Slow relaxation and diffusion in holography

Blaise Goutéraux

École Polytechnique, Palaiseau, France

Friday Nov 29, 2018

D-ITP Holography meeting, Utrecht, The Netherlands

References

Mostly based on

- Ongoing work and 'Slow relaxation and diffusion in holographic quantum critical phases' [arXiv:1808.05659] with Richard Davison (DAMTP, Cambridge, UK) and Simon Gentle (formerly Leiden/Utrecht)

but also on

- 'Incoherent transport in clean quantum critical metals' [arXiv:1507.07137] with Richard Davison and Sean Hartnoll (Stanford),
- 'Momentum dissipation and effective theories of coherent and incoherent transport' [arXiv:1411.1062] with Richard Davison,
- 'Hydrodynamic theory of quantum fluctuating superconductivity' [arXiv:1602.08171] with Richard Davison, Luca Delacrétaz (Stanford) and Sean Hartnoll
- and 'Theory of hydrodynamic transport in fluctuating electronic charge density wave states' [arXiv:1702.05104] with Luca Delacrétaz, Sean Hartnoll and Anna Karlsson (IAS).

Diffusivities: early days of AdS/CFT

- Diffusivities have played an important role in the history and development of applications of AdS/CFT [Policastro, Son & Starinets'01, '02, Herzog'02]
- Shear viscosity of $d+2$-dimensional Schwarzschild AdS:

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - 2\eta\sigma^{\mu\nu} + O(\nabla^2)$$

$$\sigma_{\mu\nu} = P_\mu^\alpha P_\nu^\beta \nabla_{(\alpha} u_{\beta)} - \frac{1}{d} g_{\mu\nu} \nabla \cdot u$$

$$G^R_{\pi_\perp \pi_\perp} = \frac{(\epsilon + p) D_{\pi_\perp} q^2}{i\omega - D_{\pi_\perp} q^2}, \qquad D_{\pi_\perp} = \frac{\eta}{\epsilon + p} = \frac{\eta}{sT} = \frac{1}{4\pi T}$$

Diffusivities: early days of AdS/CFT

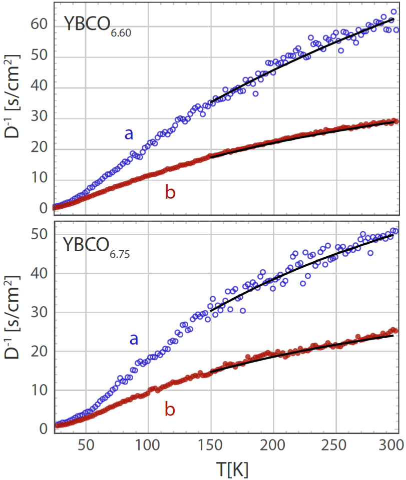
- Diffusivities have played an important role in the history and development of applications of AdS/CFT [Policastro, Son & Starinets'01, '02, Herzog'02]
- R-charge diffusion in $d+2$-dimensional Schwarzschild AdS:

$$J^{\mu} = -\sigma_o T \nabla^{\mu} \left(\frac{\mu}{T} \right) + O(\nabla^2)$$

$$G^R_{J_{\|} J_{\|}} = \frac{\chi_{nn} D_R \omega^2}{i\omega - D_R q^2}\,, \qquad D_R = \frac{\sigma_o}{\chi_{nn}} \underset{d=3}{=} \frac{1}{2\pi T}$$

Experimental evidence

How does the diffusivity behave in real, strongly-coupled systems?
Thermal diffusivity measurements in high T_c superconductors



[Zhang et al, PNAS 2017 114 (21)]

Diffusivities and Planckian dynamics

- Diffusivities are defined via Einstein relations '$\Sigma = D \cdot X$'. But, dimensional analysis

$$[D] = [v^2][\tau]$$

Interesting to express them in terms of a timescale and a velocity [Hartnoll'14].

- Reinstating units in previous results [Kovtun & Ritz'08]:

$$D \sim c^2 \tau_P \,, \qquad \tau_P \sim \hbar/(k_B T)$$

- 'Planckian' timescale [Zaanen'04]: conjectured to provides an upper bound on how fast (strongly-coupled) systems thermalize. Governs dynamics near QCPs [Sachdev].

