SYK-superconductors and their holographic duals

Strange Metals, SYK Models and Beyond; June 2-3, 2022; Collège de France

Jörg Schmalian
Karlsruhe Institute of Technology

collaborators, references, funding …

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Karlsruhe

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- I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)
- D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. **417**, 168120 (2020)
- G. A. Inkof, K. Schalm, J. S., npj-Quantum Materials **7**, 56 (2022).
- V. Stangier, I. Esterlis, and J.S. preprint

GORDON AND BETTY MOORE FOUNDATION

DFG Deutsche Forschungsgemeinschaft
German Research Foundation

CRC– TRR 288

Elasto-Q-Mat

superconductivity: from microscopics to continuum's theory

conventional s.c.

BCS theory

L. P. Gor'kov (1959)

$$\psi(\boldsymbol{r},\tau) = V_{\mathrm{ep}} F(\boldsymbol{r},\tau;\boldsymbol{r},\tau)$$

$$e^* = 2e \qquad m = m(\rho_F, V_{\mathrm{ep}})$$

Ginzburg-Landau theory

Gor'kov ~1970

anomalous propagator $F(\boldsymbol{r},\tau;\boldsymbol{r}',\tau') = -\langle T\psi_\uparrow(\boldsymbol{r},\tau)\,\psi_\downarrow(\boldsymbol{r}',\tau')\rangle$

$$F\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2},\frac{\tau+\tau'}{2};\boldsymbol{k},\epsilon\right)$$

inhomogeneous

non-eq. dynamics

internal space-time structure of the pair

superconductivity: from microscopics to continuum's theory

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<u>conventional s.c.</u>

BCS theory

↓

L. P. Gor'kov (1959)

$$\psi(\boldsymbol{r},\tau) = V_{\mathrm{ep}} F(\boldsymbol{r},\tau;\boldsymbol{r},\tau)$$

$$e^* = 2e \qquad m = m(\rho_F, V_{\mathrm{ep}})$$

Ginzburg-Landau theory

<u>quantum critical s.c.</u>

SYK-type models

↓

$$F(\boldsymbol{r},\tau;\boldsymbol{k},\epsilon)$$

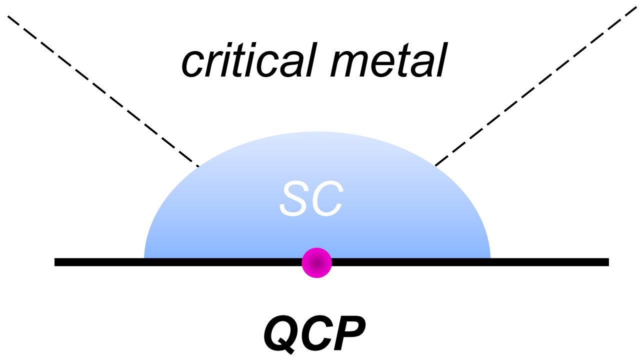
$$\updownarrow$$

$$\psi(\boldsymbol{r},\tau,\zeta)$$

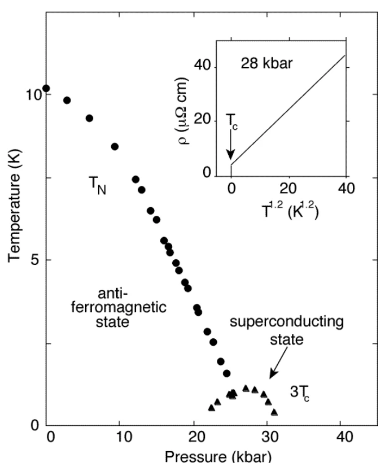
holographic s.c. in AdS_{D+1}

- mass and charge of the scalar field
- physical interpretation of the holographic dimension
- origin of gravity
- …

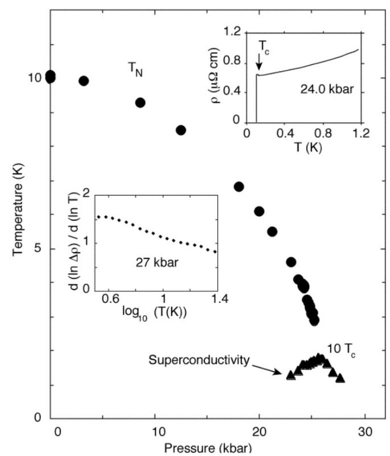
superconductivity and quantum criticality

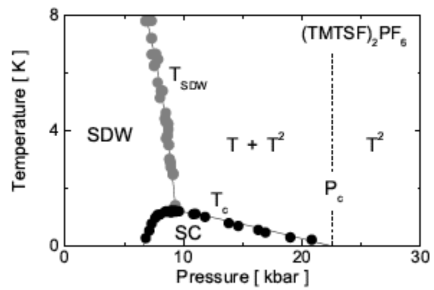


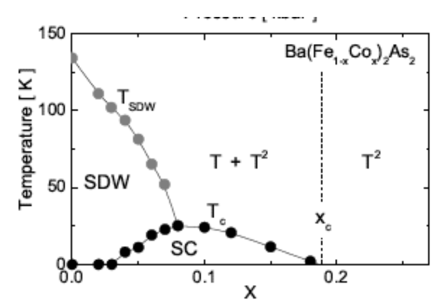
$CePd_2Si_2$



$CeIn_3$







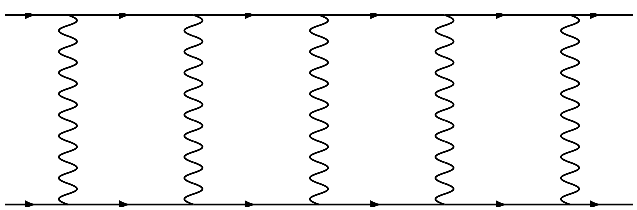
Q: Why is superconductivity in critical metals abundant?

N. D. Mathur, N. D. et al Nature **394**, 39 (1998), N. Doiron-Leyraud, et al. Phys. Rev. B **80,** 214531 (2009), S. Karahara et a. Phys. Rev B **81**, 184519 (2010).

