Effective theories of pseudo-Goldstone bosons and melting of the field-induced Wigner solid

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References:

- *'Hydrodynamic theory of quantum fluctuating superconductivity'* [arXiv:1602.08171], PRB'17 with Richard Davison, Luca Delacrétaz and Sean Hartnoll
- With Luca Delacrétaz, Sean Hartnoll and Anna Karlsson
  - *'Bad Metals from Density Waves'* [arXiv: 1612.04381], Scipost'17,
  - *'Theory of hydrodynamic transport in fluctuating electronic charge density wave states'* [arXiv:1702.05104], PRB'17,
  - *'Theory of the collective magnetophonon resonance and melting of the field-induced Wigner solid'* [arXiv:1904.04872], PRB'19.
- See also 'Theory of the supercyclotron resonance and Hall response in anomalous 2d metals', Luca V. Delacrétaz and Sean A. Hartnoll, [arXiv1803.01116], PRB'18.

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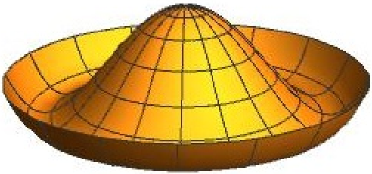
- **Spontaneous breaking** of a continuous symmetry leads to the appearance of **extra gapless degrees of freedom**, the Nambu-Goldstone bosons.

- Superfluid: spontaneous breaking of a U(1) symmetry.
  Order parameter: complex scalar. Mexican hat potential.
  The vev of the condensate is given by the modulus, the NGB by its phase.



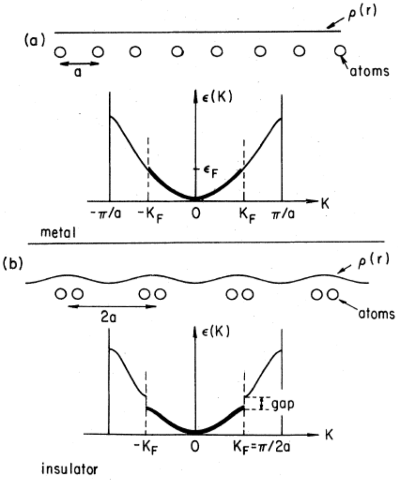
- At weak coupling, a CDW develops due to a electron-(lattice) phonon interactions which gaps out the Fermi surface [PEIERLS'55].

- Formation of a **collective electron-hole mode** at $k = 2k_F$.

- Order parameter: **complex scalar**

$$\rho(\vec{x}) = \rho_0 + \rho_1 \cos\left(2\vec{k_F} \cdot \vec{x} + \phi\right)$$

ϕ: CDW sliding mode, NGB.



[GRÜNER'88]

2d Wigner crystals: all translations are broken, longitudinal and transverse phonon $\lambda_{\parallel} = \nabla \cdot \phi$, $\lambda_{\perp} = \nabla \times \phi$, ϕ_i, $i = \{x, y\}$.

- An important property of Goldstones is that they are **shift-symmetric**: they realize non-linearly the broken symmetry. More concretely, take broken translations along x

$$x \to x + c \quad \Rightarrow \quad \phi \to \phi + c$$

- Shift symmetry: **only gradient terms** in the effective IR action:

$$f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \dots$$

- ρ_ϕ is the 'stiffness' of the order parameter:
  Superfluids, $\rho_\phi = \rho_s$ the superfluid density;
  CDW, ρ_ϕ: CDW modulus.
  2d Wigner crystal, ρ_ϕ: bulk K and shear G moduli.

- Low energy effective field theory constructed based on symmetry principles. Describes late time, long wavelength dynamics: $\omega \ll 1/\tau_{eq}$ **extreme dominance of interactions**.
- Basic ingredients:
    - Write down **conservation equations**
    - Give **constitutive relations** to conserved currents in a gradient expansion.
- Compute retarded Green's functions using [KADANOFF & MARTIN'63]
- Example: charge diffusion

$$\partial_t \rho + \nabla \cdot j = 0 j\,, \quad = -\sigma_o \nabla \mu + O(\nabla^2)\,, \quad \rho_0(k) = \chi \mu_0(k)$$

$$\langle \rho(\omega, k) \rangle = \frac{\rho_0(k)}{-i\omega + \sigma_o \chi k^2} = \frac{\chi \mu_0(k)}{-i\omega + Dk^2}$$

$$G^R_{\rho\rho}(\omega, k) - G^R_{\rho\rho}(\omega = 0, k) = -i\omega \frac{\langle \rho(\omega, k) \rangle}{\mu_0(k)} \Rightarrow G^R_{\rho\rho}(\omega, k) = \frac{\chi D k^2}{-i\omega + Dk^2}$$

