Effective theories of phases with slowly fluctuating broken symmetries

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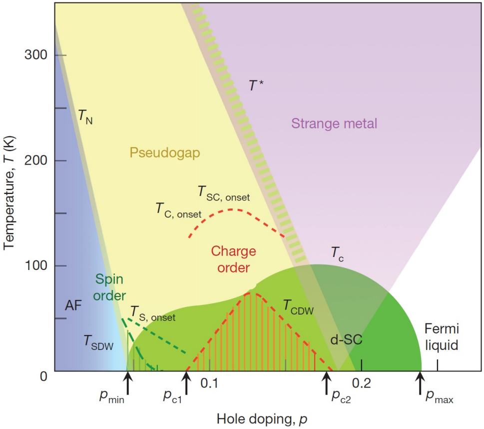
Acknowledgments and references:

- *'Hydrodynamic theory of quantum fluctuating superconductivity'* [arXiv:1602.08171] with Richard Davison, Luca Delacrétaz and Sean Hartnoll
- *'Bad Metals from Density Waves'* [arXiv: 1612.04381],
  'Theory of hydrodynamic transport in fluctuating electronic charge density wave states' [arXiv:1702.05104]
  and ongoing work on magnetotransport with Luca Delacrétaz, Sean Hartnoll and Anna Karlsson.
- *'DC resistivity of quantum critical, charge density wave states from gauge-gravity duality'* [arXiv:1712.07994],
  'Effective holographic theory of charge density waves' [arXiv:1711.06610]
  and *'A holographic strange metal with slowly fluctuating translational order'* [arXiv:1812.08118]
  with Andrea Amoretti, Daniel Areán, Daniele Musso.

- The phase diagram of high T_c superconductors reveals the presence of many phases with spontaneously broken symmetries.



[Keimer et al, Nature (2015)]

- It is plausible that these phases exist also as fluctuating phases in other parts of the phase diagram. What are their experimental signatures in transport?

- Interplay between fluctuating, broken symmetry phases and quantum criticality.

- The low energy dynamics of the ordered phase differ from those of the disordered phase by the necessity to include **new gapless degrees of freedom** (the Goldstones): eg fluid hydrodynamics vs superfluid hydrodynamics.
- An important property of Goldstones is that they are **shift-symmetric**: they realize non-linearly the broken symmetry. More concretely, take broken translations along x

$$x \to x + c \quad \Rightarrow \quad \phi \to \phi + c$$

- Shift symmetry: **only gradient terms** in the effective IR action:

$$f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \dots$$

- ρ_ϕ is the 'stiffness' of the order parameter: in superfluids, $\rho_\phi = \rho_s$ the superfluid density; in phases with translational order, ρ_ϕ will be related to the the bulk and shear moduli.

- If Q is the charge that generates the symmetry, then

$$[\phi(x), Q(y)] = i\delta(x - y) + \dots$$

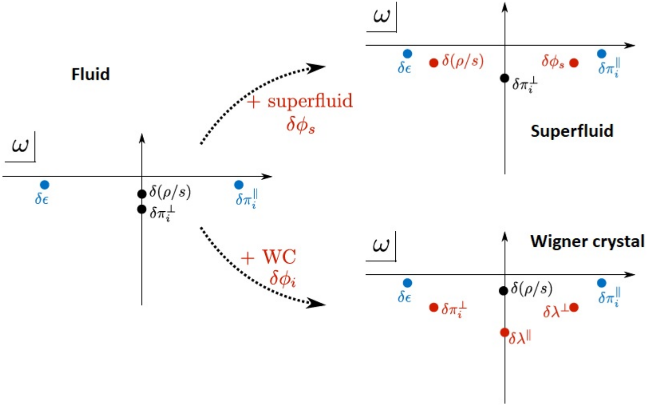
- The effective Hamiltonian contains a term

$$H \sim \int d^d x \, \mu(x) Q(x)$$

which leads to the 'Josephon' relation

$$\nabla\dot{\phi} \equiv \partial_t \nabla\phi = [H, \nabla\phi] = \nabla\mu$$

- Examples: in superfluids, Q is the U(1) charge density and μ the chemical potential; in translationally-ordered phases, Q is the momentum density and μ the velocity along the direction with broken symmetry.

