Bad Metals, Black Holes and Density Waves

Blaise Goutéraux

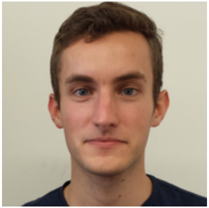
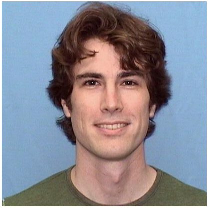
NORDITA, Stockholm

Tuesday July 18, 2017

SCES 2017, Prague

Acknowledgments

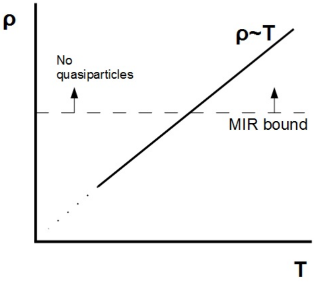
- Based on
  'Bad Metals from Fluctuating Density Waves', [arXiv:1612.04381], and
  'Hydrodynamic transport in fluctuating charge density waves ',
  [arXiv:1702.05104]
  together with Luca Delacrétaz, Sean Hartnoll and Anna Karlsson

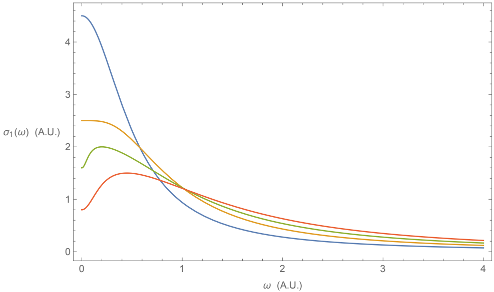




- My research is supported by a Marie Curie International Outgoing Fellowship, Seventh European Community Framework Programme.

European
Commission

Central motivation: bad metallic transport





Two experimental challenges for theorists [Hussey, Takenaka & Takagi'04]:

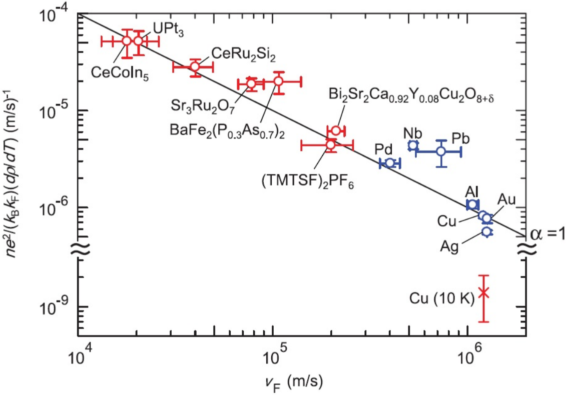
- T-linear resistivity violating the MIR bound: **no quasiparticles**

$$\ell k_F \gtrsim \hbar \quad \Rightarrow \quad \rho \equiv \sigma^{-1} = \frac{m}{ne^2\tau_{tr}} \lesssim \rho_{MIR} \sim 150\,\mu\Omega.\mathrm{cm}$$

- Optical conductivity: **far IR peak** ($\sim 10^2 cm^{-1}$) moving off axis as T increases to room temperature.

Planckian dynamics

$$\rho = \frac{m}{ne^2\tau_{tr}} \sim T \qquad \Rightarrow \qquad \tau_{tr} = \tau_P \equiv \frac{\hbar}{k_B T}$$



[Bruin et al, Science 339 804 (2013)]

The Planckian timescale

Universal scale in all systems at finite temperature which follows from dimensional analysis

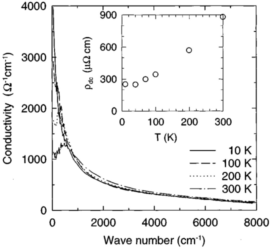
$$[\hbar] = J.s\,, \quad [k_B] = J.K^{-1}\,, \quad [T] = K \quad \Rightarrow \quad \tau_P = \frac{\hbar}{k_B T}$$

