Universal relaxation of density waves in hydrodynamics and holography

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References:

- With Luca Delacrétaz, Sean Hartnoll and Anna Karlsson
  - *'Bad Metals from Density Waves'* [arXiv: 1612.04381],
  - *'Theory of hydrodynamic transport in fluctuating electronic charge density wave states'* [arXiv:1702.05104]
  - *'Theory of the collective magnetophonon resonance and melting of the field-induced Wigner solid'* [arXiv:1904.04872].
- With Andrea Amoretti, Daniel Areán and Daniele Musso
  - *'DC resistivity of quantum critical, charge density wave states from gauge-gravity duality'* [arXiv:1712.07994],
  - *'Effective holographic theory of charge density waves'* [arXiv:1711.06610]
  - *'A holographic strange metal with slowly fluctuating translational order'* [arXiv:1812.08118]
  - *'Diffusion and universal relaxation of holographic phonons'* [arXiv:1904.11445].

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Related recent holographic work:

- Probe branes: Jarvinen, Jokela, Lippert [arXiv:1408.1397], [arXiv:1612.07323], [arXiv:1708.07837]
- Bianchi VII: Andrade, Baggioli, Krikun, Poovuttikul [arXiv:1708.08306], [arXiv: 1812.08132]
- Massive gravity: Alberte, Ammon, Baggioli, Gray, Grieninger, Jimenez, Pujolas [arXiv:1708.08477], [arXiv:1711.03100], [arXiv:1904.05785], [arXiv:1905.09164], [arXiv:1905.09488]
- Q-lattices: Donos, Martin, Pantelidou, Ziogas [arXiv:1903.05114], [arXiv:1905.00398], [arXiv:1906.03132]
- Inhomogeneous spatially modulated phases: Donos [arXiv:1303.7211], Withers [arXiv:1304.0129,1304.2011,1407.1085], Donos and Gauntlett [arXiv:1512.06861], Andrade, Krikun, Schalm and Zaanen [arXiv:1710.05791], Cai, Li, Wang and Zaanen [arXiv:1706.01470], Cremonini, Li and Ren [arXiv:1612.04385,arXiv:1705.05390], Donos, Gauntlett, Griffin and Ziogas [arXiv:1801.09084], Goutéraux, Jokela and Ponni [arXiv:1803.03089]

- Rich dialogue between holography and effective theories: eg KSS bound, anomalies.

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi}$$

- First established in the context of asymptotically AdS black branes: Schwarzschild, RN, etc.
- Sharp definition in the context of relativistic hydrodynamics: η quantifies the diffusion of transverse momentum

$$G^R_{\pi_\perp \pi_\perp}(\omega, q) = \frac{(\epsilon + p) D_\perp q^2}{i\omega - D_\perp q^2}\,, \qquad D_\perp = \frac{\eta}{\epsilon + p}$$

- Weaker bound $\eta > 0$ for positivity of entropy production.
- The KSS bound relates a transport coefficient to the area of the horizon (holographic 'membrane paradigm'). Universality?
- Less symmetric cases?

- The low energy dynamics of the ordered phase differ from those of the disordered phase by the necessity to include **new gapless degrees of freedom** (the Goldstones).
- An important property of Goldstones is that they are **shift-symmetric**: they realize non-linearly the broken symmetry. More concretely, take broken translations along x

$$x \to x + c \quad \Rightarrow \quad \varphi \to \varphi + c$$

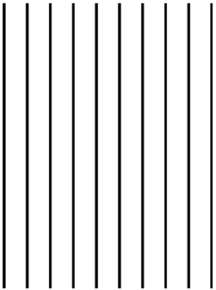
$$f = \frac{1}{2}(K+G)\lambda_{\parallel}^2 + \frac{1}{2}G\lambda_{\perp}^2 + \dots$$

where $\lambda_{\parallel} = \nabla \cdot \vec{\varphi}$, $\lambda_{\perp} = \nabla \times \vec{\varphi}$.

Shift symmetry: **only gradient terms** in the effective IR action:

$$f = \frac{1}{2}(K+G)\lambda_{\parallel}^2 + \frac{1}{2}G\lambda_{\perp}^2 + \dots$$

where $\lambda_{\parallel} = \nabla \cdot \vec{\varphi}$, $\lambda_{\perp} = \nabla \times \vec{\varphi}$.