- Lower bound on diffusivities [Hartnoll'14]?

$$D \gtrsim v^2 \tau_P$$

Main questions addressed in this talk

$$D \sim c^2 \tau_P \,, \qquad \tau_P \sim \frac{\hbar}{k_B T}$$

- Should we generally expect that $D \sim 1/T$ in strongly-coupled systems? (depends)
- What are the timescale and velocity naturally appearing in the diffusivity?
- When are lower bounds on diffusivities useful (ie tight)?

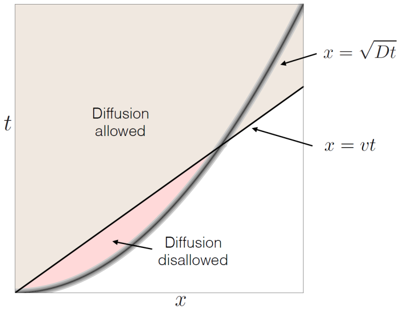
Plan of the talk

Two qualitatively different cases:

1. Diffusivities in systems with a long lived operator: eg slow momentum relaxation, systems with slowly fluctuating Goldstone bosons (superconductors, charge density waves), probe branes, higher form global symmetries...

2. Diffusivities in incoherent systems without long lived operators: bounds on diffusivities, relation to chaos parameters...

How early can the onset of diffusive transport be?



[HARTMAN, HARTNOLL & MAHAJAN'17]

In relativistic theories, $v = c$. More generally, a Lieb-Robinson velocity that controls the linear-in-t growth of operators and provides a definition of an 'operator lightcone'.

How is this short time cutoff implemented in 'microscopic' theories?

Systems with a long lived operator

If an operator is long lived, its lifetime can formally be computed using the memory matrix formalism [Forster'75]

$$\dot{A} = [H, A] = \epsilon[\Delta H, A]\,, \qquad \epsilon \ll 1$$

$$\frac{1}{\tau} = \frac{\epsilon^2}{\chi_{AA}} \lim_{\omega\to 0} \lim_{\epsilon\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{[\Delta H, A]\,[\Delta H, A]}(\omega)$$

The memory matrix formalism relates τ to microscopic parameters of the theory, eg [Goetze & Woelfle'72, Rosh & Andrei'00, Hartnoll & Hofman'12].

τ also governs the late time dynamics of any operator B overlapping with A, ie $\chi_{AB} \neq 0$.

Example: if $A = P$, $\tau = \tau_{mr}$ appears in the phenomenological 'conservation' equation

$$\dot{\pi} = -\frac{1}{\tau_{mr}}\pi$$

Slow momentum relaxation [Davison & BG'14]

At high temperatures, assume momentum relaxes slowly:

$$\partial_t \delta\epsilon + \partial^i \pi^i = 0\,, \qquad \partial_t \pi^i + \partial^j \tau^{ij} = -\frac{1}{\tau_{mr}}\pi^i\,, \qquad \tau_{mr} T \gg 1$$

The collective excitations (assuming relativistic, conformal hydrodynamics) are qualitatively different at small and large ω, q:

$$\omega, q \ll 1/\tau_{mr} \ll T : \quad \omega = -iDq^2 + \ldots\,, \quad \omega = -\frac{i}{\tau_{mr}} + iq^2\left(D - \frac{\eta}{\epsilon + p}\right)$$

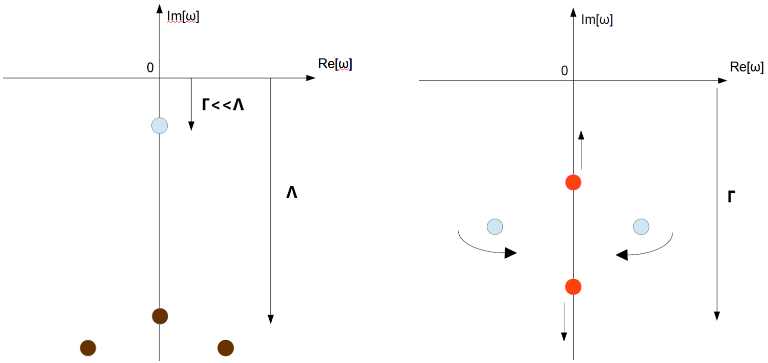
$$1/\tau_{mr} \ll \omega, q \ll T : \quad \omega = \pm c_s q - \frac{i}{2\tau_{mr}} + \ldots$$

There is a crossover between diffusive and (damped) ballistic propagation from long to short distances.