In strongly-coupled, quantum systems, expected to be the **fastest equilibration time** allowed by Nature and Quantum Mechanics [Sachdev,Zaanen]. At room temperature

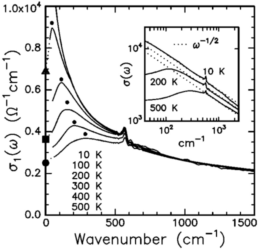
$$\tau_P \sim 25 fs$$

Off-axis peaks in optical conductivity data (1)

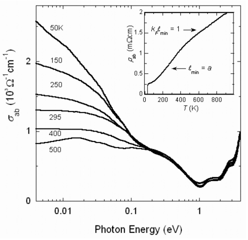
$Bi_2Sr_2CuO_6$
[PRB 55 14152 (1997)]



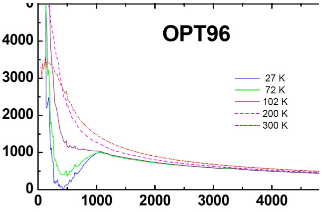
Ca_2RuO_3
[PRB 66 041104 (2002)]



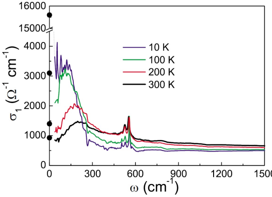
$La_{1.9}Sr_{0.1}CuO_4$
[Phil Mag 84 2847 (2004)]



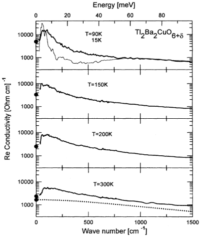
$Bi_2Sr_2CaCu_2O_{8+\delta}$
[J. of Phy: Cond Mat 19 125208 (2007)]



$Na_{0.7}CoO_2$
[PRL 93 237007 (2004)]

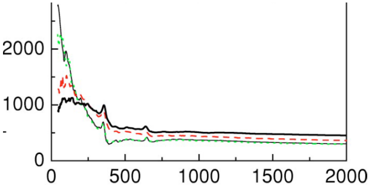


$Ti_2Ba_2CuO_{6+\delta}$
[PRB 51 3312 (1995)]

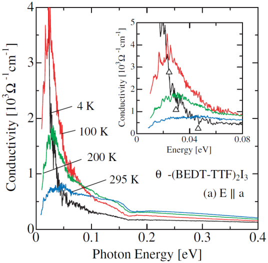


Off-axis peaks in optical conductivity data (2)

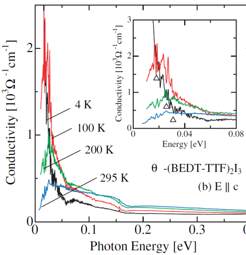
La2126
[PRB 67 134526 (2003)]



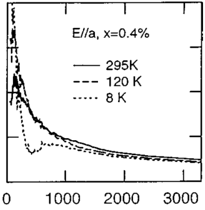
θ-(BEDT-TTF)$_2$I$_3$ (a)
[PRL 95 227801 (2005)]



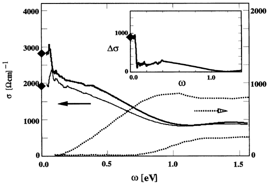
θ-(BEDT-TTF)$_2$I$_3$ (c)
[PRL 95 227801 (2005)]



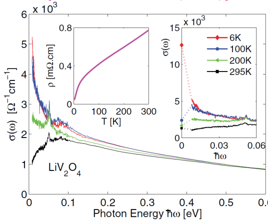
YBa$_2$(Cu$_{1-x}$Zn$_x$)$_3$O$_{7-\delta}$
[PRB 57 081 (1998)]



V$_2$O$_3$
[PRL 75 105 (1995)]