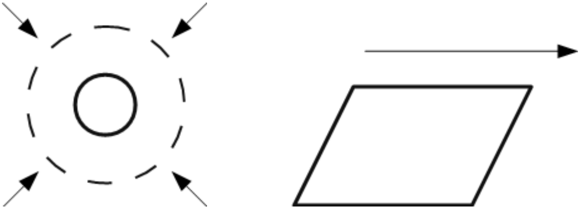


$$f = \frac{1}{2}(K+G)\lambda_{\parallel}^2 + \frac{1}{2}G\lambda_{\perp}^2 + \dots$$

- K and G are the bulk and shear moduli:

$$\chi_{\lambda_{\parallel}\lambda_{\parallel}} \equiv \frac{\delta\lambda_{\parallel}}{\delta s_{\parallel}} = \frac{1}{K+G}, \qquad \chi_{\lambda_{\perp}\lambda_{\perp}} \equiv \frac{\delta\lambda_{\perp}}{\delta s_{\perp}} = \frac{1}{G}$$

They are the static response to bulk compression and shear stress.



- Since π^i is the charge that generates the symmetry, then

$$[\varphi_i(x), \pi_j(y)] = i\delta_{ij}\delta(x-y) + \dots$$

- The effective Hamiltonian contains a term

$$H = \int d^d x \, v_i(x)\pi^i(x) + \dots$$

which leads to the 'Josephon' relations

$$\dot{\lambda}_{\parallel} = \nabla \cdot v + O(\nabla^2), \qquad \dot{\lambda}_{\perp} = \nabla \times v + O(\nabla^2)$$

- At higher order in gradients (+relativistic symmetry), linear, diffusive couplings

$$\dot{\lambda}_{\parallel} = \nabla \cdot v + \gamma_1 T \nabla^2 \left(\frac{\mu}{T}\right) + \xi_{\parallel} \nabla^2 \lambda_{\parallel} + O(\nabla^3),$$

$$\dot{\lambda}_{\perp} = \nabla \times v + \xi_{\perp} \nabla^2 \lambda_{\perp} + O(\nabla^3)$$

- Constitutive relation for the electric current

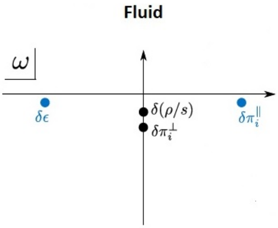
$$j = \rho v - \sigma_o T \nabla \left(\frac{\mu}{T} \right) - \gamma_1 \nabla \lambda_{\|}$$

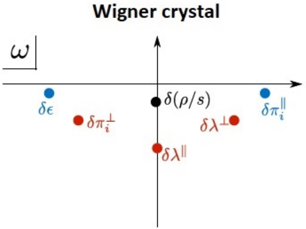
- Isotropic crystal

$$\frac{\xi_{\|}}{K + G} = \frac{\xi_{\perp}}{G} \equiv \Xi$$

- Bound ensuring positivity of entropy

$$\gamma_1^2 \leq \sigma_o \frac{\xi_{\|}}{K + G} = \sigma_o \Xi$$





- In a fluid: conservation of energy, charge and longitudinal momentum lead to two longitudinal sound poles and one longitudinal diffusion pole; conservation of transverse momentum to a shear diffusion pole.
- In a Wigner crystal: one extra 'phonon' longitudinal diffusion pole; the transverse phonon mixes with transverse momentum, leading to two transverse sound poles.

$$\omega_\perp = \sqrt{\frac{G}{\chi_{\pi\pi}}}\,q - \frac{i}{2}\left(\frac{\eta}{\chi_{\pi\pi}} + \xi_\perp\right) q^2 + O(q^3)$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{Z(\phi)}{4}F^2 - V(\phi) - Y(\phi)\left(\partial\psi_x^2 + \partial\psi_y^2\right)\right]$$

$$Y(\phi) = \phi^2 + O(\phi^3)\,, \quad Z(\phi) = 1 + O(\phi)\,, \quad V(\phi) = -6 + \phi^2 + O(\phi^3)$$

- Homogeneous generalized Q-lattice Ansatz [Andrade & Withers'13, Donos & Gauntlett'13]: $\psi_i = kx^i$. Breaks Translations$\times$Global shifts to a diagonal U(1).