Cooper instability

superconductivity:
the natural ground state of a good metal

Fermi liquid $$G_k\left(\omega\right) \sim \frac{Z_{\text{qp}}}{i\omega - \varepsilon_k} + \cdots \qquad \Sigma\left(\omega\right) \sim \left(Z_{\text{qp}}^{-1} - 1\right) i\omega$$



$$\chi^0_{\text{pair}} = \frac{\chi^0_{\text{pair}}}{1 - \lambda_{\text{pair}}\chi^0_{\text{pair}}}$$

$$\chi^0_{\text{pair}} = \int d\omega \int d\epsilon_k G_k(\omega) G_{-k}(-\omega) \sim \int d\omega \frac{1}{|\omega|} \sim \log\frac{D}{T}$$

→ instability $\quad T_c \sim D e^{-1/\lambda_{\text{pair}}}$

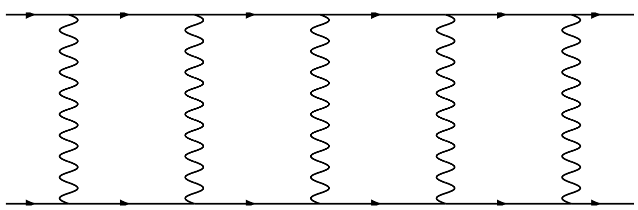
L. N. Cooper, Phys. Rev. **104**, 1189 (1956),
J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev **106**,162 (1957); **108**,1175 (1957)

Cooper problem and quantum criticality

instantaneous pairing: superconductivity at QCPs should be the exception, not the rule

quantum critical NFL (strange metal)

$$\Sigma\left(\omega\right)\sim -i\mathrm{sign}\left(\omega\right)\left|\omega\right|^{1-\gamma}$$



$$\chi^0_{\mathrm{pair}}=\frac{\chi^0_{\mathrm{pair}}}{1-\lambda_{\mathrm{pair}}\chi^0_{\mathrm{pair}}}$$

$$\chi^0_{\mathrm{pair}}=\int d\omega\int d\epsilon_k G_k(\omega)G_{-k}(-\omega)=\int d\omega\frac{1}{\left|\omega+g\omega^{1-\gamma}\right|}\sim \mathrm{finite}$$

superconductivity only if $\lambda_{\mathrm{pair}}>\lambda^*_{\mathrm{pair}}$

A. Balatsky, Philos. Mag. Lett. 68, 251 (1993); A. Sudbo, Phys. Rev. Lett. 74, 2575 (1995);
B. L. Yin and S. Chakravarty, Int. J. Mod. Phys. B 10, 805 (1996).

two quite different answers…

A: superconductivity by critical bosons $\Longleftrightarrow$ B: holographic superconductivity

A: superconductivity by critical bosons

$$S_{\mathrm{int}} \sim g_{ijk} \int \phi_k \psi_i^\dagger \psi_j$$

objections

Is this allowed, controlled … ?

(Migdal theorem, only perturbation theory …)

$$\Sigma\left(\omega\right) \sim -i \mathrm{sign}\left(\omega\right)\left|\omega\right|^{1-\gamma}$$

singular pairing interaction

$$\lambda_{\mathrm{pair}} \to \lambda_{\mathrm{pair}}(\omega) \propto \left|\omega\right|^{-\gamma}$$

generalized Cooper instability

D. T. Son, Phys. Rev. D **59**, 094019 (1999)
Ar. Abanov, A. Chubukov, and A. Finkel'stein, EPL **54**, 488 (2001)
Ar. Abanov, A. Chubukov, and J. S. EPL **55**, 369 (2001)
A. V. Chubukov and J. S., PRB **72**, 174520 (2005)
J.-H. She and J. Zaanen, PRB **80**, 184518 (2009)
M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015)

B: holographic superconductivity

$$S_{\mathrm{AdS}_{d+2}} = \int d^{d+2}x\sqrt{g}\left(D_a\psi^* D^a\psi + V\left(\psi\right)\right)$$

duality between QFT and gravity

objections

Is this relevant, quantitative … ?

(real life is not conformally invariant …)

critical modes, powerlaws …

pairing near AdS-black holes is possible; can be enhanced

Cooper pairs form in holographic metals

S. S. Gubser, *Phys. Rev.* D **78**, 065034 (2008)
S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, PRL **101**, 031601 (2008)

two quite different answers…

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A: superconductivity by critical bosons

$$S_{\text{int}} \sim g_{ijk} \int \phi_k \psi_i^\dagger \psi_j$$

bosonic dynamics gets dominated by fermions

critical fermions

$$\Sigma(\omega) \sim -i \text{sign}(\omega) |\omega|^{1-\gamma}$$

singular pairing interaction

$$\lambda_{\text{pair}} \to \lambda_{\text{pair}}(\omega) \propto |\omega|^{-\gamma}$$

generalized Cooper instability

D. T. Son, Phys. Rev. D **59**, 094019 (1999)
Ar. Abanov, A. Chubukov, and A. Finkel'stein, EPL **54**, 488 (2001)
Ar. Abanov, A. Chubukov, and J. S. EPL **55**, 369 (2001)
A. V. Chubukov and J. S., PRB **72**, 174520 (2005)
J.-H. She and J. Zaanen, PRB **80**, 184518 (2009)
M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015)

B: holographic superconductivity

$$S_{\text{AdS}_{d+2}} = \int d^{d+2}x \sqrt{g} \left(D_a \psi^* D^a \psi + V(\psi) \right)$$

duality between QFT and gravity theory in one extra dimension

$$Z_{\text{QFT}} \sim \int D\psi e^{-N^2 S_{\text{AdS}_{d+2}}[\psi]}$$

critical modes, powerlaws …

pairing near AdS-black holes is possible; can be enhanced

Cooper pairs form in holographic metals

S. S. Gubser, *Phys. Rev.* D **78**, 065034 (2008)
S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, PRL **101**, 031601 (2008)

Sachdev-Ye-Kitaev (SYK) model

model of non-dispersive fermions with random interactions

$$H = -\mu \sum_{i=1}^{N} c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{ijkl}^{N} U_{ij,kl} c_i^\dagger c_j^\dagger c_k c_l \qquad \overline{|U_{ij,kl}|^2} = U^2$$

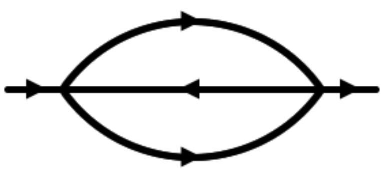
replicas, average, introduce bi-local fields (propagators, self energies)…

$$Z = \int DG(\tau,\tau')\, D\Sigma(\tau,\tau')\, e^{-S} \qquad S = -N\left(\mathrm{tr}\log(-\partial_\tau - \Sigma) + \int G\Sigma - U^2 \int G^2 G^2\right)$$

saddle point (large N):

$$\Sigma(\tau) = -U^2 G(\tau)^2\, G(-\tau)$$

$$(\partial_\tau + \Sigma)\, G = -\delta(\tau - \tau')$$

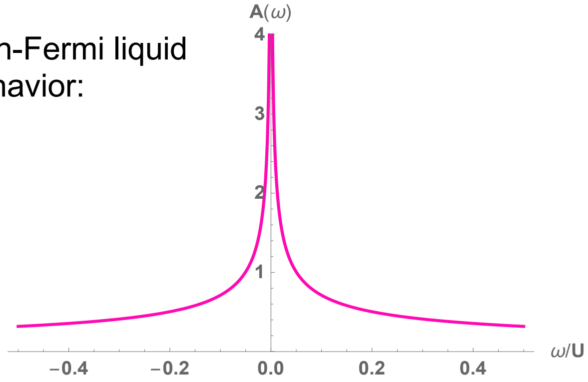


low-energy powerlaw solution $G(\tau,\tau') \sim \dfrac{\mathrm{sign}(\tau-\tau')}{|\tau-\tau'|^{2\Delta}}$ $\quad \Delta = \dfrac{1}{4}$

S. Sachdev and J. Ye, PRL, **70** 3339, (1993), A. Georges, O. Parcollet, and S. Sachdev PRL **85**, 840 (2000),
A. Kitaev, talk at KITP http://online.kitp.ucsb.edu/online/joint98/kitaev/, February, 2015.

