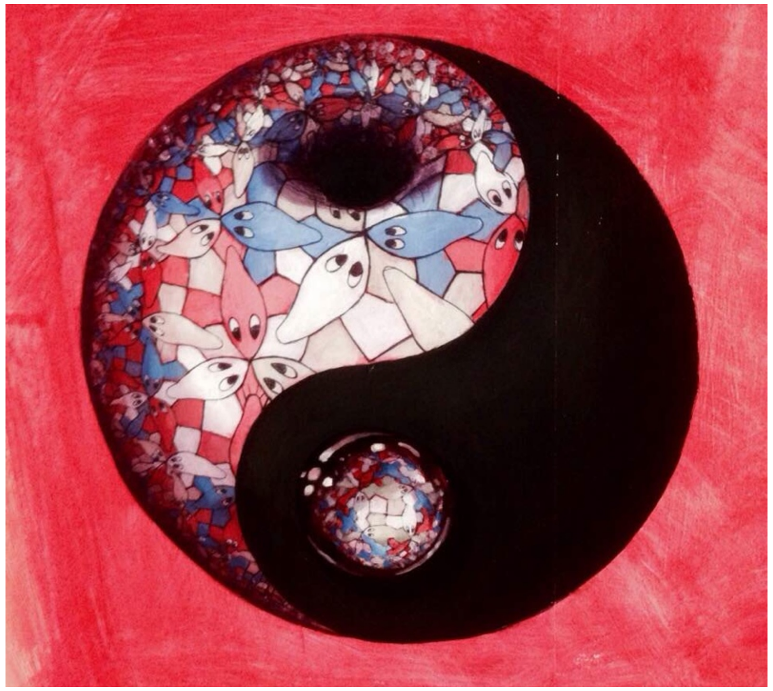
Scrambling: classical/quantum, micro/macro, early/late

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



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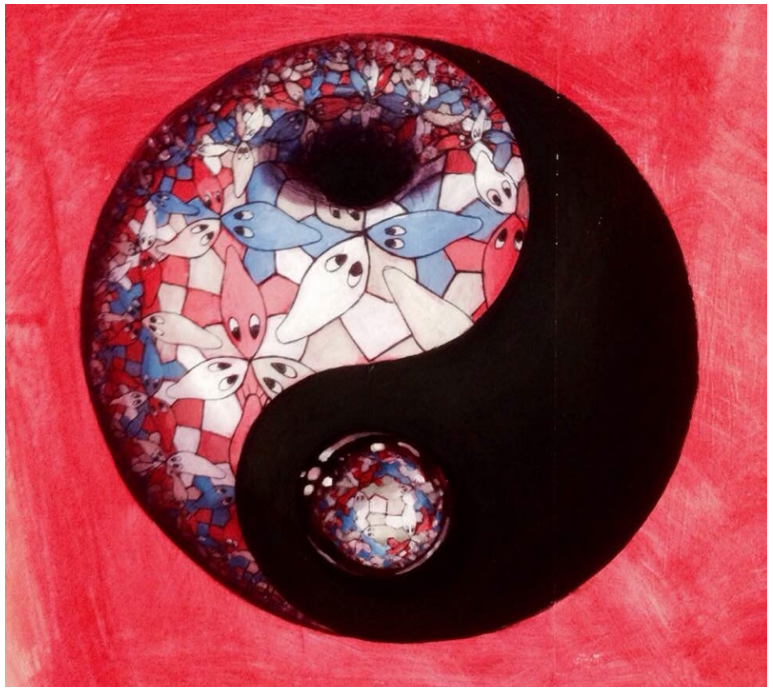

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Institute for Theoretical Physics

- From chaos in hydrodynamics to a kinetic theory for scrambling
  - Saso Grozdanov
  - Vincenzo Scopelliti

arXiv:1710.00921
arXiv:1804.09182
arXiv:22summer/PhD Thesis Scopelliti 2019

- Energy dynamics, information and heat flow and the transition from quantum to classical thermodynamics
  - Vladimir Ohanesjan
  - Zhenya Cheipesh
  - Andrei Pavlov
  - Nikolay Gnezdilov

arXiv:2011.05238
arXiv:2108.12031
arXiv:2204.12411

Chaos and hydrodynamics

- Transport from the Boltzmann equation: a dilute gas

Maxwell

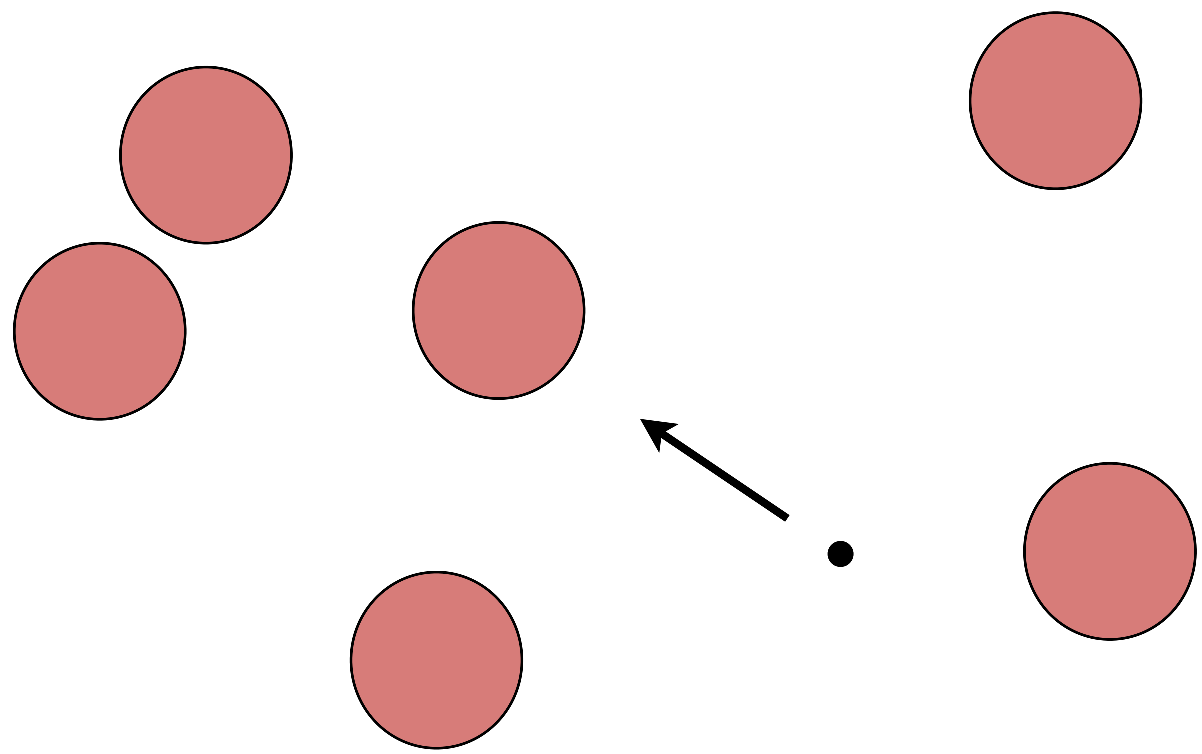
$$\eta = \frac{1}{3} m \rho \ell_{\mathrm{m.f.p.}} \sqrt{\langle v^2 \rangle}$$

- Transport from the Boltzmann equation

Maxwell

$$\eta = \frac{1}{3} m \sqrt{\langle v^2 \rangle} \frac{1}{\sigma_{2-to-2}}$$

$$\sigma_{\text{2-to-2}} = \int_{\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4} \sigma(\mathbf{p},\mathbf{p}_2|\mathbf{p_3},\mathbf{p_4})$$



Boltzmann is based on successive 2-2 collisions
This microscopic picture is *also* what encodes chaotic trajectories

- A very special feature of dilute gases

Maxwell

$$\eta = \frac{1}{3} m \sqrt{\langle v^2 \rangle} \frac{1}{\sigma_{2-to-2}}$$

van Zon, van Beijeren, Dellago

$$\lambda = \frac{1}{\tau_{\text{ave}}} \langle \frac{1}{2} \ln(\Delta \vec{v})^2 \rangle \simeq \frac{\sqrt{\langle v_{\text{rel}}^2 \rangle}}{\ell_{\text{m.f.p.}}} \simeq \rho \sqrt{\langle v^2 \rangle} \sigma_{\text{2-to-2}}$$

- Transport follows from the Boltzmann equation

$$\frac{d}{dt} f(\mathbf{p}, t) = \int_{\mathbf{k}} (R^{in}(\mathbf{p}, \mathbf{k}) - R^{out}(\mathbf{p}, \mathbf{k})) f(\mathbf{k}, t)$$

- A very special feature of dilute gases

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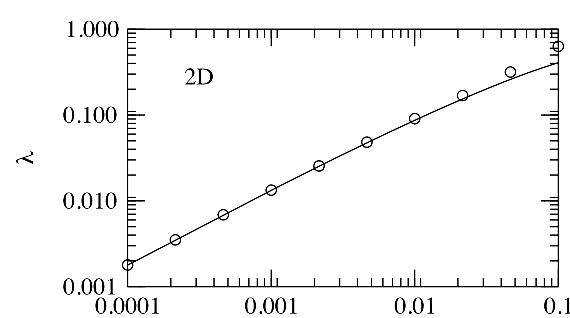
- Can we understand chaos from a kinetic-like equation?

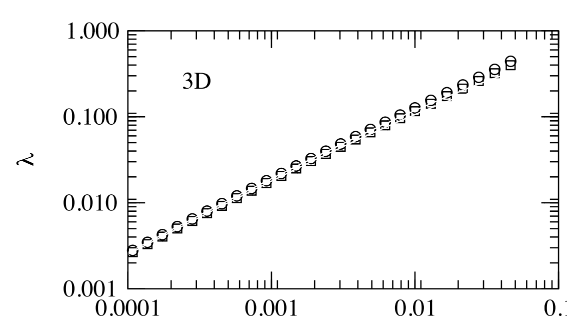
Ad hoc: clock equation

$$\frac{d}{dt} f_k = -f_k + f_{k-1}^2 + 2 f_{k-1} \sum_{\ell=0}^{k-2} f_\ell$$

van Zon, van Beijeren, Dorfman; Saarloos



f_k the fraction of particles which have experienced k collisions

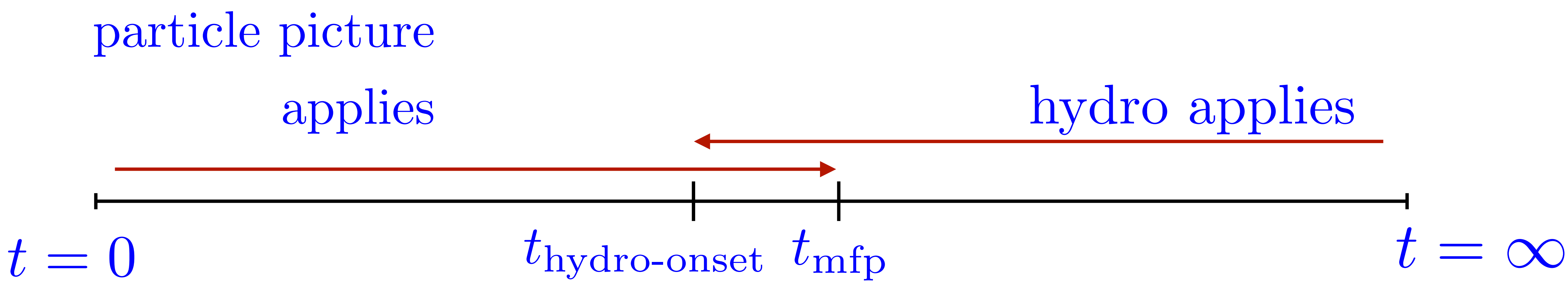


- Scrambling rate/Chaos is a microscopic “particle” property

- Transport diffusion is a macroscopic collective property

- A generic system

- Special case: weakly coupled dilute gas



$$\eta = \frac{1}{3} m \sqrt{\langle v^2 \rangle} \frac{1}{\sigma_{2-to-2}}$$

Implies hydro/Boltzmann/kinetic theory should also know about chaos!

$\text{scrambling=chaos=ergodicity}$ is very different from $\text{local therm.=equilibration}$

There is a connection:

In classical thermalization chaos is the source of ergodicity

In special situations (weakly coupled dilute gas) they are set by the same physics

~~Quantum~~ chaos from an out-of-time correlation function

Semi-classical

- A QFT way to detect chaos

$$C(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle$$

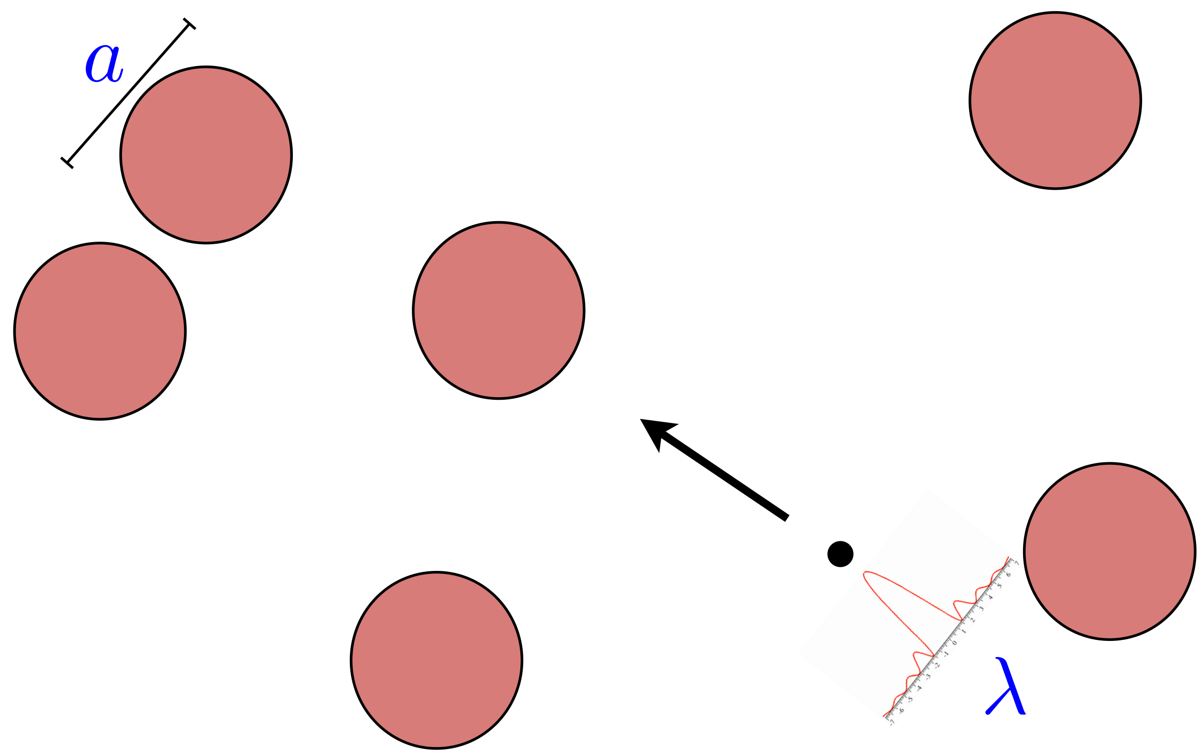
  - Choose

$$W = q(t) \quad V = p(0)$$

$$[W(t), V(0)] = [q(t), p(0)] = i\hbar\{q(t), p(0)\} = i\hbar \frac{\partial q(t)}{\partial q(0)}$$

$$\text{Chaos}: \; q(t) \sim \delta q(0) e^{\lambda_L t} \qquad C(t) \sim \hbar^2 e^{2\lambda t} \text{ with } \lambda = \lambda_{\text{Lya}}$$

- Semi-classical computation of conductivity in weak disorder

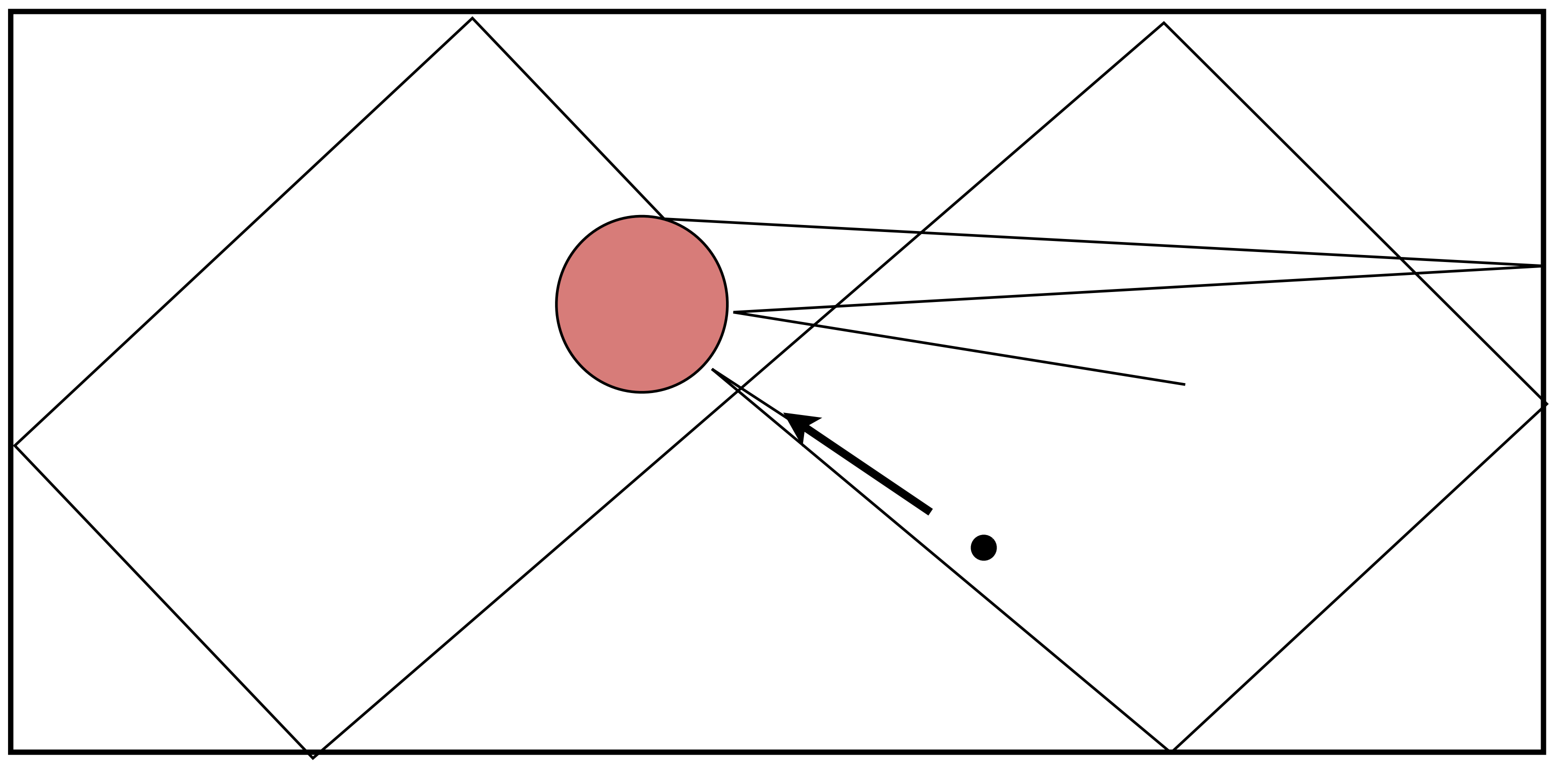


- Semiclassical regime $\lambda \ll a$

Larkin, Ovchinnikov

$$C(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle \sim \hbar^2 e^{2\lambda t}$$