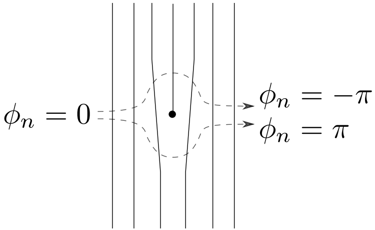


The new sound poles (propagating modes) give rise to $\omega = 0$ poles in the 'conductivity' of the current associated to the broken density

$$\text{superfluid:} \qquad \sigma_{jj} = \frac{i}{\omega} G^R_{jj} = \sigma_o + \frac{\rho_n^2}{\chi_{\pi\pi}} \frac{i}{\omega} + \frac{\rho_s}{\mu} \frac{i}{\omega}$$

$$\text{cdw:} \qquad \sigma_{xy} = \frac{i}{\omega} G^R_{T^{xy}T^{xy}} = \eta + G\frac{i}{\omega}$$

- There are obstructions to the existence of true long range order for continuous symmetries in $d \leq 2$ [Coleman-Mermin-Wagner] ($d \leq 4$ in the presence of random couplings, [Imry & Ma'75]).
- The destruction of long range order occurs via the **proliferation of topological defects**, [Berezenski-Kosterlitz-Thouless]:



- Concretely, the defects **relax the phase gradients**

$$\nabla \dot{\phi} = \nabla \mu - \Omega \nabla \phi$$

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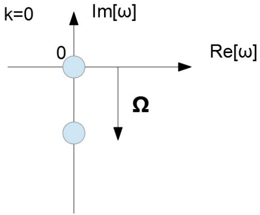
The Goldstone relaxation rate **gaps out** the $\omega = 0$ poles discussed above, $\omega = -i\Omega + \ldots$.



- This gives to large diffusivities

$$\text{superfluid:} \qquad \sigma_{jj} = \frac{i}{\omega} G^R_{jj} = \sigma_o + \frac{\rho_n^2}{\chi_{\pi\pi}} \frac{i}{\omega} + \frac{\rho_s}{\mu} \frac{1}{\Omega - i\omega} \Rightarrow D \sim \sigma_o + \frac{\rho_s}{\mu\Omega}$$

$$\text{cdw:} \qquad \sigma_{xy} = \frac{i}{\omega} G^R_{\tau^{xy}\tau^{xy}} = \eta + \frac{G}{\Omega - i\omega} \Rightarrow D \sim \eta + \frac{G}{\Omega}$$

- Important phenomenological consequences: **destruction of superconductivity** in two-dimensional films, **melting of Abrikosov lattices** in a magnetic field.
- Occurs because the Goldstones become **shorter and shorter lived** as Ω increases: gradual loss of phase coherence.

- These Goldstone relaxation rates can be computed using the **memory matrix formalism**. Crucial technical crutch: consider a Hamiltonian deformation involving the square of the density.

- Disordered superfluid: recovers flux-flow resistance [Bardeen & Stephens'65]

$$\Omega = 2\rho_s \frac{n_f \pi r_v^2}{\sigma_n}$$

- Clean CDW: [Halperin & Nelson'80]

$$\Omega = 2G \frac{n_f \pi r_v^2}{\eta_n}$$

- Can these rates be computed in a holographic setup? Motivation: eg vortex contribution to the resistivity in a critical metallic state.

- Spacetime symmetries can be explicitly broken: focus on the case of broken translations. Momentum **relaxes slowly**

$$\dot{\pi} = -\Gamma\pi + \dots$$

- Impact on the Goldstones: 'tilts the Mexican potential', the Goldstones become **massive**, which breaks their shift symmetry

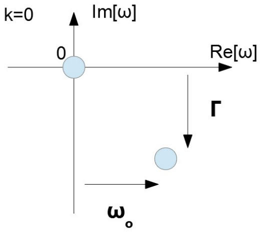
$$f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \dots \to f \sim \frac{1}{2}\rho_\phi \nabla\phi^2 + \frac{1}{2}m^2\phi^2 + \dots$$

  Also contributes to the momentum conservation equation

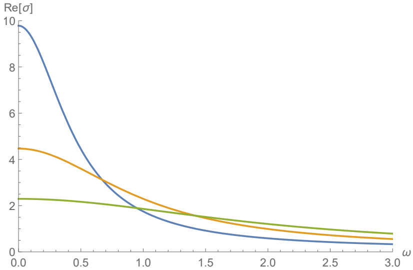
$$\dot{\pi} = -\Gamma\pi - Gm^2\phi \dots$$

The Goldstones now **resonate at a frequency** $\omega_o = m\sqrt(G/\chi_{\pi\pi})$.