Sachdev-Ye-Kitaev (SYK) model

Non-Fermi liquid behavior:



$$G\left(\omega\right) = -A_0 \frac{\mathrm{sign}\left(\omega\right) e^{\mathrm{sign}(\omega) i\pi\Delta}}{\left|\omega\right|^{1-2\Delta}}$$

IR-solution is re-parametrization invariant

$$\tau \to f(\tau)$$

explicitly broken by UV physics

$$Z = Z_0 \int Df e^{\alpha \frac{N}{U\beta} \int d\tau Sch[f,\tau]}$$

$$Sch\left[\phi,\tau\right] = \frac{\phi'''\left(\tau\right)}{\phi'\left(\tau\right)} - \frac{3}{2}\left(\frac{\phi''\left(\tau\right)}{\phi'\left(\tau\right)}\right)^2$$

closely related to gravity theories in AdS_2 (+dilaton fields)

$$S_{AdS_2} = -\frac{1}{8\pi G}\int d\tau Sch\left[\phi,\tau\right]$$

D. Stanford, J. Maldacena, Phys. Rev D 94 (2016)

S. Sachdev and J. Ye, PRL, **70** 3339, (1993), A. Georges, O. Parcollet, and S. Sachdev PRL **85**, 840 (2000), A. Kitaev, talk at KITP http://online.kitp.ucsb.edu/online/joint98/kitaev/, February, 2015.

Our model: Yukawa-SYK-model of electron-boson coupling

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$$H = -\mu \sum_{i=1}^{N} \sum_{\sigma=\pm} c_{i\sigma}^{\dagger} c_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \left(\pi_k^2 + \omega_0^2 \phi_k^2\right) + \frac{\sqrt{2}}{N} \sum_{ij,\sigma}^{N} \sum_{k}^{M} g_{ij,k} c_{i\sigma}^{\dagger} c_{j\sigma} \phi_k,$$

I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)

related models:

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, Phys. Rev. D **95**, 026009 (2017) → SUSY SYK-model
E. Marcus and S. Vandoren, Journal of High Energy Physics, **166** (2019) → Majorana fermions
Y. Wang, Phys. Rev. Lett. **124**, 017002 (2020). → superconductivity due to 1/N corrections
D. Chowdhury and E. Berg, Phys. Rev. Research **2**, 013301 (2020) → s.c. from purely fermionic couplings

extensions to finite dimensions:

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021) → Dirac systems
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021) → compressible Fermi systems

Our model: Yukawa-SYK-model of electron-boson coupling

$$H = -\mu \sum_{i=1}^{N} \sum_{\sigma=\pm} c_{i\sigma}^{\dagger} c_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \left(\pi_k^2 + \omega_0^2 \phi_k^2\right) + \frac{\sqrt{2}}{N} \sum_{ij,\sigma}^{N} \sum_{k}^{M} g_{ij,k} c_{i\sigma}^{\dagger} c_{j\sigma} \phi_k,$$

random electron-phonon coupling $g_{ij,k} = g'_{ij,k} + i g''_{ij,k}$

$$\overline{g'_{ij,k} g'_{i'j',k'}} = \left(1 - \frac{\alpha}{2}\right) \bar{g}^2 \delta_{k,k'} \left(\delta_{ii'} \delta_{jj'} + \delta_{ij'} \delta_{ji'}\right)$$

$$\overline{g''_{ij,k} g''_{i'j',k'}} = \frac{\alpha}{2} \bar{g}^2 \delta_{k,k'} \left(\delta_{ii'} \delta_{jj'} - \delta_{ij'} \delta_{ji'}\right)$$

$\alpha = 0$	Gaussian orthogonal ensemble → time reversal symmetry for each configuration	**superconductivity**
$\alpha = 1$	Gaussian unitary ensemble → max. TRS-breaking	**no s.c.**
$0 < \alpha < 1$	"partial" TRS-breaking	**pair breaking**

I. Esterlis and J.S., Phys. Rev. B **100**, 115132 (2019)

D. Hauck, M. J.Klug, I. Esterlis, J.S., Ann. of Phys. **417**, 168120 (2020)

replicas, averaging, bilocal fields,…

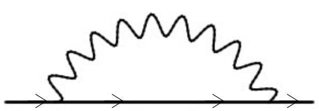
$$Z = \int DG D\Sigma DF D\Phi D\mathcal{D} D\Pi e^{-NS}$$

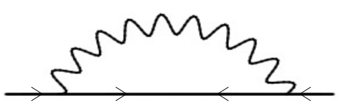
additional bilocal pairing fields: $F(\tau,\tau') \sim c_\uparrow(\tau) c_\downarrow(\tau')$ and $\Phi(\tau,\tau')$

solution at $N \to \infty$

Nambu-Gor'kov propagator, self energy

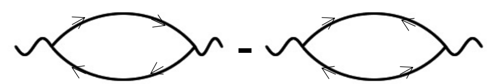
$$\hat{G}(i\omega)^{-1} = \begin{pmatrix} i\omega - \Sigma(i\omega) & \Phi(i\omega) \\ \Phi(i\omega) & i\omega + \Sigma(-i\omega) \end{pmatrix}$$

