Dynamics, Transport and Chaos through a Strange Quantum Bath

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Strange Metals, SYK Models and Beyond, June 2-3 2022

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- Steven Thomson (CdF—> Free U Berlin)

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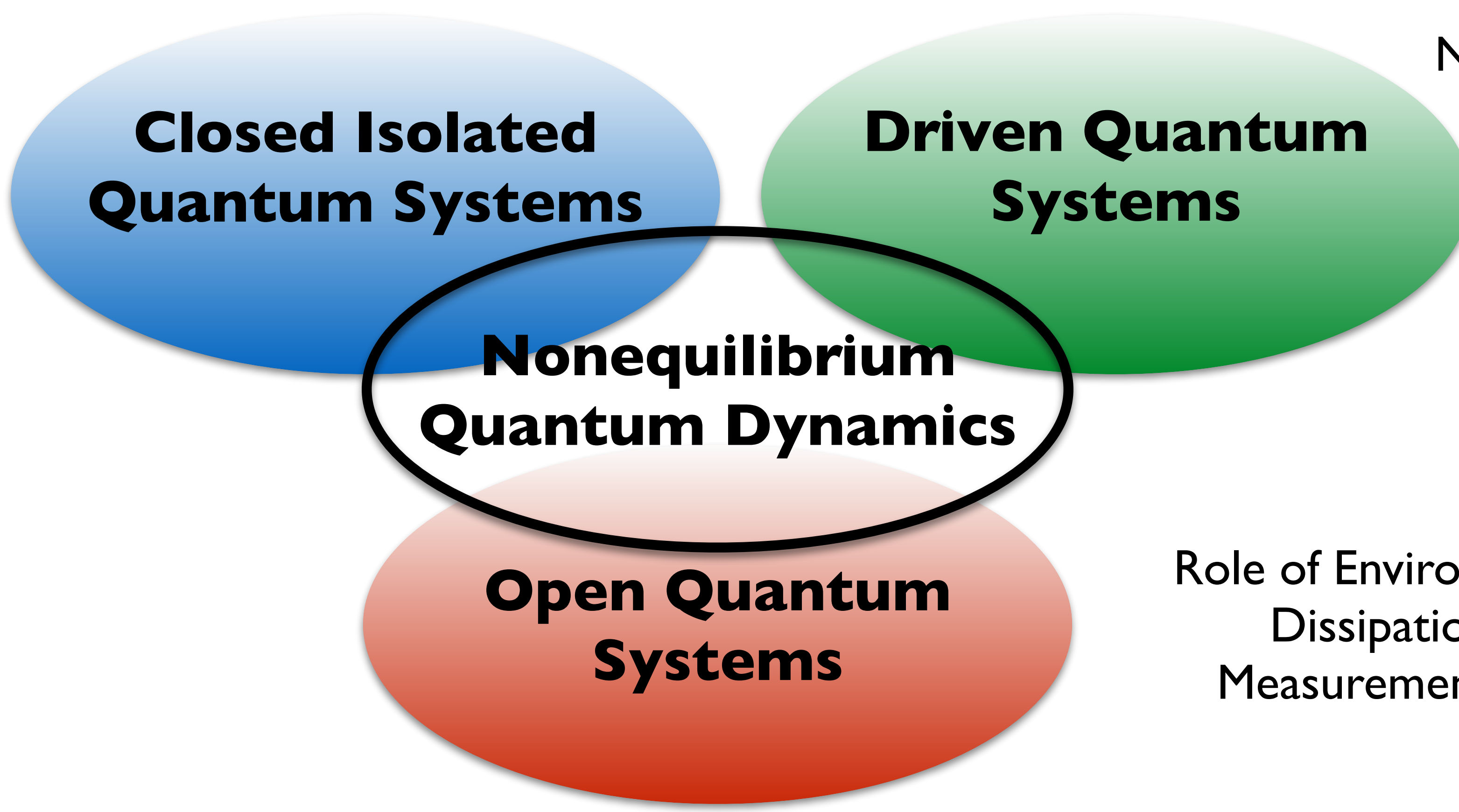
- *A. Larzul, M. Schiro', Phys. Rev. B 105 045105 (2022)*
- *A. Larzul, S. J. Thomson, M. Schiro', arXiv:2204.06434*
- *A. Larzul, M. Schiro', arXiv.2206.XXX (to appear)*

Quantum Matter Out of Equilibrium

General principles to classify **dynamical** behaviour of quantum many-body systems?

Ergodicity, Thermalization and their Breakdown

Non-Equilibrium Transport, Heating Dynamics

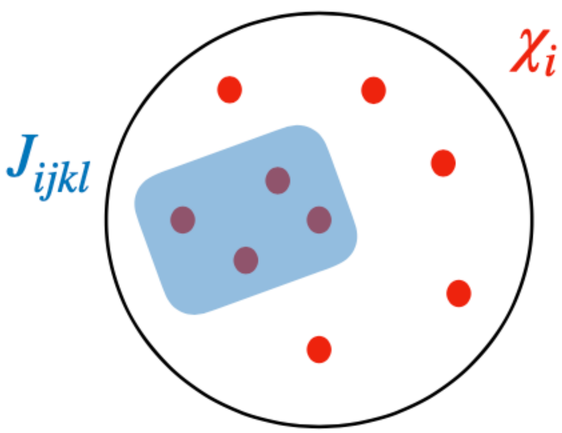


Role of Environment, Dissipation, Measurements, ..

Simple models that capture "generic" behavior?

Sachdev-Ye-Kitaev Model

Sachdev&Ye,
Georges, Parcollet&Sachdev,
Kitaev, Stanford&Maldacena,..



$$H_4 = -\frac{1}{4!}\sum_{ijkl} J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

$$\langle J_{ijkl}\rangle = 0, \qquad \langle J_{ijkl}^2\rangle = 6\frac{J^2}{N^3}$$

- Exactly Solvable in Large N: Minimal Model for Non-Fermi-Liquid (gapless, but no quasiparticles)
- Fluctuations beyond Large N described by Schwartzian Action (gravity analogue)
- Maximally Chaotic — Saturates the Bound on Chaos

◦ Bound on chaos $\lambda_L \leq \frac{2\pi}{\beta}$
Maldacena&Stanford

- Can this model give us insights about the behaviour of quantum matter far from equilibrium?

Outline

- Thermalisation in closed, isolated SYK models
- SYK as a "Strange Quantum Bath": Equilibration Dynamics
- SYK as a "Strange Quantum Bath": Energy Transport& Chaos

Dynamics and Thermalization of Isolated SYK Models

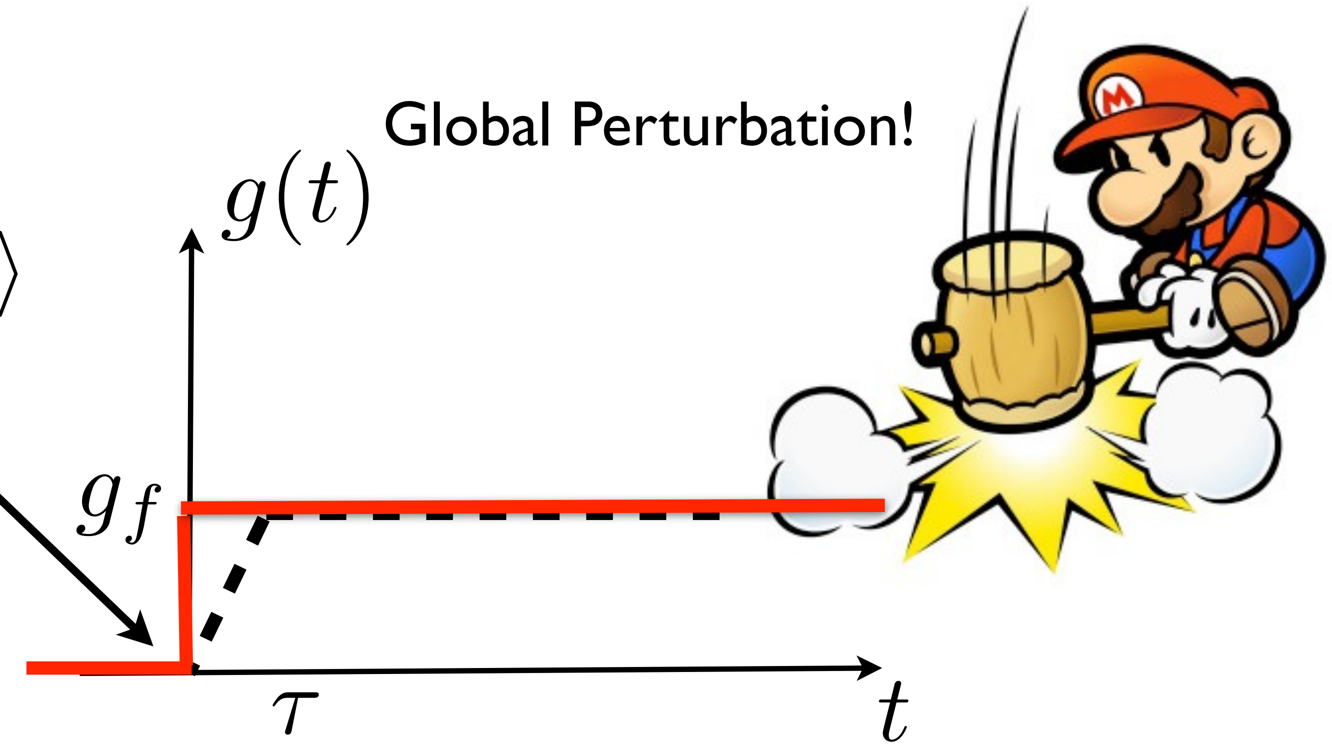
- *A. Larzul, M. Schiro', Phys. Rev. B 105 045105*

Quantum Thermalization

$$H[g(t)] = H_0 + g(t)\, H_1$$

$$|\Psi(t=0)\rangle = |\Psi_0\rangle$$

$$i\partial_t |\Psi(t)\rangle = H|\Psi(t)\rangle$$



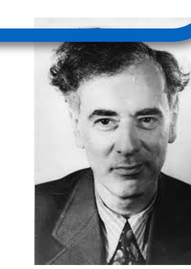
Unitary Quantum Dynamics (isolated system, no external bath)

- Pure states remain pure $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ $\quad \mathrm{Tr}\rho^2(t) = \mathrm{Tr}\rho_0 = 1$
- Quantum Mechanics is linear!

- **How generic quantum systems approach Thermal Equilibrium after a perturbation?**

Interactions induce scattering, loose memory initial condition…

$$\frac{dn_{\mathbf{k}}}{dt} = I_{coll}[n_{\mathbf{k}}(t)]$$

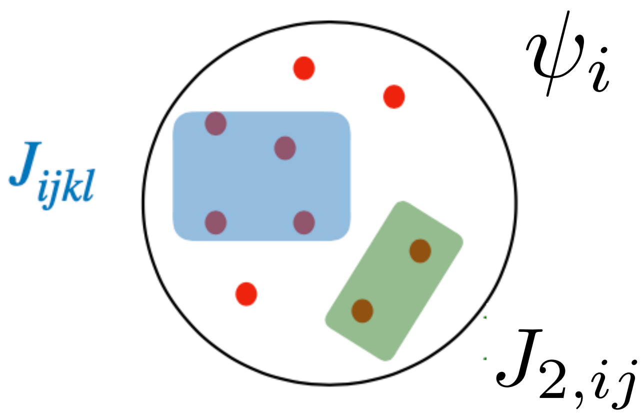


- In modern terms: "Typical" (High-Energy) Excited States are "Thermal" (ETH)

Deutsch, Srednicki, ..

Dynamics of SYK Models

- "Mixed" SYK:



$$H = \frac{i}{2}\sum_{ij} J_{2,ij}\psi_i\psi_j - \frac{1}{4!}\sum_{ijkl} J_{4,ijkl}\psi_i\psi_j\psi_k\psi_l$$

"SYK_2" "SYK_4"

- Equilibrium: Crossover from NFL to FL scaling at $T^* \sim J_2^2/J_4$

Parcollet&Georges('99),..,Song et al (2017),..