- Semi-classical computation of conductivity in weak disorder

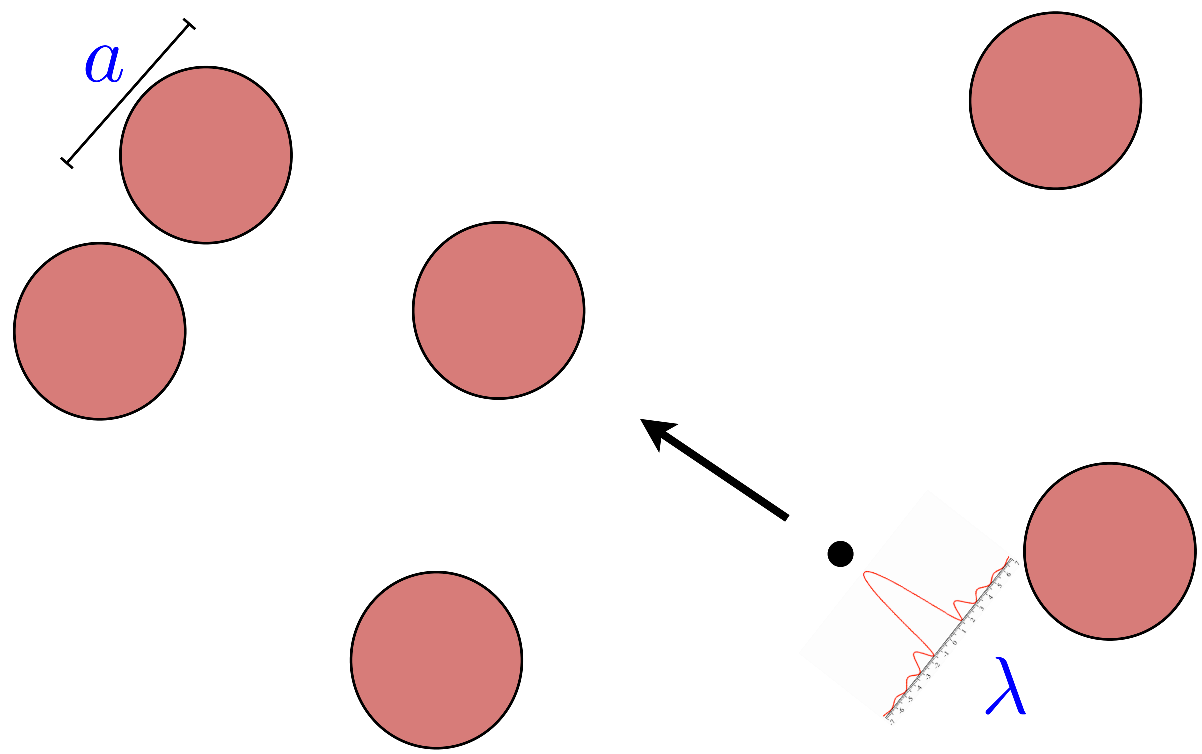


- Semiclassical regime $\lambda \ll a$ variation on Sinai billiards

Larkin, Ovchinnikov

$$C(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle \sim \hbar^2 e^{2\lambda t}$$

- Semi-classical computation of conductivity in weak disorder



- Semiclassical regime $\lambda \ll a$
- Nevertheless: quantum physics takes over when

Larkin, Ovchinnikov

$$C(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle \sim \hbar^2 e^{2\lambda t} \sim 1$$

Ehrenfest time: $t_{Ehr} = \frac{1}{\lambda} \ln \frac{1}{\hbar}$

- Careful:

  In the quantum regime chaotic behavior is hard.

  i.e. most quantum analogues of classical systems with chaos do not exhibit exponential growth in this OTOC correlator.

e.g. Grozdanov, Kukuljan, Prosen
Bertini, Kos, Prosen

  - Need a small parameter
  - In semi-classical systems $\hbar$ $C(t) \sim \hbar^2 e^{2\lambda t}$
  - In holography/in SYK: $\frac{1}{N}$ $C(t) \sim \frac{1}{N^2} e^{2\lambda t}$

    Semi-classical single-trace lumps: large N classicalization/ master field

A bound on chaos = a bound on diffusion?

- A bound on chaos

Maldacena, Shenker, Stanford

  - Related regulated function:

$$F(t) = \langle W(t) y V(0) y W(t) y V(0) y \rangle \sim 1 - e^{2\lambda t}$$

$$y^4 = \frac{e^{-\beta H}}{Z}$$

  - *Not time ordered:* but $|TFD\rangle = \sum_n e^{-\frac{\beta}{2}E}|n\rangle|n\rangle$

$$F(t) = \sum \langle TFD|(W(t)V(0) \otimes \mathbb{1})(1 \otimes W(t)V(0))|TFD\rangle$$

$$F(t) \sim \sum \langle W(t)V(0)\rangle^\dagger \langle W(t)V(0)\rangle$$

  - Analyticity in QFT demands

$$\lambda \leq 2\pi T$$

Careful: Answer depends on regulating. This one encodes chaos correctly

Romero-Bermudez, Schalm, Scopelliti

- A refined version

$$C(t,x) = -\langle [W(t,x),V(0)]^\dagger [W(t,x),V(0)]\rangle \sim \hbar^2 e^{\xi(x-v_{LR}t)}$$

gives you a "scrambling" velocity

$$\xi v_{LR} = 2\lambda$$

  - First pioneered in 1+1 dimension systems
  - Lieb-Robinson proved:

    The velocity v_{LR} is an absolute upper bound on information spreading.
  - v_{LR} acts as en emergent lightcone.
- Idea: also in other systems this butterfly/Lieb-Robinson velocity is the maximum "speed" at which information spreads

- Diffusion is characterized by a velocity

$$D \sim \frac{v^2}{T} \sim \frac{v^2}{\lambda}$$

Assumption: Planckian dissipation / Maximal Chaos

$$\tau \sim \frac{1}{T} \sim \frac{1}{\lambda}$$

- Long sought goal: a fundamental quantum bound on diffusion

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Kovtun, Son, Starinets

$$D \geq \frac{v_{inc}^2}{T} \quad \text{or} \quad D \leq \frac{v_{inc}^2}{T}$$

Hartnoll
Hartman, Hartnoll, Mahajan
Lucas,
......

- (Unstated) Hypothesis: v_{LR} provides this fundamental velocity

- Semi-classical chaos in weakly coupled systems

  "Surprisingly a relation of the form $D \sim v_{LR}^2 \tau$ shows up in a number of non-holographic contexts"

- Most of these are weakly coupled zero density field theory results.

  This should not be a surprise. This is the classical dilute gas computation.

- Semi-classical chaos in weakly coupled systems

"Surprisingly a relation of the form $D \sim v_{LR}^2 \tau$ shows up in a number of non-holographic contexts"

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From the point of view what you compute it is a *surprise*

Scrambling in weakly coupled QFT is classical dilute gas

- Object of interest for λ, v_{LR}

$$C(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle \quad \sim e^{2\lambda(t - \frac{x}{v_{LR}})}$$

growing mode

- Object of interest for $D = \dfrac{\eta}{\chi}$

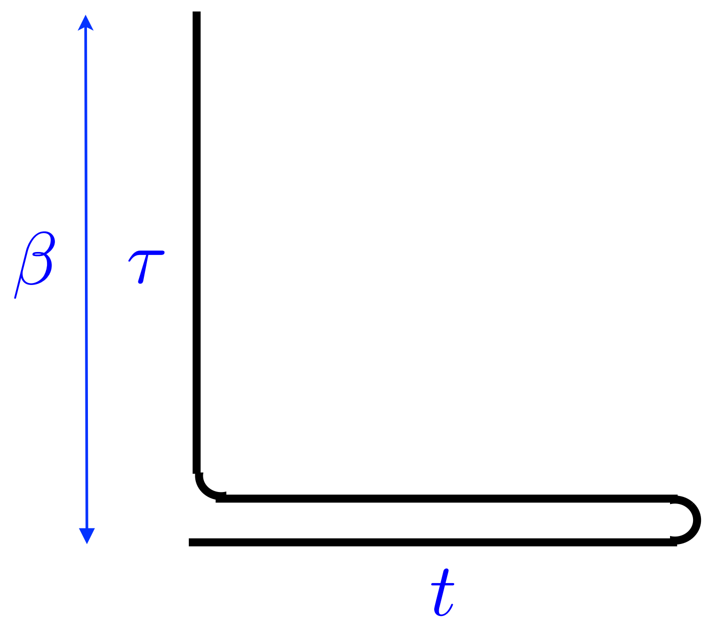
$$\eta = \lim_{\omega \to 0} \frac{1}{i\omega} \mathrm{Im} \langle T_{xy}(\omega), T_{xy}(-\omega) \rangle_R$$

Boltzmann transport only supports decaying modes:
viscosity set by smallest decay mode — relaxation time approximation

- Transport

$$G_R(t) \sim p_x p_y q_x q_y \langle [\Phi^{ab}\Phi^{ab}, \Phi^{cd}\Phi_{cd}] \rangle_\beta$$

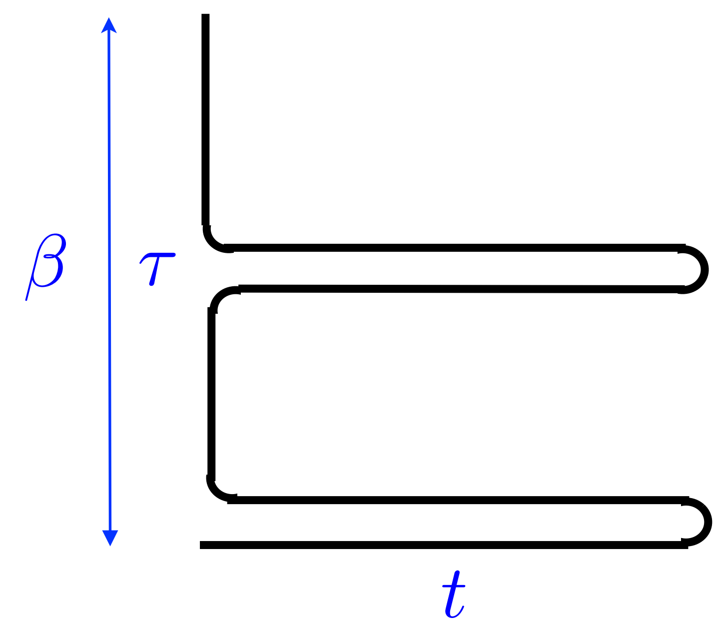
Schwinger-Keldysh contour



- Scrambling/Chaos

$$C(t) \sim \langle [\Phi^{ab}, \Phi^{cd}][\Phi_{ab}, \Phi cd] \rangle_\beta$$

OTOC contour



- Transport

$$G_R(t) \sim p_x p_y q_x q_y \langle [\Phi^{ab}\Phi^{ab}, \Phi^{cd}\Phi_{cd}] \rangle_\beta$$

Schwinger-Keldysh contour

- Scrambling/Chaos

$$C(t) \sim \langle [\Phi^{ab}, \Phi^{cd}][\Phi_{ab}, \Phi cd] \rangle_\beta$$

OTOC contour

- In free field theory

$$C(t) \sim G_R(t) = -2G_R^{\Phi\Phi}(t) + \mathcal{O}(\lambda)$$

Stanford, Jeon

- In perturbation theory Transport and Scrambling sum the same ladder diagrams

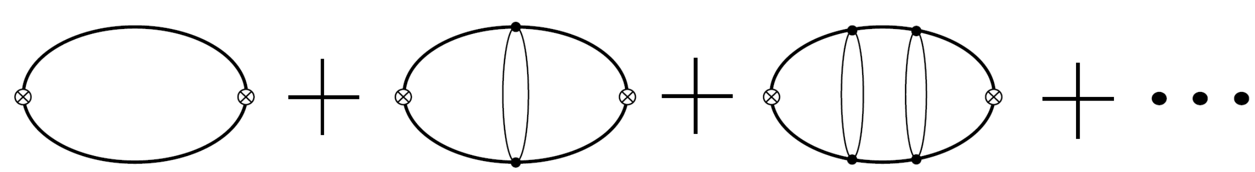
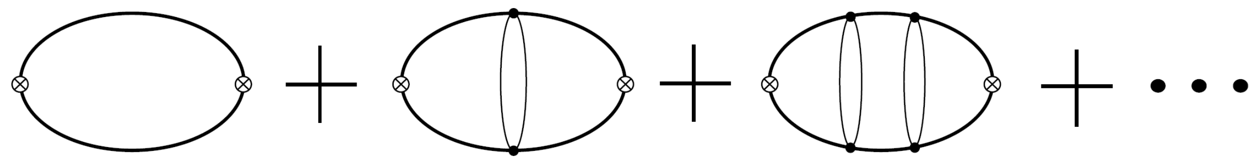


FIG. 2: Resummation of ladder diagrams. The insertions of the energy-momentum tensor operator $\hat{T}^{xy}$ is denoted by the crossed dots and black dots are the vertices with the coupling constant λ.

Schwinger Keldysh Contour

This Bethe-Salpeter eqn is the QFT version of the Boltzmann equation



$$\widetilde{G}(p|k) = \frac{\pi}{E_{\mathbf{p}}} \frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}} \left[1 + \int \frac{d^4\ell}{(2\pi)^4} R(\ell - p) \widetilde{G}(\ell|k) \right].$$

- Ansatz

$$\widetilde{G}(p|k) = \delta(p_0^2 - E_{\mathbf{p}}^2) f(\mathbf{p}|k)$$

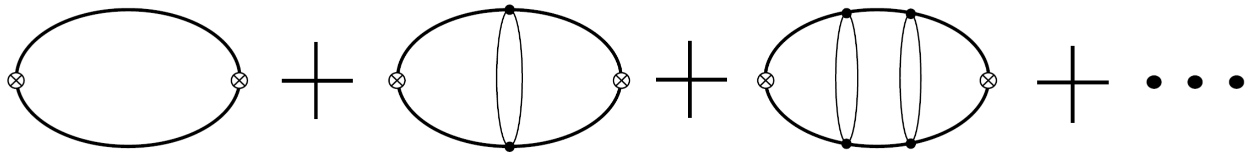
$$(-i\omega + 2\Gamma_{\mathbf{p}}) f(\mathbf{p}|k) = \frac{\pi}{E_{\mathbf{p}}} \left[1 + \int_{\mathbf{l}} (R(E_{\mathbf{l}} - E_{\mathbf{p}}, \mathbf{l} - \mathbf{p}) + R(E_{\mathbf{l}} + E_{\mathbf{p}}, \mathbf{l} - \mathbf{p})) f(\mathbf{l}|k) \right].$$

gives

$$\frac{d}{dt} f(\mathbf{p}, t) = \int_{\mathbf{k}} (R^{in}(\mathbf{p}, \mathbf{k}) - R^{out}(\mathbf{p}, \mathbf{k})) f(\mathbf{k}, t)$$

Schwinger Keldysh vs OTOC Contour

This Bethe-Salpeter eqn is the QFT version of the Boltzmann equation

- SchwKeld 

$$\widetilde{G}(p|k) = \frac{\pi}{E_{\mathbf{p}}} \frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}} \left[1 + \int \frac{d^4\ell}{(2\pi)^4} R(\ell - p)\widetilde{G}(\ell|k)\right].$$

- OTOC

$$\widetilde{\mathcal{G}}(p|k) = \frac{\pi}{E_{\mathbf{p}}} \frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}} \left[1 + \int \frac{d^4\ell}{(2\pi)^4} \frac{\sinh(\beta p^0/2)}{\sinh(\beta \ell^0/2)} R(\ell - p)\widetilde{\mathcal{G}}(\ell|k)\right].$$

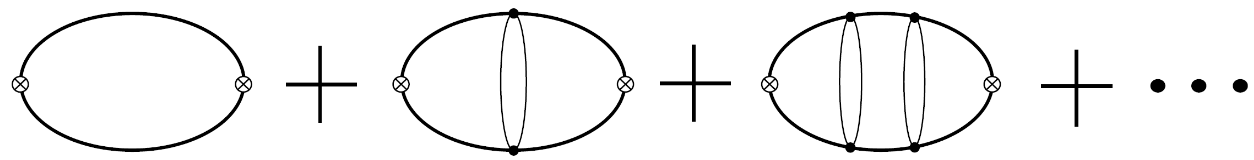
- Ansatz

$$\widetilde{\mathcal{G}}(p|k) = \delta(p_0^2 - E_{\mathbf{p}}^2)\mathsf{f}(\mathbf{p}|k)$$

$$(-i\omega + 2\Gamma_{\mathbf{p}})\mathsf{f}(\mathbf{p}|k) = \int_{\mathbf{l}} \frac{\sinh(\beta p^0/2)}{\sinh(\beta \ell^0/2)} \left(R(l_+) - R(l_-)\right) \mathsf{f}(\mathbf{k}|k)$$