In this example, τ_{mr} is the short time cutoff of diffusive transport.

Motion of poles

The crossover occurs through a collision between the collective excitations in the lower half complex frequency plane:



Diffusivity in the presence of a long lived operator

The collision is coherent and can be modeled by solving for the zeros of the denominator of G^R:

$$i\omega\left(i\omega + \frac{1}{\tau_{mr}} + \frac{\eta q^2}{\epsilon + p}\right) - q^2 c_s^2 = 0$$

which automatically implies the relation

$$D = c_s^2 \tau_{mr}$$

- The natural velocity that appears is that of the sound mode at short distances.
- The natural timescale that appears is the lifetime of the long-lived mode.
- Can be modeled [Davison & BG'14] using a holographic toy model based on [Andrade & Withers'13]: $D \sim T$ at high T.

The diffusivity associated to a phase disordered Goldstone mode will obey the same relation. Examples:

- In the presence of vortices, the superfluid phase gets a (small) gap and becomes long-lived. Superfluid sound becomes at long distances a pair of diffusive/pseudo-diffusive modes, [Davison, Delacrétaz, B.G. & Hartnoll'16].
- Dislocations also gap out phonons in crystals. Shear sound becomes gapped at long distances and turns into a pair of diffusive/pseudo-diffusive modes [Delacrétaz, B.G., Hartnoll & Karlsson'17].

Holographic realizations of pinned WC/CDW phases in [Jokela, Järvinen & Lippert'17, Andrade, Baggioli, Krikun & Poovuttikul'17, Amoretti, Arean, B.G. & Musso'18 (to appear), Andrade & Krikun'18 (to appear)]

Thermal diffusivity

The diffusivities just discussed all characterize coherent transport, ie due to a long-lived operator (see eg [Chen & Lucas'17, Grozdanov & Poovuttikul'18, Grozdanov, Lucas & Poovuttikul'18] for more examples).

However, diffusivities also characterize incoherent transport, ie transport at long distances of quickly relaxing operators. Two distinct cases

- No long-lived operator (see later).
- There are operators without any overlap with long-lived operators. For instance, the thermal diffusivity with open circuit boundary conditions is not sensitive to momentum relaxation [Mahajan & al'14]

$$D_{th} = \frac{\kappa}{c_n} \sim O(\tau_{mr}^0)\,, \qquad \kappa = \bar{\kappa} - \alpha^2/(\sigma T)$$

Incoherent diffusivity

Define an incoherent charge that characterizes transport that doesn't carry momentum [Davison, B.G. & Hartnoll'15, Davison, Gentle & B.G.'18]:

$$\delta n_{inc} = s^2 T \delta \left(\frac{n}{s} \right) , \quad J_{inc} = (\epsilon + p) J - nP \, , \quad \chi_{J_{inc} P} = 0$$

It obeys a conservation equation

$$\partial_t \delta n_{inc} + \partial^i j^i_{inc} = 0$$

Define an incoherent susceptibility and conductivity

$$\chi_{inc} = \frac{\delta n_{inc}}{\delta \mu_{inc}} \, , \qquad j_{inc} = -\sigma_{inc} \nabla \delta n_{inc}$$

The retarded Green's function is purely diffusive

$$G^R_{J_{inc} J_{inc}}(\omega, q) = \frac{\omega^2 \sigma_{inc}}{i\omega - D_{inc} q^2}$$

$$D_{inc} = \frac{\sigma_{inc}}{\chi_{inc}} \, , \quad \sigma_{inc} = \lim_{\omega \to 0, q \to 0} \frac{i}{\omega} G^R_{J_{inc} J_{inc}} = (\epsilon + p) \sigma_o$$

Low temperatures

- The thermal diffusivity and the incoherent diffusivity are both insensitive to slow momentum relaxation.

