Density waves, bad metals and black holes

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NORDITA, Stockholm

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Workshop on Strange Metals
Radboud University, Nijmegen, The Netherlands

Goal of the talk

- Discuss the (potential) relevance of (pseudo-)spontaneous symmetry breaking to transport across the phase diagram of cuprate high T_c superconductors.
- Discuss the impact of (pseudo-)Goldstone dynamics on hydrodynamic transport and how this leads to bad metallic transport.
- Present an effective holographic toy-model of transport in cdw states and compute the dc resistivity in holographic critical phases.

References

This talk is mainly based on

- *Bad Metals from Density Waves*, Luca Delacrétaz, Blaise Goutéraux, Sean Hartnoll and Anna Karlsson, [arXiv:1612.04381].

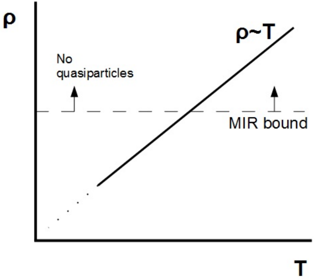
- *Theory of hydrodynamic transport in fluctuating electronic charge density wave states*, Luca Delacrétaz, Blaise Goutéraux, Sean Hartnoll and Anna Karlsson, [arXiv:1702.05104].

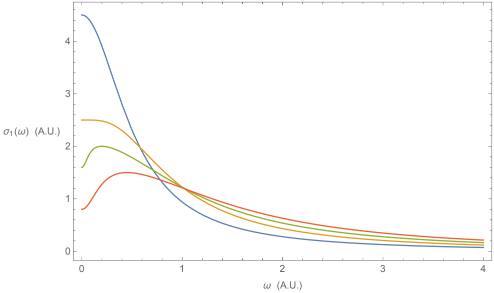
- *Effective holographic theory of charge density waves*, Andrea Amoretti, Daniel Areán, Blaise Goutéraux and Daniele Musso, [arXiv: 1711.06610].

- *DC resistivity at holographic charge density wave quantum critical points*, Andrea Amoretti, Daniel Areán, Blaise Goutéraux and Daniele Musso, [arXiv:1712.07994].

- And ongoing work.

Central motivation: bad metallic transport





Two experimental challenges for theorists [Hussey, Takenaka & Takagi'04]:

- T-linear resistivity violating the MIR bound: **no quasiparticles**
- Optical conductivity: **far IR peak** ($\sim 10^2 cm^{-1}$) moving off axis as T increases to room temperature.

Planckian dynamics

$$\rho = \frac{m}{ne^2\tau_{tr}} \sim T \qquad \Rightarrow \qquad \tau_{tr} = \tau_P \equiv \frac{\hbar}{k_B T}$$

Universal scale in all systems at finite temperature which follows from dimensional analysis

$$[\hbar] = J.s\,, \quad [k_B] = J.K^{-1}\,, \quad [T] = K \quad \Rightarrow \quad \tau_P = \frac{\hbar}{k_B T}$$

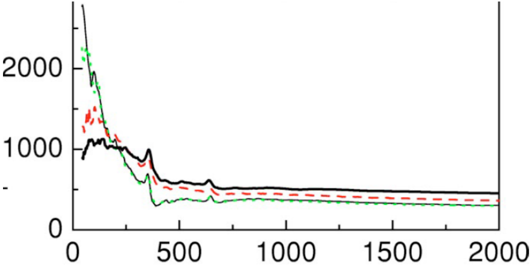
In strongly-coupled, quantum systems, expected to be the **fastest equilibration time** allowed by Nature and Quantum Mechanics [Sachdev,Zaanen]. At room temperature

$$\tau_P \sim 25 fs$$

Off-axis peaks in optical conductivity data

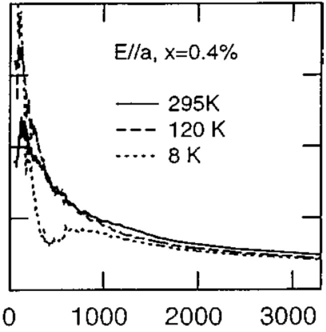
La2126

[PRB 67 134526 (2003)]



