N=2 to N=1 with one hypermultiplet

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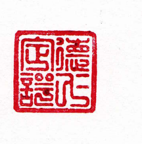

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Partially breaking supersymmetry

Partial breaking of susy had a poor start:

Two no-go claims, for global and local supersymmetry

- Global: simply wrong. Based on the superalgebra, but at the level of ill-defined Noether (super)charges, and a notion of vacuum energy which is not significant.
  (classical and quantum mechanics, QFT, only know about energy *differences*, the value of energy is meaningless)
  Disproved at the current algebra level (before $\int d^3x$ to charges)
  (Polchinski + Hugues, Liu)
- Local: the no-go claim is correct (Cecotti, Girardello, Porrati, 84-86)
  But based on too strong restrictions (use of e.-m. duality not well understood at the time)

Few really cared, phenomenology likes chiral fermion representation $N = 1$ (or $N = 0$)

String compactifications / branes / fluxes then produced a variety of breaking patterns. Called for interest.

Partially breaking N=2 supersymmetry

- A class of $N=2$ theories broken into $N=1$ with a single Maxwell multiplet in global susy. Depends on a holomorphic function $F(X)$, non canonical $F_{XXX} \neq 0$. Goldstino partner of the massless photon field. A chiral multiplet with mass $\sim \langle F_{XXX} \rangle$
  (APT model: Antoniadis, Partouche, Taylor, 1996)

- A class of $N=2$ theories broken into $N=1$ with a single single-tensor multiplet (dual to a hypermultiplet) in global susy. Depends on a holomorphic function $W(\Phi)$, non canonical $W_{\Phi\Phi} \neq 0$. Goldstino partner of the $B_{\mu\nu}$ gauge field. A chiral multiplet with mass $\sim \langle W_{\Phi\Phi} \rangle$
  (ADM model: Antoniadis, JPD, Markou, 2017)

- Some generalisations to several multiplets

- An example of spontaneous breaking of $N=2$ supergravity into $N=1$ Minkowski. Field content: supergravity, a Maxwell multiplet, a single hypermultiplet on $SO(4,1)/SO(4)$
  (FGP model: Ferrara, Girardello, Porrati, 1996)

Partially breaking N=2 supergravity

- No other example worked out with one hypermultplet.
- Seems relatively common with several hypermultiplets, but explicit examples hard to find
  (Needed: quaternion-Kähler metrics in $4n_H \geq 8$ dimensions ??)
  (Louis, Smyth, Triendl, 2009-2010)
- We decided to explicitly find and to classify all $N = 2$ supergravity theories with a single hypermultiplet admitting vacua with $N = 1$ supersymmetry in Minkowski space-time ...
- ... we found that the FGP model is indeed unique
  (Antoniadis, JPD, Petropoulos, Siampos, 2018)

Partially breaking N=2 supergravity

Ingredients for partial breaking:

- A Maxwell multiplet: the massive spin 3/2 gravitino in $N = 1$ needs two massive vectors in its $6_B + 6_F$ multiplet.
- A hypermultiplet to have scalars able to break the $SU(2)$ R–symmetry. To generate gravitino masses $0, m_{3/2} \neq 0$
- A hypermultiplet quaternion-Kähler space with two commuting isometries. A gauging of these isometries to generate the scalar potential, the scalar vev's, the cosmological constant $\langle V \rangle$
- Stability is automatic if $N = 1$ unbroken, but $\langle V \rangle = 0$ is of course not. (Louis, Smyth, Triendl)
- The six-scalar V should have a (stable) ground state with $\langle V \rangle = 0$ generating only one goldstino. . .
- However: use fermion shifts and peculiarities of a single hypermultiplet

One hypermultiplet N=2 supergravity

Supergravity hypermultiplet scalars live on quaternion-Kähler manifolds (in general not Kähler)

(Bagger, Witten)