- Dynamics: Keldysh field theory is "exactly solvable" in Large-N

- Kadanoff-Baym Equations for Green's functions

$$G^{\alpha\beta}(t_1,t_2) = -\frac{i}{N}\sum_i \langle \psi^\alpha(t_1)\psi^\beta(t_2)\rangle$$

Large N Kadanoff-Baym Equations for Mixed SYK

Larzul&MS PRB(2022)

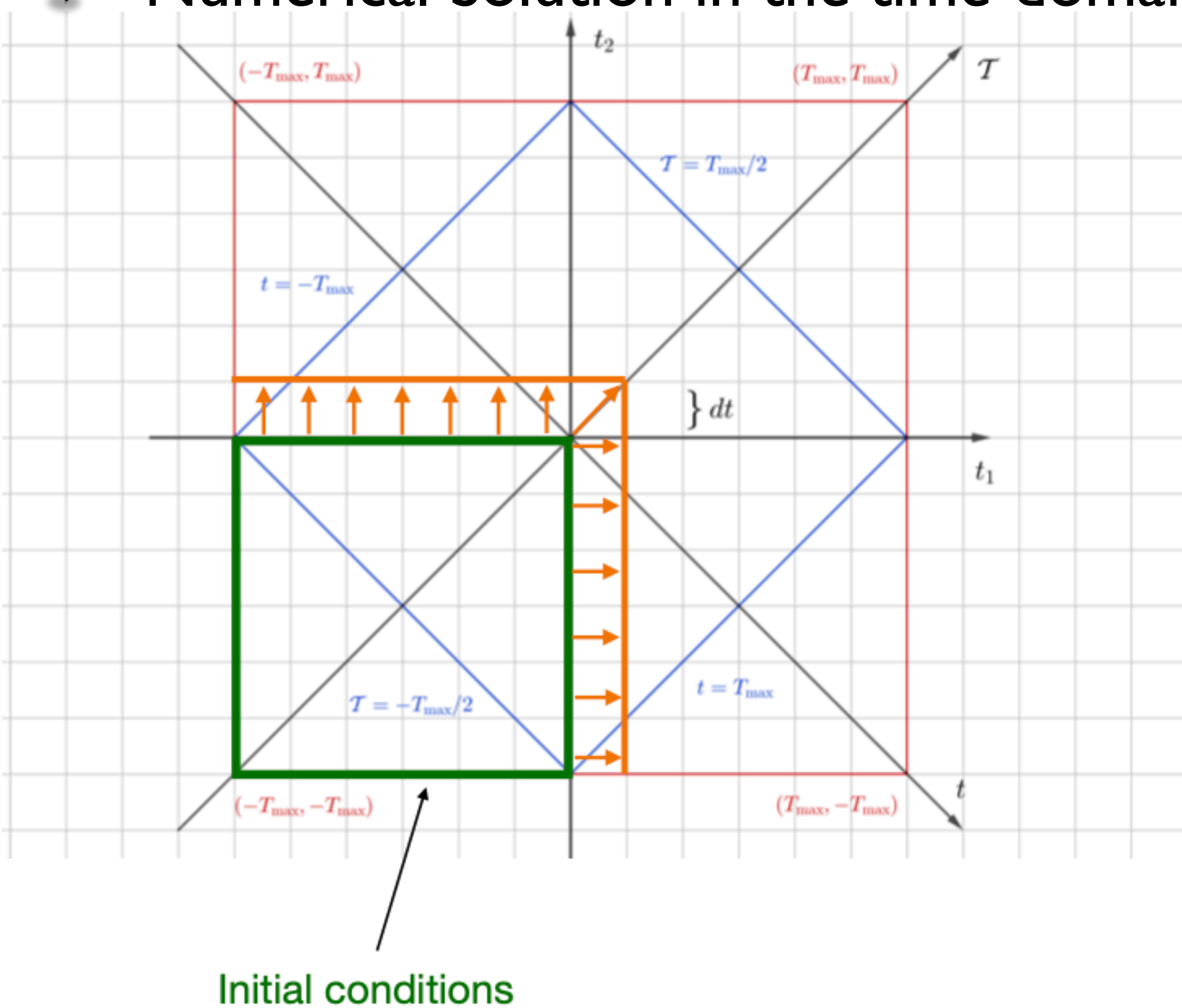
$$i\partial_{t_1} G^{>,<}(t_1,t_2) = \int_{-\infty}^{+\infty} dt \Big[\Sigma^R(t_1,t)G^{>,<}(t,t_2) + \Sigma^{>,<}(t_1,t)G^A(t,t_2)\Big]$$

$$-i\partial_{t_2} G^{>,<}(t_1,t_2) = \int_{-\infty}^{+\infty} dt \Big[G^R(t_1,t)\Sigma^{>,<}(t,t_2) + G^{>,<}(t_1,t)\Sigma^A(t,t_2)\Big]$$

- Self-Energy term

$$\Sigma^{\alpha\beta}(t_1,t_2) = -\alpha\beta J_4(t_1)J_4(t_2)G^{\alpha\beta}(t_1,t_2)^3 + J_2(t_1)J_2(t_2)G^{\alpha\beta}(t_1,t_2)$$

- Numerical Solution in the time-domain (similar to Nonequilibrium DMFT)



- Initial Condition+Propagation(predictor/corrector)

- Wigner Transform

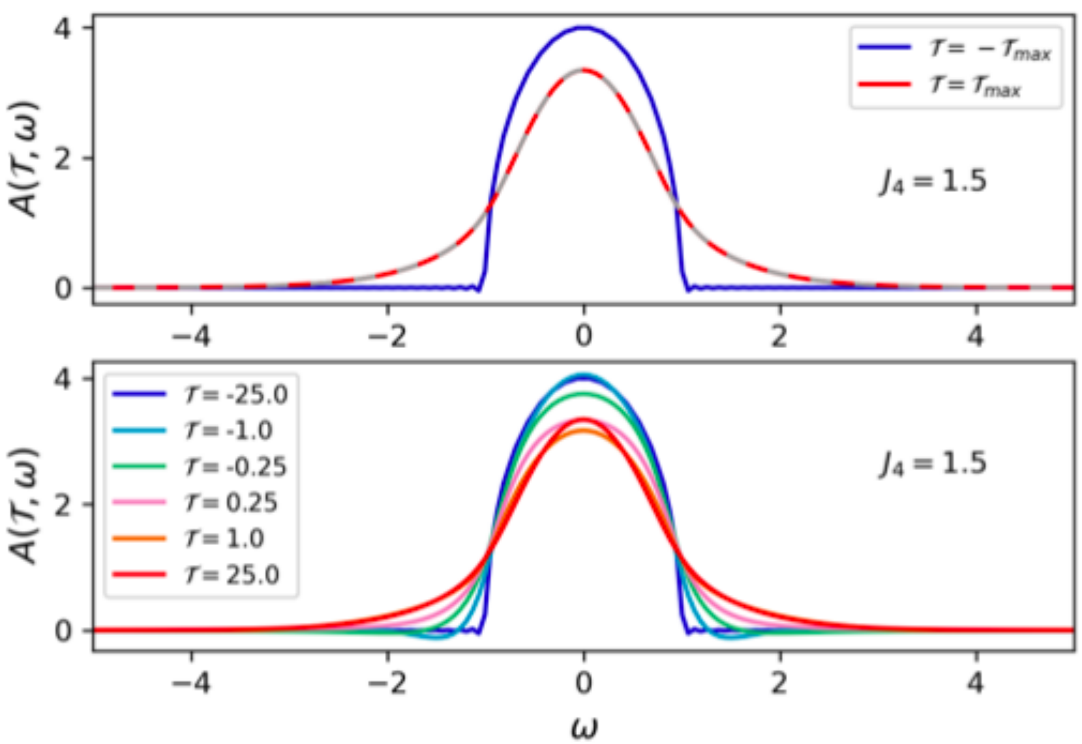
$$G(\mathcal{T},\omega) = \int dt\, e^{i\omega t} G\Big(t_1 = \mathcal{T} + \frac{t}{2}, t_2 = \mathcal{T} - \frac{t}{2}\Big)$$

Thermalization of Mixed SYK Model

Larzul&MS PRB(2022)

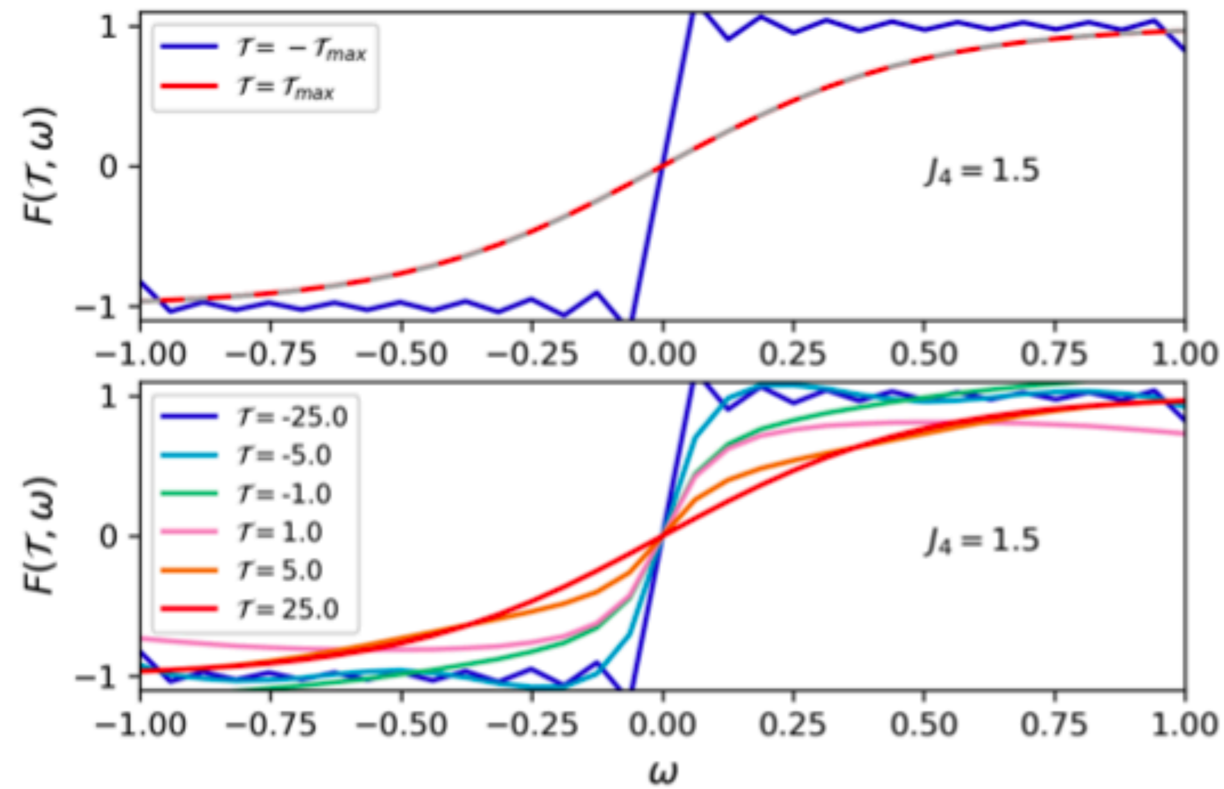
- Spectral Function

$$A(\mathcal{T},\omega) = -2\mathrm{Im}G^R(\mathcal{T},\omega)$$



- Distribution Function

$$iG^K(\mathcal{T},\omega) = F(\mathcal{T},\omega)A(\mathcal{T},\omega)$$



- Exactly solvable Large N model that shows thermalisation !

