Density waves, bad metals and black holes

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Goal of the talk

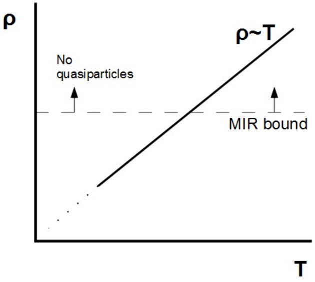
- Discuss the (potential) relevance of (pseudo-)spontaneous symmetry breaking to transport across the phase diagram of cuprate high T_c superconductors.
- Discuss the impact of (pseudo-)Goldstone dynamics on hydrodynamic transport and how this leads to bad metallic transport.
- Present an effective holographic toy-model of transport in cdw states and compute the dc resistivity in holographic critical phases.

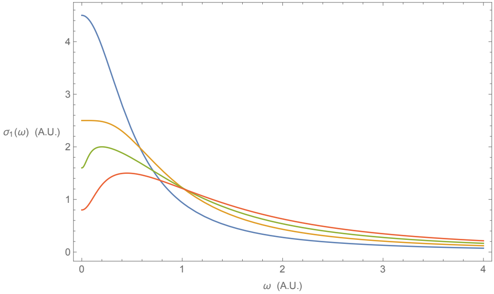
References

- *Bad Metals from Density Waves*, Luca Delacrétaz, Blaise Goutéraux, Sean Hartnoll and Anna Karlsson, [arXiv:1612.04381].
- *Theory of hydrodynamic transport in fluctuating electronic charge density wave states*, Luca Delacrétaz, Blaise Goutéraux, Sean Hartnoll and Anna Karlsson, [arXiv:1702.05104].
- *Effective holographic theory of charge density waves*, Andrea Amoretti, Daniel Areán, Blaise Goutéraux and Daniele Musso, [arXiv: 1711.06610].
- *DC resistivity at holographic charge density wave quantum critical points*, Andrea Amoretti, Daniel Areán, Blaise Goutéraux and Daniele Musso, [arXiv:1712.07994].

- And ongoing work.

Central motivation: bad metallic transport





Two experimental challenges for theorists [Hussey, Takenaka & Takagi'04]:

- T-linear resistivity violating the Mott-Ioffe-Regel bound: **no quasiparticles**
- Optical conductivity: **far IR peak** ($\sim 10^2 cm^{-1}$) moving off axis as T increases to room temperature.

Planckian dynamics

$$\rho = \frac{m}{ne^2 \tau_{tr}} \sim T \qquad \Rightarrow \qquad \tau_{tr} = \tau_P \equiv \frac{\hbar}{k_B T}$$

Universal scale in all systems at finite temperature which follows from dimensional analysis

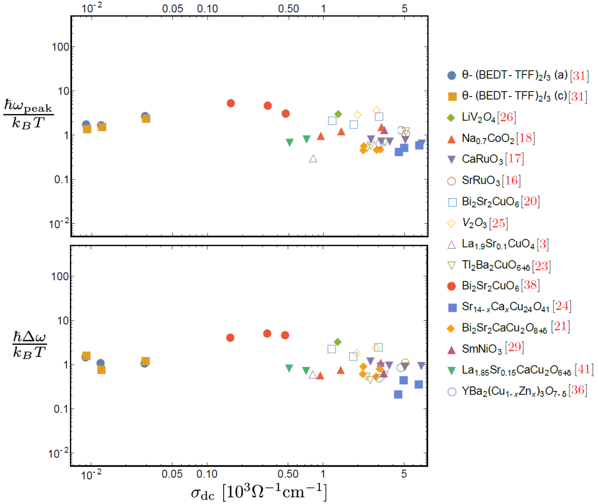
$$[\hbar] = J.s\,, \quad [k_B] = J.K^{-1}\,, \quad [T] = K \quad \Rightarrow \quad \tau_P = \frac{\hbar}{k_B T}$$