- Hypermultiplet: helicities $2 \times (\pm 1/2, 0, 0)$, on-shell realisation only $(4_B + 4_F)$
- $4n_H$–dimensional, holonomy in $SU(2n_H) \times SU(2)$
- Not pertinent for $n_H = 1$, $SU(2) \times SU(2) \sim SO(4)$.
  Replaced by Weyl self-duality
- Rich literature in maths, for $n_H = 1$: generic explicit metrics exist for spaces with one or more isometries. In contrast with $n_H > 1$.
- General analysis is then feasible

Analysis: fermion shifts

Susy variations of fermions include a fermion shift (2×2) matrix function of scalar fields (after eliminating auxiliary fields if an off-shell formulation is available):

Gravitinos: $\delta \psi_\mu^i = \frac{1}{2}\kappa^2 S^{ij}\gamma_\mu \epsilon_j + \cdots$

$$S^{ij} = \frac{1}{\kappa}\mathrm{e}^{\mathcal{K}/2} P_a^{ij} U^I \,\Theta_I{}^a = S^{ji}$$

Gauginos: $\delta \lambda_i^\alpha = \kappa^2 g^{\alpha\overline{\beta}} \overline{W}_{\overline{\beta} ij}\epsilon^j + \cdots$

$$W_\alpha{}^{ij} = -\frac{1}{\kappa}\,\mathrm{e}^{\mathcal{K}/2} P_a^{ij} \nabla_\alpha U^I \,\Theta_I{}^a$$

Hyperinos: $\delta \zeta^A = \overline{N}_i{}^A \epsilon^i + \cdots$

$$N^i{}_A = \frac{i}{\kappa}\,\mathrm{e}^{\mathcal{K}/2} f^{iB}{}_u U^I \,\Theta_I{}^a \xi_a^u \,\Omega_{BA}$$

Vev's indicate breaking pattern (+ space-time geometry if partial breaking)

Fermion shifts

To write fermion shifts:

- Hypermultiplet QK metric h_{uv} and 4-bein $f^{iB}{}_{u}$: Killing vectors ξ_a^u, prepotentials P_a^{ij}
- Vector multiplet Kähler metric $g^{\alpha\overline{\beta}}$ and Kähler potential $\mathcal{K}$ from sections U^I (electric and magnetic)
- Always use prepotential frame and dyonic gauging.
  Prepotential frames exist everywhere along electric-magnetic duality orbits.
  (Exception: original FGP model).
- Embedding tensor Θ_I^a (numbers and gauge couplings): which gauge field for each isometry.

$$\boxed{\begin{aligned} \Rightarrow \quad e^{-1}V \quad = \quad & -\frac{1}{2}\,(\mathrm{Im}\mathcal{N})^{-1IJ}\,\Theta_I{}^a\Theta_J{}^b P_a^x P_b^x \\ & +\overline{V}^I V^J \Theta_I{}^a\Theta_J{}^b\left(-4\,\kappa^2 P_a^x P_b^x + \frac{2}{\kappa^2}\,h_{uv}\,\xi_a^u\xi_b^v\right) \end{aligned}}$$

Fermion shifts

We need:

- One Goldstino only, *i. e.*
- A common zero eigenvector, for one unbroken supersymmetry in Minkowski space-time.
- (A common zero eigenvector of $\boldsymbol{W}$ and $\boldsymbol{N}$ for one unbroken supersymmetry in AdS space-time. Easier then. . .)

Straightforward if generic expressions for the vector multiplet and the hypermultiplet geometries are available.

Getting these expressions right is not straightforward, if more than 30% of published equations have misprints, every equation is suspect. . .