A peek ahead: shouldn't we also expect that $m \neq 0 \Rightarrow \Omega \neq 0$? Ie, explicit breaking also gives the Goldstones a finite lifetime.



Weakly-disordered metal

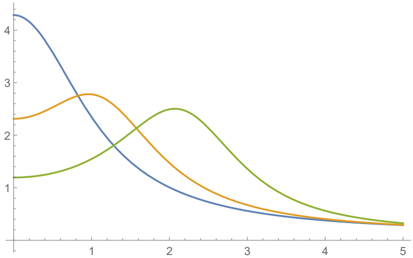


$$\sigma(\omega) = \sigma_o + \frac{\omega_p^2}{\Gamma - i\omega}$$

$$\sigma_{dc} \sim \frac{1}{\Gamma}$$

The dc conductivity is dominated by **momentum relaxation**

Weakly-pinned CDW



$$\sigma(\omega) = \sigma_o + \omega_p^2 \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

$$\sigma_{dc} = \sigma_o + \frac{\omega_p^2}{\Gamma_{cdw}}, \quad \Gamma_{cdw} = \Gamma + \frac{\omega_o^2}{\Omega}$$

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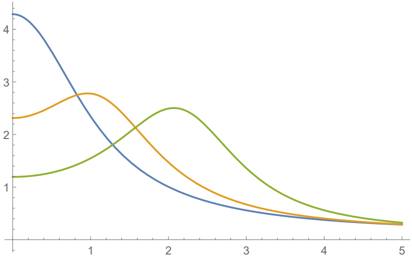


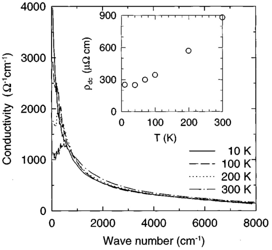
- Insulating phase: Γ, $1/\Omega$ increase as T decreases; the low temperature dc conductivity is dominated by **diffusive coefficients** computed in the clean theory.

- Metallic phase: Γ, $1/\Omega$ decrease as T decreases; the dc conductivity is set by Γ_{cdw} and is **large**.

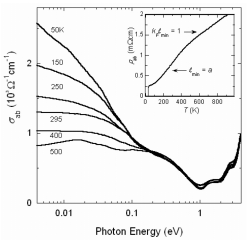
- **A peak at** $\omega = 0$ is recovered if Ω becomes sufficiently large: 'destruction of translational order'.

Interplay between off-axis and Drude-like peaks also observed in cuprates:

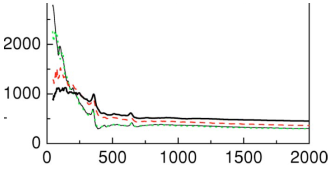
$Bi_2Sr_2CuO_6$
[PRB 55 14152 (1997)]



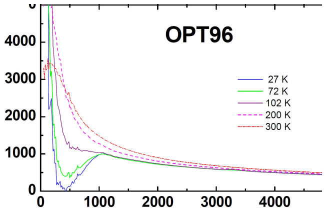
$La_{1.9}Sr_{0.1}CuO_4$
[Phil Mag 84 2847 (2004)]



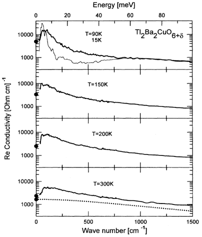
La2126
[PRB 67 134526 (2003)]



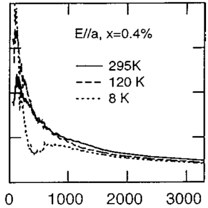
$Bi_2Sr_2CaCu_2O_{8+\delta}$
[J. of Phy: Cond Mat 19 125208 (2007)]



$Tl_2Ba_2CuO_{6+\delta}$
[PRB 51 3312 (1995)]