$\Sigma(i\omega) =$ 

$\Phi(i\omega) =$ 

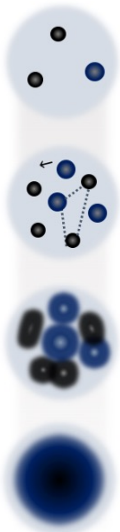
boson propagator, self energy

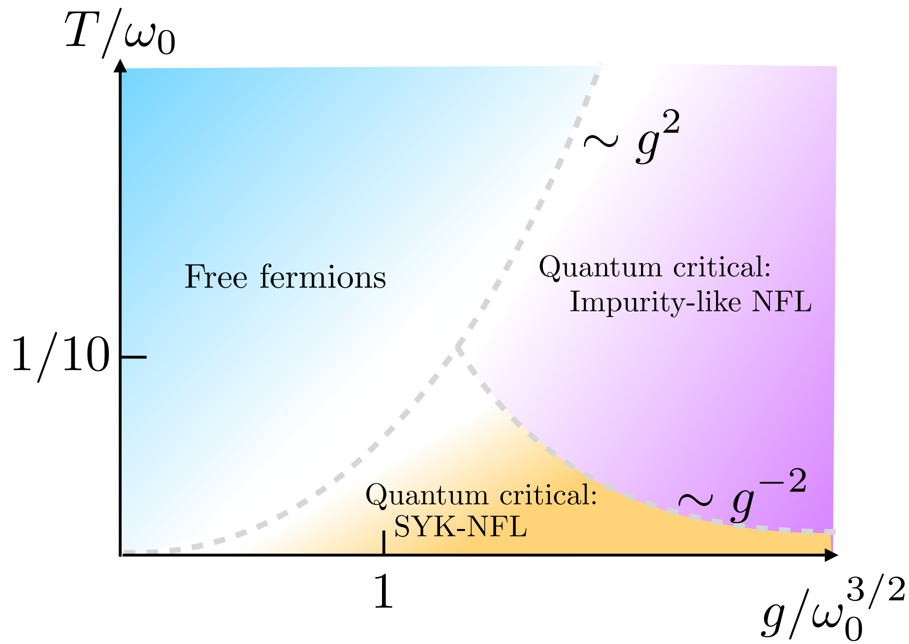
$$D^{-1}(i\omega) = \omega^2 + \omega_0^2 - \Pi(i\omega)$$

$\Pi(i\omega) =$ 

Eliashberg equations of superconductivity become exact!

normal state: critical ground state





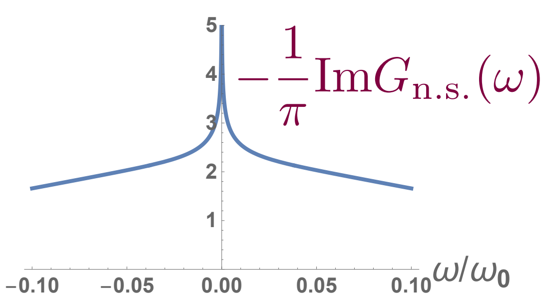
$$G_{\text{n.s.}}(\tau_1-\tau_2) \propto \frac{\tanh(\pi q \mathcal{E}) + \text{sign}(\tau_1-\tau_2)}{|\tau_1-\tau_2|^{\frac{1+\gamma}{2}}}$$

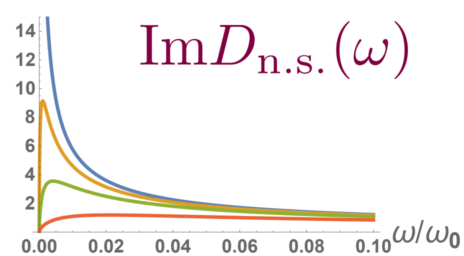
$$D_{\text{n.s.}}(\tau_1-\tau_2) \propto \frac{1}{|\tau_1-\tau_2|^{1-\gamma}}$$

$\gamma \approx 0.68$

q fermion charge

$\mathcal{E}(n)$ spectral asymmetry

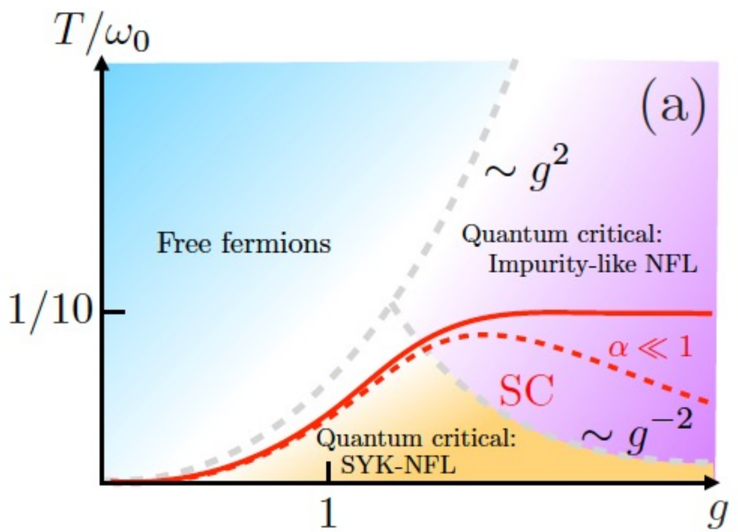


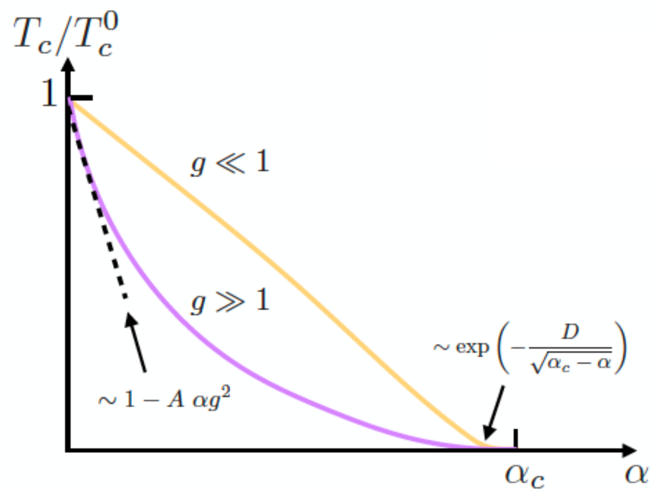


boson mass

$$\omega_r^2 = \omega_0^2 - \Pi(0)$$
$$\propto T^\gamma$$

pair breaking (partial time reversal symmetry breaking)





$$T_c\left(\alpha \sim Z_B\right) \ll T_c^0$$

small coherent weight makes s.c. fragile against pair breaking

$$T_c\left(\alpha \approx \alpha_c\right) = T^* \exp\left(-\frac{D}{\sqrt{\alpha_c - \alpha}}\right)$$

spontaneously broken conformal invariance

like in holographic models in AdS_2

D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. **417**, 168120 (2020)

superconducting transition temperature

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linearized gap equation of the Yukawa-SYK model

generalized Cooper instability

$$\Phi\left(\epsilon\right)=g^{2}\int_{T}^{\Lambda}\frac{d\epsilon'}{2\pi}\frac{\Phi\left(\epsilon'\right)}{\left|\epsilon-\epsilon'\right|^{\gamma}\left|\epsilon'\right|^{1-\gamma}}$$