Thanks to Freedman, Van Proeyen

QK metric: Calderbank-Pedersen

In the derivation of the type IIB dilaton supergravity, a particular Weyl self-dual metric with two commuting has been used. It is defined in terms of a function $F(\rho, \eta)$ such that

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{\partial^2 F}{\partial \eta^2} = \frac{3F}{4\rho^2}$$

A larger isometry exists for a function of ρ only: $F(\rho) = \rho^{3/2} + C\rho^{-1/2}$

The resulting metric

- is $SU(2,1)/SU(2) \times U(1)$ for $C = 0$. Tree-level
- has isometry (X, Y, Z, M) when $C \neq 0$ with $[X, Y] = Z$, $[M, X] = Y$, $[M, Y] = -X$, $[Z, X] = [Z, Y] = [Z, M] = 0$ Loop-corrected

Antoniadis, Minasian, Theisen, Vanhove, 2003

The Calderbank-Pedersen metric (2001)

Calderbank-Pedersen

Coordinates: ρ, η, ψ, ϕ, two commuting shift isometries $\delta\psi = c$, $\delta\phi = d$.

For $F(\rho, \eta)$ such that $\frac{\partial^2 F}{\partial\rho^2} + \frac{\partial^2 F}{\partial\eta^2} = \frac{3F}{4\rho^2}$ quaternion-Kähler metric:

$$ds^2 = \frac{4\rho^2(F_\rho^2 + F_\eta^2) - F^2}{4F^2}\, d\ell^2 + \frac{((F - 2\rho F_\rho)\alpha - 2\rho F_\eta \beta)^2 + ((F + 2\rho F_\rho)\beta - 2\rho F_\eta \alpha)^2}{F^2(4\rho^2(F_\rho^2 + F_\eta^2) - F^2)}$$

$$\alpha = \sqrt{\rho}\, d\varphi \qquad \beta = (d\psi + \eta d\varphi)/\sqrt{\rho} \qquad d\ell^2 = \rho^{-2}(d\rho^2 + d\eta^2)$$

Calderbank-Pedersen: (arXiv:math/0105253)

“Any selfdual Einstein metric of nonzero scalar curvature with two linearly independent commuting Killing fields arises locally in this way (i.e., in a neighbourhood of any point, it is of the form (1.1) up to a constant multiple)."

Analysis: Calderbank-Pedersen metric

Deriving the fermion shifts from this metric, one easily proves that

the partial breaking $N = 2 \to N = 1$ (Minkowski) never occurs.

Contradiction: CP theorem $\iff$ the existence of the FGP example.

Actually:

Calderbank-Pedersen coordinates and metric do not exist for a single case, the QK space $SO(4,1)/SO(4)$ with a pair of translation isometries.

And all other inequivalent pairs of isometries in $SO(4,1)$ admit CP ccordinates and metric.

CP's claim is then inaccurate and

The FGP model breaking $N = 2$ supergravity into $N = 1$ with one hypermultiplet is unique

To prove these statements, start with Przanovski-Tod coordinates and metric.

Przanowski-Tod

$N_H = 1$ $(4d)$ quaternion-Kähler metric in coordinates (X, Y, Z, ψ) and shift isometry $\delta\psi = C$.

- For a solution $\Psi(X, Y, Z)$ of Toda equation, define U:

$$\Psi_{XX} + \Psi_{YY} + \left(e^{\Psi}\right)_{ZZ} = 0 \quad \text{(Toda)} \qquad 2U = 2 - Z\,\Psi_Z$$

- The Przanowski-Tod quaternion-Kähler metric is

$$ds^2 = \frac{1}{Z^2}\left[\frac{1}{U}(d\psi + \omega)^2 + U\left(dZ^2 + e^{\Psi}\left(dX^2 + dY^2\right)\right)\right]$$

- The one-form ω is defined by

$$d\omega = U_X\, dY \wedge dZ + U_Y\, dZ \wedge dX + \left(U\, e^{\Psi}\right)_Z\, dX \wedge dY$$

Not the most convenient for a general treatment. For each space and pair of isometries (for partial breaking), find the Toda solution in the appropriate coordinates . . .