Schwinger Keldysh vs OTOC Contour

This Bethe-Salpeter eqn is the QFT version of the Boltzmann equation

- SchwKeld 

$$\widetilde{G}(p|k) = \frac{\pi}{E_{\mathbf{p}}}\frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}}\left[1 + \int \frac{d^4\ell}{(2\pi)^4} R(\ell - p)\widetilde{G}(\ell|k)\right].$$

- OTOC

$$\widetilde{\mathcal{G}}(p|k) = \frac{\pi}{E_{\mathbf{p}}}\frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}}\left[1 + \int \frac{d^4\ell}{(2\pi)^4} \boxed{\frac{\sinh(\beta p^0/2)}{\sinh(\beta \ell^0/2)}} R(\ell - p)\widetilde{\mathcal{G}}(\ell|k)\right].$$

- Ansatz

$$\widetilde{\mathcal{G}}(p|k) = \delta(p_0^2 - E_{\mathbf{p}}^2)\mathsf{f}(\mathbf{p}|k)$$

$$(-i\omega + 2\Gamma_{\mathbf{p}})\mathsf{f}(\mathbf{p}|k) = \int_{\mathbf{l}} \boxed{\frac{\sinh(\beta p^0/2)}{\sinh(\beta \ell^0/2)}} \left(R(l_+) - R(l_-)\right)\mathsf{f}(\mathbf{k}|k)$$

Grozdanov, Schalm, Scopelliti,

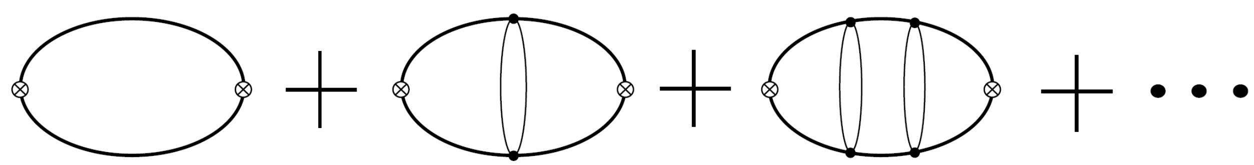
- Transport
- Scrambling/Chaos

$$G_R(t) \sim p_x p_y q_x q_y \langle [\Phi^{ab}\Phi^{ab}, \Phi^{cd}\Phi_{cd}]\rangle_\beta$$

$$C(t) \sim \langle [\Phi^{ab}, \Phi^{cd}][\Phi_{ab}, \Phi cd]\rangle_\beta$$

Schwinger-Keldysh contour

OTOC contour



Boltzmann equation (net density)

Kinetic equation (gross collisions)*

$$\frac{d}{dt} f(\mathbf{p},t) = \int_{\mathbf{k}} (R^{in}(\mathbf{p},\mathbf{k}) - R^{out}(\mathbf{p},\mathbf{k})) f(\mathbf{k},t)$$

$$\frac{d}{dt} \mathsf{f}(\mathbf{p},t) = \int_{\mathbf{k}} \frac{\epsilon(\mathbf{p})}{\epsilon(\mathbf{k})} (R^{in}(\mathbf{p},\mathbf{k}) + \widehat{R^{out}}(\mathbf{p},\mathbf{k})) \mathsf{f}(\mathbf{k})$$

purely relaxational

front propagation into unstable states

$$f(\mathbf{p},t) \sim e^{\lambda t} \text{ with } \lambda \leq 0$$

$$\mathsf{f}(\mathbf{p},t) \sim e^{\lambda t} \text{ with } \lambda \leq \lambda_{max} > 0$$

Saarloos, vBeijeren,
Aleiner, Faoro, Ioffe
Gu, Kitaev

$$* : \widehat{R^{out}}(\mathbf{p},\mathbf{k}) = R^{out}(\mathbf{p},\mathbf{k}) - 2\delta(\mathbf{p}-\mathbf{k})R^{out}(\mathbf{k},\mathbf{k})$$

- Chaos follows from kinetic equation for *gross* energy exchange

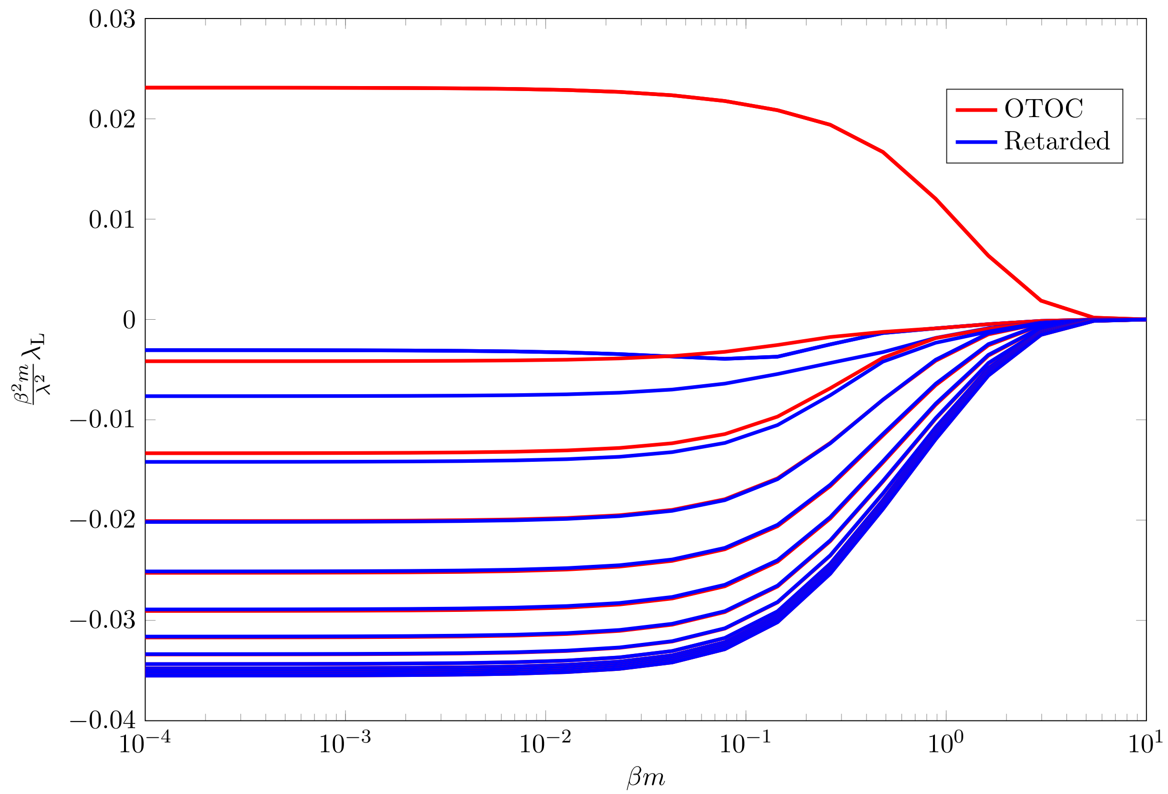
$$\frac{d}{dt}\mathsf{f}(\mathbf{p},t) = \int_{\mathbf{k}} \frac{\epsilon(\mathbf{p})}{\epsilon(\mathbf{k})} \left(R^{in}(\mathbf{p},\mathbf{k}) + R^{out}(\mathbf{p},\mathbf{k}) - 2\delta(\mathbf{p}-\mathbf{k})R^{out}(\mathbf{k},\mathbf{k})\right) \mathsf{f}(\mathbf{k})$$

  - This is derived as opposed to ad hoc clock model

$$\frac{d}{dt}f_k = -f_k + f_{k-1}^2 + 2f_{k-1}\sum_{\ell=0}^{k-2} f_\ell$$

Qualitatively physics is similar (unstable front dynamics) blue: eigenvalues λ for SchwKeld/Boltzmann

red: eigenvalues λ for OTOC/Energy-exchange



- This explicitly shows in weakly coupled dilute QFT scrambling and diffusion are set by the same dynamics --- even though they are not identical.

$$\eta = \frac{1}{3} m \sqrt{\langle v^2 \rangle} \frac{1}{\sigma_{2-to-2}}$$

$$\lambda = \frac{1}{\tau_{\rm ave}} \langle \frac{1}{2} \ln(\Delta \vec{v})^2 \rangle \simeq \frac{\sqrt{\langle v^2_{\rm rel} \rangle}}{\ell_{\rm m.f.p.}} \simeq \rho \sqrt{\langle v^2 \rangle} \sigma_{\text{2-to-2}}$$

- Chaos follows from kinetic equation for *gross* (energy) exchange

$$\frac{d}{dt}\mathsf{f}(\mathbf{p},t) = \int_{\mathbf{k}} \frac{\epsilon(\mathbf{p})}{\epsilon(\mathbf{k})} \left(R^{in}(\mathbf{p},\mathbf{k}) + R^{out}(\mathbf{p},\mathbf{k}) - 2\delta(\mathbf{p}-\mathbf{k})R^{out}(\mathbf{k},\mathbf{k})\right)\mathsf{f}(\mathbf{k})$$

- We have now shown that this holds in general:
  - For bosonic and fermionic systems (Gross-Neveu model)
  - Models near a QCP approached from perturbative regime (Wilson-Fisher $O(N)$ model)
  - Shorter derivation using 2PI formalism
- In all cases *off-shell* Bethe-Salpeter contains both chaos and Boltzmann transport.
  - One solution ansatz: Boltzmann. Complement: Chaos
  - pQFT analogue of Maxwell relation: weakly coupled dilute gas.
  - Pole-skipping….

Grozdanov, Schalm, Scopelliti,
arXiv:22summer.xxxx

Ultra strongly correlated systems are similar to dilute gases

- Semi-classical chaos in weakly coupled systems

“Surprisingly a relation of the form $D \sim v_{LR}^2 \tau$ shows up in a number of non-holographic contexts”

- Most of these are weakly coupled zero density field theory results.

  This should not be a surprise. This is the classical dilute gas computation.

- Is scrambling rate related to diffusion?

$$D \sim \frac{v^2}{T} \sim \frac{v_{\mathrm{LR}}^2}{\lambda}$$

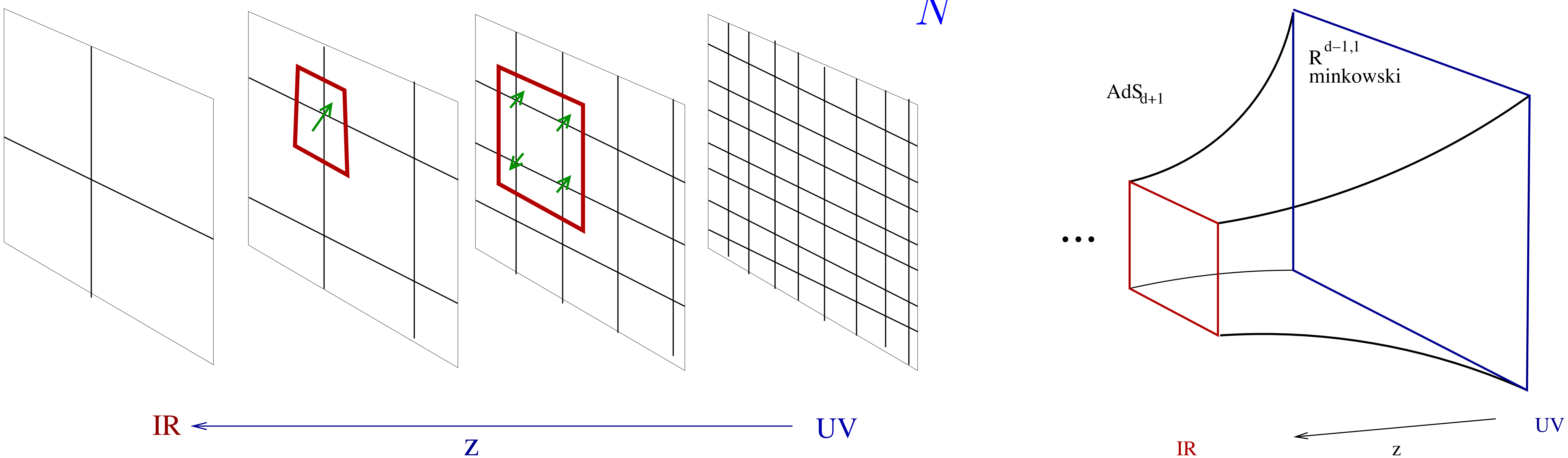
Assumption:
Planckian
dissipation
/
Maximal
Chaos

$$\tau \sim \frac{1}{T} \sim \frac{1}{\lambda}$$

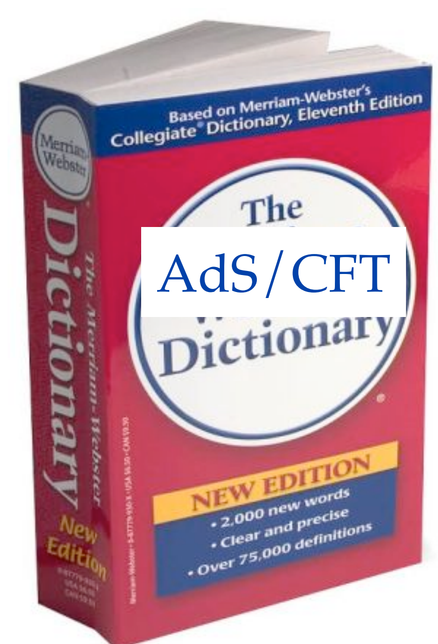
Holography for Strongly coupled systems

works best when d.o.f. are matrices Φ_{ij} $i,j = 1 \ldots N$ with $N \gg 1$

semi-classical limit $\frac{1}{N} \to 0$



$$Z_{CFT}(J) = \exp i S_{AdS}^{on-shell}(\phi(\phi_{\partial AdS} = J))$$



Quantum numbers
Finite Temp
Finite Density
Conserved Current
Energy dynamics

Quantum numbers
AdS Black hole
Extremal AdS black hole
Gauge field
Gravity dynamics

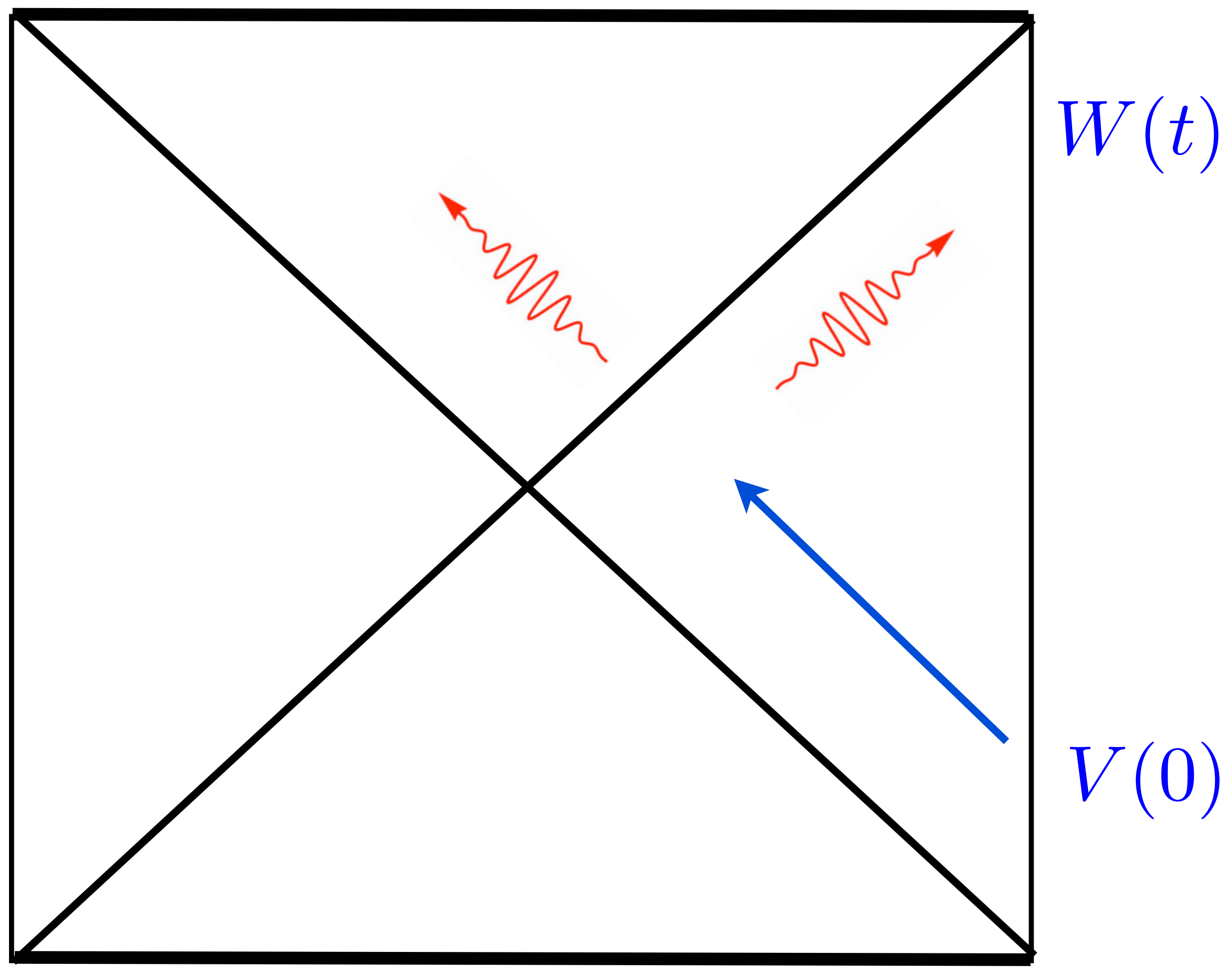
- Is scrambling rate related to diffusion?

$$D \sim \frac{v^2}{T} \sim \frac{v_{\rm LR}^2}{\lambda}$$

Roberts, Stanford, Susskind

- Gravitational shockwave calculation in AdS BH computes OTOC

$$F(t) = \sum \langle TFD|(W(t)V(0)\otimes \mathbb{1})(1\otimes W(t)V(0))|TFD\rangle$$

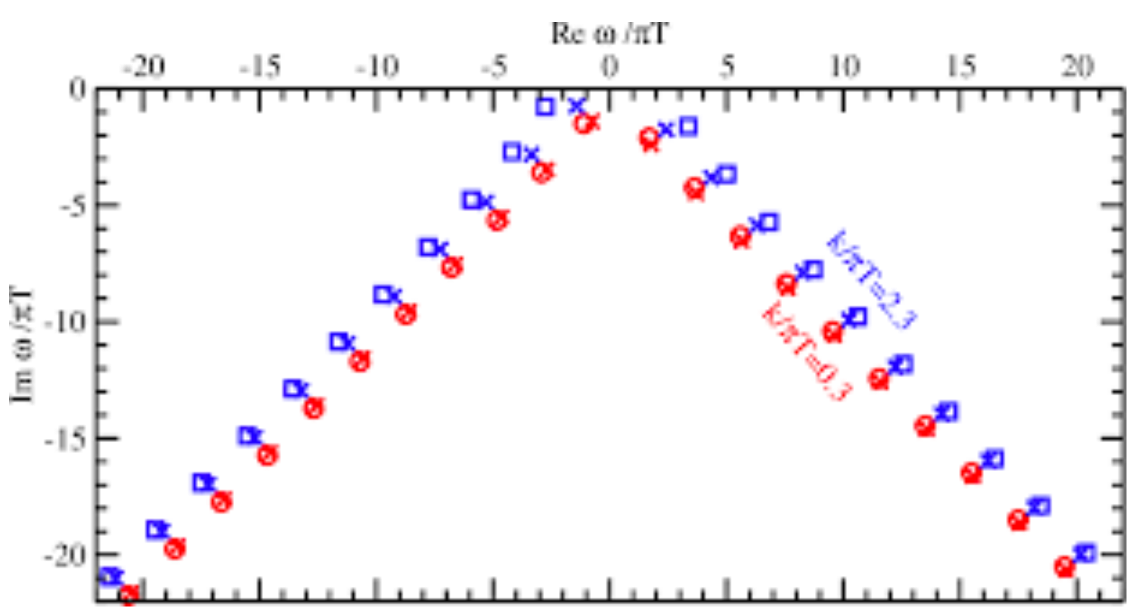


- Energy-Momentum transport is computed from the spectrum of linearized metric perturbations around the black hole (quasi-normal-modes).