γ-model

A. Abanov, and A. V. Chubukov, Phys. Rev. B **102**, 024524 (2020),
Y.-M. Wu, A. Abanov, and A. V. Chubukov, Phys. Rev. B 102, 094516 (2020)

same gap equation occurs in theories with pairing due to critical:

- antiferromagnetic + ferromagnetic spin fluctuations
- gauge-field induced composite fermion pairing
- nematic fluctuations
- massless gluons in high-density quark matter
- U(1) and Z_2 gauge fluctuations in spin liquids

N. E. Bonesteel, I. A. McDonald, and C. Nayak, PRL 77, 3009 (1996), D. T. Son, Phys. Rev. D **59**, 094019 (1999),. Ar. Abanov, A. Chubukov, and A. Finkel'stein, EPL **54**, 488 (2001) , Ar. Abanov, A. Chubukov, and J. S. EPL **55**, 369 (2001), R. Roussev and A. J. Millis, Phys. Rev. B 63, 140504R (2001), A. V. Chubukov and J. S., PRB **72**, 174520 (2005), J.-H. She and J. Zaanen, PRB **80**, 184518 (2009), M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015)

Eliashberg equation

$$\Phi(\epsilon) = g^2 \int_T^\Lambda \frac{d\epsilon'}{2\pi} \frac{\Phi(\epsilon')}{|\epsilon-\epsilon'|^\gamma |\epsilon'|^{1-\gamma}} \quad \overset{\gamma \ll 1}{\Longrightarrow} \quad \frac{d}{d\epsilon}\epsilon^{1-\gamma}\frac{d}{d\epsilon}\epsilon^\gamma \Phi(\epsilon) = -\frac{\gamma g^2}{\pi}\frac{\Phi(\epsilon)}{\epsilon}$$

scalar field $\psi(\zeta) = \zeta^{\frac{1-\gamma}{2}} \Phi(1/\zeta)$ with coordinate $\zeta = 1/\epsilon$

$$-\partial_\zeta^2 \psi + \frac{m^2}{\zeta^2}\psi = 0 \qquad m^2 = -\frac{1}{4} + \frac{\gamma^2}{4} - \frac{g^2\gamma}{\pi}$$

static Klein-Gordon equation of pairs with mass m in AdS$_2$

D. T. Son, Phys. Rev. D **59**, 094019 (1999), A. V. Chubukov and J. S., PRB **72**, 174520 (2005), D. Hauck, M. J. Klug, I. Esterlis, J. S., Ann. of Phys. 417, 168120 (2020), A. Abanov, and A. V. Chubukov, Phys. Rev. B **102**, 024524 (2020), Y.-M. Wu, A. Abanov, and A. V. Chubukov, Phys. Rev. B 102, 094516 (2020)

On the level of the action

SYK= AdS_2 gravity + other fluctuations

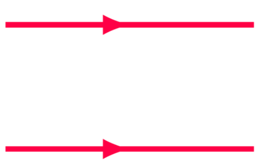
fluctuating field: anomalous Gor'kov Green's function

$$F\left(\tau,\tau'\right) \rightarrow F\left(\varepsilon,\omega\right)$$

FT of the relative time — FT of the absolute time

Gaussian fluctuations near the quantum critical normal state

$$S^{(\mathrm{sc})}/N = \int_{\omega,\epsilon} \frac{F^{\dagger}\left(\omega,\epsilon\right)F\left(\omega,\epsilon\right)}{\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right)} - \frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'} F^{\dagger}\left(\omega,\epsilon\right)D_{\mathrm{n.s.}}\left(\epsilon-\epsilon'\right)F\left(\omega,\epsilon'\right)$$

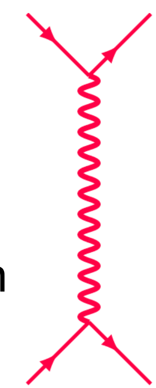


particle-particle propagator

$$\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right) = G_{\mathrm{n.s.}}\left(\frac{\omega}{2}-\epsilon\right)G_{\mathrm{n.s.}}\left(\epsilon+\frac{\omega}{2}\right)$$

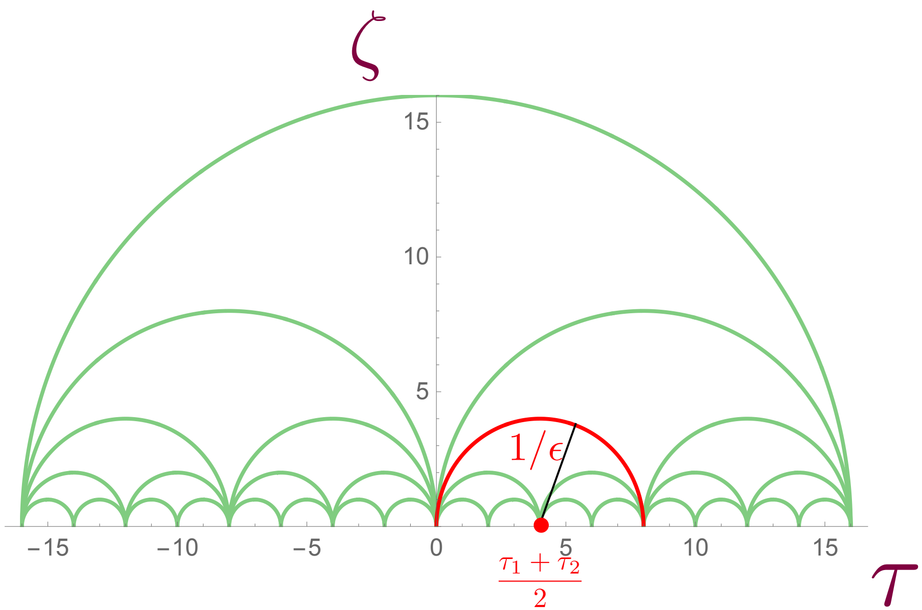
boson propagator
= singular paring interaction

$$D_{\mathrm{n.s.}}(\epsilon) \propto |\epsilon|^{-\gamma}$$



holographic map $F(\omega, \epsilon) \to F\left(\frac{\tau_1+\tau_2}{2}, \epsilon\right)$

$$F\left(\frac{\tau_1+\tau_2}{2}, \epsilon\right) = |\epsilon|^{\frac{\gamma-1}{2}} \int_\Gamma \psi(\tau, \zeta)\, dl$$