- UV boundary conditions on ϕ

$$\phi = \lambda r + \phi_v r^2 + \dots$$

- If $\lambda = 0$, then $\psi_i = kx^i$ is a vev: **spontaneous breaking**.

- If $\lambda \neq 0$, then $\psi_i = kx^i$ is a source: **explicit breaking**.

- But if $\lambda/\mu \ll \phi_v/\mu^2$, **pseudo-spontaneous breaking**.

Let us first set $\lambda = 0$: **purely spontaneous breaking**

- The phase does not minimize the free energy: describes the low energy dynamics of phonons coupled to conserved densities, not the phase transition. We can choose k, but ultimately this would be fixed in a UV-complete model.

- The **phonon**: act with Lie derivative along ∂/∂_x, find that $\varphi \sim \delta\psi_{(0)}$ where $\delta\psi_i = \delta\psi_{(-1)}/r + \delta\psi_{(0)} + O(r)$.

- Numerically recover the **WC hydro retarded Green's functions**:

$$G^R_{\tau^{xy}\tau^{xy}} = G - i\omega\eta\,, \quad G^R_{jj} = \frac{\rho^2}{\chi_{\pi\pi}} - i\omega\sigma_o\,, \quad G^R_{j\pi} = \rho\,,$$

$$G^R_{j\varphi} = \gamma_1 + \frac{\rho}{\chi_{\pi\pi}}\frac{i}{\omega}\,, \quad G^R_{\pi\varphi} = \frac{i}{\omega}\,, \quad G^R_{\varphi\varphi} = \frac{1}{\chi_{\pi\pi}\omega^2} - \Xi\frac{i}{\omega}\,.$$

Moreover all the diffusive couplings can be computed from a membrane paradigm analysis (and match the numerics)

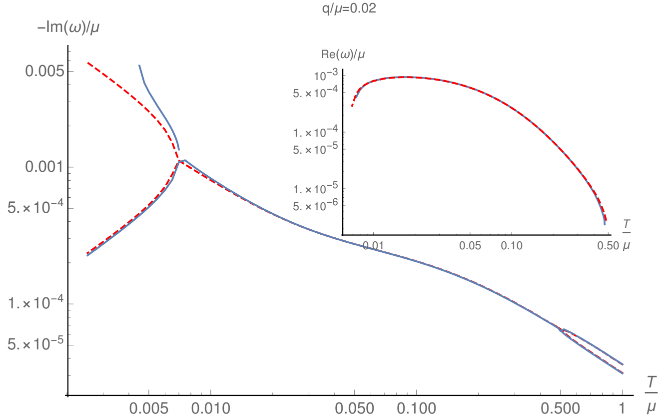
$$\sigma_o = \frac{\left(sT + k^2 I_Y\right)^2}{\left(\chi_{\pi\pi}\right)^2} Z_h + \frac{4\pi k^2 (I_Y)^2 \rho^2}{s Y_h \left(\chi_{\pi\pi} Y\right)^2} \,,$$

$$\gamma_1 = -\frac{4\pi I_Y \rho \left(sT + \mu\rho\right)}{s Y_h \left(\chi_{\pi\pi}\right)^2} - \frac{\mu Z_h \left(sT + k^2 I_Y\right)}{\left(\chi_{\pi\pi}\right)^2} \,,$$