- So we don't expect that they can be expressed as

$$D = c_s^2 \tau \,, \qquad \tau T \gg 1$$

  where τ is a long timescale compared to temperature, and c_s the velocity of a sound-like mode in the system.

- Does it mean that they are governed by processes $\tau \sim \tau_P \sim 1/T$? To test this, we need to go to low temperatures.

- For simplicity, we will consider nonzero density, translation invariant states.

Diffusion near QCPs

- In general, the thermal diffusivity and the incoherent diffusivity are not equal to each other:

$$\sigma_{inc} = n^2 T \kappa$$

- At low temperature near a QCP, there is a simplification, since

$$\chi_{inc} \sim n^2 T c_n$$

  Then, at low temperature

$$D_{th} \sim D_{inc}$$

Holographic quantum critical phases

Take a simple bottom-up holographic action

$$S = \int d^4x \left[R - \frac{1}{2}\partial\phi^2 - \frac{Z(\phi)}{4}F^2 - V(\phi) \right]$$

Impose UV boundary conditions on V, Z such that spacetime is asymptotically AdS.

We are interested in setups where the flow goes in the IR to a scaling geometry, dual to an IR quantum critical phase.

The simplest possibility is for the scalar to minimize its effective potential

- At zero density, we have an AdS_4 to AdS_4 domain-wall
- At nonzero density, the IR geometry is $AdS_2 \times R^2$.

Runaway flows

- Assume that the scalar couplings have runaway branches

$$V(\phi \to \infty) \to V_o e^{-\delta\phi}\,, \qquad Z(\phi \to \infty) \to Z_o e^{\gamma\phi}$$

- The IR endpoint of the flow violates hyperscaling ($\theta \neq 0$) and possibly time and space isotropy ($z \neq 1$):

$$ds^2 = u^\theta \left(-\frac{L_t^2}{u^{2z}} dt^2 + \frac{L^2}{u^2} du^2 + \frac{L_x^2}{u^2}(dx^2 + dy^2) \right)$$

$$\phi \sim \ln u$$

- L, θ and z are determined by the values of V_o, γ and δ. L_t and L_x are length scales setting the IR units of time and space.

- At small nonzero T, $s \sim T^{(d-\theta)/z}$: hyperscaling violation.

IR scaling of the gauge field

Generally,

$$A_t \sim A_o r^{\alpha}$$

Depending on the choice of γ, δ exponents, this sources a marginal or an irrelevant deformation of the $T = 0$ IR geometry:

- Marginal deformation: $\alpha = \theta - 2 - z$, $z \neq 1$
- Irrelevant deformation: $\alpha = 2\Delta_{A_o} + \theta - 3$, $z = 1$ and $\Delta_{A_o} < 0$.
  The gauge field does not backreact on the $T = 0$ IR geometry, but sources corrections that vanish in the IR as $u \to +\infty$

$$1 + \# A_o^2 u^{2\Delta_{A_o}} + O(u^{4\Delta_{A_o}})$$

Thermal diffusion near $z \neq 1$ QCPs

- [Blake, Davison & Sachdev'17] studied D_{th} near holographic QCPs. Following [Blake'16], they observed that when $z \neq 1$,

$$D_{th} = \frac{z}{2(z-1)} v_B^2 \tau_L$$

where $\tau_L = 2\pi/T$ and v_B are 'chaos parameters' characterizing the early time growth of bounday OTOC [Shenker & Stanford'13].

$$\langle [W(0,0), V(x,t)]^2 \rangle_\beta \sim \frac{1}{N^2} e^{\frac{1}{\tau_L}\left(t - \frac{x}{v_B}\right)} + \dots$$

- These $z \neq 1$ QCPs are 'incoherent' systems without long-lived operators, that is $\tau \sim 1/T$ (eg [Sybesma & Vandoren'15]).
- More on this relation between chaos and diffusion: 'pole-skipping' [Grozdanov, Schalm & Scopelliti'17, Blake, Lee & Liu'18, Blake, Davison, Grozdanov & Liu'18].