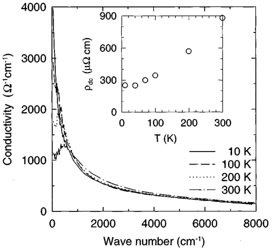
$YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$

[PRB 57 081 (1998)]

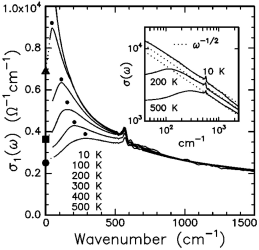


Off-axis peaks in optical conductivity data (2)

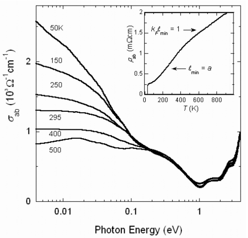
$Bi_2Sr_2CuO_6$
[PRB 55 14152 (1997)]



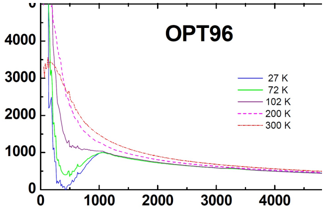
Ca_2RuO_3
[PRB 66 041104 (2002)]



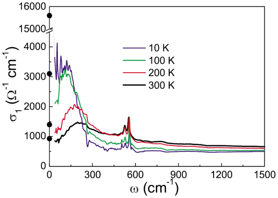
$La_{1.9}Sr_{0.1}CuO_4$
[Phil Mag 84 2847 (2004)]



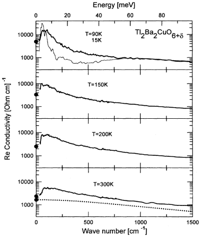
$Bi_2Sr_2CaCu_2O_{8+\delta}$
[J. of Phy: Cond Mat 19 125208 (2007)]



$Na_{0.7}CoO_2$
[PRL 93 237007 (2004)]

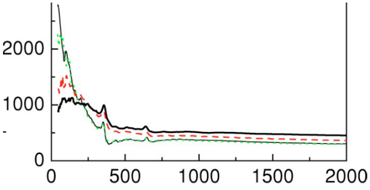


$Tl_2Ba_2CuO_{6+\delta}$
[PRB 51 3312 (1995)]

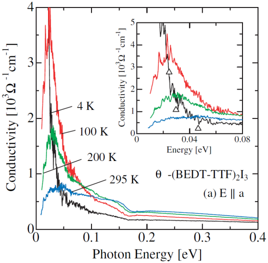


Off-axis peaks in optical conductivity data (3)

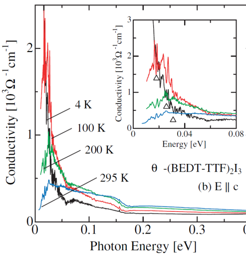
La2126
[PRB 67 134526 (2003)]



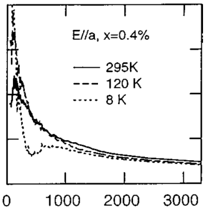
θ-(BEDT-TTF)$_2$I$_3$ (a)
[PRL 95 227801 (2005)]



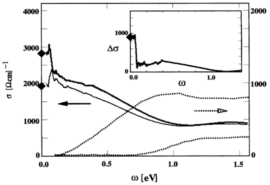
θ-(BEDT-TTF)$_2$I$_3$ (c)
[PRL 95 227801 (2005)]



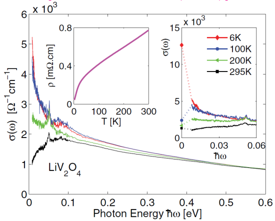
YBa$_2$(Cu$_{1-x}$Zn$_x$)$_3$O$_{7-\delta}$
[PRB 57 081 (1998)]



V$_2$O$_3$
[PRL 75 105 (1995)]