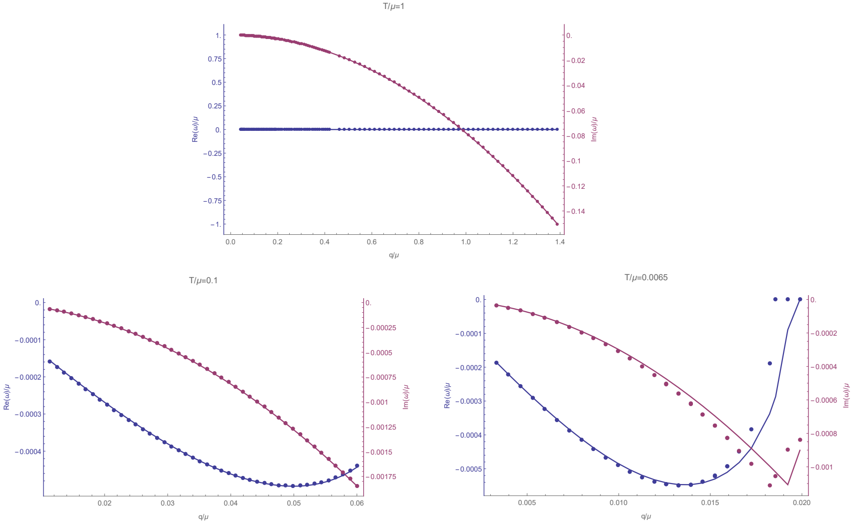
$$\Xi = \frac{4\pi \left(sT + \mu\rho\right)^2}{k^2 s Y_h \left(\chi_{\pi\pi}\right)^2} + \frac{\mu^2 Z_h}{\left(\chi_{\pi\pi}\right)^2} \,,$$

$$\chi_{\pi\pi} = sT + \mu\rho + k^2 I_Y \,, \quad I_Y = \int_0^{r_h} Y(\phi)$$

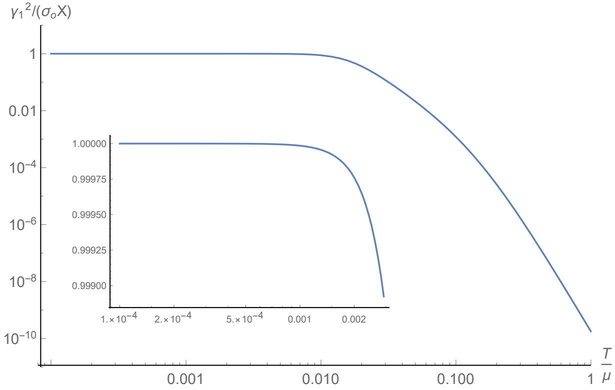
Transverse QNMs at nonzero q:



Very good match to the hydrodynamic dispersion relation (no fitting involved).



Hydrodynamics at all temperatures (no fitting involved).



The positivity of entropy production bound is obeyed

$$\sigma_o \Xi - \gamma_1^2 = \frac{4\pi s T^2 Z_h}{k^2 (\chi_{\pi\pi})^2 Y_h} \geq 0$$

Even saturates at low T. Why?

From

$$
\begin{aligned}
G^R_{j\varphi_\parallel}(\omega, q=0) &= \gamma_1 + \frac{i\rho}{\chi_{\pi\pi}\,\omega}\,, \\
G^R_{\varphi_\parallel\varphi_\parallel}(\omega, q=0) &= \frac{1}{\chi_{\pi\pi}\,\omega^2} - \Xi\frac{i}{\omega}\,,
\end{aligned}
$$

we can write Kubo formulæ

$$
\begin{aligned}
\gamma_1 &= \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{j\dot\varphi}(\omega, q=0)\,, \\
\Xi &= \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{\dot\varphi\dot\varphi}(\omega, q=0)\,.
\end{aligned}
$$

All we need is a mechanism producing a nonzero $\partial_t\varphi$.

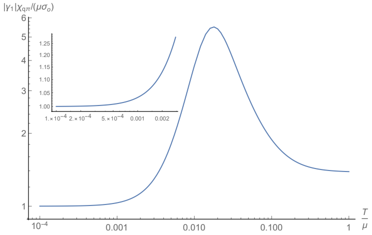
The saturation at low temperatures can be explained by universal relaxation into the heat current

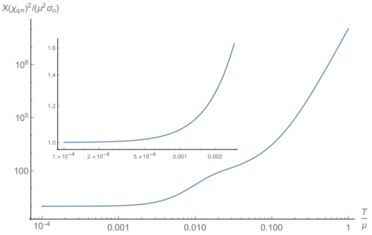
$$\Delta H = \int dx \frac{\pi \cdot j_q}{\chi_{\pi j_q}} \quad \Rightarrow \quad \dot{\varphi} = \frac{j_q}{\chi_{\pi j_q}}$$

$$\gamma_1 = \qquad \frac{1}{\chi_{\pi j_q}} \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im} G^R_{j j_q}(\omega, q=0) \qquad = -\frac{\mu}{\chi_{\pi j_q}} \sigma_o \,,$$

$$\Xi = \quad \left(\frac{1}{\chi_{\pi j_q}})\right)^2 \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im} G^R_{j_q j_q}(\omega, q=0) \quad = \left(\frac{\mu}{\chi_{\pi j_q}}\right)^2 \sigma_o \,.$$