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Black Hole}} + h_{\mu\nu}$$

$$\text{Obtain and Solve } \Box h_{\mu\nu} = 0$$



- Is scrambling rate related to diffusion?

Blake;
Davison, Fu, Georges, Gu, Jensen, Sachdev.

For “relevant diffusion” (=irrelevant suscep) in holographic theories

$$D = \frac{d-\theta}{\Delta_\chi}\frac{v_{LR}^2}{2\pi T}$$

$$\Delta_\chi \equiv [\rho] - [\mu] > 0$$

..similar results for massive gravity (mean-field disorder), but fails in general

Lucas, Steinberg;
Gu, Lucas, Qi

- Refinement: charged systems with mean-field disorder
  - Thermal diffusivity set by horizon properties only

$$D_P = \eta/sT$$

Policastro, Son, Starinets

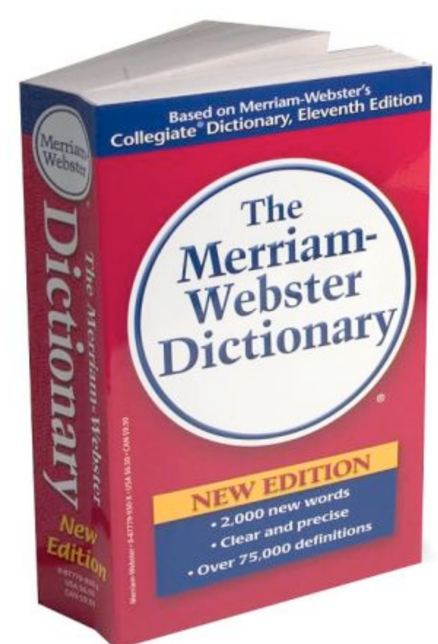
$$D_T = \frac{z}{2z-2}\frac{v_{LR}^2}{\lambda_L}$$

Blake, Davison, Sachdev

- From a physics perspective these are puzzling results:

$$Z_{CFT}(J) = \exp i S_{AdS}^{\mathrm{on-shell}}(\phi(\phi_{\partial AdS} = J))$$

Quantum numbers
Finite Temp
Finite Density
Conserved Current
Energy dynamics



Quantum numbers
AdS Black hole
Extremal AdS black hole
Gauge field
Gravity dynamics

- Shock waves are sound
  - General metric

$$ds^2_{d+2} = A(UV)dUdV + B(UV)g_{ij}dx^i dx^j - A(U,V)h(U,\vec{x})dUdU$$

  - Shock wave equation

$$\delta(U)\left(\Delta_g h - d\frac{B'}{A}h\right) = 32\pi E A \delta^d(\vec{x})\delta(U)$$

  - Sound perturbation from AdS/CFT

$$\Delta_g h(U,\vec{x}) - 2d\frac{B}{A}h(U,\vec{x}) - d\frac{B'}{A}U\frac{\partial}{\partial U}h(U,\vec{x}) = 0$$

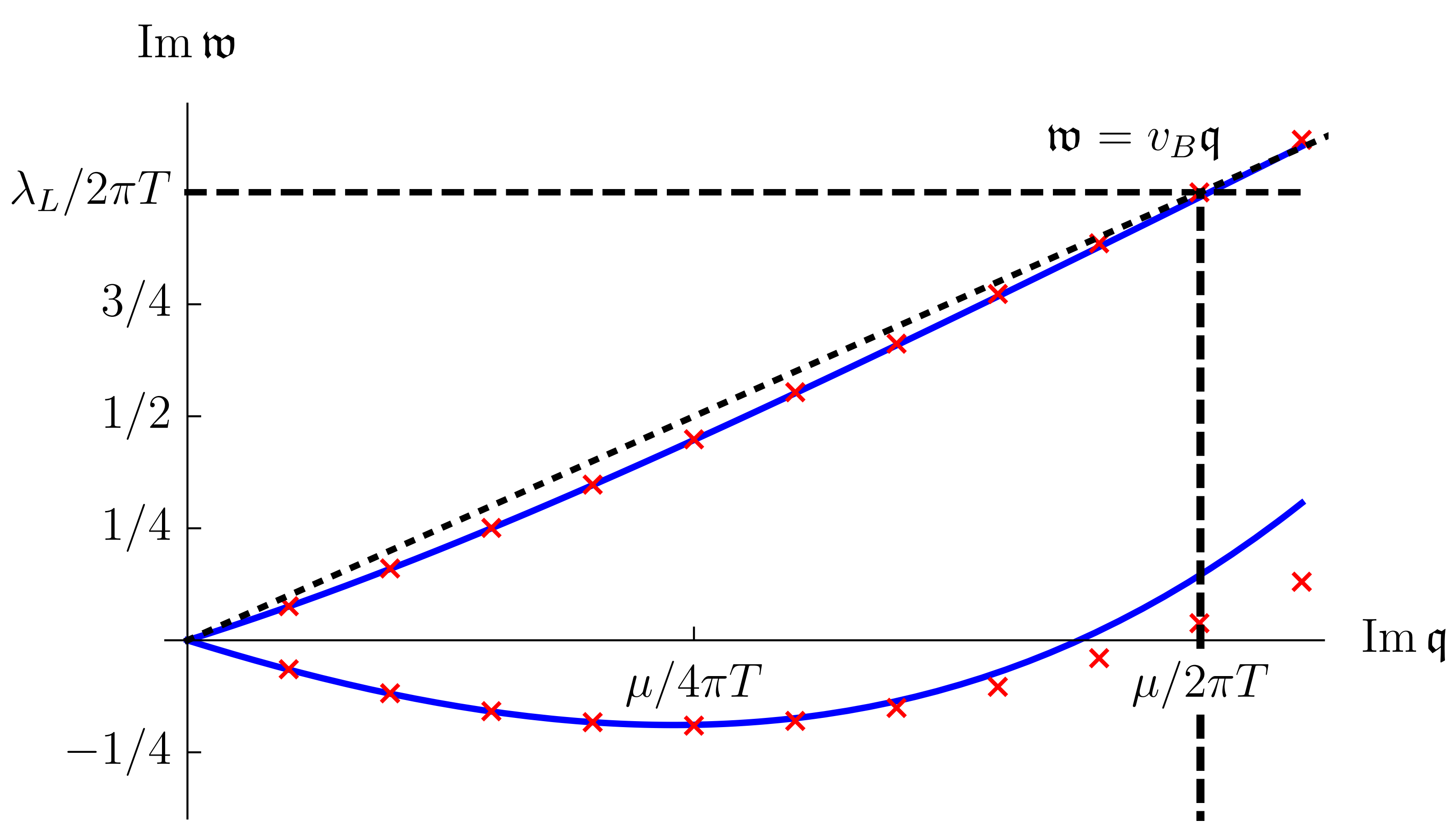
$$\text{for } h(U,\vec{x}) \sim \delta(U)h(\vec{x}) \text{ reduces to shock}$$

- OTOC Shockwave = Sound at *imaginary* values of freq. and momentum

$$\omega = 2\pi i T = i\lambda \quad , \quad k^2 = -\mu^2 = -6\pi^2 T^2 = -\frac{\lambda^2}{v_B^2}$$

- Hydrodynamical sound (known up to 3rd order analytically)

$$\omega(k) = \pm\frac{1}{\sqrt{3}}k - \frac{i}{6\pi T}k^2 + \ldots$$



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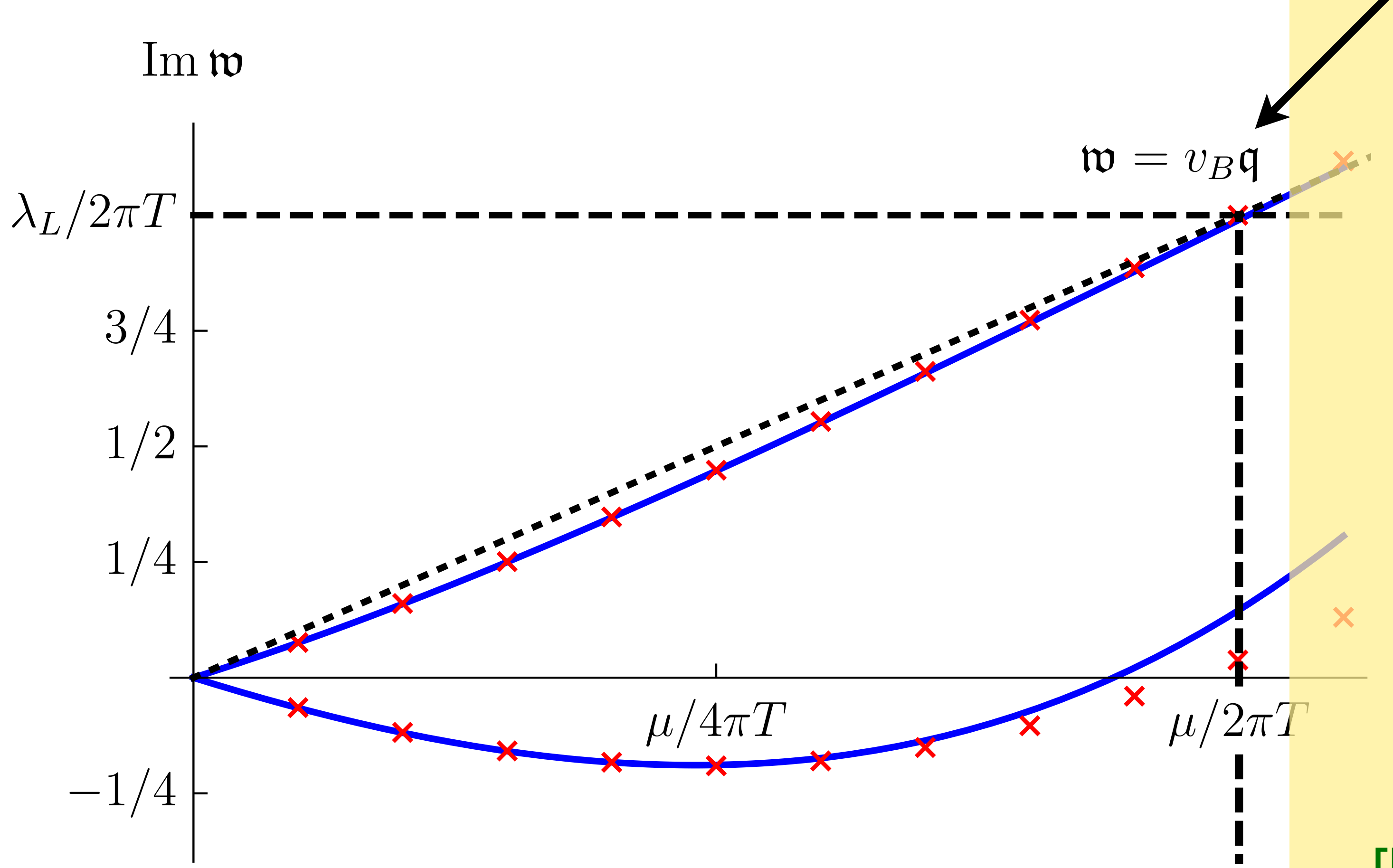


Pole-skipping:

QNM mode residue vanishes precisely at

$$\omega = 2\pi i T$$

Also happens in SYK.

[Gu, Qi, Stanford]

Direct consequence of the existence of the shockwave solution.

[Blake, Lee, Liu]

Beautiful GR story: non-unique BC at the horizon

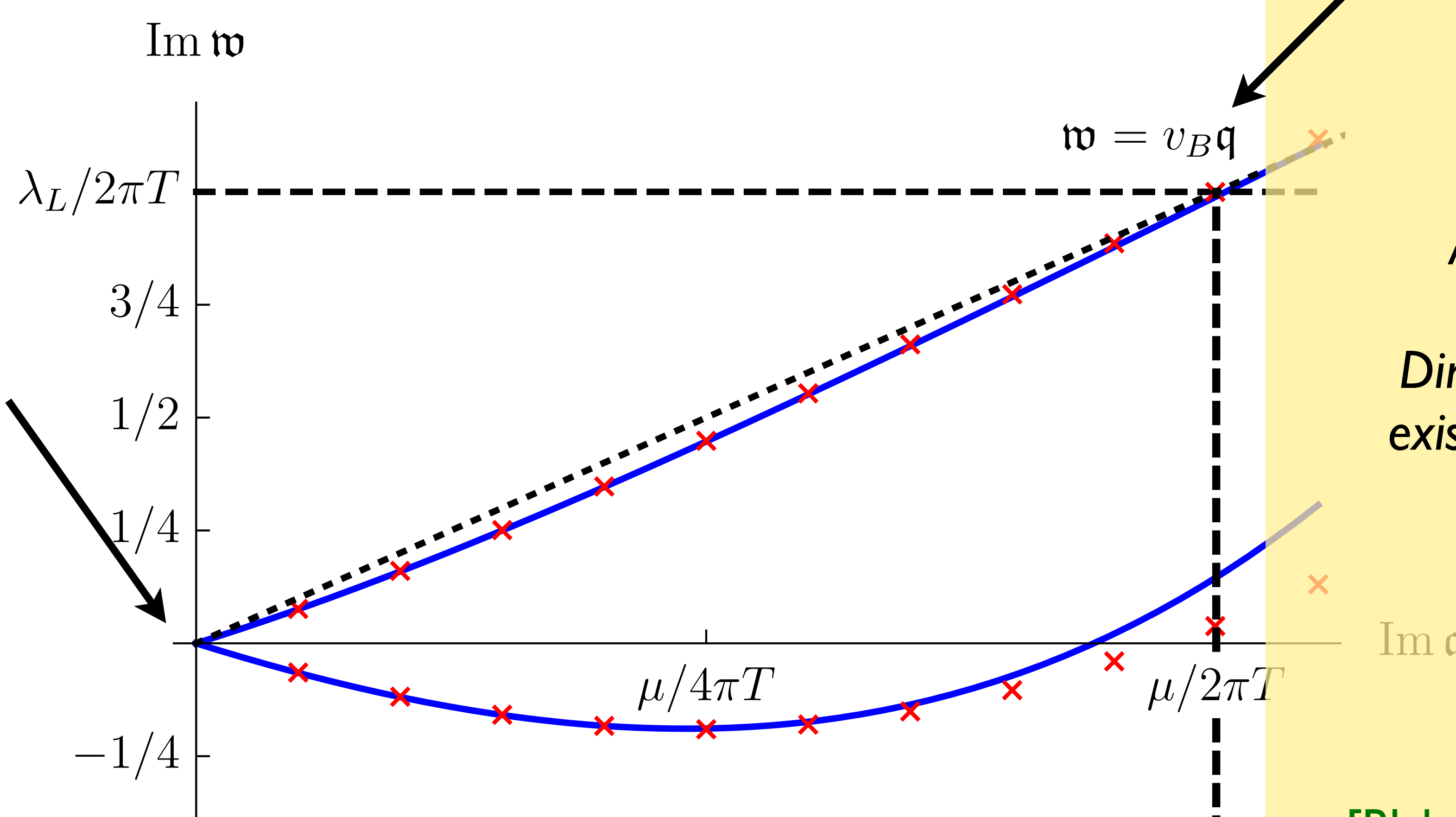
[Blake, Davison, Grozdanov, Liu]

Physical diffusion is given by the behavior near

$$\omega \ll 1$$

by now verified in many models

[Blake, Davison, Grozdanov,Liu]