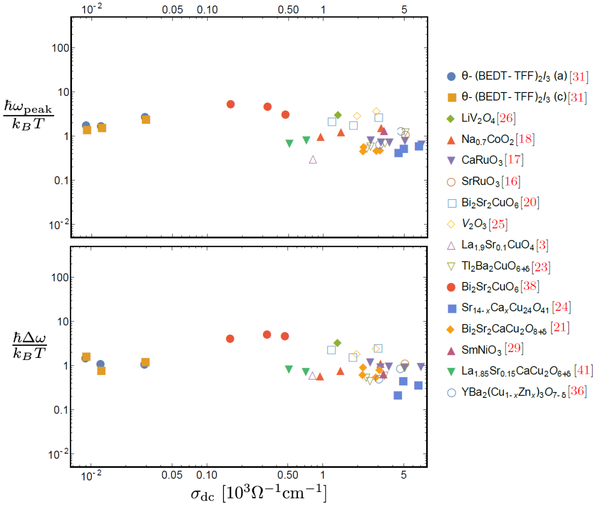


LiV$_2$O$_4$
[PRL 99 167402 (2007)]



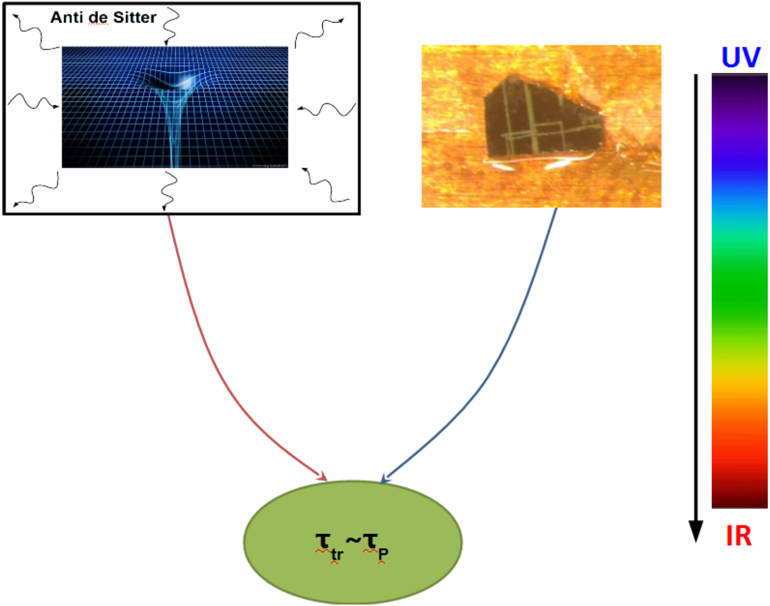
Planckian dynamics in the optical conductivity [arXiv:1612.04381]

$$\hbar\omega_{\text{peak}} \sim k_B T\,, \qquad \hbar\Delta\omega \sim k_B T\,,$$

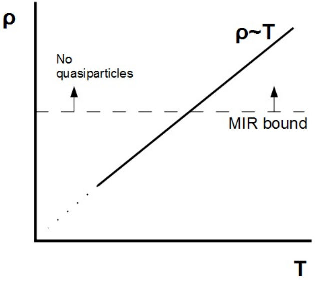


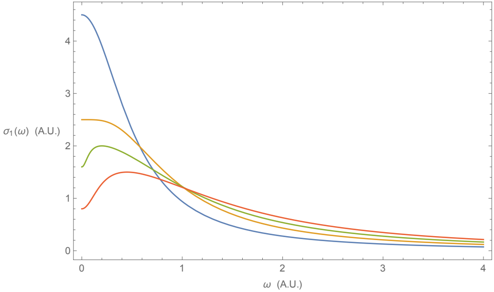
- These observations suggest that **Planckian dynamics** is a defining feature of both **ac and dc transport** in bad metals.
- Planckian dynamics also emerge in the **low energy effective description** of strongly-coupled (holographic) quantum matter.
- **Universal** low energy effective theory?