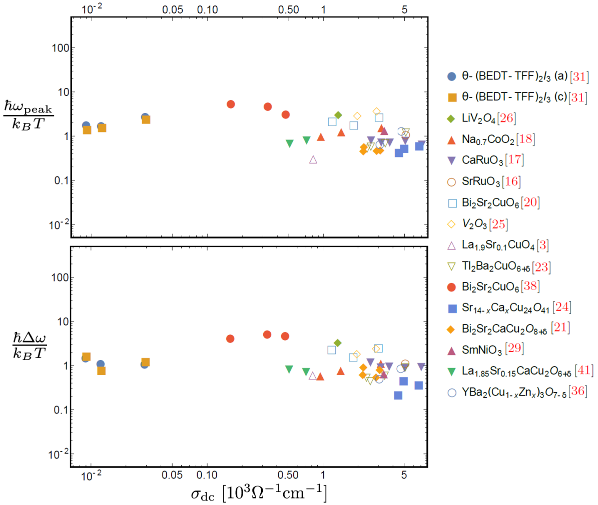


LiV$_2$O$_4$
[PRL 99 167402 (2007)]

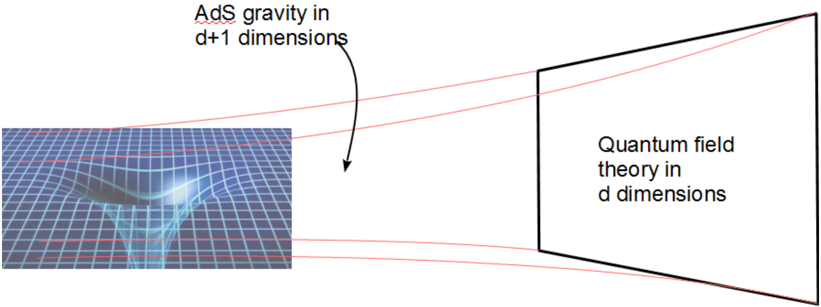


Planckian dynamics in the optical conductivity [arXiv:1612.04381]

$$\hbar\omega_{\mathrm{peak}} \sim k_B T\,, \qquad \hbar\Delta\omega \sim k_B T\,,$$



- These observations suggest that **Planckian dynamics** is a defining feature of both **ac and dc transport** in bad metals.
- Planckian dynamics also emerge in the **low energy effective description** of strongly-coupled (holographic) quantum matter.
- **Universal** low energy effective theory?



- Gravity in Anti de Sitter is dual to certain strongly-coupled Quantum Field Theories in one spatial dimension less [Maldacena'97].

- The complicated **dynamics of strongly-coupled quantum matter** can be described non-perturbatively by **solving Einstein's equations in Anti de Sitter.**

Planckian dynamics in AdS black holes

- Perturb the horizon of an AdS black hole: **linear hydrodynamics**.
- The **shear viscosity** is bounded from below [Kovtun, Son & Starinets'05]

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{4\pi k_B} \quad \Leftrightarrow \quad D_\perp \gtrsim c^2 \tau_P$$

- [Hartnoll'14]: charge and energy **diffusivities** are **bounded**

$$\frac{D}{v^2} \gtrsim \tau_P$$

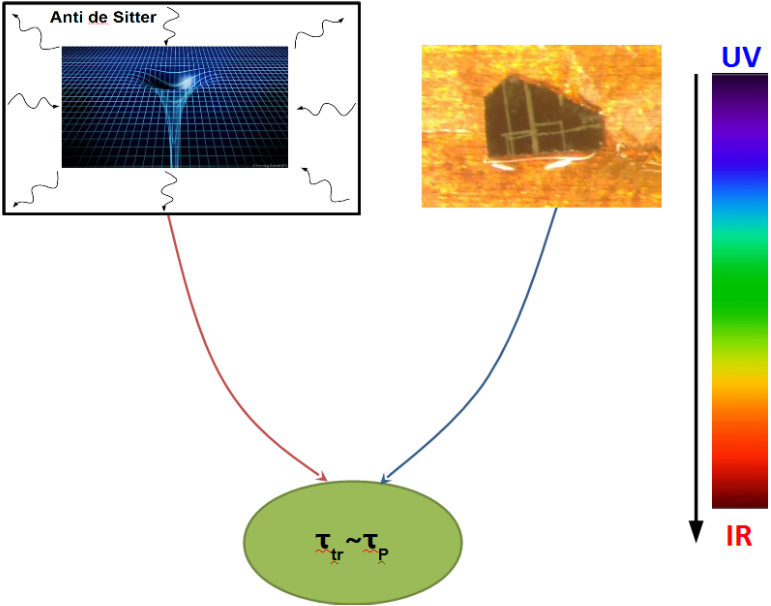
  and the bound is **saturated at strong coupling**. Combine with the Einstein relation $\sigma = D/\chi$:

$$\rho \sim T$$

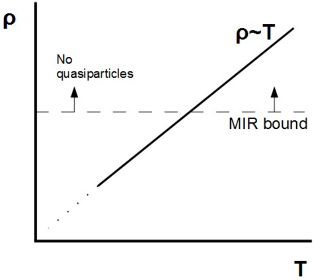
- [Blake'16]: v is related to the **spread of quantum chaos** $v = v_B$. The improved bound holds in many gravitational duals.

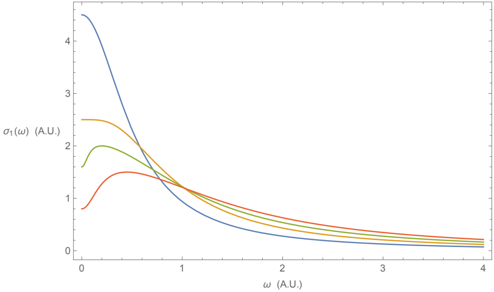
- Similar results by now in field theory calculations: [Gu et al'16, Davison et al'16], [Werman et al'17], [Chowdhury & Swingle'17], [Bohrdt et al'16], [Patel & Sachdev'16], [Patel et al'17]. . .

Universal low energy Planckian dynamics



Remainder of this talk: Back to bad metals



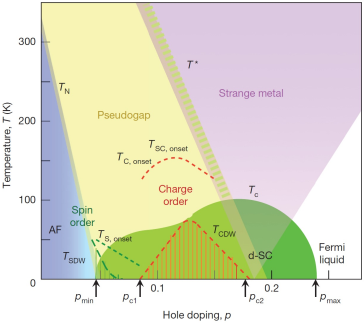


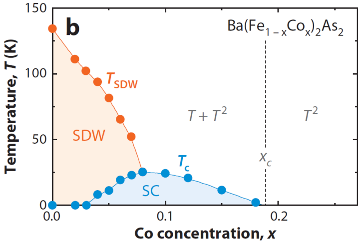
I will offer a theory based on hydrodynamics and spontaneous translation symmetry breaking which

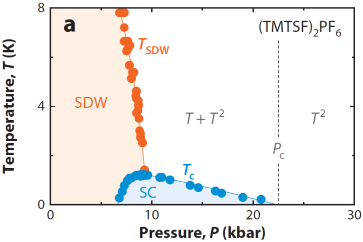
- leads to **small dc conductivities**;
- accounts for the **far IR off-axis peak** in $\sigma(\omega)$;
- naturally **relates** the dc and ac transport timescales.

Disclaimer: effective low energy theory of transport, not a microscopic theory.

Spontaneous translation symmetry breaking







Late time dynamics from hydrodynamics

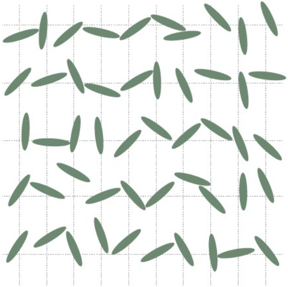
Short-lived quasiparticles: **conserved quantities** are more fundamental for late-time transport

$$\partial_t \epsilon + \vec{\nabla} \vec{\pi} = 0$$

$$\partial_t \pi^i + \nabla_k \tau^{ik} = 0$$

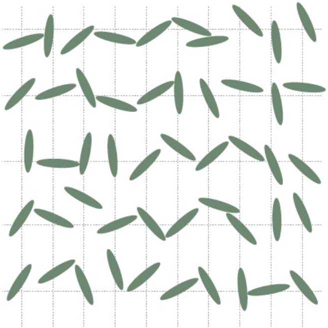
$$\partial_t \rho + \vec{\nabla} \vec{j} = 0$$