In strongly-coupled, quantum systems, expected to be the **fastest equilibration time** allowed by Nature and Quantum Mechanics [Sachdev,Zaanen]. At room temperature

$$\tau_P \sim 25 fs$$

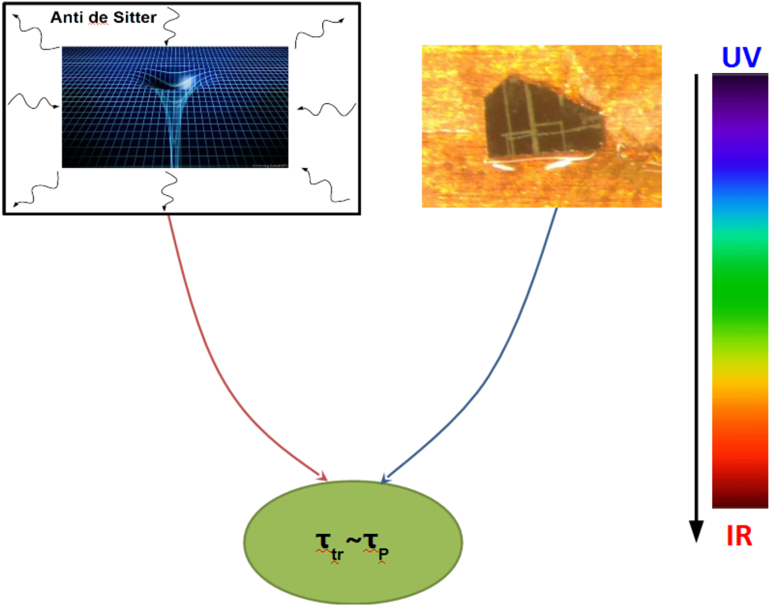
Planckian dynamics in the optical conductivity [arXiv:1612.04381]

$$\hbar\omega_{\text{peak}} \sim k_B T\,, \qquad \hbar\Delta\omega \sim k_B T\,,$$

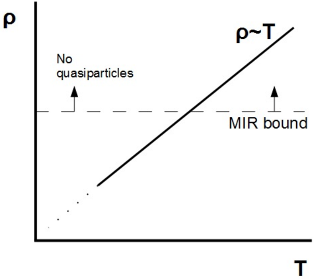


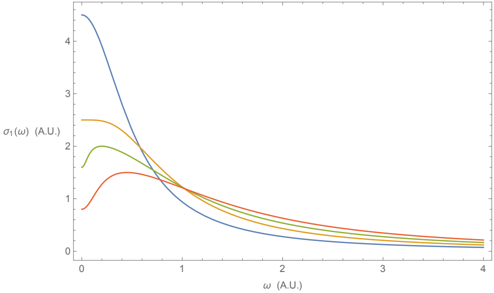
- These observations suggest that **Planckian dynamics** is a defining feature of both **ac and dc transport** in bad metals.
- Planckian dynamics also emerge in the **low energy effective description** of strongly-coupled (holographic) quantum matter.
- **Universal** low energy effective theory?

Universal low energy Planckian dynamics



Next part of this talk



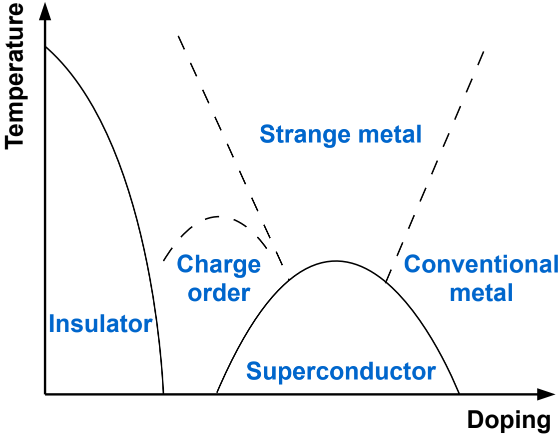


I will offer a theory based on hydrodynamics and spontaneous translation symmetry breaking which

- leads to **small dc conductivities**, ie bad metal;
- accounts for the **far IR off-axis peak** in $\sigma(\omega)$;
- naturally **relates** the dc and ac transport timescales.

Disclaimer: effective low energy theory of transport, not a microscopic theory.