- If Q is the charge that generates the symmetry, then

$$[\phi(x), Q(y)] = i\delta(x-y) + \dots$$

- The effective Hamiltonian contains a term

$$H \sim \int d^d x \, s_Q(x) Q(x)$$

which leads to the **'Josephon' equation**

$$\nabla\dot{\phi} \equiv \partial_t \nabla\phi = [H, \nabla\phi] = \nabla s_Q$$

- Superfluids: $Q = \rho$ (U(1) charge) and $s_Q = -\mu$ (chemical potential);
- Translationally-ordered phases: $Q = \pi$ (momentum) and $s_Q = v$ (velocity) along the direction with broken symmetry.

Conservation of Q + Josephson equation for ϕ:

- new sound modes with velocity $v_s^2 \sim \rho_\phi$ (superfluid sound = NGB + U(1), shear sound = NGB + $\pi_\perp$)
- enhances existing sound velocity + new diffusive mode ($\lambda_\parallel$)

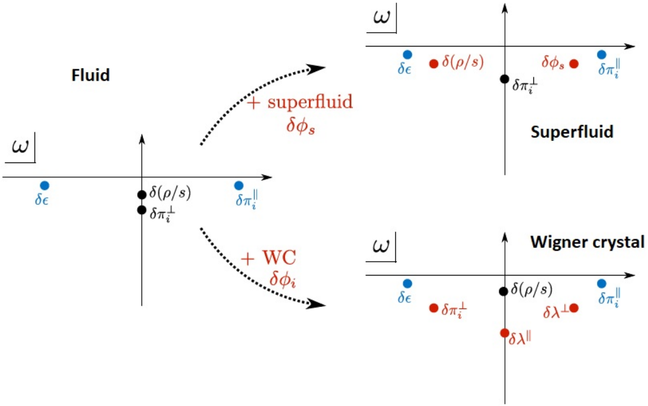


Figure: Credit: Luca V. Delacrétaz

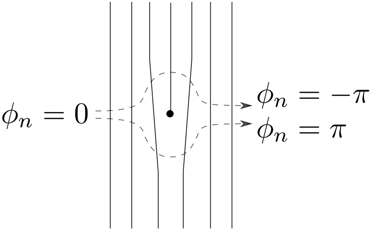
The new sound poles (propagating modes) give rise to $\omega = 0$ poles in the 'conductivity' of the current associated to the broken density

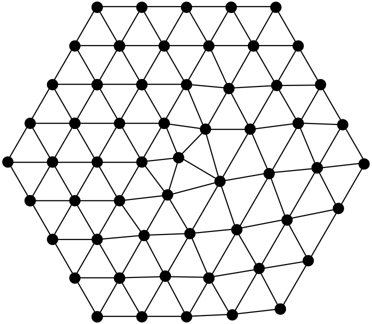
$$\text{superfluid:} \qquad \sigma_{jj} = \frac{i}{\omega} G^R_{jj} = \sigma_o + \frac{\rho_n^2}{\chi_{\pi\pi}} \frac{i}{\omega} + \frac{\rho_s}{\mu} \frac{i}{\omega}$$

$$\text{2d WC:} \qquad \sigma_{xy} = \frac{i}{\omega} G^R_{T^{xy} T^{xy}} = \eta + G \frac{i}{\omega}$$

Reflects that the new dofs are **gapless**.

- There are obstructions to the existence of true long range order for continuous symmetries in $d \leq 2$ [Coleman-Mermin-Wagner] ($d \leq 4$ in the presence of random couplings, [Imry & Ma'75]).
- The destruction of long range order occurs via the **proliferation of topological defects**, [Berezenski-Kosterlitz-Thouless]:





- Concretely, the defects **relax the phase gradients**

$$\nabla \dot{\phi} = \nabla \mu - \Omega \nabla \phi$$

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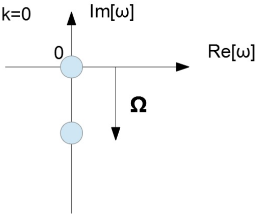
The Goldstone relaxation rate **gaps out** the $\omega = 0$ poles discussed above, $\omega = -i\Omega + \dots$.