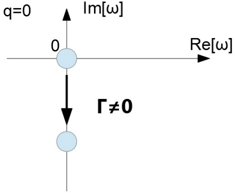
These values verify $\gamma_1^2 = \sigma_o \Xi$ and match our numerics





Spacetime symmetries can be explicitly broken: focus on the case of broken translations. Momentum **relaxes slowly**

$$\dot{\pi} = -\Gamma\pi + \dots$$



The Goldstones become **massive**, which breaks their shift symmetry

$$f = \frac{1}{2}(K+G)(\nabla\cdot\phi)^2 + \frac{1}{2}G(\nabla\times\phi)^2 + \frac{1}{2}m^2\phi^2 + \dots$$

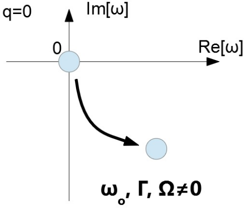
and damps them at a rate Ω. Also relaxes momentum

$$\dot{\pi} = -\Gamma\pi - Gm^2\phi + \dots$$

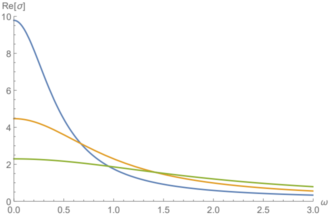
All poles are gapped

$$(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2 = 0$$

with $\omega_o \equiv m\sqrt(G/\chi_{\pi\pi})$ the pinning frequency.



Weakly-disordered metal

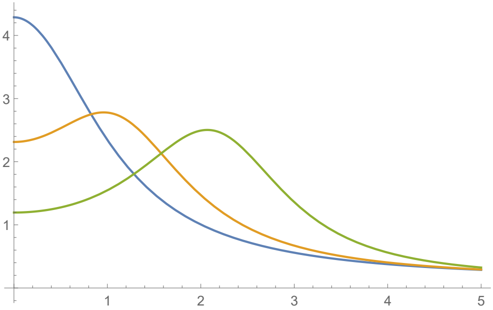


$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}} \frac{1}{\Gamma - i\omega}$$

$$\sigma_{dc} \sim \frac{1}{\Gamma}$$

The dc conductivity is dominated by **momentum relaxation**

Weakly-pinned Wigner crystal



$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

$$\sigma_{dc} = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}\Gamma_{WC}}\,, \quad \Gamma_{WC} = \Gamma + \frac{\omega_o^2}{\Omega}$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{Z(\phi)}{4}F^2 - V(\phi) - Y(\phi)\left(\partial\psi_x^2 + \partial\psi_y^2\right)\right]$$

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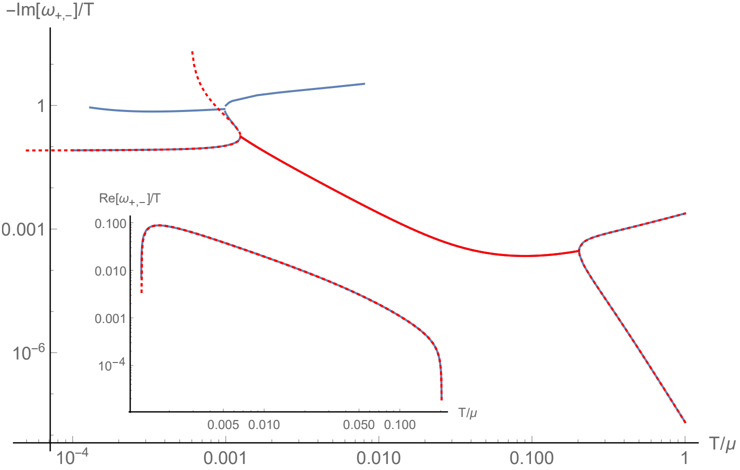
- UV boundary conditions on ϕ

$$\phi = \lambda r + \phi_v r^2 + \dots$$

- If $\lambda \neq 0$, then $\psi_i = kx^i$ is a source: **explicit breaking**.
- But if $\lambda/\mu \ll \phi_v/\mu^2$, **pseudo-spontaneous breaking**.