Spontaneous translation symmetry breaking



Late time dynamics from hydrodynamics

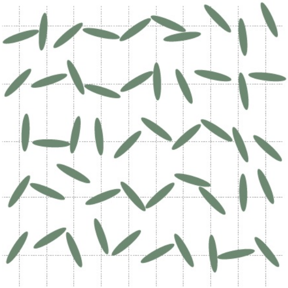
Short-lived quasiparticles: **conserved quantities** are more fundamental for late-time transport

$$\partial_t \epsilon + \vec{\nabla}\vec{j}_\epsilon = 0$$

$$\partial_t \pi^i + \nabla_k \tau^{ik} = 0$$

$$\partial_t \rho + \vec{\nabla}\vec{j} = 0$$

Hydrodynamics: long wavelength description of the system



[credit: Beekman et al'16]

Electronic crystal

We also wish to include a CDW [Grüner'88, Chaikin & Lubensky]:

$$\rho(x) = \rho_0 \cos\left[Qx + \Psi(x,t)\right]$$

The phase $\Psi(x,t)$ is a new dof coming from the SSB of translations (Goldstone): **'phonon' of the electronic crystal**.

$$[\pi, \Psi] = -i\delta$$

$$\Rightarrow \dot{\Psi} = i[H, \Psi] = i\left[\int \pi v, \Psi\right] = v$$

Josephson relation for the phonon

[credit: Beekman et al'16]

CDW hydrodynamics [Grüner'88, Chaikin-Lubensky,1612.04381]

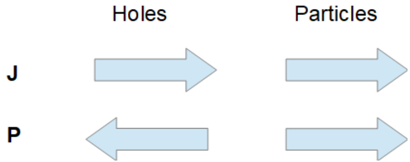
- Constitutive relation for the current

$$j = \rho v - \sigma_o \nabla \mu + \dots ,$$

- σ_o is a **diffusive** transport coefficient: charge transport **without momentum drag** [Davison, Goutéraux & Hartnoll'15].

- An analogy: particle-hole creation in a CFT.

$$P = P_p + P_h = 0 \qquad J = J_p + J_h = 2J_p$$



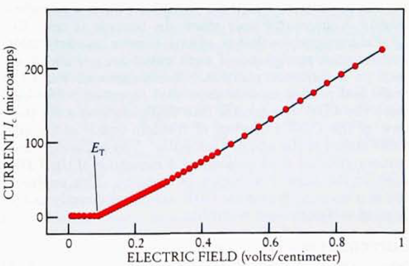
Pinning of CDW

- A CDW is '**pinned**' by impurities: sliding only occurs beyond a threshold electric field.

- More formally: the Goldstones acquire a **small mass** ω_o.

- **Momentum is relaxed** by impurities

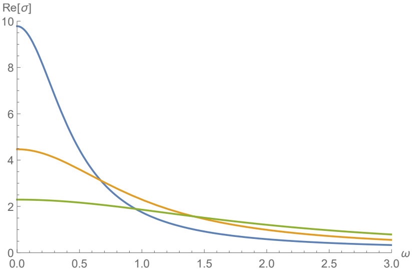
$$\partial_t \pi^i + \nabla^j T^{ij} = -\Gamma \pi^i$$



[THORNE'96]

Consequences on charge transport

Weakly-disordered metal

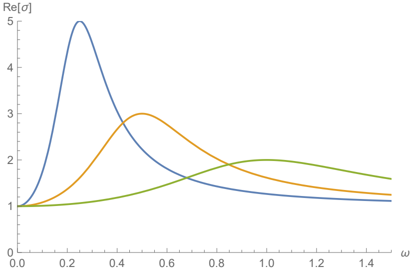


$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma_\pi - i\omega}$$

$$\sigma_{dc} \sim \Gamma_\pi^{-1}$$

The dc conductivity is dominated by **momentum relaxation**

Weakly-pinned CDW



$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{-i\omega(\Gamma_\pi - i\omega) + \omega_o^2}$$

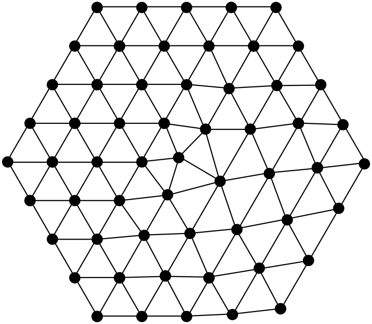
$$\sigma_{dc} = \sigma_o + O(\Gamma_\pi)$$

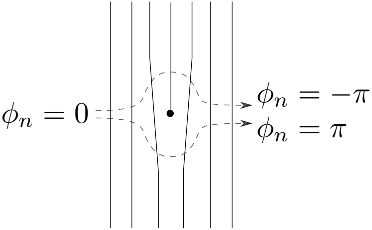
The dc conductivity is set by the **incoherent conductivity** computed in the clean theory.