- This gives to large diffusivities

$$\text{superfluid:} \qquad \sigma_{jj} = \frac{i}{\omega} G^R_{jj} = \sigma_o + \frac{\rho_n^2}{\chi_{\pi\pi}} \frac{i}{\omega} + \frac{\rho_s}{\mu} \frac{1}{\Omega - i\omega} \Rightarrow D \sim \sigma_o + \frac{\rho_s}{\mu\Omega}$$

$$\text{2d WC:} \qquad \sigma_{xy} = \frac{i}{\omega} G^R_{\tau^{xy}\tau^{xy}} = \eta + \frac{G}{\Omega_\perp - i\omega} \Rightarrow D \sim \eta + \frac{G}{\Omega_\perp}$$

- Important phenomenological consequences: **destruction of superconductivity** in two-dimensional films, **melting of Abrikosov lattices** in a magnetic field.
- Occurs because the Goldstones become **shorter and shorter lived** as Ω increases: gradual loss of phase coherence.

- These Goldstone relaxation rates can be computed from the following Kubo formula:

$$\Omega = \lim_{\omega\to 0}\lim_{\Omega\to 0}\frac{1}{\omega}\mathrm{Im}G^R_{j_\phi j_\phi}(\omega, k=0)\,,\quad J_\phi = \int_{T^2/\{d.c.\}}\nabla\phi$$

- Crucial technical crutch: consider a Hamiltonian deformation involving the square of the density.

$$\Delta H = \frac{1}{2\chi}\int dx\, Q(x)^2 \Rightarrow$$

$$j_\phi = \partial_t J_\phi + i[\Delta H, J_\phi] = -\frac{2}{\chi}\int_{T^2/\{d.c.\}}\nabla Q = \frac{2}{\chi}\int_{\{d.c.\}}\nabla Q$$

$$\Omega = \frac{4}{\chi^2}\lim_{\omega\to 0}\frac{1}{\omega}\int_{\{d.c.\}}dx\int_{\{d.c.\}}dy\nabla_x\nabla_y\mathrm{Im}G^R_{QQ}(\omega; x-y)\,,$$

Assumption: Large enough cores that the hydro expression for G^R_{QQ} can be used. Assume that Q diffuses inside cores

$$G^R_{QQ} = -\frac{\chi D k^2}{i\omega - D k^2}$$

- Disordered superfluid: recovers flux-flow resistance [Bardeen & Stephens'65]

$$\Omega = 2\rho_s \frac{n_f \pi r_v^2}{\sigma_n}$$

- Clean 2d WC: [Halperin & Nelson'80]

$$\Omega = 2G \frac{n_f \pi r_v^2}{\eta_n}$$

- Ω is controlled by the 'conductivity' of the unbroken phase inside the cores.

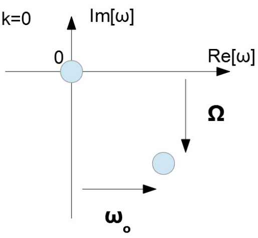
- Spacetime symmetries can be explicitly broken: focus on the case of broken translations.
- Impact on the Goldstones: 'tilts the Mexican potential', the Goldstones become **massive**, which breaks their shift symmetry

$$f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \dots \to f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \frac{1}{2}m^2\phi^2 + \dots$$

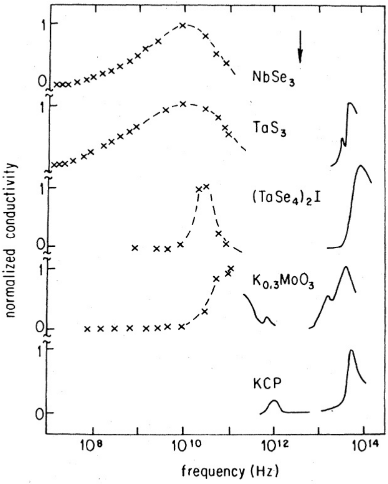
The Goldstones now resonate at **a pinning frequency** $\omega_o = m\sqrt{(}G/\chi_{\pi\pi})$.