Universal low energy Planckian dynamics



Next part of this talk



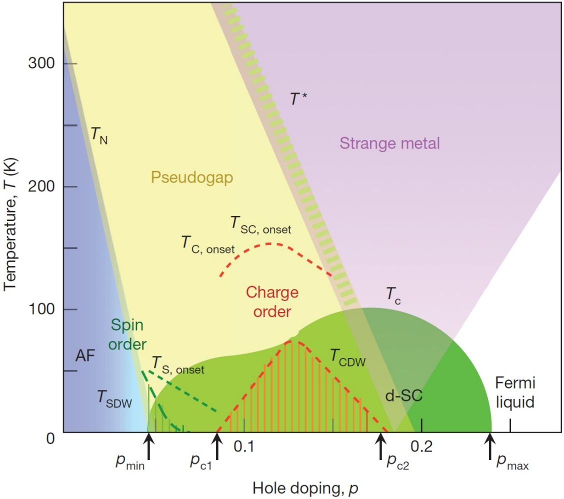


I will offer a theory based on hydrodynamics and spontaneous translation symmetry breaking which

- leads to **small dc conductivities**;
- accounts for the **far IR off-axis peak** in $\sigma(\omega)$;
- naturally **relates** the dc and ac transport timescales.

Disclaimer: effective low energy theory of transport, not a microscopic theory.

Spontaneous translation symmetry breaking



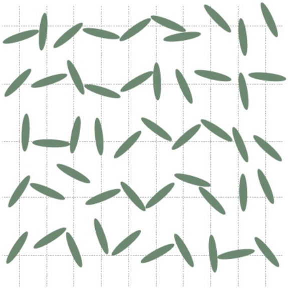
[Keimer et al, Nature 518 179 (2015)]

Late time dynamics from hydrodynamics

Short-lived quasiparticles: **conserved quantities** are more fundamental for late-time transport

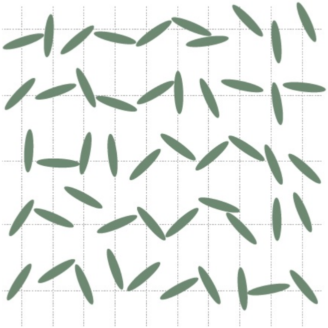
$$\partial_t \epsilon + \vec{\nabla} \vec{j}_e = 0$$
$$\partial_t \pi^i + \nabla_k \tau^{ik} = 0$$
$$\partial_t \rho + \vec{\nabla} \vec{j} = 0$$

Hydrodynamics: long wavelength description of the system



[credit: Beekman et al'16]

Electronic crystal



[credit: Beekman et al'16]

[credit: Beekman et al'16]

We also wish to include a CDW [Grüner'88, Chaikin & Lubensky]:

$$\rho(x) = \rho_0 \cos\left[Qx + \phi(x,t)\right]$$

The phase $\phi(x,t)$ is a new dof coming from the SSB of translations (Goldstone): **'phonon' of the electronic crystal**.

CDW hydrodynamics [Grüner'88, Chaikin-Lubensky,1612.04381]

- Constitutive relation for the current and the Goldstone

$$j = \rho v - \sigma_o \nabla \mu + \dots , \qquad \dot{\phi} = v + \dots$$

- σ_o is a **diffusive** transport coefficient encoding charge transport **without momentum drag**.

- $$\rho(x) = \rho_0 \cos\left[Qx + \phi(x,t)\right]$$

  Define $v = Q\dot{x}$. Then the 'Josephson' relation comes from the shift symmetry of the low energy dynamics.

Conductivity of a pinned, non-Galilean CDW

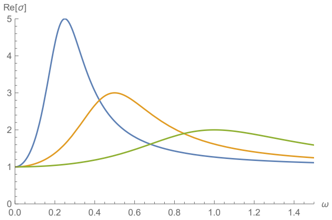
- Standard procedure to extract retarded Green's functions [Kadanoff & Martin'63].

- **Weak disorder**: finite momentum lifetime $1/\Gamma_\pi$, **pins the Goldstone** ϕ with a small mass ω_o.