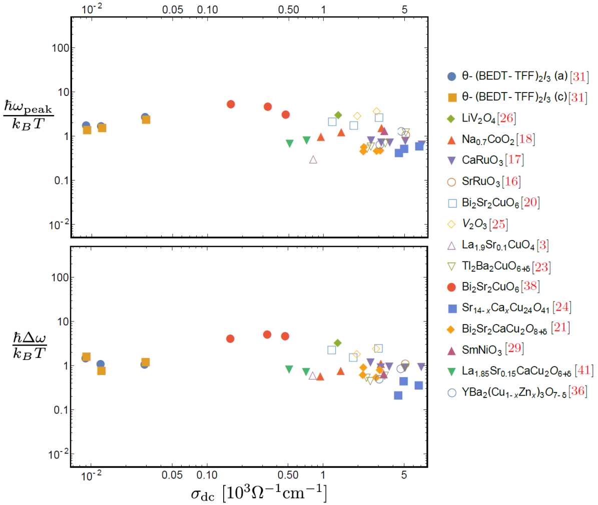


$YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$
[PRB 57 081 (1998)]



Planckian dynamics in the optical conductivity:

$$\hbar\omega_{\text{peak}} \sim k_B T\,, \qquad \hbar\Delta\omega \sim k_B T\,,$$



I will now turn to holographic techniques to study this problem:

- Construct a model **breaking translations homogeneously**, easier to analyze than inhomogeneous models (but not suitable for 'UV' questions).

- Describes the **low energy dynamics of phonons coupled to conserved densities**: check of cdw hydrodynamics, including in the presence of weak explicit translation breaking.

- What happens when the phase becomes **critical**?

- Based on [arXiv:1711.06610, arXiv:1712.07994, arXiv:1812.08118] with Andrea Amoretti, Daniel Areán and Daniele Musso.

- See also [Andrade, Krikun et al, arXiv:1708.08306], [Andrade & Krikun, arXiv:1812.08132], [A. Donos' talk] yesterday, [Alberte et al', arXiv:1708.08477, arXiv:1711.03100], [Jokela et al, arXiv:1612.07323, arXiv1708.07837]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{Z(\phi)}{4}F^2 - V(\phi) - Y(\phi)\left(\partial\psi_x^2 + \partial\psi_y^2\right)\right]$$

$$Y(\phi) = \phi^2 + O(\phi^3)\,, \quad Z(\phi) = 1 + O(\phi)\,, \quad V(\phi) = -6 + \phi^2 + O(\phi^3)$$

- Homogeneous Ansatz [Andrade & Withers'13, Donos & Gauntlett'13]: $\psi_i = kx^i$.
- UV boundary conditions on ϕ

$$\phi = \phi_s r + \phi_v r^2 + \dots$$

- If $\phi_s = 0$, then $\psi_i = kx^i$ is a vev: **spontaneous breaking**.
- If $\phi_s \neq 0$, then $\psi_i = kx^i$ is a source: **explicit breaking**.
- But if $\phi_s/\mu \ll \phi_v/\mu^2$, **pseudo-spontaneous breaking**.

Let us first set $\phi_s = 0$: **purely spontaneous breaking**

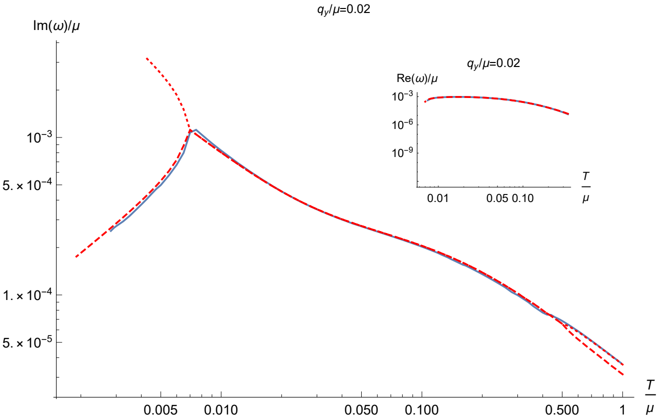
- The phase does not minimize the free energy: describes the low energy dynamics of phonons coupled to conserved densities, not the phase transition. We can choose k, but ultimately this would be fixed in a UV-complete model.

