Out-of-equilibrium, Tilings & the Conformal Bootstrap

École polytechnique, Centre de Physique Théorique

Organisers: A. Fiorucci, M. Petropoulos, M. Sanchez (CPHT)

Monday March 31, 2025 — Salle Louis Michel

Programme of the day

10:00 – 10:30	Participant welcome and coffee
10:30 – 11:30	Talk by **Kirone Mallick** (IPhT, CEA Saclay)
11:30 – 11:45	Coffee break
11:45 – 12:45	Talk by **Ada Altieri** (MSC, Université Paris Cité)
12:45 – 14:15	Lunch time at the *Magnan*
14:15 – 15:15	Talk by **Balt van Rees** (CPHT, École polytechnique)
15:15 – 16:15	Talk by **Grégory Schehr** (LPTHE, Sorbonne Université)
16:15 – 16:30	Coffee break
16:30 – 17:30	Talk by **Jean-René Chazottes** (CPHT, École polytechnique)

Titles & abstracts of the scientific talks

1 Kirone Mallick (IPhT, CEA Saclay)

Title: *An exact solution of the macroscopic fluctuation theory*

Abstract: Interacting diffusive particle systems are considered as paradigms for non-equilibrium statistical physics. Their macroscopic behaviour follows a variational principle, proposed by G. Jona-Lasinio and his collaborators, known as the Macroscopic Fluctuation Theory (MFT). Optimal fluctuations far from equilibrium are thus determined at a coarse-grained scale by two coupled non-linear hydrodynamic equations. However, for a long time, only few exact solutions of the MFT equations were known.

In this talk, we shall show that, for the exclusion process, the MFT equations are classically integrable and can be solved with the help of the inverse scattering method, originally used to study solitons in the KdV or the NLS equations. This exact solution will allow us to calculate the large deviations of the current and the optimal profiles that generates a given fluctuation, both at initial and final times. This macroscopic solution matches previous results that were derived, by applying the Bethe Ansatz, at the microscopic level.

2 Ada Altieri (MSC, Université Paris Cité)

Title: *Disordered ecologicald dynamics and Moment-matching Inference for gut microbial communities*

Abstract: The remarkable biodiversity observed in natural ecosystems has recently attracted increasing attention, particularly among theoretical physicists.

In this talk, I will address timely questions in theoretical ecology by focusing on a Generalized Lotka-Volterra (GLV) model, which incorporates random species interactions and demographic fluctuations.[1] By leveraging disordered systems and random matrix theory techniques, I will uncover a rich, and ultimately hierarchical, organization of the equilibria to be associated with a glass-like slowdown of the dynamics.

I will then provide a proof of concept showing how this framework can effectively capture the complexity of the human gut microbiota. Analyzing metagenomic data from both healthy individuals and patients suffering from inflammatory bowel diseases, I will map distinct physiological states of the gut microbiome to different disorder- and noise-driven regimes of the GLV model.[2] Finally, I will discuss a few generalizations beyond the well-mixed approximation to account for spatial heterogeneity across discrete patches.[3]

[1] **A. Altieri**, F. Roy, C. Cammarota, G. Biroli, *Phys. Rev. Lett.* 126 (2021)

[2] J. Pasqualini, A. Maritan, A. Rinaldo, S. Facchin, E. V. Savarino, **A. Altieri*** and S. Suweis*, 2406.07465, *to appear in eLife* (2025)

[3] G. Garcia Lorenzana, **A. Altieri*** and G. Biroli*, *PRX Life 2* (2024)

3 Balt van Rees (CPHT, École polytechnique)

Title: *Bootstrapping frustrated magnets*

Abstract: We study multiscalar theories with $O(N) \times O(2)$ symmetry in three dimensions which describe the critical behavior of certain magnetic systems with frustration. These models have a stable fixed point if N is greater than some critical value N_c. Previous estimates of this critical value from perturbative and non-perturbative renormalization group methods have produced mutually incompatible results. We use numerical conformal bootstrap methods to show that $N_c > 3.78$, which favors the scenario that the physically relevant models with $N = 2, 3$ do not have a stable fixed point and undergo a first-order transition. Our result exemplifies how conformal windows can be rigorously constrained with modern numerical bootstrap algorithms.

4 Grégory Schehr (LPTHE, Sorbonne Université)

Title: *Universal distribution of the number of minima for random walks and Lévy flights*

Abstract: We compute exactly the full distribution of the number m of local minima in a one-dimensional landscape generated by a random walk or a Lévy flight. We consider two different ensembles of landscapes, one with a fixed number of steps N and the other till the first-passage time of the random walk to the origin. We show that the distribution of m is drastically different in the two ensembles (with a Gaussian tail in the former case, while having a power-law tail with exponent $-3/2$ in the latter case). However, the most striking aspect of our results is that, in each case, the distribution is completely universal for all m (and not just for large m), *i.e.*, independent of the jump distribution in the random walk. This means that the distributions are exactly identical for Lévy flights and random walks with finite jump variance. Our analytical results are in excellent agreement with our numerical simulations.

5 Jean-René Chazottes (CPHT, École polytechnique)

Title: *Thermodynamic Formalism, Symbolic Dynamics, and Quasicrystal Models*

Abstract: The study of simple statistical physics models on the lattice $\mathbb{Z}^d$, aimed at understanding the transition from a disordered state to periodic or quasi-periodic order at low temperatures, reveals a natural connection between the formalism of Gibbs measures and equilibrium states, multidimensional symbolic dynamics, aperiodic tilings, and theoretical computer science. This talk offers an introductory overview of this research field.