See Also Numerical/Analytical Studies of ETH
(for example J.Sonner)

- Allows to extract dynamics of Effective Temperature

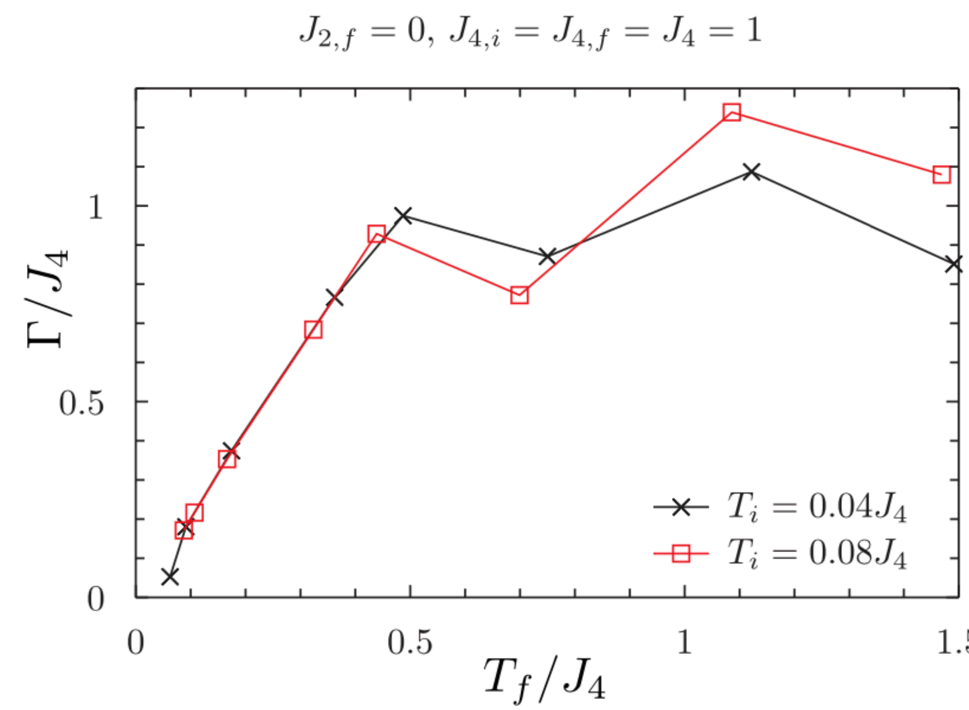
$$F(\mathcal{T},\omega) \equiv \tanh\left(\frac{\beta_{eff}(\mathcal{T})\omega}{2}\right)$$

- Effective temperature matches the energy injected

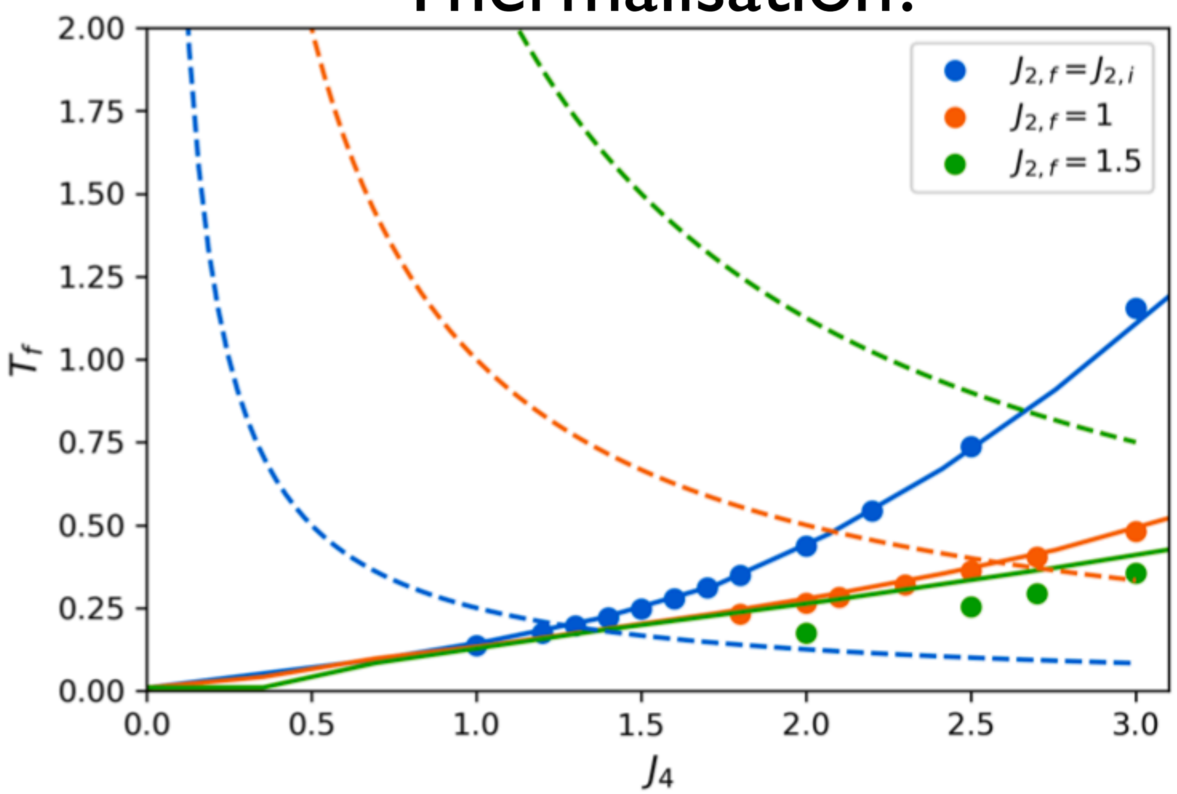
Thermalization time?

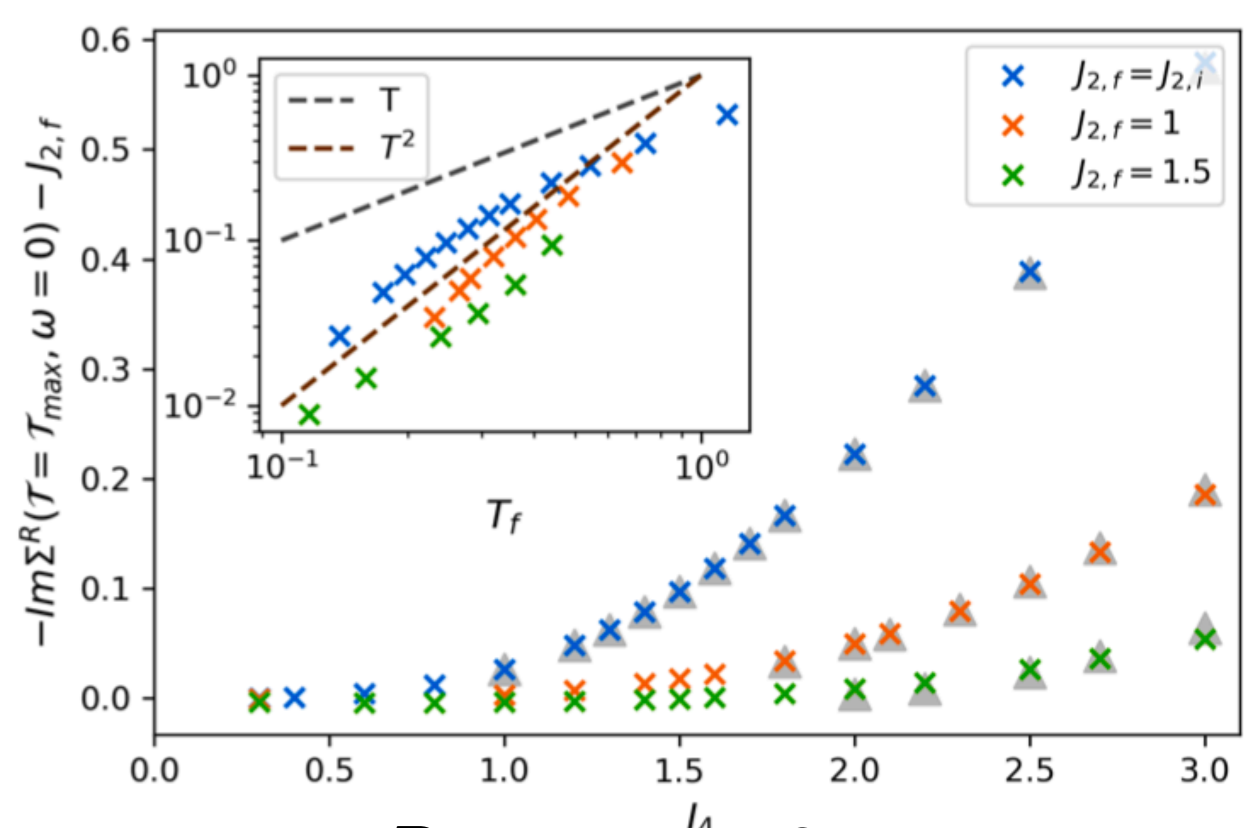
- Quenches in pure SYK4:
  “Planckian” Rate $\Gamma \sim T_f$

  Eberlein et al PRB(2017)



- Mixed SYK Role of Quasiparticles for Thermalisation?





- Slow down of thermalisation rate $\Gamma \sim \mathrm{Im}\Sigma^R(T) \sim T_f^2$

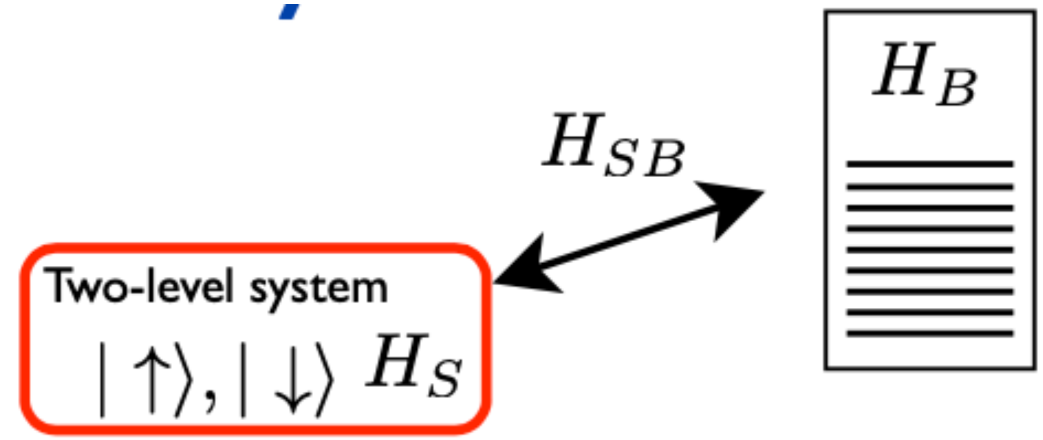
Larzul&MS PRB(2022)

SYK Model as a “Strange” Quantum Bath

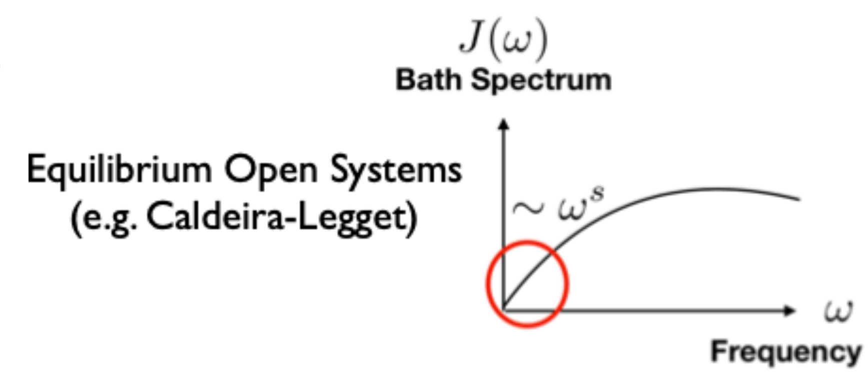
- *A. Larzul, S. J. Thomson, M. Schiro’, arXiv:2204.06434*

Open Quantum Systems

- System+Bath Picture cft. Breuer&Petruccione



- Bath is usually treated as a set of non-interacting degrees of freedom (electrons, phonons, photons,..)



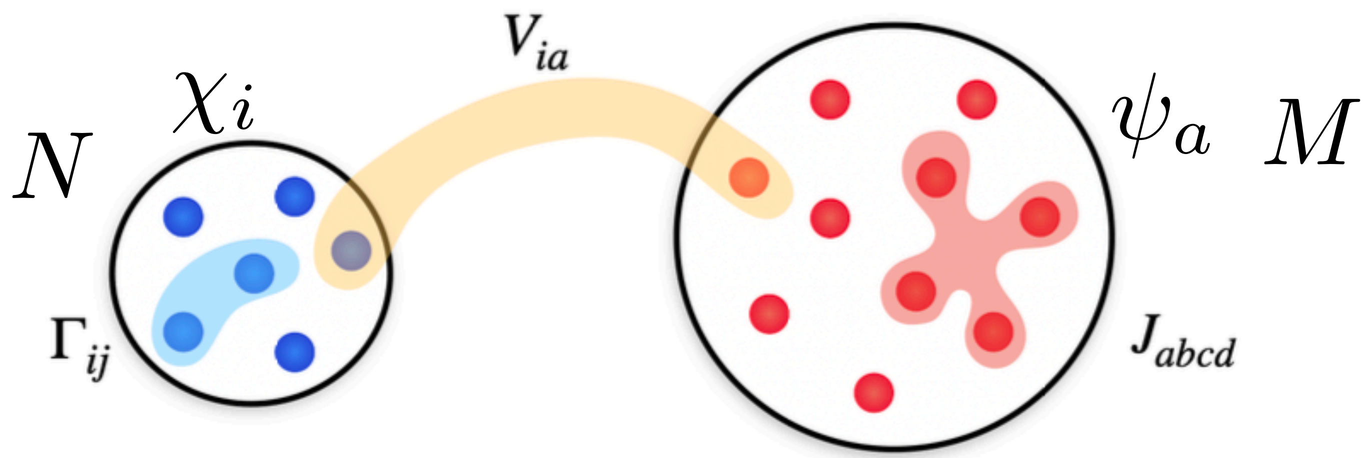
- Physics of "interacting" quantum baths?