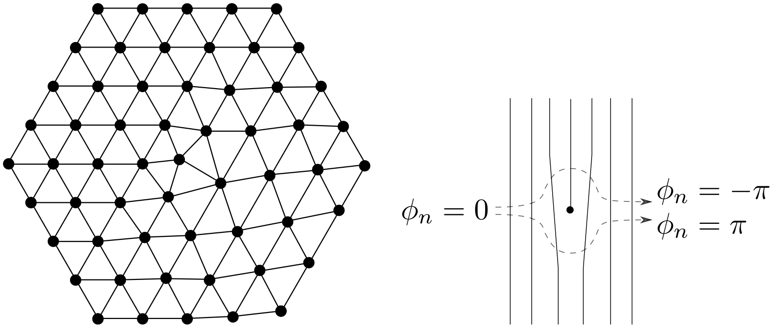
$$\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}$$

- Non-zero dc conductivity $\sigma_{dc} = \sigma_o + O(\Gamma_\pi)$

- Can be **small** even for weak momentum relaxation: **bad metal**.



Phase disordering



- In 2d, crystals can **melt by proliferation of topological defects** in the crystalline structure [Nelson & Halperin'79].
- At $T = 0$: quantum melting [Kivelson et al'98, Beekman et al'16].
- The phase gets disordered ($\sim$ BKT) at a rate Ω: **flow of mobile dislocations** [arXiv:1702.05104].

Conducting, phase-disordered CDWs [arXiv:1612.04381]

- Now the conductivity reads

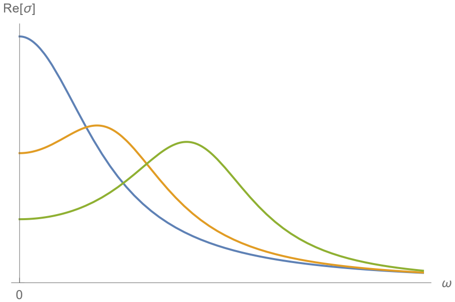
$$\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}\,, \qquad \sigma_{dc} = \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma_{CDW}}$$

$$\Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}$$

New transport inverse timescale, **non-zero** even if $\Gamma_\pi \sim 0$.

- **Off-axis peak** for sufficiently small Ω or large pinning ω_o

$$\omega_o \geq \frac{\Omega^3}{\Gamma_\pi + 2\Omega}$$



Bad metallic transport from fluctuating CDWs

- Neglect momentum relaxation $\Gamma_\pi \ll \omega_0, \Omega$ + Galilean $\sigma_o = 0$:

$$\sigma_{dc} = \frac{n\,e^2}{m}\frac{\Omega}{\omega_o^2}$$

- The width and position of the peak are controlled by Ω, ω_o.
  The data shows $\Omega \sim \omega_o \sim k_B T/\hbar$

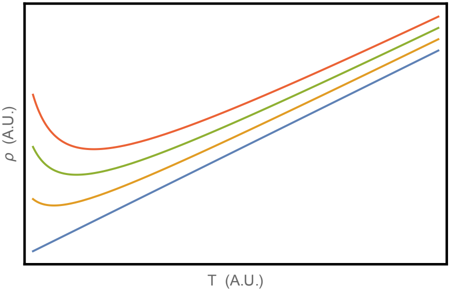
$$\Rightarrow \rho_{dc} = \frac{1}{\sigma_{dc}} \sim \frac{m}{n\,e^2}\frac{k_B\,T}{\hbar}$$

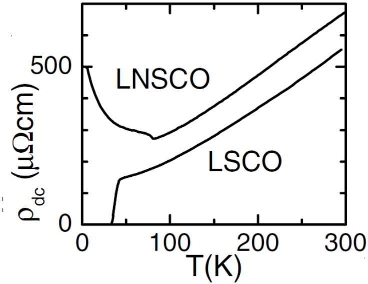
  T**-linear resistivity**!

- Hydrodynamics of fluctuating CDWs provide a natural mechanism whereby the ac and dc conductivities are controlled by **the same Planckian timescale**.