- The **phonon**: act with Lie derivative along ∂/∂_x, find that $\varphi \sim \delta\psi_{(0)}$ (normalization factor!) where $\delta\psi_i = \delta\psi_{(-1)}/r + \delta\psi_{(0)} + O(r)$.

- Recovers the **cdw hydro retarded Green's functions**:

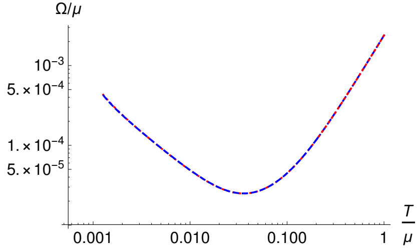
$$G^R_{T^{xy}T^{xy}} = -G + i\omega\eta\,, \quad G^R_{jj} = \frac{\rho^2}{\chi_{\pi\pi}} - i\omega\sigma_o\,, \quad G^R_{j\pi} = \rho\,,$$

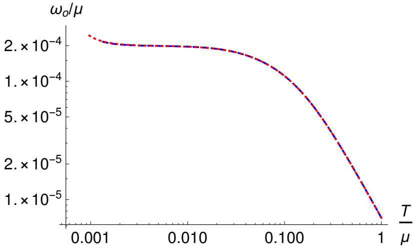
$$G^R_{j\varphi} = \gamma_1 + \frac{\rho}{\chi_{\pi\pi}}\frac{i}{\omega}\,, \quad G^R_{\pi\varphi} = \frac{i}{\omega}\,, \quad G^R_{\varphi\varphi} = \frac{1}{\chi_{\pi\pi}\omega^2} - \frac{\xi}{G}\frac{i}{\omega}\,.$$



Solids support transverse sound waves (or do they?)

$$\omega_{shear} = \frac{1}{2}\left[-iq_y^2\left(\xi + \frac{\eta}{\chi_{\pi\pi}}\right) \pm q_y\sqrt{4\frac{G}{\chi_{\pi\pi}} - q_y^2\left(\frac{\eta}{\chi_{\pi\pi}} - \xi\right)^2}\,\right].$$



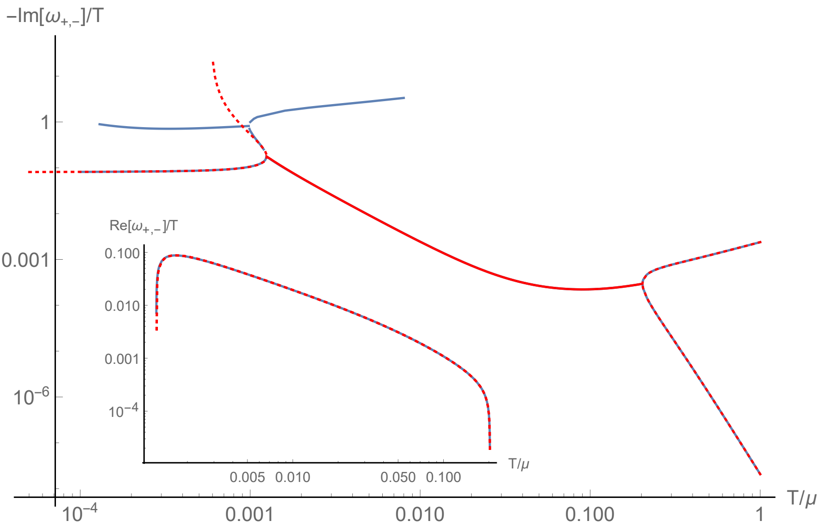


From the memory matrix

$$\Gamma_{cdw} \equiv M_{PP} = \Gamma + \frac{\omega_o^2}{\Omega} \underset{holography}{=} \frac{k^2 s Y_h}{4\pi \chi_{\pi\pi}}$$