Numerically compute vector QNMs at $q = 0$

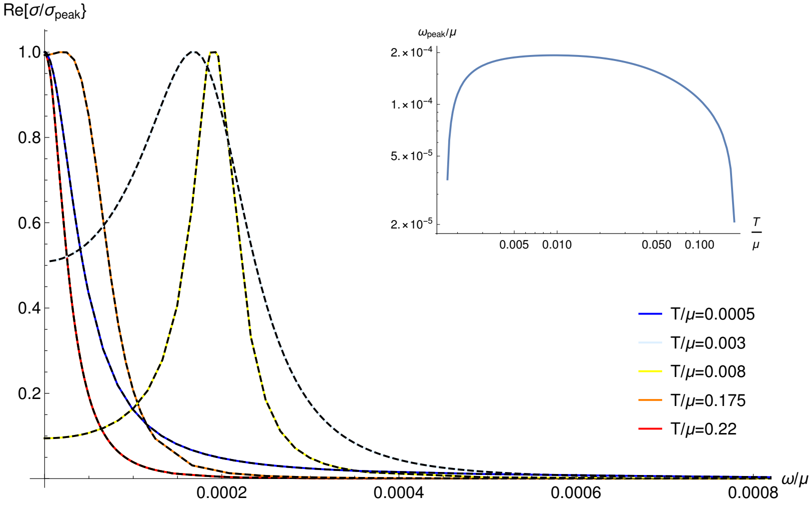


Red lines: plot of the solutions of $(\Omega - i\omega)(-i\omega) + \omega_o^2 = 0$ together with

$$\Omega \simeq \chi_{\pi\pi}\omega_o^2\Xi \simeq -Im[\omega_{QNM,k=0}]$$

Phonon relaxation dominated by $k = 0$ dynamics.

The small ω behavior of the ac conductivity is in excellent agreement with the WC/Drude hydro predictions (no fit).

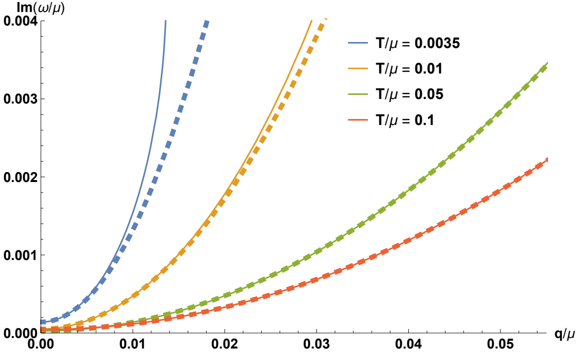


- At $k = 0$, no breaking of translations. For $\lambda = 0$ ($\lambda \neq 0$), it breaks the global shift symmetry spontaneously (explicitly).
- Accordingly, for small λ, expect a pseudo-Goldstone boson

$$\omega = -i\Omega - D_\varphi q^2 + O(q^3)\,, \quad (\chi_{\varphi\varphi})^{-1} = m^2 + q^2$$

- Confirmed numerically, and analytically for small ω, q, with

$$D_\psi \simeq \Omega/m^2 \simeq G\Xi$$



$$\Omega \simeq m^2 D_\varphi \simeq m^2 G \Xi$$

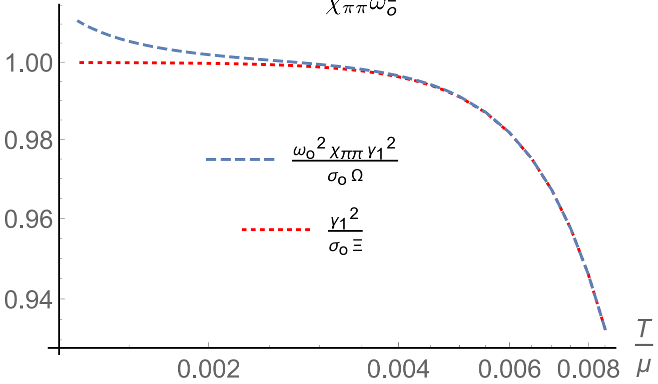
- This relation still holds at $k \neq 0$, at all Ts where the WC hydro picture is valid.
- Consequence of the fact that G, K are always 'small'.

$$G = 2K = k^2 I_Y + O(k^4)$$

- Suggest that perhaps true for non-holographic systems with small bulk/shear moduli? Eg close to the translation-ordering phase transition or in phases with fluctuating charge density waves (eg cuprate high T_c superconductors).