Resistivity upturns from fluctuating cdws

$$\rho_{dc} = \frac{m}{ne^2}\Gamma_{CDW}\ , \quad \Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}$$



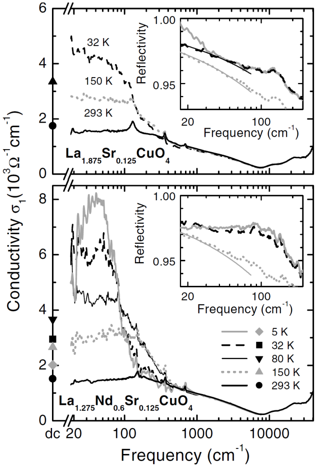


[Dumm et al, PRL 88 14 (2002)]

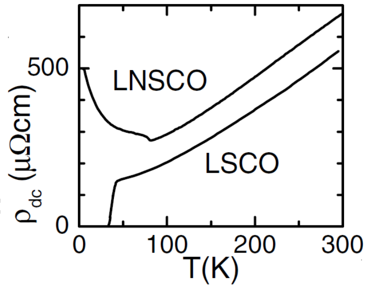
An **upturn** occurs as Ω decreases and phase fluctuations dominate Γ_{CDW}: relation to underdoped cuprates and static charge order?

Wiedeman-Franz law violation?

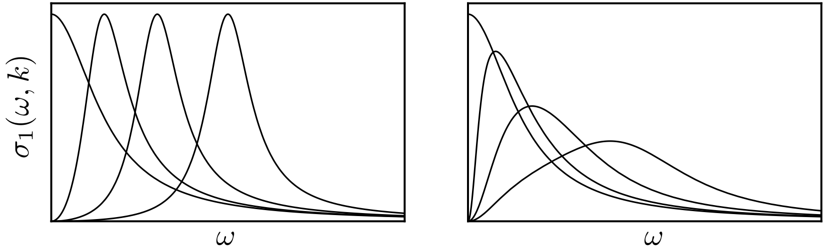
$$\frac{\kappa}{T\sigma} \sim \frac{1}{\Omega} \gg \mathcal{L}_o$$



[Dumm et al, PRL 88 14 (2002)]



Spatially-resolved conductivity

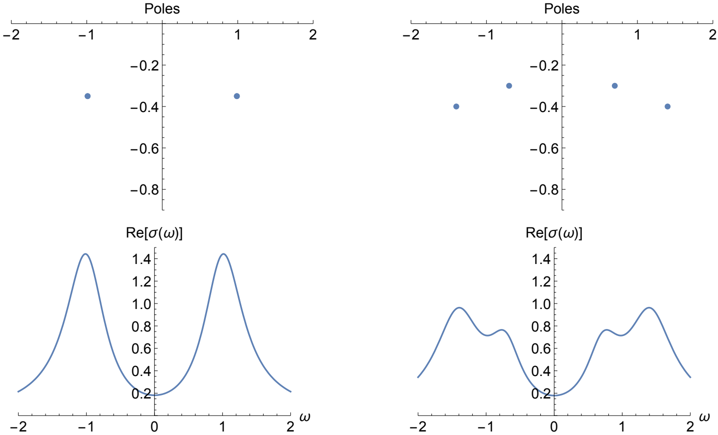


This is a case where Ω is large enough that $\sigma(\omega, 0)$ only displays a Drude peak.

The presence of fluctuating CDWs (right) is signaled by the broadening of the peak as k increases.

In contrast to the structure factor, the spectral weight is ne^2/m.

Magnetotransport



We estimate $B > B_c \sim m^\star / m_e \cdot 7 - 9\,T$ in NdLSCO at $x = 1/8$.

Some open questions

- Typical frequency scales of order T: at the **edge of validity of hydrodynamics** $\omega \ll T$.
- The role played by the Planckian timescale is indicative of quantum criticality: **quantum critical computation**.
- Work in progress: use Gauge/Gravity duality to compute non-hydrodynamic transport in phases with spontaneously broken translation symmetry.