Hydrodynamics: long wavelength description of the system



[credit: Beekman et al'16]

Electronic crystal



[credit: Beekman et al'16]

[credit: Beekman et al'16]

We also wish to include a CDW [Grüner'88, Chaikin & Lubensky]:

$$\rho(x) = \rho_0 \cos\left[Qx + \phi(x,t)\right]$$

The phase $\phi(x,t)$ is a new dof coming from the SSB of translations (Goldstone): **'phonon' of the electronic crystal**.

Conductivity of a pinned CDW [Grüner'88]

- Constitutive relation for the current and the Goldstone

$$j = nev + \dots\,, \qquad \dot\phi = v + \dots$$

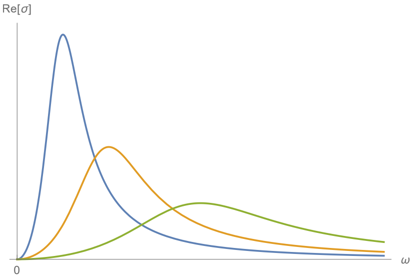
- Standard procedure to extract retarded Green's functions [Kadanoff & Martin'63].

- **Weak disorder**: finite momentum lifetime $1/\Gamma_\pi$, **pins the Goldstone** ϕ with a small mass ω_o.

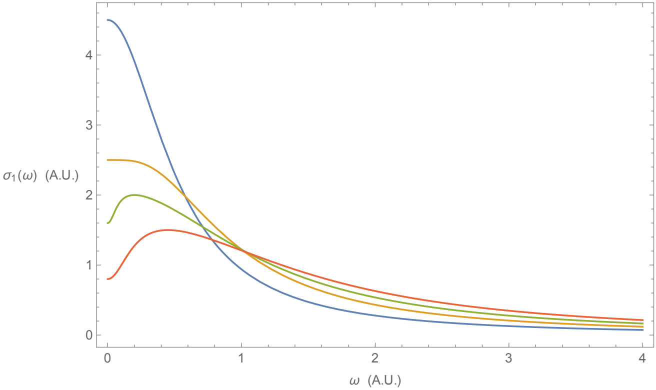
- Conductivity

$$\sigma = \frac{ne^2}{m}\frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}$$

- **Dc insulator** due to Galilean invariance.



Conducting CDWs?



We wish to describe conducting CDWs. Two mechanisms

1. Relax Galilean symmetry;
2. Introduce phase disordering by mobile dislocations.

Conducting, non-Galilean invariant CDWs [arXiv:1612.04381]

- Modified constitutive relation for the current

$$j = qv - \sigma_o \nabla \mu + \ldots , \qquad \dot{\phi} = v + \ldots$$

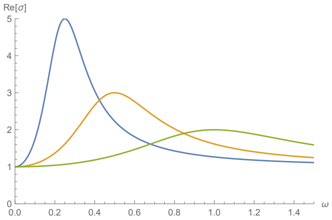
σ_o is a **diffusive** transport coefficient encoding charge transport **without momentum drag**.

- Conductivity

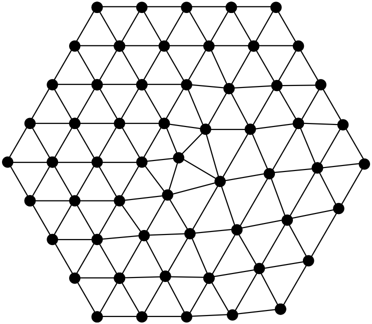
$$\sigma = \sigma_o + \frac{q^2}{\chi_{\pi\pi}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}$$

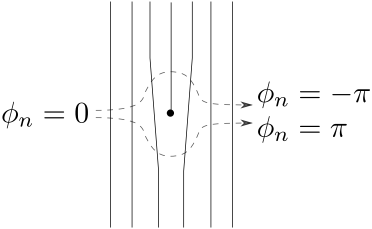
- Non-zero dc conductivity $\sigma_{dc} = \sigma_o + O(\Gamma_\pi)$

- Can be **small** even for weak momentum relaxation: **bad metal**.