Radon transformation

$$\Gamma: \quad |\epsilon|^{-1} = \sqrt{\left(\tau - \frac{\tau_1+\tau_2}{2}\right)^2 + \zeta^2}$$

geodesics of Eucledian AdS_2

$$ds^2 = g_{ab}dx^a dx^b = \frac{1}{\zeta^2}\left(d\tau^2 + d\zeta^2\right)$$

SYK superconductor = holographic superconductor

$$F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)=|\epsilon|^{\frac{\gamma-1}{2}}\int_\Gamma \psi\left(\tau,\zeta\right)dl$$

holographic superconductor in AdS_2 with Euclidean signature

$$S^{(\mathrm{sc})}=N\int d\tau d\zeta\left(\frac{m^2}{\zeta^2}\left|\psi\right|^2+\left|\partial_\tau\psi\right|^2+\left|\partial_\zeta\psi\right|^2\right)$$

dynamics

RG scale

$$S^{(\mathrm{sc})}/N=\int_{\omega,\epsilon}\frac{F^\dagger\left(\omega,\epsilon\right)F\left(\omega,\epsilon\right)}{\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right)}-\frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'}F^\dagger\left(\omega,\epsilon\right)D_{\mathrm{n.s.}}\left(\epsilon-\epsilon'\right)F\left(\omega,\epsilon'\right)$$

SYK superconductor = holographic superconductor

$$F\left(\frac{\tau_1+\tau_2}{2},\epsilon\right)=|\epsilon|^{\frac{\gamma-1}{2}}\int_\Gamma \psi\left(\tau,\zeta\right)dl$$

holographic superconductor in AdS_2 with Euclidean signature at T=0

$$S^{(\mathrm{sc})}=N\int d\tau d\zeta\left(\frac{m^2}{\zeta^2}\left|\psi\right|^2+\left|\partial_\tau\psi\right|^2+\left|\partial_\zeta\psi\right|^2\right)$$

positive contribution to the mass (no Cooper instability in NFL with instantaneous pairing)

negative contribution to the mass (generalized Cooper instab.)

$$S^{(\mathrm{sc})}/N=\int_{\omega,\epsilon}\frac{F^\dagger\left(\omega,\epsilon\right)F\left(\omega,\epsilon\right)}{\Pi_{\mathrm{n.s.}}\left(\omega,\epsilon\right)}-\frac{\bar{g}^2\lambda_p}{2}\int_{\omega,\epsilon,\epsilon'}F^\dagger\left(\omega,\epsilon\right)D_{\mathrm{n.s.}}\left(\epsilon-\epsilon'\right)F\left(\omega,\epsilon'\right)$$

holographic instability $m^2=m_{\mathrm{BF}}^2=-1/4$ (Breitenlohner Freedman condition)

=

Eliashberg instability

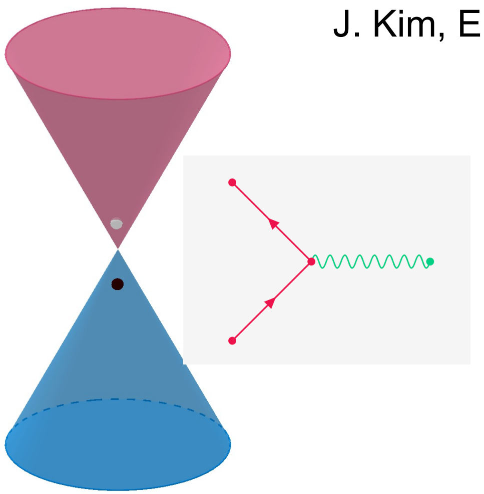
P. Breitenlohner and D. Z. Freedman, Ann. Phys. **144**, 249 (1982)

finite-dimensions: Dirac systems

Karlsruhe Institute of Technology

$$S=\int d^dx d\tau \left(\sum_{l=1}^{N}\overline{\psi_l} i\partial_\mu\gamma^\mu\psi_l+\frac{1}{2}\sum_{r=1}^{M}\phi_r\left(m_0^2-\partial_\mu\partial^\mu\right)\phi_r+\frac{1}{N}\sum_{lm,r}^{N,M}g_{lm,r}\overline{\psi_l}\psi_m\phi_r\right)$$

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021)



critical solutions for d=0,1,2

fermions $G\left(k\right)\propto k_\mu\gamma^\mu\left|k\right|^{2\Delta-d-2}$

phonons $D\left(q\right)\propto\left|q\right|^{d+1-4\Delta}$

superconductivity for zero density (real coupling constants)

anomalous self energy condenses in l=1 longitudinal channel

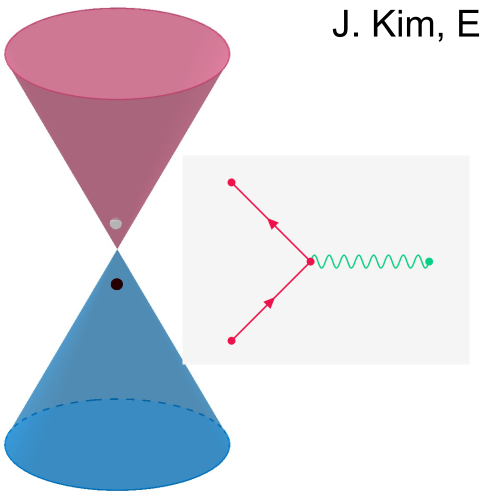
$$\Phi\left(k\right)=\sum_{m=-1}^{1}\phi_m\left(\left|k\right|\right)Y_{1,m}\left(\hat{k}\right)\frac{k_\mu\gamma^\mu}{\left|k\right|}+\cdots$$

V. Stangier, I. Esterlis, J. S., unpublished

finite-dimensions: Dirac systems

$$S=\int d^d x d\tau\left(\sum_{l=1}^{N}\overline{\psi_l} i\partial_\mu\gamma^\mu\psi_l+\frac{1}{2}\sum_{r=1}^{M}\phi_r\left(m_0^2-\partial_\mu\partial^\mu\right)\phi_r+\frac{1}{N}\sum_{lm,r}^{N,M}g_{lm,r}\overline{\psi_l}\psi_m\phi_r\right)$$

J. Kim, E. Altman, and X. Cao, Phys. Rev. B **103**, L081113 (2021)



holographic superconductor in AdS$_{d+2}$

$$F_m\left(\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2},\frac{\tau_1+\tau_2}{2},|k|\right)=|k|^{2\Delta-\frac{d}{2}-1}\int_\Gamma\psi_m\left(\boldsymbol{x},\tau,\zeta\right)dl$$

holographic variable $\zeta\sim\left(\boldsymbol{k}^2+\epsilon^2\right)^{-1/2}$

superconductivity for zero density (real coupling constants)

anomalous self energy condenses in l=1 longitudinal channel

$$\Phi\left(k\right)=\sum_{m=-1}^{1}\phi_m\left(|k|\right)Y_{1,m}\left(\hat{k}\right)\frac{k_\mu\gamma^\mu}{|k|}+\cdots$$