An effective holographic model of spontaneous symmetry breaking

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - Y(\phi)\sum_{i=1}^{d}\partial\psi_I^2\right]$$

- Inspired by [Donos & Gauntlett'13, Andrade & Withers'13].
- Static Ansatz: only radial dependence

  $$\mathrm{d}s^2 = -D(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + C(r)\mathrm{d}\vec{x}^2\,, \quad A = A(r)\mathrm{d}t\,, \quad \phi = \phi(r)$$

  except for $\psi_I = k\delta_{Ij}x^j$.
- Internal shift and rotation symmetry of the ψ_I combines with spatial translations and rotations to preserve the translation and rotation symmetry of the Ansatz.

UV deformation by complex scalar operators

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - Y(\phi)\sum_{i=1}^{d}\partial\psi_I^2\right]$$

- For simple choices of $Y = \phi^2$, $Y = (\sinh\phi)^2$, the real scalars can be rewritten as complex scalars $\Phi_I = \phi\,\mathrm{e}^{i\psi_I}$ [Donos & Gauntlett'13], $\Phi_I = \tanh\phi\,\mathrm{e}^{i\psi_I}$ [Donos & al'14].

- Not possible to do explicitly in general, but **still true asymptotically**

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} - \frac{1}{2}\left(\lambda^I\mathcal{O}_I^* + \lambda_I^*\mathcal{O}^I\right)$$

  Same as in mean-field treatments of CDWs [Grüner'88].

- If $\lambda_I = 0$, spontaneous breaking.

The incoherent conductivity: computation

- Recall that the conductivity of a static, pinned CDW is

$$\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}$$

$$\sigma_{dc} = \sigma_o + O(\Gamma_\pi)$$

- We computed the incoherent conductivity analytically (see A. Donos' talk on Friday for more on how to compute dc conductivities in holography).

- At low temperatures:

$$\sigma_o(T \to 0) = \frac{4B^2}{(\mu\rho - 2B)^2} \left(Z_h + \frac{4\pi\rho^2}{sk^2 Y_h} \right)$$

CDW quantum critical point

- Long story short: RG flows between a UV CFT ($\phi = 0$) and a **hyperscaling violating** IR ($\phi \to \infty$) [Goutéraux'14]

$$V_{IR} = V_0 e^{-\delta\phi}\,, \quad Z_{IR} = Z_0 e^{\gamma\phi}\,, \quad Y_{IR} = Y_0 e^{\lambda\phi}$$

$$ds^2 = r^\theta \left[-\frac{dt^2}{r^{2z}} + \frac{L^2 d^2 r}{r^2} + \frac{d\vec{x}^2}{r^2} \right], \quad A = A_0\, r^{\zeta - z} dt$$

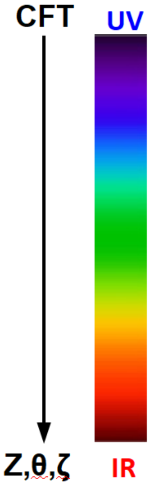
$$\psi_i = k x^i\,, \quad \phi = \kappa \log r$$

- The solution is **scale covariant**

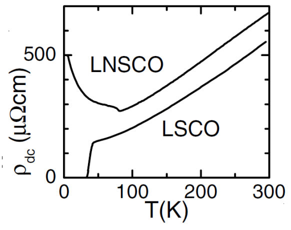
$$t \to \lambda^z t\,, \quad r \to \lambda r\,, \quad \vec{x} \to \lambda \vec{x}$$

- Typical observables (entropy density) **scale**

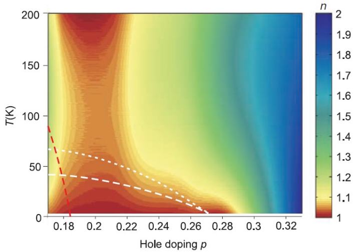
$$s \sim T^{\frac{d-\theta}{z}}$$



Two interesting cases



[Dumm et al, PRL 88 14 (2002)]



[Cooper et al'09]

- $z \to \infty$: $AdS_2 \times R^2$

$$\sigma_o(T \to 0) \to T^0$$

  Underdoped cuprates?

- $z \to \infty$, $\theta \to \infty$, $\theta = -z$: conformal to $AdS_2 \times R^2$

$$\sigma_o(T \to 0) \to T^{-1}$$

  Optimally doped cuprates? ([Davison, Schalm & Zaanen'13])