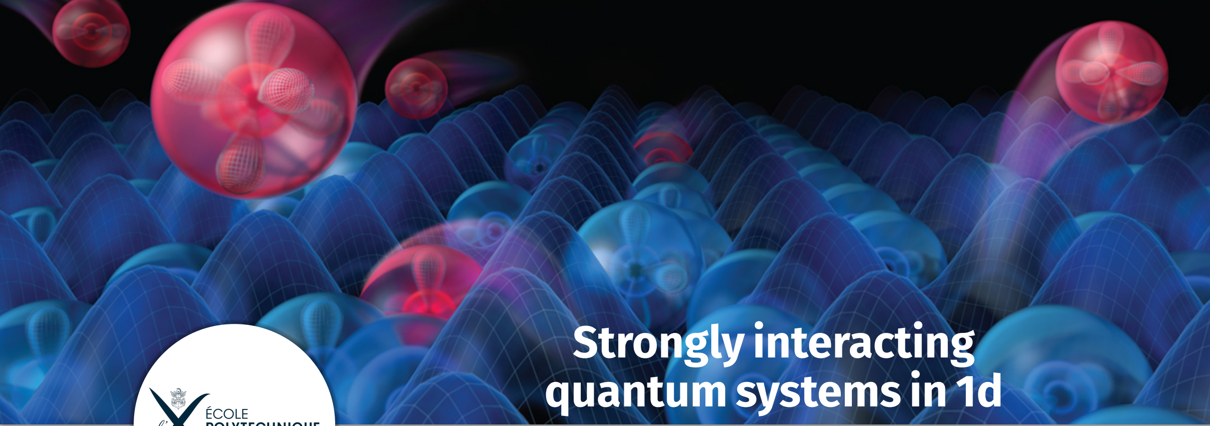

Strongly interacting quantum systems in 1d

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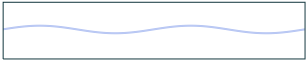
The one dimensional world

Very different from 3D, unintuitive results.

$$\mathcal{H}_{\text{LL}} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{i<j}\delta(x_i - x_j) \qquad \text{(Lieb-Liniger model)} \tag{1}$$

(a) $E_a \sim E_{\text{kin}} = N\frac{\hbar^2 n^2}{2m}$



(b) $E_b \sim E_{\text{int}} = Ngn$

The 1d system will be in the strongly interacting regime if:

$$\gamma \equiv \frac{mg}{\hbar^2 n} = \frac{E_{\text{int}}}{E_{\text{kin}}} \gg 1 \quad \Leftrightarrow \quad \boxed{n \ll \frac{\hbar^2}{mg}} \tag{2}$$

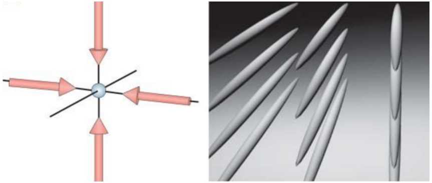
1d $\longrightarrow$ **study strong interactions with ultracold gases (small density)**

Study 1d systems

- Theoretically: specific approaches in 1d
  ▶ Analytical: Bethe Ansatz (integrable systems), Luttinger liquid theory (low-energy, gapless systems)
  Pros of the Bethe Ansatz: exact and analytical, valid no matter the interaction

  ▶ Numerical: Density Matrix Renormalization Group (DMRG), Quantum Monte-Carlo (QMC) ...

- Experimentally: 1D tubes of atoms are formed using counterpropagating lasers trapping atoms using light = optical lattice



The spirit of the Bethe Ansatz

2 particles Lieb-Liniger model: $\mathcal{H}_{\text{LL}} = -\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} + g\,\delta(x_1 - x_2)$ $\boxed{E = ? \quad \psi = ?}$

Bethe Ansatz = **make a 'plane-wave' guess** for ψ:

$$\psi = A(k_1, k_2)\,\mathrm{e}^{i(k_1 x_1 + k_2 x_2)} + A(k_2, k_1)\,\mathrm{e}^{i(k_2 x_1 + k_1 x_2)} \equiv \sum_{\mathcal{P}} A_{\mathcal{P}}\,\mathrm{e}^{ik_{\mathcal{P}} x_{\mathcal{P}}} \tag{3}$$

If $\frac{A(k_1, k_2)}{A(k_2, k_1)} = e^{i\theta(k_1 - k_2)}$ with $\theta(k) \equiv -2\arctan\left(\frac{2k}{g}\right)$ then the dispersion relation becomes equivalent to the one of a free system.

▶ **Interactions are included in the quasimomentum density of state in k-space**

Computing exactly thermodynamical functions in the thermodynamic limit

For macroscopic systems, rewrite the constraint on the amplitudes with the density of quasimomentum $\rho(k)$:

$$2\pi \rho(k) = 1 + \int \frac{g}{(g/2)^2 + (k-k')^2} \rho(k') \, \mathrm{d}k' \tag{4}$$

- contains all the thermodynamic information on the system

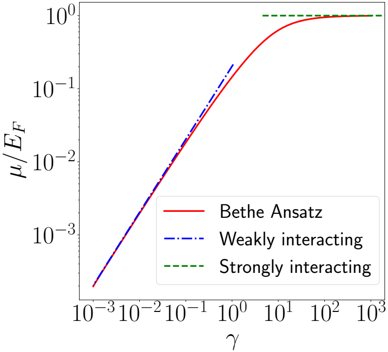


Figure: Exact equation of state (chemical potential as a function of the interaction parameter) obtained by solving numerically the Bethe equations for the Lieb-Liniger gas (log scale).

Perspectives

- Excitation spectrum, and propagation of correlations (dynamics)
- Fermionic systems and quasi-integrable systems

- Longer term: hydrodynamical description of out-of-equilibrium dynamics