- SYK4 provides an example of critical (gapless) bath without quasiparticles

- Is it SYK4 a good "thermalizer"?

Are Fast Scramblers Good Thermal Baths?

- Couple a small SYK2 "System" to a large SYK4 "Bath"



$$M \gg N$$

$$H(t) = \frac{i}{2}\sum_{i,j=1}^{N}\Gamma_{i,j}\,\chi_i\chi_j + H_B[\psi] + \theta(t)H_{SB}[\chi,\psi]$$

Related Models (Banjeree&Altman'17, Zhang '19, Cheipesh'20, Can'20,..)

- Bath fully interacting and described by SYK4

$$H_B^{(4)} = -\frac{1}{4!}\sum_{a,b,c,d=1}^{M} J_{abcd}\,\psi_a\psi_b\psi_c\psi_d$$

- Sudden switching of linear system-bath coupling

$$\overline{J_{abcd}^2} = \frac{3!J^2}{M^3},\ \overline{V_{ia}^2} = \frac{V^2}{M}$$

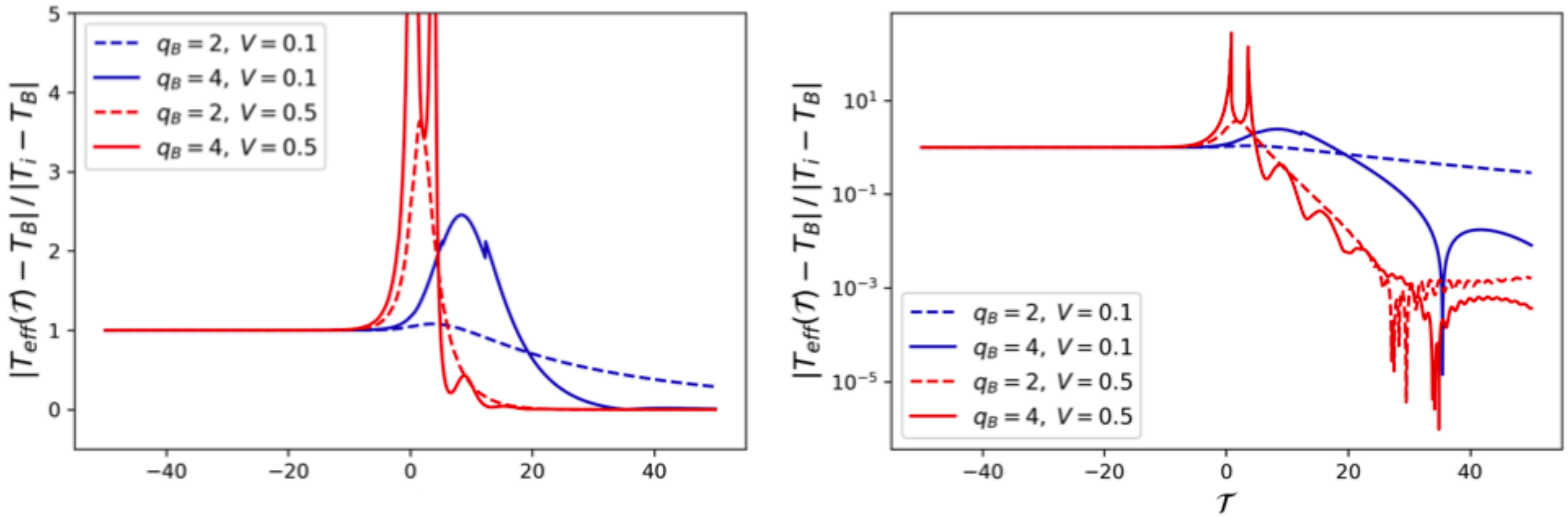
$$H_{SB} = i\sum_{i=1}^{N}\sum_{a=1}^{M} V_{ia}\,\chi_i\psi_a$$

$$\overline{\Gamma_{ij}^2} = \frac{\Gamma^2}{N}$$

- Compare with the case in which both system and bath are SYK2

Dynamics of Effective Temperature

- Initial Condition: System hotter than the bath $T_S > T_B$
- Compare Thermalization Rate for SYK4 and SYK2 baths



$$T_{eff}(\mathcal{T}) = T_B + (T_S - T_B)e^{-\lambda \mathcal{T}}$$

- Thermalization to the bath temperature
- Thermalization rate $\lambda(V)$ at weak and strong system-bath coupling V
- SYK4 bath is a <u>better thermalizer at small V</u> than large V

Weak-Coupling: Fast thermalisation to a "Strange" Bath

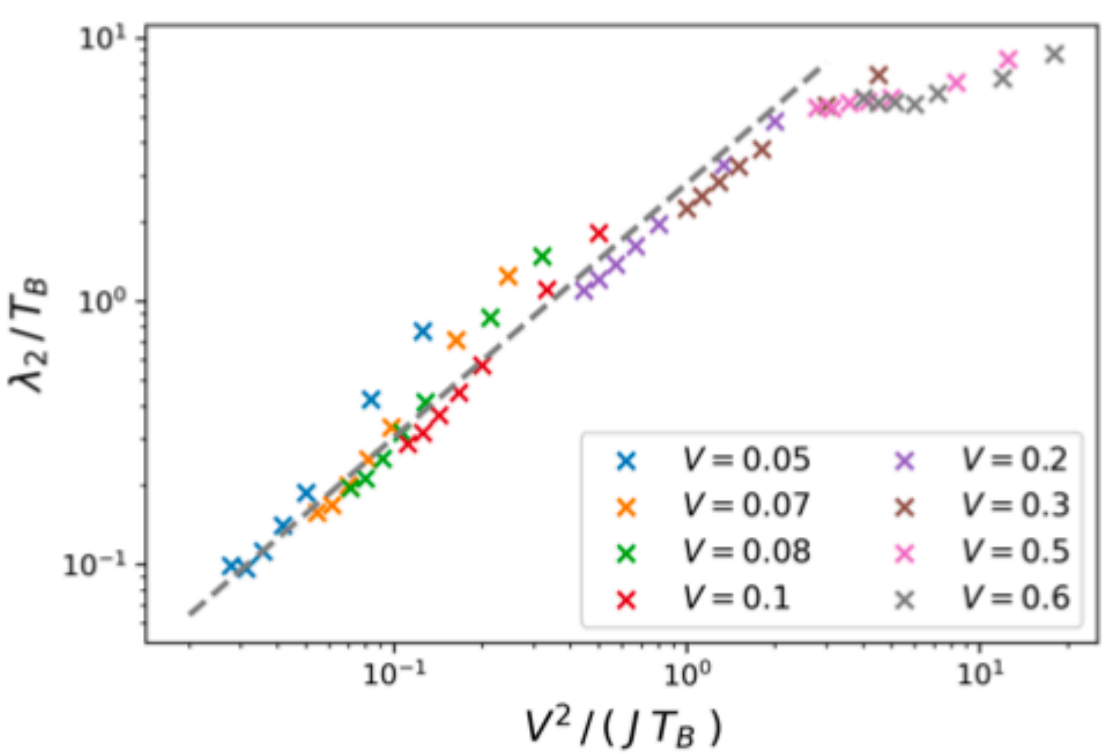
Kadanoff-Baym Equation
(Gradient expansion)

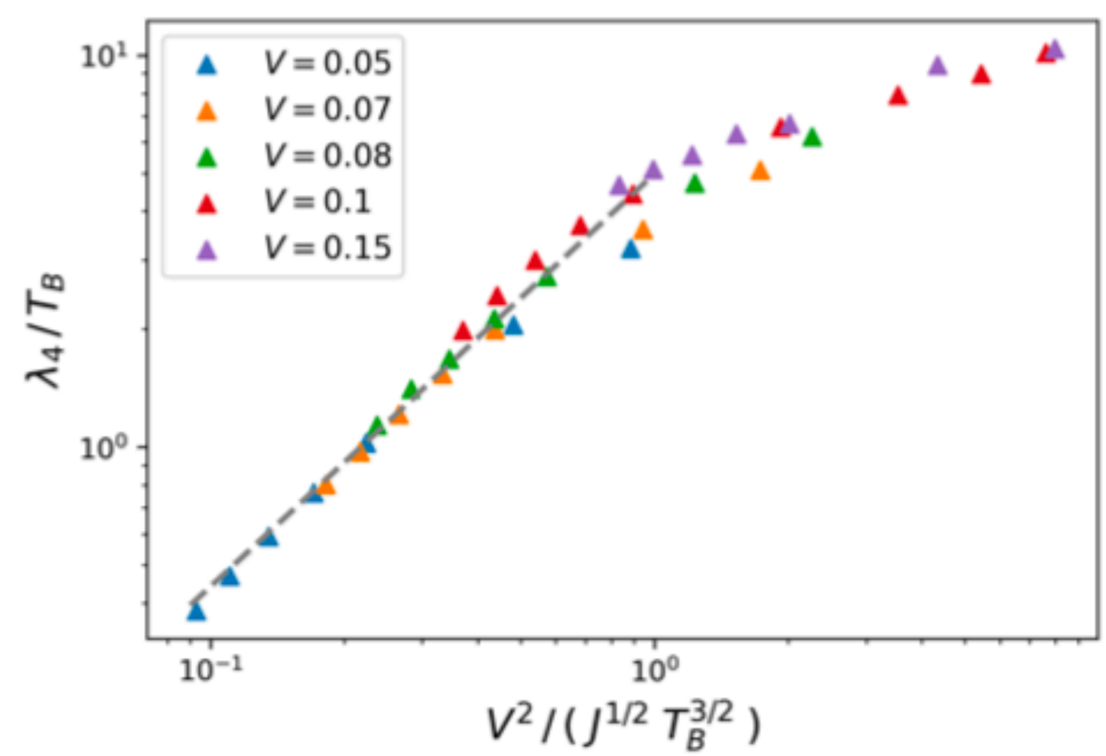
$$\partial_{\mathcal{T}} F_S(\mathcal{T},\omega) = i\Sigma_S^K(\mathcal{T},\omega) - iF_S(\mathcal{T},\omega)\left(\Sigma_S^R(\mathcal{T},\omega) - \Sigma^R(\mathcal{T},\omega)\right)$$

$$F_S(\mathcal{T},\omega) = \tanh\left(\frac{\beta_f\omega}{2}\right) + \left[F_S(\mathcal{T}_0,\omega) - \tanh\left(\frac{\beta_f\omega}{2}\right)\right]e^{-\lambda(\omega)(\mathcal{T}-\mathcal{T}_0)}$$

- Thermalization time for SYK2 bath vs SYK4 bath

$$\lambda_2 \sim V^2/J \qquad \lambda_4 \sim V^2/\sqrt{JT_B} \qquad \lambda_4 \gg \lambda_2$$





- SYK4 bath is parametrically more effective than a FL bath as "thermaliser"