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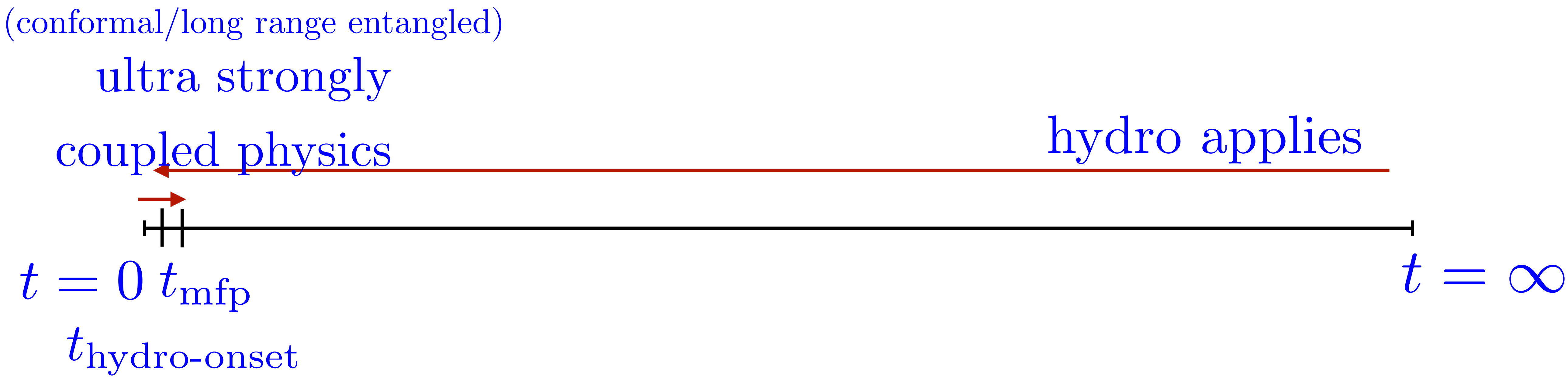
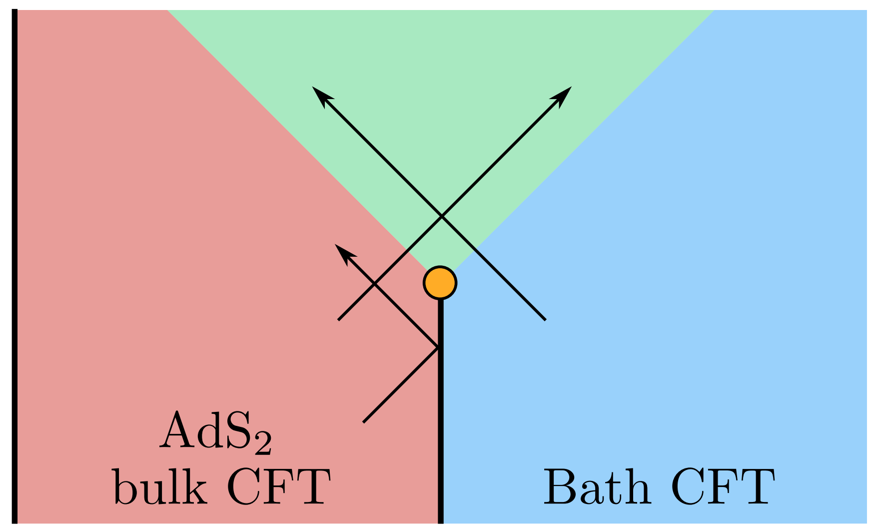
- A generic system


(conformal/long range entangled)
ultra strongly
coupled physics
hydro applies
$t = 0$
t_{mfp}
$t_{\text{hydro-onset}}$
$t = \infty$

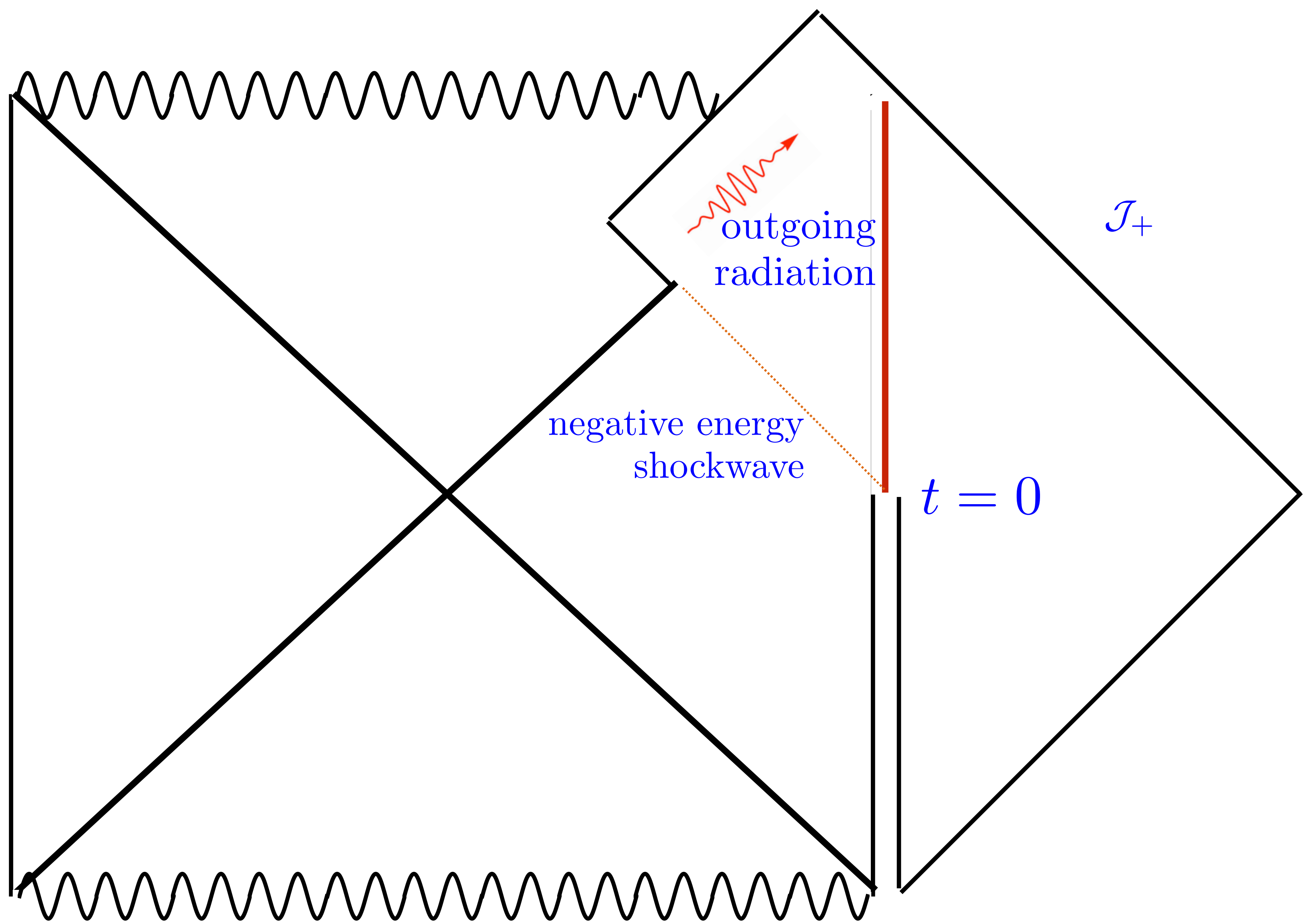
From microscopic scrambling to macroscopic scrambling

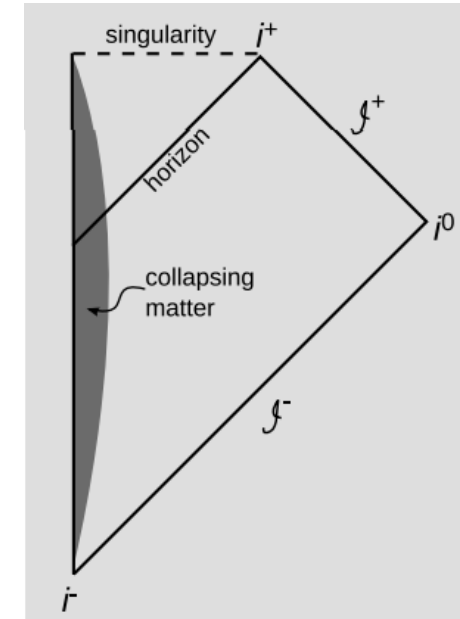
In classical physics microscopic scrambling is responsible for macroscopic ergodicity.
What about quantum physics?

- AdS-CFT: Can model black hole evaporation (quenched cooling) with conventional quantum systems.



- A black hole quantum quench





A careful computation reproduces the Page curve.

Almheiri;
Almheiri, Engelhardt, Marolf, Maxfield;
Penington

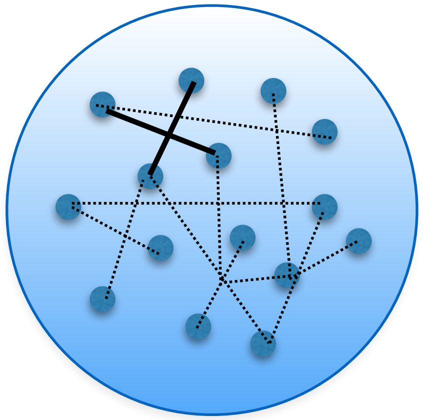
- AdS-CFT: Can model black hole evaporation (quenched cooling) with conventional quantum systems.
  - We will use two coupled SYK models:

Sachdev-Ye-Kitaev model: N complex/real fermions with $q = 2p$-point interactions

$$H = J_{i_1 i_2 \ldots i_p j_1 j_2 \ldots j_p} c^\dagger_{i_1} c^\dagger_{i_2} \ldots c^\dagger_{i_p} c_{j_1} c_{j_2} \ldots c_{j_p}$$

with random disorder averaged interactions

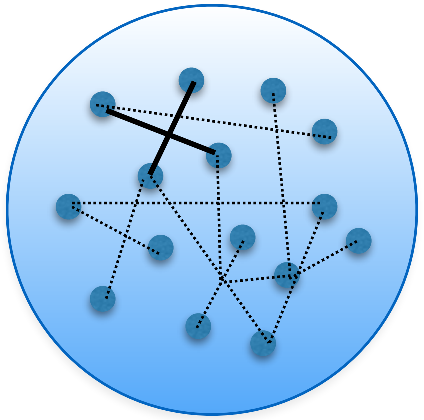
$$\langle J_{i_1 i_2 \ldots i_p j_1 j_2 \ldots j_p} J_{i'_1 i'_2 \ldots i'_p j'_1 j'_2 \ldots j'_p} \rangle = \frac{(p!)^2}{N^{2p-1}} J^2 \delta_{i_1 i'_1} \ldots \delta_{j_1 j'_1}$$



- AdS-CFT: Can model black hole evaporation (quenched cooling) with conventional quantum systems.
  - We will use two coupled SYK models:

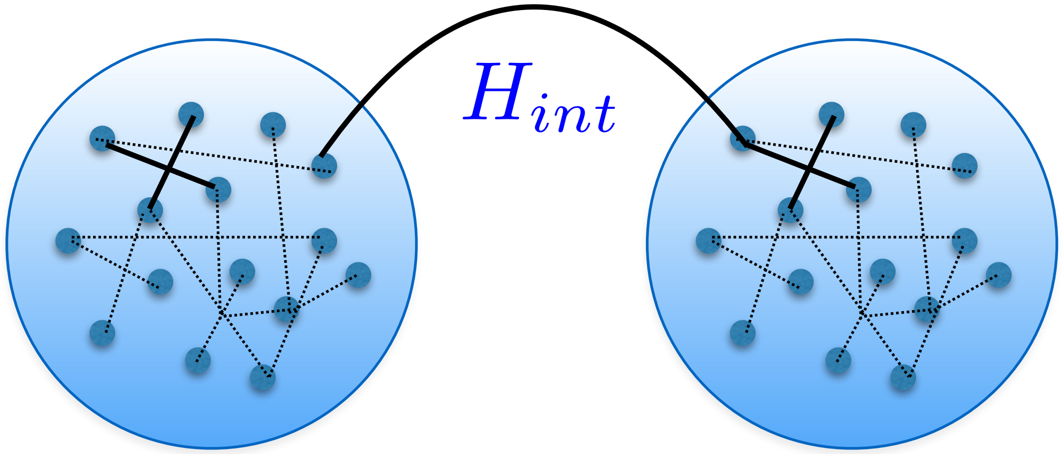
Sachdev-Ye-Kitaev model: N complex/real fermions with $q = 2p$-point interactions

  - This has a quantum spin liquid (long range entangled strongly correlated) ground state, which is exactly solvable in the large N limit, and dual to an AdS2 gravity theory.



Zhang: 1909.10637

- Quenched cooling in two coupled SYK quantum dots



$$H_{int} = g^{ij_1\dots j_n}(c_i^\dagger \psi_{j_1}\dots\psi_{j_n} + \psi_{j_n}^\dagger \dots \psi_{j_1}^\dagger c_i)\theta(t)$$

Zhang: 1909.10637

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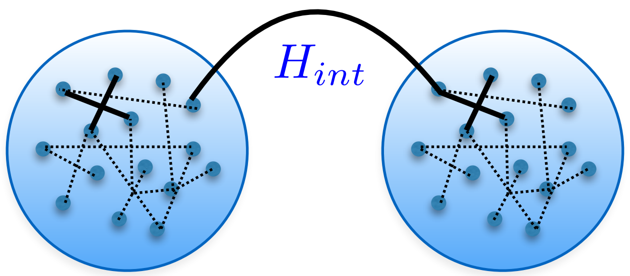

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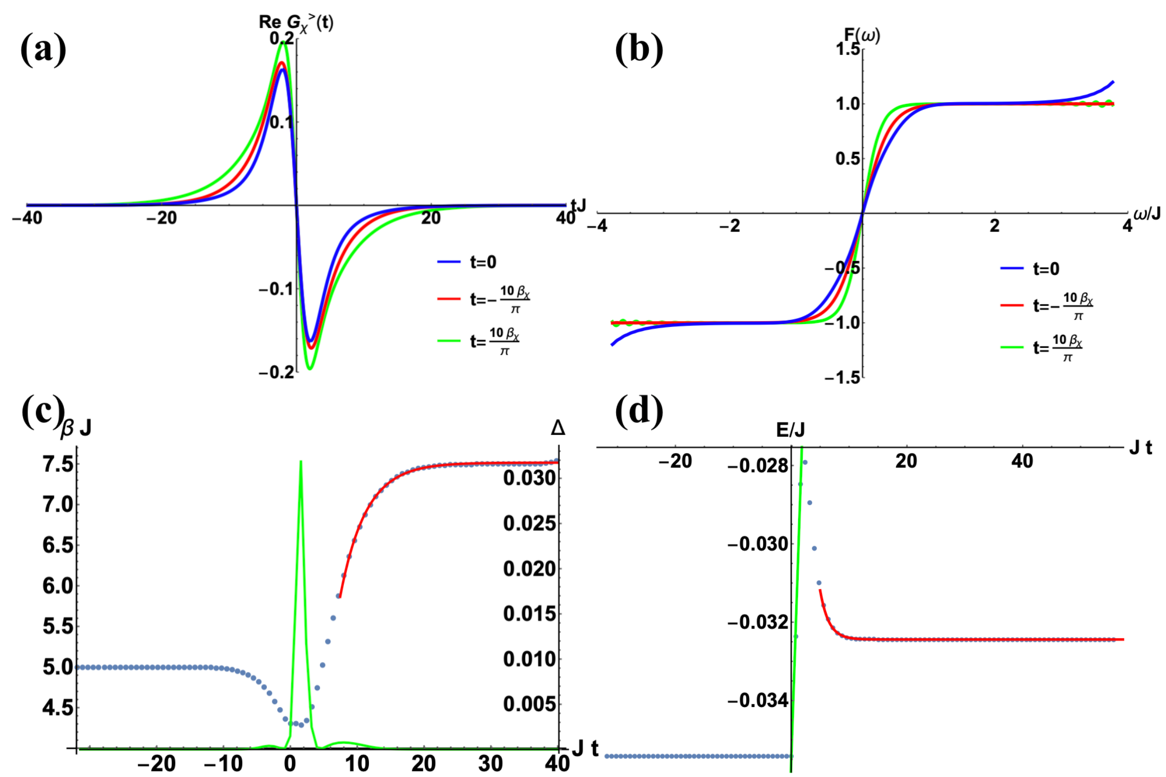
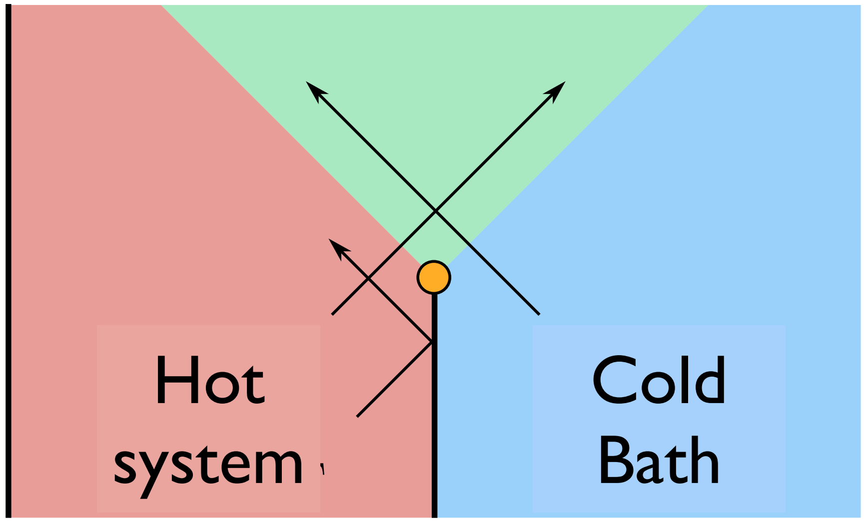


FIG. 2. The result of quench dynamics for $n = 3$ with $V/J = 0.6$, $T_\chi = 0.2J$ and $T_\chi = 1.5T_\psi$. (a). The real part of $G^>_\chi(t + \frac{t_r}{2}, t - \frac{t_r}{2})$ as a function of t_r for different t. (b). $F(\omega, t) = G_{\chi,K}(\omega, t)/\left(G_{\chi,R}(\omega, t) - G_{\chi,A}(\omega, t)\right)$ for different time t. (c). The evolution of effective temperature $T(t)$. The red line shows the result of exponential fitting of the late-time behavior. The green line represents the distance between $F(\omega)$ and $1 - 2n_F(\omega, T(t))$ defined by (38), we take the cutoff Λ by requiring $F(\Lambda) = 0.8$. (d). The evolution of energy $E(t)$ determined by (39). The green line is a fit for the short-time linear increase of energy and the red line is a late-time exponential fit for the relaxation of energy.