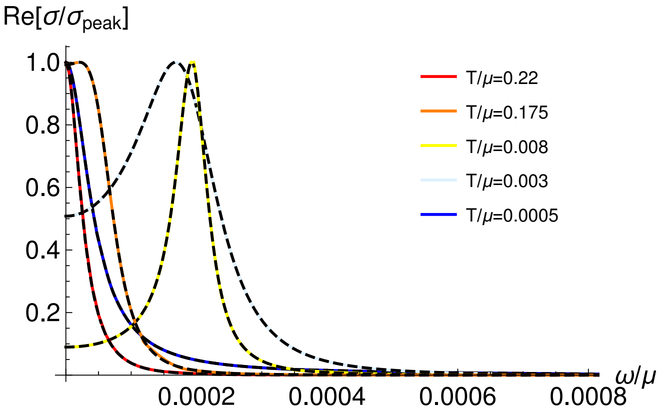
Analytical approximate expressions

$$\Omega^{-1} = \frac{1}{4\pi T} \int_0^{r_h} dr \left(\frac{s T Y_h \sqrt{g_{rr}}}{g_{xx} Y \sqrt{g_{tt}}} - \frac{1}{r_h - r} \right), \quad \frac{\omega_o^2}{\Omega} = \frac{k^2 s Y_h}{4\pi \chi_{\pi\pi}}, \quad \Gamma = 0$$



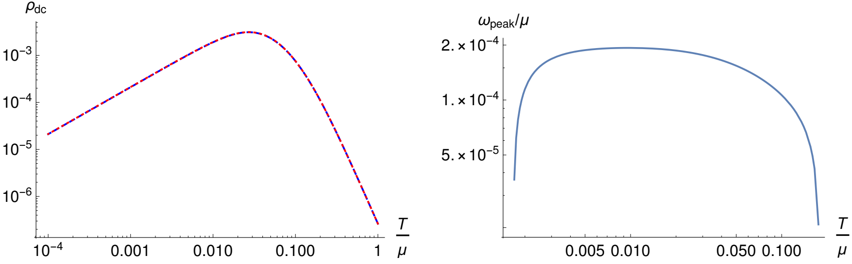
Hydro prediction for the location of the poles:

$$\omega_{\pm} = -\frac{i}{2}\Omega \pm \frac{1}{2}\sqrt{4\omega_o^2 - \Omega^2}\,,$$



Hydro prediction for the ac conductivity:

$$\sigma(\omega) = \sigma_o + \frac{\omega_o^2 \gamma_1 (2\rho - i\gamma_1 \chi_{\pi\pi}\omega) - \frac{\rho^2}{\chi_{\pi\pi}}(\Omega - i\omega)}{\omega^2 - \omega_o^2 + i\omega\Omega}$$



The resistivity is linear in T at low T (same reason as in [Davison, Schalm & Zaanen '13]) but is **always dominated by phonon dissipation**

$$\rho_{dc} \sim \omega_o^2/(\Omega\omega_p^2)$$

The peak turns around when the system becomes metallic, but not with a T-linear dependence. Instead

$$\omega_{peak} = \sqrt{\omega_o^2 - \frac{1}{2}\Omega^2} \underset{T \ll \mu}{\sim} \sqrt{a - \frac{b}{T}}$$

What have we learned?

- At the price of considering thermodynamically unstable phases, **the coupled low energy dynamics of phonons and conserved densities can be modeled holographically without actually constructing inhomogeneous spatially modulated backgrounds**.

- The low energy dynamics precisely matches the expectations from cdw hydrodynamics, including in the presence of **weak explicit translation breaking**.

- Holographic metal where relaxation is completely **dominated by phonon dissipation**.

- **Drude-like peaks** from slowly-fluctuating cdws when the ground state is metallic: interplay with off-axis peaks from pinning. $O(1)$ effect on ac/dc transport.

- First part of the talk: $\Omega \Leftarrow$ topological defects. But the holographic calculation reveals that **explicit breaking generates both a mass and a relaxation rate for the phonons** (see also [Andrade & Krikun'18]).

- Moreover, $\Omega = m^2 \xi$. Can we prove this from field theory? Does it also hold in other holographic models? (eg Bianchi VII [Andrade & Krikun'18], [A. Donos' talk]).

- In a quantum critical phase, we might expect $\Omega \sim m \sim T$. This would imply $\xi \sim 1/T$. ξ is a diffusivity.

- Diffusivities $\sim 1/T$ are expected on general grounds in incoherent metals [Hartnoll'14]. In both electron and hole-doped cuprates, the thermal diffusivity $D_T \sim 1/T$ [Zhang et al'16,'18].