Phase disordering

- In 2d, crystals can **melt by proliferation of topological defects** in the crystalline structure [Nelson & Halperin'79].
- At $T = 0$: quantum melting [Kivelson et al'98, Beekman et al'16].
- The phase gets disordered ($\sim$ BKT) at a rate Ω: **flow of mobile dislocations** [arXiv:1702.05104].

$$\Rightarrow \dot{\Psi} + \Omega\Psi = v$$





Conducting, phase-disordered CDWs [arXiv:1612.04381]

- Now the conductivity reads

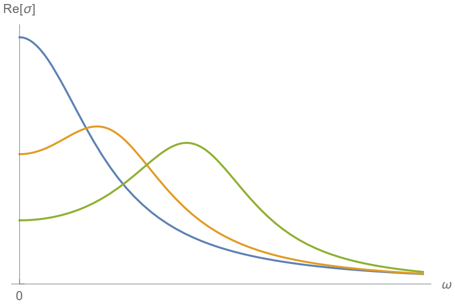
$$\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}, \qquad \sigma_{dc} = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma_{CDW}}$$

$$\Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}$$

New transport inverse timescale, **non-zero** even if $\Gamma_\pi \sim 0$.

- **Off-axis peak** for sufficiently small Ω or large pinning ω_o

$$\omega_o \geq \frac{\Omega^3}{\Gamma_\pi + 2\Omega}$$



Strange metallic transport from fluctuating CDWs

- Neglect momentum relaxation $\Gamma_\pi \ll \omega_0, \Omega$ + Galilean $\sigma_o = 0$:

$$\sigma_{dc} = \frac{n\,e^2}{m}\frac{\Omega}{\omega_o^2}$$

- The width and position of the peak are controlled by Ω, ω_o. **The data shows $\Omega \sim \omega_o \sim k_B T/\hbar$**

$$\Rightarrow \rho_{dc} = \frac{1}{\sigma_{dc}} \sim \frac{m}{n\,e^2}\frac{k_B\,T}{\hbar}$$

T-linear resistivity!

- Hydrodynamics of fluctuating CDWs provide a natural mechanism whereby the ac and dc conductivities are controlled by **the same Planckian timescale**.

Some open questions

- Typical frequency scales of order T: at the **edge of validity of hydrodynamics** $\omega \ll T$.
- The role played by the Planckian timescale is indicative of quantum criticality: **quantum critical computation**.
- Work in progress: use Gauge/Gravity duality to compute non-hydrodynamic transport in phases with spontaneously broken translation symmetry.

Holographic model of spontaneous symmetry breaking

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - Y(\phi)\sum_{i=1}^{2}\partial\psi_I^2\right]$$

- Inspired by [Donos & Gauntlett'13, Andrade & Withers'13].
- Static Ansatz: only radial dependence

  $$\mathrm{d}s^2 = -D(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + C(r)\mathrm{d}\vec{x}^2\,, \quad A = A(r)\mathrm{d}t\,, \quad \phi = \phi(r)$$

  except for $\psi_I = k\delta_{Ij}x^j$.
- Internal shift and rotation symmetry of the ψ_I combines with spatial translations and rotations to preserve the translation and rotation symmetry of the Ansatz.

UV deformation by complex scalar operators

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - Y(\phi)\sum_{i=1}^{2}\partial\psi_I^2\right]$$

- For simple choices of $Y = \phi^2$, $Y = (\sinh\phi)^2$, the real scalars can be rewritten as complex scalars $\Phi_I = \phi\,\mathrm{e}^{i\psi_I}$ [Donos & Gauntlett'13], $\Phi_I = \tanh\phi\,\mathrm{e}^{i\psi_I}$ [Donos & al'14].