Maldacena, Milekhin: 1912.03276
Almheiri, Milekhin, Swingle: 1912.04912

- Follow-ups also show counterintuitive early energy rise in hot system.



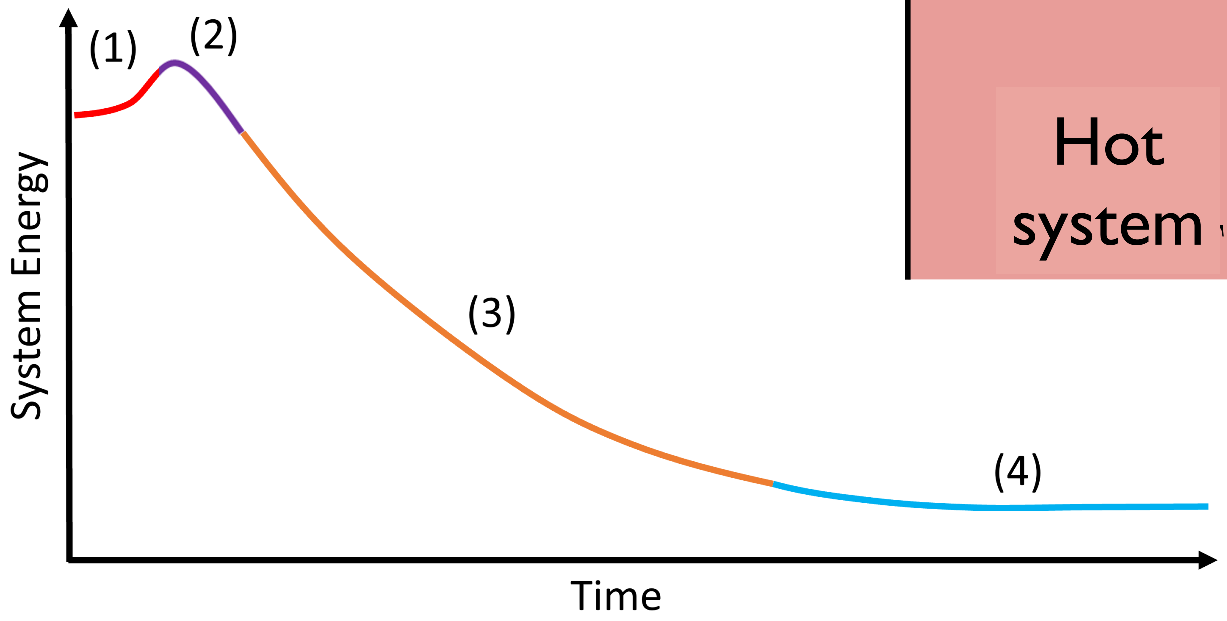


Figure 1: Typical behavior of system energy as a function of time for a large bath at lower temperature. We distinguish four dynamical regimes, labeled (1), (2), (3), and (4), which are discussed in detail in the text. Roughly they correspond to the early time energy rise, the subsequent turnover to energy loss, a sustained period of energy loss, and the final approach to global equilibrium.

Maldacena, Milekhin: 1912.03276
Almheiri, Milekhin, Swingle: 1912.04912

- Follow-ups also show counterintuitive early energy rise in hot system.

  - Almheiri, Milenkhin, Swingle give a non-proof proof

$$\frac{d^2}{dt^2} E(t)|_{t=0} > 0$$

$$\left(\frac{d}{dt} E(t)|_{t=0} = 0 \text{ generically}\right)$$

    of the energy in a hot system coupled to a cold bath with a counter-example: two coupled two-level systems

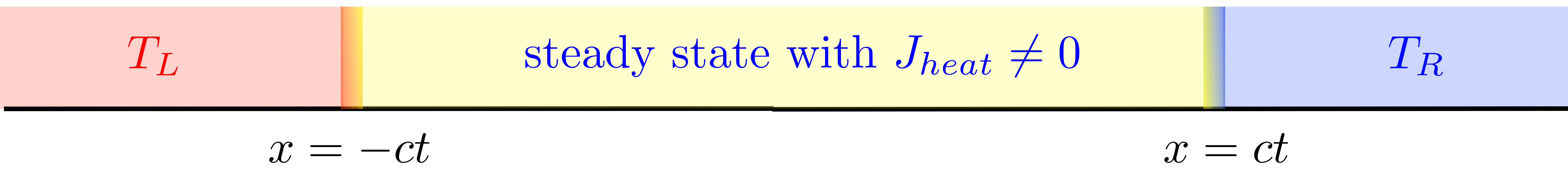
  - It is in fact easy to show in two coupled two-level systems

$$\frac{d^2}{dt^2} E(t)|_{t=0} \sim \left(e^{-\beta_B \Delta E_B} - e^{-\beta_A \Delta E_A}\right)$$

  - This counterintuitive rise must be due to some special effect.

- Thermal Quench in 1+1 CFTs

Bernard, Doyon



$$\langle J \rangle = \frac{c\pi}{12}(T_L^2 - T_R^2)$$

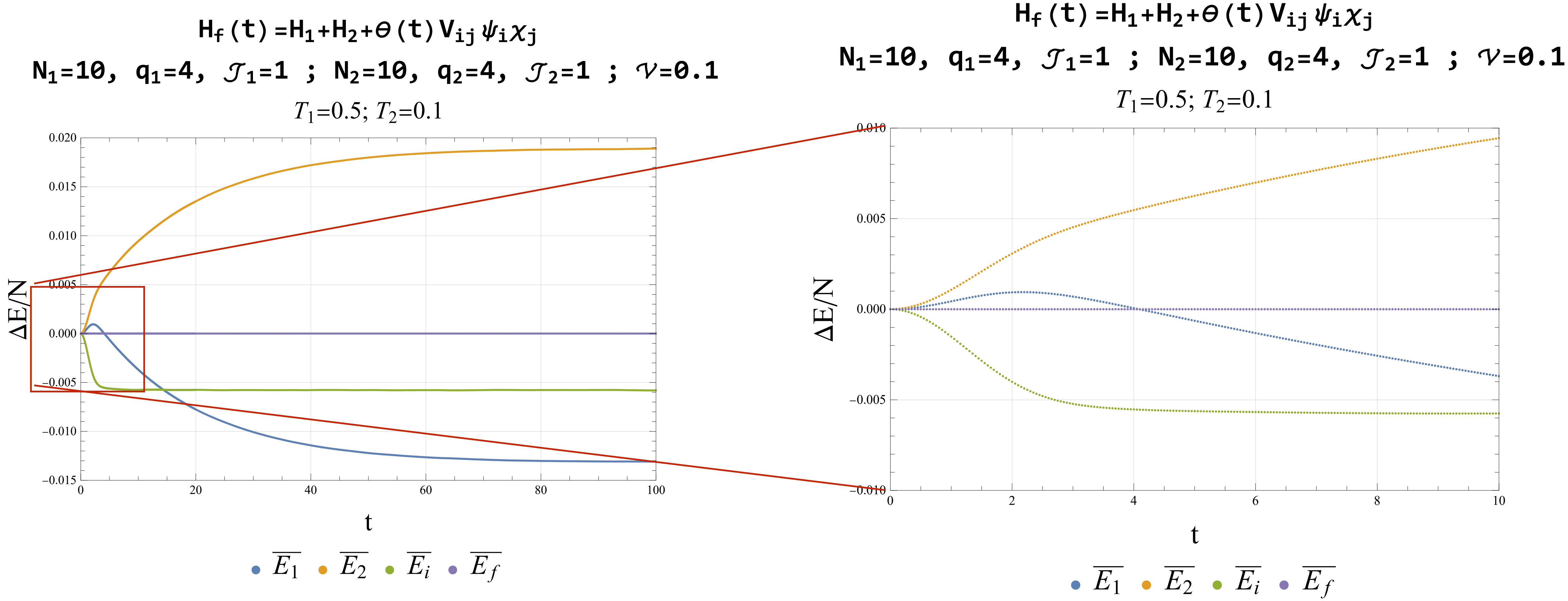
- Quantum version of hydrodynamic Riemann problem where hydrodynamic (classical) intuition is exact. (free field representation of 1+1 CFTs)

Bhaseen, Doyon, Lucas, KS

- AdS Gravity dual is known. No "rise in energy" in the hot bath.

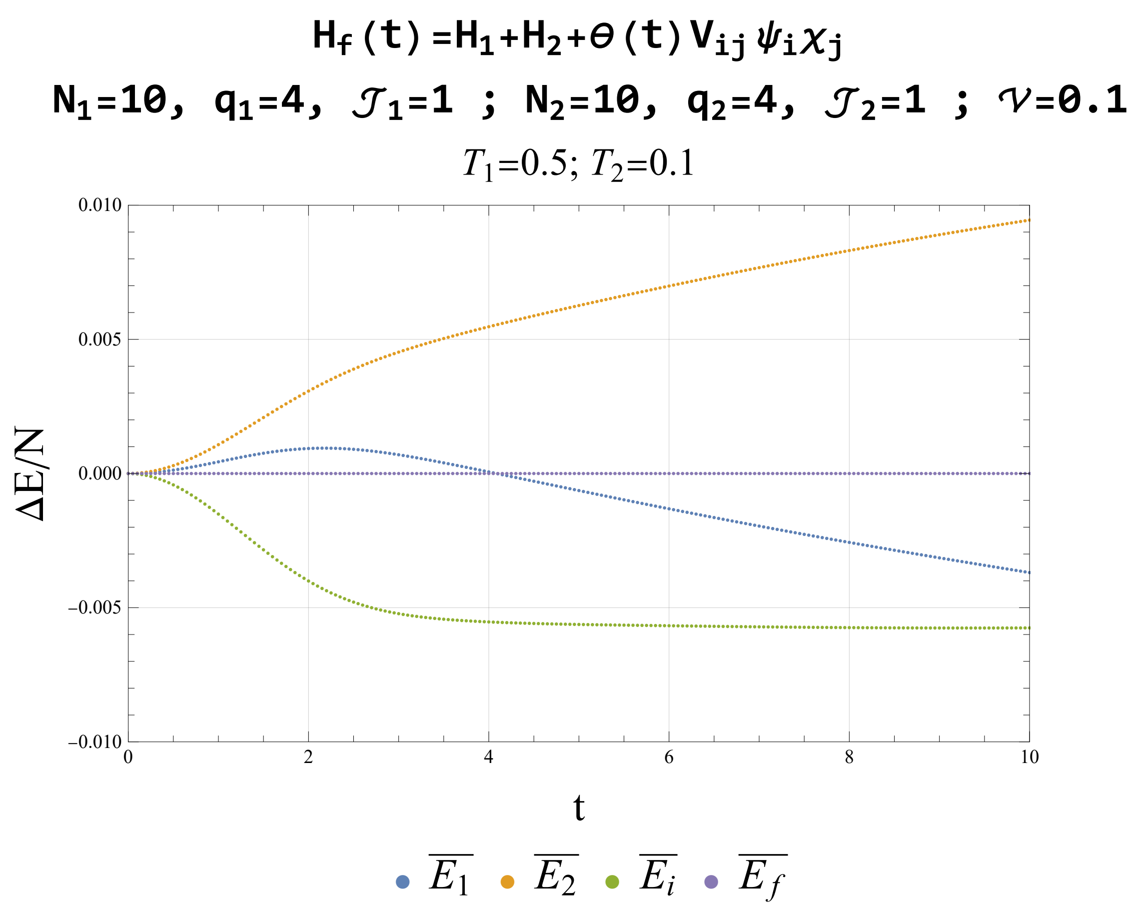
- Exact diagonalization in Majorana SYK

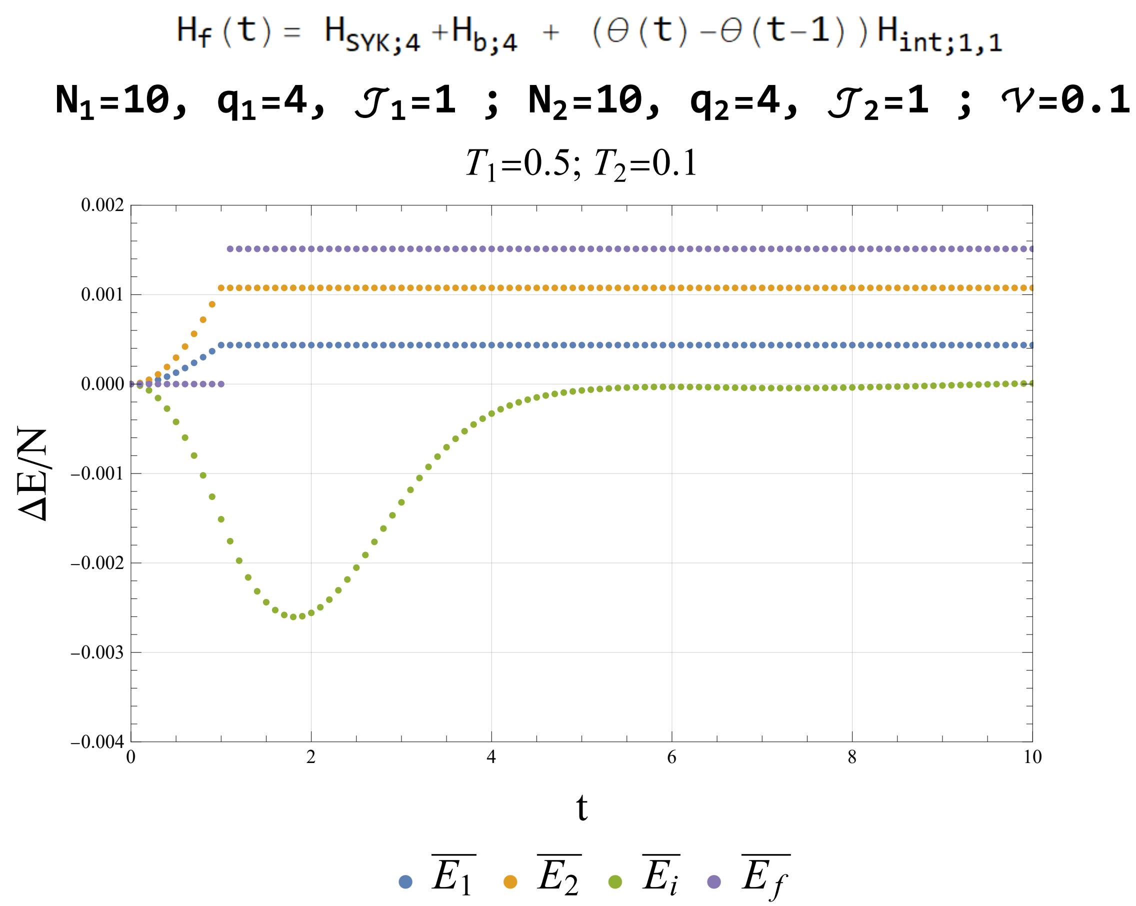
Cheipesh, Gnezdilov, Ohanesjan, Pavlov, KS



$$E_1 = \mathrm{Tr}(H_1\rho)\ ,\quad E_2 = \mathrm{Tr}(H_2\rho)\ ,\quad E_{\mathrm{int}} = \mathrm{Tr}(H_{\mathrm{int}}\rho)\ ,\quad E_f = \mathrm{Tr}((H_1+H_2+H_{\mathrm{int}})\rho)$$

- Exact diagonalization in Majorana SYK





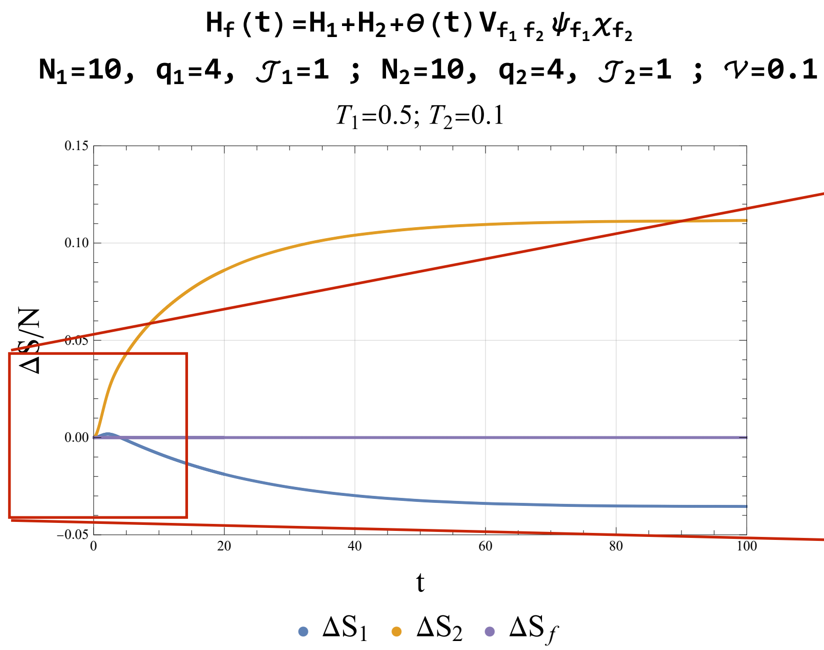
Subsequent decoupling quench pumps in energy