Should also expect that the Goldstone are **damped** at a rate Ω (distinct from the contribution from defects).

Combined, these effects **gap the sound modes**.



If $\omega_o \ll 1/\tau_{eq}$, the pinned Goldstones remain **light** and must be kept in the EFT: **pinning peak** in eg ac conductivity:



[GRUENER'88]

- In real materials, if disorder is too strong, ω_o does not remain in the IR and no collective peak is observed.
- Strong magnetic fields offer a way to bring the peak back to the IR.
- Application to the Wigner solid phase of 2d electron systems (GaAs/GaAlAs heterostructures). Vicinity to Quantum Hall phases.

- Now turn on a magnetic field: the longitudinal and transverse sound modes hybridize into (gapless) **magnetophonons** and gapped **magnetoplasmons** $\omega \sim \omega_c \sim B$ [Fukuyama & Lee'78].
- Upon turning on disorder, the magnetophonons are pinned at $\omega_{\rm pk} \equiv \omega_o^2/\omega_c \sim O(1/B)$: within hydrodynamics at large magnetic fields, **even for strong disorder**.
- So if $\omega_{\rm pk} \ll 1/\tau_{eq} \ll \omega_c$, **hydro theory of the magnetophonon alone**.
- Write down a similar hydrodynamic theory as before: conservation of charge, Josephson equation for magnetophonon, constitutive relations, solve and get conductivity.
- Also positivity of entropy production bound.
- Compute the relaxation rate due to mobile defects.
- **new**: relaxation due to dissipation into currents.

- Magnetic translations in a transverse B field

$$\sqrt{\rho B}\mathcal{P}_i = P_i + \int d^2x\rho A_i\,, \quad \vec{A} = (-By, 0)\,.$$

$$\Rightarrow [\mathcal{P}_i, \mathcal{P}_j] = -iRB\epsilon_{ij}$$

- We want to **break magnetic translations**. When broken generators do not commute, reduction on the number of expected Goldstones [Watanabe & Muruyama'12].
- Under **magnetic translations**, Goldstones $\varphi_i \to \varphi_i + \delta x_i$. Leads to

$$\mathcal{L} = \epsilon^{ij}\varphi_i\dot{\varphi}_j$$

  Upon quantizing

$$[\varphi_i(x), \varphi_j(y)] = -i\epsilon_{ij}\delta(x-y)$$

  The Goldstones are **not** independent fields!
- From Noether, conserved densities are $\pi^i \sim \epsilon^{ij}\varphi_j$, which leads to the magnetic translation algebra.

- Extend the Lagrangian to include pinning and spatial gradients

$$\mathcal{L} = \epsilon^{ij}\varphi_i\dot{\varphi}_j - \varphi_j\left[\delta^{ij}\omega_{\mathrm{pk}} + \left(Kk^i k^j + Gk^2\delta^{ij}\right) + \ldots\right]\varphi_j$$

- Leads to the modes [Fukuyama & Lee'78]

$$\omega(k) = \pm\sqrt{\left(\omega_{\mathrm{pk}} + Gk^2\right)\left(\omega_{\mathrm{pk}} + (K+G)k^2\right)}$$

$$\begin{cases} \omega_{\mathrm{pk}} = 0 & \Rightarrow \quad \omega(k) = \pm k^2 \\ k = 0 & \Rightarrow \quad \omega = \pm\omega_{\mathrm{pk}} \end{cases}$$

- Enter relaxation

$$\begin{pmatrix} j^i \\ \varphi^i \end{pmatrix} = \begin{pmatrix} \sigma_o^{ij} & \gamma^{ij} \\ \gamma^{ij} & \Omega^{ij}/\omega_{\rm pk} \end{pmatrix} \begin{pmatrix} E_j \\ s_j - \omega_{\rm pk}\varphi_j \end{pmatrix}$$

$$\sigma_o^{ij} = \sigma_o \delta^{ij} + \sigma_o^H \epsilon^{ij} \, , \gamma^{ij} = \gamma \delta^{ij} + \sqrt{\nu}\epsilon^{ij} \, , \Omega^{ij} = \Omega \delta^{ij} + \omega_{\mathsf{pk}} \epsilon^{ij}$$