Energy Transport Through a Strange Quantum Bath

- *A. Larzul, M. Schiro', arXiv.2206.XXX (to appear)*

Thermal Transport of Quasiparticles

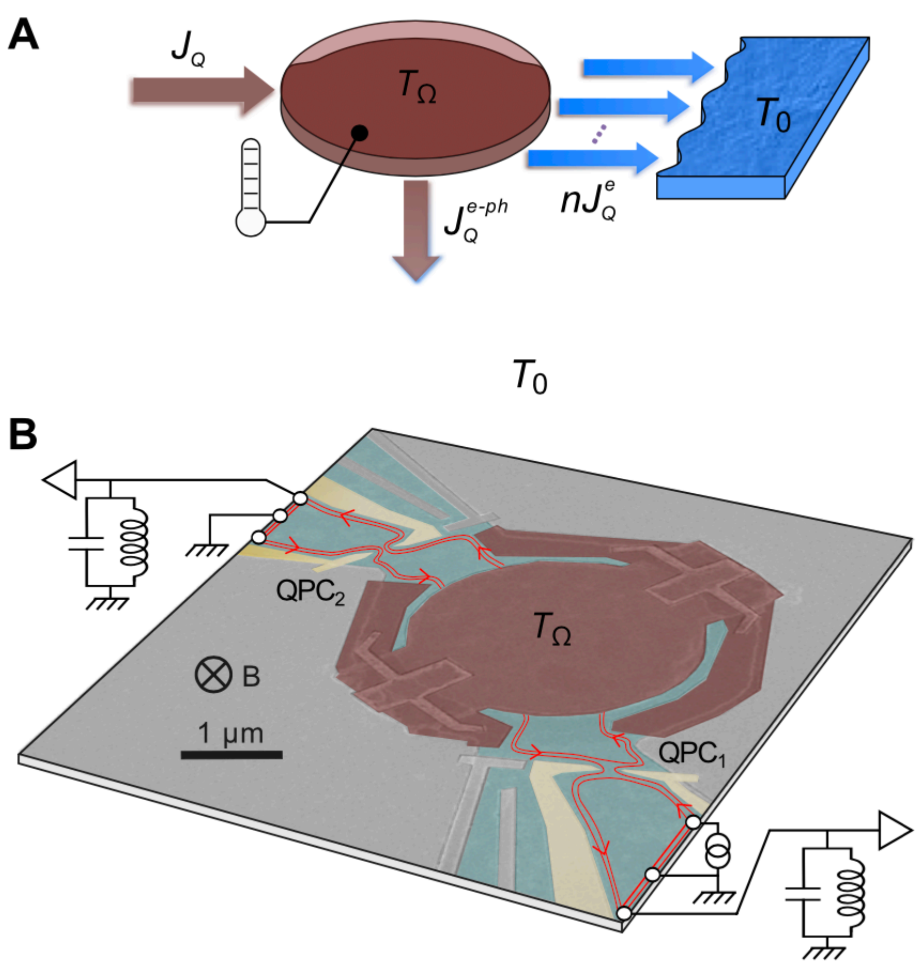
- Example: Ballistic channel (Landauer-Buttiker)

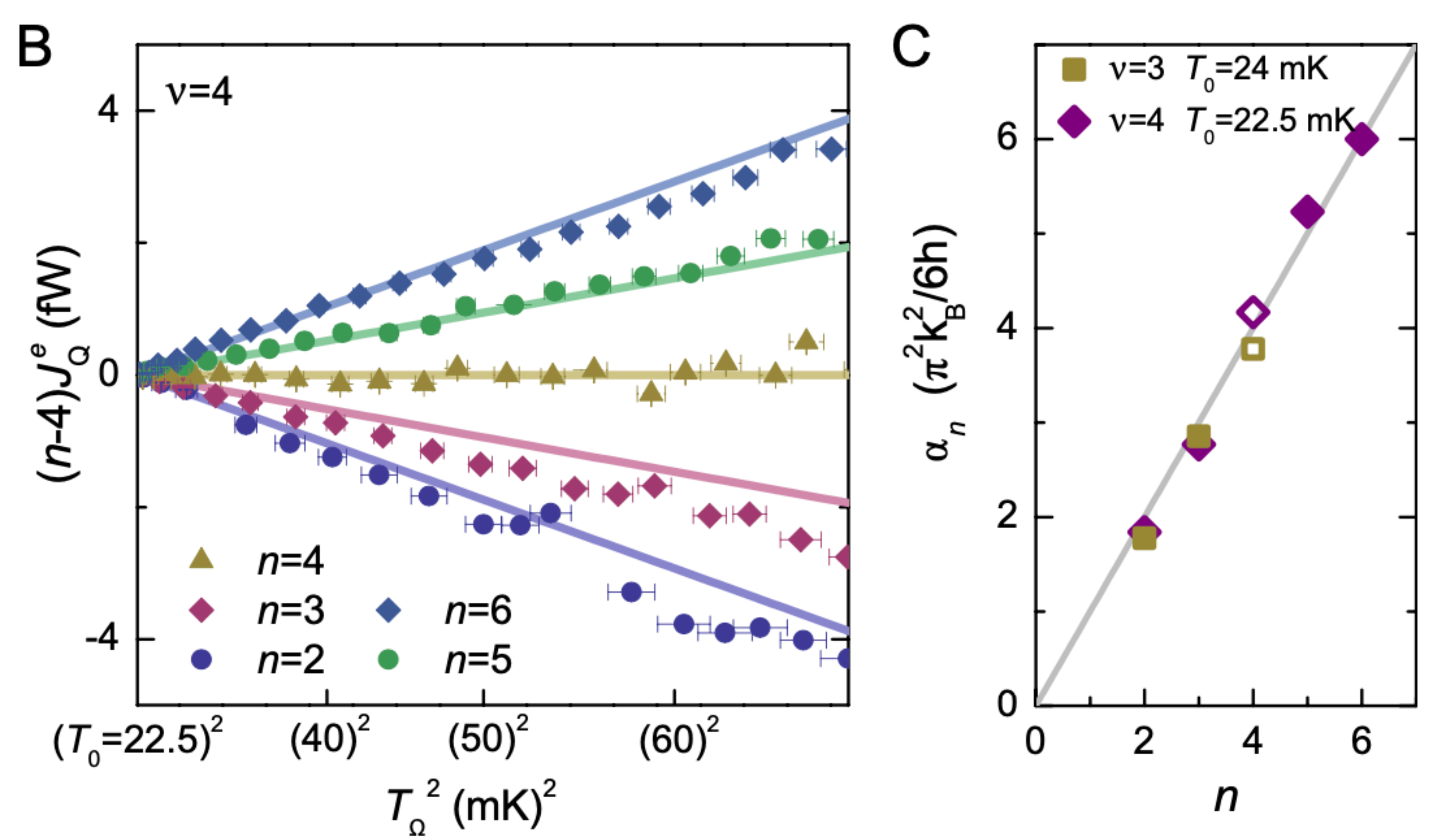
J. Pekola&B. Karimi RMP(2021)

$$J_Q^e(T_\Omega, T_0) = \frac{\pi^2 k_B^2}{6h}(T_\Omega^2 - T_0^2).$$

$$G(T) = \frac{\pi^2 k_B^2 T}{3h}$$

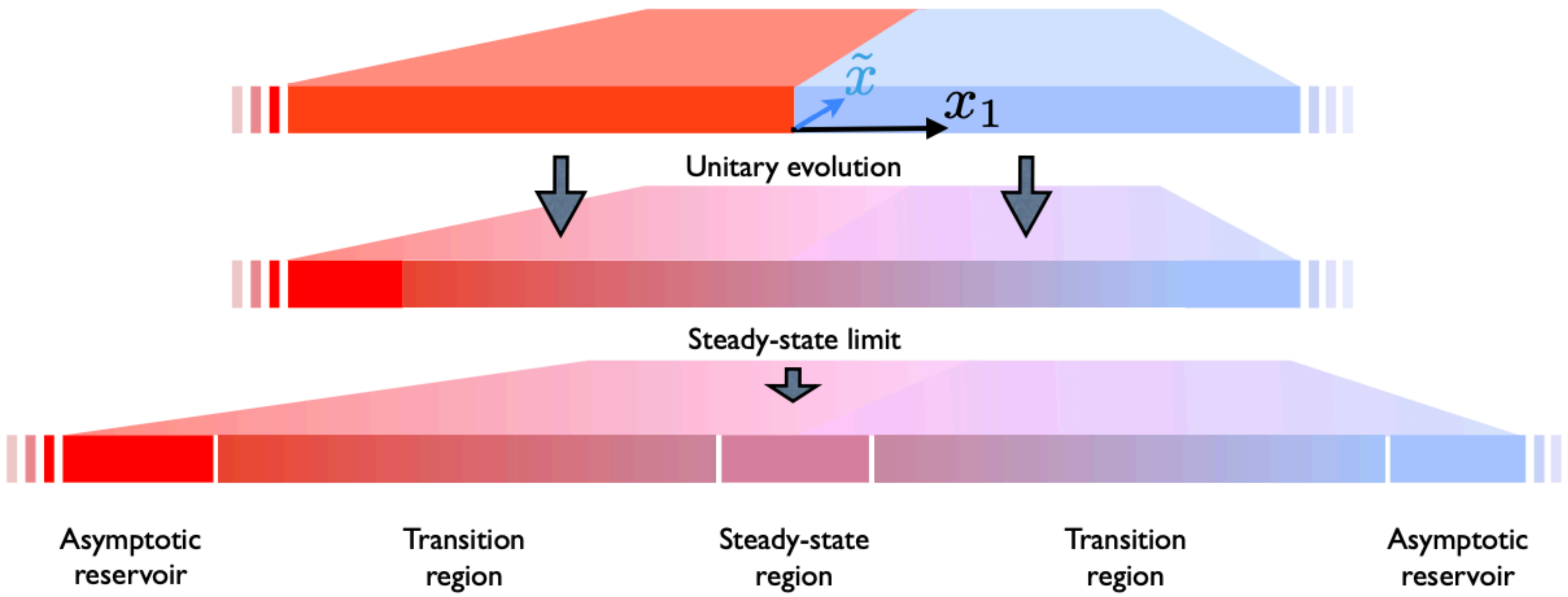
Quantum of Thermal Conductance





S. Jezouin,…,F. Pierre, Science (2013)

Thermal Transport from CFT



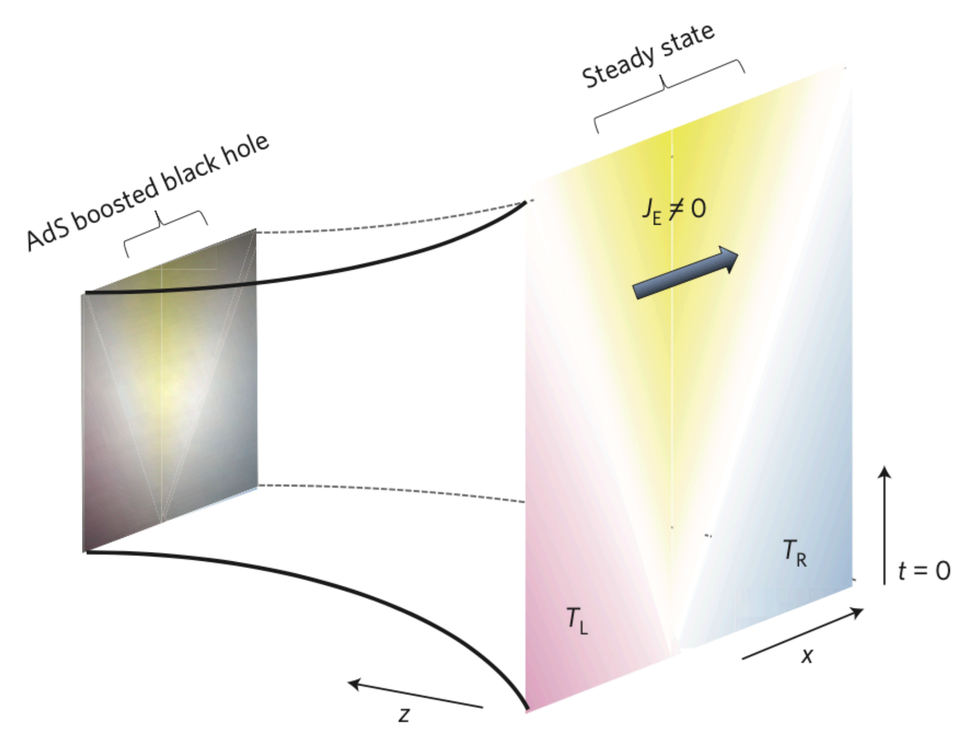
- Steady-State Energy Current from merging CFTs (d=1)