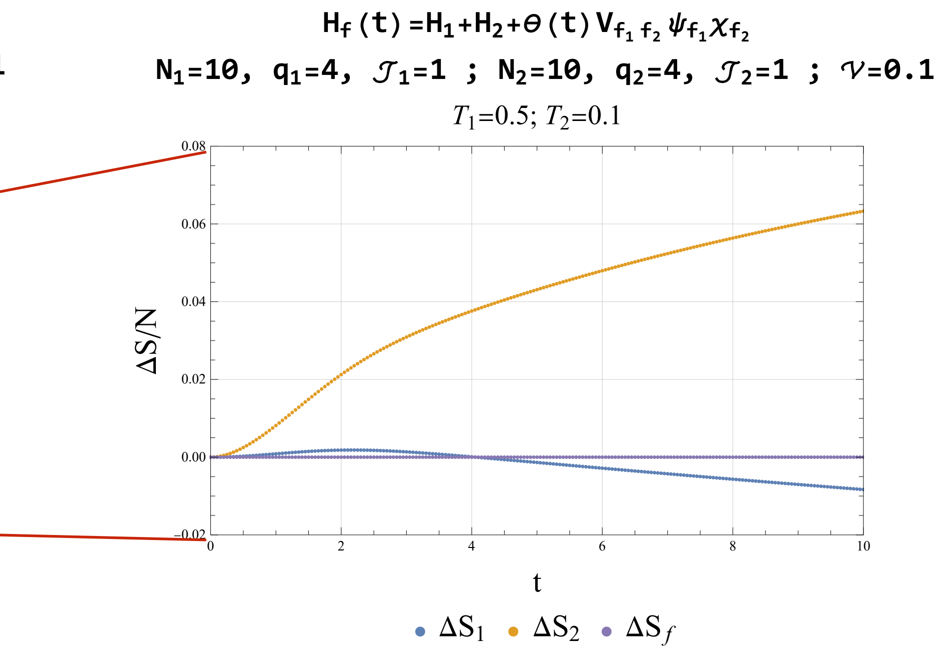
$$E_1 = \mathrm{Tr}(H_1\rho)\ ,\quad E_2 = \mathrm{Tr}(H_2\rho)\ ,$$
$$E_{\rm int} = \mathrm{Tr}(H_{\rm int}\rho)\ ,\quad E_f = \mathrm{Tr}((H_1 + H_2 + H_{\rm int})\rho)$$

This is related to the two-time measurement protocol in quantum thermodynamics studies

Exact diagonalization in Majorana SYK

- Von Neumann Entropy





Exact diagonalization in Majorana SYK

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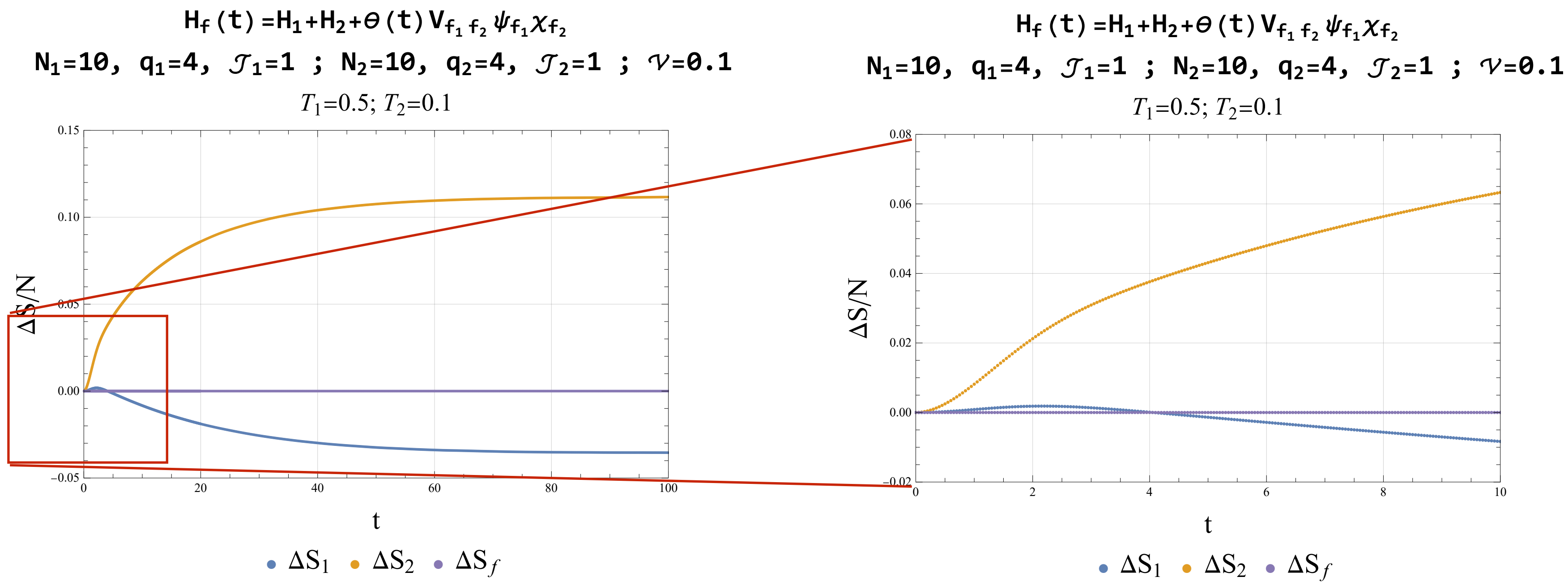
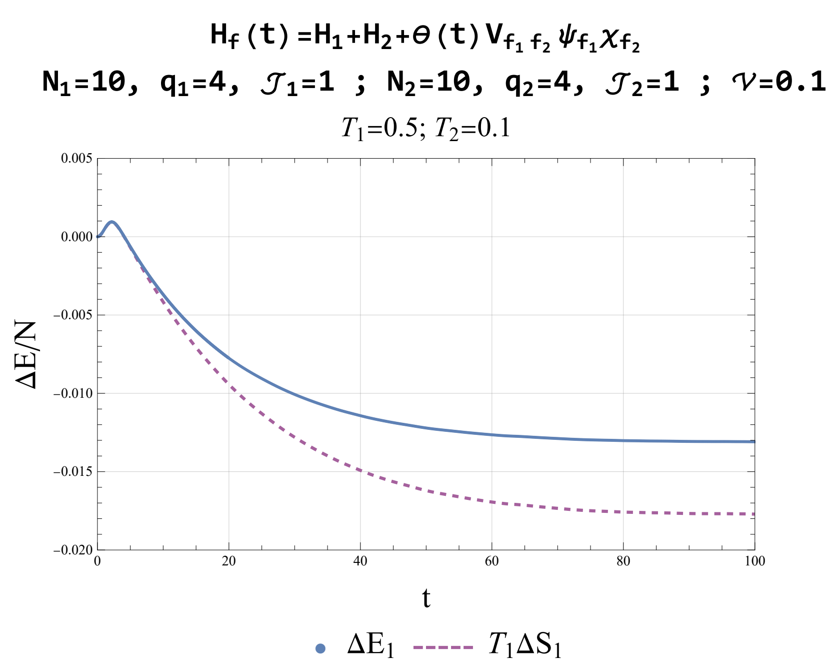
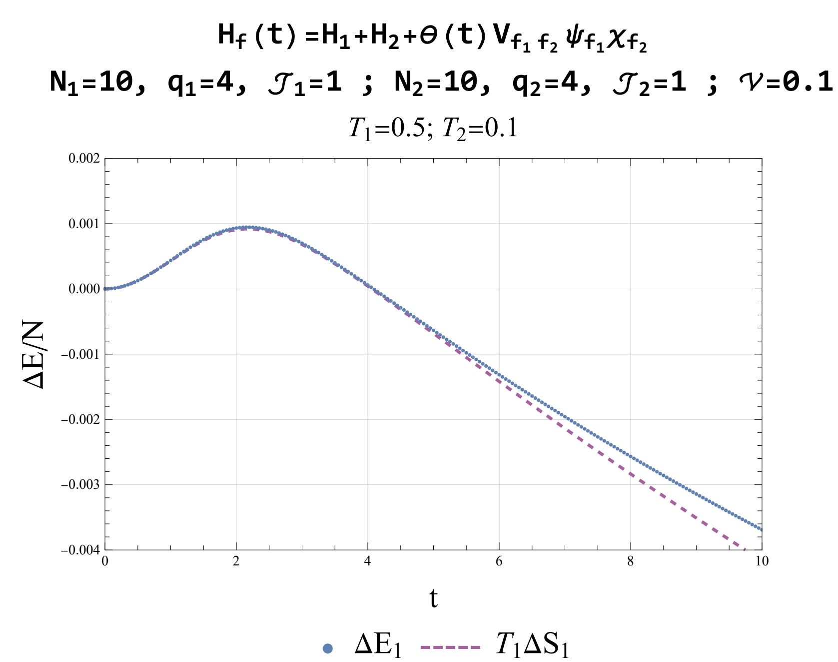


- "Ohanesjan's" First law

$$\text{For } t \ll \frac{1}{J} \quad : \quad E_1(t) = T_1 S_{\mathrm{vN},1}(t)$$





- This intrigued us greatly …. path to a universal understanding of the behavior of

$$\frac{d^2}{dt^2} E(t)|_{t=0}$$

- Information Free energy improvement on thermal equilibrium free energy

cf. Deffner, Campbell

$$F_{\rm info}(\rho(t)||\rho_{T_1}) = E(t) - T_1 S_{\rm vN}(t) = F + T_1 D(\rho(t)||\rho_{T_1})$$

$$D(\rho_A||\rho_B) = {\rm Tr}\rho_A \log \rho_A - {\rm Tr}\rho_A \log \rho_B$$

- Easy to show

$$\Delta E_1(t) = T_1 \Delta S_1(t) + T_1 D(\rho_1(t)||\rho_{T_1})$$

- Quenched cooling of a thermal state: decoupled at time t

essential to make E well defined

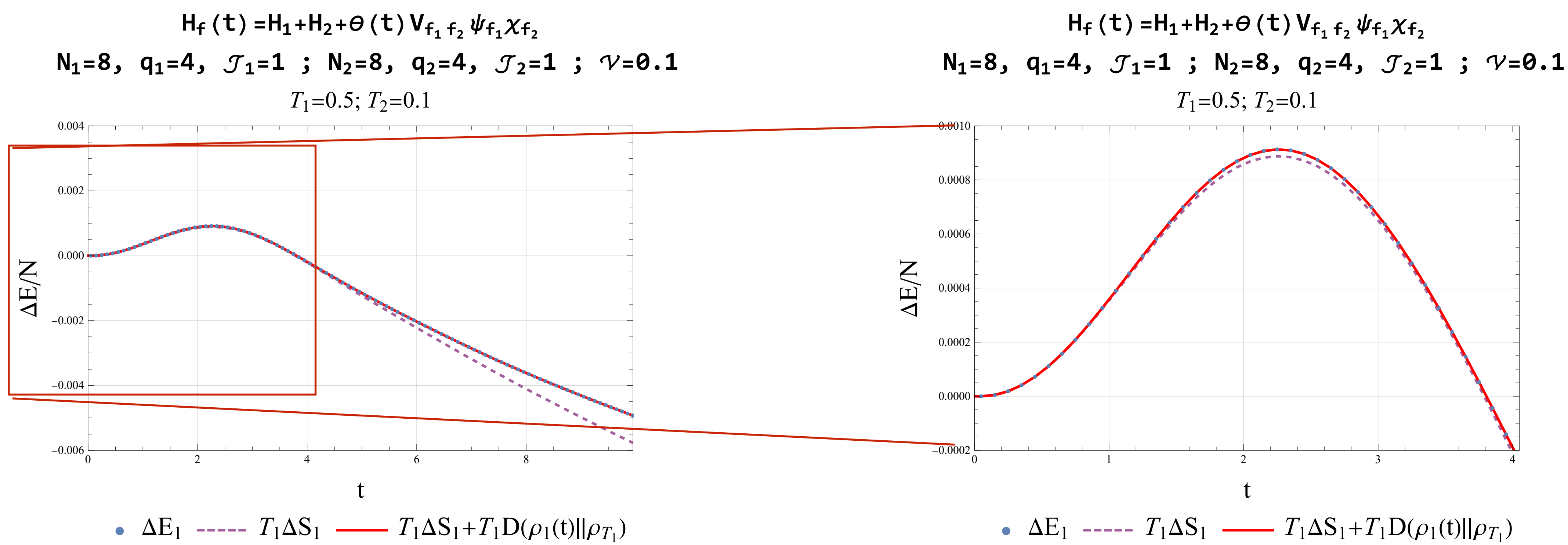
$$\Delta E_1(t) = T_1 \Delta S_1(t) + T_1 D(\rho_1(t)||\rho_{T_1})$$

$$D(\rho_1(t)||\rho_{T_1}) \geq 0 \text{ strictly true}$$

implies the following quantum "non-equilibrium" inequality for quenched cooling

$$\Delta E_1(t) \geq T_1 \Delta S_{\mathrm{vN},1}(t)$$

$$\Delta E_1(t) = T_1 \Delta S_1(t) + T_1 D(\rho_1(t)||\rho_{T_1})$$



Note:

$$\Delta E_1(t) \geq T_1 \Delta S_{\mathrm{vN},1}(t)$$

- Intuitively at low T_1, $\Delta S_{\mathrm{vN},1}(t) > 0$ is dominated by/ completely determined by the growth in quantum correlations:
  - This explains qualitatively the "counterintuitive rise in energy"
  - But it also prompts the question:

"Can one direcly detect quantum correlations between subparts of a quantum composite system by measuring the resulting energy increment?"

- “Can one direcly detect quantum correlations between subparts of a quantum composite system by measuring the resulting energy increment?”

$$\Delta E_1(t) = T_1 \Delta S_{\mathrm{vN},1}(t) \quad ?$$

- Yes, perturbative Fermi systems with local tunneling quench contact:

$$H_A = \sum_p \xi_p a_p^\dagger a_p$$

$$H_B = \sum_p \xi'_p b_p^\dagger b_p$$

$$V_{AB} = \lambda \delta(x)(a^\dagger(x) b(x) + b^\dagger(x) a(x))$$

$$H(t) = H_A + H_B + (\theta(t) - \theta(t - t_0)) V_{AB}$$

Essential Ingredient:

Perturbation Theory:

$$\Delta E \sim \lambda^2$$

$$\Delta S_{\mathrm{vN},1} \sim \lambda^2$$

$$D(\rho(t)||\rho_{T_1}) \sim \lambda^3$$

[assuming $T_{A,B} < \mu_{A,B}$]

- "Can one direcly detect quantum correlations between subparts of a quantum composite system by measuring the resulting energy increment?"

$$\Delta E_1(t) = T_1 \Delta S_{\mathrm{vN},1}(t) \quad ?$$

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$$H_A = \sum_p \xi_p a_p^\dagger a_p$$

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$$H(t) = H_A + H_B + (\theta(t) - \theta(t - t_0)) V_{AB}$$

One of by now several ways to "measure" S_{vN}

Abanin, Demler; Beenakker, Emary, Kinderman, van Velzen; Cardy; Islam et al; Klich, Levitov

- Quenched cooling of two thermal quantum systems

$$\Delta E_1(t) = T_1 \Delta S_1(t) + T_1 D(\rho_1(t)||\rho_{T_1})$$

$$D(\rho_1(t)||\rho_{T_1}) \geq 0 \text{ strictly true}$$

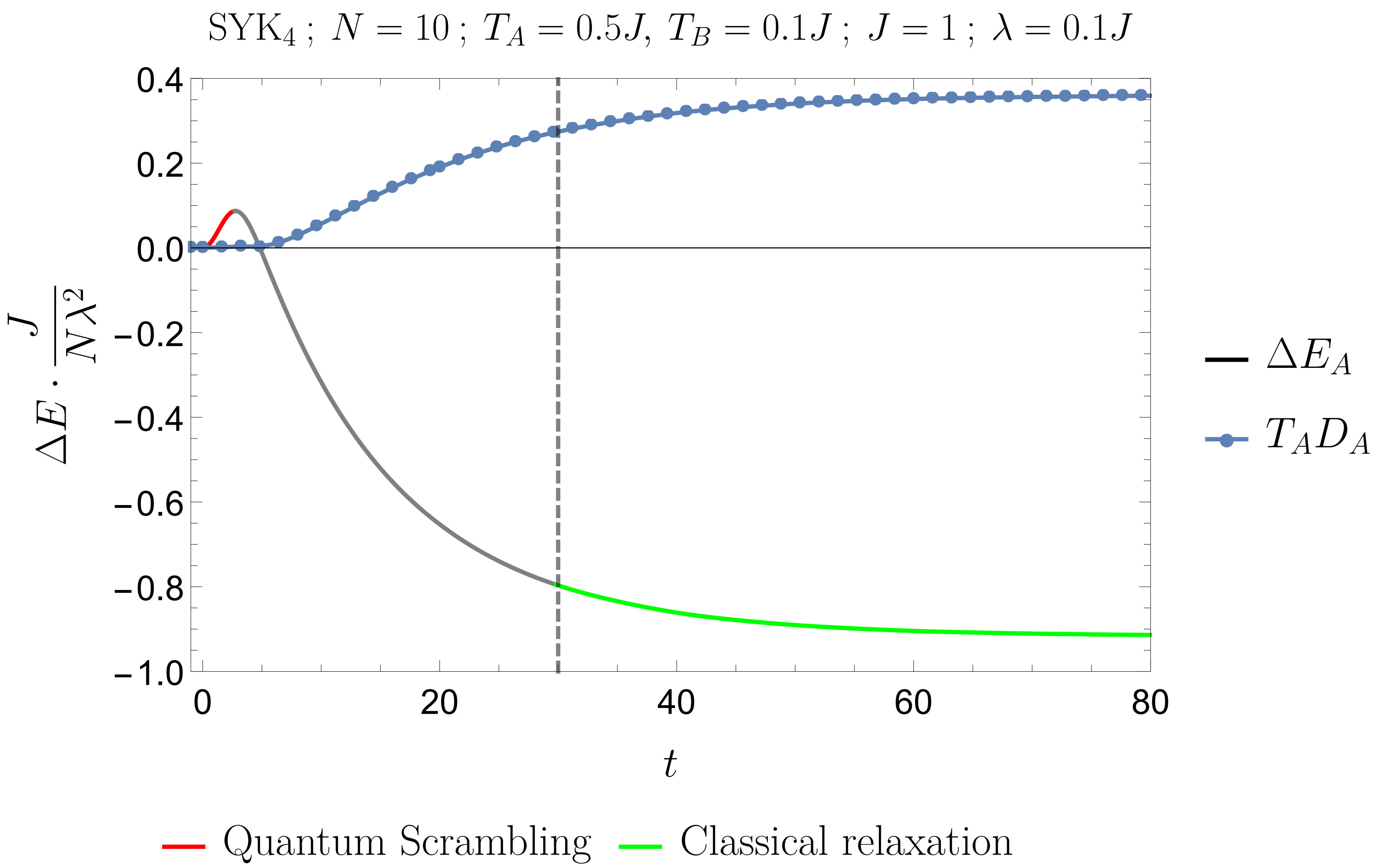
$$\Delta E_1(t) \geq T_1 \Delta S_{\mathrm{vN},1}(t)$$

  - This explains qualitatively the "counterintuitive rise in energy" seen in many cases. Though this does not automatically imply

$$\frac{d^2}{dt^2} E(t)|_{t=0} > 0$$

  - When do we recover intuitive classical physics with heat flowing from hot to cold?