Charge response near $z = 1$ QCPs

- [Blake, Davison & Sachdev'17] observed that the relation $D_{th} \sim v_B^2 \tau_L$ broke down near $z = 1$ QCPs. The same occurs for D_{inc} [Davison, Gentle & B.G.'18]. This is due to the presence of the irrelevant deformation sourced by A_o.

- To understand this better, we have computed the ac charge conductivity for these $z = 1$ QC states

$$\sigma(\omega) \equiv \frac{i}{\omega} G^R_{JJ}(\omega, q = 0)$$

In the low frequency limit, it reads

$$\sigma(\omega) = \frac{\sigma_o}{1 - i\omega\tau} + \frac{n^2}{\epsilon + p}\frac{i}{\omega}\,, \quad \tau \sim \frac{1}{T}\left(\frac{T^{\Delta_{A_o}}}{A_o}\right)^2\,,$$

$$\Delta_{A_o} < 0 \quad \Rightarrow T\tau \underset{T\to 0}{\to} \infty$$

Breakdown of relativistic hydrodynamics near $z = 1$ QCPs

$$\sigma(\omega) = \frac{\sigma_o}{1 - i\omega\tau} + \frac{n^2}{\epsilon + p}\frac{i}{\omega}, \quad \tau \sim \frac{1}{T}\left(\frac{T^{\Delta_{A_o}}}{A_o}\right)^2,$$

$$\Delta_{A_o} < 0 \quad \Rightarrow T\tau \underset{T\to 0}{\to} \infty$$

- Emergent long lived collective mode at low T: $\Rightarrow$ relativistic hydrodynamics breaks down

$$\partial_t \delta n_{inc} + \partial^i j^i_{inc} = 0$$

- Simple improvement of relativistic hydro which produces the desired pole in σ_{inc}:

$$\partial_t j^i_{inc} = -\frac{j^i_{inc}}{\tau}$$

This improved EFT would be valid when $T^{-1} \lesssim t \ll \tau$.
At times $t \gg \tau$, relativistic hydro would apply.

Thermal/incoherent diffusion near $z \neq 1$ QCPs

- τ provides a natural timescale to express diffusivities near these $z = 1$ QCPs. Indeed:

$$D_{th,inc} = \frac{2}{d+1-\theta} v_B^2 \tau$$

This is consistent with the arguments of [Hartman, Hartnoll & Mahajan'17]: the correct timescale is the equilibration timescale of the system, which in this case is τ, not τ_L or τ_P.

- It is known that asymptotically $z = 1$, $\theta \neq 0$ spacetimes possess a sound mode with $c_{s,\theta}^2 = 1/(d-\theta)$, [Kanitscheider & Skenderis'09, B.G., Skenderis, Smolic, Smolic & Taylor'11]. Then

$$D_{th,inc} = c_{s,\theta}^2 \tau$$

But is there such a sound mode in the full, asymptotically AdS spacetime? (see [Betzios, Gürsoy, Järvinen & Policastro'17,'18])

Summary and Outlook

- Two types of long distance transport: coherent transport of long lived excitations of the system; incoherent transport of quickly relaxing operators.
- Signature of coherent transport: pole collision between a diffusive and a pseudo-diffusive pole, with (gapped) ballistic transport at short distances.

$$D_{coh} = c_s^2 \tau$$

- Coherent diffusivities are large, and typically conjectured lower bounds [Hartnoll'14, Blake'16] on diffusivities are not tight.

Summary and Outlook

- In contrast, incoherent diffusivities appear to be governed by 'Planckian' dynamics and 'chaos parameters'

$$\frac{D_{inc}}{v_B^2 \tau_L} \sim O(1)$$

- It would be nice to understand this last statement better in terms of an EFT (see [Blake, Lee & Liu'18] for a proposal).

- Incoherent diffusivities are not large, and this is wehere conjectured lower bounds [Hartnoll'14, Blake'16] on diffusivities are helpful.