V. Stangier, I. Esterlis, J. S., unpublished

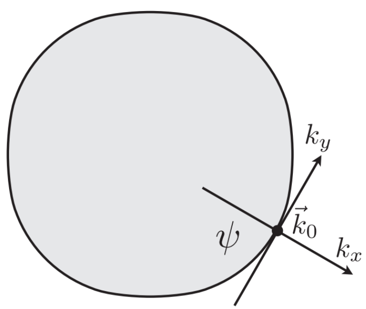
finite-dimensions: compressible fermions

KIT Karlsruhe Institute of Technology

$$S=\int d^dkd\tau\sum_{l=1}^{N}\psi^{\dagger}_{\boldsymbol{k}l}\left(\partial_\tau-\varepsilon\left(\boldsymbol{k}\right)\right)\psi_{\boldsymbol{k}l}+\frac{1}{2}\int d^dqd\tau\sum_{r=1}^{M}\phi_{\boldsymbol{q}r}\left(m_0^2-\partial_\tau^2+q^2\right)\phi_{\boldsymbol{q}r}$$

$$+\frac{1}{N}\int d^dkd^dqd\tau\sum_{lm,r}^{N,M}g_{lm,r}\left(\boldsymbol{k}\right)\psi^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}l}\psi_{\boldsymbol{k}m}\phi_{\boldsymbol{q}r}$$

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021)



critical point

$$\Sigma\left(\boldsymbol{k},\omega\right)\sim i\text{sign}\left(\omega\right)\left|\omega\right|^{2/3}$$

$$\Pi\left(\boldsymbol{q},\omega\right)\sim\frac{\left|\omega\right|}{\left|\boldsymbol{q}\right|}$$

holographic superconductor in $AdS_2\otimes R_2$

holographic variable: $\zeta\sim\left|\epsilon\right|^{-1}$

charged black holes at low energies → AdS_2

J. Maldacena, J. Michelson, and A. Strominger, JHEP **9902**, 011 (1999).

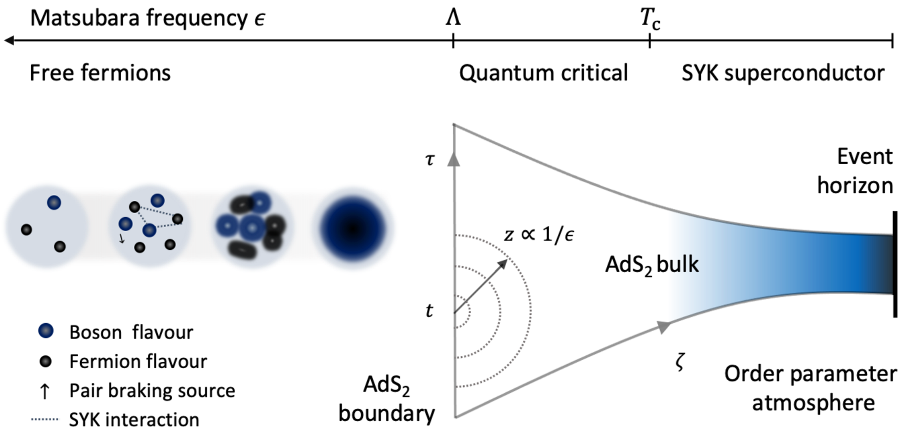
back to SYK, T>0, finite particle number

the holographic map can be extended

$$S^{(\mathrm{sc})} = S_{\mathrm{AdS}_2} = N \int d^2x \sqrt{g} \left(D_a \psi^* D^a \psi + m^2 |\psi|^2 \right)$$

- black hole horizon at finite T $\zeta_T^{-1} = 2\pi T$

$$ds^2 = g_{ab} dx^a dx^b = \frac{1}{\zeta^2} \left((1 - \zeta^2/\zeta_T^2) d\tau^2 + \frac{1}{(1 - \zeta^2/\zeta_T^2)} d\zeta^2 \right)$$



$$\frac{\delta S_{\mathrm{AdS}_2}}{\delta \psi} = 0$$

Eliashberg equations

back to SYK, T>0, finite particle number

Karlsruhe Institute of Technology

the holographic map can be extended

$$S^{(\mathrm{sc})} = S_{\mathrm{AdS}_2} = N \int d^2x \sqrt{g} \left(D_a \psi^* D^a \psi + m^2 |\psi|^2 \right)$$

- black hole horizon at finite T $\zeta_T^{-1} = 2\pi T$

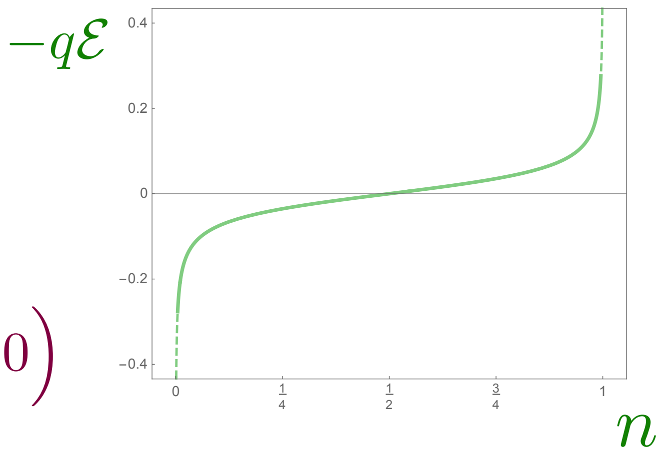
$$ds^2 = g_{ab} dx^a dx^b = \frac{1}{\zeta^2} \left((1 - \zeta^2/\zeta_T^2) d\tau^2 + \frac{1}{(1 - \zeta^2/\zeta_T^2)} d\zeta^2 \right)$$

- away from half filling

$$D_a = \partial_a - i q^\star A_a$$

Cooper pair charge: $q^\star = 2q$

boundary electric field: $A_a = \left(\frac{i\mathcal{E}}{\zeta} \left(1 - \zeta/\zeta_T\right), 0 \right)$