D. Bernard, B. Doyon (2012)

$$J = \frac{c\pi}{12\hbar} k_B^2 (T_l^2 - T_r^2).$$

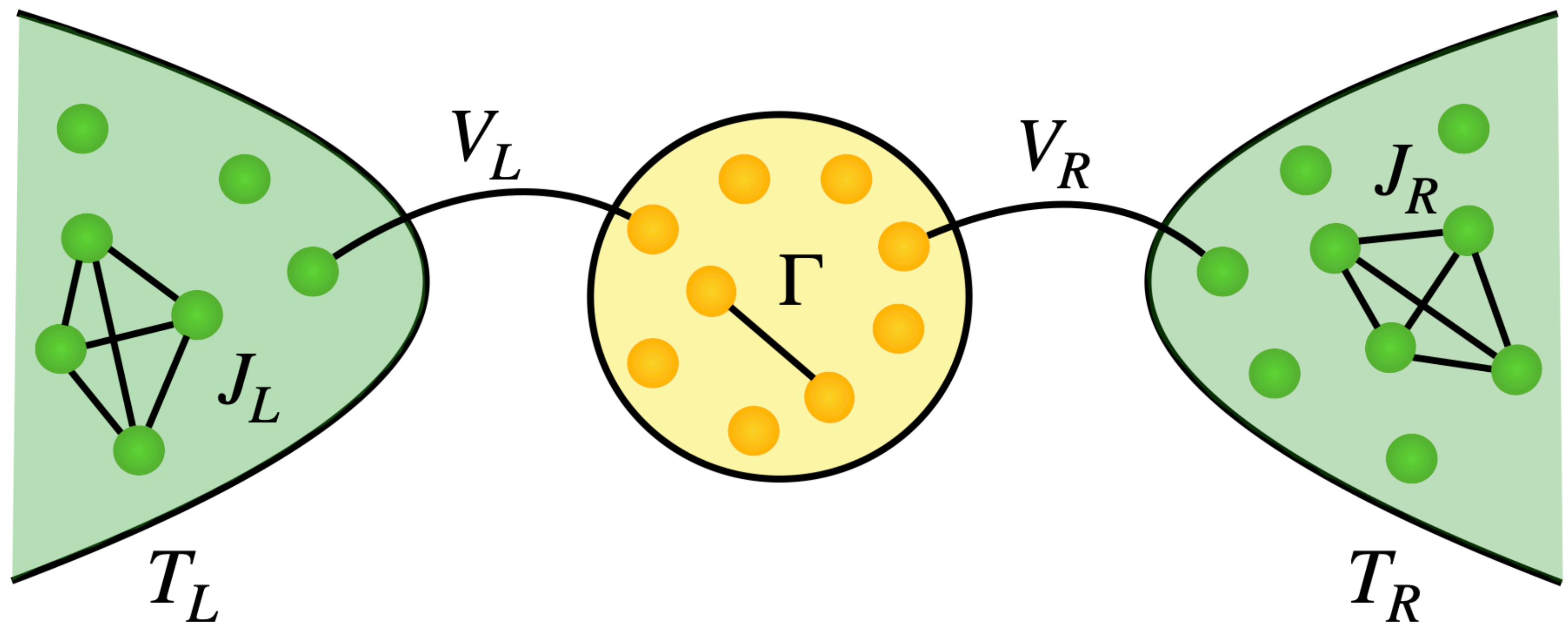
- Insights from AdS for d>1: "boosted Black Hole"

Bhaseen,…,Schalm, Nature Phys 2015



SYK Energy Transport Setup

- Two SYK4 bath at different temperatures coupled by a SYK2 island



$$H = \sum_{\alpha=L,R} H_4^\alpha + H^S + \sum_{\alpha=L,R} H^{S\alpha}$$

$$H_4^\alpha = -\frac{1}{4!}\sum_{a,b,c,d=1}^{M} J_{abcd}\, \psi_a^\alpha \psi_b^\alpha \psi_c^\alpha \psi_d^\alpha \qquad \overline{J_{abcd}^2} = \frac{3!J^2}{M^3}, \quad \overline{V_{ia}^2} = \frac{V^2}{M}$$

$$H^S = \frac{i}{2}\sum_{i,j=1}^{N} \Gamma_{ij}\, \chi_i \chi_j \qquad H^{S\alpha} = i\sum_{i=1}^{N}\sum_{a=1}^{M} V_{ia}\, \chi_i \psi_a^\alpha \qquad \overline{\Gamma_{ij}^2} = \frac{\Gamma^2}{N}$$

- Exactly solvable in large N,M at fixed $p = N/M$

Energy Current Between SYK

- Focus here: Energy Current in the stationary state $\mathcal{J}_\alpha = \overline{\dot{E}_\alpha(t)} = i\overline{\langle [H, H_\alpha] \rangle}(t)$

- Exact Formula in large N,M limit

$$\mathcal{J} = -\frac{NV^2}{2} \int \frac{d\omega}{2\pi} \omega \left(G_L^<(\omega) - G_R^<(\omega) \right) G_S^>(\omega)$$

- Current depends on Green's functions of L/R Baths and System (coupled through Dyson Equation)

- Analogous to Meir-Wingreen Formula for charge transport in quantum dots: here <u>interacting reservoirs</u>!

- Here: assume N<<M (large bath, not renormalised by the system)

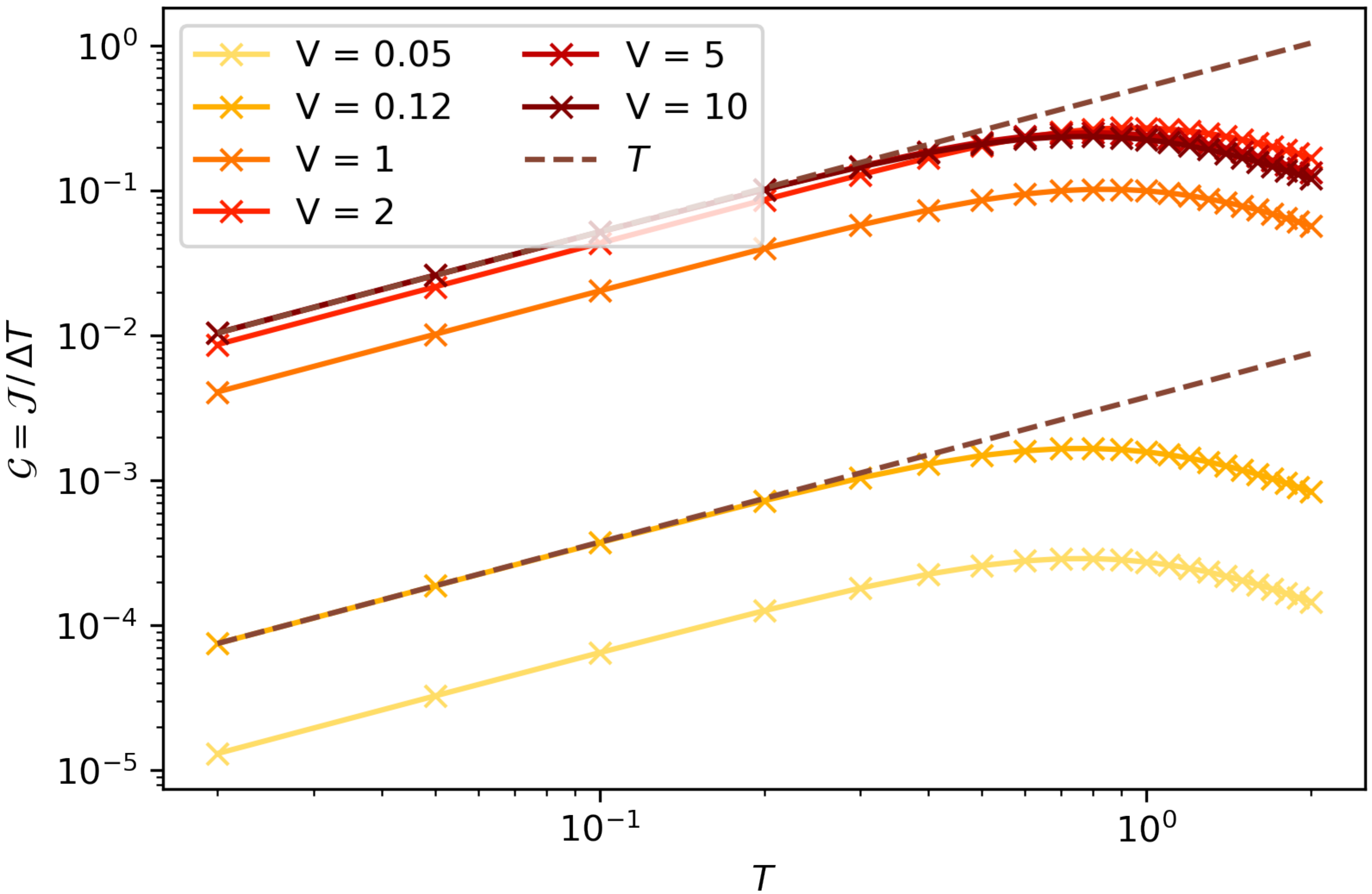
- Linear Thermal Conductance

$$\mathcal{G}(T) = \frac{NV^2}{2} \int \frac{d\omega}{2\pi} \omega \, A_S^{eq}(\omega) f_{eq}(-\omega) \frac{\partial}{\partial T} \left(A_B^{eq}(\omega) f_{eq}(\omega) \right)$$

Warmup: Energy Transport by Quasiparticles

Consider SYK2 baths $T_{L/R} = T \pm \Delta T$

Thermal Conductance $\mathcal{G} \sim \mathcal{J}/\Delta T$

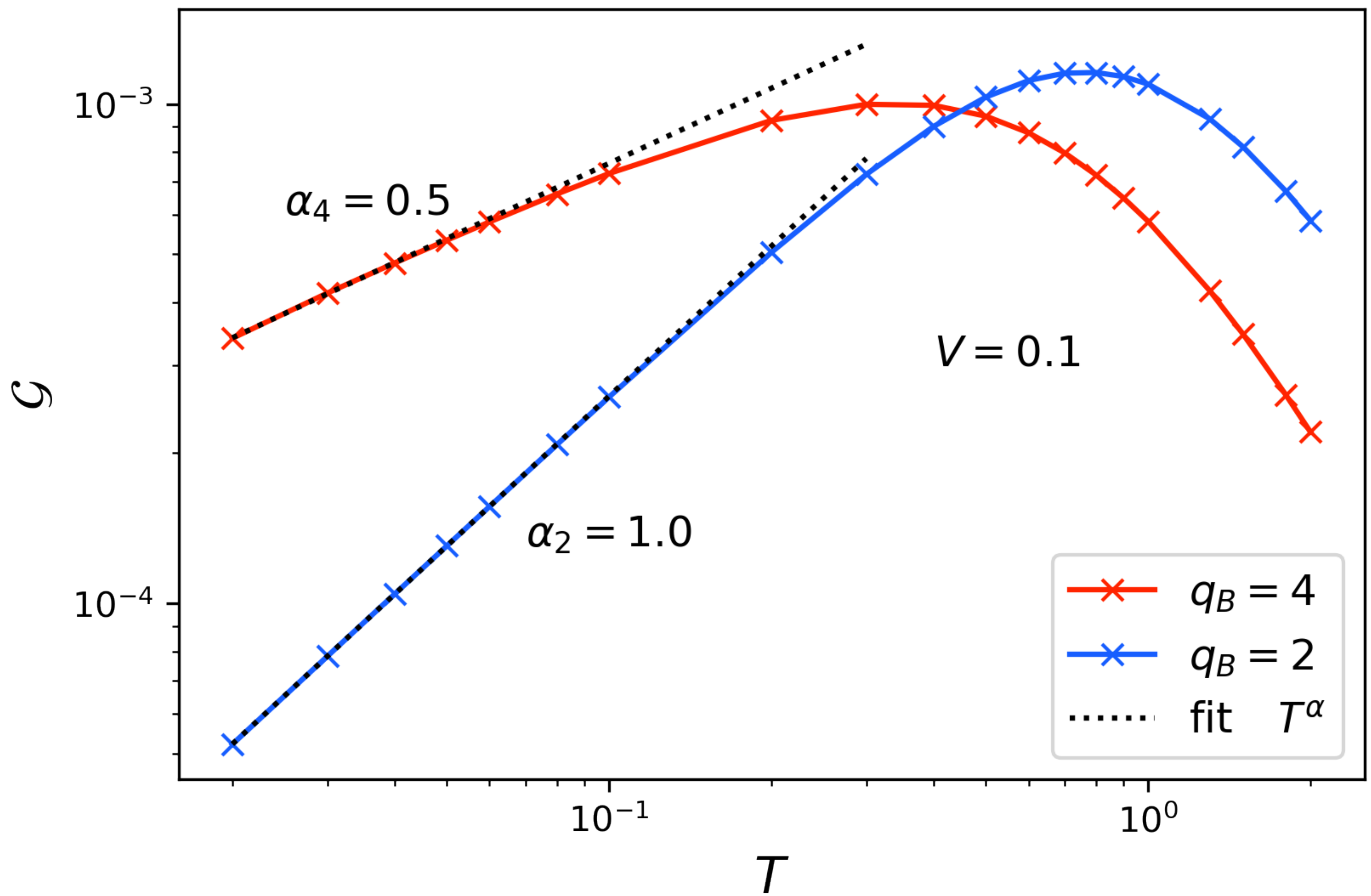


- Thermal Conductance is linear in T, for all couplings V

$$\mathcal{G}_2 \sim \mathcal{G}_Q(T) \sim \frac{\pi^2 k_B^2}{3h} T$$

SYK4 Baths: Weak System-Bath Coupling

Thermal Conductance $\mathcal{G} \sim \mathcal{J}/\Delta T \qquad T_{L/R} = T \pm \Delta T$