But it can be done for each pair of isometries of $SO(4,1)/SO(4)$.

Przanowski-Tod

$$ds^2 = \frac{1}{Z^2}\left[\frac{1}{U}(d\psi+\omega)^2 + U\left(dZ^2 + e^{\Psi}\left(dX^2+dY^2\right)\right)\right]$$

The simplest example is of course $\Psi =$ constant

and then $U = 1$, $d\omega = 0$

Redefining X, Y, ψ into z^1, z^2, z^3 (and $Z = z^0$):

$$ds^2 = \frac{1}{(z^0)^2}dz^a dz^a \qquad a = 0,1,2,3 \qquad SO(4,1)/SO(4)$$

$\Rightarrow$ three commuting isometries, shifts of z^1, z^2, z^3

$\Rightarrow$ several inequivalent pairs of commuting isometries (Cartan elements of $SO(4)$, one element of $SO(4)$ commuting with a shift, ...

$\Rightarrow$ For all of them, one can find coordinates for a PT metric.

CP versus PT

Assume to have a solution $V(\rho,\eta)$ of the equation

$$\boxed{I: \quad \frac{1}{\rho}\left(\rho V_\rho\right)_\rho + V_{\eta\eta} = 0}$$

Take ∂_ρ to get for $F(\rho,\eta) = \sqrt{\rho}\, V_\rho$:
$$\boxed{\frac{\partial^2 F}{\partial\rho^2} + \frac{\partial^2 F}{\partial\eta^2} = \frac{3F}{4\rho^2}}$$

and $F(\rho,\eta)$ generates a quaternion-Kähler metric in Calderbank–Pedersen coordinates.

Trade (ρ,η) for (X, Z) by a double Legendre transformation:

$$\boxed{V(\rho,\eta) - X\eta - 2\,Z\ln\rho = -K(X,Z)}$$

$$\Rightarrow \qquad \rho V_\rho = 2\,Z \qquad V_\eta = X \qquad \eta = K_X \qquad 2\,\ln\rho = K_Z$$

$$\Rightarrow \qquad \frac{\partial Z}{\partial\rho} = \tfrac{1}{2}\left(\rho V_\rho\right)_\rho \qquad \frac{\partial Z}{\partial\eta} = \tfrac{\rho}{2}V_{\rho\eta} \qquad \frac{\partial X}{\partial\rho} = V_{\rho\eta} \qquad \frac{\partial X}{\partial\eta} = V_{\eta\eta}$$

$$\frac{\partial\rho}{\partial X} = \tfrac{\rho}{2}K_{XZ} \qquad \frac{\partial\rho}{\partial Z} = \tfrac{\rho}{2}K_{ZZ} \qquad \frac{\partial\eta}{\partial X} = K_{XX} \qquad \frac{\partial\eta}{\partial Z} = K_{XZ}$$

CP versus PT

As usual, $\frac{\partial x^i}{\partial x^j} = \delta^i_j$ for each set of coordinates delivers the relations between the second derivatives of V and K.

$\Rightarrow$ the "Legendre partner" of eq. I is $\boxed{II: \qquad K_{XX} + \frac{\rho^2}{4} K_{ZZ} = 0}$

Define finally $\Psi(X, Z) = \ln\left(\frac{1}{4}\rho^2\right)$ $\qquad e^{\Psi} = \frac{1}{4}\rho^2 = \frac{1}{4}e^{K_Z}$

Legendre relations and eq. II lead to $\boxed{\mathbf{Toda}: \qquad \Psi_{XX} + \left(e^{\Psi}\right)_{ZZ} = 0}$

See Ward and Calderbank-Pedersen

This procedure allows to find CP coordinates for a metric with two isometries expressed in PT coordinates, for a given Toda solution Ψ.

But: $\Psi =$ constant is clearly excluded
$SO(4,1)/SO(4)$ with two translation isometries has PT coordinates (trivial).
It does not have CP coordinates