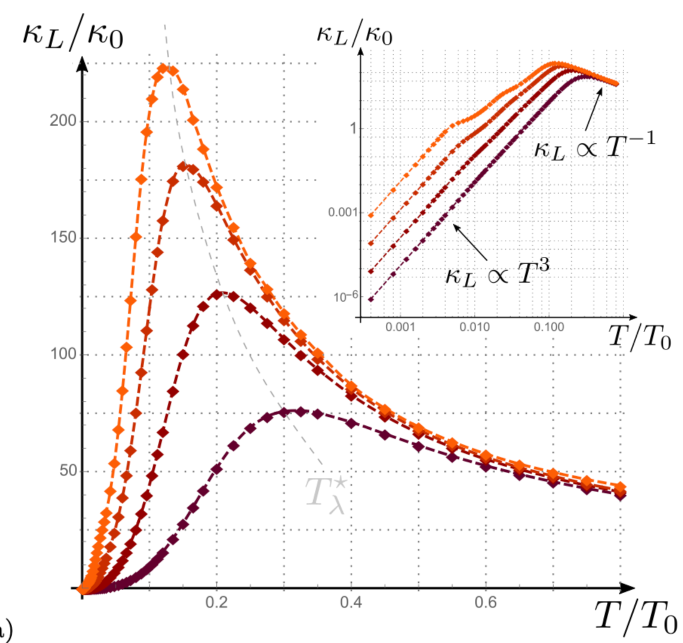
coming soon

positions (PhD and postdoc) open in the group!

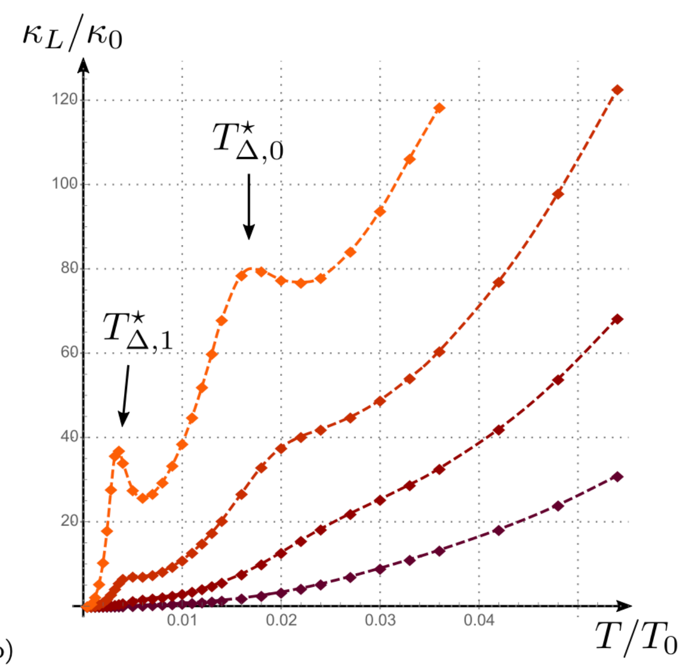
Merci !







(a)



(b)