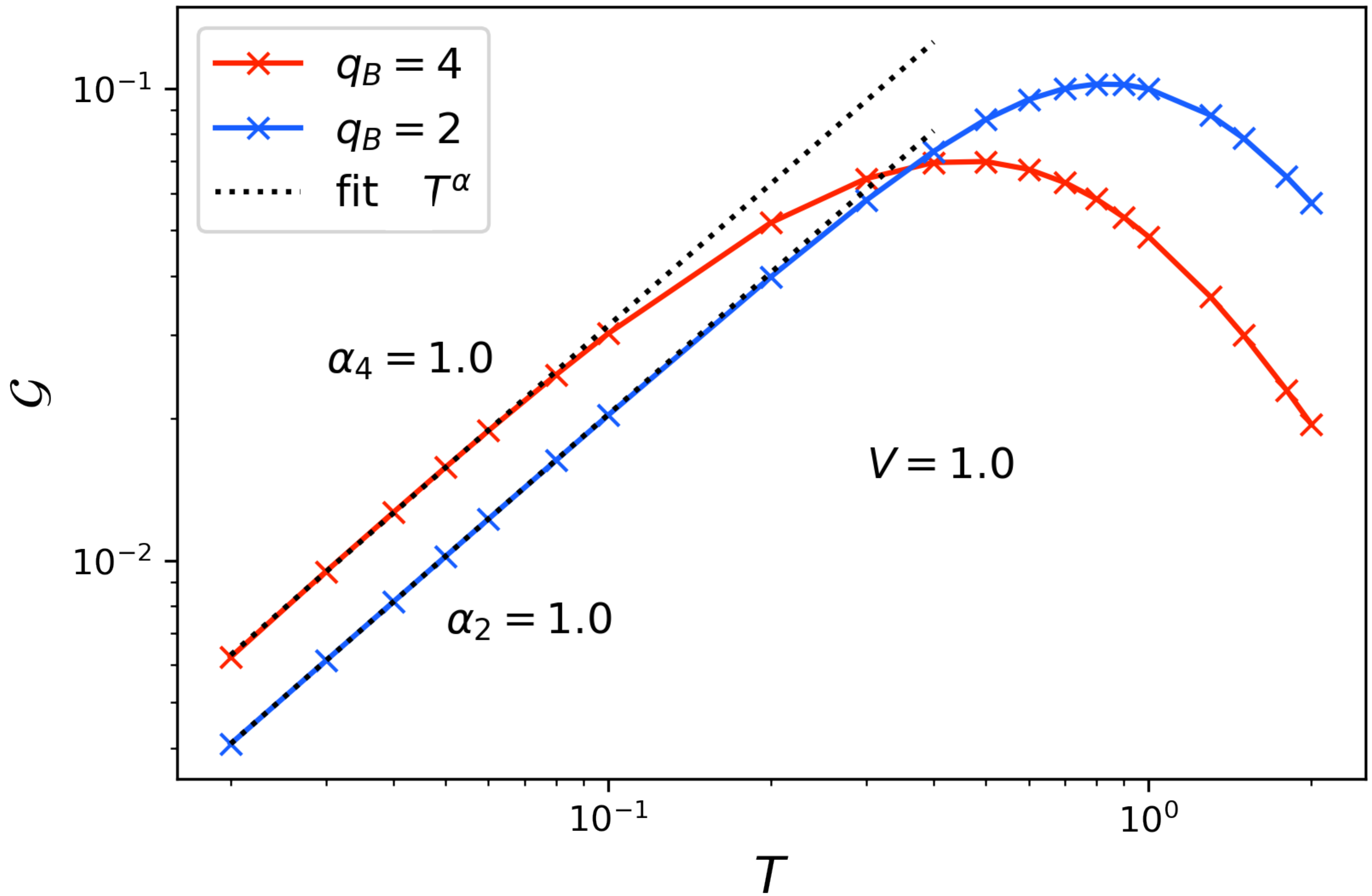
- Thermal conductance acquires anomalous scaling $\mathcal{G}_4(T) \sim \sqrt{T}$

SYK+FL Bath: Kruchov, Patel, Kim, Sachdev'20

- Enhancement of Thermal Conductance due to the interacting bath

SYK4 Baths: Strong System-Bath Coupling

Thermal Conductance $\mathcal{G} \sim \mathcal{J}/\Delta T$ $\quad T_{L/R} = T \pm \Delta T$



- Linear-T behaviour is restored $\mathcal{G}_4(T) \sim T$
- FL? No! Rather a different mechanism at play!

Insights from Spectral Functions

- Weak-Coupling: system is "decoupled" at low frequency…

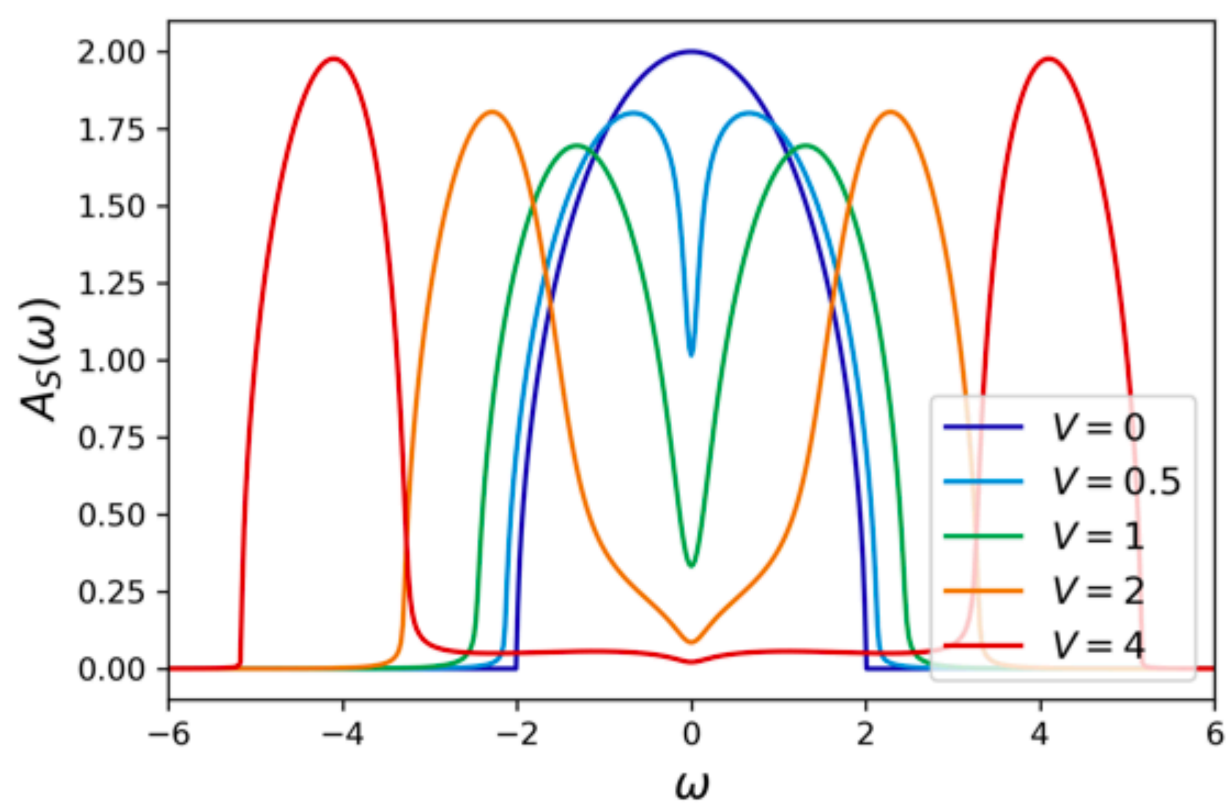
$$A_S^{eq}(\omega \to 0) \sim 2/\Gamma \qquad A_B^{eq}(\omega) = 2\left(\frac{\pi}{J^2}\right)^{1/4} \frac{1}{\sqrt{2\pi T}} \mathrm{Re}\left(\frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T}\right)}\right)$$

…Enhanced Bath Spectral function leads to enhanced thermal conductance

- Strong Coupling: System spectral function is largely renormalised

$$A_S^{eq}(\omega) = \frac{1}{V^2}\left(\frac{J^2}{\pi}\right)^{1/4} \sqrt{2\pi T}\, \mathrm{Re}\left(\frac{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T}\right)}{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T}\right)}\right)$$

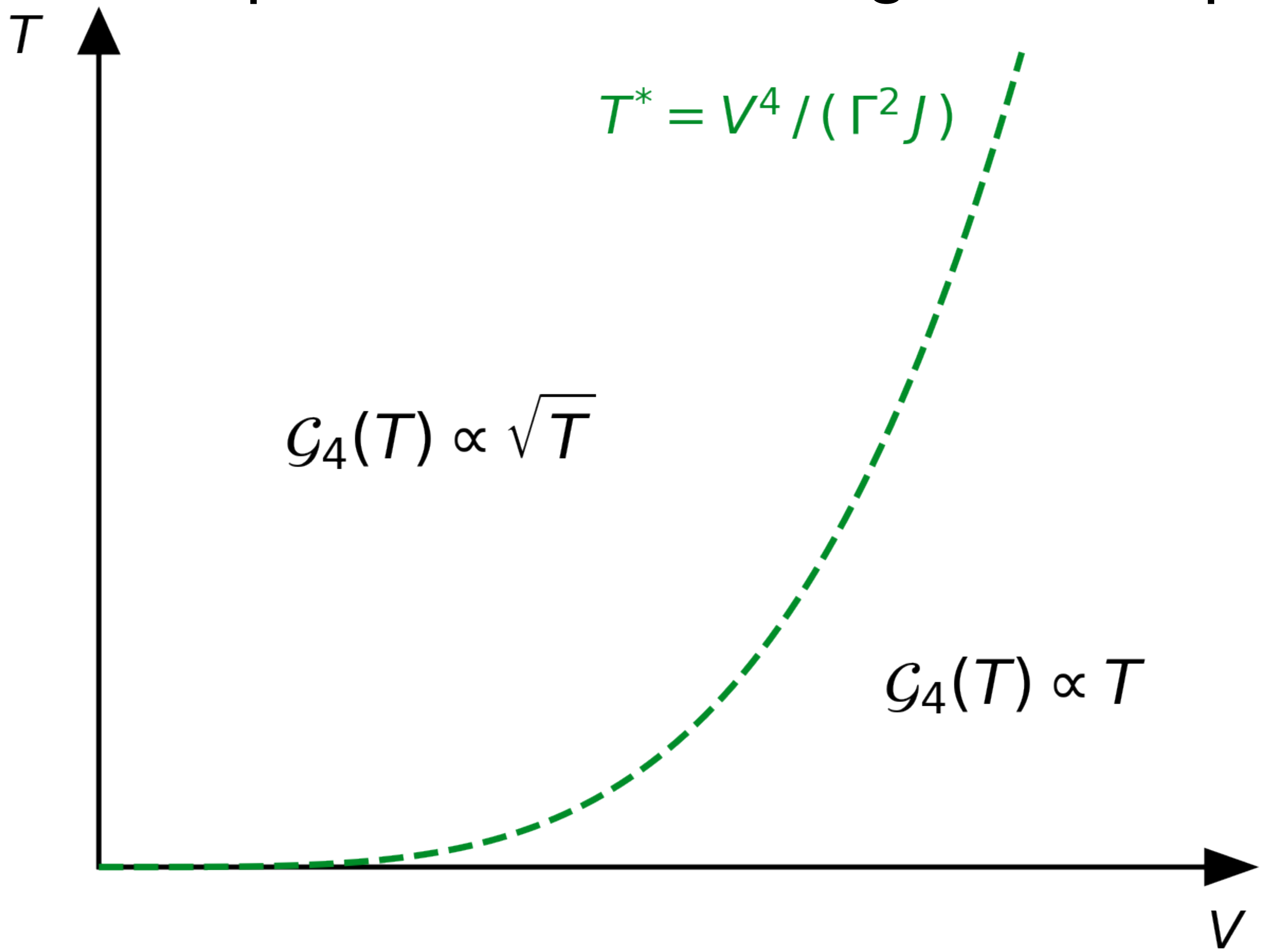
$$A_S^{eq}(\omega \to 0) \sim \sqrt{\omega}$$