Phase disordering





- In 2d, crystals can **melt by proliferation of topological defects** in the crystalline structure [Nelson & Halperin'79].
- At $T = 0$: quantum melting [Kivelson et al'98, Beekman et al'16].
- The phase gets disordered ($\sim$ BKT) at a rate Ω: **flow of mobile dislocations** [arXiv:1702.05104].

Conducting, phase-disordered CDWs [arXiv:1612.04381]

- Now the conductivity reads

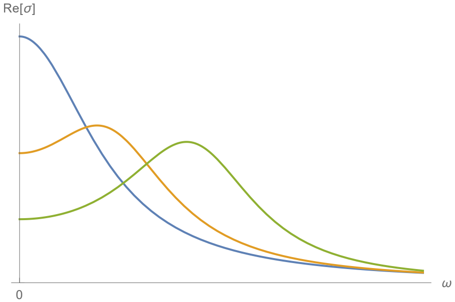
$$\sigma = \frac{ne^2}{m} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}, \qquad \sigma_{dc} = \frac{ne^2}{m} \frac{1}{\Gamma_{CDW}}$$

$$\Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}$$

New transport inverse timescale, **non-zero** even if $\Gamma_\pi = 0$.

- **Off-axis peak** for sufficiently small Ω or large pinning

$$\omega_0 \geq \frac{\Omega^3}{\Gamma_\pi + 2\Omega}$$



Bad metallic transport from fluctuating CDWs

- Neglect momentum relaxation $\Gamma_\pi \ll \omega_0, \Omega$:

$$\sigma_{dc} = \frac{n\,e^2}{m} \frac{\Omega}{\omega_o^2}$$

- The width and position of the peak are controlled by Ω, ω_o. **The data shows** $\Omega \sim \omega_o \sim k_B T/\hbar$

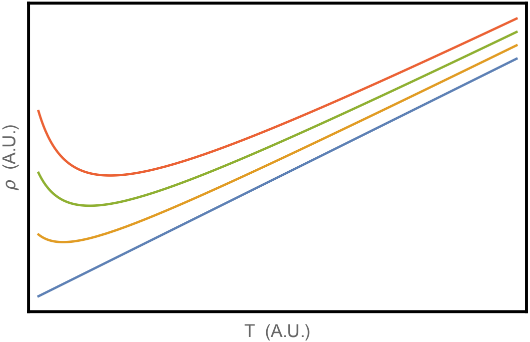
$$\Rightarrow \rho_{dc} = \frac{1}{\sigma_{dc}} \sim \frac{m}{n\,e^2} \frac{k_B\,T}{\hbar}$$

  ***T*-linear resistivity**!

- Hydrodynamics of fluctuating CDWs provide a natural mechanism whereby the ac and dc conductivities are controlled by **the same Planckian timescale**.

Resistivity upturns from fluctuating cdws

$$\rho = \frac{m}{ne^2}\Gamma_{CDW}\,, \quad \Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}$$



An **upturn** occurs as Ω decreases and phase fluctuations dominate Γ_{CDW}: relation to underdoped cuprates and static charge order?

Violation of the Wiedeman-Franz law: $\rho\kappa/T \sim 1/\Omega \gg L_o$.

Some open questions

- Typical frequency scales of order T: at the **edge of validity of hydrodynamics** $\omega \ll T$.
- The role played by the Planckian timescale is indicative of quantum criticality [SACHDEV]: **quantum critical computation**. Quantum critical metals [HERTZ-MILLIS], Gauge/Gravity duality...
- Data suggests phase relaxation Ω increases as $T \to T_c$: metallic phase without CDW
  $\Rightarrow$ indicative of **competition** with superconductivity?