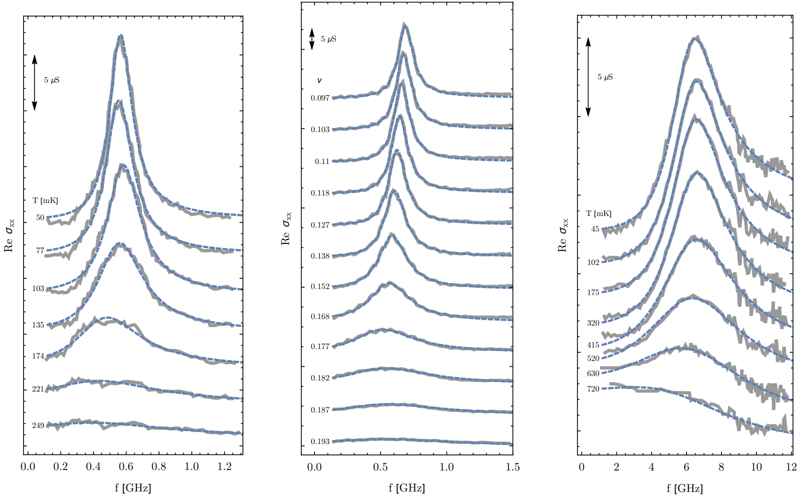
- Conductivity

$$\sigma_{xx}(\omega) = \sigma_o + \nu\, \omega_{\mathsf{pk}} \frac{(1-a^2)(-i\omega + \Omega) - 2a\omega_{\mathsf{pk}}}{(-i\omega+\Omega)^2 + \omega_{\mathsf{pk}}^2} \qquad \nu = \frac{\rho}{B} \, .$$

- **new:** $a \equiv \gamma/\sqrt{\nu}$ asymmetry parameter.
- Positivity of entropy production:

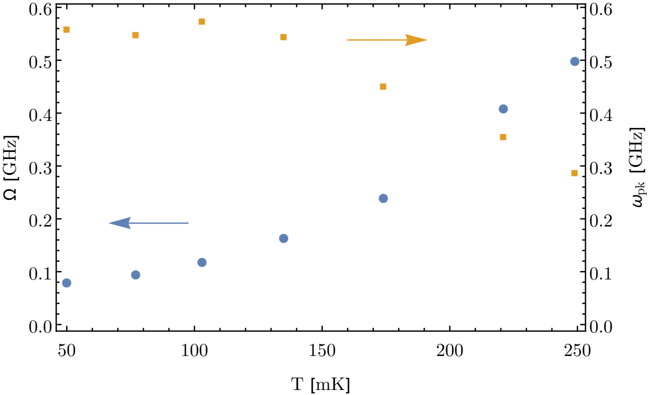
$$\gamma^2 \leq \frac{\sigma_o \Omega}{\omega_{\mathsf{pk}}}$$

Fit to data on GaAs heterojunctions (2DEG) [YP Chen at al, Nature Physics'06], [YP Chen et al, International Journal of Modern Physics B'07], [YP Chen, PhD thesis'05]



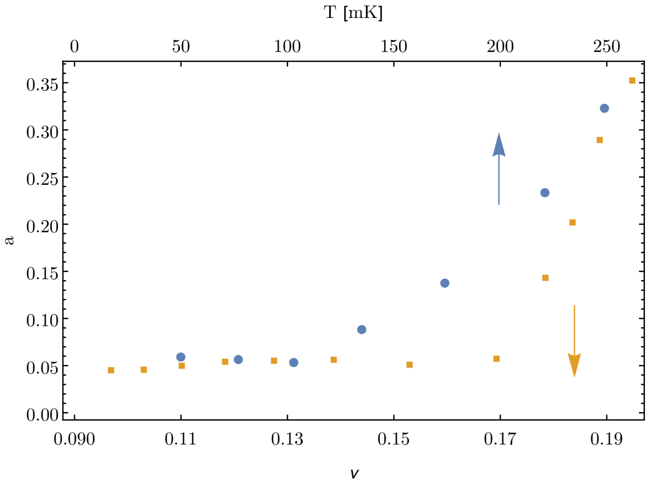
Sample A	Sample B	Sample C
Thermal melting	Quantum melting	Thermal melting (more disordered)

Ω increases as melting is approached: **shorter-lived magnetophonon**.



sample A

The fits require a **nonzero asymmetry parameter** $a \neq 0$:

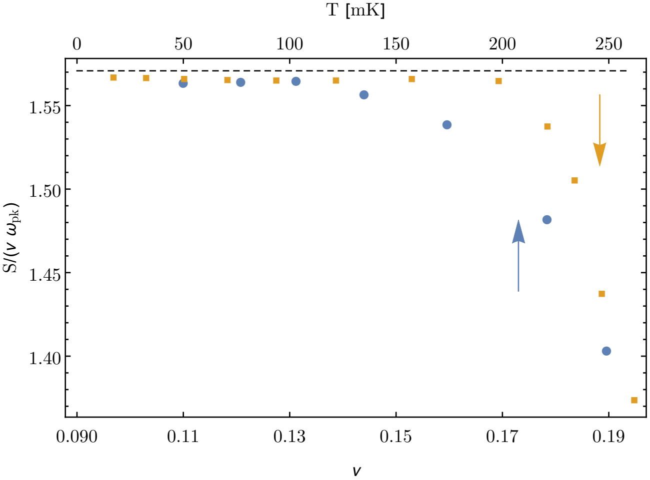


samples A and B

$a \neq 0$ leads to a **violation of the Fukuyama-Lee sum rule**

$$S = \frac{\pi}{2}(1 - a^2)\nu\omega_{\text{pk}}$$

Consistent with previous violations reported in [YP Chen et al PRL'03].



- Compute relaxation parameters? Use Kubo formulas

$$\Omega = \omega_{\rm pk} \lim_{\omega\to 0} \lim_{\Omega,\gamma\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{\dot\varphi_x \dot\varphi_x}(\omega)\,,$$
$$\gamma = \lim_{\omega\to 0} \lim_{\Omega,\gamma\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{j_x \dot\varphi_x}(\omega)\,,$$

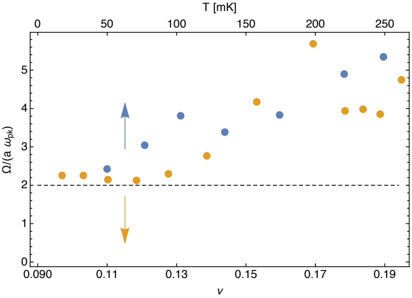
- Now need to compute $\dot\varphi$.
  - **Mobile dislocations**

$$\Omega_{\rm vor} = \frac{2x}{\sigma_{\rm n}} \nu \omega_{\rm pk}\,, \quad \gamma_{\rm vor} = x\sqrt{\nu}\frac{\sigma_{\rm n}^H}{\sigma_{\rm n}} \quad \Rightarrow \quad \frac{\Omega}{a\omega_{\rm pk}} = 2\,.$$

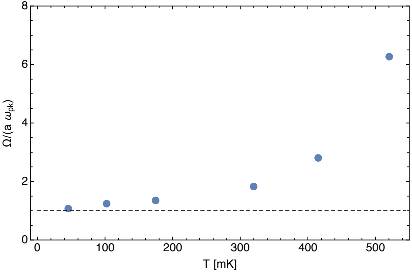
  - **Relaxation into current** $H_{\rm dis} = \frac{1}{\sqrt{\nu}} \int d^2x \epsilon_{ij} \varphi_i(x) j_j(x)\,.$

$$\Omega_{\rm dis} = \frac{\omega_{\rm pk}\sigma_0}{\nu}\,, \quad \gamma_{\rm dis} = \frac{\sigma_0}{\sqrt{\nu}} \quad \Rightarrow \quad \frac{\Omega}{a\omega_{\rm pk}} = 1\,.$$

Different microscopic relaxation mechanisms appear to be at play in the different samples:



Samples A and B
mobile dislocations



Sample C (more disordered)
universal dissipation
into hydrodynamic currents

- I have described how to construct EFTs of pseudo-Goldstones (weak explicit symmetry breaking): applications to superfluids, CDWs, 2d Wigner crystals.

- Hydrodynamic theory of the magnetophonon: quantitatively accounts for experimental data on 2DEG.

- Distinct relaxation mechanisms observed in data: mobile defects or universal relaxation into currents.

- At zero field, detailed checks of the hydrodynamic theory of relaxed density waves using Gauge/Gravity duality.

- Similar universal relaxation into currents observed in [arXiv:1812.08118], [arXiv:1904.11445] with Andrea Amoretti, Daniel Areán and Daniele Musso.