…Suppressed System Spectral function cancels Bath enhancement and gives back to linear-T thermal conductance

Energy Transport Phase-Diagram

- Crossover in transport from weak to large bath coupling



- Weak-Coupling, T>T* — broad T range where conductance is enhanced by SYK4 bath

- Strong Coupling: conductance is linear in T…but <u>the system is not a FL!</u>

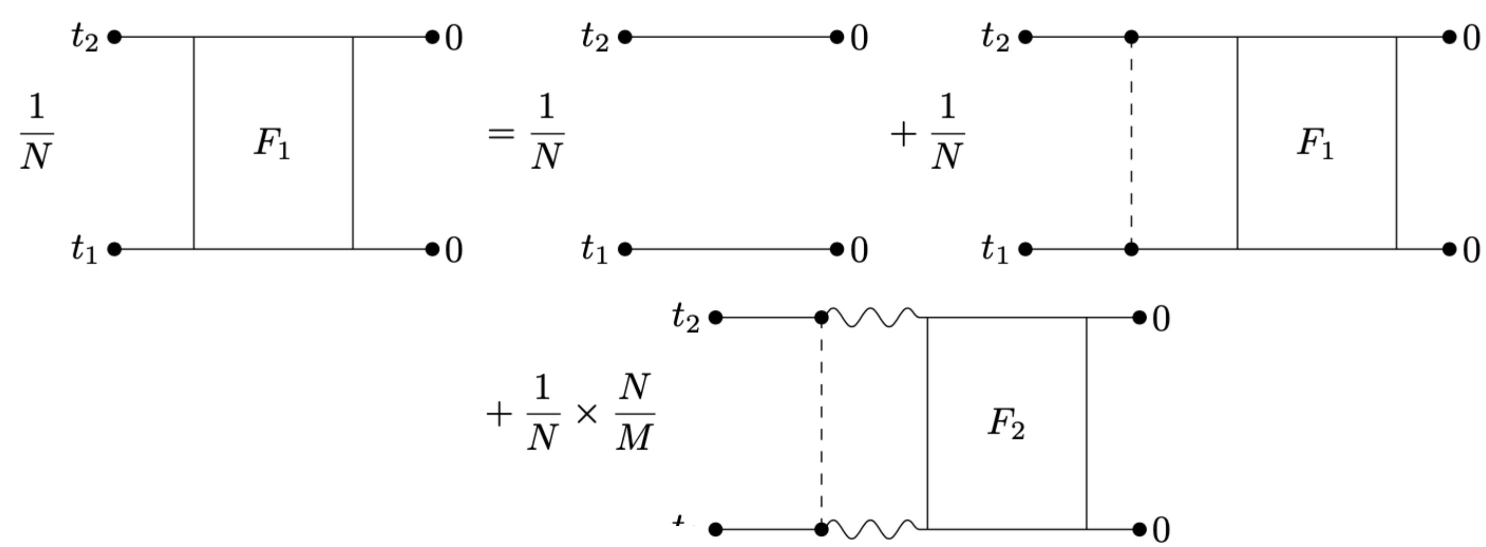
- T* signals a crossover in the Liapunov exponent

Chaos between System and Strange Bath

$$H = \frac{i}{2}\sum_{i,j=1}^{N}\Gamma_{ij}\chi_i\chi_j - \frac{1}{4!}\sum_{m,n,p,q=1}^{M}\psi_m\psi_n\psi_p\psi_q + i\sum_{i=1}^{N}\sum_{m=1}^{M}V_{im}\chi_i\psi_m$$

- Isolated system SYK2 is non chaotic (Liapunov Exponent=0)
- Coupling to the SYK4 bath induce a non trivial correction to OTOC

$$C_1(t_1,t_2) = \frac{1}{N^2}\sum_{i,j=1}^{N}\left\langle \chi^r_{2,i}(t_1)\chi^a_{2,j}(0)\chi^r_{1,i}(t_2)\chi^a_{1,j}(0)\right\rangle_S = \frac{1}{N}F_1(t_1,t_2) + ..$$



Mixed-Bath OTOC: Chaos between system&bath

$$C_2(t_1,t_2) = \frac{1}{NM}\sum_{i}^{N}\sum_{m=1}^{M}\left\langle \psi^r_{2,m}(t_1)\chi^a_{2,i}(0)\psi^r_{1,m}(t_2)\chi^a_{1,i}(0)\right\rangle_S = \frac{1}{N}F_2(t_1,t_2)$$

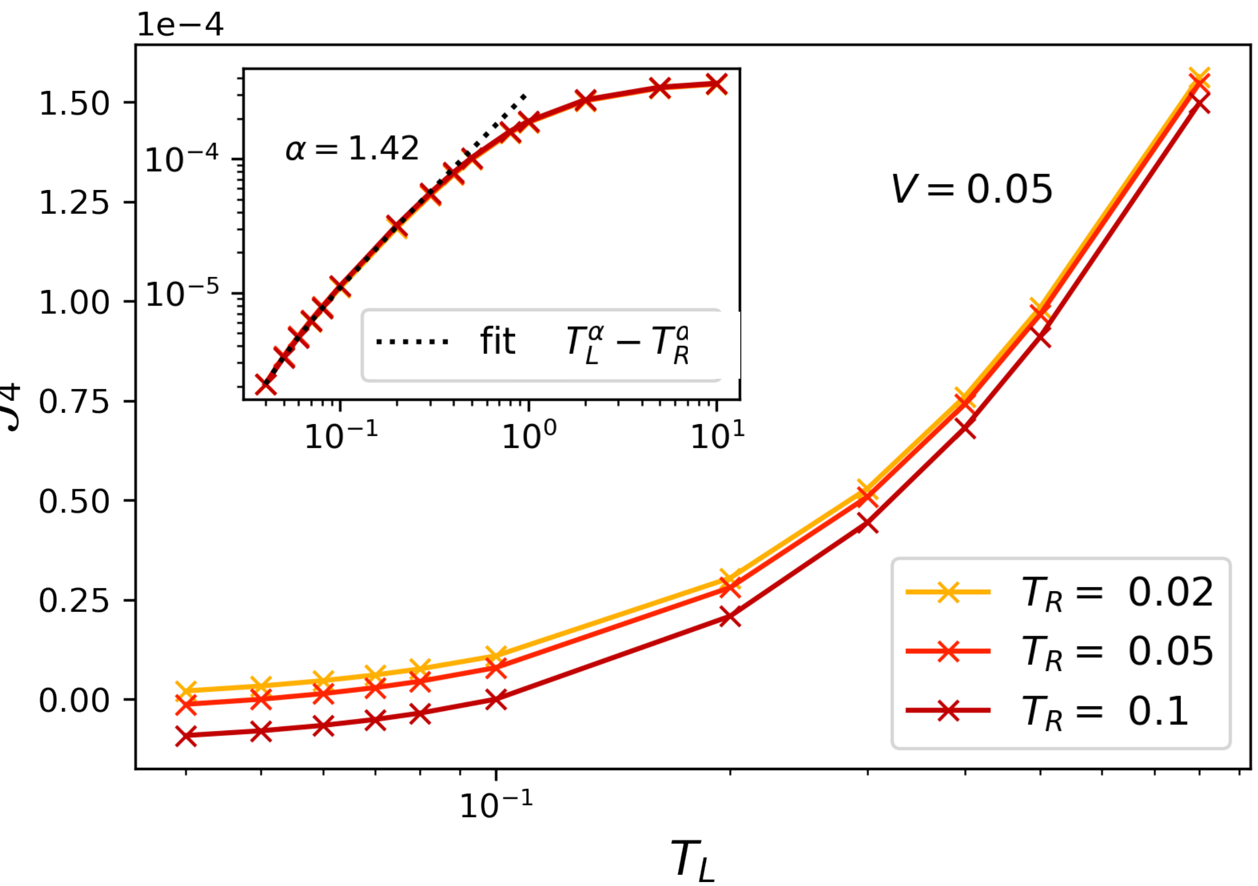
- Below T* Liapunov Exponent crosses over to the maximal value

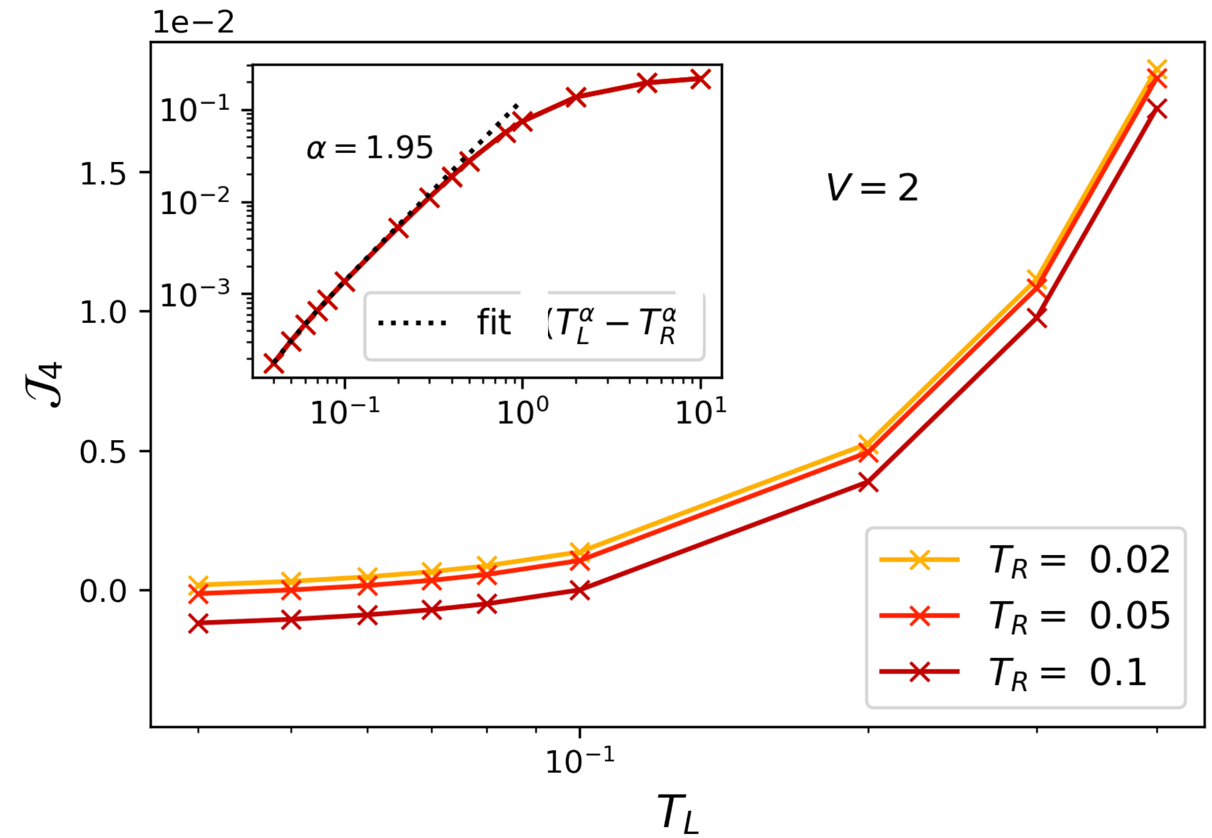
Non-Equilibrium Energy Transport

- Energy current beyond linear response: $\Delta T \sim T_L, T_R$
- SYK2 Baths $\mathcal{J} = \Phi(T_L) - \Phi(T_R)$ $\Phi(T) \sim T^2$

D. Bernard, B. Doyon (2012)

- SYK4 Baths:





- Scaling collapse support still the structure $\mathcal{J} = \Phi(T_L) - \Phi(T_R)$
- For weak-coupling: $\Phi(T) \sim T^{3/2}$
- For large-coupling: $\Phi(T) \sim T^2$

Conclusions&Perspectives

- [x] "Strange" Quantum Matter Far From Equilibrium
- [x] SYK Model as "Strange Quantum Bath": fast thermalisation dynamics
- [x] Energy Transport: Enhanced Conductance at weak coupling, cross-over to linear T scaling but still no QP (chaos crossover)

- [ ] Distribution function in non-equilibrium steady-state: boosting?
- [ ] Interplay of energy and particle transport (charged SYK)? Energy Current Fluctuations?
- [ ] Scrambling and Thermalisation of Non-Fermi Liquids?

Thanks!