- Classical physics should emerge in the high temperature, perturbative particle regime.



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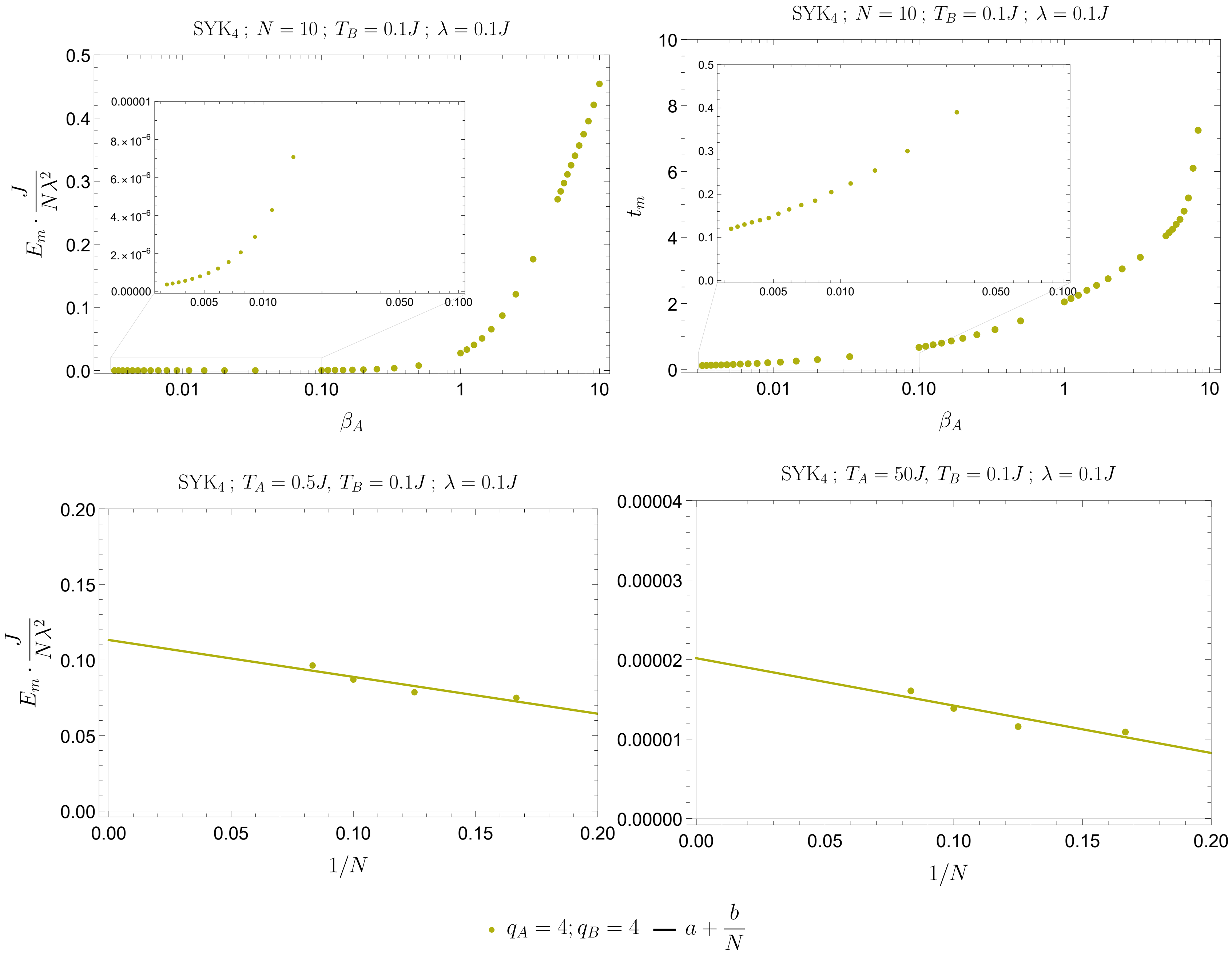

$E_m =$ bump height

$t_m =$ bump time

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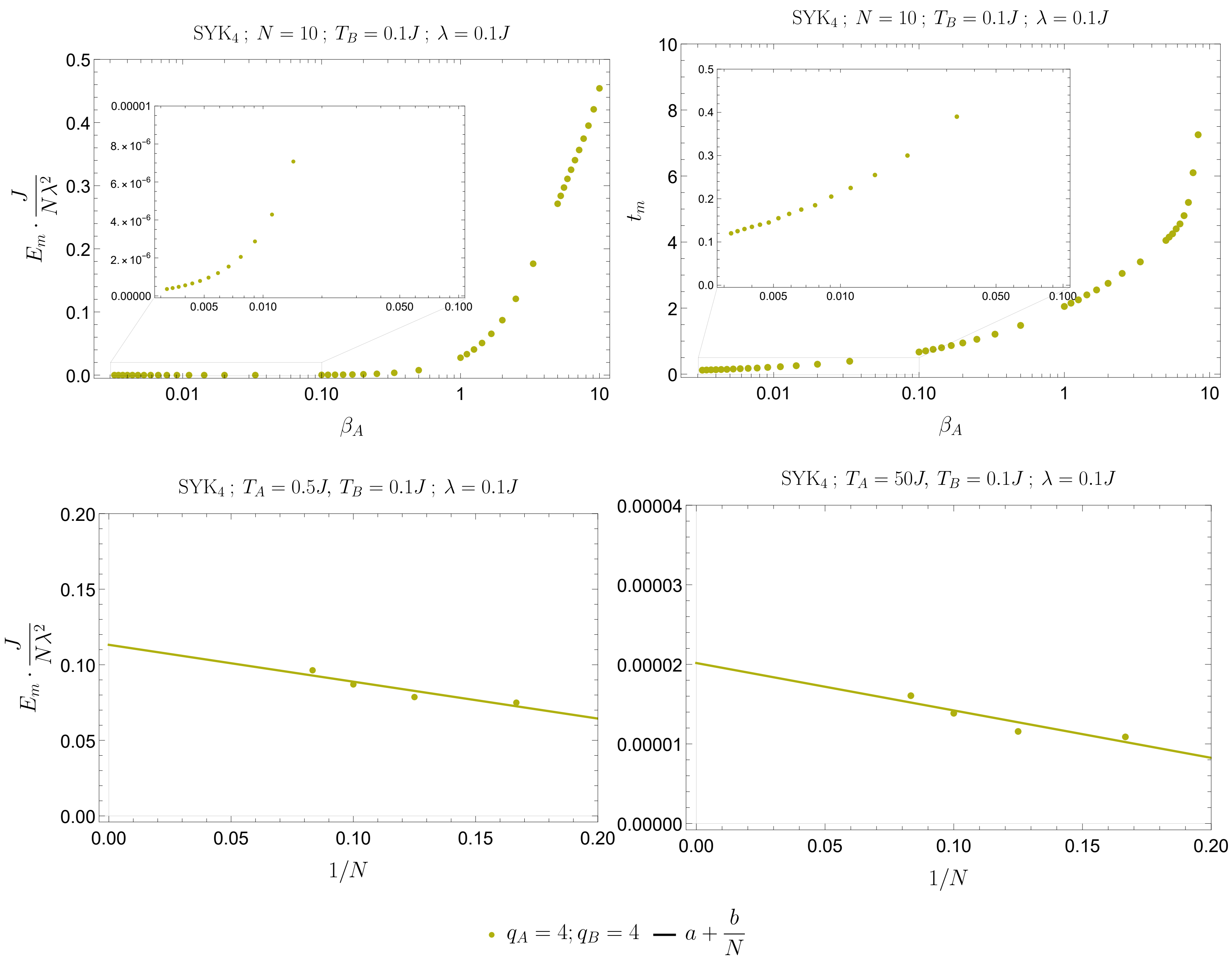


E_m = bump height

t_m = bump time

- Two SYK dots are "too quantum". There is never a classical regime

- Classical physics should emerge in the high temperature, perturbative particle regime.
    - Two Ising half-lines

      Non-extensive interaction energy
      (Similar to perturbative fermions but for $T \gg \mu$)

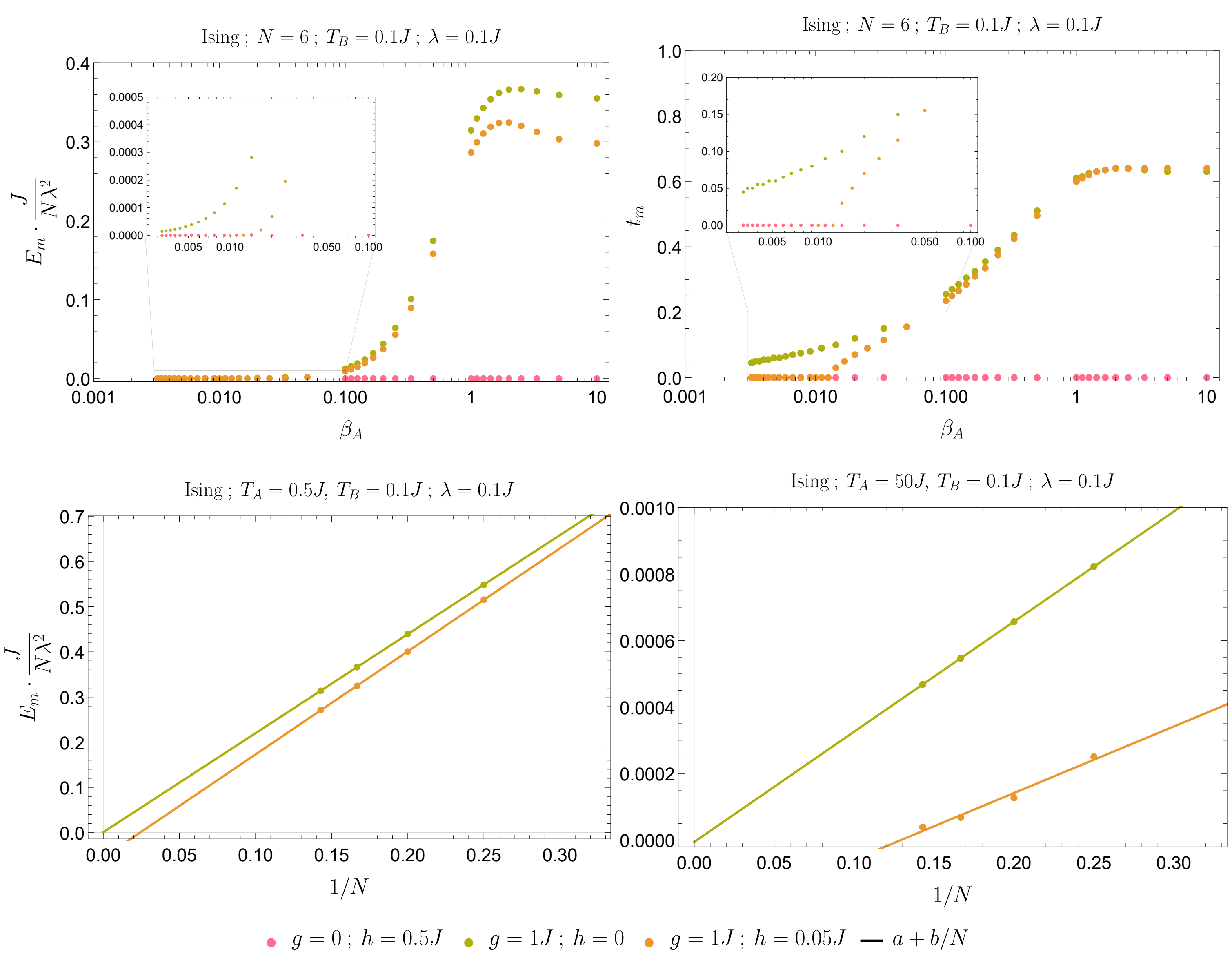
$$H_1 = -\sum Z_i Z_{i+1} - g X_i - h Z_i$$

$$H_2 = -\sum Z'_i Z'_{i+1} - g X'_i - h Z'_i$$

$$H_{int} = Z_{last} Z'_{first}$$

- Classical physics should emerge in the high temperature, perturbative particle regime.
  - Two Ising half-lines

    Non-extensive interaction energy



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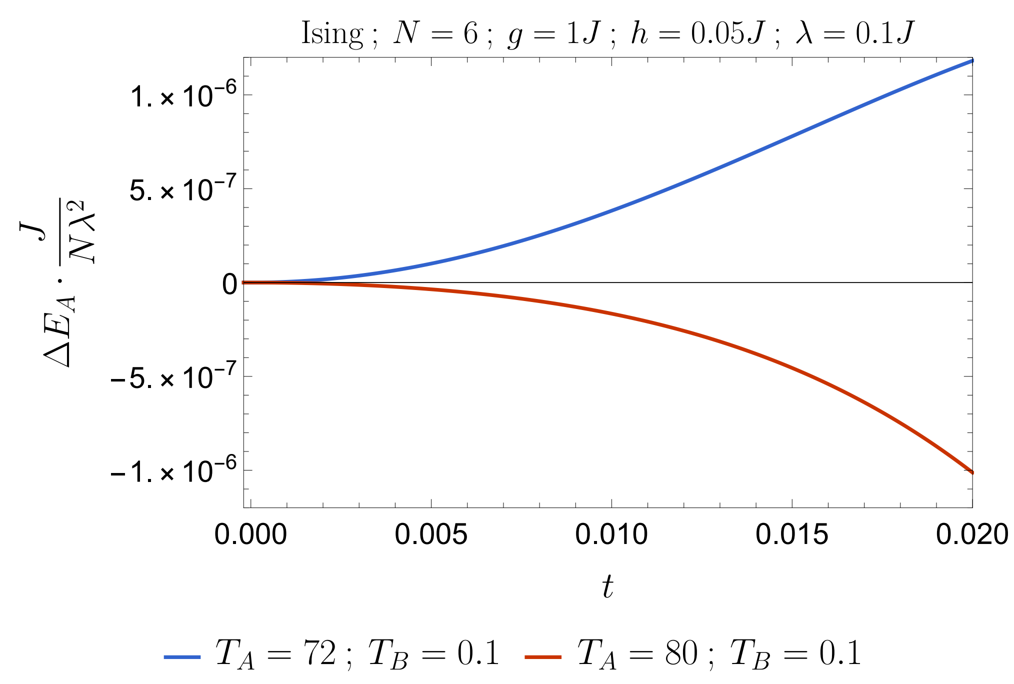
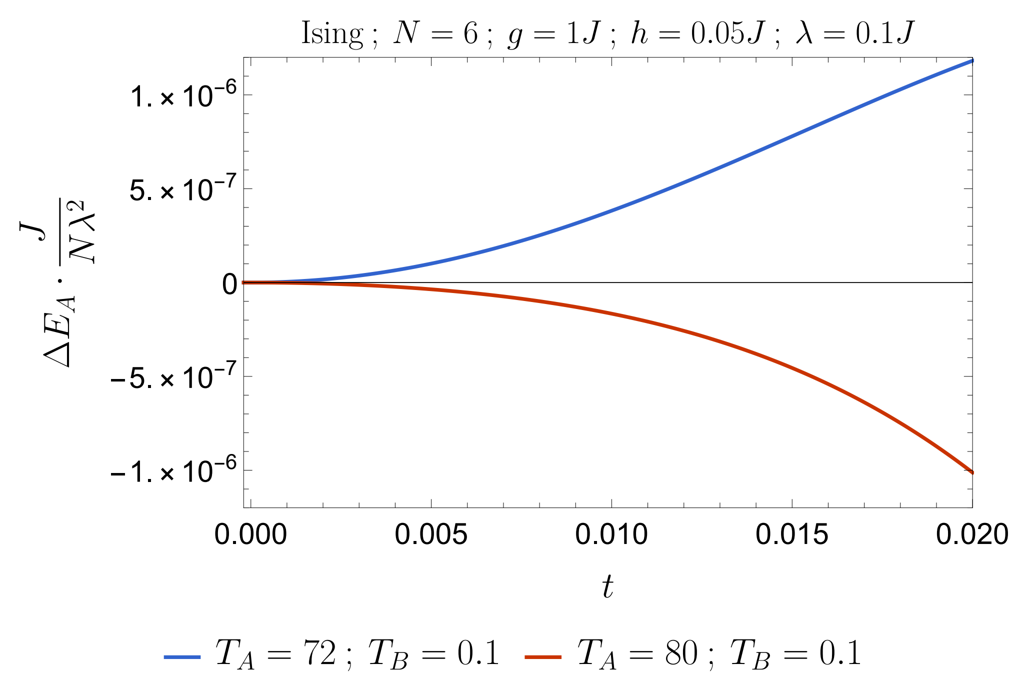


FIG. 10. Quenched cooling in two Ising half lines. For $T < T_c \simeq 77.845J$ one still observes the counterintuitive rise in the hotter system A, but for $T > T_c$ one transitions to a regime where classical intuition is restored and the system cools instantaneously upon contact.

- Classical physics should emerge in the high temperature, perturbative particle regime.
  - Two Ising half-lines

    Non-extensive interaction energy



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Conclusions

- From chaos in hydrodynamics to a kinetic theory for scrambling
  - Scrambling and diffusion:
    a priori set by different timescales

arXiv:1710.00921
arXiv:1804.09182
arXiv:22summer

Except in a dilute gas: a kinetic theory for chaos

Except in a ultra strongly correlated system: pole skipping in hydrodynamics

- Energy dynamics, information and heat flow and the transition from quantum to classical thermodynamics
  - In quenched cooling, a non-equilibrium first law

arXiv:2011.05238
arXiv:2108.12031
arXiv:2204.12411

$$\Delta E_1(t) = T_1 \Delta S_1(t) + T_1 D(\rho_1(t) || \rho_{T_1})$$

  - Quantum correlation growth and energy relaxation:
    a priori set by different timescales

Except in high T dilute particle limit: classical relaxation dominates

Thank you