- Can always be done **asymptotically** provided $Y_{UV} \sim \phi^2$

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} - \frac{1}{2}\left(\lambda^I\mathcal{O}_I^* + \lambda_I^*\mathcal{O}^I\right)$$

  Same as in mean-field treatments of CDWs [Grüner'88].

- If $\lambda_I = 0$, spontaneous breaking.

Match to crystal stress tensor

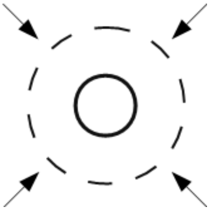
Dual stress-tensor: equilibrium stress-tensor for an isotropic crystal

$$\langle T^{ij}_{eq} \rangle = [p - (G+K)\,\partial\cdot\Psi]\,\delta^{ij} - 2G\left[\partial^{(i}\Psi^{j)} - \delta^{ij}\partial\cdot\Psi\right], \quad \Psi^i = x^i$$

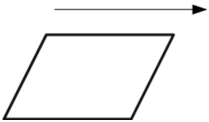
with the **bulk modulus** (elastic resistance to compression)

$$K = -\frac{k^2}{2}\int_{r_h}^{0} \mathrm{d}r\sqrt{BD}\,Y$$

Isotropy $\Rightarrow$ The **shear modulus** G does not appear at equilibrium.



Bulk modulus



Shear modulus

The incoherent conductivity: computation

- Recall that the conductivity of a static, pinned CDW is

$$\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}$$

$$\sigma_{dc} = \sigma_o + O(\Gamma_\pi)$$

- We computed the incoherent conductivity analytically.

- At low temperatures:

$$\sigma_o(T \to 0) = \frac{4K^2}{(\mu\rho - 2K)^2} \left(Z_h + \frac{4\pi\rho^2}{sk^2 Y_h} \right)$$

CDW quantum critical point

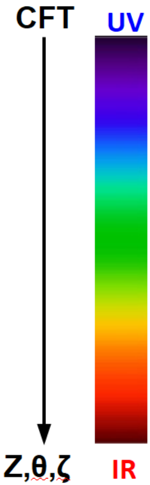
- Long story short: RG flows between a UV CFT ($\phi = 0$) and a **hyperscaling violating** IR ($\phi \to \infty$) [Goutéraux'14]

$$V_{IR} = V_0 e^{-\delta\phi}\,, \quad Z_{IR} = Z_0 e^{\gamma\phi}\,, \quad Y_{IR} = Y_0 e^{\lambda\phi}$$

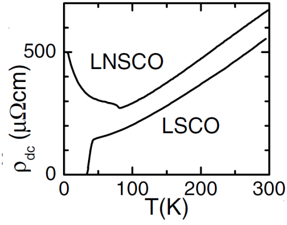
$$ds^2 = r^\theta \left[-\frac{dt^2}{r^{2z}} + \frac{L^2 d^2 r}{r^2} + \frac{d\vec{x}^2}{r^2} \right], \quad A = A_0\, r^{\zeta - z} dt$$

$$\psi_i = kx^i\,, \quad \phi = \kappa \log r$$

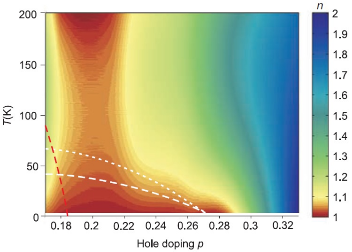
- The solution is **scale covariant**
  $t \to \lambda^z t\,, \quad (r, x) \to \lambda(r, x)$
- Typical observables **scale**, eg $s \sim T^{\frac{d-\theta}{z}}$



Two interesting cases



[Dumm et al, PRL 88 14 (2002)]



[Cooper et al'09]

- $z \to \infty$: $AdS_2 \times R^2$, underdoped cuprates?

$$\sigma_o(T \to 0) \to T^0$$

- $z, \theta \to \infty$, $\theta = -z$: conformal to $AdS_2 \times R^2$

$$\sigma_o(T \to 0) \to T^{-1}$$

Optimally doped cuprates? ([Davison, Schalm & Zaanen'13])