A. Georges, O. Parcollet, S. Sachdev, PRB **63,** 134406 (2001)

Source fields

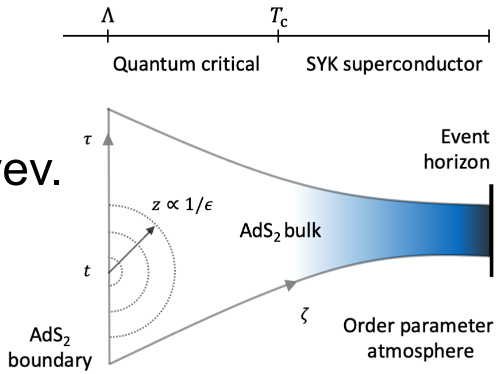
add an external pairing field
e.g. via Josephson coupling to another superconductor

$$S_J = -\int d\tau J_0(\tau) \frac{1}{N} \sum_i c_{i\uparrow}(\tau) c_{i\downarrow}(\tau) + h.c.$$

holographic map $$J(\zeta,\omega) = 2J_0(\omega)\zeta^{\frac{1-\gamma}{2}} \int_1^\infty \frac{dx}{x^{\frac{1+\gamma}{2}}} \frac{\cos\left(\omega\zeta\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}$$

$$S_{J,\mathrm{AdS}_2} = -\int d^2x \sqrt{g}\left(J^*(x)\psi(x) + h.c.\right)$$

combination of source and vev.
acts only on
the boundary



dynamic pairing susceptibility

not easy to calculate within Eliashberg approach, but easy in holography

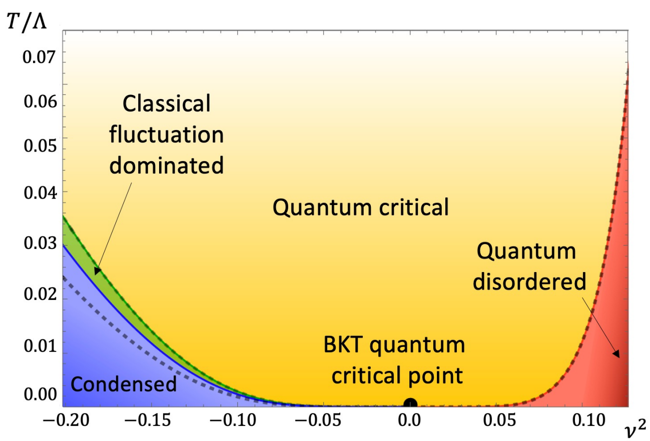
N. Iqbal, H. Liu, M. Mezei, Phys. Rev. D91, 025024 (2015)

$$\chi_{\mathrm{AdS}_2} = \frac{1 - g\mathcal{G}}{1 - f\mathcal{G}}. \quad \mathcal{G}(T,\omega) = \frac{2\nu - \gamma}{2\nu + \gamma} T^{2\nu} \frac{\Gamma(u-\nu)\,\Gamma(v+\nu)}{\Gamma(u+\nu)\,\Gamma(v-\nu)}$$

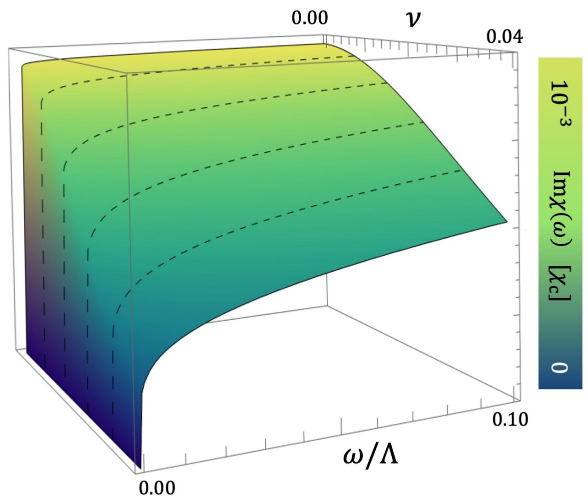
$$u = \tfrac{1}{2} + i2q\mathcal{E}$$

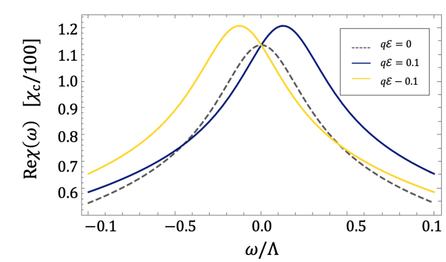
$$v = \tfrac{1}{2} - i\frac{\omega - 4\pi T q\mathcal{E}}{2\pi T}$$

phase diagram



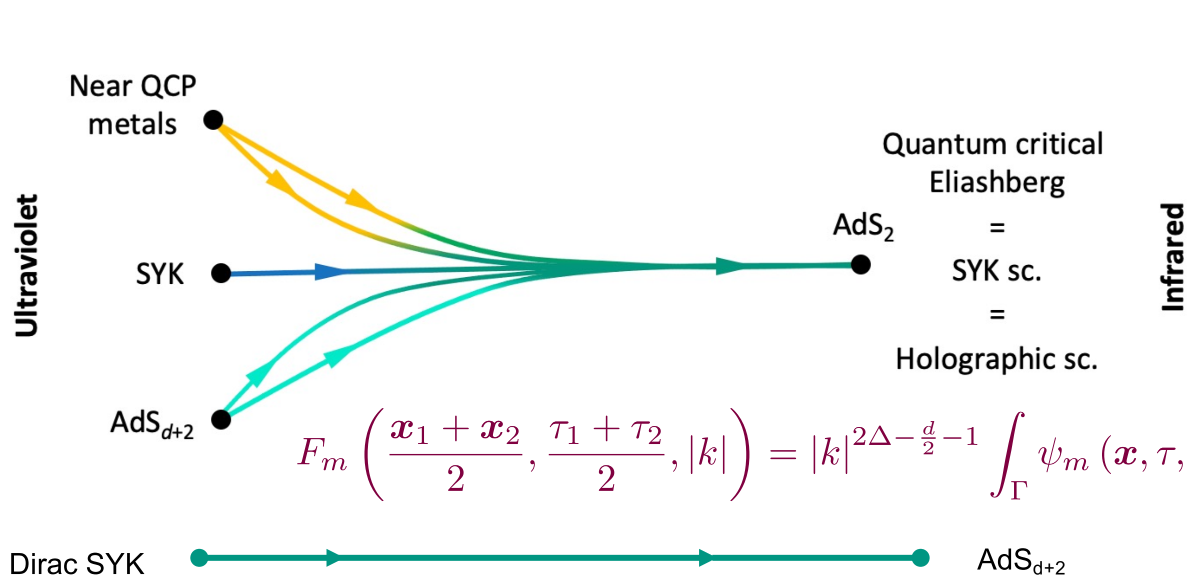
dynamic susceptibility





dynamic response, non-equilibrium behavior … fluctuations beyond Eliashberg, …

Conclusions:



$$F_m\left(\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2},\frac{\tau_1+\tau_2}{2},|k|\right)=|k|^{2\Delta-\frac{d}{2}-1}\int_\Gamma \psi_m\left(\boldsymbol{x},\tau,\zeta\right)dl$$

superconductivity via critical bosons ↔ Yukawa-SYK superconductor ↔ holographic superconductor