- There is another positivity of entropy production bound

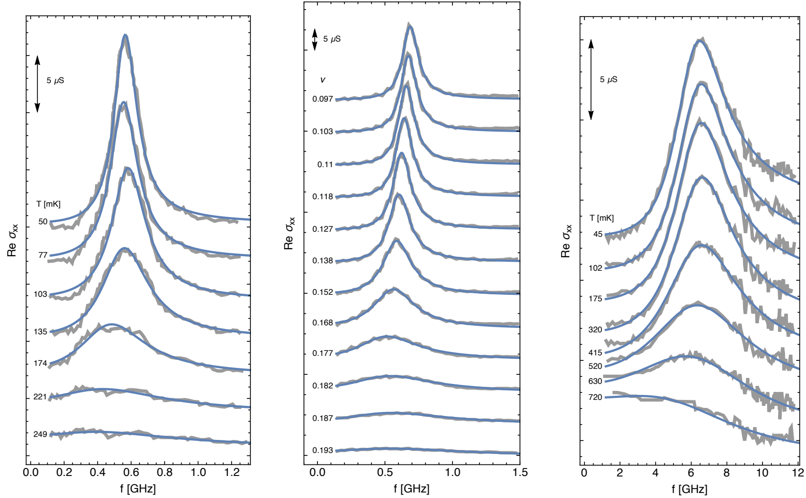
$$\gamma_1^2 \leq \frac{\Omega \sigma_o}{\chi_{\pi\pi} \omega_o^2}$$



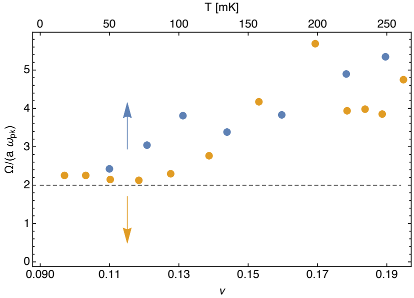
- Almost saturates: relaxation of phonons into the heat current.
- Violation when we get close to pole collision: breakdown of WC hydro picture.

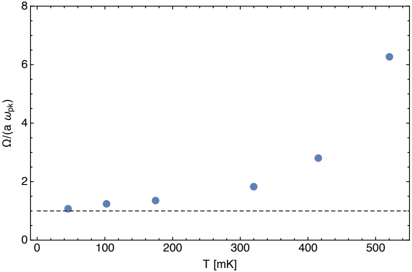
- Now turn on a magnetic field: the longitudinal and transverse sound modes hybridize into (gapless) magnetophonons and gapped magnetoplasmons.
- Upon turning on disorder, the magnetophonons are pinned at $\omega_o^2/\omega_c \sim O(1/B)$: within hydrodynamics at large magnetic fields.
- Write down a similar hydrodynamic theory as before: conservation of charge, Josephson for magnetophonon, constitutive relations, solve and get conductivity.
- Also positivity of entropy production bound.

Fit to data on GaAs heterojunctions (2DEG)



Different microscopic mechanisms appear to be at play in less/more disordered samples: phase relaxation by mobile dislocations or universal dissipation into hydrodynamic currents.





Summary and Outlook

- Effective field theories (hydro) predict specific functional forms for conductivities, dispersion relation for modes, etc. in terms of a few transport coefficients and given an equation of state.
- The EoS and the transport coefficients must be computed from miscroscopics.
- When symmetries are explicitly broken weakly, the departures from hydrodynamics can be encapsulated in a few extra relaxation parameters.
- We have seen several examples in holography and real systems where it appears that relaxation is universal, in the sense that it is controlled by dissipation into a hydrodynamic operator.
- Deep statement? Artifact of the toy model? Overinterpretation of the data? Jury still out...