Clarification to “Zero Temperature Limits of Gibbs States”

In Theorem 1.2, the term “sequence” is poorly chosen. It should be understood as continuous parameter family, as indicated by the notation $(\mu_{\beta\varphi})_{\beta>0}$. The content of this theorem is that if one chooses for ***each*** β an equilibrium state $\mu_{\beta\varphi}$, then necessarily $\lim_{\beta\to\infty}\mu_{\beta\varphi}$ will not exist. 1 Since there has been some confusion about this result, we would like to make some additional remarks.

- If there were a unique Gibbs measure for each β then there would be a unique choice $\mu_{\beta\varphi}$, and the result could be formulated more cleanly as simply $\lim_{\beta\to\infty}\mu_{\beta\varphi}$ does not exist; but in our example we believe that uniqueness does not hold at low temperatures.

- By compactness of the space of probability measures, for any such choice there will always exist subsequences $(\beta_i)_{i\in\mathbb{N}}$ such that the limit $\lim_{i\to\infty}\mu_{\beta_i\varphi}$ exists. The result is about continuous parameter families. Note that continuous families are more natural physically, since the process of cooling a system is a continuous one.

- There is nothing new in the fact that one can choose ***some*** divergent family $\beta\mapsto\mu_{\beta\varphi}$ of equilibrium states. For example in the Ising model one can choose a family which alternates, when β is large, between the ergodic + and - phases. However, it is also possible to choose families which converge to one of the ground states, and these are the families one expects to observe “experimentally” if one cools such a system slowly: it will converge to a ground state. On the other hand, in our example it is ***not possible*** to choose ***any*** family which converges to a ground state: if you cool this system sufficiently slowly, there will be some observable which fluctuates significantly and continues to do so at arbitrarily low temperatures.